Lattice QCD's flavour physics results for phenomenologists - a mini review

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La Thuile, Italy

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Introduction

Why lattice QCD?

- perturbation theory works well for weak coupling
- bound state observables like the proton mass or the B-decay constant for example cannot be predicted by perturbation theory

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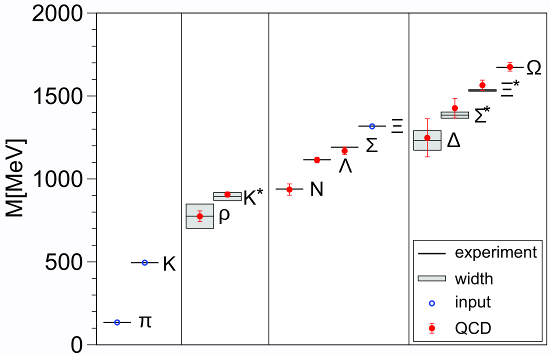
but simulations of lattice QCD can do this:



BMW Collaboration, Science 322 (2008) 1224-1227

Introduction

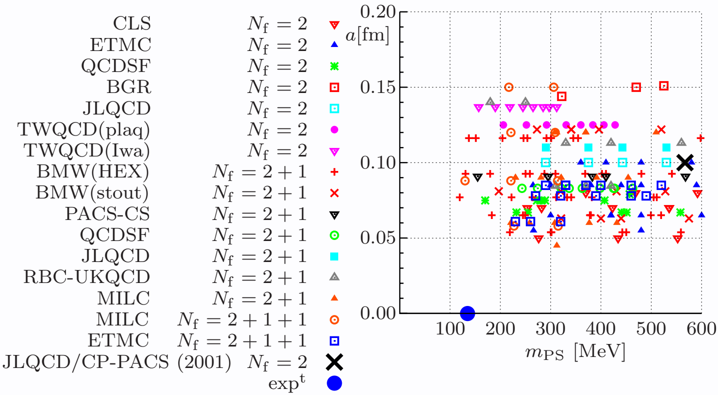
What is . . . ?

	QCD
N_c	3
N_f, fundamental	1+1+1+1+1+1
$SU(2)$ iso-spin brk.	✔
$m_\pi^\pm$	135MeV
V	∞
a	0

What is . . . ?

	QCD	Lattice QCD
N_c	3	3
N_f, fundamental	1+1+1+1+1+1	0, 2, 2+1, 2+1+1
$SU(2)$ iso-spin brk.	✓	✗ *see Taku's talk*
$m_\pi^\pm$	135MeV	$\lesssim m_\pi^{\rm sim}$
V	∞	2-3fm
a	0	0.05-0.1fm

Status of simulations



Plot kindly provided by G. Herdoiza

- dynamical simulations are standard by now
- first simulations now include dynamical charm quark
- simulations with phyiscal pion masses have bcome feasible
- cut-off $a^{-1} = 1/0.05fm \approx 4GeV$

Simulations

- discretise space-time in euclidean QCD path integral

$$\langle \mathcal{O}[\bar\psi,\psi,A]\rangle_{\mathrm{QCD}} = \frac{1}{Z}\int D\bar\psi D\psi DA\, \mathcal{O}(\bar\psi,\psi,A)\, e^{-S_G(U)-S_q(\bar\psi,\psi,U)}$$

  and do numerical MC-integration
  (discretisations: Wilson, staggered, twisted mass, domain wall, overlap, . . .)

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- biggest machine is good but ≠ best lattice QCD
  even more important are fundamental understanding and usage of
  - field theory
  - algorithms

IBM BlueGene/Q

Systematics

- most current results systematics-dominated
- extrapolation of lattice data to the physical point very often tricky (it’s easy if you already know the experimental number ... but what if not?)

