

# Recent results of CKM physics, quark masses from RBC/UKQCD

Taku Izubuchi  
for Riken-BNL-Columbia/UKQCD collaboration



Deepest Condolences to those who suffer from Tohoku-Pacific Ocean Earthquake, and all my best wishes for their earliest recovery.

- Lattice QCD computation :  
“Accelerator” and “Detector”
- CKM physics : continuum limit of  $B_K$
- QCD+QED simulation : up, down and strange quark masses
- Conclusion
- 
- ( Current situation of Japanese community )

# RBC/UKQCD collaboration

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A. Soni, R. Van de Water,  
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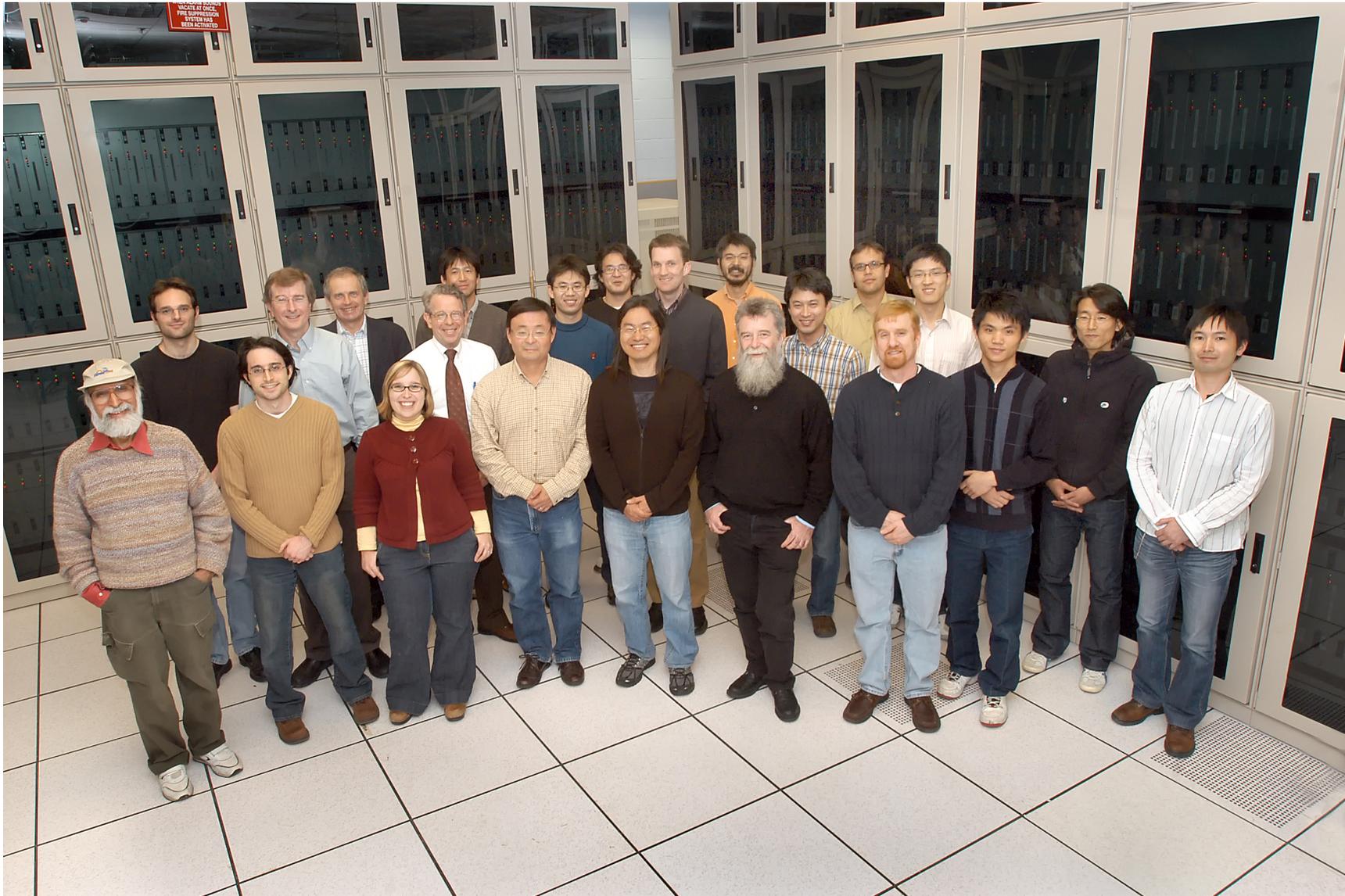
D. Brommel, J. Flynn, P. Fritzsich,  
E. Goode, C. Sachrajda

Univ. Regensburg E. Scholz

CERN A. Juttner

**11 students, 18 PhD theses**

# RBC/UKQCD collaboration



# Lattice QCD Advertisements

- Lattice QCD ( LQCD, or LGT ) is a perfect way to calculate physical quantities of QCD based on the first principle.
- Systematic errors could be, in principle, completely removable.
  - Perturbation theory is reliable for QED, and high energy QCD
  - Unknown truncation errors ( $\alpha_s^n$ ).
  - **Non-perturbative effects** of, *e.g.* a form

$$\exp[-1/\alpha_s^2] = 0 + 0\alpha_s^2 + 0\alpha_s^4 \dots$$

has no perturbative expansion in  $\alpha_s$ , which could be important for QCD's phenomena : bound states (**Hadron**), **spontaneously breaking chiral symmetry**, **confinement of colored**,.....

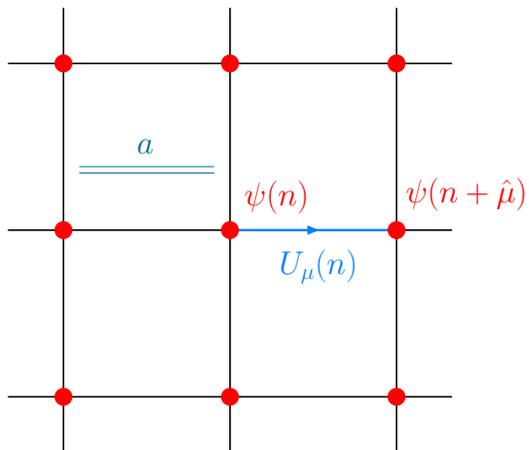
LGT computing has two ingredients :

- QCD vacuum ensemble generation  $\sim$  **Accelerator**
- Physical observable measurements  $\sim$  **Detector**

# Lattice Gauge Theory

- Analysis of Quantum Field Theory such as Quantum Chromo Dynamics, needs **non-perturbative** calculation.

