

The proton size, current status and perspectives

Paul Indelicato for the CREMA collaboration

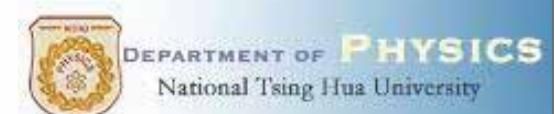


CREMA: Muonic Hydrogen Collaboration

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1. Introduction

- proton properties
- experimental proton size determinations
- muonic hydrogen

2. Experiment

- muonic hydrogen source
- laser system
- analysis

3. Results

- signal
- proton radius from muonic hydrogen spectroscopy

4. Perspectives



2 quarks up ($2/3 e$) + 1 quark down ($-1/3 e$) + strong interaction (gluons)

Vertex EM interaction: Dirac and Pauli Form factors

(S, P : spin and 4-momentum of nucleon, f : quark flavor)

$$\langle P', S' | V_{(f)}^\mu | P, S \rangle = \bar{U}(P', S') \left[\gamma^\mu F_1^{(f)}(Q^2) + i \sigma^{\mu\nu} \frac{q_\nu}{2M_N} F_2^{(f)}(Q^2) \right] U(P, S),$$

$$V_{(f)}^\mu = \bar{\psi}_{(f)} \gamma^\mu \psi_{(f)},$$

Physical charge density are derived from the Sachs Form factors

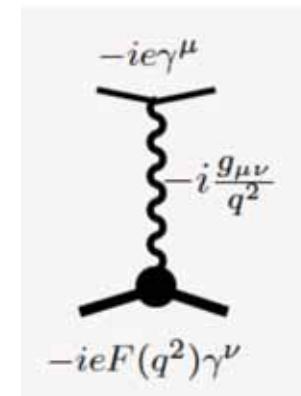
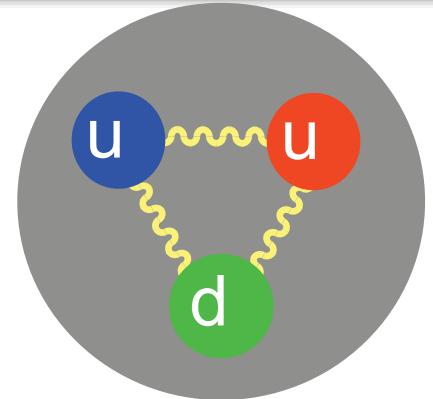
$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{(2M_N)^2} F_2(Q^2),$$

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2).$$

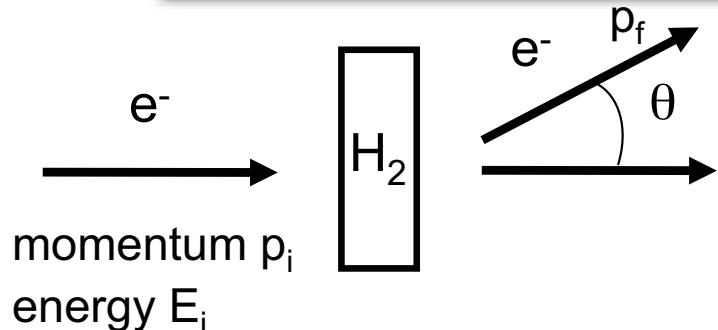
Measure the moments of the charge distribution:

$$G_N(q^2) = \int d\mathbf{r} e^{-i\mathbf{q} \cdot \mathbf{r}} \frac{\rho_N(\mathbf{r})}{4\pi},$$

$$\langle r^n \rangle = \int_0^\infty r^{2+n} \rho(r) dr,$$



Electron-proton elastic scattering



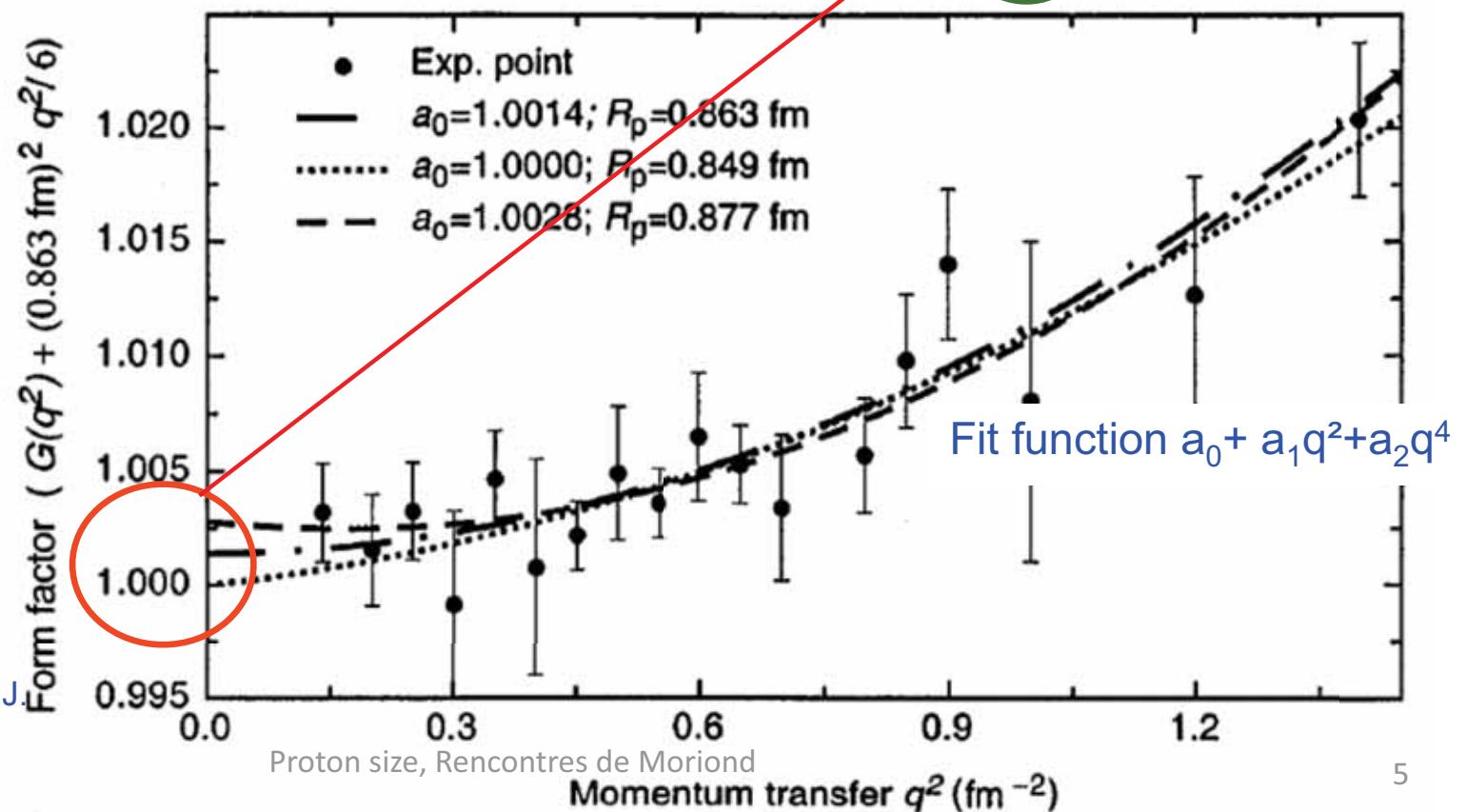
$$\frac{d\sigma(E_i, \theta)}{d\omega} = \frac{d\sigma_{\text{Rut.}}(E_i, \theta)}{d\omega} G_E(q^2)$$

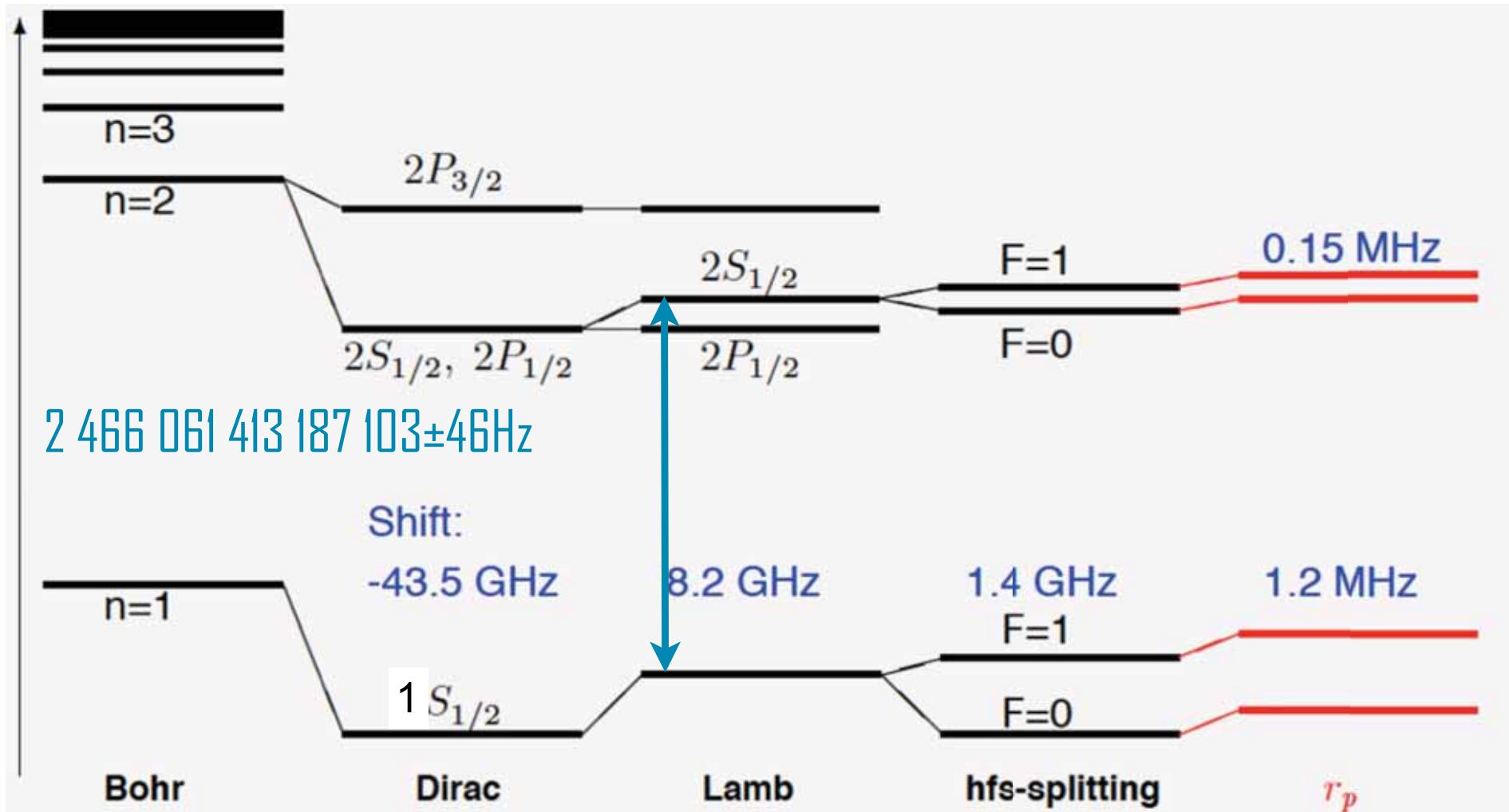
$$q = 2p_f \sin\left(\frac{\theta}{2}\right)$$

$$\vec{q} = \vec{p}_f - \vec{p}_i$$

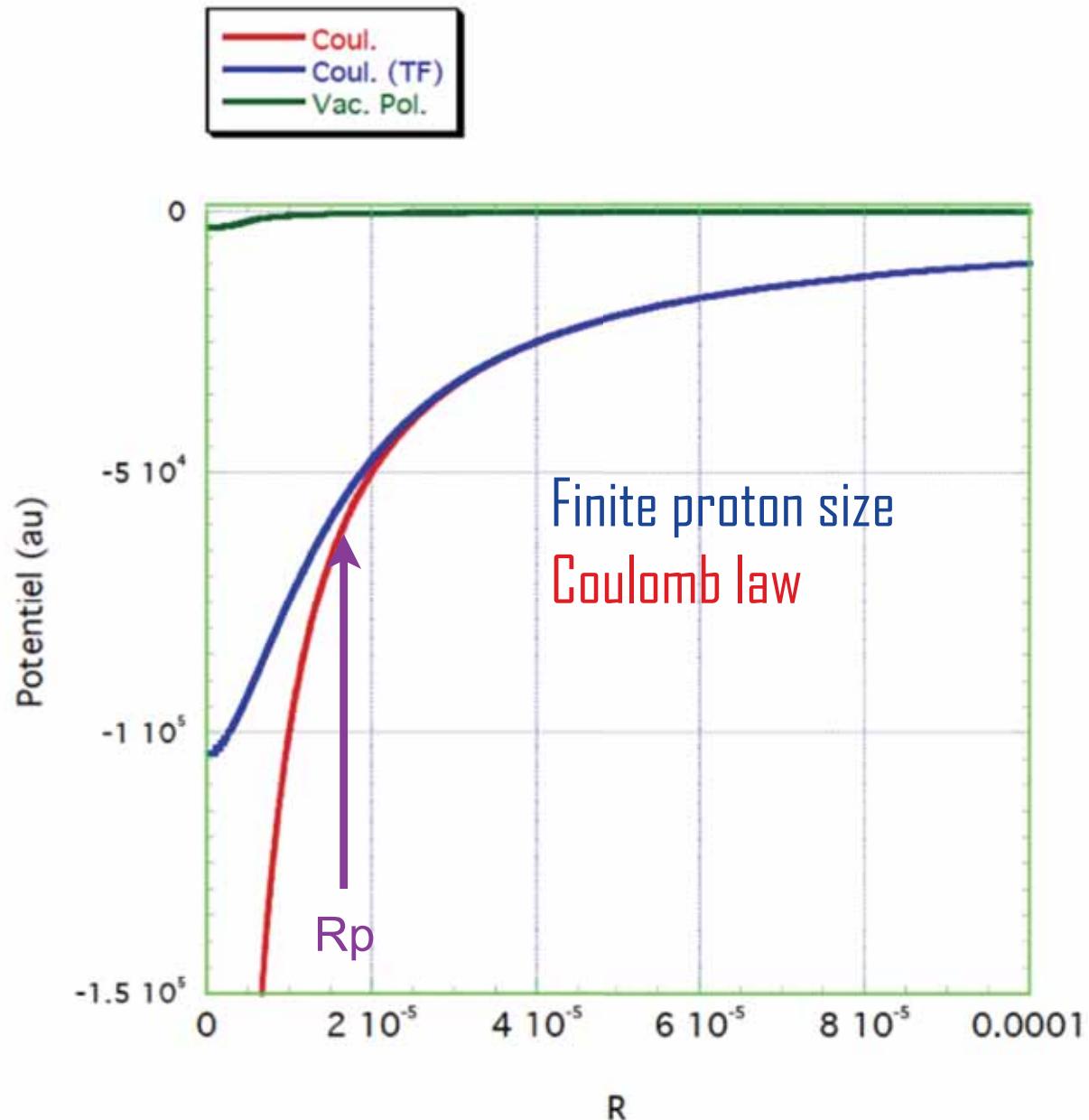
$$G_N(q^2) = \frac{1}{\left(1 + \frac{R^2 q^2}{12}\right)^2} \approx 1 - \frac{R^2}{6} q^2 + \frac{R^4}{48} q^4 + \dots$$

$$G_N(q^2) = e^{-\frac{1}{6} R^2 q^2} \approx 1 - \frac{R^2}{6} q^2 + \frac{R^4}{72} q^4 + \dots$$



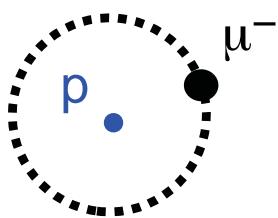


Proton size effect and VP potential



Muonic hydrogen spectroscopy

Exotic atom



$$\frac{m_i}{m_e} = 207$$

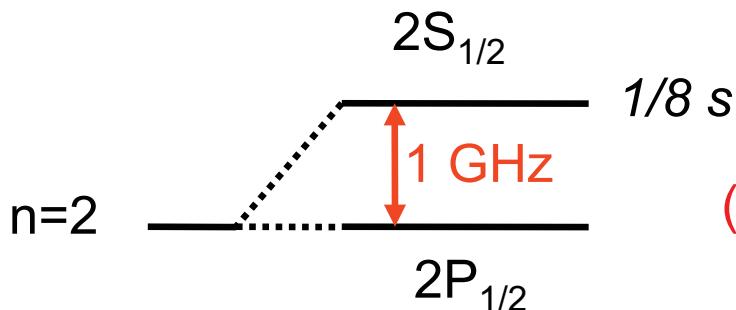
radius : $\sim a_0/200$

$$\mathcal{E}_{\text{NS}} = \frac{2}{3} \left(\frac{m_r}{m_e} \right)^3 \frac{(Z\alpha)^2}{n^3} m_e c^2 \left(\frac{Z\alpha R_N}{\chi_C} \right)^2$$