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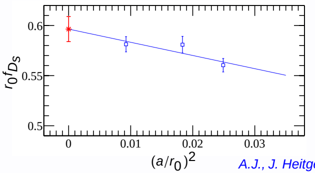

A.J., J. Heitger, JHEP 0905 (2009) 101

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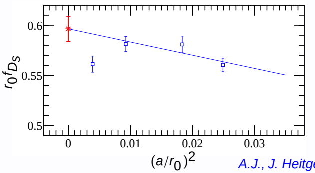

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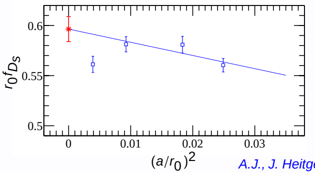

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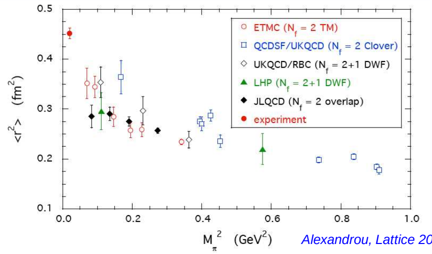
$m_q \rightarrow m_q^{\rm phys}$ chiral eff. th.



Alexandrou, Lattice 2010

Systematics continued

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- scale setting
- finite size errors
- chosen discretisation
- . . .

Lattice phenomenology

Experiment	=	**SM $\perp$ NPQCD**	$\times$	**NPQCD**
ΔM_d	=	const. $\times\, \lvert V_{tb} V_{td}\rvert^2 S\left(\frac{\bar{m}_t^2}{M_W^2}\right)$	$\times$	$f_{B_d}^2 B_{B_d}$
		perturbation theory		lattice QCD, sum rules

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Other NP SM observables or parameters:

- meson and baryon spectra
- matrix elements relevant for phenomenology:
  - decay constants $(f_\pi, f_K, f_{D_{(s)}}, f_{B_{(s)}})$
  - form factors $(f^{\pi\pi}, f^{K\pi}, f^{D\to K}, f^{B\to\pi}, \ldots)$
  - mixing matrix elements $(B, B_{B_{(s)}}, \ldots)$
  - hadr. K-decays (A_0, A_2)

Quality of lattice results

Very strong claims are made based on lattice QCD results:

- "We find a (2-3)σ tension in the unitarity triangle"
  Laiho, Lunghi, Van de Water, PRD 81 (2010) 034503
- "... confirming CKM unitarity at the permille level"
  FLAG arXiv:1011.4408
- "... we find evidence of new physics in both B_d and B_s systems ..."
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How to deal with lattice results?

Are these statements water-proof?

FLAG

→ Flavia Net Lattice Averaging Group (**FLAG**) was founded to allow also to an outsider to judge the quality and 'state-of-the-art'-fulnes of lattice results relevant to flavor physics

People: G. Colangelo, S. Dürr, A. J., L. Lellouch, H. Leutwyler, V. Lubicz, S. Necco, C. Sachrajda, S. Simula, A. Vladikas, U. Wenger, H. Wittig

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→ criteria:
- publication status
- chiral extrapolation
- continuum extrapolation
- finite volume errors
- renormalisation
- renormalisation scale running

→ quantities: $m_{u,d}$, m_s, $f_+^{K\pi}(0)$, f_K/f_π, B_K, NLO LEC's, potentially more in the future

→ planned periodic updates of *arXiv:1011.4408, http://itpwiki.unibe.ch/flag*

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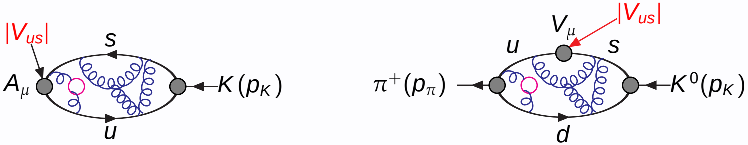
Other efforts by Laiho, Lunghi, Van de Water:
Lattice QCD inputs to the CKM unitarity triangle analysis, Phys.Rev. D81 (2010) 034503 http://www.latticeaverages.org