$\Psi(x), A_\mu(x), x \in \mathcal{R}^4$ : **continuous infinity**  
**quantum divergences**: needs **regularization and renormalization**



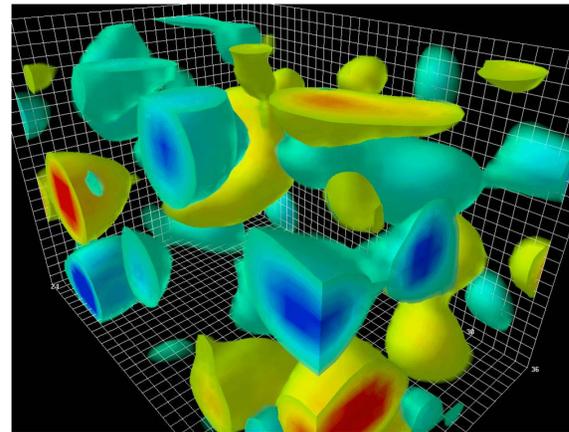
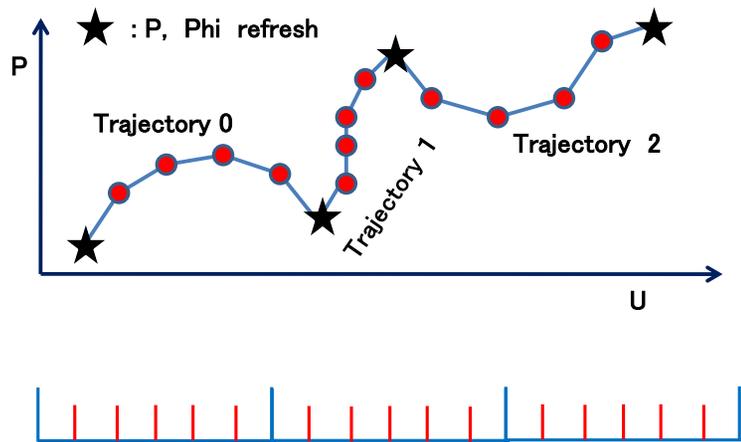
- Discretized Euclidean space-time
- lattice spacing  $a \sim 0.1 \text{ fm}$   
(UV cut-off  $|p| \leq \pi/a$ )
- $\psi(n)$  : Fermion field (Grassmann number)
- $U_\mu(n)$  : Gauge field

- Feynman's path integral for **Huge** dimensional variables  $32^3 \times 64 \sim 150\text{M}$   
Number of states (for simplest  $4^4$  Ising)  $2^{4^4} \sim 10^{77}$  needs more than  $10^{35}$  years !

# Hybrid Monte Carlo (LGT's "Accelerator")

- Monte Carlo to Sample Important configurations of QCD action  $e^{-S_{\text{QCD}}}$
- Accumulate samples of **QCD vacuum**, typically  $\mathcal{O}(100) \sim \mathcal{O}(1,000)$  files of gauge configuration  $U_\mu(n)$  on disk (1  $\sim$  10 GB/conf).
- By solving a classical QCD, with an occasional stochastic "hit": **exactly**  $\propto e^{-S_{\text{QCD}}}$
- Must generate sequentially  $\{U_\mu^{(0)} \rightarrow U_\mu^{(1)} \rightarrow \dots\}$ , which **needs capable machines**.

$$\text{Prob}(U_\mu) \propto \det D_{u,d,s}[U] e^{-S_g}$$



[D. Leinweber]

- RHMC for odd flavor [Clark Kennedy]
- Solve **short** (**long**) modes **more** (**less**) frequently [Hasenbush's trick]

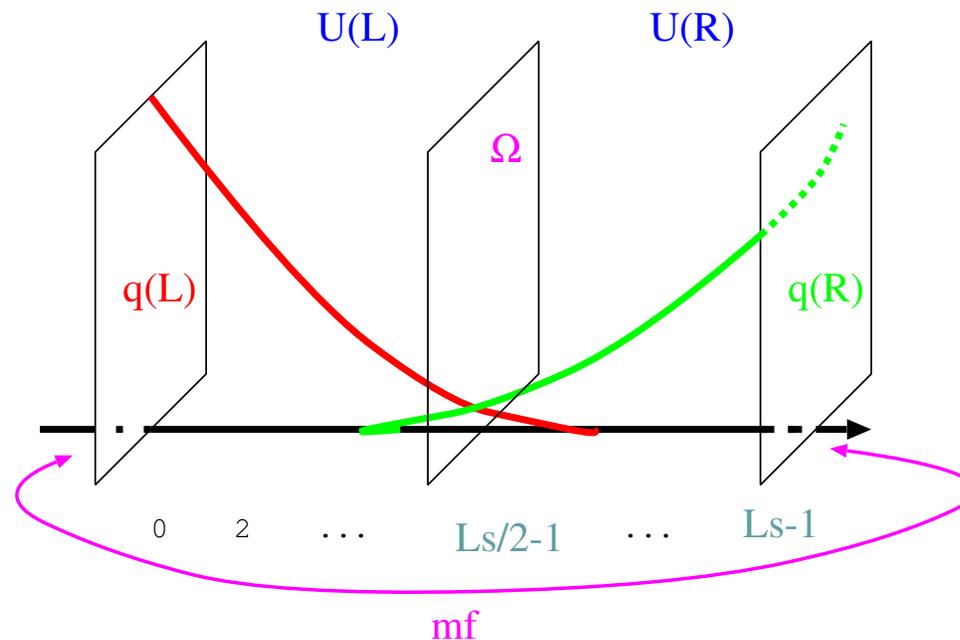
# Domain Wall Quarks (for up, down, and strange)

[Kaplan, Shamir, Blum & Soni]

- 4D lattice quark utilizing an “extra dimension”,  $L_s$ . (expensive)
- Almost perfect chiral symmetry

*Small unphysical mixing for the Weak Matrix Elements*  
*Error from discretization is small  $\mathcal{O}(a^2 \Lambda_{\text{QCD}}^2) \sim \text{a few \%}$ .*  
*Chiral extrapolation is simpler, continuum like.*

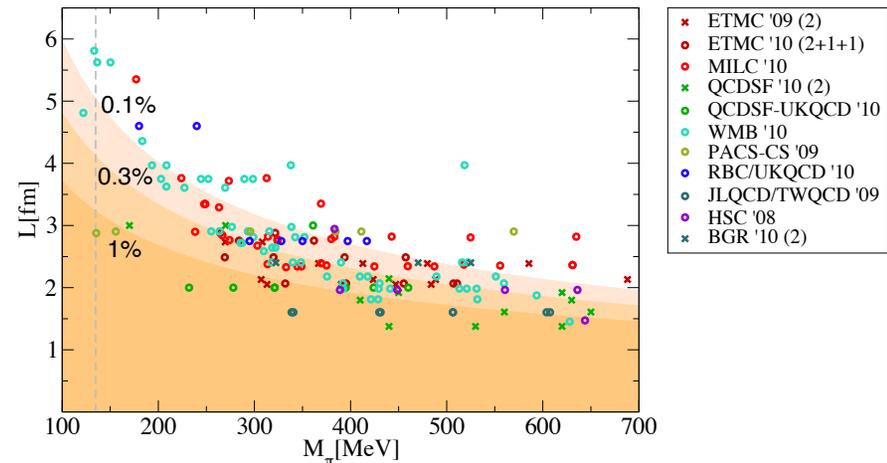
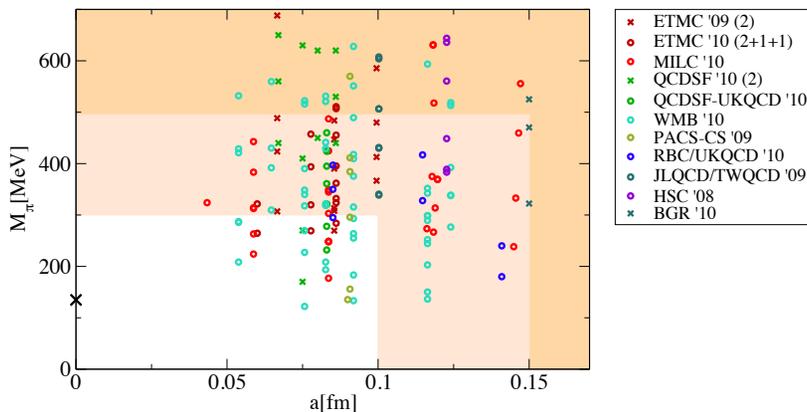
- Unitary theory (at long distance).



