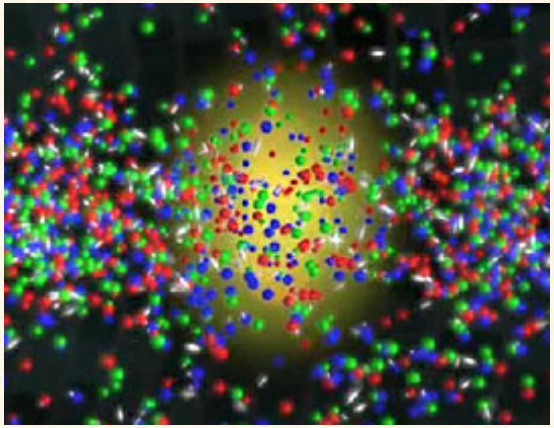
Parton showers as a source of energy-momentum deposition in the QGP and the implication for shockwave formation

Bryon Neufeld, LANL

Based on the preprint

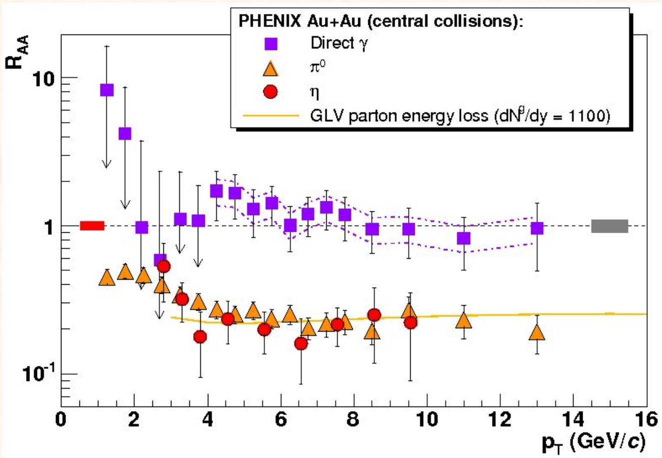
R.B. Neufeld and Ivan Vitev: arXiv:1105.2067 [hep-ph]

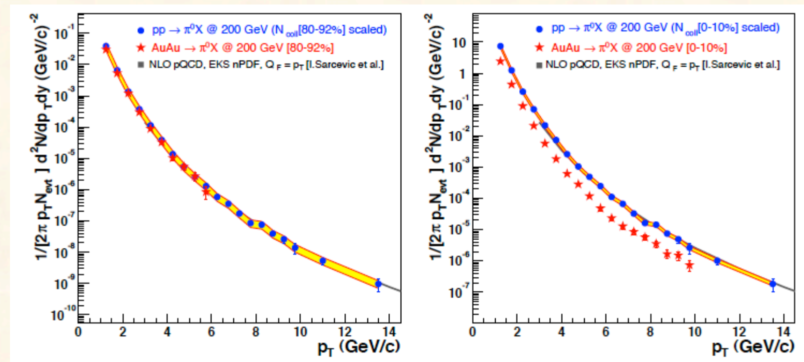


High pT Probes of High Density QCD at the LHC, May 30-June 1 2011

One of the most striking results from RHIC:

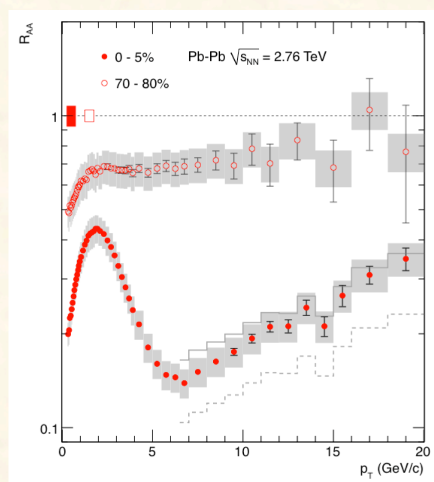
Large leading particle suppression or ‘jet quenching’

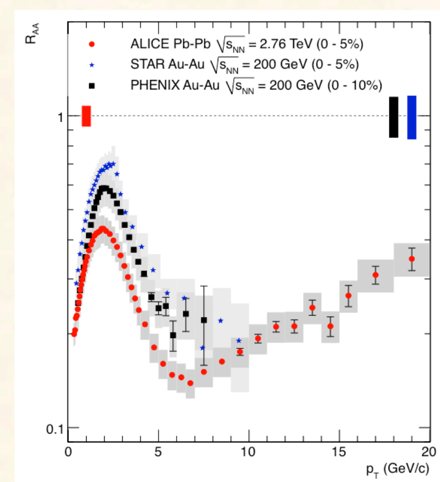




Now one of the most striking results from LHC:

First results from heavy-ion program at LHC confirm results on jet quenching, and extend to higher pT.





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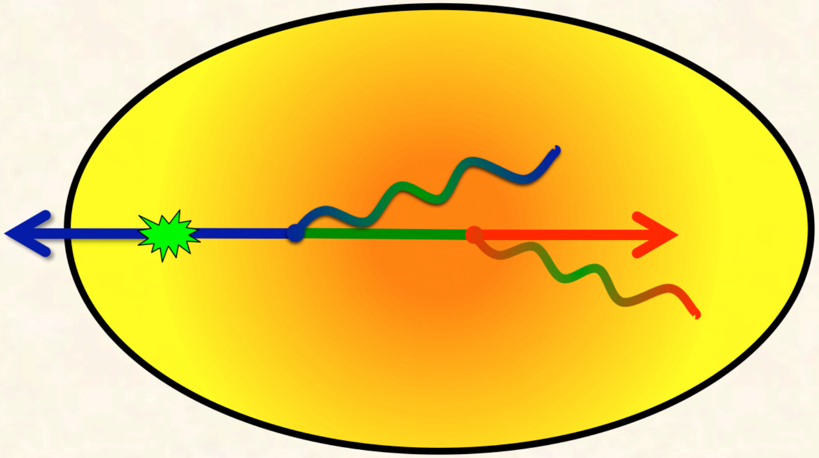

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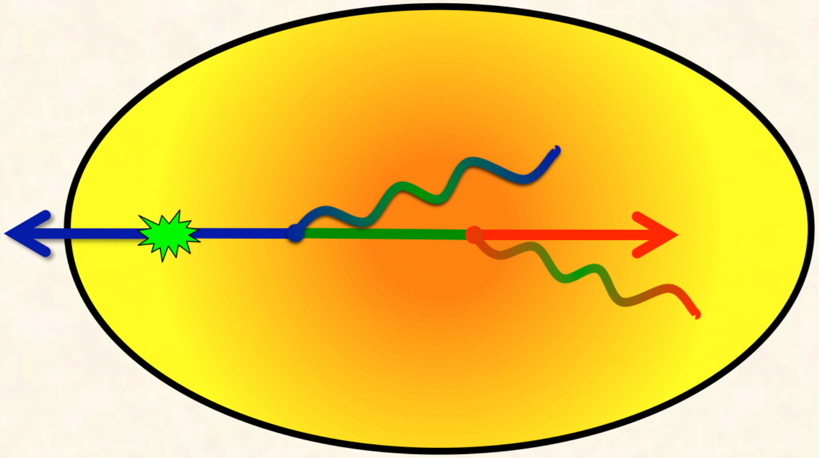

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Energetic partons lose energy primarily through medium-induced radiation and thus evolve into an in-medium parton shower.

