ON THE POTENTIAL OF MINIMAL FLAVOUR VIOLATION

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Within the scheme of Minimal Flavour Violation, the possibility of spontaneous flavour symmetry breaking is explored by analyzing the scalar potential compatible with the symmetries. In this setup the Yukawa couplings arise from the vacuum expectation value (vev) of fields that transform under the flavour group. The outcome of the analysis of the potential depends much, especially for the mixing angles, on the field content.

1 Minimal Flavour Violation

The hypothesis of Minimal Flavour Violation [1] (MFV) accounts for the suppression of flavour changing neutral currents as well as for CP violating processes in any theory beyond the Standard Model. The reason for the flavour alignment of all processes is, in this hypothesis, assumed to stem in an underlying symmetry. In the limit of vanishing Yukawa couplings the quark sector of the Standard Model presents and extended symmetry group that is, aside from $U(1)$ factors:

$$\mathcal{G}_F \equiv SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R} \,. \tag{1}$$

Under this group the left-handed quark doublet Q_L would transform as $(3,1,1)$, the right-handed up-type quarks U_R as $(1,3,1)$ and the right-handed down type quarks D_R as $(1,1,3)$. This symmetry is assumed to be an exact symmetry at some high scale ($\Lambda_{fl} \gg v$). The introduction of the Yukawa couplings must then be accompanied by the assignment of transformation properties under $\mathcal{G}_F$, such that the Yukawa interaction is made invariant,

$$\mathscr{L}_Y = \overline{Q}_L Y_D D_R H + \overline{Q}_L Y_U U_R \tilde{H} + h.c. \,, \qquad Y_U \sim (3,\overline{3},1) \,, \qquad Y_D \sim (3,1,\overline{3}) \,. \tag{2}$$

Besides the flavour symmetry proposed, the other assumption of MFV is that these are the only flavour carrying structures. With this hypothesis, any operator arising from new physics in an effective Lagrangian formalism with the Standard Model fields has it's flavour structure determined by the imposition of invariance under $\mathcal{G}_F$, which is achieved by the proper insertion of Yukawa couplings.

2 The dynamical origin of MFV

Through all this reasoning there is the implicit assumption of a dynamical origin of these Yukawa couplings. The first consequence of exploring this assumption seriously is regarding the Yukawa interaction as an effective one [4] involving flavons, that is, the fields whose vevs will fix the

Yukawa couplings. The immediate extension is a dimension 5 *Yukawa operator*:

$$\mathscr{L}_Y = \overline{Q}_L \frac{\Sigma_d}{\Lambda_{fl}} D_R H + \overline{Q}_L \frac{\Sigma_u}{\Lambda_{fl}} U_R \tilde{H} + h.c. \,, \qquad \Sigma_u \sim (3,\overline{3},1)\,, \qquad \Sigma_d \sim (3,1,\overline{3})\,. \tag{3}$$

The transformation properties are fixed by the imposition of invariance under $\mathcal{G}_F$, and therefore the Σ scalar fields transform as *bi-fundamental* representations. From an effective Lagrangian point of view the next possibility is a dimension 6 *Yukawa operator*,

$$\begin{aligned} \mathscr{L}_{Y_D} &= \overline{Q}_L \frac{\chi_d^L \chi_d^{R\dagger}}{\Lambda_{fl}^2} D_R H + h.c.\,, & \chi_d^L &\sim (3,1,1)\,, & \chi_d^R &\sim (1,1,3)\,, \\ \mathscr{L}_{Y_U} &= \overline{Q}_L \frac{\chi_u^L \chi_u^{R\dagger}}{\Lambda_{fl}^2} U_R \tilde{H} + h.c.\,, & \chi_u^L &\sim (3,1,1)\,, & \chi_u^R &\sim (1,3,1)\,. \end{aligned} \tag{4}$$

The scalar fields χ transform as *fundamental* representations. The dimension 7 operator could contain a fermion condensate as Georgi and Chivukula suggested [2] but here only the two first cases will be discussed.

The scalar fields must acquire a vev through a scalar potential, and such potential must be invariant under $\mathcal{G}_F$ transformations. The discussion is now turned to whether a general scalar potential invariant under $\mathcal{G}_F$ will naturally fix the actual masses and mixing angles.

