Sigma term and strange content of the nucleon
[BMW + Regensburg]

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Rencontre de physique de particules 2011

BMW + Regensburg.

Dark matter

- Discrepancy between measurements of the mass of structures larger than galaxies made trough dynamical (GR) means and measurements based on the "luminous" matter these objects contains.

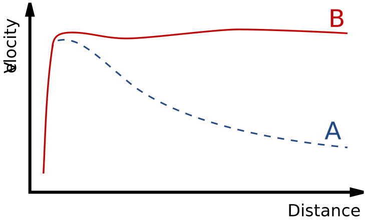

Figure: Rotation curve of a typical spiral galaxy: predicted (A) and observed (B). Dark matter can explain the velocity curve having a 'flat' appearance out to a large radius.

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Dark mater detection is one of the challenges of the decade.

Direct DM detection

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- Measures the recoil of the scattering of a DM particle with the nuclei.

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$$\mathcal{L} = \lambda_N \overline{\chi} \chi \overline{n} n$$

The squared amplitude is

$$|A_N|^2 = 64 \left(\lambda_N M_\chi M_N\right)^2$$

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Direct DM detection

Hadronic Uncertainties in the Elastic Scattering of Supersymmetric Dark Matter

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(Dated: February 7, 2008)

Abstract

We review the uncertainties in the spin-independent and -dependent elastic scattering cross sections of supersymmetric dark matter particles on protons and neutrons. We propagate the uncertainties in quark masses and hadronic matrix elements that are related to the π-nucleon σ term and the spin content of the nucleon. By far the largest single uncertainty is that in spin-independent scattering induced by our ignorance of the $\langle N|\bar{q}q|N\rangle$ matrix elements linked to the π-nucleon σ term, which affects the ratio of cross sections on proton and neutron targets as well as their absolute values. This uncertainty is already impacting the interpretations of experimental searches for cold dark matter. *We plead for an experimental campaign to determine better the π-nucleon σ term.* Uncertainties in the spin content of the proton affect significantly, but less strongly, the calculation of rates used in indirect searches.

Motivation

Definitions

$$
\begin{aligned}
\sigma_{\pi N} &= \hat{m}\langle N(p) \mid (\overline{u}u + \overline{d}d)(0) \mid N(p)\rangle \\
\sigma_{\bar{s}sN} &= m_s\langle N(p) \mid (\bar{s}s)(0) \mid N(p)\rangle \\
y &= \frac{2\langle N \mid \bar{s}s \mid N\rangle}{\langle N \mid \overline{u}u + \overline{d}d \mid N\rangle}
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Important for

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- The quark mas ratio $m_s/\hat{m}$
- $\pi - N$ and $K - N$ scattering amplitudes
- Counting rates in searches of the Higgs boson

Motivation

The nucleon mass is given by

$$M_N = \langle N \mid T_{\mu\mu} \mid N\rangle = \sum_q m_q \langle N \mid \overline{q}q \mid N\rangle + \text{Gluonic contribution}$$

then

$$\begin{aligned} m_{ud}\frac{\partial M_N}{\partial m_{ud}} = m_{ud}\langle N \mid \overline{u}u + \overline{d}d \mid N\rangle &= \sigma_{\pi N} \\ m_s\frac{\partial M_N}{\partial m_s} = m_s\langle N \mid \overline{s}s \mid N\rangle &= \sigma_{\overline{s}sN} \end{aligned}$$

- The sigma terms measures how much the nucleon mass changes when you change quark masses
- Is this useful ??

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"There has been sensational progress in calculating quantum electrodynamics, but very little progress in understanding it; and strong interactions are neither calculable nor understood."
– K. G. Wilson and J. Kogut (1974) –

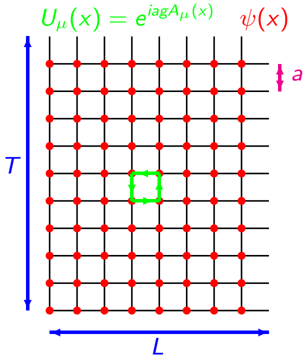
Motivation

YES!!

Quark masses are constant in the real world, but can be changed in the lattice.