Parton showers carry more information about the underlying QCD dynamics than leading particles alone and form the basis for using full jet observables as a new and powerful probe of the QGP.

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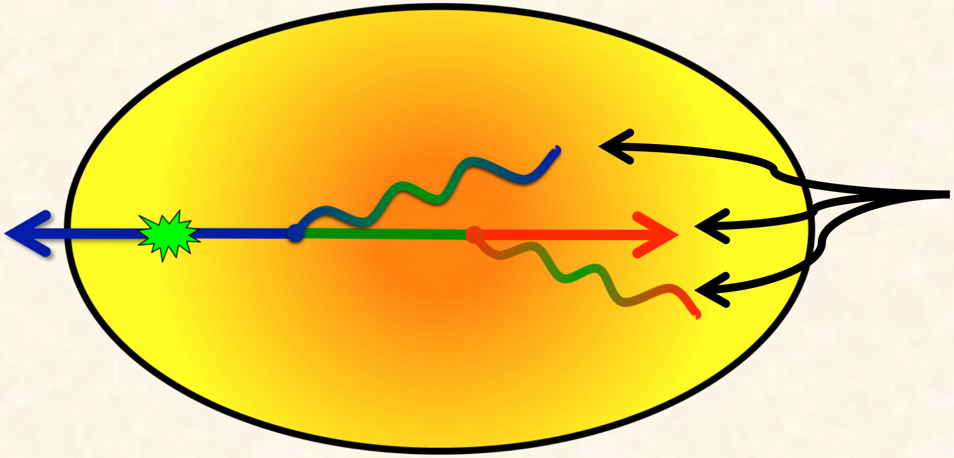

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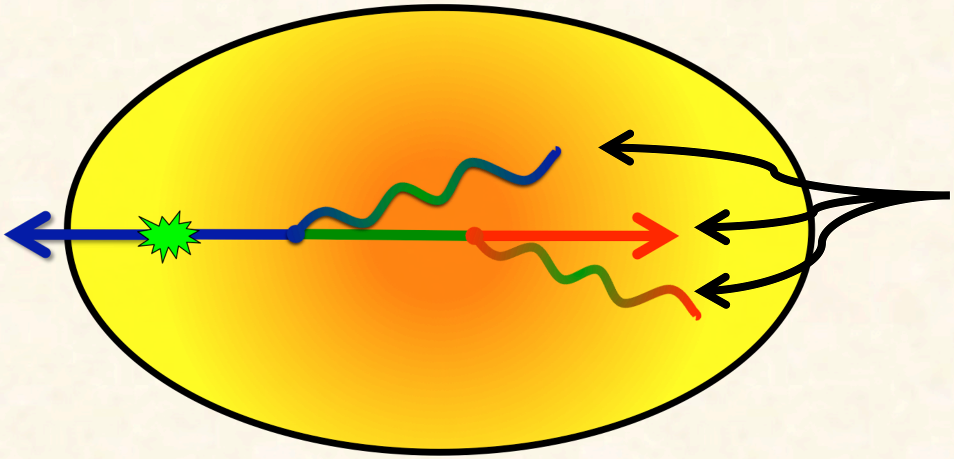

Although the collisional energy loss associated with a single parton is thought to be relatively small, the cumulative effect associated with a full parton shower can become quite large because the radiated gluons themselves are sources of energy loss in the medium.

Thus, the question of how much energy a parton shower transmits to the medium is of fundamental interest in the description of full jet observables, and, as will be shown, is sensitive to the number and angle of gluon radiation.

A related question to how much energy is lost: How does the medium respond to parton showers?

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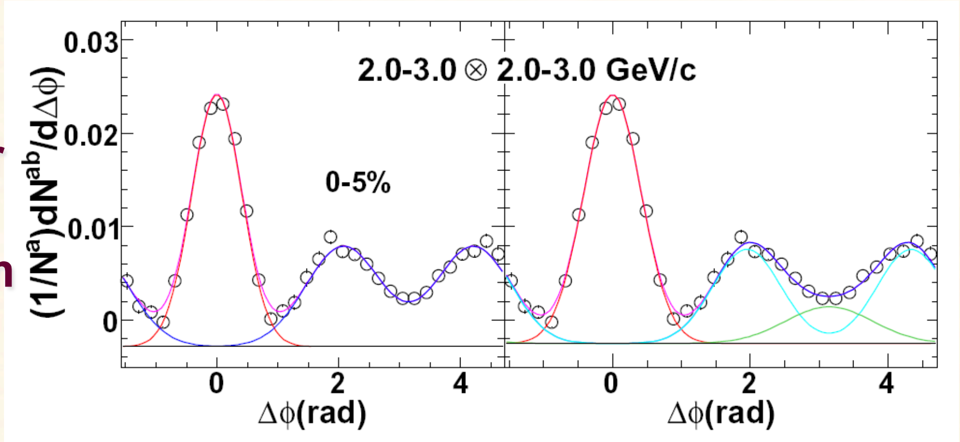
Dihadron correlations at RHIC may show conical flow indicative of a shockwave induced by energetic partons. Whether or not these measurements reflect shockwave phenomena, there is a medium response to energetic partons and parton showers.

PHENIX



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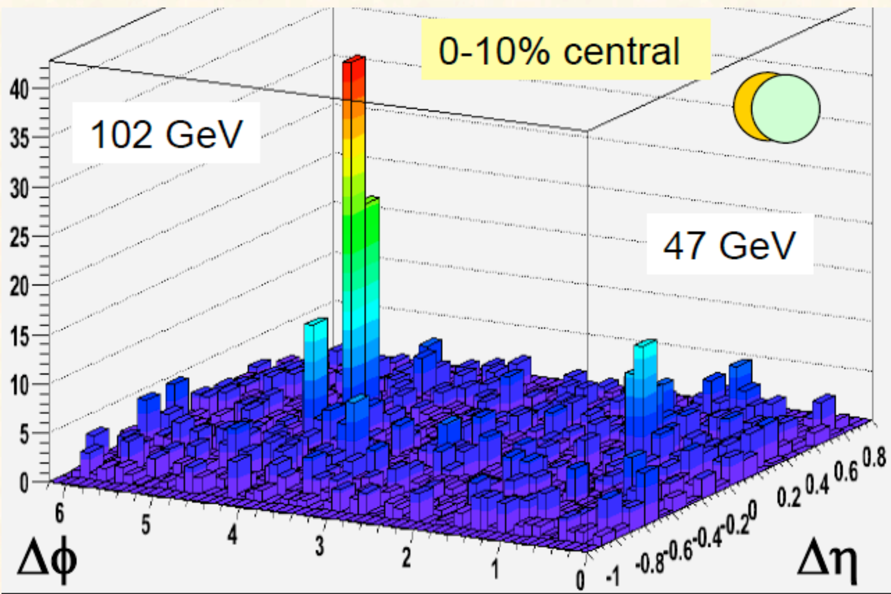
Dihadron correlations at RHIC may show conical flow indicative of a shockwave induced by energetic partons. Whether or not these measurements reflect shockwave phenomena, there is a medium response to energetic partons and parton showers.