Lamb shift = self-energy + vacuum polarization + proton radius

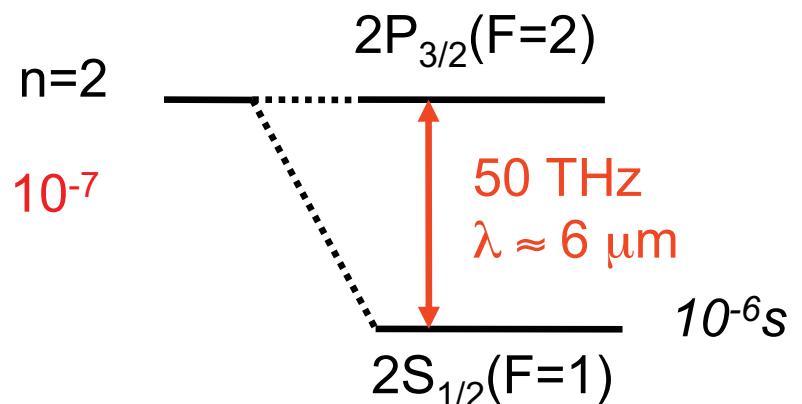
2S-2P	self-energy	vacuum pol.	r_p	total
e-p	1084.1 MHz	-26.9 MHz	0.146 MHz	1057.8 MHz
μ-p	0.17 THz	-50.94 THz	0.96 THz	-49,81 THz

electronic hydrogen : e-p

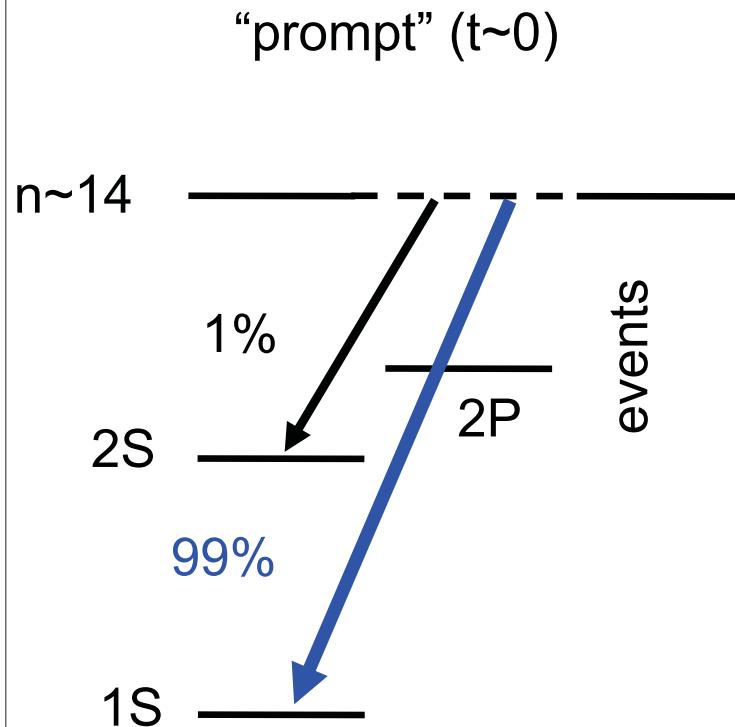


$$(f_{\mu p}/f_{ep}) \propto 1/(200)^3 \approx 10^{-7}$$

muonic hydrogen : μ-p



Principle of the experiment

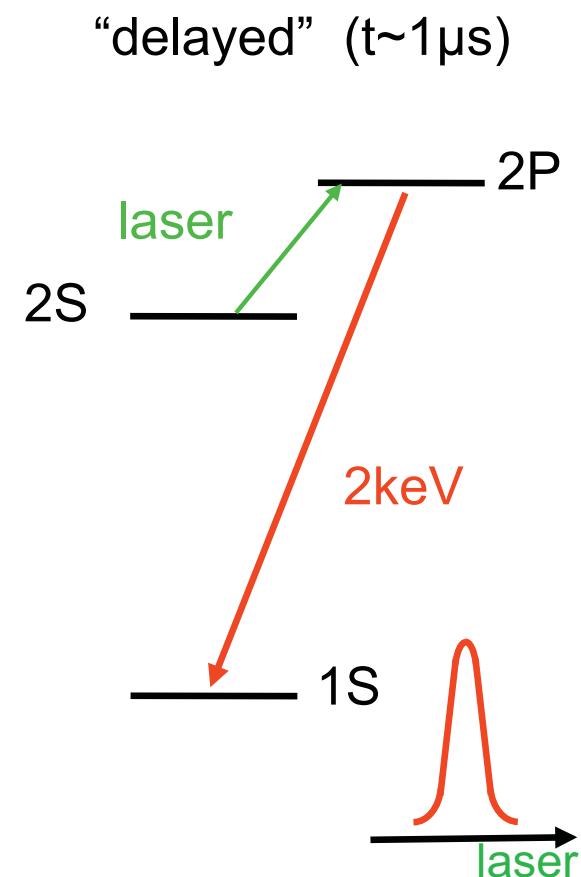
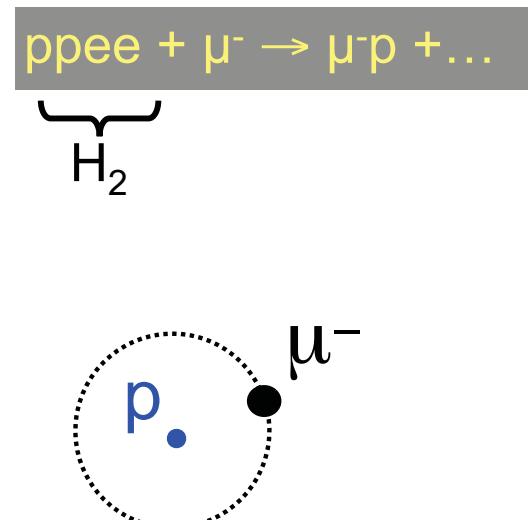


μ^- stop in H_2 gas
 $\Rightarrow \mu p^*$ atoms formed ($n \sim 14$)

99%: cascade to 1S emitting
 prompt K α , K β , ...

1%: long lived 2S state ($\tau \sim 1\mu s$ at 1mbar)

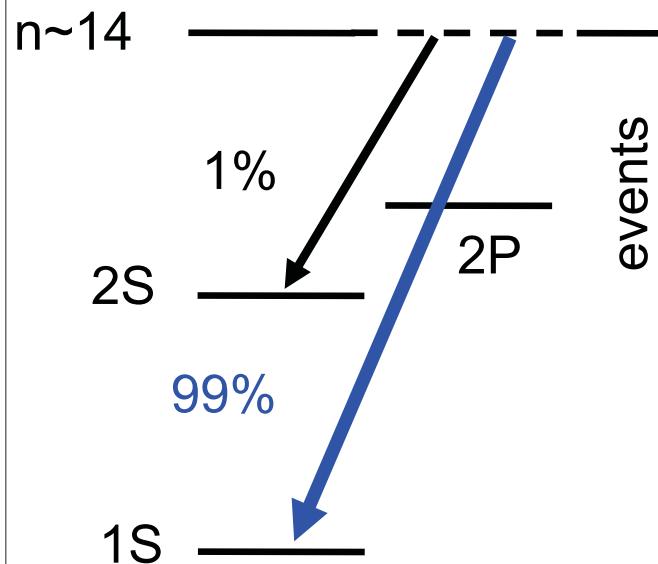
March 2011



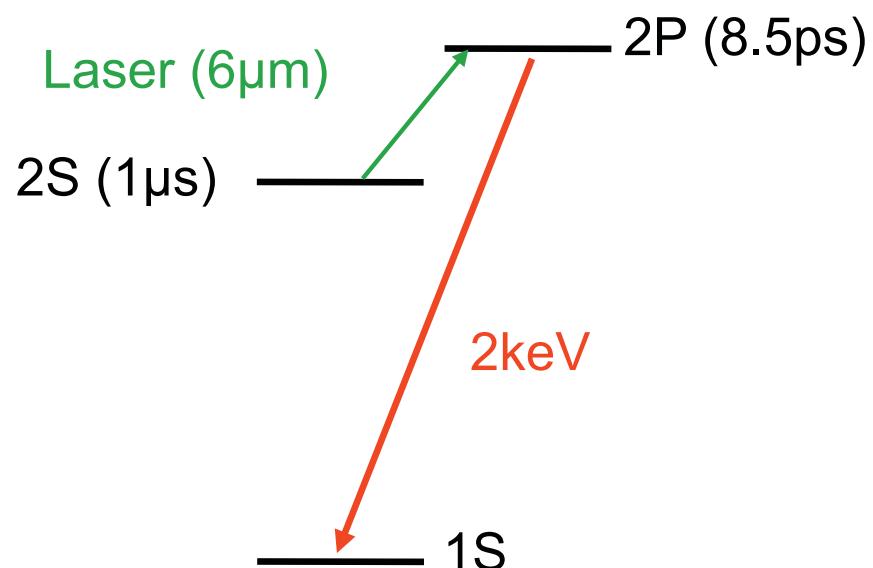
Proton size, Rencontres de Moriond

Principle of the experiment

“prompt” ($t \sim 0$)



“delayed” ($t \sim 1\mu s$)



Challenges

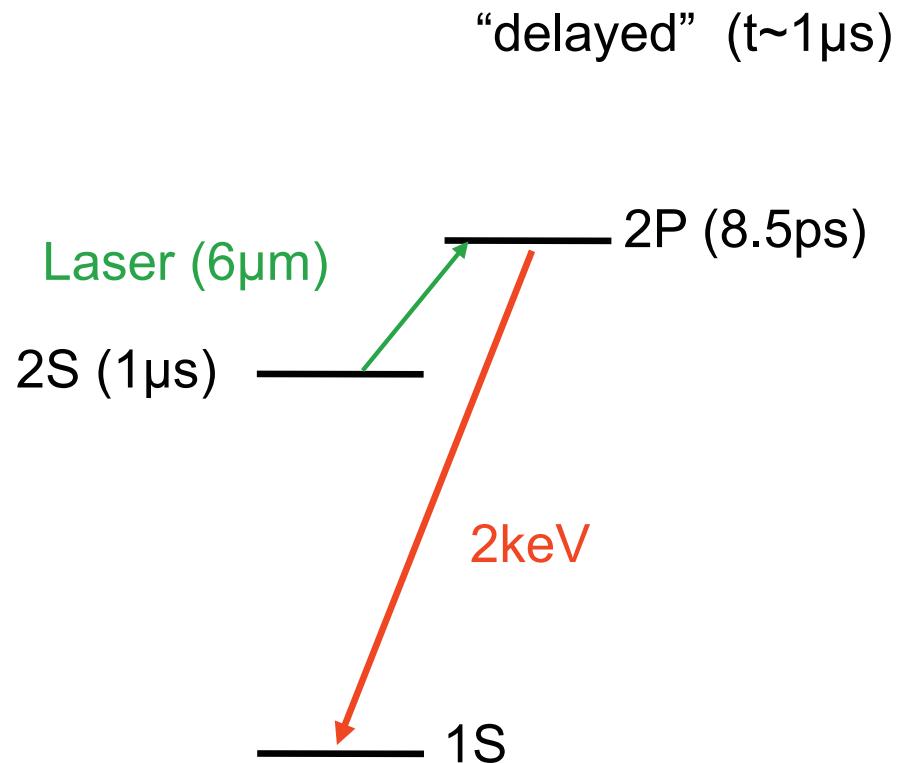
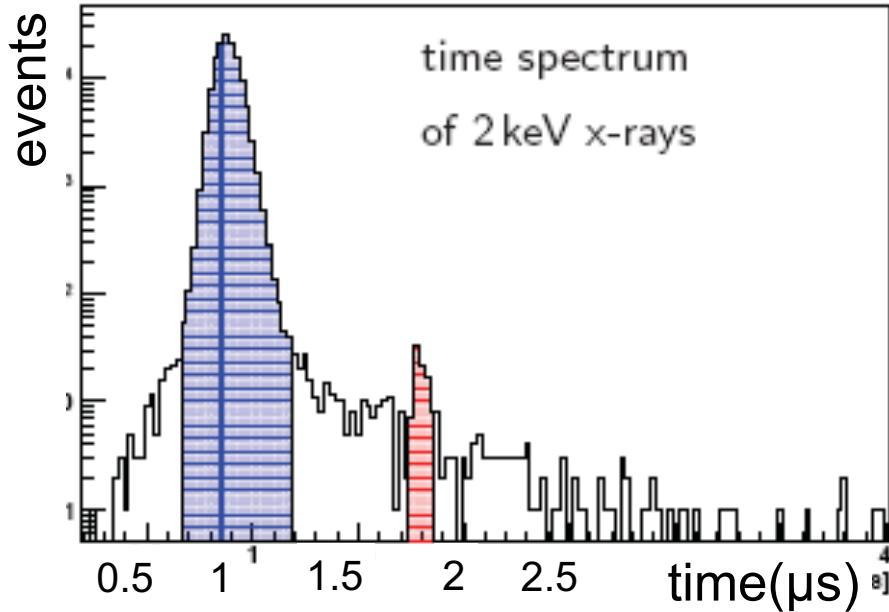
- production of muonic hydrogen in 2S
- powerful triggerable $6\mu m$ laser
- small signal analysis

Fire laser ($\lambda \sim 6\mu m$, $\Delta E \sim 0.2\text{eV}$)
 \Rightarrow induce $\mu p(2S-2P)$

\Rightarrow observe delayed K α x-rays

\Rightarrow normalize $\frac{\text{delayed K}\alpha}{\text{prompt K}\alpha}$ x-rays

Principle of the experiment



Challenges

- production of muonic hydrogen in 2S
- powerful triggerable $6\mu\text{m}$ laser
- small signal analysis

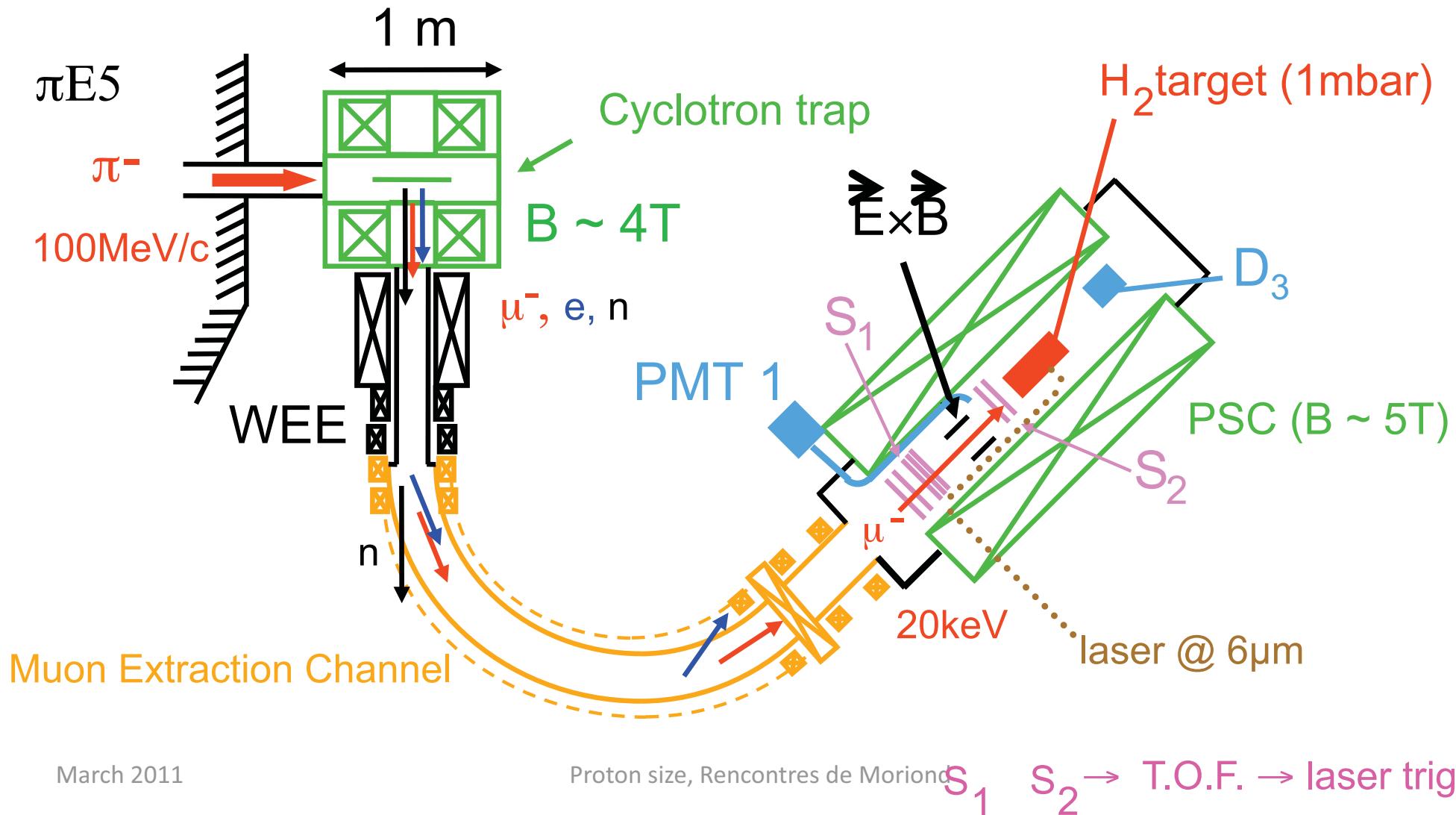
Fire laser ($\lambda \sim 6\mu\text{m}$, $\Delta E \sim 0.2\text{eV}$)
 \Rightarrow induce $\mu p(2\text{S}-2\text{P})$
 \Rightarrow observe **delayed K α x-rays**
 \Rightarrow normalize $\frac{\text{delayed K}\alpha}{\text{prompt K}\alpha}$ x-rays

muonic hydrogen source

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p (500Mev, 2mA) \rightarrow carbon target $\rightarrow \pi^- \rightarrow \mu^- + \bar{\nu}_\mu^-$ (300 μ /s)