2.1 Dimension 5 Yukawa Operator

The construction of an invariant scalar potential for the fields Σ requires first the identification of the invariant magnitudes that can be constructed with the fields. Such a list was first made by Feldmann *et al.* [3], here we use a different notation:

$$\begin{aligned} A_u &= \mathrm{tr}\left(\Sigma_u \Sigma_u^\dagger\right)\,, & B_u &= \det\left(\Sigma_u\right)\,, \\ A_d &= \mathrm{tr}\left(\Sigma_d \Sigma_d^\dagger\right)\,, & B_d &= \det\left(\Sigma_d\right)\,, \\ A_{uu} &= \mathrm{tr}\left(\Sigma_u \Sigma_u^\dagger \Sigma_u \Sigma_u^\dagger\right)\,, & A_{dd} &= \mathrm{tr}\left(\Sigma_d \Sigma_d^\dagger \Sigma_d \Sigma_d^\dagger\right)\,, \\ A_{ud} &= \mathrm{tr}\left(\Sigma_u \Sigma_u^\dagger \Sigma_d \Sigma_d^\dagger\right)\,. \end{aligned} \tag{5}$$

The potential then has the form, for the three family case and to the renormalizable level,

$$V^{(4)}\left(\Sigma\right) = \sum_{i=u,d} \left(-\mu_i^2 A_i + \tilde{\mu}_i B_i + \lambda_i A_i^2 + \lambda_i' A_{ii}\right) + g_{ud} A_u A_d + \lambda_{ud} A_{ud}\,. \tag{6}$$

As the vevs of the Σ fields are related to quark masses and mixing through Eq. 3 ,

$$\frac{\langle \Sigma_u \rangle}{\Lambda_{fl}} = Y_U = V_{CKM}^\dagger \cdot Diag\left(y_u, y_c, y_t\right)\,, \qquad \frac{\langle \Sigma_d \rangle}{\Lambda_{fl}} = Y_D = Diag\left(y_d, y_s, y_b\right)\,, \tag{7}$$

substitution of these relations in Eq. 5 allows for the analysis of the potential as a function of quark masses and mixing parameters.

The study of the potential in Eq. 6 reveals that the complete pattern of masses and mixing angles cannot arise for any value of the potential parameters; one massive quark per sector and no mixing is the closest to the actual values achievable at this level. For the two family case a renormalizable potential can yield the hierarchy $y_u - y_c$ and $y_d - y_s$ for a certain set of fine-tuned parameters but again no non-zero angle is obtained. To obtain nonzero angles one can go to the non-renormalizable level, as, after all, ours is an effective Lagrangian with cut-off

Λ_{fl}. To illustrate how to fix the Cabibbo angle to it's actual value through a non-renormalizable potential, let's examine the angle dependence of the potential at the renormalizable level for two families:

$$V^{(4)}_{\theta_c} \equiv \lambda_{ud} A_{ud} = \lambda_{ud} \Lambda^4_{fl} \left(\cos 2\theta_c \left(y_c^2 - y_u^2 \right) \left(y_s^2 - y_d^2 \right) + \left(y_c^2 + y_u^2 \right) \left(y_s^2 + y_d^2 \right) \right) /2 \,, \tag{8}$$

This term, regarded as a potential for θ_c, will have it's minimum at either $\theta_c = 0$ or $\theta_c = \pi/2$, so some different dependence on θ_c must be added for nonzero angle. The next relevant term appearing in the series of increasing dimension invariants depending on θ_c is A^2_{ud}. With these two terms one can construct a 'mexican hat' , $V^{(8)} \supset \lambda_{ud} A_{ud} + \lambda_{udud} A^2_{ud}/\Lambda^4_{fl} \sim \lambda_{udud}(A_{ud} - \alpha)^2/\Lambda^4_{fl}$ that will fix the Cabibbo angle to the experimental value provided the fine tunned ratio $\lambda_{ud}/\lambda_{udud} \sim 10^{-10}$. This type of fine-tuning illustrates the difficulty within this approach for obtaining the actual masses and mixing parameters.

2.2 Dimension 6 Yukawa Operator

This case requires careful connection of the vev of the fields with the Yukawa couplings,

$$\frac{\left\langle \chi^L_u \right\rangle \left\langle \chi^{R\dagger}_u \right\rangle}{\Lambda^2_{fl}} = Y_U \,, \qquad\qquad \frac{\left\langle \chi^L_d \right\rangle \left\langle \chi^{R\dagger}_d \right\rangle}{\Lambda^2_{fl}} = Y_D \,. \tag{9}$$

The structure on the left of each term is not a general mass matrix but one composed of two 'vectors'. One finds the result of this fact by looking at the eigenvalues and eigenvectors of the matrix

$$Y_U Y_U^\dagger = \frac{\left\langle \chi^{R\dagger}_u \chi^R_u \right\rangle}{\Lambda^4_{fl}} \left\langle \chi^L_u \right\rangle \left\langle \chi^{L\dagger}_u \right\rangle \,. \tag{10}$$