A FLAG example - the kaon sector



$$\frac{\Gamma(K \to \mu\bar{\nu}_\mu(\gamma))}{\Gamma(\pi \to \mu\bar{\nu}_\mu(\gamma))} = \frac{|V_{us}|^2}{|V_{ud}|^2}\left(\frac{f_K}{f_\pi}\right)^2 \frac{m_K(1-m_\mu^2/m_K^2)}{m_\pi(1-m_\mu^2/m_\pi^2)} \times 0.9930(35)$$

(Marciano, Phys.Rev.Lett. 2004)

$$\Gamma_{K\to\pi l\nu} = C_K^2 \frac{G_F^2 m_K^5}{192\pi^2} I\, S_{\rm EW}[1+\Delta_{SU(2)}+\Delta_{\rm EM}] \times |V_{us}|^2 |f_+^{K\pi}(0)|^2$$

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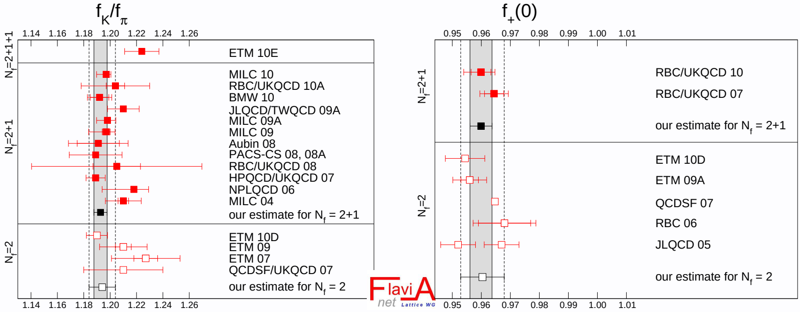

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Collaboration	N_f	publication status	chiral extrapolation	continuum extrapolation	finite volume errors	f_K/f_π
ETM 10E	2+1+1	C	●	●	●	$1.224(13)_{stat}$
MILC 10	2+1	C	●	★	★	$1.197(2)(^{+3}_{-7})$
RBC/UKQCD 10A	2+1	P	●	●	★	1.204(7)(25)
BMW 10	2+1	A	★	★	★	1.192(7)(6)
JLQCD/TWQCD 09A	2+1	C	●	■	■	$1.210(12)_{\rm stat}$
MILC 09A	2+1	C	●	★	★	$1.198(2)(^{+6}_{-8})$
MILC 09	2+1	A	●	★	★	$1.197(3)(^{+6}_{-13})$
Aubin 08	2+1	C	●	●	●	1.191(16)(17)
PACS-CS 08, 08A	2+1	A	★	■	■	1.189(20)
RBC/UKQCD 08	2+1	A	●	■	★	1.205(18)(62)
HPQCD/UKQCD 07	2+1	A	●	★	●	1.189(2)(7)
NPLQCD 06	2+1	A	●	■	■	$1.218(2)(^{+11}_{-24})$
ETM 10D	2	C	●	★	●	$1.190(8)_{stat}$
ETM 09	2	A	●	★	●	1.210(6)(15)(9)
QCDSF/UKQCD 07	2	C	●	●	★	1.21(3)

FLAG arXiv:1011.4408

A FLAG example - the kaon sector

For both $N_f = 2+1$ and $N + f = 2$ FLAG identified high quality lattice results and provides averages:

FLAG averages *FLAG arXiv:1011.4408*:

N_f	$f_+^{K\pi}(0)$	f_K/f_π
2+1	0.9560(57)(62)	1.193(5)
2	0.9599(34)(41)	1.210(6)(17)

Together with experimental input

$\lvert V_{us} f_+^{K\pi}(0)\rvert$	=	0.2163(5)
$\left\lvert \frac{f_K V_{us}}{f_\pi V_{ud}} \right\rvert$	=	0.2758(5)
$\lvert V_{ud}\rvert$	=	0.97425(22)