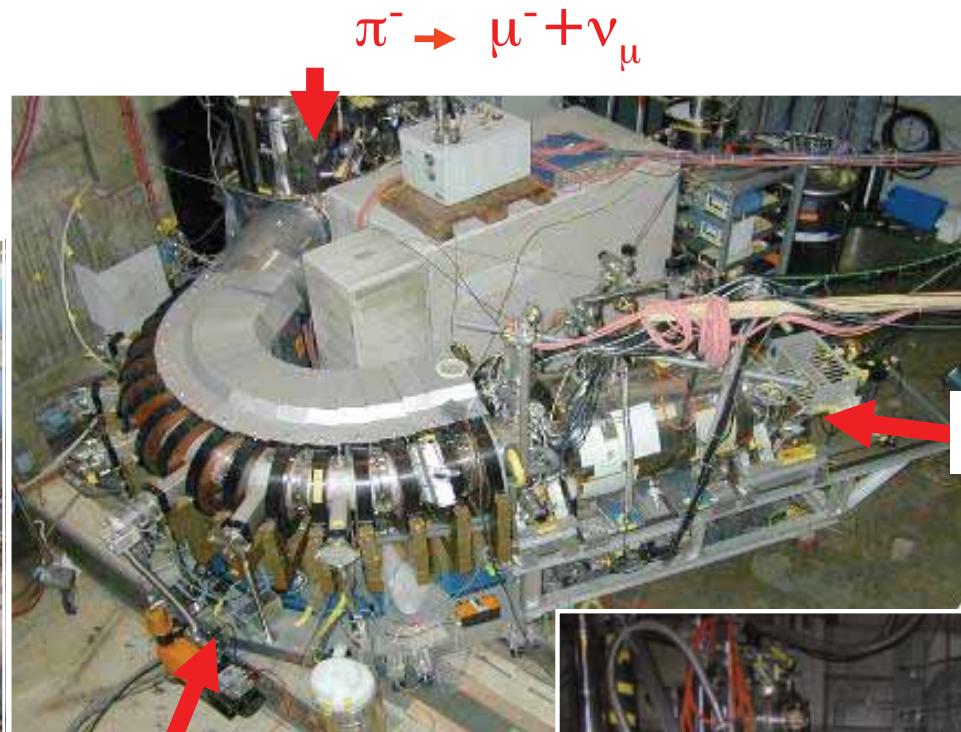
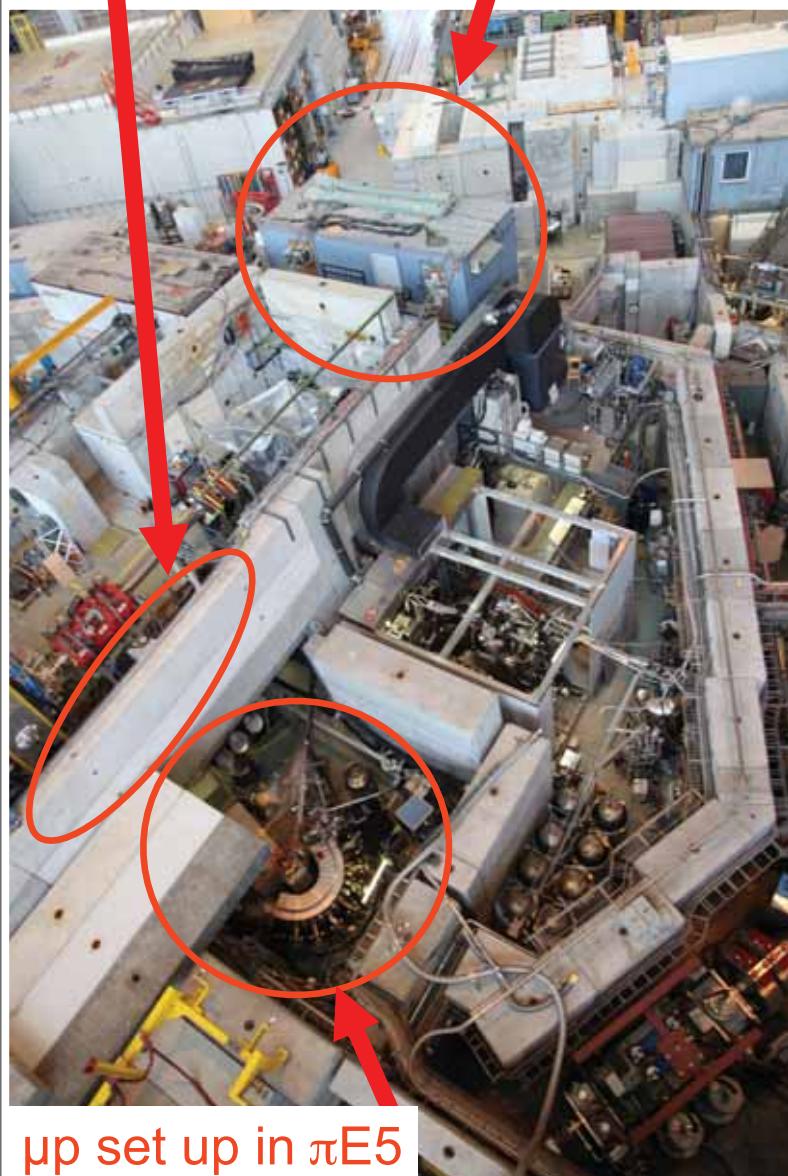
muon beam apparatus

laser hut

below

concrete blocks

counting room

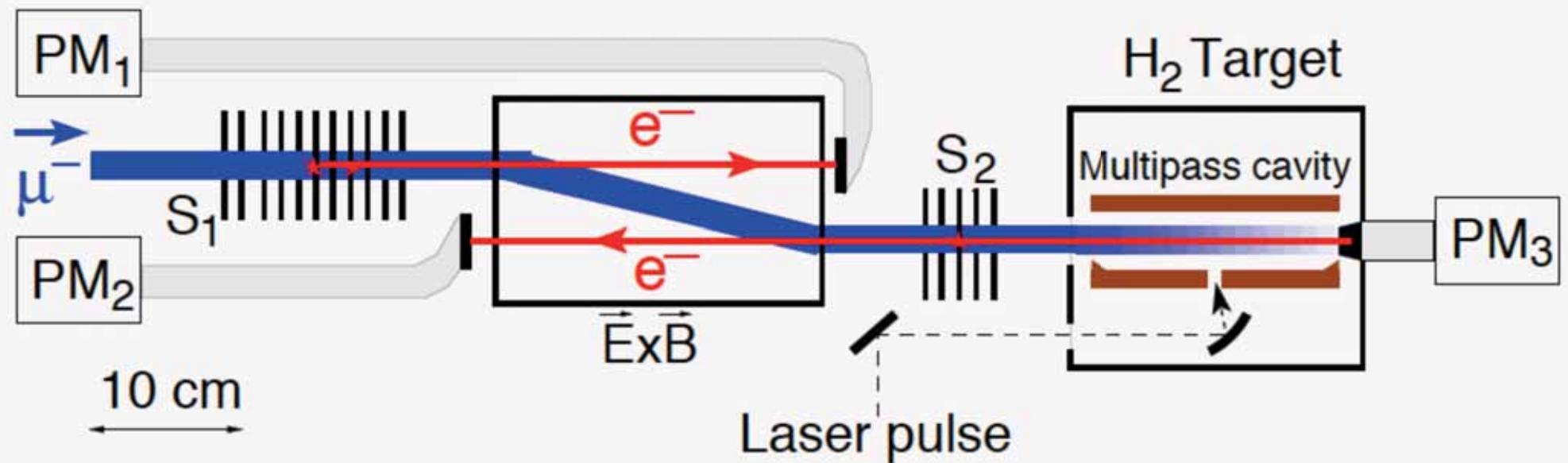


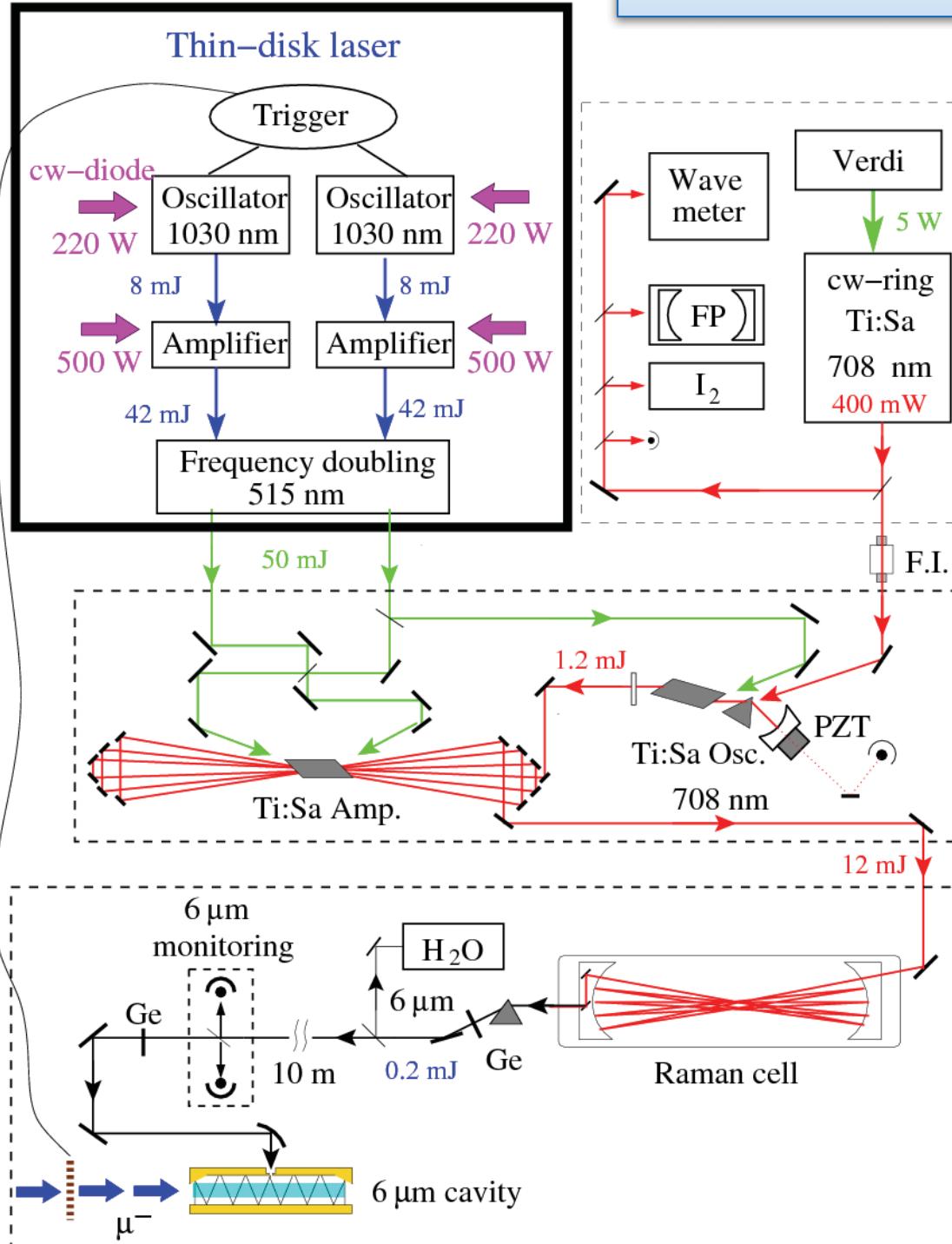
Muon extraction
channel



H2 target, laser cavity,
detectors

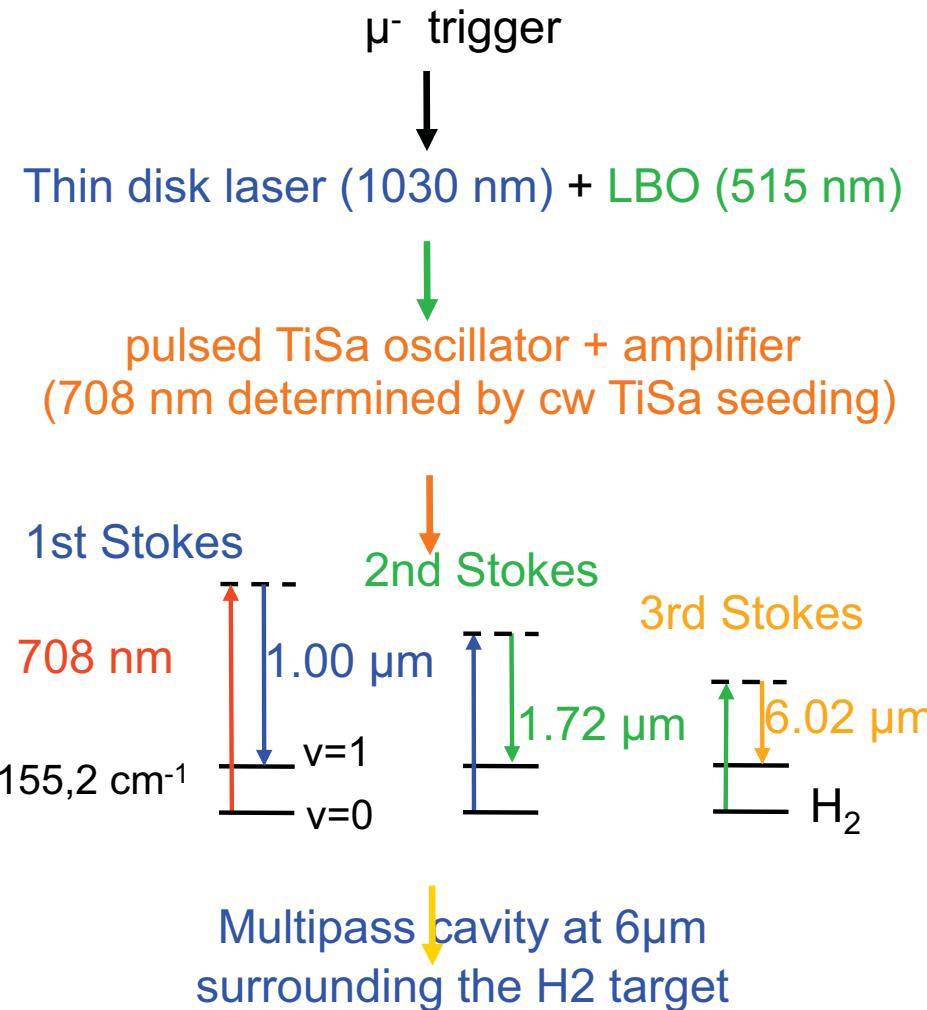
The laser trigger signal



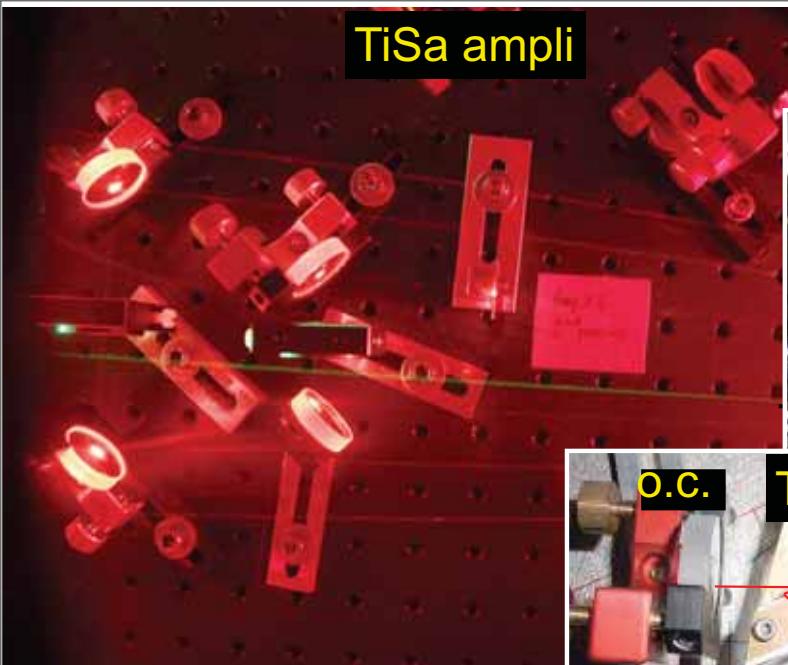


Laser chain

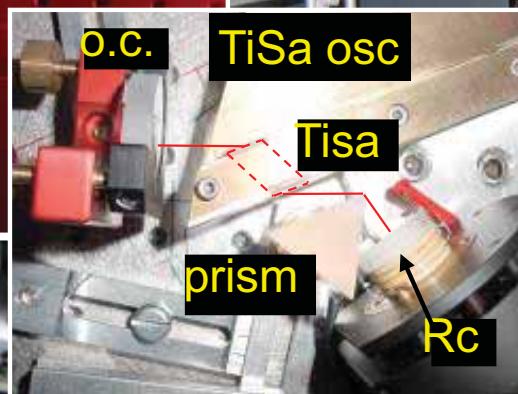
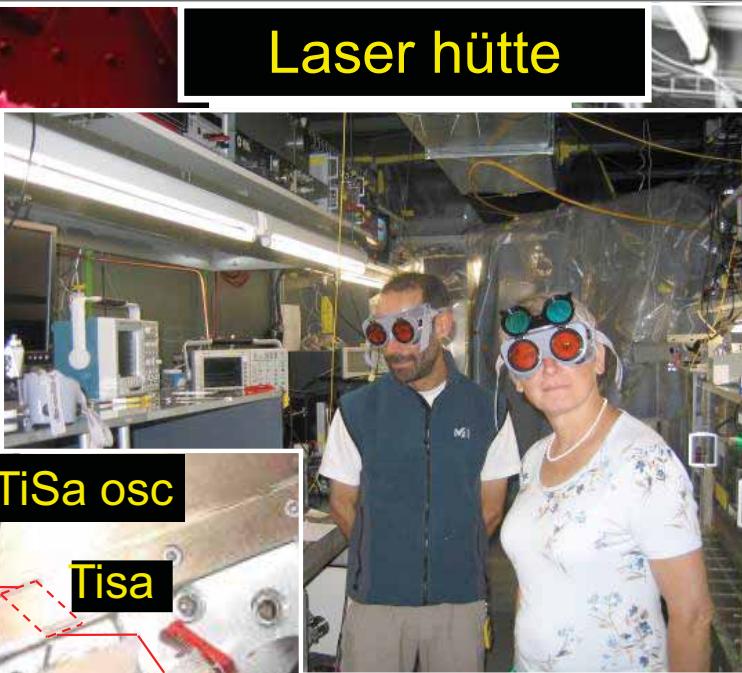
- Each single muon triggers the laser system (random trigger)
- 2S lifetime $\sim 1\mu\text{s} \rightarrow$ short laser delay (disk laser)
- 6 μm tunable laser pulse (0.2mJ)