Lattice QCD in one slide

Lattice field theory $\longrightarrow$ Non Perturbative definition of QFT.



$$\langle O \rangle = \int \mathcal{D}[U]\mathcal{D}\overline{\psi}\mathcal{D}\psi\, O(U,\overline{\psi},\psi) e^{-S_G[U]-S_F[U,\overline{\psi},\psi]}$$
$$= \int \mathcal{D}[U]\, O(U)_{\text{Wick}} e^{-S_G[U]} \det(D)$$

- Compute the integral numerically $\rightarrow$ Monte Carlo sampling of $e^{-S_G[U]}\det(D) \geq 0$.
- Observable computed averaging over samples

$$\langle O \rangle = \frac{1}{N_{\text{conf}}} \sum_{i=1}^{N_{\text{conf}}} O(U_i) + \mathcal{O}(1/\sqrt{N_{\text{conf}}})$$

NOT A MODEL: Lattice QCD IS real world QCD ($a \to 0$, $L \to \infty$, ...)

BMW 2008 landscape

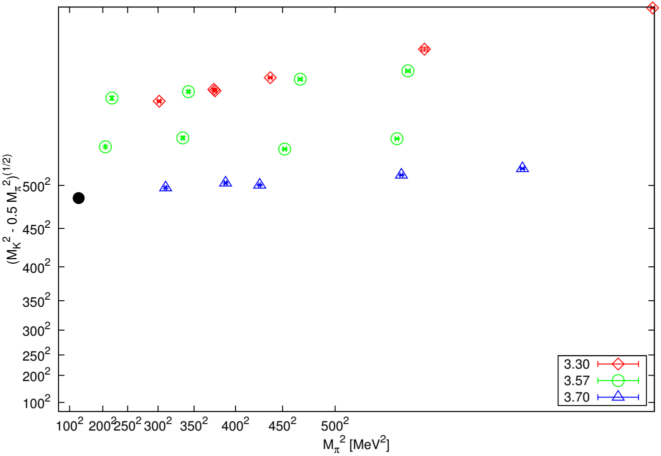


Figure: Simulation points in the "m_{ud}, m_s" plane.

Taylor extrapolations

Any physical quantity is analytic in the quark masses if you do not expand around $m_q = 0$.

Expansion variables

$$
\begin{aligned}
(M_\pi^{\mathrm{exp}})^2 &= \frac{1}{2}[(M_\pi^{\Phi})^2 + (M_\pi^{\mathrm{max}})^2] \\
M_{\bar{s}s}^2 &= 2M_K^2 - M_\pi^2
\end{aligned}
$$

Nucleon mass dependence

$$
M_N = M_0 + \sum_{i=1}^{N_\pi} \alpha_i' \left[M_\pi^2 - (M_\pi^{\mathrm{exp}})^2 \right]^i + \sum_{i=1}^{N_\pi} \beta_i' \left[M_{\bar{s}s}^2 - \left(M_{\bar{s}s}^{\Phi} \right)^2 \right]^j
$$

Really you are making a polynomial fit.

To fit to lattice data

More suitable to use with lattice data

$$(aM_N) = (aM_N^\Phi)\left\{1+\sum_{i=1}^{N_\pi}\alpha_i\left[\left(\frac{aM_\pi}{aM_N^\Phi}\right)^2-\left(\frac{M_\pi^\Phi}{M_N^\Phi}\right)^2\right]^i + \right.$$

$$\left. + \sum_{j=1}^{N_K}\beta_j\left[\left(\frac{aM_{\bar{s}s}}{aM_N^\Phi}\right)^2-\left(\frac{M_{\bar{s}s}^\Phi}{M_N^\Phi}\right)^2\right]^j\right\}$$

Or Pade approximations

$$(aM_N) = (aM_N^\Phi)\left\{1+\sum_{i=1}^{N_\pi}\alpha_i\left[\left(\frac{aM_\pi}{aM_N^\Phi}\right)^2-\left(\frac{M_\pi^\Phi}{M_N^\Phi}\right)^2\right]^i + \right.$$

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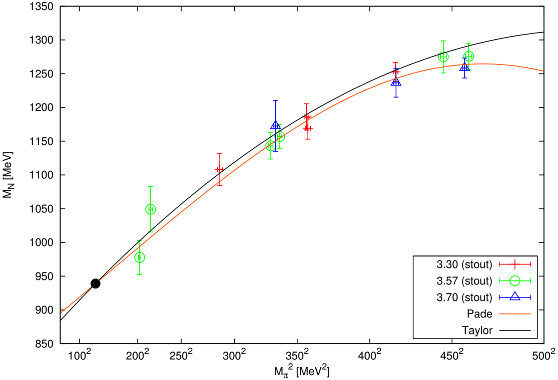
Some trial fits