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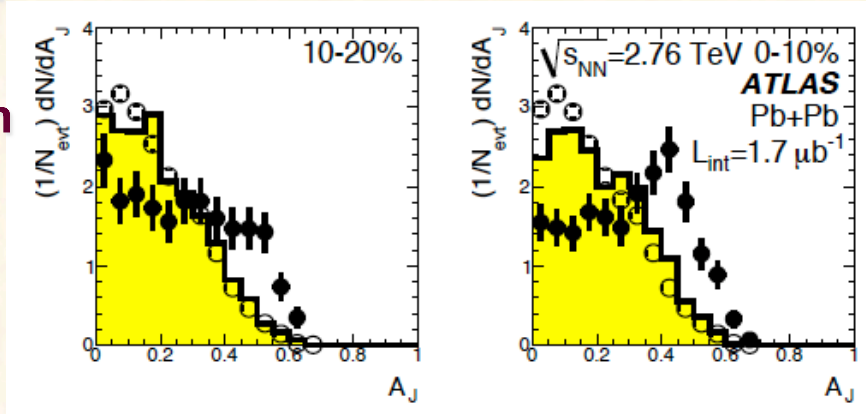

Understanding the medium response to a parton shower is essential for predicting how the energy associated with a jet is redistributed in the medium and where it can be recovered

Both the question of how much energy a parton shower transmits to the medium and how the medium responds to a parton shower can be addressed in the same framework by calculating the source term associated with the parton shower.

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Energy loss to the medium ✔

Source for evolution of bulk QGP ✔

The source term is the space-time distribution of energy and momentum flowing between a parton shower and the underlying medium. It carries information about the rate of energy transfer to the medium and acts as a source for the evolution of the underlying medium in the presence of the parton shower.

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- In rest of presentation will present general formalism for evaluating source term from thermal field theory. Will then apply formalism to case of parton shower being careful to include appropriate quantum interference effects.

Deriving the source term from the fundamental EMT of QCD:

Evaluate $\partial_\mu T^{\mu\nu} \equiv J^\nu$ where $T^{\mu\nu}$ is the fundamental EMT of QCD

- **Begin with a medium of quarks only, to simplify the calculation**

$$T^{\mu\nu} = \frac{i}{4}\bar{\psi}\left(\gamma^\mu \overleftrightarrow{D}^\nu + \gamma^\nu \overleftrightarrow{D}^\mu\right)\psi - g^{\mu\nu}\mathscr{L}$$

$$\mathscr{L} = \frac{i}{2}\bar{\psi}\,\overleftrightarrow{\not{D}}\,\psi,\ D^\mu = \partial^\mu - ig\,A_a^\mu\,t^a$$

Neufeld, Phys. Rev. D83, 065012 (2011)

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Advantage of going directly to underlying field theory is that one can obtain the full spatial distribution for both soft and hard momentum exchange with the medium. Also, this is the natural approach for incorporating quantum interference effects between correlated color charges, as will be shown in what follows.

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- The energetic parton couples to the medium via an interaction term in the Lagrangian

$$-i\int d^4z\, j_a^\alpha(z)\, e^{iz\cdot p_4}$$

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This coupling can be made quite general to consider hard partons created in an initial hard scattering or to incorporate medium induced parton showering. First will consider an asymptotic fast parton, then generalize to parton shower.

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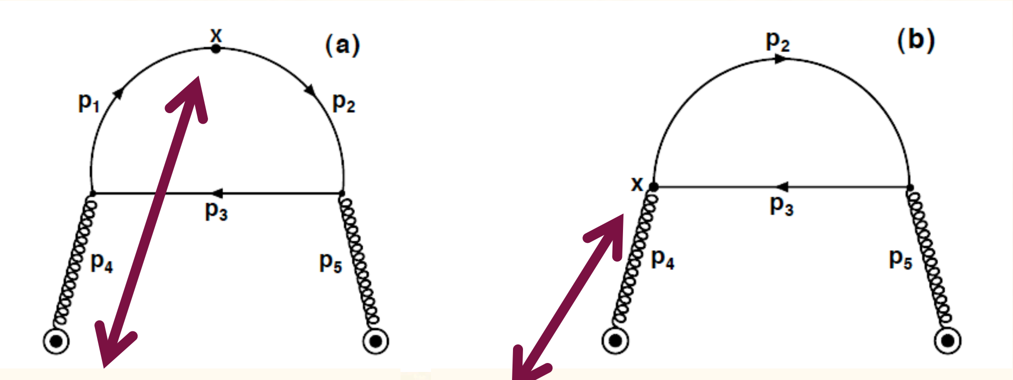
- **Additionally, must incorporate Feynman rule for EMT:**



$$\frac{ie^{-ix\cdot(p_1-p_2)}}{4} \times \left((p_2^2 - p_1^2)\gamma^\nu + p_1^\nu(3\not{p}_2 + \not{p}_1) - p_2^\nu(3\not{p}_1 + \not{p}_2)\right)$$

$$-ig\, e^{-ix\cdot(p_4+p_3-p_2)}\,(p_4+p_3-p_2)_\mu \times \frac{(\gamma^\nu j_a^\mu + \gamma^\mu j_a^\nu - 2g^{\mu\nu}\not{j}_a)\, t^a}{2}$$

Neufeld, Phys. Rev. D83, 065012 (2011)

Deriving the source term from the fundamental EMT of QCD:

- Once the rule for the source and EMT are extracted, the calculation proceeds via standard rules for real-time finite T perturbation theory (must extract T dependent part)

- The result is shown here in integral form:

$$\langle \partial_\mu T^{\mu\nu}(x) \rangle = -4i\, N_F\, C_2\, g^4 \int \frac{d^4p_3\, d^4p_4\, d^4p_5}{(2\pi)^9} e^{-ix\cdot(p_4+p_5)} \delta(p_3^2) n_F(p_3) G_R(p_4) G_R(p_3+p_4) G_R(p_5) \delta(p_4\cdot U)\delta(p_5\cdot U)$$
$$\times \left[2\,(p_3\cdot U)^2\, p_5^\nu - U^2\, p_3\cdot p_4\, p_5^\nu - U^\nu\,(p_3\cdot U)(2p_3\cdot p_5 + p_4\cdot p_5) \right]$$

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- The result is complicated looking, but the underlying physics can be seen in the Green's functions