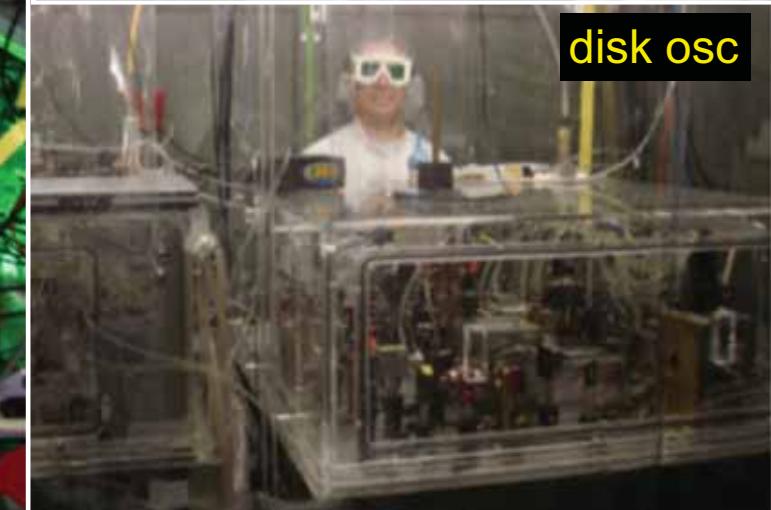
TiSa ampli



Laser hütte



disk osc

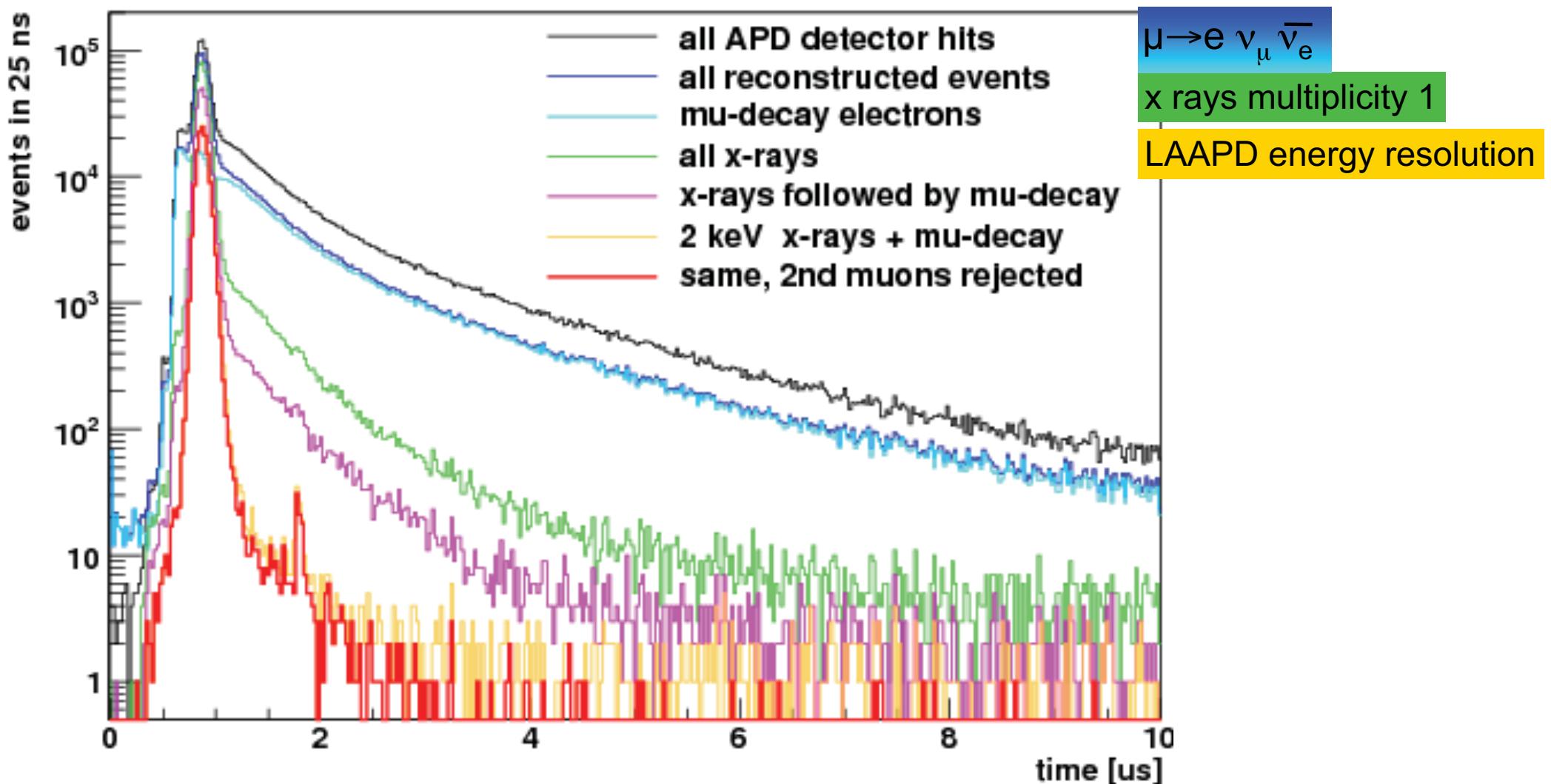


X-rays analysis → noise rejection

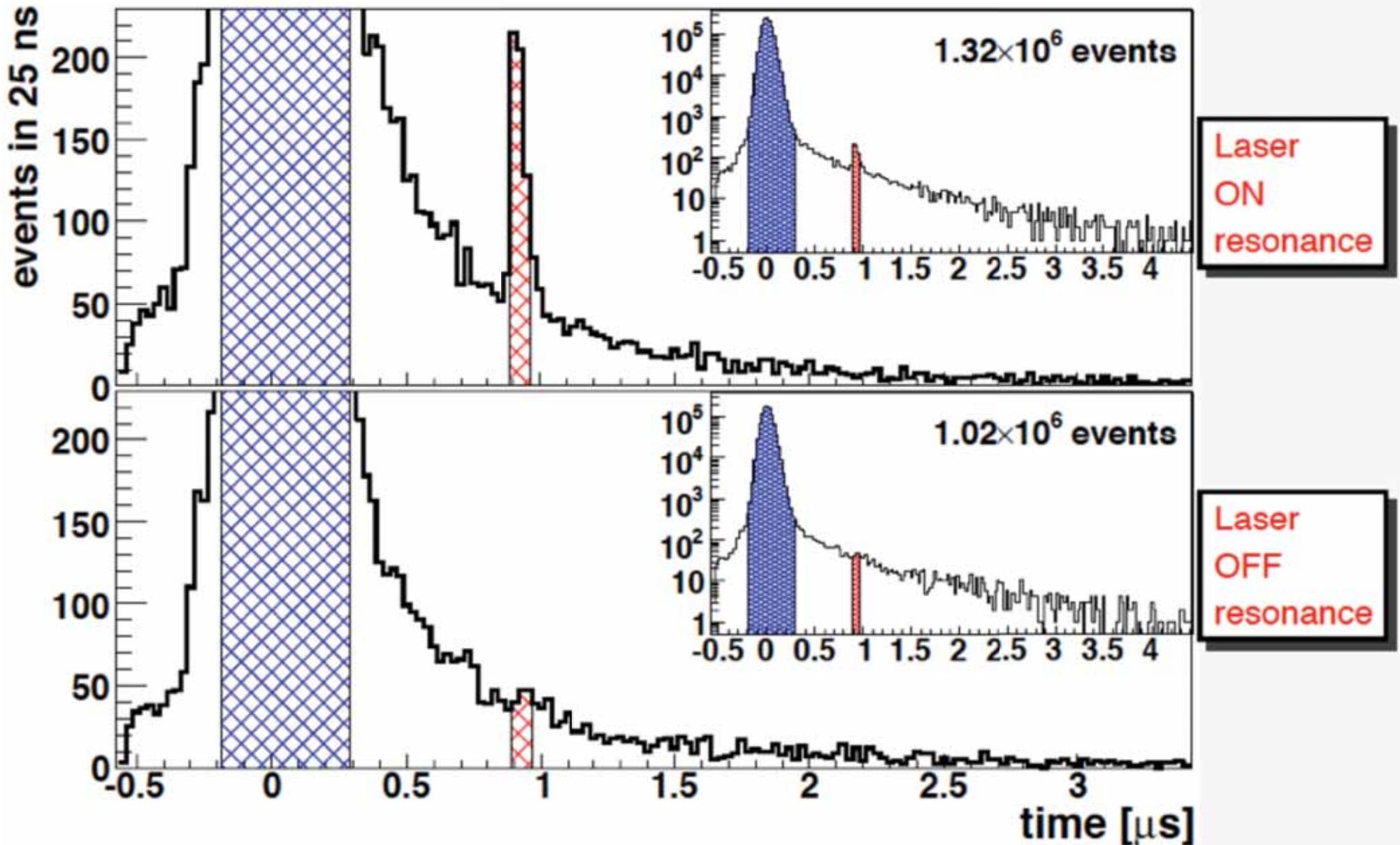


Example : FP 900 - 11 hrs meas.

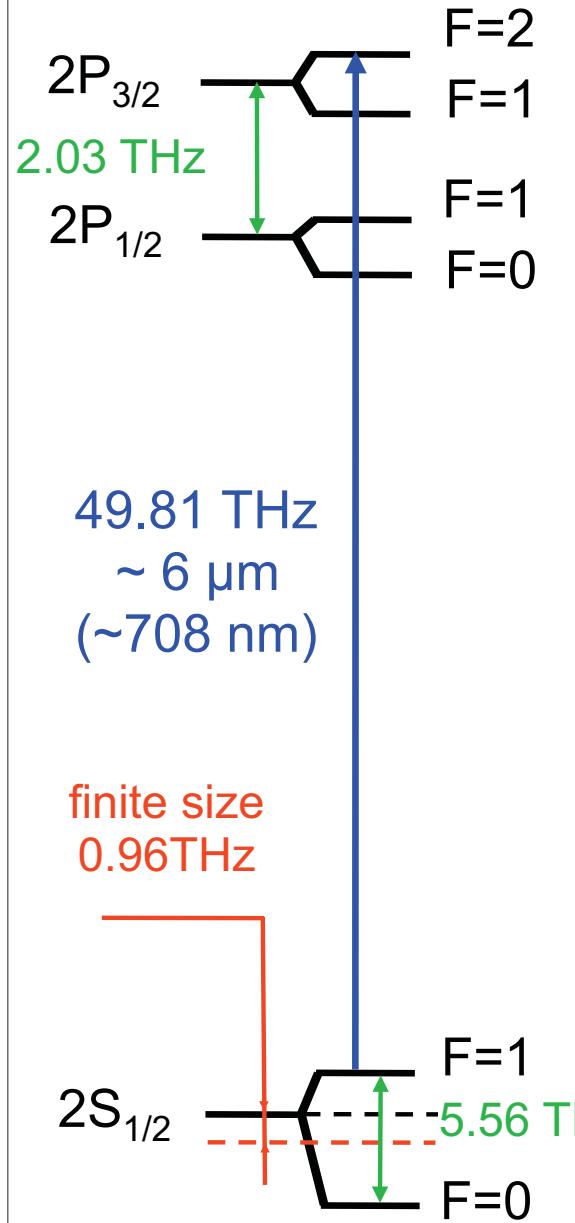
- 400 μs
- 240 laser shot/s
- 860 000 laser shot/hour
- 1.56 million detector clicks
- 19600 clicks in the laser region
- expected 2-3 laser induced events/hour !



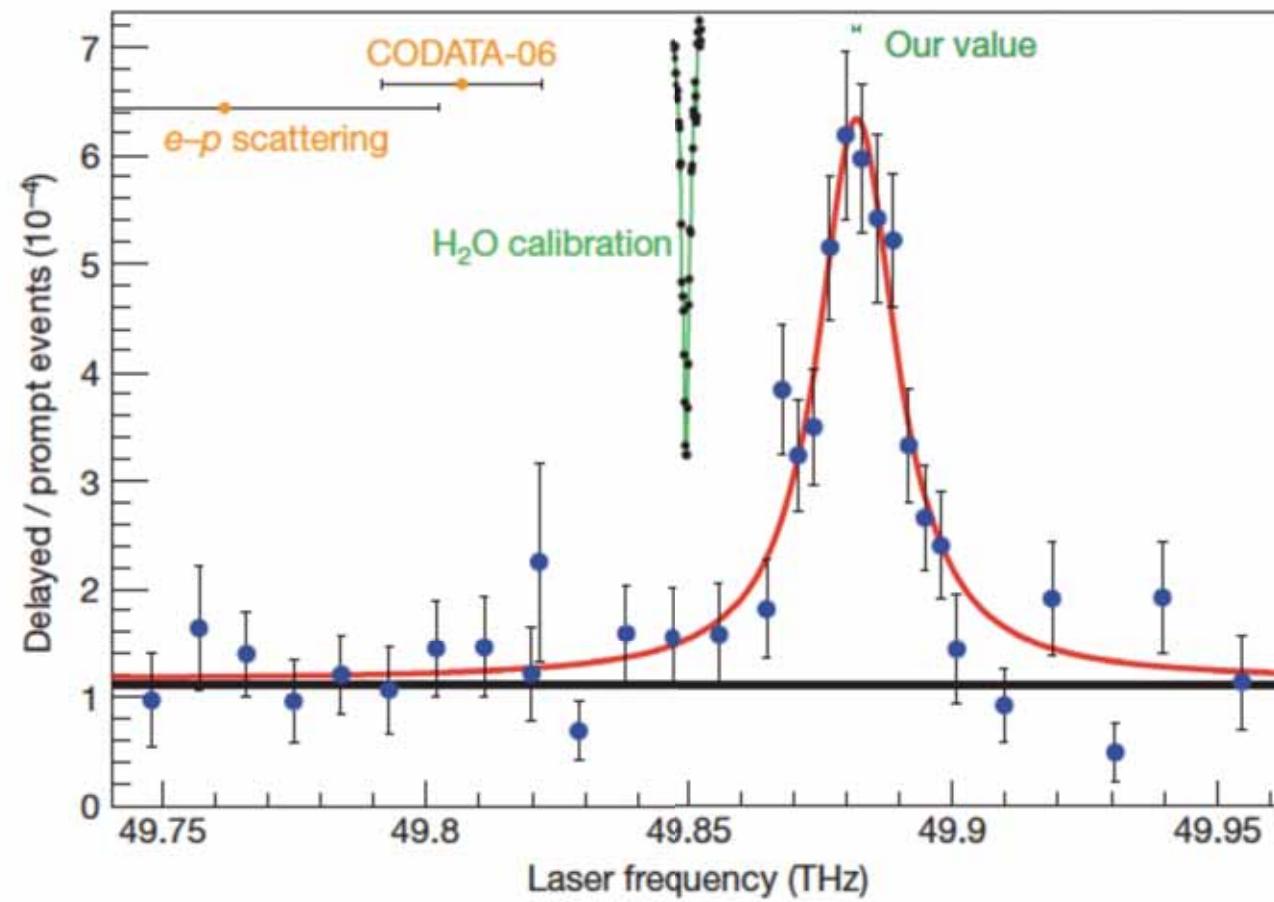
Time spectra



Same if laser on resonance but blocked

muonic hydrogen : $2S_{1/2}(F=1)$ - $2P_{3/2}(F=2)$ 

- 550 events measured
- 155 backgrounds
- 31 FP fringes
- 250 hours



R. Pohl, A. Antognini, F. Nez, et al., Nature 466, 213 (2010).

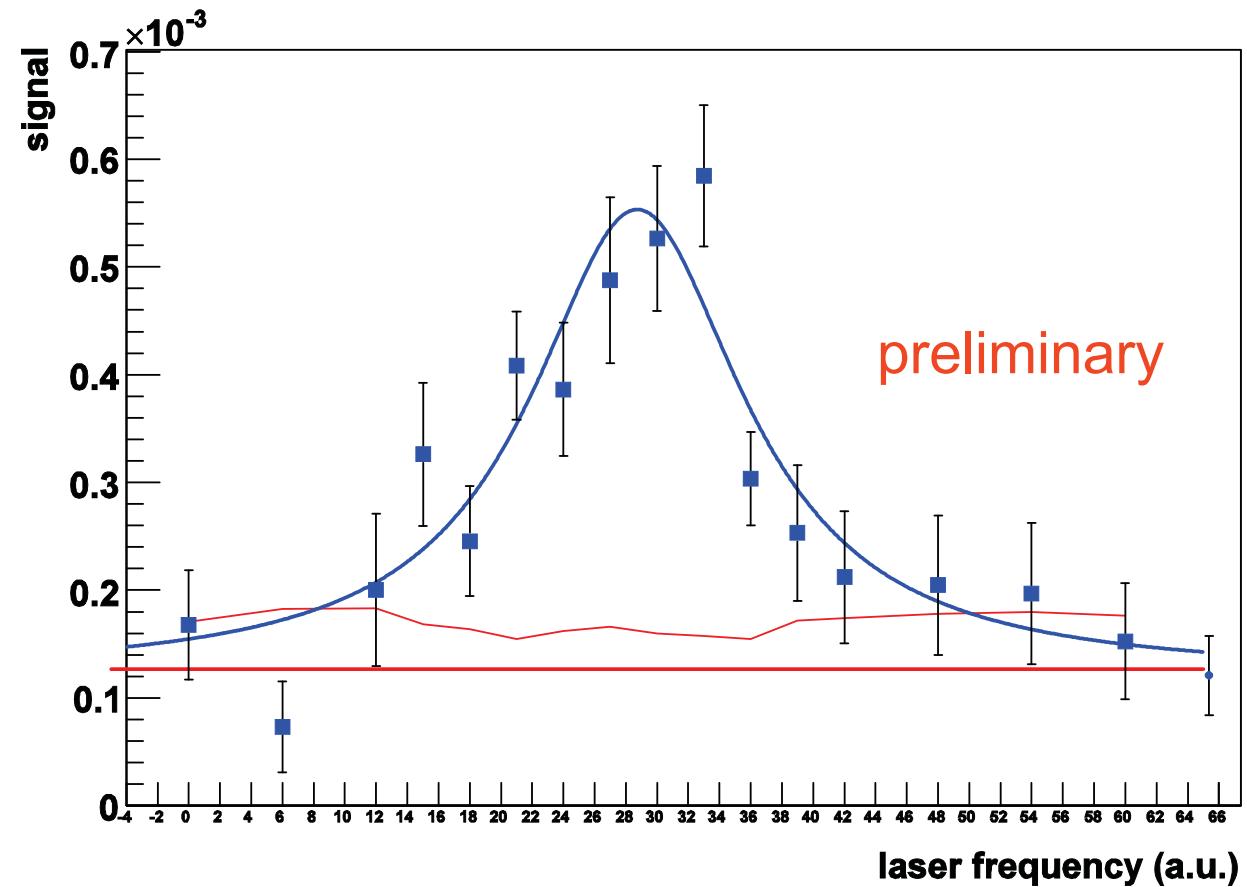
March 2011

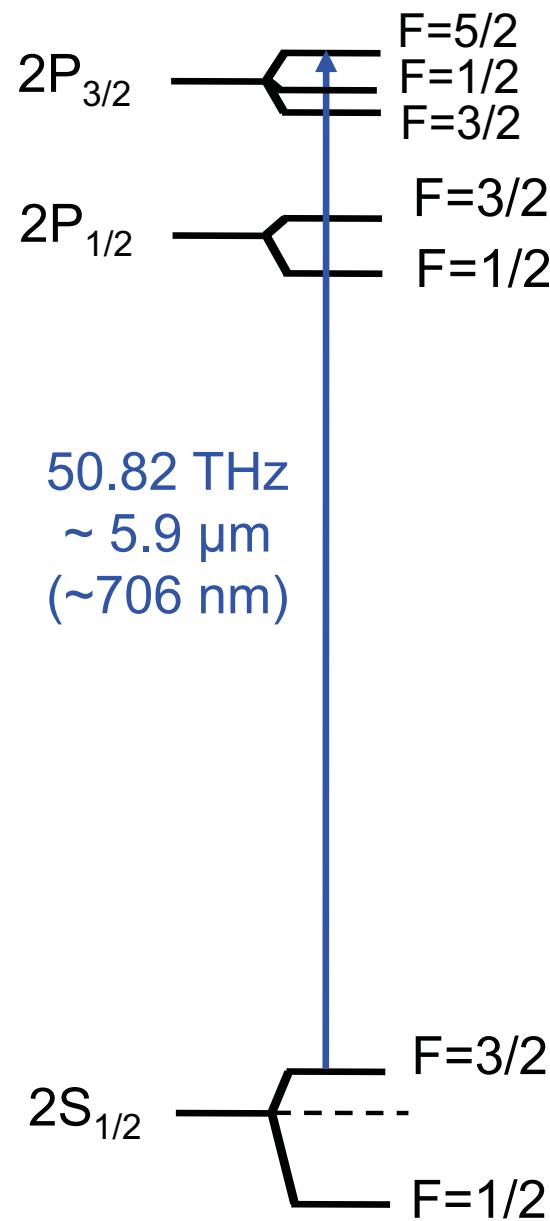
Proton size, Rencontres de Moriond

→ proton charge radius (~0.1%)