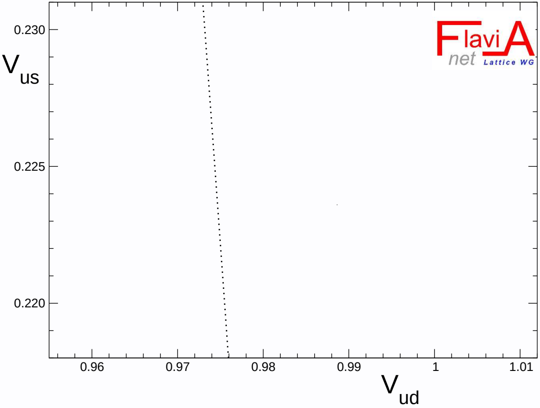
FLAVIA KAON WG Eur.Phys.J. C69 (2010) 399-424
Hardy, Towner, Phys. Rev., C79, 2009,05550

FLAG did two kind of analysis:

a) test the SM: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \stackrel{?}{=} 1$

b) using SM-correlations

FLAG tests the SM: $|V_u|^2 \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$



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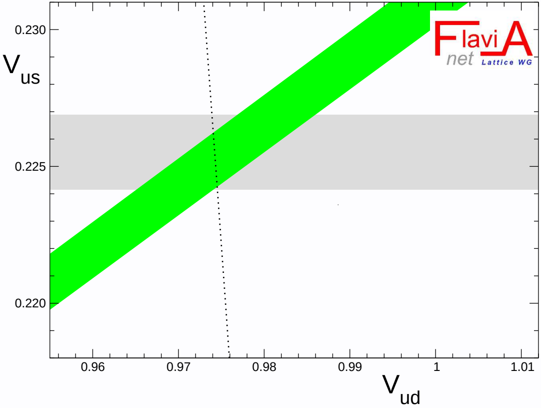

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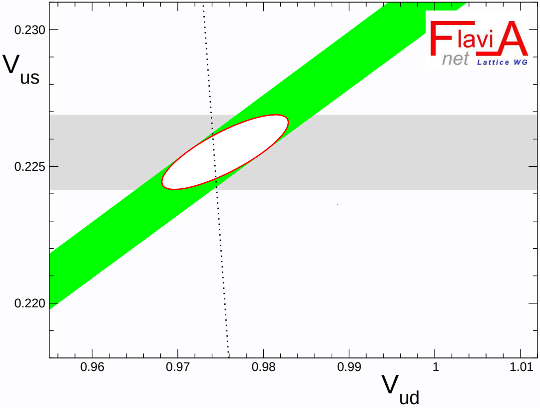

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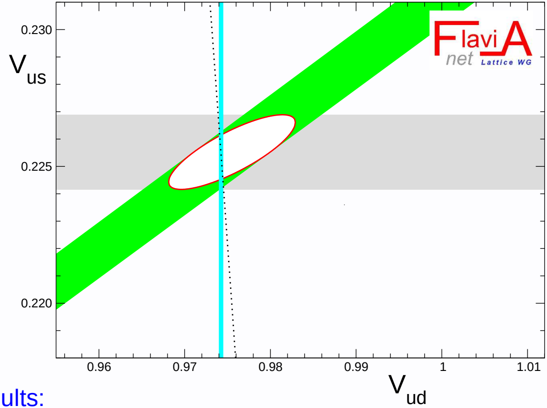

Results:

N_f	$\lvert V_u\rvert^2\lvert_{no\lvert V_{ud}\rvert}$	$\lvert V_u\rvert^2\lvert_{\lvert V_{ud}\rvert}$ from $f_+^{K\pi}(0)$	$\lvert V_u\rvert^2\lvert_{\lvert V_{ud}\rvert}$ from f_K/f_π
2+1	1.002(15)	1.0000(7)	0.9999(6)
2	1.037(36)	1.0004(10)	0.9985(16)

So indeed, CKM-unitarity confiremd at the per-mil level

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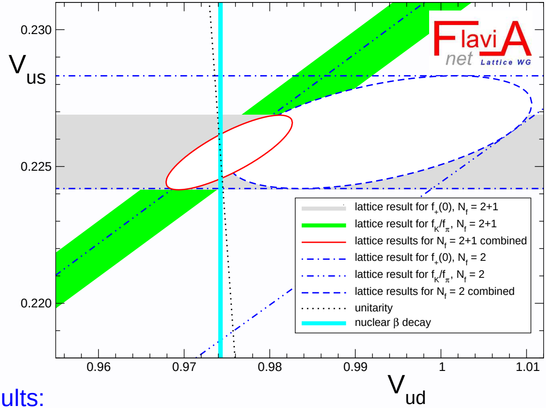

