

# The Potential of Minimal Flavour Violation

Rodrigo Alonso

Departamento de Física Teórica  
Universidad Autónoma de Madrid  
& IFT-CSIC

*based on the work with  
Belén Gavela, Luca Merlo  
& Stefano Rigolin  
arXiv:1103.2915 [hep-ph]*

Rencontres de Moriond, 2011

- 1 Introduction**  
Minimal Flavour Violation  
The Dynamics Behind MFV
- 2 The Potential**  
Bifundamental Approach  
Construction of the Potential  
Minimum of the Potential  
Fundamental Approach  
The Potential
- 3 Conclusions**

# Minimal Flavour Violation (MFV)

MFV is a symmetry approach to the flavour problem

- The MFV hypothesis: *The Yukawa couplings are the only sources of flavour violation in and **beyond** the Standard Model<sup>1</sup>.*
- Generations are distinguished by masses and mixing angles; in the limit of zero masses the SM presents an extended symmetry group:

$$G_f = SU(3)_{Q_L} \times SU(3)_{D_R} \times SU(3)_{U_R} \times \dots$$


$$D_R = (d_R, s_R, b_R)^T \quad D_R \sim (1, 3, 1 \dots)$$

This is the symmetry of the high scale theory. At this scale the Yukawa couplings are also invariant:

$$\bar{Q}_L Y_D D_R H \qquad Y_D \sim (3, \bar{3}, 1)$$

At low energies  $Y_D$  acquires a v.e.v. and breaks the flavour symmetry.  $Y_U$  &  $Y_D$  are the *only* directions of flavour breaking.

Such an hypothesis is strongly supported by current data

<sup>1</sup> G. D'Ambrosio, G. Giudice, G. Isidori, and A. Strumia, Nucl. Phys. B645(2002) 

# The Dynamics Behind MFV

In this scheme  $Y_U$  &  $Y_D$  have a *dynamical origin*. At some regime the Yukawa Interaction involves extra fields increasing the dimension of the 'Yukawa Operator':

- **Dimension 3** Group Representation

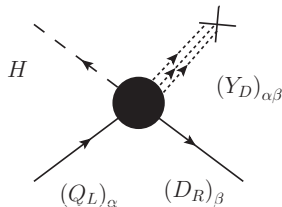
- **Dimension 5**  $\leftrightarrow$  Bifundamental Fields

$$\overline{Q}_L \frac{\Sigma_U}{\Lambda} D_R H \quad \Sigma_d \sim (3, \bar{3}, 1)$$

- **Dimension 6**  $\leftrightarrow$  Fundamental Fields

$$\overline{Q}_L \frac{\chi_d^L \chi_d^{R\dagger}}{\Lambda^2} D_R H \quad \chi_d^L \sim (3, 1, 1) \\ \chi_d^R \sim (1, 3, 1)$$

- **Dimension 7** ; Georgi & Chivukula's original idea: a fermion condensate<sup>1</sup>



The next question is how these fields acquire a v.e.v.  
Through a **Scalar Potential** ...

<sup>1</sup> Phys. Lett. B188 (1987) 99.

# Construction of the Potential

## Dimension 5 Yukawa Operator

The potential will also respect the flavour symmetry, so it will be built with the  $G_f$  invariants:

$$\begin{aligned} & \text{Tr} \left( \Sigma_u \Sigma_u^\dagger \right), \quad \det \left( \Sigma_u \right), \quad \text{Tr} \left( \Sigma_u \Sigma_u^\dagger \Sigma_d \Sigma_d^\dagger \right), \\ & \text{Tr} \left( \Sigma_d \Sigma_d^\dagger \right), \quad \det \left( \Sigma_d \right), \quad \dots \end{aligned}$$

These invariants can be expressed in terms of masses and mixing angles (2 generations):

$$\begin{aligned} \langle \Sigma_u \rangle &= \Lambda \cdot V^\dagger \text{Diag} \{ y_{u_i} \}, & \langle \Sigma_d \rangle &= \Lambda \cdot \text{Diag} \{ y_{d_i} \}; \\ \text{Tr} \left( \langle \Sigma_u \Sigma_u^\dagger \Sigma_d \Sigma_d^\dagger \rangle \right) &= \frac{1}{2} \Lambda^4 \left[ \left( y_c^2 - y_u^2 \right) \left( y_s^2 - y_d^2 \right) \cos 2\theta_c + \dots \right] \end{aligned}$$