# Parameters of “Accelerator” (HMC)

- **Number** of QCD vacuum ensembles, controls statistical accuracies (“Luminosity”).  
 $N_{\text{conf}} \sim \mathcal{O}(100 - 1,000)$ ,
- **lattice spacings**  $a$  controls discretization errors  $a \sim 0.05\text{-}0.15$  fm
- up, down, and strange ( $N_F = 2+1$ ) **quark masses**:  $m_u, m_d, m_s < \Lambda_{\text{QCD}} < m_c, m_b, m_t$   
 lighter the more computationally demanding  $\propto m_q^{-3}$ .  $M_\pi : 190 \sim 500$  MeV
- lattice **volume**  $V : (2 \sim 6\text{fm})^3$

[C. Hoelbling, Lattice 2010]

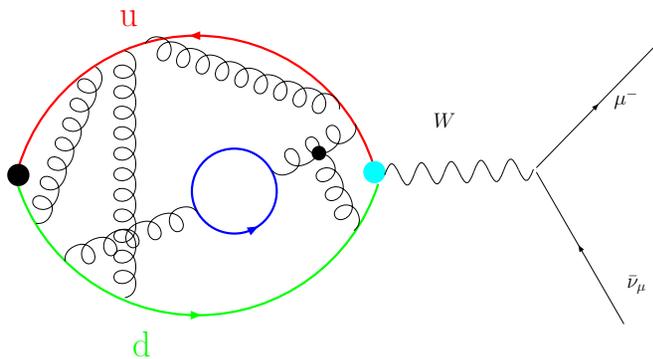


# Physics measurements “Detectors”

- Measurements **physical observables** on the vacuum ensemble.

$$\langle \mathcal{O} \rangle = \int \mathcal{D}U_\mu \text{Prob}[U_\mu] \times \mathcal{O}[U_\mu]$$

- Could do Analysis on many configurations **independently** (trivial parallel jobs) → could also use PC Clusters
- We made hadron **operator** (EW operators) from quark, and let the quark propagates on each of the generated QCD configuration (by solving the Dirac Eq)
- Obtain **hadron mass** or **QCD matrix elements** of operators



$$\langle 0 | \bar{d} \gamma_5 u(0) | \pi \rangle \frac{e^{i\vec{p}\cdot\vec{x}}}{\sqrt{2E}} \langle \pi | \bar{u} \gamma_\mu u \gamma_5 d | 0 \rangle \times G_F V_{ud} m_\mu \bar{\nu}(1 - \gamma_5) \mu$$

$$\begin{aligned} \mathcal{M}(\pi \rightarrow \mu \bar{\nu}) &\sim i f_\pi q_\mu \times G_F V_{ud} m_\mu (\bar{\nu} \mu)_L \\ &= \langle \pi(q) | \bar{u} \gamma_\mu \gamma_5 d(0) | 0 \rangle \times G_F V_{ud} m_\mu (\bar{\nu} \mu)_L \end{aligned}$$

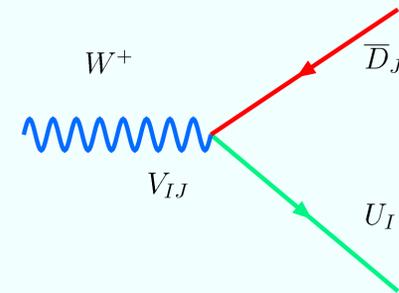


# CKM physics on DWQCD [ Ruth Van de Water lat09 ]



- In 1963, N. Cabbibo, then M. Kobayashi and T. Maskawa in 1973 introduced a mixing between quark flavors.

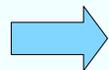
$$V_{CKM} = \begin{bmatrix} V_{ud}, V_{us}, V_{ub} \\ V_{cd}, V_{cs}, V_{cb} \\ V_{td}, V_{ts}, V_{tb} \end{bmatrix} \approx \begin{bmatrix} 1 - \lambda^2/2, & \lambda, & A\lambda^3(\rho - i\eta) \\ -\lambda, & 1 - \lambda^2/2, & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta), & -A\lambda^2, & 1 \end{bmatrix}$$



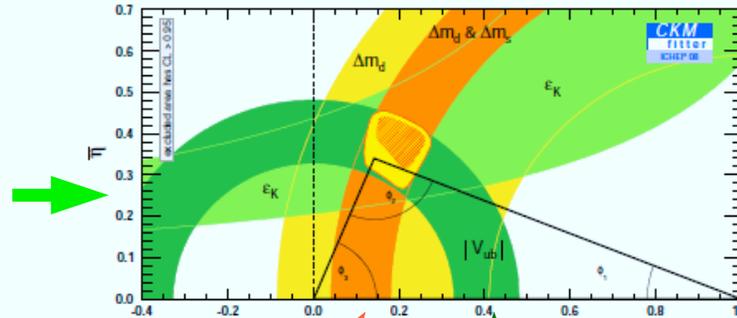
- Unitary condition CKM matrix includes: the unitarity triangle

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$$



$K_0 - \bar{K}_0$   $B_K$



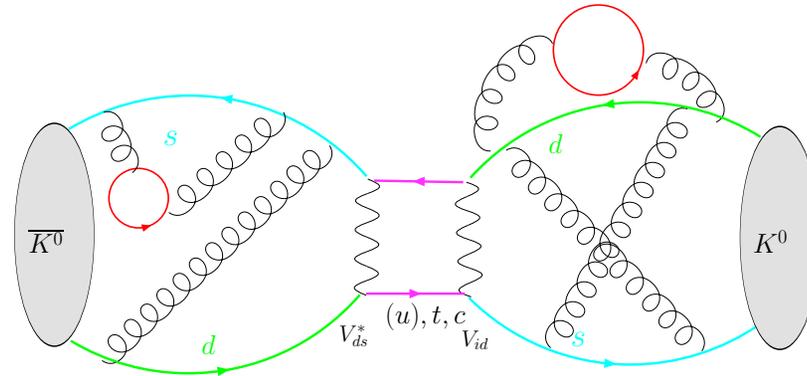
- To pin down the CKM, and to search for the new physics, the Hadron matrix elements is needed: **EXPT. = CKM x LATTICE**