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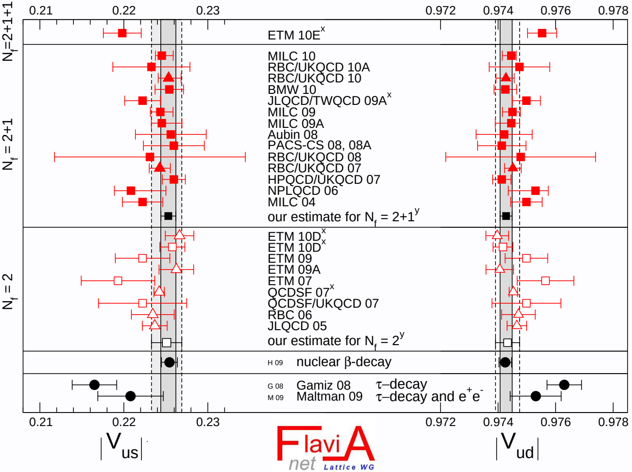

Nice result: Assuming first row unitarity lattice QCD makes prediction for $|V_{ud}|$ with same precision as super-allowed nuclear beta decays and fully compatible with it

ϵ_K, $f_{B_{(s)}}$ and $B_{B_{(s)}}$

lattice QCD input to SM-tests *(cf. Soni's talk in this session)*

K-observables	B-observables	
$\epsilon_K \propto B_K$	$\Delta M_d \propto f_{B_d}^2 B_{B_d}$	$\Delta\Gamma_d \propto (f_{B_d} B_{B_d})^2$
	$\Delta M_s \propto f_{B_s}^2 B_{B_s}$	$\Delta\Gamma_s \propto (f_{B_s} B_{B_{(s)}})^2$

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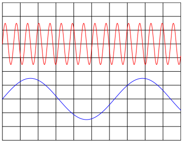
ϵ_K	$\propto$	B_K	ΔM_d	$\propto$	$f_{B_d}^2 B_{B_d}$	$\Delta\Gamma_d$	$\propto$	$(f_{B_d} B_{B_d})^2$
			ΔM_s	$\propto$	$f_{B_s}^2 B_{B_s}$	$\Delta\Gamma_s$	$\propto$	$(f_{B_s} B_{B_{(s)}})^2$

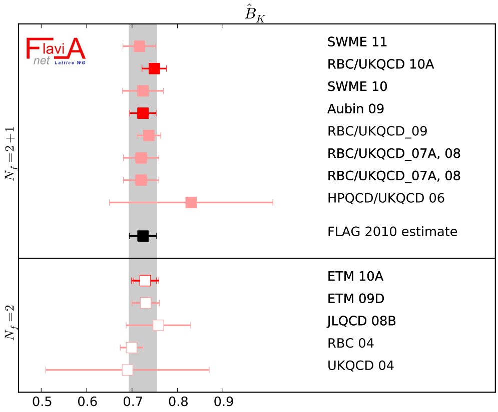
b-quarks on the lattice not straight forward! Reason: $m_b \approx 4\text{GeV} \approx a^{-1}$



This causes head-aches and huge cut-off effects $\rightarrow$ naively impossible
Techniques for *b*-quarks on the lattice (separation of scales):

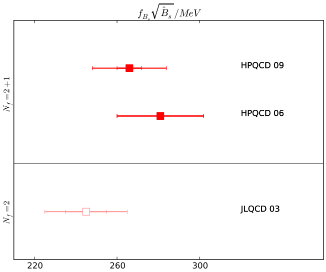
- static approx (leading HQET) *(INF-TOV, ALPHA, ETM)*
- HQET *(ALPHA)*
- NRQCD *(HPQCD)*
- relativistic heavy quark actions *(Fermilab, RBC/UKQCD , PACS-CS)*