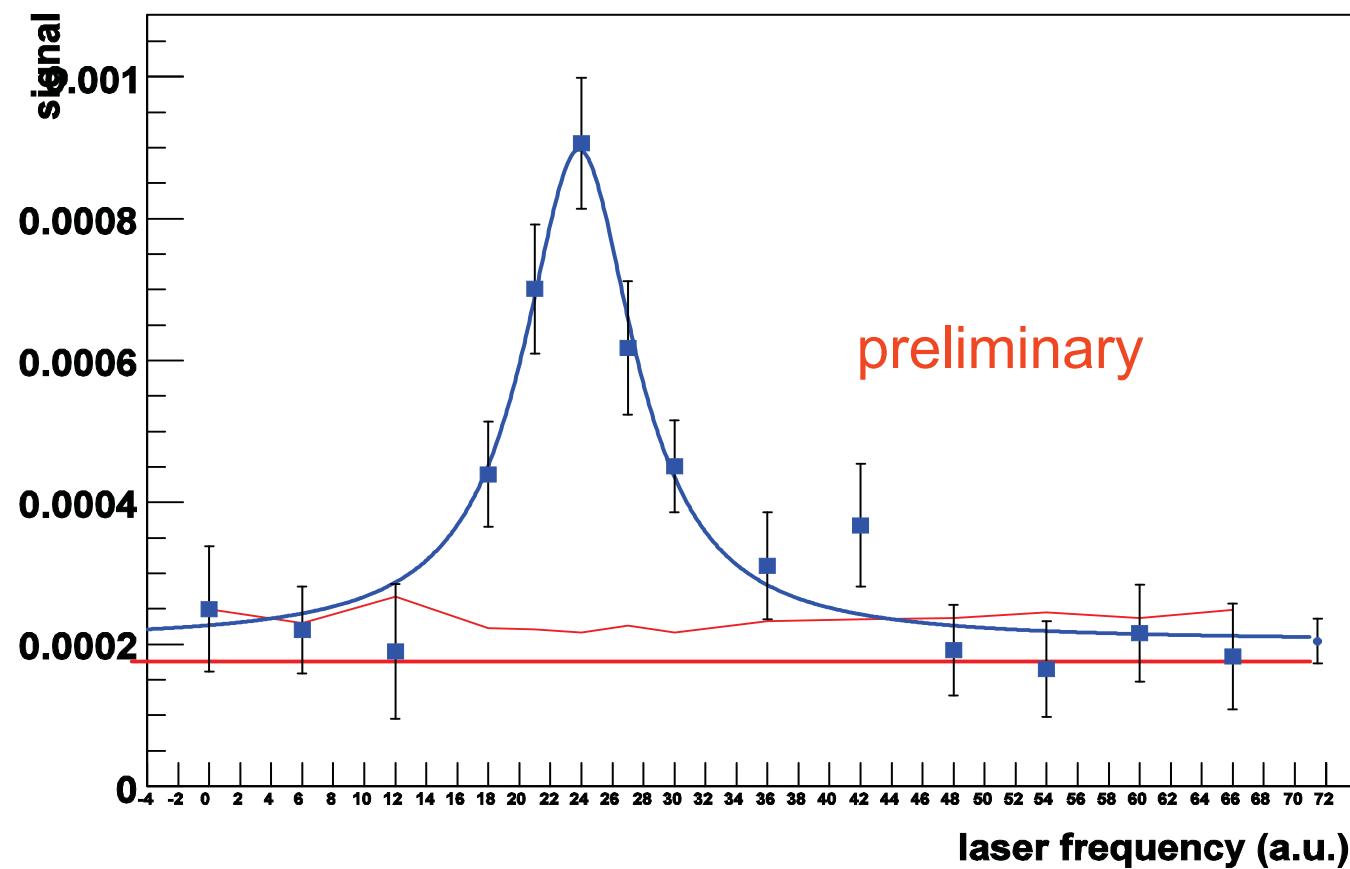


- measured position fits with our proton radius (**preliminary**)
- laser worked even better at 5.5 μm

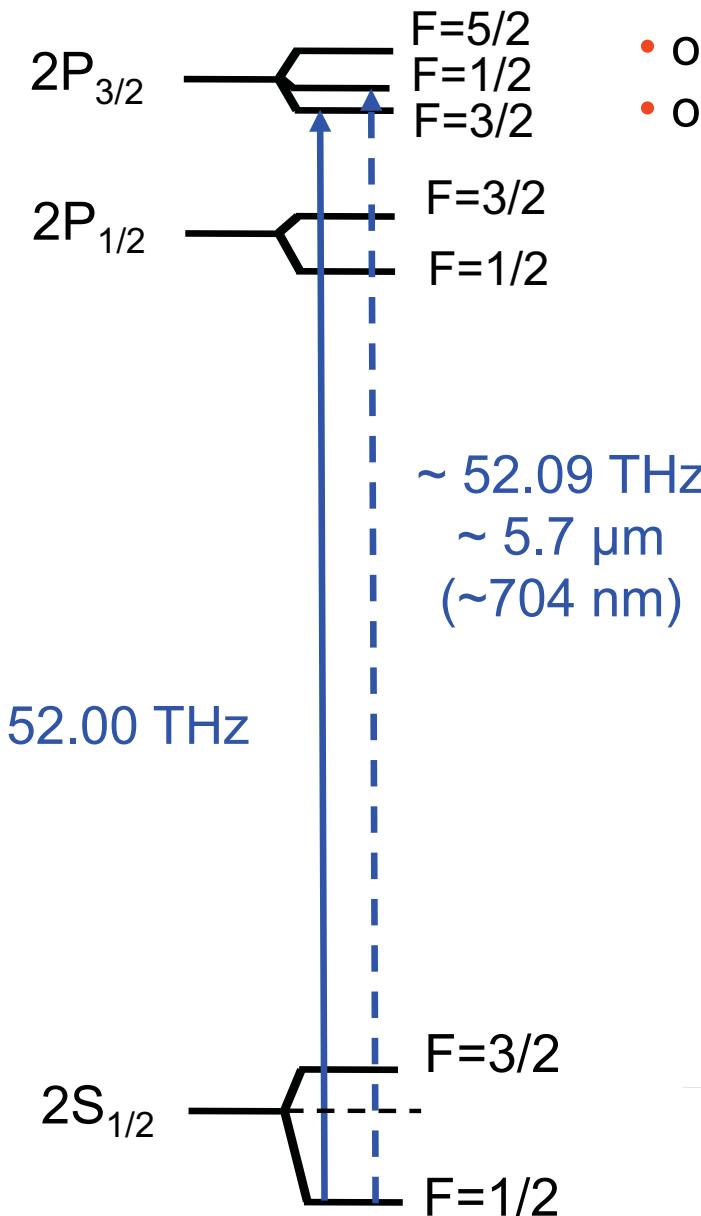


muonic deuterium : $2S_{1/2}(F=3/2)$ - $2P_{3/2}(F=5/2)$ 

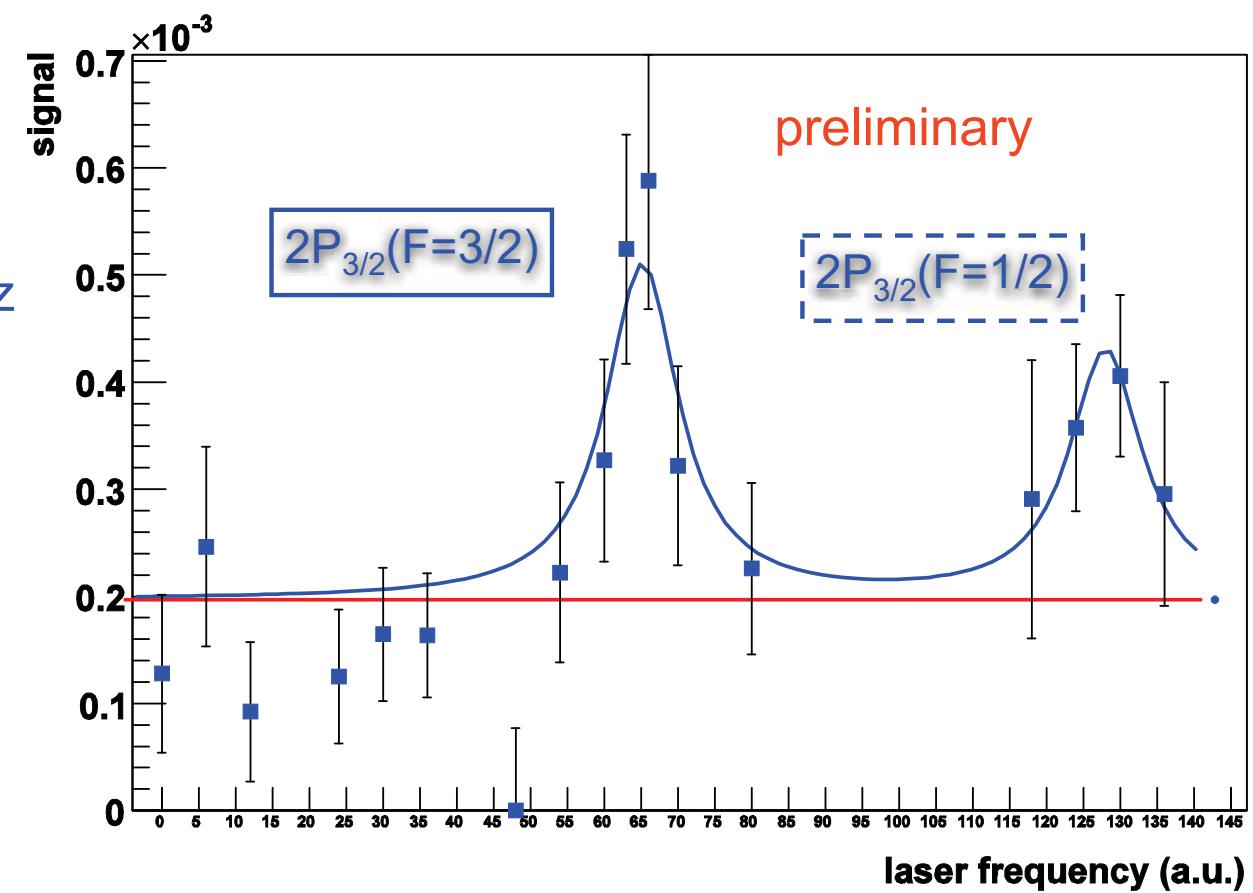
- on line signal, **preliminary**
- frequency position again off



Not at the position estimated with new r_p and isotopic shift → deuteron polarizability



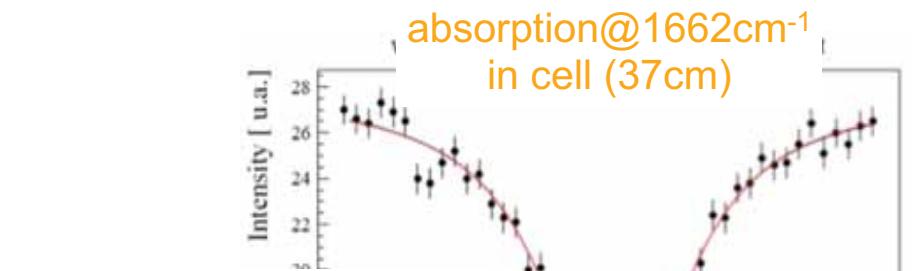
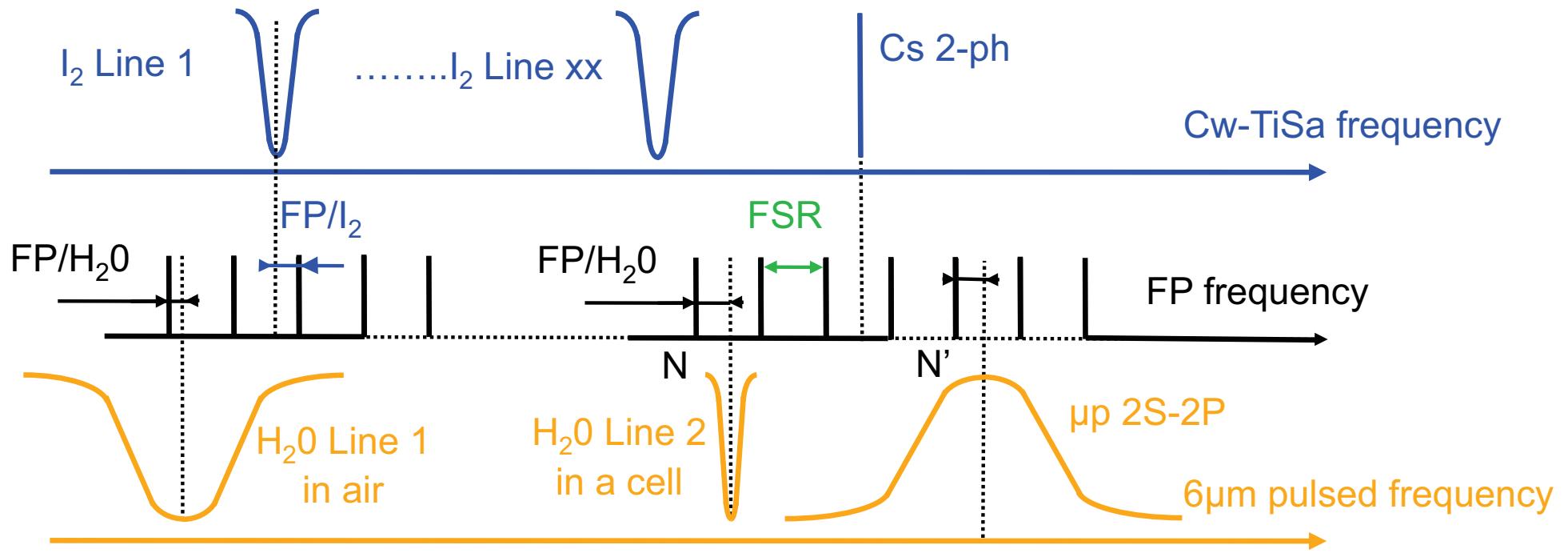
- on line signal, **preliminary**
- observation of 2 more lines in the last 2 days of beam line



"observation" → check calculation in μd

Laser chain : frequency calibration

FSR measured/controlled in cw with I_2 (1 ph abs), Cs (2 ph fluo), Rb (2 ph fluo), lines



FP absolute frequency
calibration @ 6μm
with H_2O lines

March 20

$$\nu(\mu\text{p}:2S-2P) = \nu(H_2O \text{ Line 2}) + (N - N') \text{ FSR}$$

size, Rencontres de Moriond

Statistics

- uncertainty on position (fit) 700 MHz ($\sim 4\%$ of Γ_{nat})

$$\Delta\nu_{\text{experimental}} = 20 \text{ (1) GHz} \quad (\Gamma_{\text{nat}} = 18.6 \text{ GHz})$$

Sources :

- Laser frequency (H_2O calibration) 300 MHz
- AC and DC stark shift < 1 MHz
- Zeeman shift (5 Telsa) < 30 MHz
- Doppler shift < 1 MHz
- Collisional shift 2 MHz

TOTAL UNCERTAINTY ON FREQUENCY 762 MHz

Broadening :

- 6 μm laser line width ~ 2 GHz
- Doppler Broadening < 1 GHz

$$\nu (\mu p : 2S_{1/2}(F=1)-2P_{3/2}(F=2)) = 49\ 881.88 \text{ (76) GHz} \quad (1.6 \cdot 10^{-5})$$

We need QED including Hyperfine structure

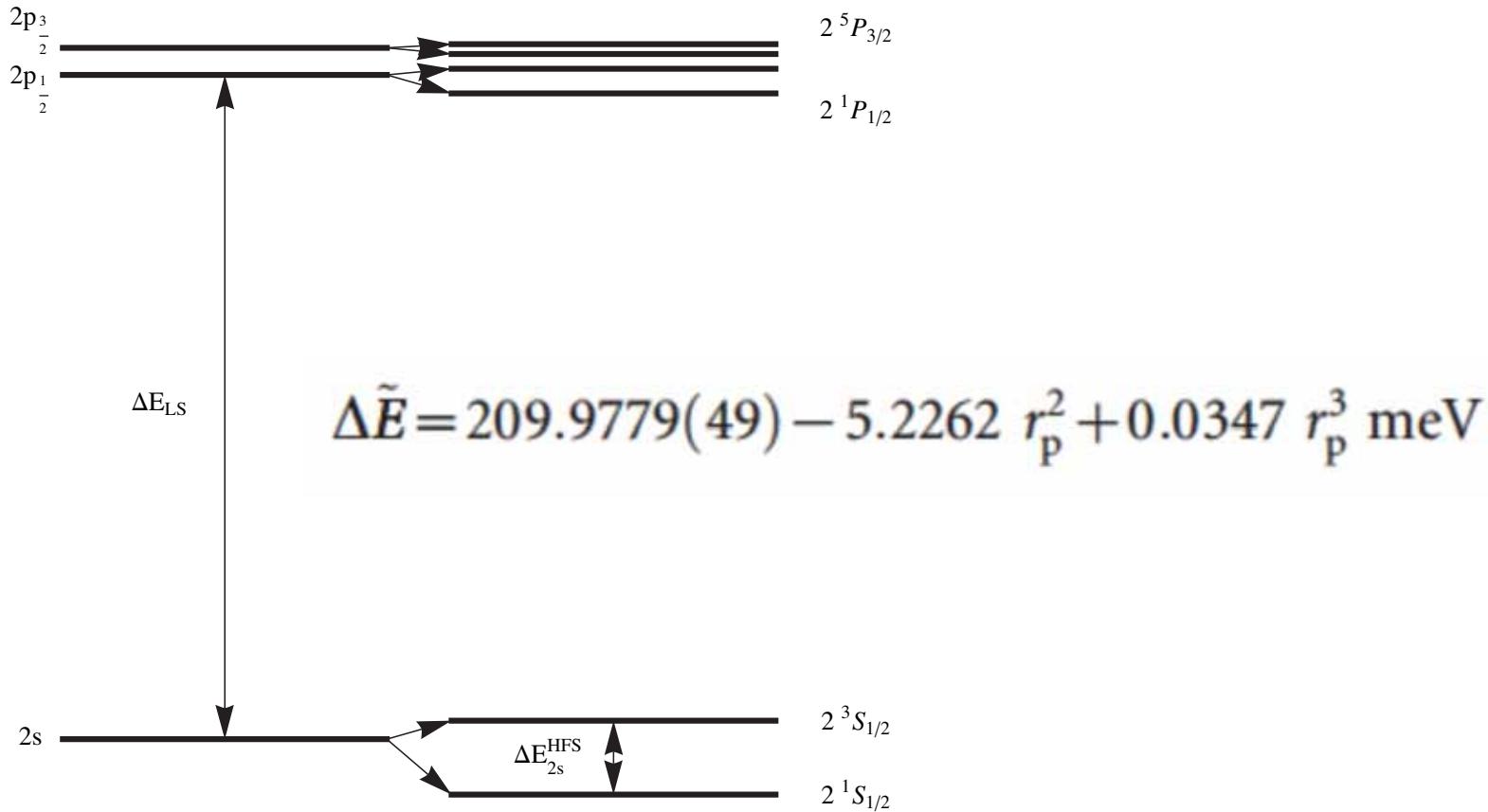


Fig. 19. The energy structure of the $n = 2$ level in muonic hydrogen. The drawing is to scale. Details of the $2p$ level structure are presented in Fig. 20.