B decay,  
 $B_0 - \bar{B}_0$

$B \rightarrow \pi$  | nu

# $K^0 - \overline{K^0}$ mixing, Indirect CP violation

- $K_{L,S}^0$  are the superpositions of CP eigen states  $|K_{\pm}^0\rangle$   
 $|K_{L,S}^0\rangle \sim \frac{1}{\sqrt{1+\epsilon_K^2}} (|K_{\mp}^0\rangle + \epsilon_K |K_{\pm}^0\rangle)$
- To compared with experimentally measured  $\epsilon_K$ , one needs to evaluate the matrix



elements of EW Hamiltonian

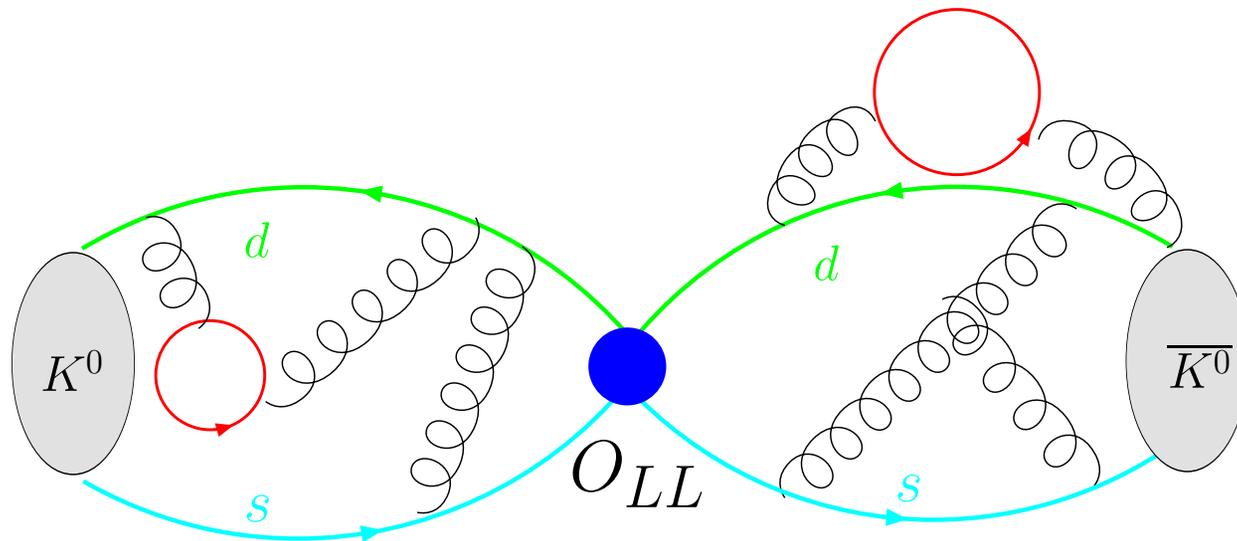
$$H^{\Delta S=2} = \frac{G_F^2 M_W^2}{16\pi^2} [ (V_{cd}^* V_{cs})^2 F(x_c) + (V_{td}^* V_{ts})^2 F(x_t) + 2(V_{td}^* V_{ts} V_{cd}^* V_{cs}) \bar{F}(x_c, x_t) ] O_{LL}$$

Besides known perturbative functions, this leads to **Kaon's bag parameter**  $B_K$

$$O_{LL} = \bar{s}\gamma_\mu(1 - \gamma_5)d\bar{s}\gamma_\mu(1 - \gamma_5)d$$

$$\epsilon_K = \mathcal{A} - \mathcal{B}\eta\rho\hat{B}_K$$

$$B_K = \frac{\hat{B}_K}{\alpha(\mu)^{-\gamma_0/(2\beta_0)}} = \frac{\langle \bar{K} | O_{LL} | K \rangle}{\langle \bar{K} | O_{LL} | K \rangle_{VS}} = \frac{\langle \bar{K} | O_{LL} | K \rangle}{\frac{8}{3}f_K^2 M_K^2}$$



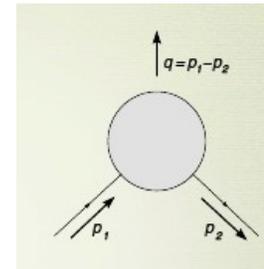
# Calibration of “Detector”

## New Renormalization Schemes

[09 C. Sturm, Y. Aoki, N. Christ, T. I. C. Sachrajda, A. Soni]

[10 L. Almeida, C. Sturm]

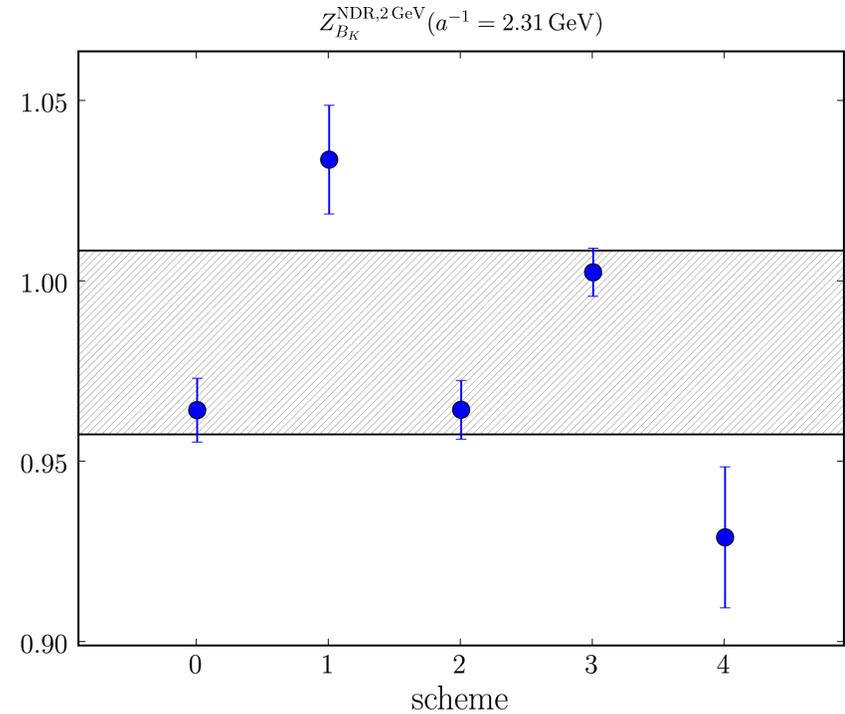
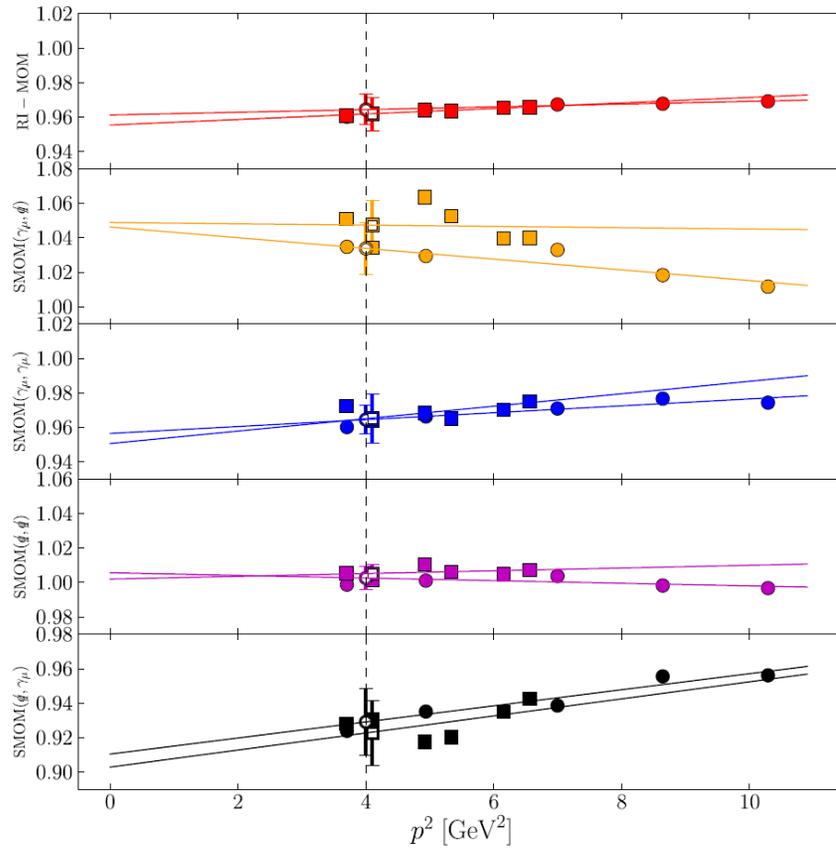
- Match the normalization of operator on lattice and in continuum theory (MS) via RI/SMOM schemes  
calculate the 3 pt amplitudes for a momentum configuration (SMOM) on lattice non-perturbatively, and in continuum theories.