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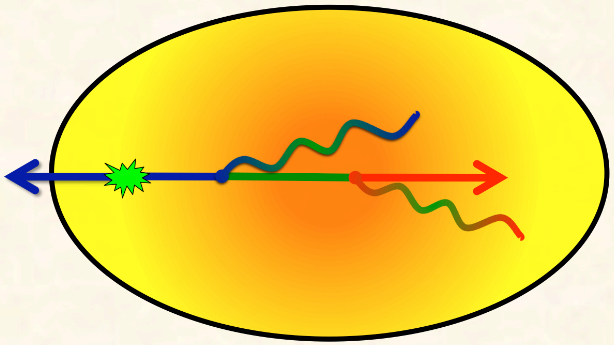

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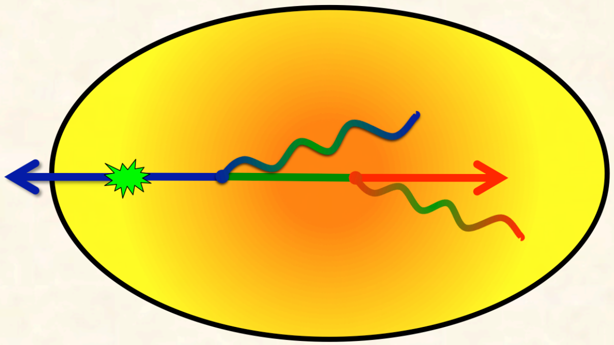

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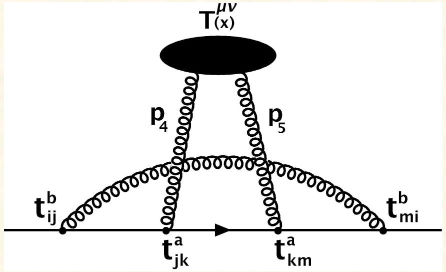
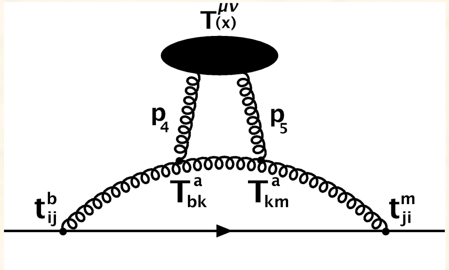
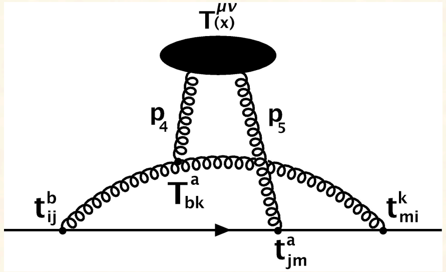
- The Feynman diagrams, along with their explicit color structure, are shown here:







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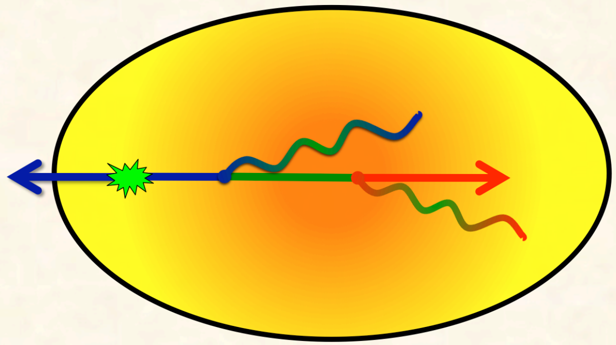

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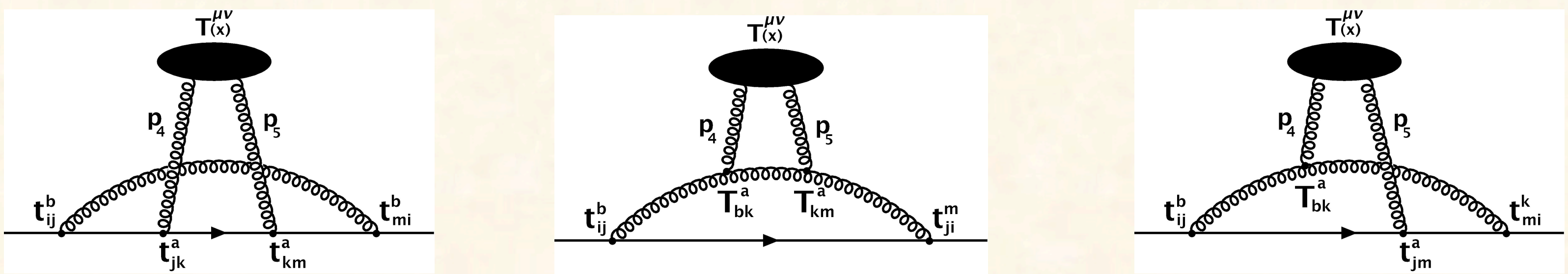

R.B. Neufeld and Ivan Vitev: arXiv:1105.2067 [hep-ph]

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One obtains the above structure, where the dark blobs are the operator insertion described in the previous slides.

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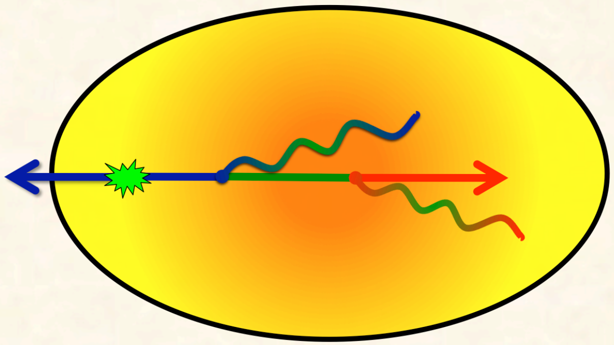

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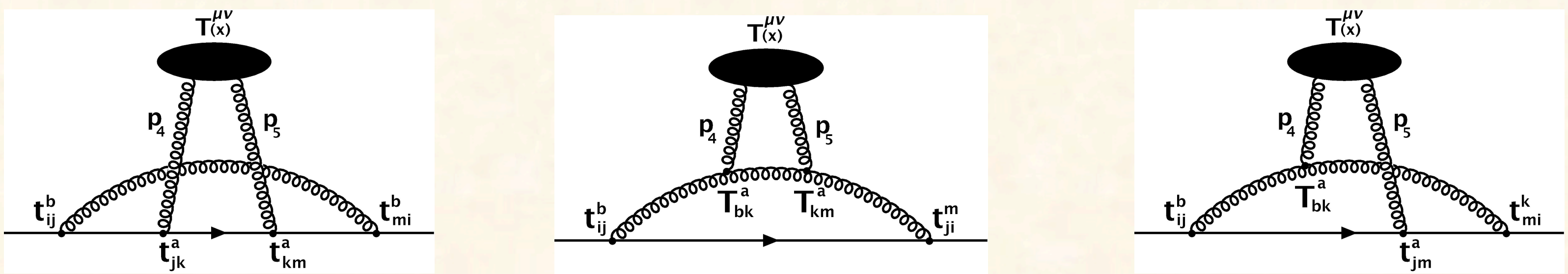

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If the radiation is truly collinear, the medium cannot resolve the two partons – their color structure is the same before and after emission

Now generalize to consider parton shower:

R.B. Neufeld and Ivan Vitev: arXiv:1105.2067 [hep-ph]

- A lengthy calculation is performed to evaluate the integral expression, suitably modified for a parton shower
- Derived result in closed analytic form
- Includes time dependence of radiated gluons (which are formed at an initial time)

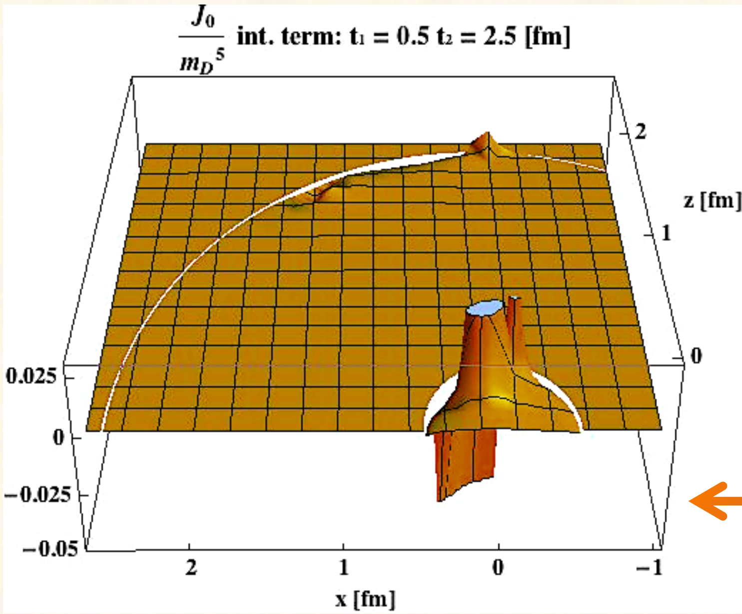
$$J^\nu(x,u_1,u_2) = \Theta(t^2-r^2)\frac{m_D^2\,\alpha_s}{8\pi}\left[-\frac{\gamma\, w_1^\nu\left(\delta u\cdot w_2 + u_1\cdot u_2\gamma_2\sqrt{-w_2^2}\right)}{\left(w_2^0+\gamma_2\sqrt{-w_2^2}\right)^2(-w_1^2)^{3/2}}\right.$$
$$+\frac{u_1^\nu\gamma_1\left(\delta u\cdot w_1\left(t-w_2^0-\gamma_2\sqrt{-w_2^2}\right)+w_1^2\right)}{\left(w_2^0+\gamma_2\sqrt{-w_2^2}\right)^2(-w_1^2)^{3/2}} - \frac{u_1\cdot u_2\gamma_2\gamma_1 w_1^\nu}{\sqrt{-w_2^2}(-w_1^2)^{3/2}}$$
$$+\frac{u_1^\nu\gamma_1}{(-w_1^2)^{3/2}}\left(1-\frac{\gamma_1^2(t-\mathrm{u}_1\cdot\mathrm{r})}{(\mathrm{r}-\mathrm{u}_2 t)^2}\right.$$
$$\left.\times\,\delta\mathrm{u}\cdot\left((\mathrm{r}-\mathrm{u}_2 t)+\frac{\gamma_2\left(\mathrm{u}_2(r^2-\mathrm{u}_2\cdot\mathrm{r}t)+\mathrm{r}(u_2^2 t-\mathrm{u}_2\cdot\mathrm{r})\right)}{\sqrt{-w_2^2}}\right)\right)$$
$$+\frac{u_1^\nu\gamma_1}{(-w_1^2)^{3/2}}\left(\frac{u_2^2 t-\mathrm{u}_2\cdot\mathrm{r}+\delta\mathrm{u}\cdot\mathrm{u}_2\gamma_1^2(t-\mathrm{u}_1\cdot\mathrm{r})}{t}\left(\frac{\tanh^{-1}u_2-u_2}{u_2^3}\right)\right.$$
$$\left.\left.+\frac{r-t\tanh^{-1}\frac{r}{t}}{r}\left(\frac{\gamma_1^2(\mathrm{u}_1\cdot\mathrm{r}-u_1^2 t)}{t}-1+\frac{(\mathrm{r}\cdot\mathrm{u}_1)\gamma_1^2(t-\mathrm{u}_1\cdot\mathrm{r})}{r^2}\right)\right)\right]$$

$$J^\nu_{\mathrm{total}} = C_p J_a^\nu(x,u_1,u_1) + C_A J^\nu(x,u_2,u_2) - \frac{C_A}{2}\left[J^\nu(x,u_1,u_2)+J^\nu(x,u_2,u_1)\right]$$

$$w_i^\alpha = x^\alpha - \gamma_i^2(x\cdot u_i)u_i^\alpha$$

Now generalize to consider parton shower:

A plot of the interference contribution:



A 3D representation of the extent and strength of the appropriately scaled interference term at two different times, t=0.5 fm and t=2.5 fm, for a primary parton that propagates in the z direction.

R.B. Neufeld and Ivan Vitev: arXiv:1105.2067 [hep-ph]

$$J^\nu(x,u_1,u_2) = \Theta(t^2-r^2)\frac{m_D^2\,\alpha_s}{8\pi}\Bigg[-\frac{\gamma\, w_1^\nu\left(\delta u\cdot w_2 + u_1\cdot u_2\gamma_2\sqrt{-w_2^2}\right)}{\left(w_2^0+\gamma_2\sqrt{-w_2^2}\right)^2(-w_1^2)^{3/2}}$$

$$+\frac{u_1^\nu\gamma_1\left(\delta u\cdot w_1\left(t-w_2^0-\gamma_2\sqrt{-w_2^2}\right)+w_1^2\right)}{\left(w_2^0+\gamma_2\sqrt{-w_2^2}\right)^2(-w_1^2)^{3/2}} - \frac{u_1\cdot u_2\gamma_2\gamma_1 w_1^\nu}{\sqrt{-w_2^2}(-w_1^2)^{3/2}}$$

$$+\frac{u_1^\nu\gamma_1}{(-w_1^2)^{3/2}}\left(1-\frac{\gamma_1^2(t-\mathbf{u}_1\cdot\mathbf{r})}{(\mathbf{r}-\mathbf{u}_2 t)^2}\right.$$

$$\left.\times\,\delta\mathbf{u}\cdot\left((\mathbf{r}-\mathbf{u}_2 t)+\frac{\gamma_2\left(\mathbf{u}_2(r^2-\mathbf{u}_2\cdot\mathbf{r}t)+\mathbf{r}(u_2^2 t-\mathbf{u}_2\cdot\mathbf{r})\right)}{\sqrt{-w_2^2}}\right)\right)$$

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$$\left.+\frac{r-t\tanh^{-1}\frac{r}{t}}{r}\left(\frac{\gamma_1^2(\mathbf{u}_1\cdot\mathbf{r}-u_1^2 t)}{t}-1+\frac{(\mathbf{r}\cdot\mathbf{u}_1)\gamma_1^2(t-\mathbf{u}_1\cdot\mathbf{r})}{r^2}\right)\right)\Bigg]$$