We need QED including Hyperfine structure

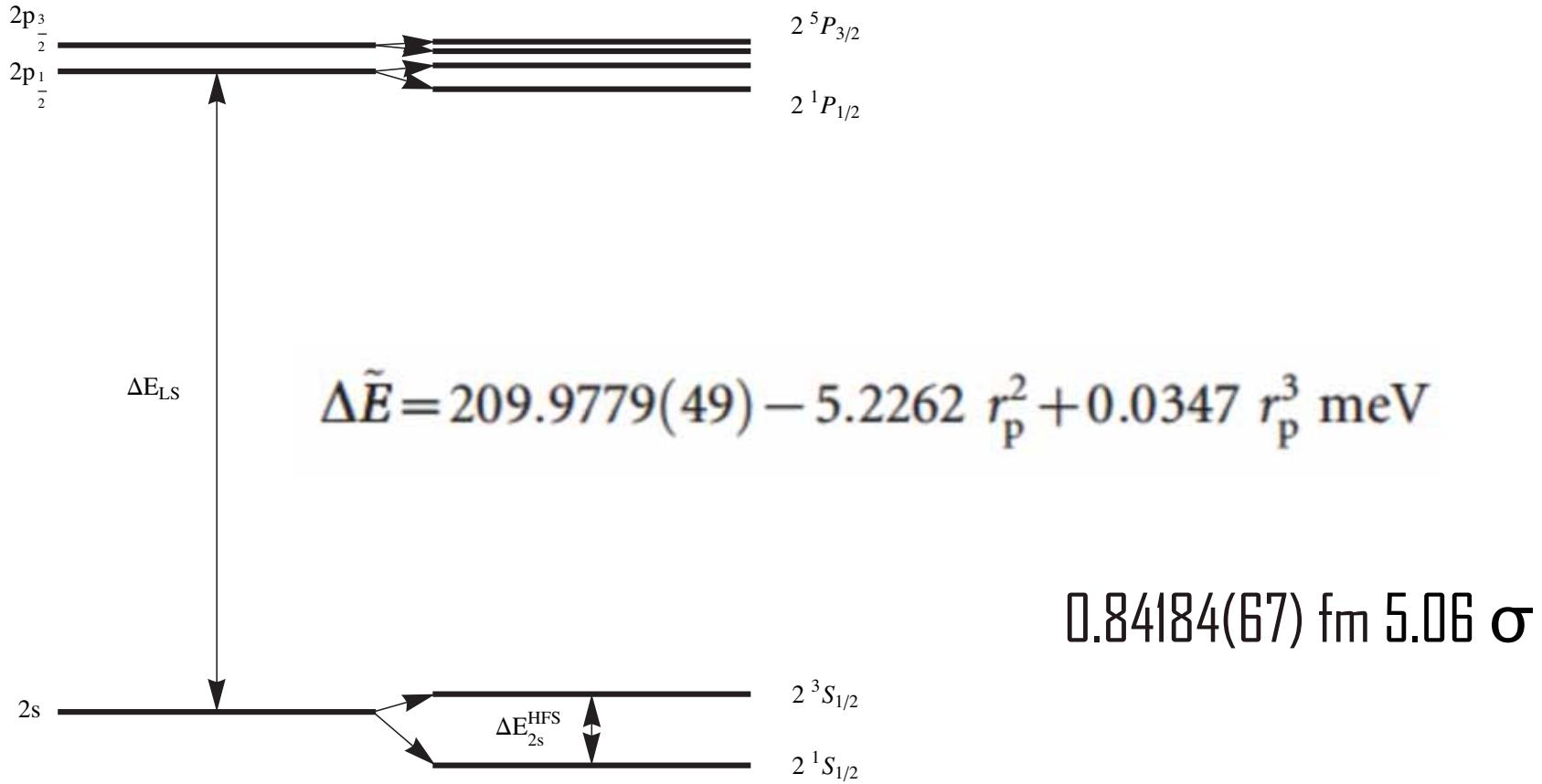
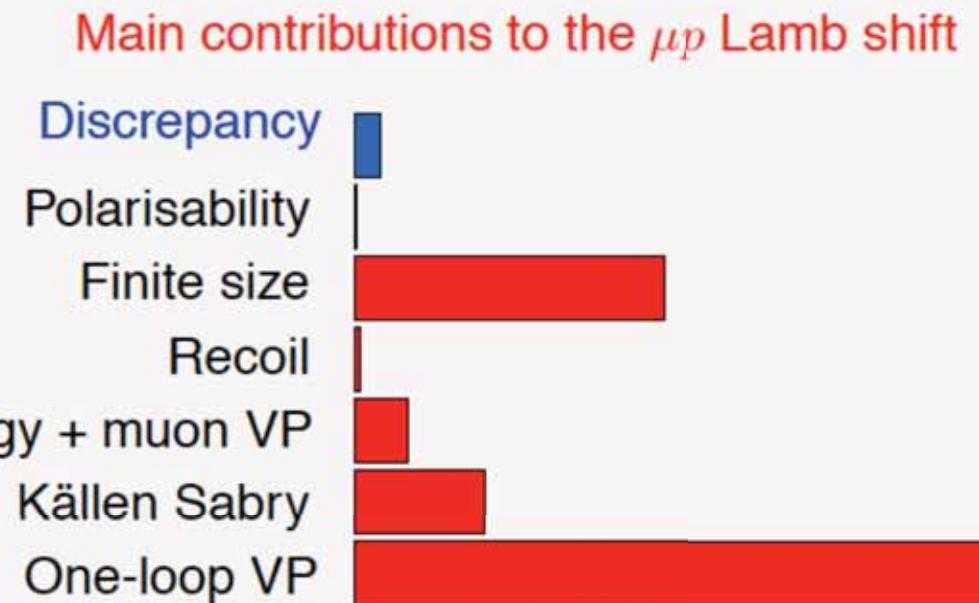
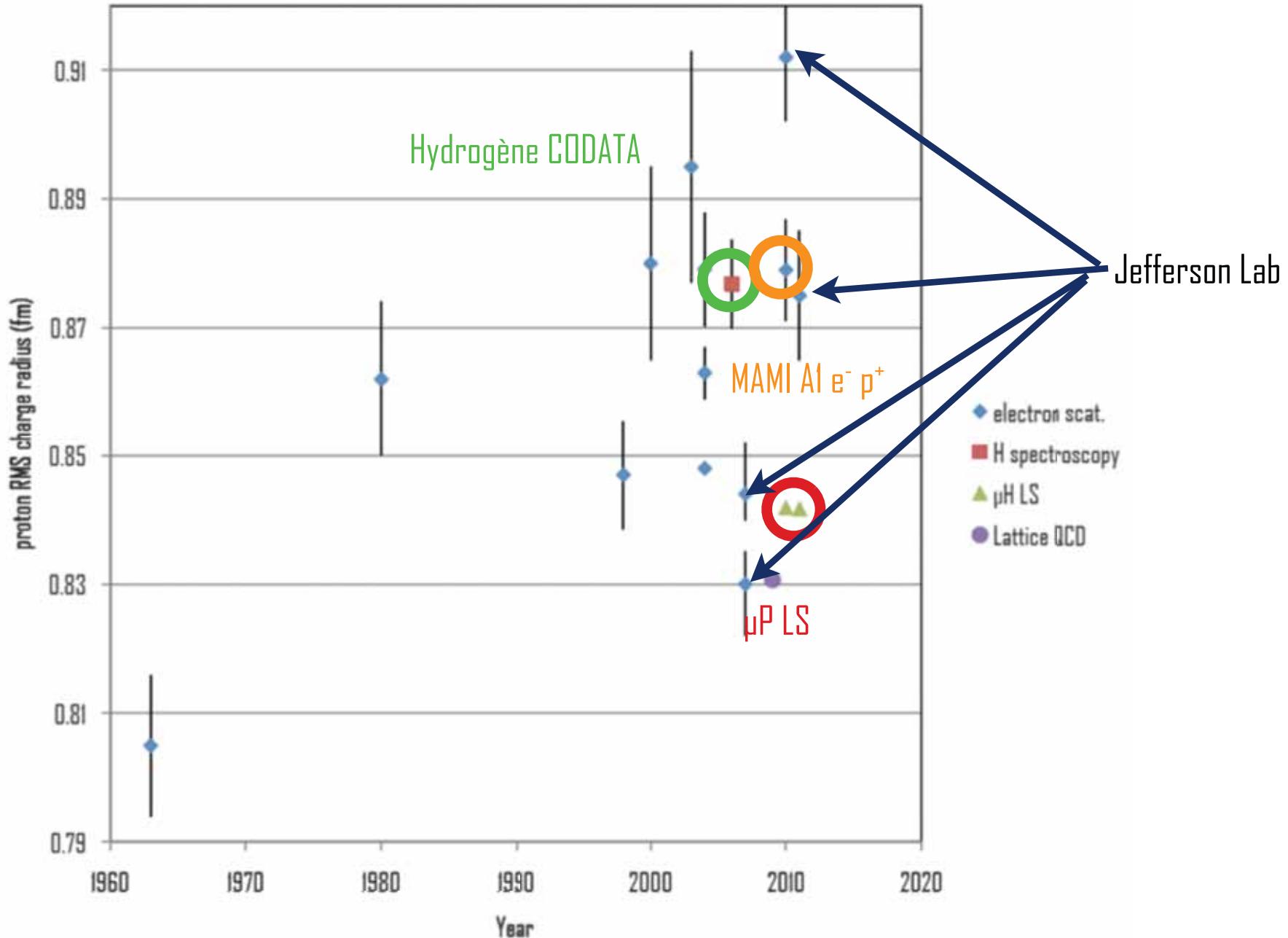


Fig. 19. The energy structure of the $n = 2$ level in muonic hydrogen. The drawing is to scale. Details of the $2p$ level structure are presented in Fig. 20.

Discrepancy=0.31 meV
Th. uncertainty=0.005 meV
 $\Rightarrow 60\delta(\text{theory})$ deviation



Proton size results summary



Direct numerical solution of Dirac equation, numerical grid, 10000 points, ~4000 inside the proton

$$(c\alpha \cdot p + \beta \mu_r c^2 + V_{\text{Nuc}}(r)) \Phi_{n\kappa\mu}(r) = \mathcal{E}_{n\kappa\mu} \Phi_{n\kappa\mu}(r),$$

$$V_{11}^{pn}(r) = -\frac{\alpha(Z\alpha)}{3\pi} \int_1^\infty dz \sqrt{z^2 - 1} \left(\frac{2}{z^2} + \frac{1}{z^4} \right) \frac{e^{-2m_e r z}}{r}$$

$$= -\frac{2\alpha(Z\alpha)}{3\pi} \frac{1}{r} \chi_1 \left(\frac{2}{\lambda_e} r \right)$$



$$\chi_n(x) = \int_1^\infty dz e^{-xz} \frac{1}{z^n} \left(\frac{1}{z} + \frac{1}{2z^3} \right) \sqrt{z^2 - 1}.$$

$$V_{11}(r) = -\frac{2\alpha(Z\alpha)}{12\pi} \frac{1}{r} \int_0^\infty dr' r' \rho(r') \times \left[\chi_2 \left(\frac{2}{\lambda_e} |r - r'| \right) - \chi_2 \left(\frac{2}{\lambda_e} |r + r'| \right) \right].$$

Analytical expression for the evaluation of vacuum-polarization potentials in muonic atoms, S. Klarsfeld. Physics Letters 66B, 86-88 (1977).

Charge radius dependence Coul.+VP

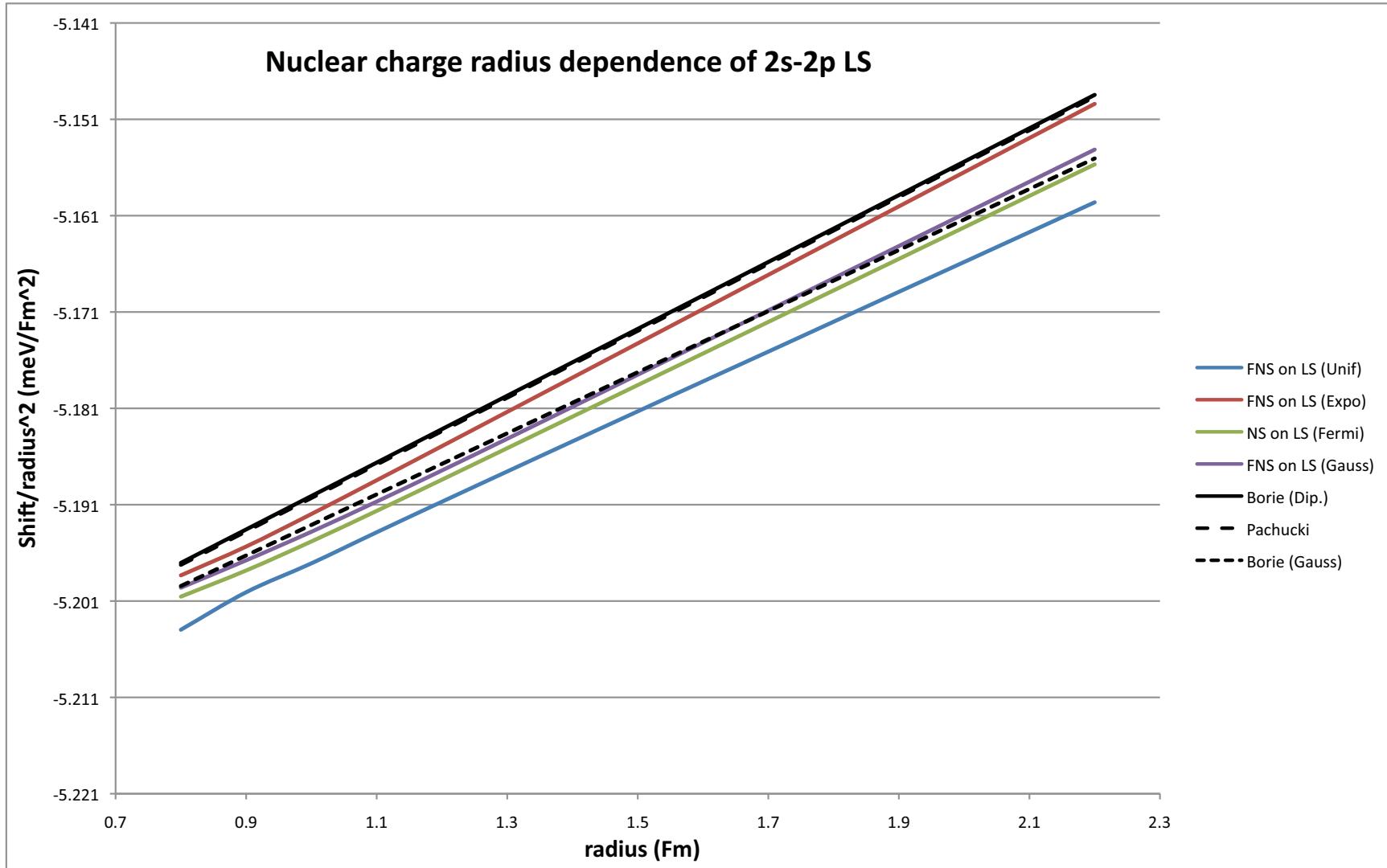
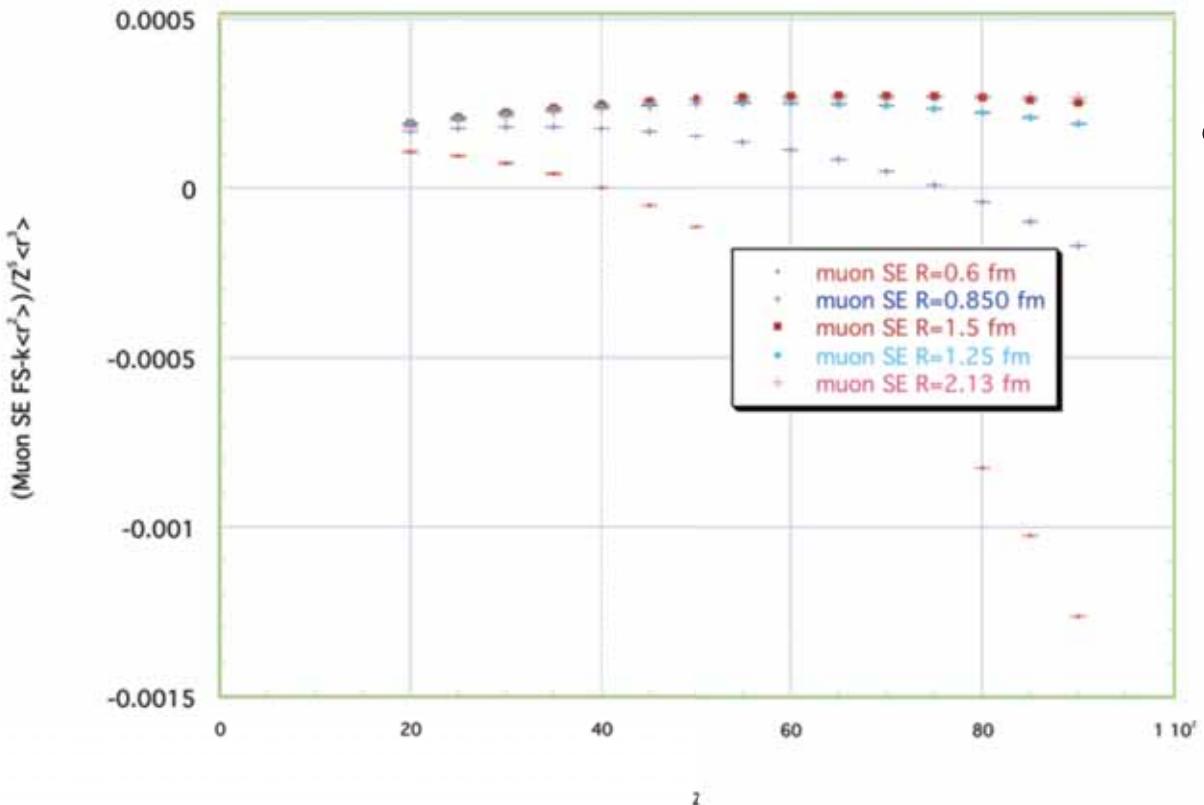


Fig. 3. Dependence of $\frac{\Delta E_{V11FN}}{R^2}$ as a function of R in meV/fm² for different charge distribution models.