$$(q^2 = p_1^2 = p_2^2)$$

- We find symmetric momentum (SMOM) configuration is useful to reduce one of the dominant systematic errors due to IR effects.
- Quark mass renormalization error  
 $\sim 10\%$  (MOM)  $\rightarrow \sim 5\%$  (SMOM)  $\rightarrow \sim 2\%$  (SMOM 2loop),
- Using different RI/SMOM schemes (using various spinor projections) to check the systematic errors
- Four quark operator for B\_K

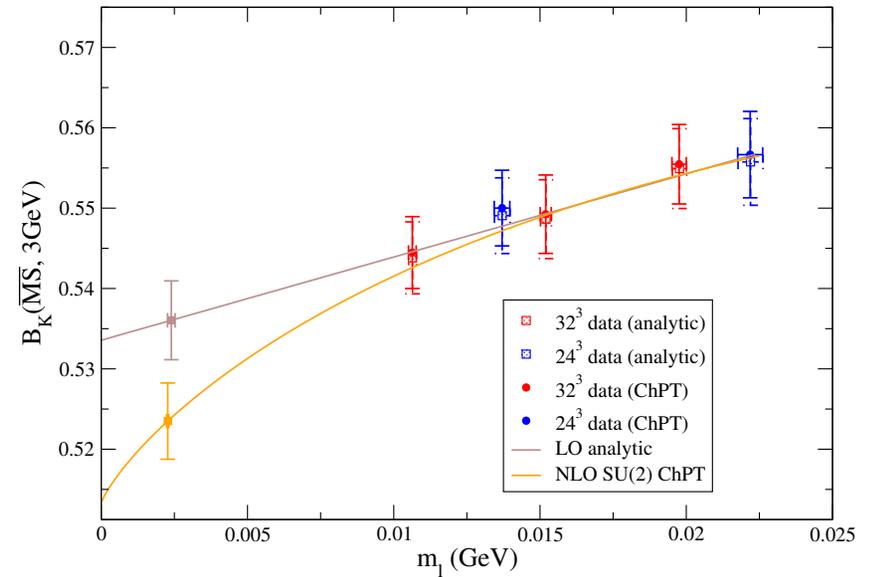
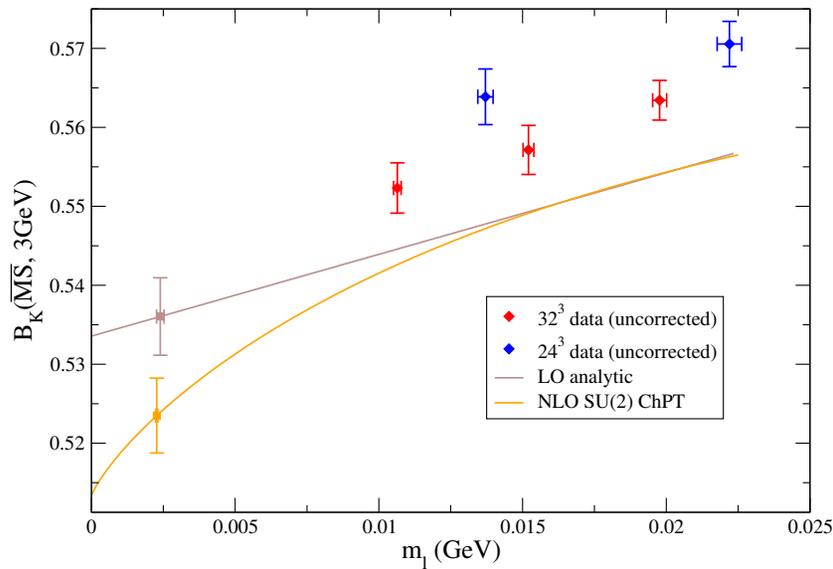
# Z(BK) systematic errors