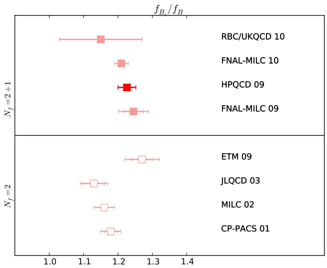
FLAG: ϵ_K

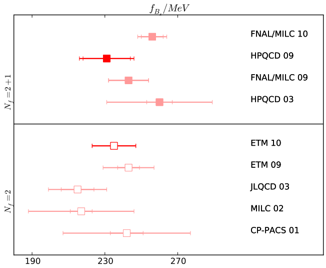


- recent breakthrough thanks to chirally symmetric lattice actions (Domain Wall Fermions, *see Taku's talk (next)*)
- well advanced lattice calculations, $\delta \approx 3-4\%$
- FLAG average currently being updated

NON-FLAG: $f_{B_{(s)}}$ and $B_{B_{(s)}}$







e.g. HPQCD-uncertainties

$$\delta(f_{B_s}\sqrt{\hat{B}_{B_s}}) \approx\ > 7\%$$
$$\delta(\xi) \approx\ > 3\%$$
$$\delta(f_{B_s}/f_B) \approx\ > 2\%$$
$$\delta(f_{B_s}) \approx\ > 6\%$$

NON-FLAG: $f_{B_{(s)}}$ and $B_{B_{(s)}}$

- given it's

  - potentially huge impact

  - non-straight forward lattice computation,

  $f_{B_{(s)}}$ and $B_{B_{(s)}}$ urgently need confirmation by other (non-staggered) groups using approaches with ideally uncorrelated systematics

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- again, cleanest would be to renormalise the operators non-perturbatively and it is very well understood how to do it
- $N_f = 2$ QCD and $N_f = 2+1$ QCD are different theories - it is not meaningful to average results
- consecutive updates of results by one and the same collaboration are highly correlated

NON-FLAG: $K \to \pi\pi$

$$\langle \pi\pi(I)|\mathcal{H}^{\text{eff}}|K\rangle$$

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current non-standard approaches allow to compute it anyway:

- indirectly via χPT *RBC PRD68, (2003), CP-PACS PRD68, (2003)*
  depends crucially on SU(3) chiral PT which turns out to converge badly
  repeat efforts with unphysically light strange quarks?

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- directly, using (partially twisted, G-parity) boundary condition tricks to project on the desired state *arXiv:0912.2917*
  - very feasible in $\Delta I = 3/2$ channel
  - very cost-intensive in $\Delta I = 1/2$ channel

  N. Christ in his Kaon 2009 write-up *arXiv:0912.2917*: "$\Delta I = 1/2, 3/2$ amplitudes A_0 and A_2 with 10-20% precision within the next 2-3 years" (RBC+UKQCD)

Outlook

- Kaons:
  - well advanced calculations
  - of f_K/f_π and $f_+^{K\pi}(0)$ the latter is most likely to improve considerably in the near future
  - tremendous progress in B_K over the last years $\rightarrow \approx 4\%$ uncertainty
  - dominant uncertainties in renormalisation procedure and running
- B-physics:
  - calculations are advanced but one could do much better (clean heavy-quark discretisation, continuum limit and non-perturbative renormalisation)
  - more dedicated efforts based on different approaches are under way
  - in the near future individual results will not be produced with much improved precision

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  - dominant uncertainties in renormalisation procedure and running
- B-physics:
  - calculations are advanced but one could do much better (clean heavy-quark discretisation, continuum limit and non-perturbative renormalisation)
  - more dedicated efforts based on different approaches are under way
  - in the near future individual results will not be produced with much improved precision
- FLAG and ALVdW started efforts to provide you with easily accessible summaries of lattice phenomenology - please make use of this service and get in touch with us if interested