Figure: $M_\pi < 420$ MeV; $\chi^2/dof \approx 4.9/7$.



$$\sigma_{\pi N} = 53(10)_{\text{stat}} \qquad \text{Taylor}$$

$$\sigma_{\pi N} = 44(6)_{\text{stat}} \qquad \text{Pade}$$

χPT

Nucleon mass as a function of quark masses

$$M_N = M_0 + \alpha M_\pi^2 + \text{Higher order terms}$$

Higer order terms

- HBχPT: $\propto g_A M_\pi^3$
- CBχPT: $\propto g_A h(M_\pi)$

$$h(M_\pi) = -\frac{M_\pi^3}{4\pi^2}\left\{\sqrt{1-\left(\frac{M_\pi}{2M_0}\right)^2}\arccos\frac{M_\pi}{2M_0} + \frac{M_\pi}{2M_0}\log\frac{M_\pi}{M_0}\right\}$$

Three flavour CBχPT [S. Dürr @ LAT10]

$$M_X = M_0 - 4c_X^{\pi} M_\pi^2 - 4c_X^{s} M_{\bar{s}s}^2 + \sum_{\alpha=\pi,K,\eta} \frac{g_X^\alpha}{F_\alpha^2} M_0^3 h\left(\frac{M_\alpha}{M_0}\right) + d^\pi M_\pi^4 + d^s M_{\bar{s}s}^4$$

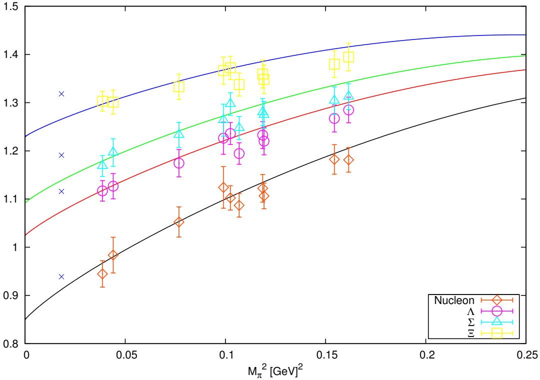
The good thing: All the constants $c_X^{\pi,s}, g_X^\alpha$ depend only on 5 independent parameters: b_0, b_D, b_F, g_A, ξ

How to fit lattice data

- First we set the scale (using Ω).
- Fit data in physical units using the formula above.

CBχPT fits

Figure: $M_\pi < 420$ MeV; χ^2/points $\approx 9/10$.



$$\sigma_{\pi N} \quad = \quad 47(9)_{\text{stat}} \qquad \text{CB}\chi\text{PT}$$

CBχPT fits

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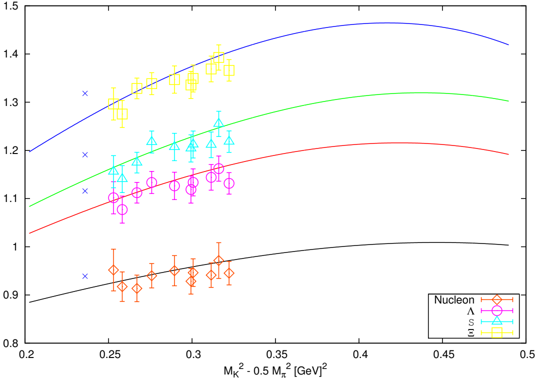


Even statistics kills you for a signal for the strangeness of the nucleon

More data [C. Hoelbling @ LAT10]

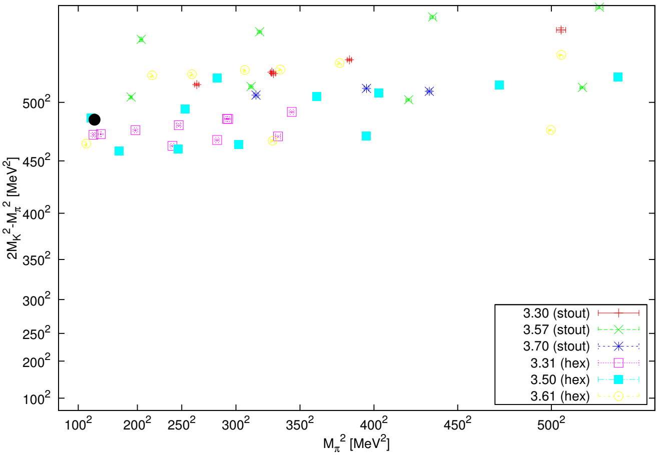
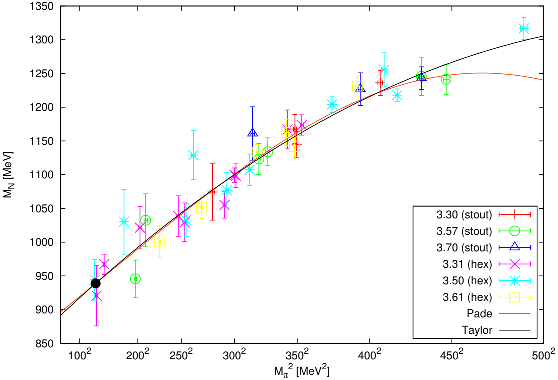