$$Z_{B_K}^{NDR}(\mu = 2\text{GeV}, a^{-1} = 2.31\text{GeV}) = 0.964(25)[2.6\%]$$

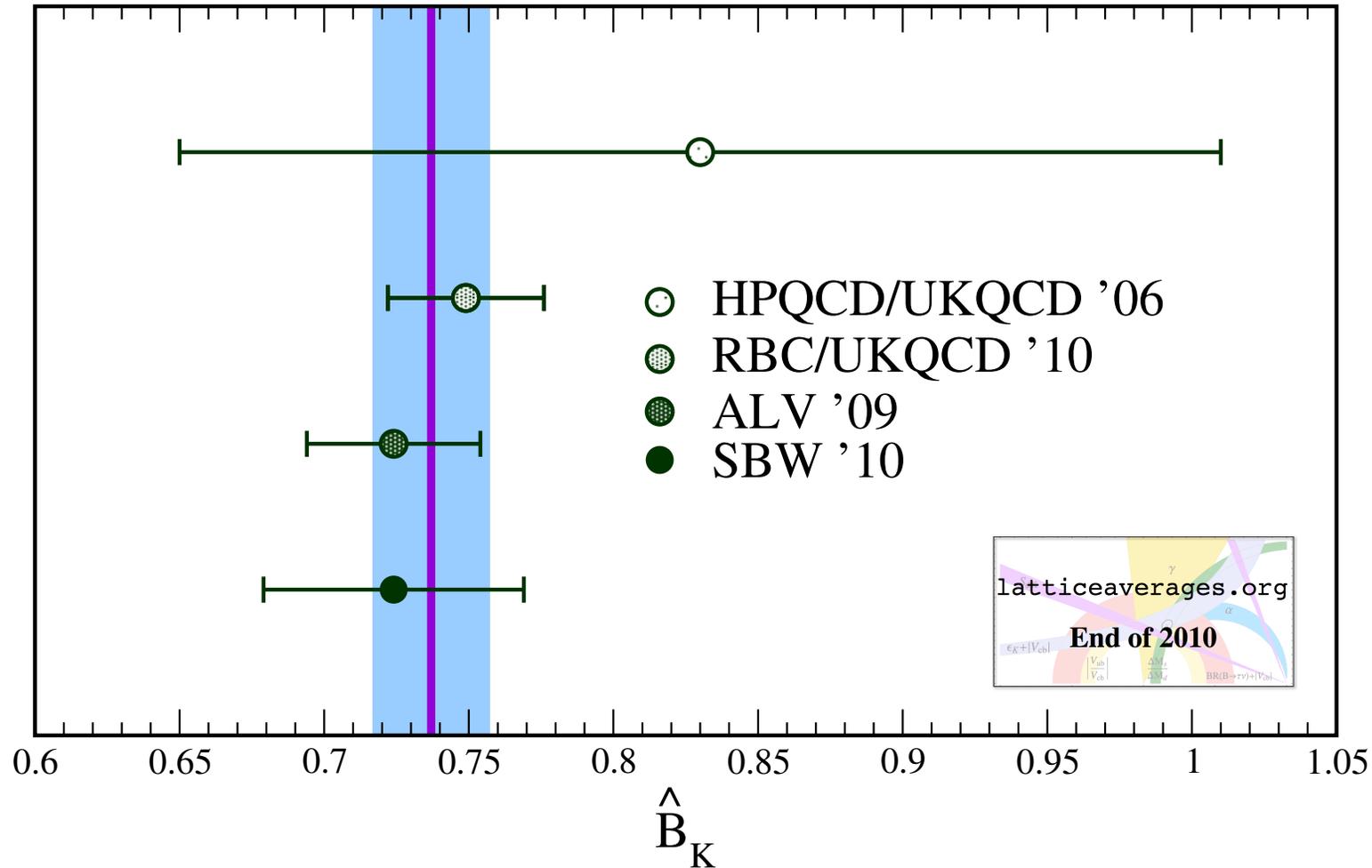
$$Z_{B_K}^{NDR}(\mu = 2\text{GeV}, a^{-1} = 1.73\text{GeV}) = 0.936(30)$$

# $a^2$ correction of $B_K$



- $a^{-1} = 1.73(3)$  GeV and  $2.28(3)$  GeV.
- $\mathcal{O}(a^2)$  correction is done to data point in right plot
- Two extrapolations: SU(2)+Kaon ChPT and the linear.

# Comparison with other results

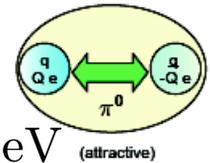
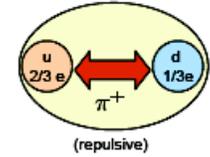


- $\hat{B}_K = 0.749(7)_{stat}(21)_\chi(3)_{FV}(15)_{NPR}$  [RBC/UKQCD10 arXiv:1012.4178]
- (Laiho, Lunghi, & Van de Water, <http://latticeaverages.org/>)

# QCD+QED simulation

[ R. Zhou, S. Uno, T. Blum, T. Doi, M. Hayakawa, TI, N. Yamada, ]

- Up down quark has different **electric charge** and **masses**  
→ Breaking of **isospin symmetry**



$$\Delta m_\pi = m_{\pi^\pm} - m_{\pi^0} = 4.5936(5)\text{MeV}, \quad m_N - m_P = 1.2933317(5)\text{MeV}$$

- Quark masses.** (Strong CP problem)
- Compute Pion/Kaon using Nf=2+1 DWF QCD + QED

$$24^3 \quad m_l = 0.01$$

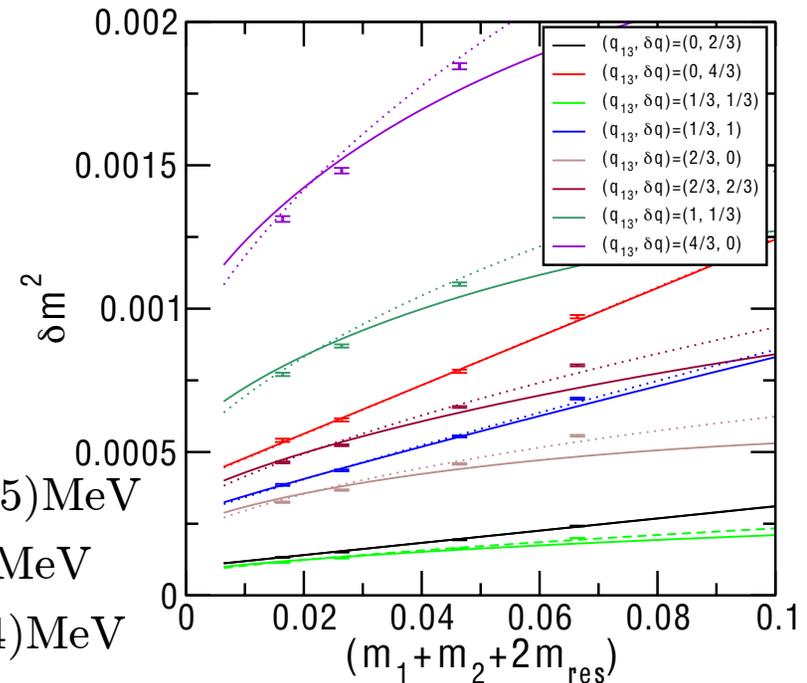
Requiring  $m_q < 40$  MeV (70MeV), 48 (120)  
partially quenched data points for PS meson survive