Finite size correction on muon self-energy

All-orders calculations



$$E_{SE-NS} = \left(4 \ln 2 - \frac{23}{4}\right) \alpha(Z\alpha) \mathcal{E}_{NS}$$

$$\mathcal{E}_{NS} = \frac{2}{3} \left(\frac{\mu_r}{m_\mu}\right)^3 \frac{(Z\alpha)^2}{n^3} m_\mu \left(\frac{Z\alpha \langle r \rangle}{\lambda_C}\right)^2$$

$$E_{SE-NS} = -0.000824 \langle r^2 \rangle$$

$$(All-orders calculations - E_{SE-NS} \langle r^2 \rangle) / Z^5 \langle r^3 \rangle \\ 1.8 \pm 1 \times 10^{-5}$$

$$\tilde{\Delta E} = 209.9779(49) - 5.2262 r_p^2 + 0.0347 r_p^3 \text{ meV} \quad 0.84184(67) \text{ fm } 5.06 \sigma$$

R. Pohl, A. Antognini, F. Nez, et al., Nature **466**, 213 (2010).

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$$E_{\text{th}} = \left(209.9974(48) - 5.2262 \frac{r_p^2}{\text{fm}^2} \right) \text{ meV} \quad 0.84169(66) \text{ fm} \\ 5.09 \sigma$$

U. D. Jentschura, Annals of Physics **326**, 500 (2011).

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$$209.9759 - 5.228880 R^2 + 0.03573760 R^3 - 0.00004500000 R^4 \quad 0.84145(66) \text{ fm} \\ 5.12 \sigma$$

P. Indelicato, P. Mohr, in preparation

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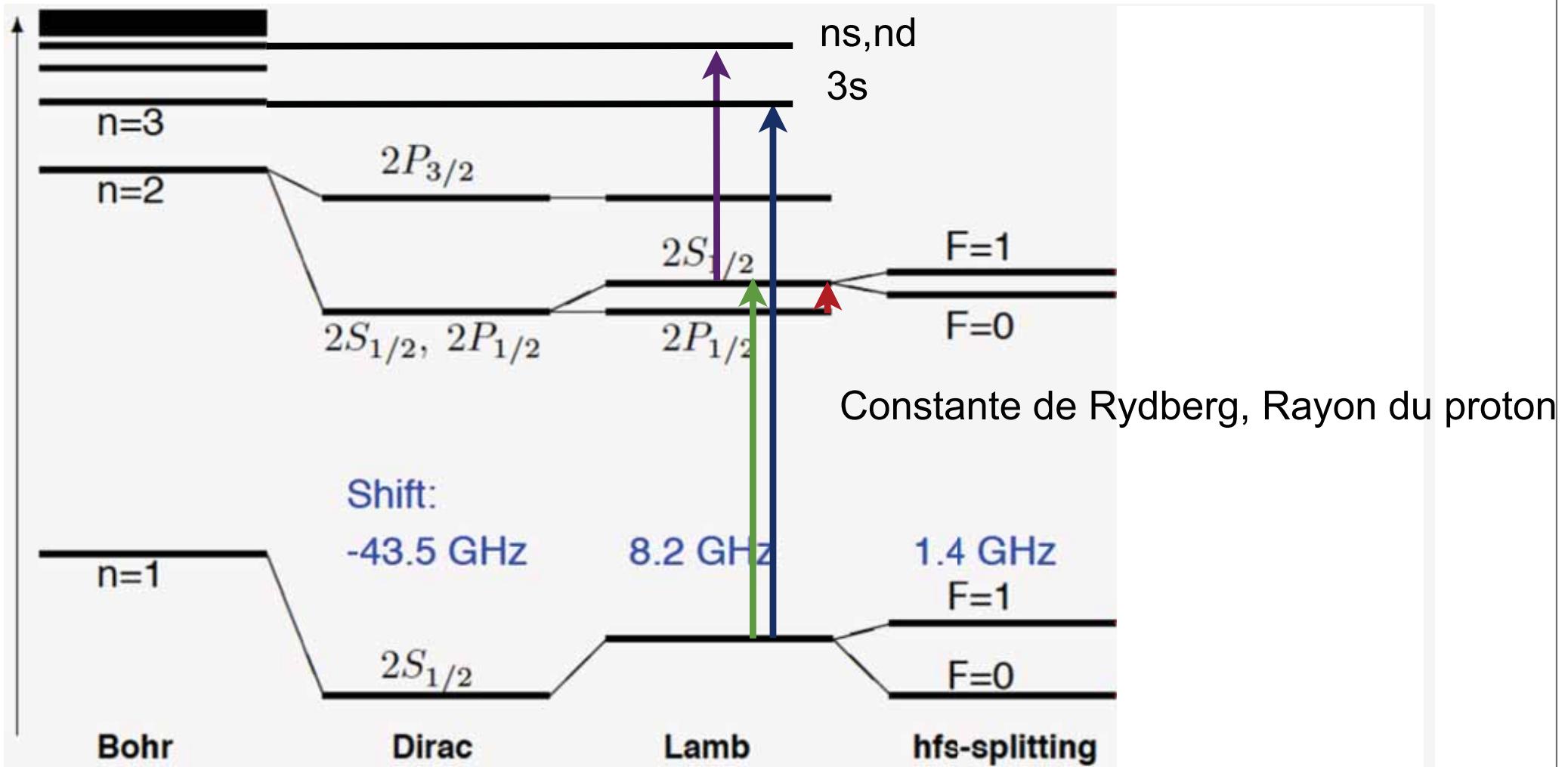
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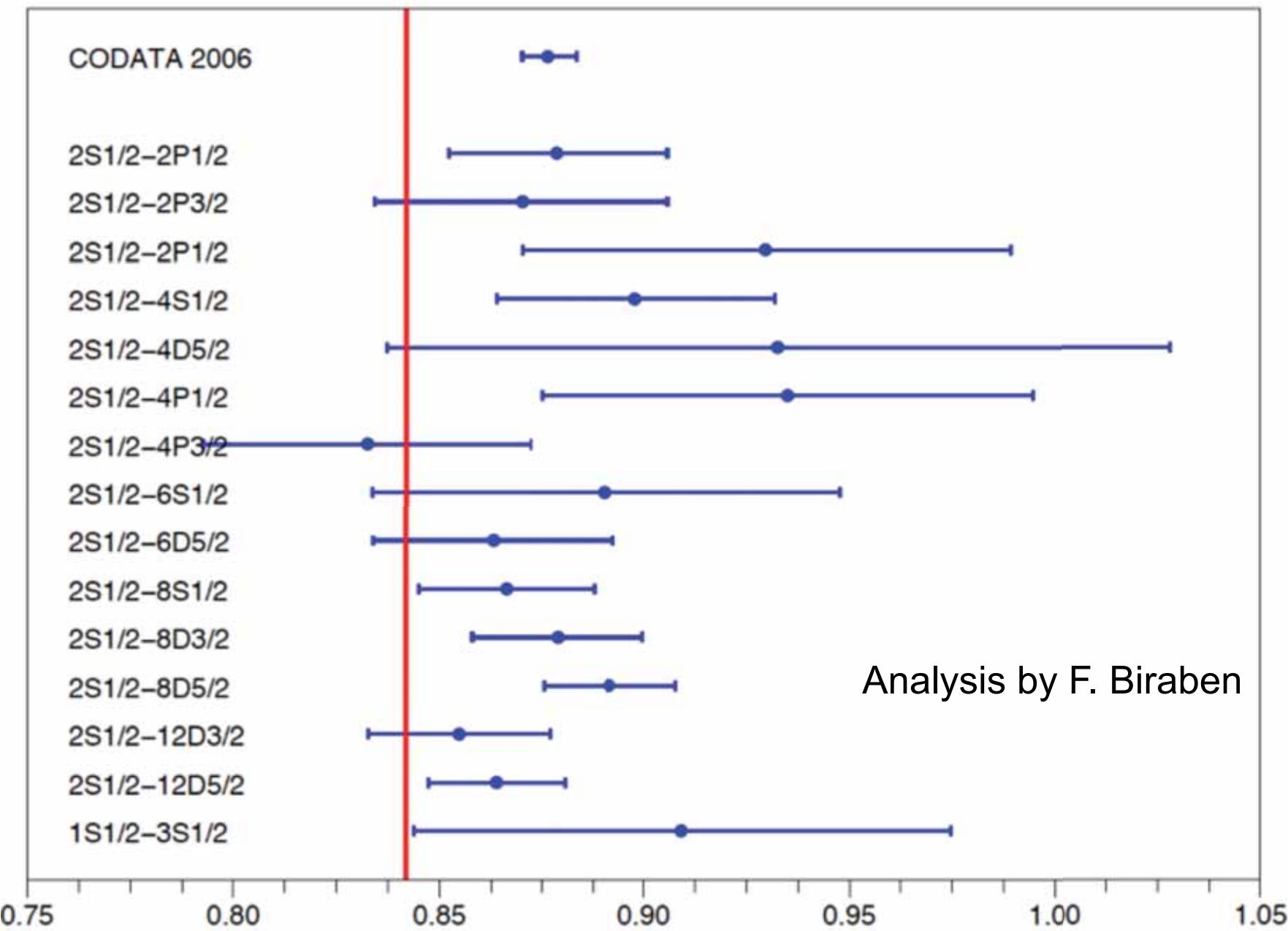
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The coefficient of r_p^3 is model dependent, using dipole model
(more on next talk)

- Electron-proton elastic scattering data analysis:
 - very probable (next talk)
- Under-estimated systematic errors in some hydrogen measurements:
 - possible, but many different experiments (microwave, 1s-3s, 2s-ns and 2s-nd)
- New physics:
 - Predictions are always difficult, in particular about the Future [N. Bohr]





- Hydrogen mean value: 0.8752(71) fm: 4.8σ
- CODATA: 0.8768(69) fm (uses scattering data): 5.1σ

- A muon edm? If $d_\mu = 2 \times 10^{-19}$ e·cm would shift the energy level < 200 MHz
- Charge equality between e^- and μ^- generation? Checked to $u_r = 10^{-8}$ (from μ^+e^-)
- Deviation from Coulomb's law: probe of hidden sector
 - Test of Coulomb law via spectroscopy is very clean and model independent probe of new particles
It is independent on stability and decay channel.
$$V(r) = -\frac{Z\alpha}{r}(1 + \alpha'e^{-mr}) \quad \text{or} \quad V(r) = -\frac{Z\alpha}{r}(1 + \alpha''(\mathbf{s}_1 \cdot \mathbf{s}_2)e^{-mr})$$
- From simple atoms there are constraints on light bosons with ultra-weak coupling:
 $m \in [1\text{eV}, \text{MeV}]$ and $\alpha' < 10^{-13}$, $\alpha'' < 10^{-17}$ [PRL 104,220406 (2010), arXiv:1008.3536v2]
- Minicharge particles? [Jaeckel and Roy (2010), Jentschura(2010)]
 Vacuum polarization with pair production of light fermions with $q = \varepsilon e$ and masses $m_\varepsilon < m_e$

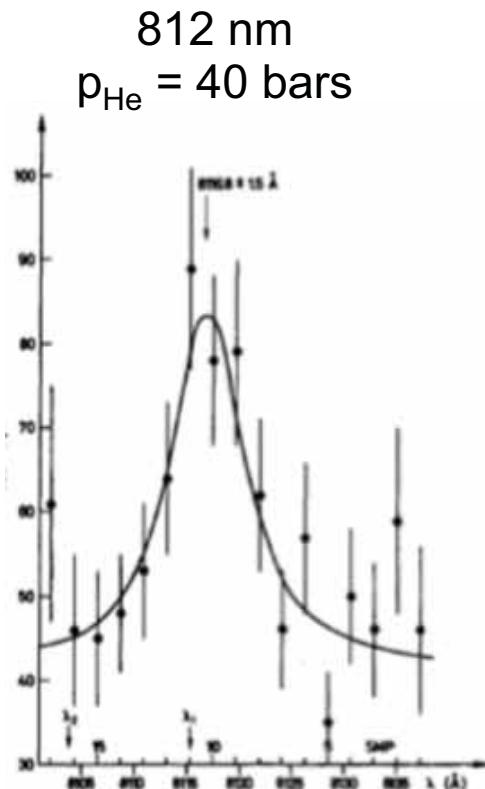
No parameter found explaining r_p puzzle without contradicting, simple atoms spectroscopy, $g_e/\mu - 2$, $\alpha...$

- We have performed a 15 ppm measurement of the Lamb-shift in muonic hydrogen
- The deduced proton radius using a Dipole model is 5 standard deviations away from the hydrogen and electron-proton elastic scattering data
- Better modeling of the proton form-factor required to confirm or reduce the disagreement
- Experiment checked with 2nd μH line and 3 μD lines
- Muonic He in 2013 (check of theory, different laser wavelength-in the red)



Proton Size Investigators thank you for your attention

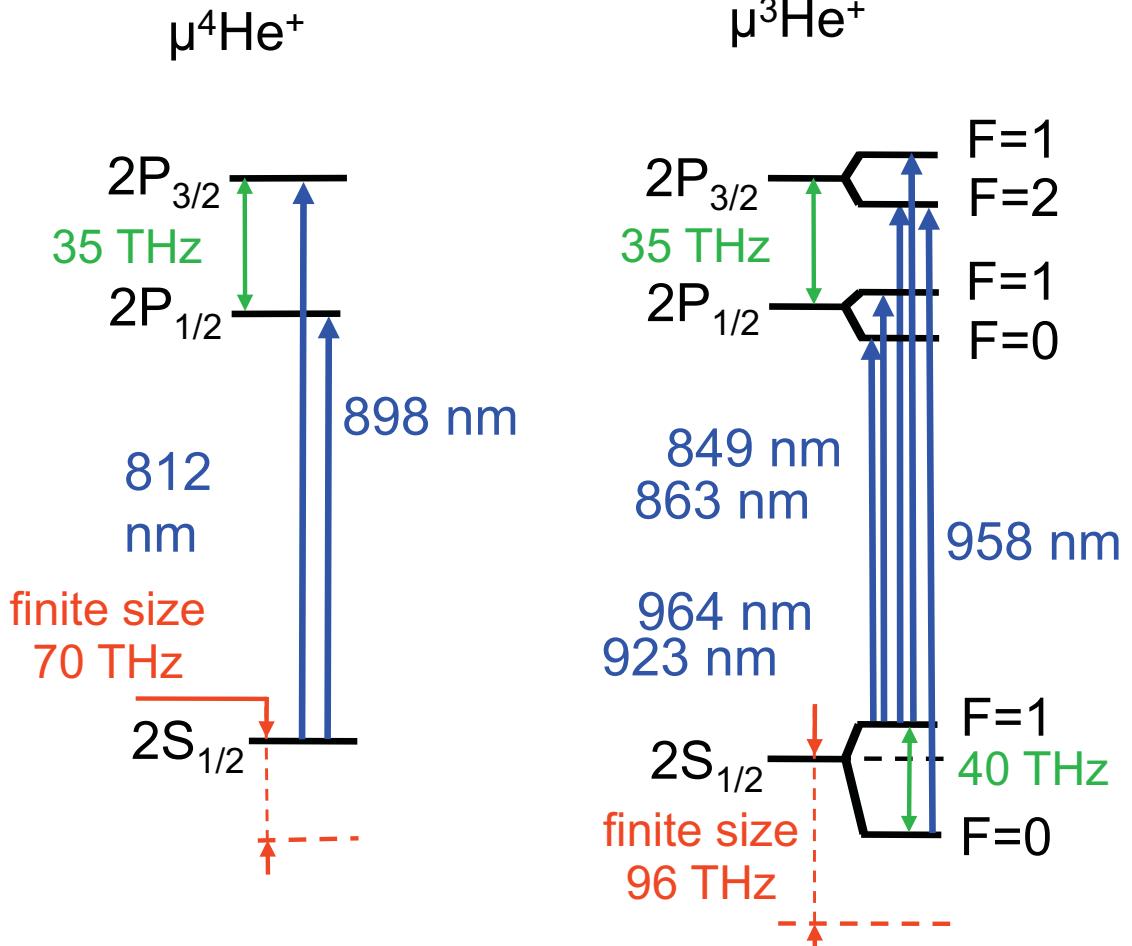




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but signal never
reproduced
(10 bars, 40 bars)

2011-2013 → muonic helium spectroscopy (4 mbar)

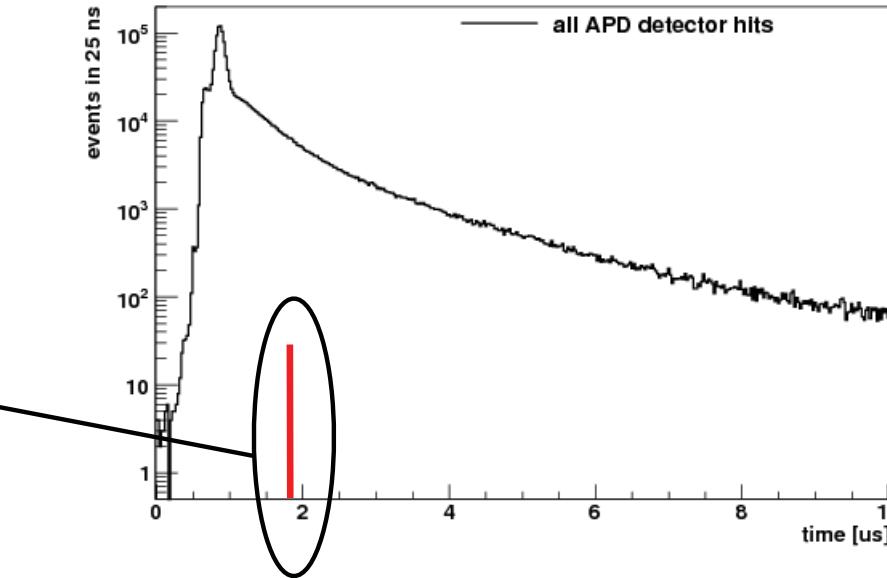


- μHe^+ spectroscopy + He^+ spectroscopy → QED test ($Z\alpha$)
- improve He spectroscopy

Example : FP 900 - 11 hrs meas.