The matrix structure is given only by the flavon χ^L_u, and has an immediate diagonalization; χ^L_u is the only eigenvector with non-zero eigenvalue, such eigenvalue being

$$y^2_{u_i} \equiv \left\langle \chi^{L\dagger}_u \chi^L_u \right\rangle \left\langle \chi^{R\dagger}_u \chi^R_u \right\rangle /\Lambda^4_{fl} \,, \tag{11}$$

where y_{u_i} is the only nonzero up-type Yukawa entry in this approach. The mixing then arises as the misalignment when diagonalizing both $Y_U Y_U^\dagger$ and $Y_D Y_D^\dagger$. Such misalignment is just the relative direction between χ^L_u and χ^L_d, but as we are talking of two 'vectors' this magnitude is described by one relative angle, which means there is one physical angle only in this scheme;

$$\cos\theta = \frac{\left\langle \chi^{L\dagger}_u \chi^L_d \right\rangle}{\left|\chi^L_u\right| \left|\chi^L_d\right|} \,, \tag{12}$$

where $\left|\chi^L_u\right|^2 \equiv \left\langle \chi^{L\dagger}_u \chi^L_u \right\rangle$. Once made the connection with masses and mixing angles we turn to constructing the potential. All the magnitudes related to masses and mixing angles have expressions in terms of the vevs of the only five possible $\mathcal{G}_F$ invariants:

$$X^2 \equiv \left(\chi^{L\dagger}_u \chi^L_u \,,\, \chi^{L\dagger}_d \chi^L_d \,,\, \chi^{R\dagger}_u \chi^R_u \,,\, \chi^{R\dagger}_d \chi^R_d \,,\, \chi^{L\dagger}_d \chi^L_u \right)^T \,. \tag{13}$$

The potential to the renormalizable level will be the sum of a linear combination of these invariants and products of two of these invariants. This can be formally written:

$$V^{(4)}(\chi) = -\mu^2 \cdot X^2 + \left(X^2\right)^\dagger \lambda \, X^2 \,. \tag{14}$$

In a first approach we neglect any CP violation effect and chose real parameters, μ^2 is an array of 5 real components and λ a symmetric matrix [a]. The minimum of this potential is then:

$$\langle X^2 \rangle = \frac{1}{2}\lambda^{-1}\mu^2\,, \tag{15}$$

provided that λ is invertible. This approach naturally accommodates the angle as it's expression in terms of the potential parameters involves the ratio of linear combinations of the entries of μ^2 given by λ^{-1}, which is naturally of $\mathcal{O}(1)$. For definiteness let us take the two family case, although the discussion this far is independent of the number of generations, and write explicitly:

$$\begin{gathered} y_c^2 = \frac{1}{4\Lambda_{fl}^4}\left(\lambda^{-1}\mu^2\right)_{uL}\left(\lambda^{-1}\mu^2\right)_{uR}\,, \qquad y_s^2 = \frac{1}{4\Lambda_{fl}^4}\left(\lambda^{-1}\mu^2\right)_{dL}\left(\lambda^{-1}\mu^2\right)_{dR}\,, \\ \cos\theta_c = \frac{\left(\lambda^{-1}\mu^2\right)_{ud}}{\sqrt{(\lambda^{-1}\mu^2)_{dL}\,(\lambda^{-1}\mu^2)_{uL}}}\,. \end{gathered} \tag{16}$$

3 Conclusions

Here the possibility of spontaneous breaking of the flavour symmetry regarded in MFV was considered. In such framework, the Yukawa couplings are fixed by the vev of flavour-carrying fields. The analysis of the potential leading to spontaneous flavour symmetry breaking differs for the field content and therefore the group representation on which the scalar fields are placed. The choice of a dimension 5 Yukawa operator, that is, the introduction of scalar fields transforming in bi-fundamental representations, does not allow for mixing among quarks at the renormalizable and classical level, it can accommodate the actual hierarchy of masses in the two family case and only a partial hierarchy for the three family case. The introduction of fundamental fields through a dimension 6 Yukawa operator allows for natural mixing among quarks and imposes the strong hierarchy of one massive quark only per up and down sector at the classical and renormalizable level. Although a dimension 6 Yukawa operator is better suited to accommodate the experimental data, none of the approaches gives the complete picture of masses and mixing angles. Such complete landscape could arise from the simultaneous consideration of both operators or addition of more scalar fields. Overall, it is remarkable that the requirement of invariance under the flavour symmetry strongly constrains the scalar potential of MFV, up to the point that the obtention of quark mass hierarchies and mixing angles is far from trivial.

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[a] The indices of λ and μ^2 run over the five values $\{uL,dL,uR,dR,ud\}$.