- Fit to chiral perturbation theory with EM (**SU(3)+EM** and **SU(2)+Kaon+EM**) to extract quark masses.
- Chiral symmetry is essential** to define quark massless points.
- Input:

$$M_{\text{PS}}(m_u, 2/3, m_d, -1/3) = 139.57018(35)\text{MeV}$$

$$M_{\text{PS}}(m_u, 2/3, m_s, -1/3) = 493.673(14)\text{MeV}$$

$$M_{\text{PS}}(m_d, -1/3, m_s, -1/3) = 493.673(14)\text{MeV}$$



# Quark mass from QCD+QED simulation

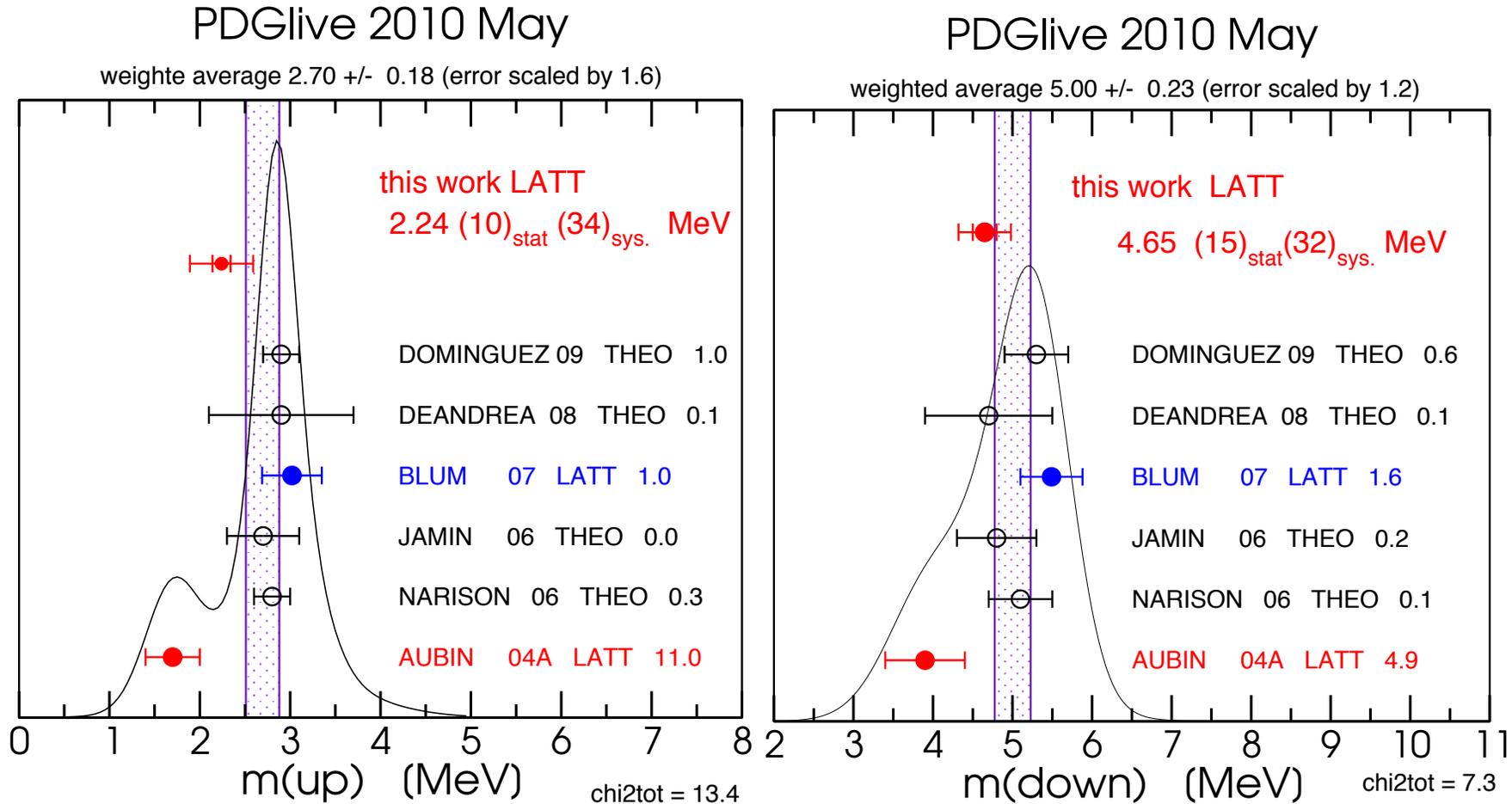
[PRD82 (2010) 094508 [47pages]]

$$\begin{aligned}m_u &= 2.24 \pm 0.10 \pm 0.34 \text{ MeV} \\m_d &= 4.65 \pm 0.15 \pm 0.32 \text{ MeV} \\m_s &= 97.6 \pm 2.9 \pm 5.5 \text{ MeV} \\m_d - m_u &= 2.411 \pm 0.065 \pm 0.476 \text{ MeV} \\m_{ud} &= 3.44 \pm 0.12 \pm 0.22 \text{ MeV} \\m_u/m_d &= 0.4818 \pm 0.0096 \pm 0.0860 \\m_s/m_{ud} &= 28.31 \pm 0.29 \pm 1.77,\end{aligned}$$

- $\overline{\text{MS}}$  at 2 GeV using NPR/SMOM scheme.
- Particular to QCD+QED, **finite volume error** is large: 14% and 2% for  $m_u$  and  $m_d$ .
- This would be due to photon's **non-confining feature** (vs gluon).
- Volume,  $a^2$ , chiral extrapolation errors are being removed.
- Applications for Hadronic contribution to  $(g - 2)_\mu$  in progress.

# Comparison with u,d masses in PDG

red  $N_F = 2 + 1$  staggered, DWF    blue  $N_F=2$  DWF (2.7 fm)<sup>3</sup>



# Conclusions

- Progress in Theory, Algorithm, and Hardware are finally bringing us the opportunity to calculate physics quantities [A. Juttner's talk]

$$\underline{m_u}, \underline{m_d}, \underline{m_s}, \underline{f_\pi}, \underline{f_K}, \underline{B_K}, \underline{f_+(q^2)}, \underline{f_B}, \underline{B_B}, \dots$$

which are highly relevant to our understanding of the fundamental constituents of matter and the forces between them with fully controlled systematic errors.

- Continuum limit of  $B_K$  is taken.
- One step towards QCD+QED computation that fully includes isospin breaking effects (quark mass and charge).
- Future prospects :
  - $K \rightarrow \pi\pi$  decay,  $\epsilon'/\epsilon$ .  $\Delta I = 3/2$  is under control,  $\Delta I = 1/2$  is begin challenged.
  - Eliminates three sources of systematic errors on the next generation machines.
  - Other applications of QCD+QED including  $(g - 2)_\mu$  Hadronic light-by-light diagram.

# 2011 Tohoku-Pacific Ocean Earthquake

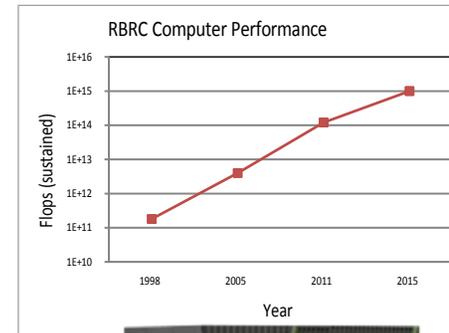
- Friday, March 11th 14:46 (JST) M9.0 + many smaller but still significant earth quakes.
- Among many other bad damages, **KEK, J-PARC, RIBF**, were shutdown after the earthquake.
- Monday, March 14th 22:50 (JST) A roll call (head count) for members of national-wide **Particle & Nuclear Theory Group (Soryushi-Ron Group; SG)**, who belong to the institutes located in **Tohoku-Kantou-Koushinetsu region**, announced on the mailing list.
- Some colleagues may be staying in shelters.