Outlook

- Kaons:
  - well advanced calculations
  - of f_K/f_π and $f_+^{K\pi}(0)$ the latter is most likely to improve considerably in the near future
  - tremendous progress in B_K over the last years $\rightarrow \approx 4\%$ uncertainty
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- B-physics:
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Lattice QCD is continously evolving and so does the quality of the results - so there is more to come

Thank you!

additional material

$\Delta S = 1, 2$ - CP-violation in the K-system

$$K_L \propto K_2 + \bar{\epsilon} K_1$$

CP odd — CP even

CP even $\pi\pi$

ϵ': direct

KTeV, NA48 1999

CP even $\pi\pi$

ϵ: indirect

Cronin&Fitch 1964

- indirect CP-violation ($\Delta S = 2$ Kaon mixing):

$$\epsilon_K = \frac{A(K_L \to (\pi\pi)_{I=0})}{A(K_S \to (\pi\pi)_{I=0})}$$

$$|\epsilon_K|_{\text{exp.}} = 2.28(2) \times 10^{-3}$$ *(PDG 2006)*

- direct CP-violation ($\Delta S = 1$ non-leptonic Kaon decay)

$$A(K^0 \to \pi^+\pi^-) = \sqrt{\tfrac{2}{3}} A_0 e^{i\delta_0} + \sqrt{\tfrac{1}{3}} A_2 e^{i\delta_2}$$

$$A(K^0 \to \pi^0\pi^0) = \sqrt{\tfrac{2}{3}} A_0 e^{i\delta_0} - \sqrt{\tfrac{1}{3}} A_2 e^{i\delta_2}$$

$$\epsilon' = \frac{\omega}{\sqrt{2}} e^{i\phi} \left(\frac{\text{Im}A_2}{\text{Re}A_2} - \frac{\text{Im}A_0}{\text{Re}A_0} \right)$$

$$\epsilon'/\epsilon|_{\text{exp.}} = 1.72(18) \times 10^{-3}$$

(PDG 2006)

$$\Delta I = \tfrac{1}{2}: \ \omega = \frac{\text{Re}A_2}{\text{Re}A_0}$$

$$\omega^{-1}|_{\text{exp.}} \approx 22$$

Results for B_K

Aubin, Laiho, van de Water Phys.Rev.D81:014507,2010

Source uncertainty/error	uncertainty/error on B_K
statistics	1.2%
chiral & continuum extrapolation	1.9%
scale and quark-mass uncertainties	0.8%
finite volume errors	0.6%
renormalization factor	3.4%
total systematic	4.0%
total	4.2%

Results for B_K

Aubin, Laiho, van de Water Phys.Rev.D81:014507,2010

Source uncertainty/error	uncertainty/error on B_K	
statistics	1.2%	
chiral & continuum extrapolation	1.9%	$a \gtrsim 0.09$fm $m_\pi \gtrsim 240$MeV
scale and quark-mass uncertainties	0.8%	
finite volume errors	0.6%	
renormalization factor	3.4%	← mainly NLO running
total systematic	4.0%	
total	4.2%	

Results for $f_+^{K\pi}(0)$

RBC+UKQCD Phys.Rev.Lett.100:141601,2008, arXiv:1004.0886

Source uncertainty/error	uncertainty/error on $f_+^{K\pi}(0)$
statistical	0.3%
chiral extrapolation	0.4%
continuum extrapolation	0.1%
total systematic	0.4%
total	0.5%

- dominant uncertainty from chiral extrapolation

Results for $f_+^{K\pi}(0)$

RBC+UKQCD Phys.Rev.Lett.100:141601,2008, arXiv:1004.0886

Source uncertainty/error	uncertainty/error on $f_+^{K\pi}(0)$	
statistical	0.3%	
chiral extrapolation	0.4%	$\gtrsim$ 330MeV
continuum extrapolation	0.1%	
total systematic	0.4%	
total	0.5%	

- dominant uncertainty from chiral extrapolation

Results for f_K/f_π

BMW Phys.Rev.D81:054507,2010

Source uncertainty/error	uncertainty/error on f_K/f_π
statistics	0.6%
chiral extrapolation	
- functional form	0.3%
- pion mass range	0.3%
continuum extrapolation	0.3%
exited states	0.2%
scale setting	0.1%
finite volume	0.1%
total syst	0.5%
total	0.8%