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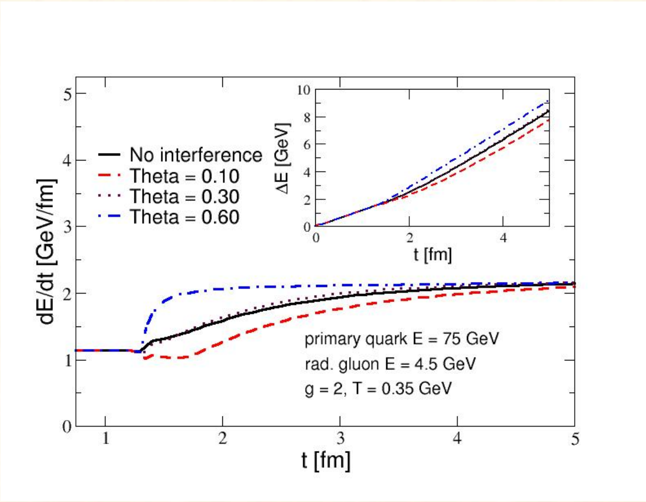
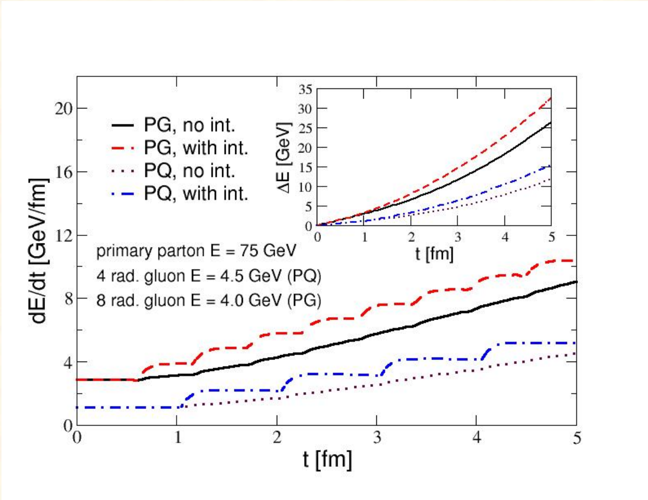
$$w_i^\alpha = x^\alpha - \gamma_i^2(x\cdot u_i)u_i^\alpha$$

Application: Parton shower energy loss

The parton shower energy loss is obtained through a spatial integration over the zero component:

$$\frac{dE}{dt} = \int dr\ \left[C_p J_a^0(x,u_1,u_1) + C_A J^0(x,u_2,u_2)\ - \frac{C_A}{2}\left(J^0(x,u_1,u_2) + J^0(x,u_2,u_1)\right)\right]$$





Left panel: The rate of energy transfer for single gluon emission at three different angles, 0.1, 0.3, 0.6, as well as for no interference.
Right panel: Rate of energy transfer for realistic in medium parton showering constrained by GLV emission spectrum. The insets show total energy transfers.

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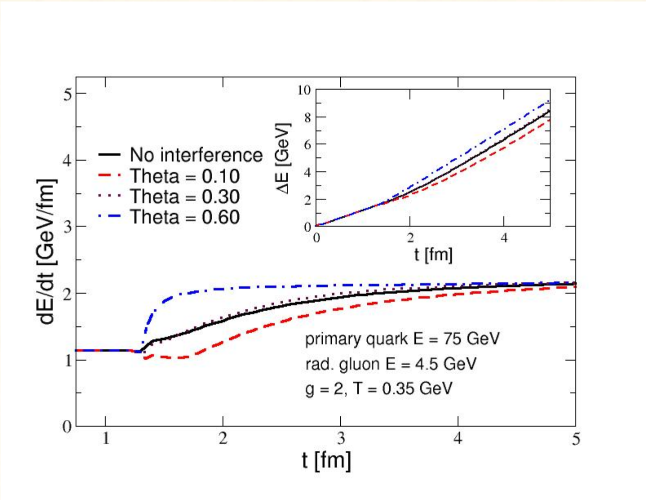
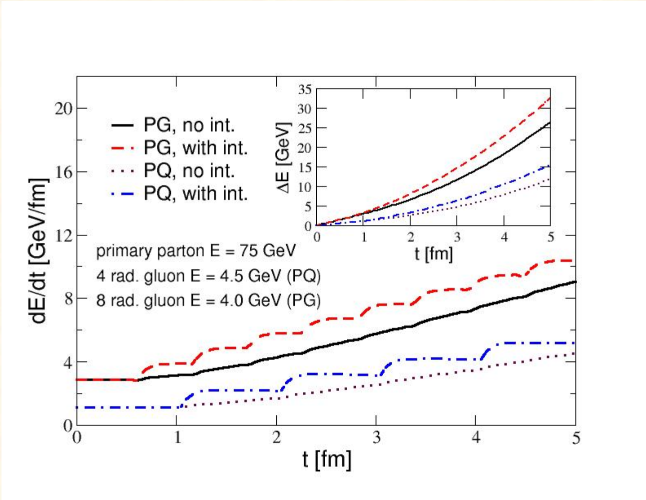


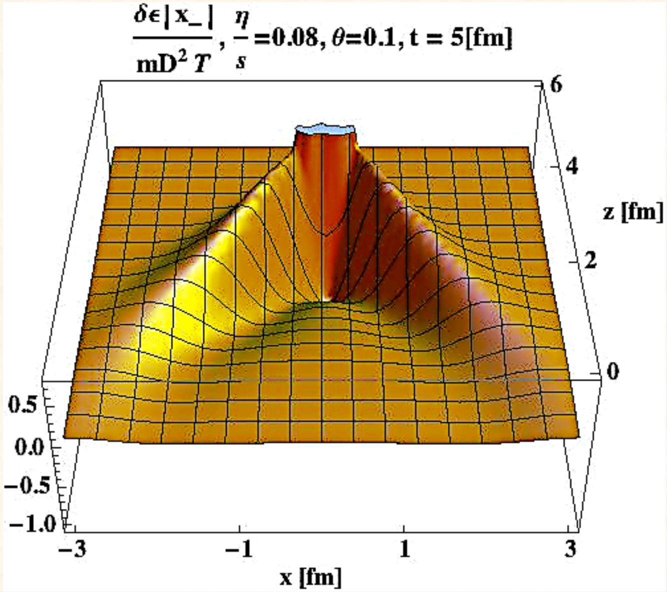


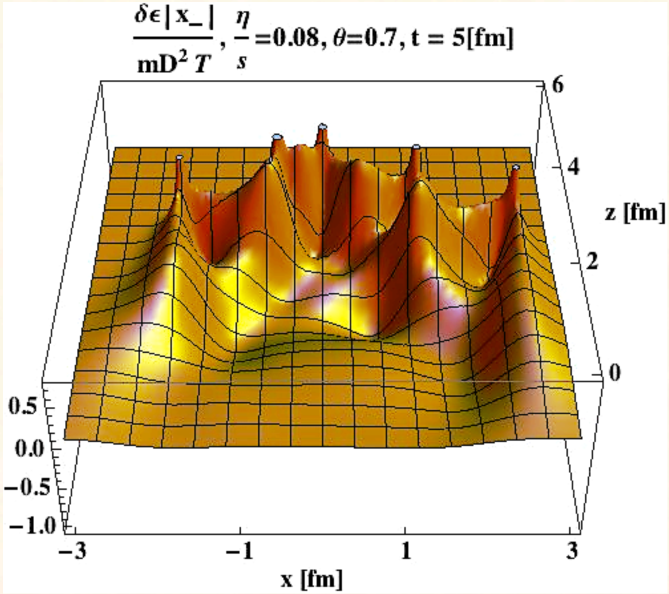
A significant amount of energy can be lost from a parton shower to the underlying medium. The amount lost is sensitive to number and angle of gluon radiation. This effect is important in the description of full jet observables.