- As of 19:00 (JST) yesterday, approximately after 43 hours from the call, **387 names, from 45 institutes** are reported to be safe on the wik. It's been rapidly growing.  
[Koji Hahsimoto (RIKEN), Tetsuji Kimura (KEK), Yuji Tachikawa (IPMU), TI]  
(Note this is NOT 100 % confirmed results (**PRELIMINARY**) ).
- No seriously injured person **from SG** are known to us so far.
- Computers, such as RICC, are still working ! (suffering once a day shutdown due to the scheduled power outage, RIBF as well).
- Some colleagues, from affected institutes, already started looking for places to continue their studies. Some already moved and started to work.
- I believe many experimentalists at KEK, RIKEN, Univ. of Tokyo and very many others are monitoring the radioactive levels of the nation.
- RIBF at RIKEN, already restarted from Monday, KEK and J-PARC will start to recover as soon as next week.

# QCD\* machines

[ Columbia Univ, RBRC, Univ. of Edinburgh ]

- QC DSP (48GF/rackpeak), TIDSPC31 (1993-1998)  
12 racks at RBRC, **600GF**
- QCDOC (0.8TF/rack) 180nm ASIC (2003-2005)  
1 core, System on Chip, 12 racks at RBRC, **10TF**,  
\$1/Mflops(sustained)
- QCDCQ (205TF/rack) 45nm (2007-2011)  
2 racks at RBRC/BNL, **400 TF**



- Low power consumptions → large number of nodes (1,024) per rack
- High memory/interconnect Bandwidth
- Gives efficient performance for lattice QCD

# Other machines



Argonne, ( IBM BG/P 557 TF peak)



RIKEN/Japan RICC (Cluster, 100 TF peak)

Via USQCD collaboration, als Clusters @ J-Lab, FNAL

## Origins of Isospin breaking in Kaon

- Reason why the iso doublet,  $(K^+, K^0)$ , has the mass splitting

$$M_{K^\pm} - M_{K^0} = -3.937(29) \text{ MeV}, \quad [\text{PDG}]$$

- ▷  $(m_{\text{down}} - m_{\text{up}})$  : makes  $M_{K^+} - M_{K^0}$  negative.
- ▷  $(q_u - q_d)$  : makes  $M_{K^+} - M_{K^0}$  positive.

- Using the determined quark masses and SU(3) LEC, we could isolate (to  $\mathcal{O}((m_{\text{up}} - m_{\text{down}})\alpha)$ ) each of contributions,

$$\begin{aligned} & M_{\text{PS}}^2(m_{\text{up}}, 2/3, m_{\text{str}}, -1/3) - M_{\text{PS}}^2(m_{\text{down}}, -1/3, m_{\text{str}}, -1/3) \\ & \simeq M_{\text{PS}}^2(m_{\text{up}}, 0, m_{\text{str}}, 0) - M_{\text{PS}}^2(m_{\text{down}}, 0, m_{\text{str}}, 0) \quad [\Delta M(m_{\text{up}} - m_{\text{down}})] \\ & + M_{\text{PS}}^2(\bar{m}_{ud}, 2/3, \bar{m}_{ud}, -1/3) - M_{\text{PS}}^2(\bar{m}_{ud}, -1/3, m_{\text{str}}, -1/3) \quad [\Delta M(q_u - q_d)] \end{aligned}$$

- ▷  $\Delta M(m_{\text{up}} - m_{\text{down}}) = -5.23(14) \text{ MeV} \quad [133(4)\% \text{ in } \Delta M^2(m_{\text{up}} - m_{\text{down}})]$
- ▷  $\Delta M(q_u - q_d) = 1.327(37) \text{ MeV} \quad [-34(1)\% \text{ in } \Delta M^2(q_u - q_d)]$

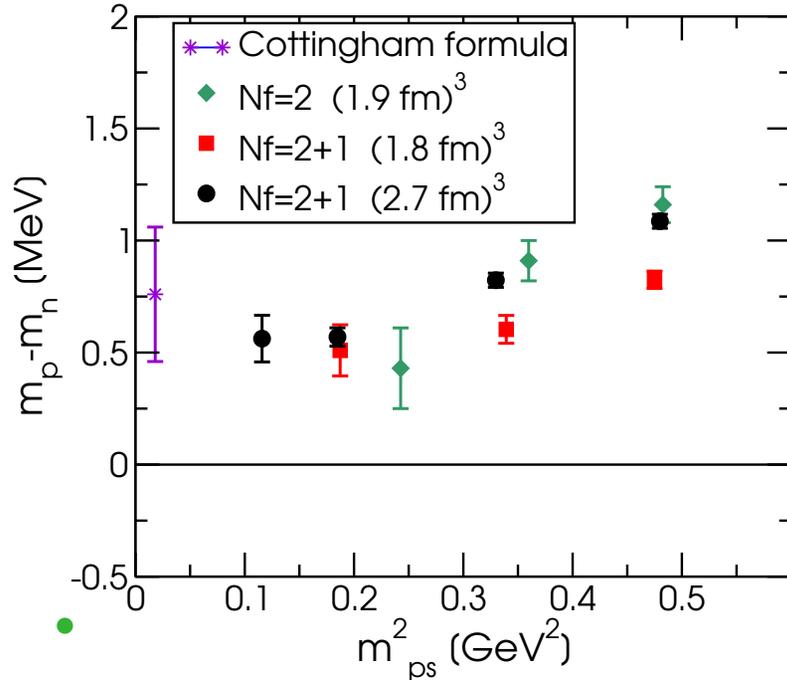
Also SU(3) ChPT,  $\Delta M(m_{\text{up}} - m_{\text{down}}) = -5.7(1) \text{ MeV}$  and  $\Delta M(q_u - q_d) = 1.8(1) \text{ MeV}$ .

- Similar analysis for  $\pi$  is possible, but facing a difficulty of isolating sea strange quark terms.  $m_{\pi^\pm} - m_{\pi^0} = 4.50(23) \text{ MeV}$

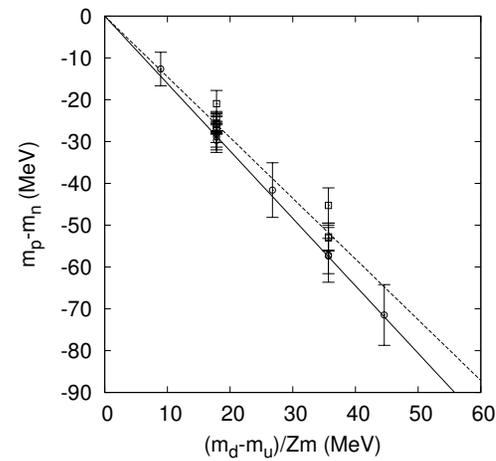
# Nucleon mass splitting in $N_F = 2, 2 + 1$

[R.Zhou, T.Doi]

$(q_u - q_d)$  effect



$(m_{up} - m_{down})$  effect



$$M_N - M_p|_{\text{QED}} = 0.383(68) \text{ MeV}$$

$$M_N - M_p|_{\text{quark mass}} = -2.24(12) \text{ MeV}$$

$$\implies M_N - M_p| = -1.86(14)(47)_{\text{FV,fit}} \text{ MeV}$$