Figure: Simulation points in the "m_{ud}, m_s" plane. 2008 dataset + partial 2010 dataset

Including data at the physical point

Figure: $M_\pi < 420$ MeV; $\chi^2/dof \approx 4.9/7$.

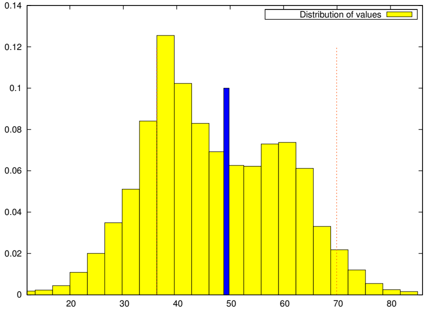


$$\sigma_{\pi N} = 46(5)_{\text{stat}} \qquad \text{Taylor}$$
$$\sigma_{\pi N} = 42(3)_{\text{stat}} \qquad \text{Pade}$$

Preliminary analysis [A. Ramos @ LAT10]

- Only Taylor + Pade fits
- 6 values of β
- $\mathcal{O}(a)$ and $\mathcal{O}(a^2)$ cutoff effects
- $M_\pi > 190$ MeV
- Two pion mass cuts
- 144 fitting intervals for correlators

Figure: Distribution of values for $\sigma_{\pi N}$.



Preliminary analysis [A. Ramos @ LAT10]

$M\pi = 134$ MeV just by chance!!

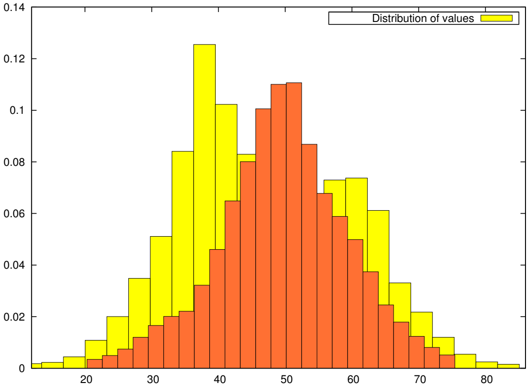
Imaginary world

Repeat the analysis in an imaginary world with a heavier m_{ud}.

- $M_\pi = 200$ MeV
- $M_K = 505$ MeV
- $M_N = 1000$ MeV

Preliminary analysis [A. Ramos @ LAT10]

Figure: Distribution of values in imaginary world.



Preliminary analysis [A. Ramos @ LAT10]

Physical point data

Reduces both statistical and systematic errors by a $\approx 40\%$

Conclusions

- Regarding CBχPT
  - Works(!?). Reasonable values for LEC (g_A). You can fix g_A to the physical value, or remove higher order terms, and data is well described.
  - Symmetries are important. Keep them in your functional form (i.e. Run F's).
- No "surprises" after the lattice data.
- Preliminary "complete" analysis, with two (similar) actions $190 < M_\pi < 460$ MeV [A. Ramos @ LAT10] (Other sources of systematic much below statistical precision).
  - $\sigma_{\pi N} = 49(10)_{\text{stat}}(11)_{\text{sys}}$ MeV
  - $\sigma_{\bar{s}sN} = 49(37)_{\text{stat}}(26)_{\text{sys}}$ MeV
  - $y = 0.08(7)_{\text{stat}}(4)_{\text{sys}}$
- Even for a quantity that you can not "read" at the physical point, having data there reduces dramatically both statistical and systematic errors ($\approx 40\%$).
- Alert: The computation is "model independent" except in the case of a composite WIMP (technicolor). In this case this is only (the easy) half of the history.