Application II: Medium response to parton shower

The source term couples to the energy momentum tensor: $\partial_\mu T^{\mu\nu} = J^\nu$

Can be used as source for hydrodynamics. Consider primary quark which emits 4 gluons (as in the energy loss):





Energy density disturbance from a parton shower for two different average angles of emission. The narrow angle generates a Mach cone similar to that expected from a single parton, but the interference suppresses the energy transfer to the medium. For the larger angle there is no longer a well defined Mach cone but rather a superposition of several distinct disturbances.

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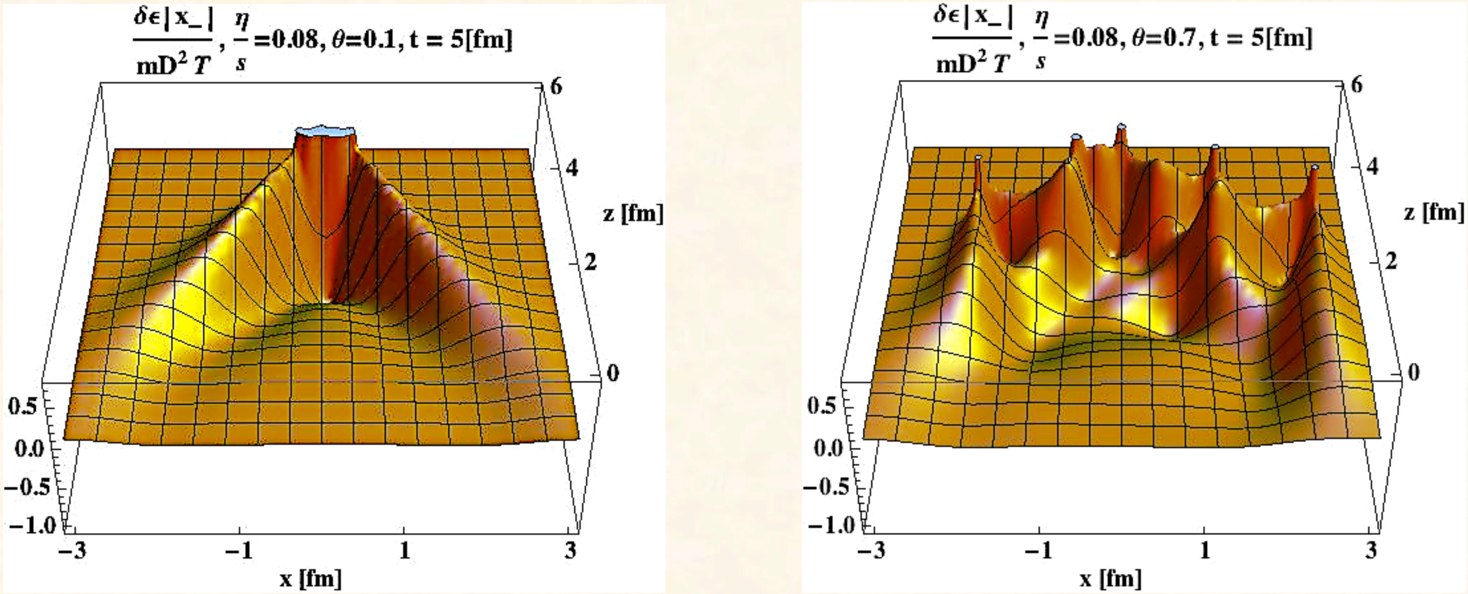

Parton showering at realistic angles alters the medium response compared to narrow or collinear emission. Schematic treatments of the parton shower provide an inaccurate description of shockwave phenomena. Also, very high pT triggers with large gluon number are not likely to have well defined shockwave structures.