- “dominant” uncertainties: chiral and continuum extrapolation (other collabs reach much smaller stat. error than BMW)

Results for f_K/f_π

BMW Phys.Rev.D81:054507,2010

Source uncertainty/error	uncertainty/error on f_K/f_π	
statistics	0.6%	
chiral extrapolation		
- functional form	0.3%	
- pion mass range	0.3%	$\gtrsim$ 190MeV
continuum extrapolation	0.3%	$\gtrsim$ 0.064fm
exited states	0.2%	
scale setting	0.1%	
finite volume	0.1%	
total syst	0.5%	
total	0.8%	

- "dominant" uncertainties: chiral and continuum extrapolation (other collabs reach much smaller stat. error than BMW)

Results for f_K/f_π

TABLE III. Errors in % for $f_{B_s}\sqrt{\hat{B}_{B_s}}$, $f_{B_d}\sqrt{\hat{B}_{B_d}}$ and ξ.

Source of error	$f_{B_s}\sqrt{\hat{B}_{B_s}}$	$f_{B_d}\sqrt{\hat{B}_{B_d}}$	ξ
Stat + chiral extrap.	2.3	4.1	2.0
Residual a^2 extrap. uncertainty	3.0	2.0	0.3
$r_1^{3/2}$ uncertainty	2.3	2.3	-
$g_{B^*B\pi}$ uncertainty	1.0	1.0	1.0
m_s and m_b tuning	1.5	1.0	1.0
Operator matching	4.0	4.0	0.7
Relativistic corr.	2.5	2.5	0.4
Total	6.7	7.1	2.6

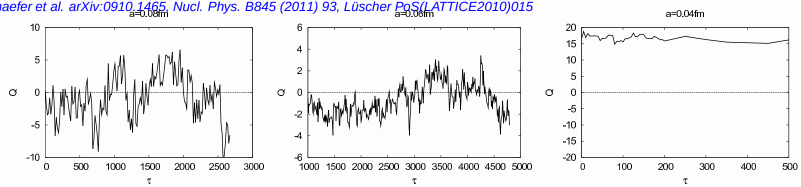
HPQCD, PRD 80 014503 (2009)

TABLE V. Errors in % for f_{B_s}, f_{B_d}, and f_{B_s}/f_{B_d}.

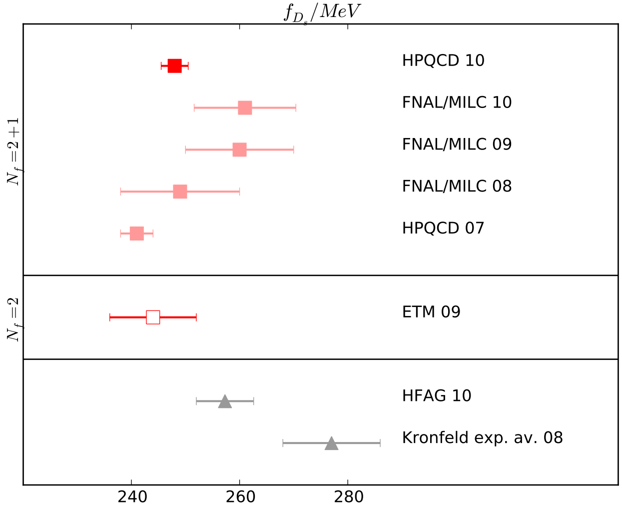
Source of error

Critical slowing down

Schaefer et al. arXiv:0910.1465, Nucl. Phys. B845 (2011) 93, Lüscher PoS(LATTICE2010)015



f_{D_s}



f_{D_s}/MeV
HPQCD 10
FNAL/MILC 10
FNAL/MILC 09
FNAL/MILC 08
HPQCD 07
ETM 09
HFAG 10
Kronfeld exp. av. 08
$N_f=2+1$
$N_f=2$
240
260
280