The most general Potential is:

$$\begin{aligned} V &= \sum_{i=u,d} \left( -\mu_i^2 \text{Tr} \left( \Sigma_i \Sigma_i^\dagger \right) - \tilde{\mu}_i^2 \det \left( \Sigma_i \right) \right) + \\ & \sum_{i,j=u,d} \left( \lambda_{ij} \text{Tr} \left( \Sigma_i \Sigma_i^\dagger \right) \text{Tr} \left( \Sigma_j \Sigma_j^\dagger \right) + \tilde{\lambda}_{ij} \det \left( \Sigma_i \right) \det \left( \Sigma_j \right) + \dots \right) \dots \end{aligned}$$

# Minimum of the Potential

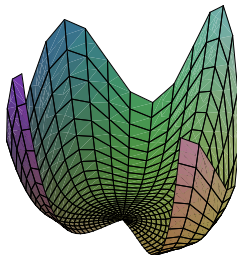
## Dimension 5 Yukawa Operator

The minimum of the Potential is given  
by:

$$\frac{\partial V}{\partial y_i} = 0 \quad \frac{\partial V}{\partial \theta_i} = 0$$

Take the angle for example:

$$\frac{\partial V}{\partial \theta_c} \propto (y_c^2 - y_u^2) (y_s^2 - y_d^2) \sin 2\theta_c = 0$$



Can the actual masses and mixings fit naturally in the minimum of  
the Potential?

The answer is worse than **NO**.

- Not only we have to input the hierarchy of masses in the potential parameters, which was to expect but ...
- To accommodate the mixing we must introduce wild fine tunings of  $O(10^{-10})$  and nonrenormalizable terms of dimension 8

# Fundamental Fields

## Dimension 6 Yukawa Operator

Let's first have a careful look at the Yukawa structure:

$$Y_D = \frac{\langle \chi_d^L \chi_d^{R\dagger} \rangle}{\Lambda^2} \quad Y_U = \frac{\langle \chi_u^L \chi_u^{R\dagger} \rangle}{\Lambda^2}$$

The Yukawas are composed of two 'vectors'. Such a structure has only one eigenvalue, **one mass**. This fact becomes evident when rotating the v.e.v.s of the fields to the form:

$$V_L^\dagger Y_D V_{D_R} = \frac{|\chi_d^L| |\chi_d^R|}{\Lambda^2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$V_L^\dagger Y_U V_{U_R} = \frac{|\chi_u^L| |\chi_u^R|}{\Lambda^2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

This means a **hierarchy** among the **masses** and **an angle only by construction!**

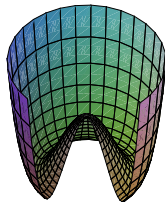
# Minimum of the Potential

## Dimension 6 Yukawa Operator

The invariants are:

$$\chi_u^{L\dagger} \chi_u^L, \quad \chi_u^{R\dagger} \chi_u^R, \quad \chi_d^{L\dagger} \chi_d^L,$$

$$\chi_d^{R\dagger} \chi_d^R, \quad \chi_u^{L\dagger} \chi_d^L = |\chi_u^L| |\chi_d^L| \cos \theta_c.$$



We can fit the angle and the masses in the  
Potential; as an example:

$$V' = \lambda_u \left( \chi_u^{L\dagger} \chi_u^L - \frac{\mu_u^2}{2\lambda_u} \right)^2 + \lambda_d \left( \chi_d^{L\dagger} \chi_d^L - \frac{\mu_d^2}{2\lambda_d} \right)^2$$

$$+ \lambda_{ud} \left( \chi_u^{L\dagger} \chi_d^L - \frac{\mu_{ud}^2}{2\lambda_{ud}} \right)^2 + \dots$$

Whose minimum sets (2 generations):

$$y_c^2 = \frac{\mu_u^2}{2\lambda_u \Lambda^2} \quad y_s^2 = \frac{\mu_d^2}{2\lambda_d \Lambda^2} \quad \cos \theta = \frac{\mu_{ud} \sqrt{\lambda_u \lambda_d}}{\mu_u \mu_d \lambda_{ud}}$$



# Conclusions

- The Potential of MFV requires further assumptions about the underlying dynamics.
- The outcome is radically varying with the assumptions.
- A first approximation to masses and mixings fits naturally in a symmetry breaking mechanism of the flavour group

Thank you for your attention

# Conclusions

- The Potential of MFV requires further assumptions about the underlying dynamics.
- The outcome is radically varying with the assumptions.
- A first approximation to masses and mixings fits naturally in a symmetry breaking mechanism of the flavour group

Thank you for your attention