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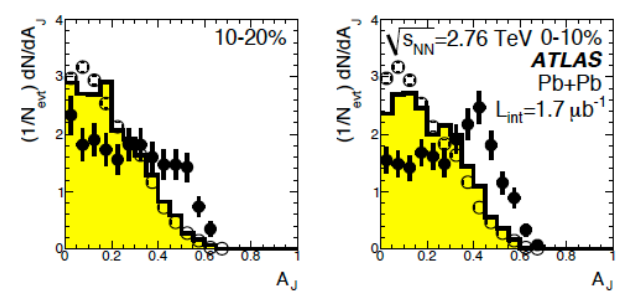
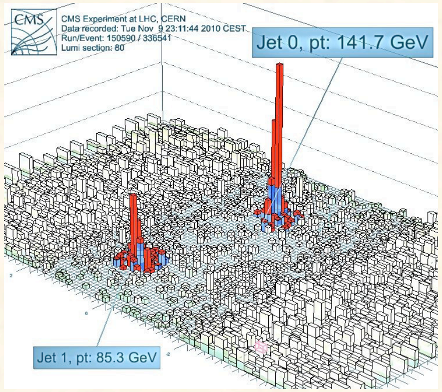
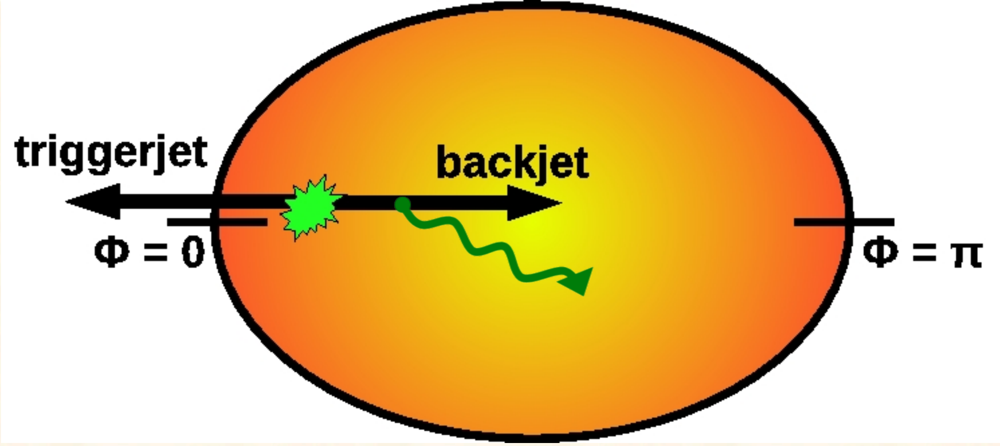
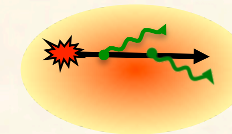
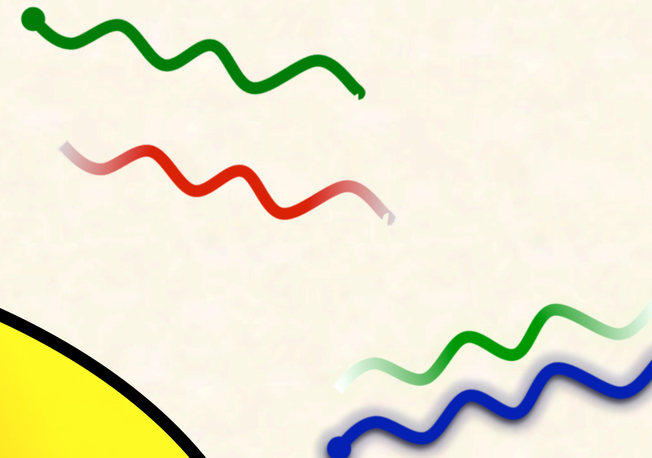
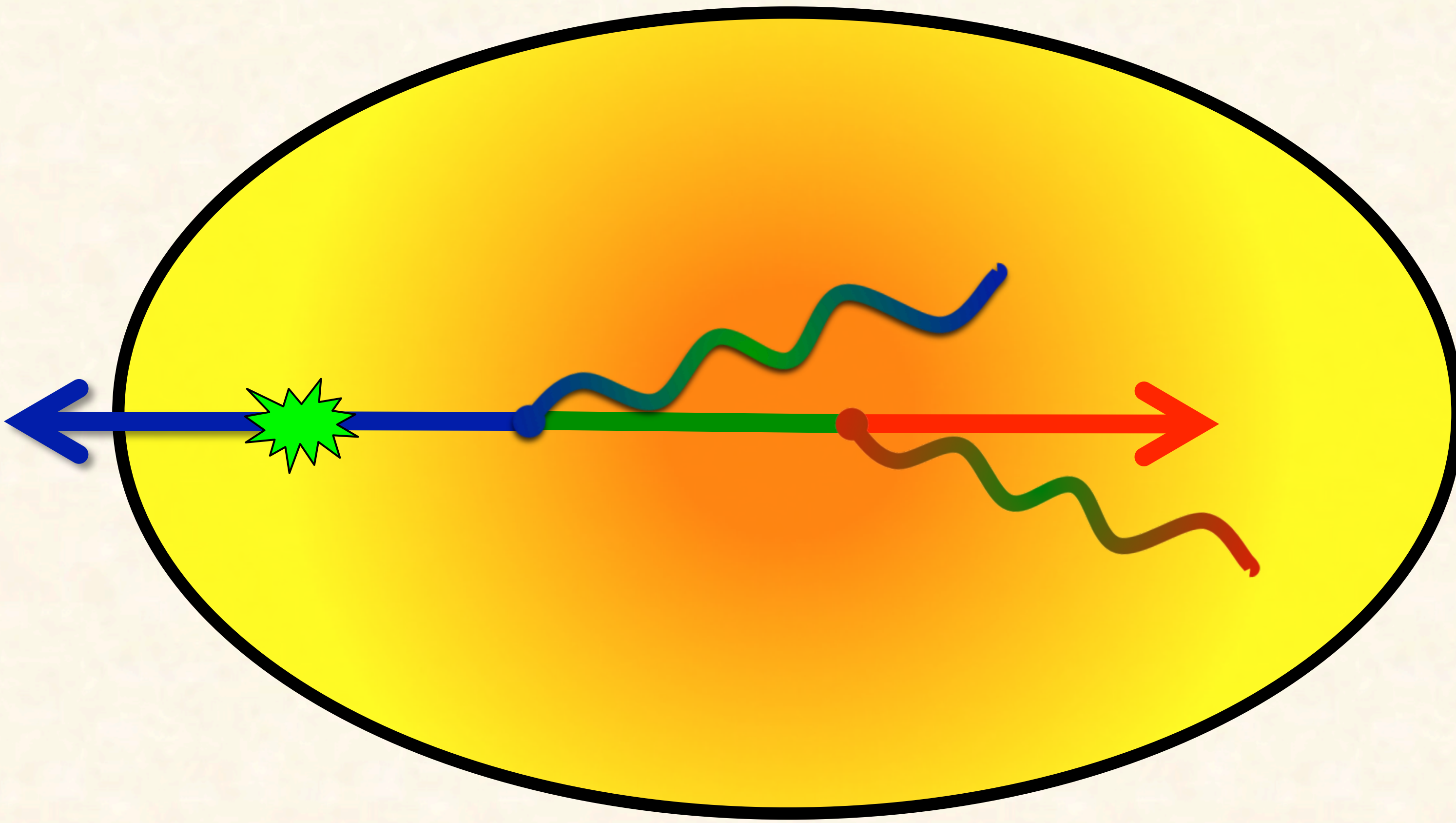
Future directions include:

consider back-to-back hard parton showers created in a hard scattering

full description of the redistribution of energy lost to the medium in an effort to describe jet observables






triggerjet
backjet
Φ = 0
Φ = π




$$
\begin{aligned}
T^{\mu\nu} &= (\epsilon + p)u^\mu u^\nu - g^{\mu\nu} p \\
&\rightarrow (\epsilon + p + \delta\epsilon + \delta p)(u + \delta u)^\mu (u + \delta u)^\nu - g^{\mu\nu}(p + \delta p) \\
&\approx T_0^{\mu\nu} + (\delta\epsilon + \delta p)u^\mu u^\nu - g^{\mu\nu}\delta p + (\epsilon + p)(\delta u^\mu u^\nu + \delta u^\nu u^\mu)
\end{aligned}
$$

$$
\begin{aligned}
\delta T^{00} &= \delta\epsilon \\
\delta T^{0i} &= (\epsilon + p)\delta u^i \equiv \mathrm{g} \\
\delta T^{ij} &= c_s^2 \, \delta\epsilon \, \delta^{ij} + \mathrm{gradients}(\delta u)
\end{aligned}
$$

$$\left(\delta_{ij} c_s^2 \delta\epsilon - \frac{\eta}{sT}\left(\partial^i g^j + \partial^j g^i - \frac{2}{3}\delta_{ij} \nabla \cdot \mathrm{g}\right) - \frac{\zeta}{sT}\delta_{ij} \nabla \cdot \mathrm{g}\right)$$

$$\zeta = 15\,\eta(1/3 - c_s^2)^2$$

$$J^\nu_{\mathrm{total}} = C_p J_a^\nu(x, u_1, u_1) + C_A J^\nu(x, u_2, u_2) - \frac{C_A}{2}\left[J^\nu(x, u_1, u_2) + J^\nu(x, u_2, u_1)\right]$$