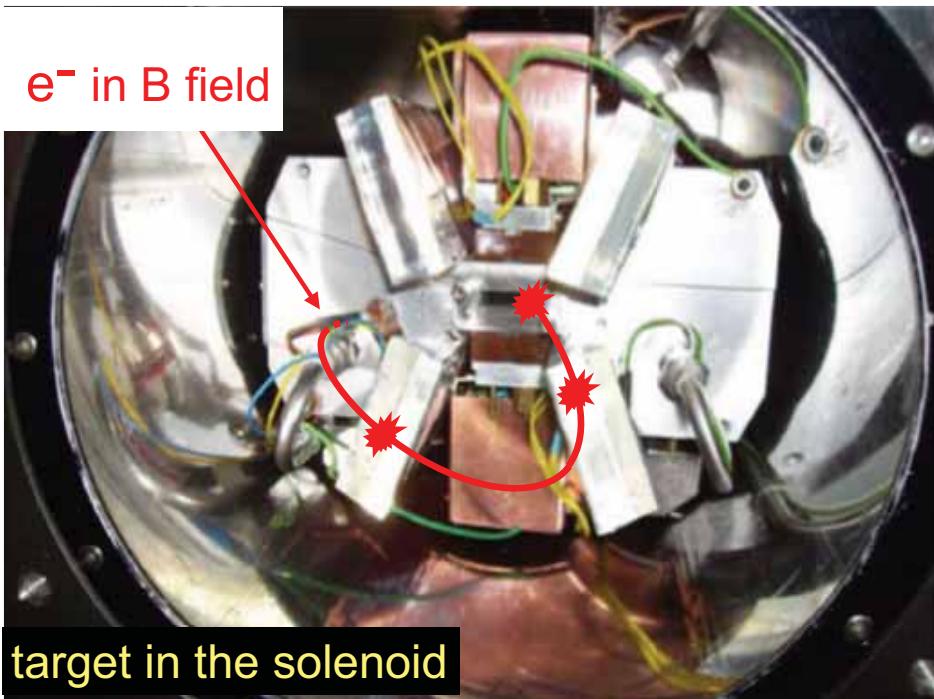
1.56 million detector events

expected 2-3 laser induced events/hour !



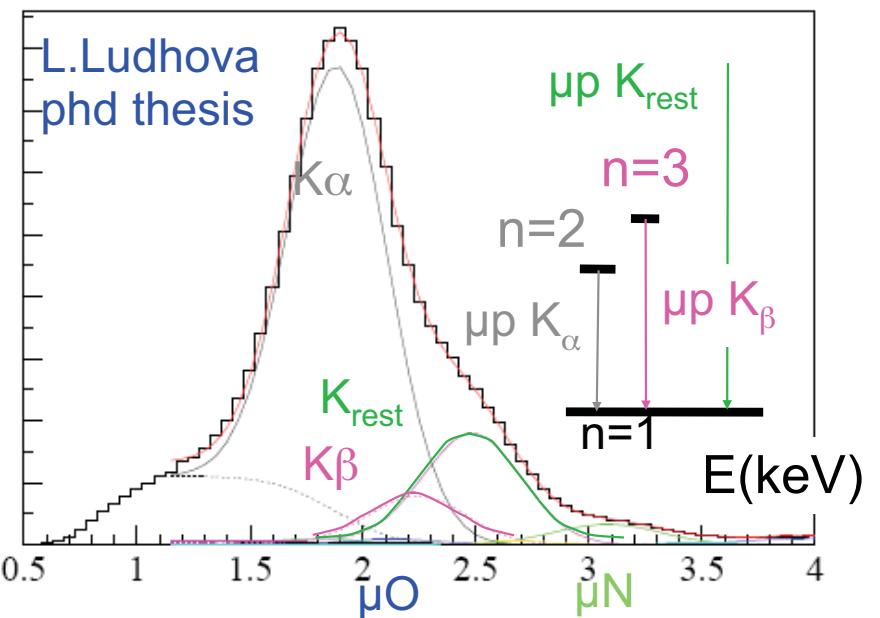
time signature in LAAPD

- photon < 10keV → 1 shot in the LAAPD
- e^- in $B = 5T$ → many counts in detectors



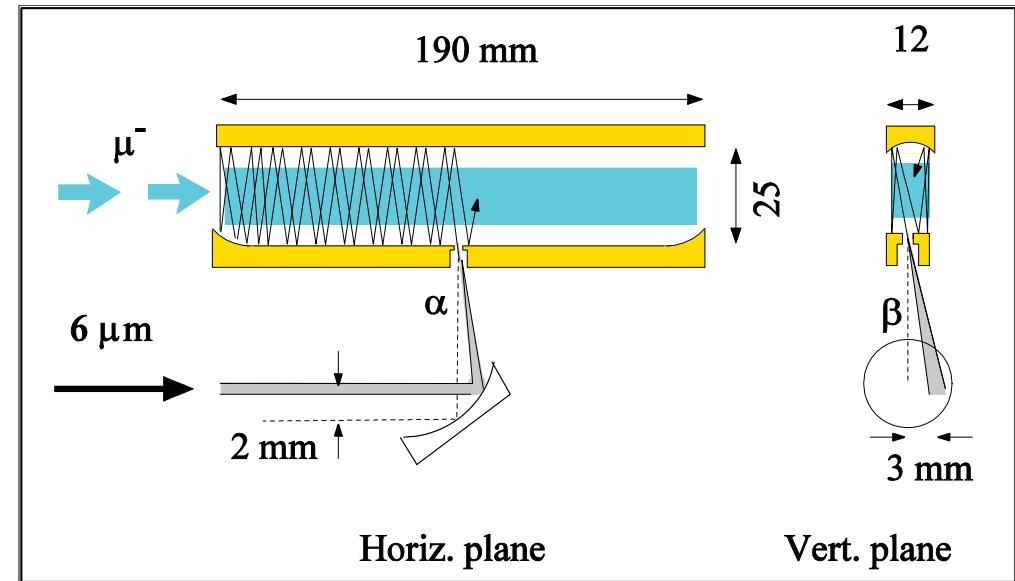
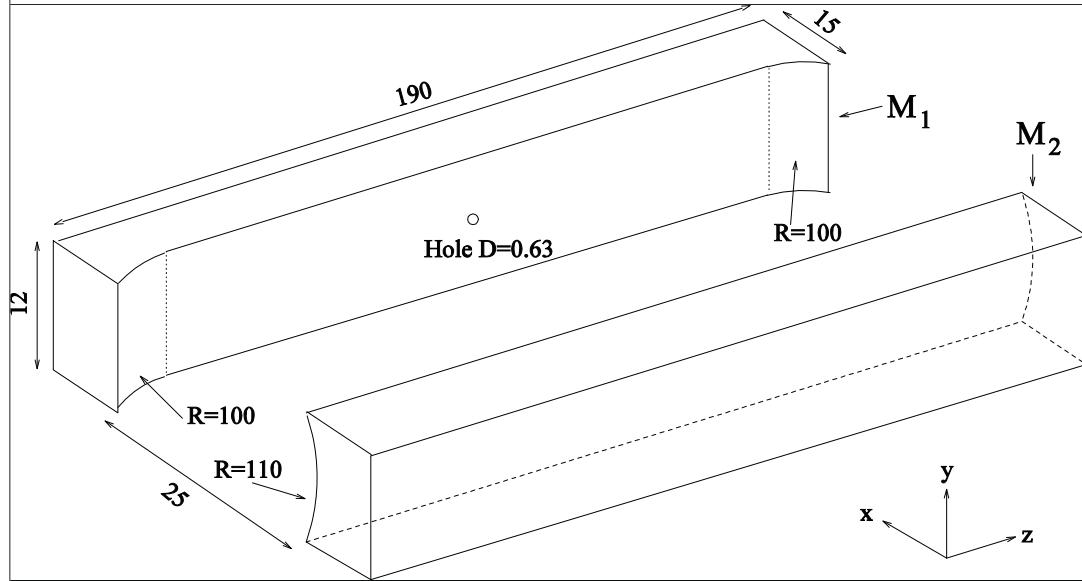
energy signature in LAAPD

- $E > 8\text{keV} \Leftrightarrow$ electron
- $1\text{keV} < E < 8\text{keV} \Leftrightarrow$ X ray
- $E < 1\text{keV} \Leftrightarrow$ neutron

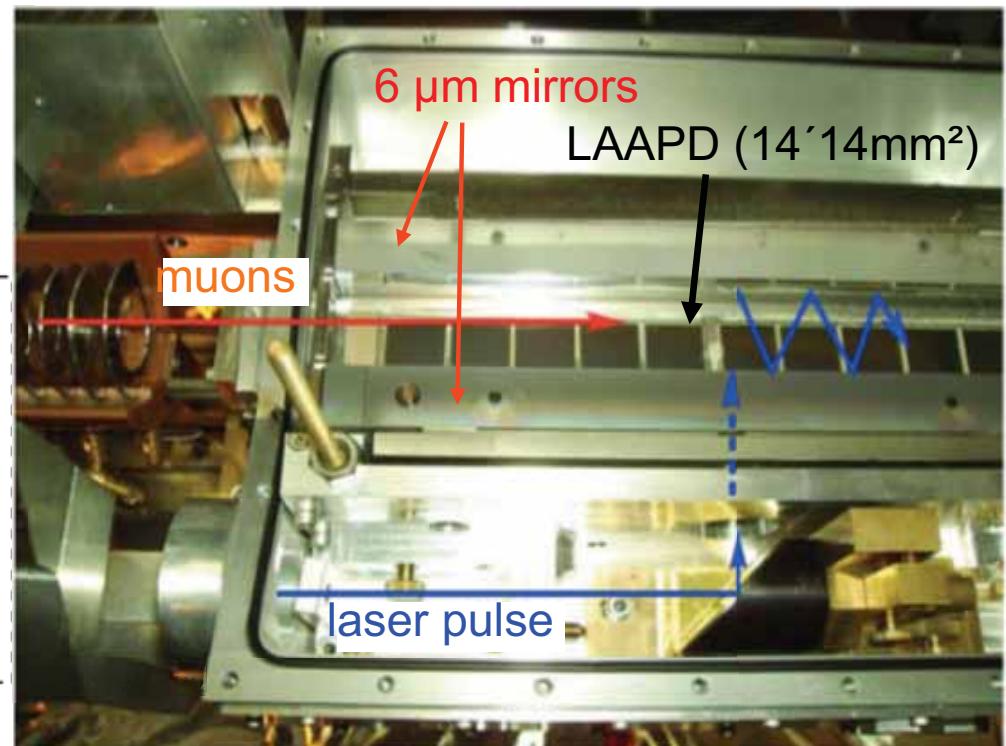
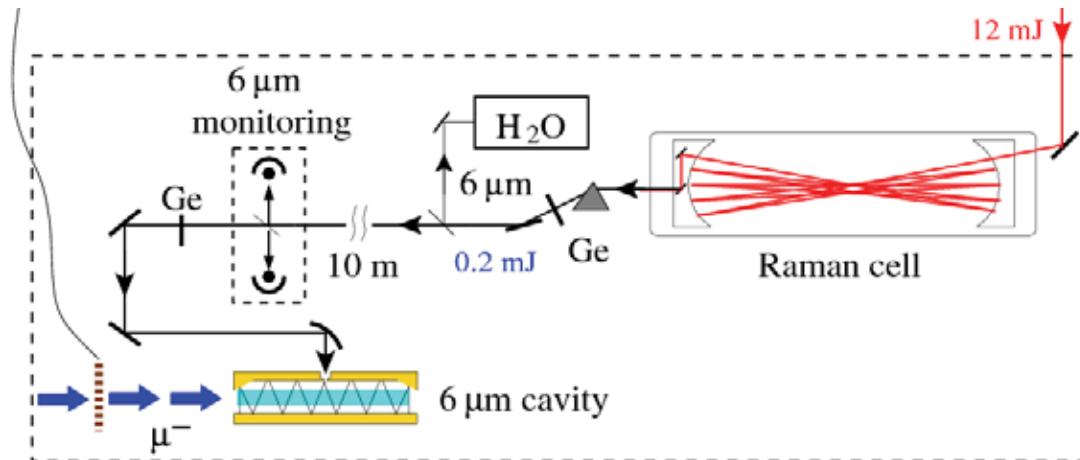


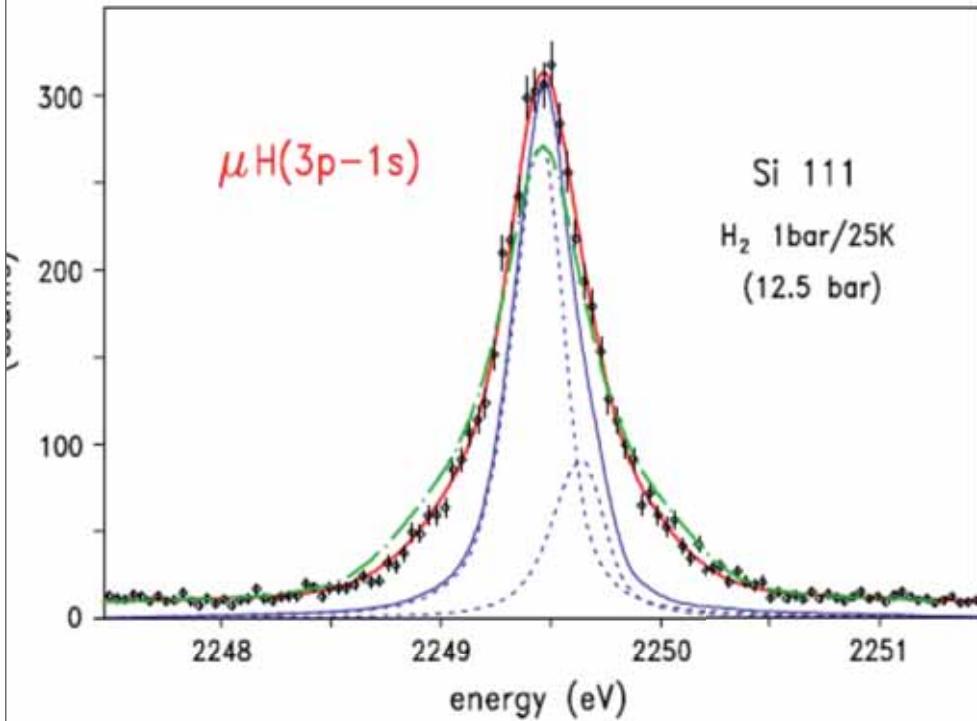
Laser chain : multipass cavity

→ illuminate at 6 μm all the muon stopping volume ($5'15'190 \text{ mm}^3$)



- coupling through a 0.63mm diameter hole
- R=99.90% at 6 μm
- 1000 reflections
- 0.15mJ injected → 2S-2P saturated





Line Shape of the mu H(3p-1s) Hyperfine Transitions,
D.S. Covita, D.F. Anagnostopoulos, H. Gorke, D. Gotta,
A. Gruber, A. Hirtl, T. Ishiwatari, P. Indelicato, E.-O.L.
Bigot, M. Nekipelov, J.M.F.d. Santos, P. Schmid, L.M.
Simons, M. Trassinelli, J.F.C.A. Veloso and J. Zmeskal.
Phys. Rev. Lett. 102, 023401 (2009).



Ref.	proton charge radius (fm)		
Hand et al. [1]	0.805	±	0.011
Simon et a/ [2]	0.862	±	0.012
Mergel et al. [11]	0.847	±	0.008
Rosenfelder [12]	0.880	±	0.015
Sick 2003 [13]	0.895	±	0.018
Angeli [14]	0.8791	±	0.0088
Kelly [15]	0.863	±	0.004
Hammer et al. [16]	0.848		
CODATA 06 [10]	0.8768	±	0.0069
Arington et al. [17]	0.850		
Belushkin et al. [18] SC approach	0.844	-0.004 +0.008	
Belushkin et al. [18] pQCD app.	0.830	-0.008 +0.005	
Wang (ChPt, 2009) [19]	0.828		
Pohl et al. [20]	0.84184	±	0.00067
Bernauer et al (2010)	0.879	±	0.008

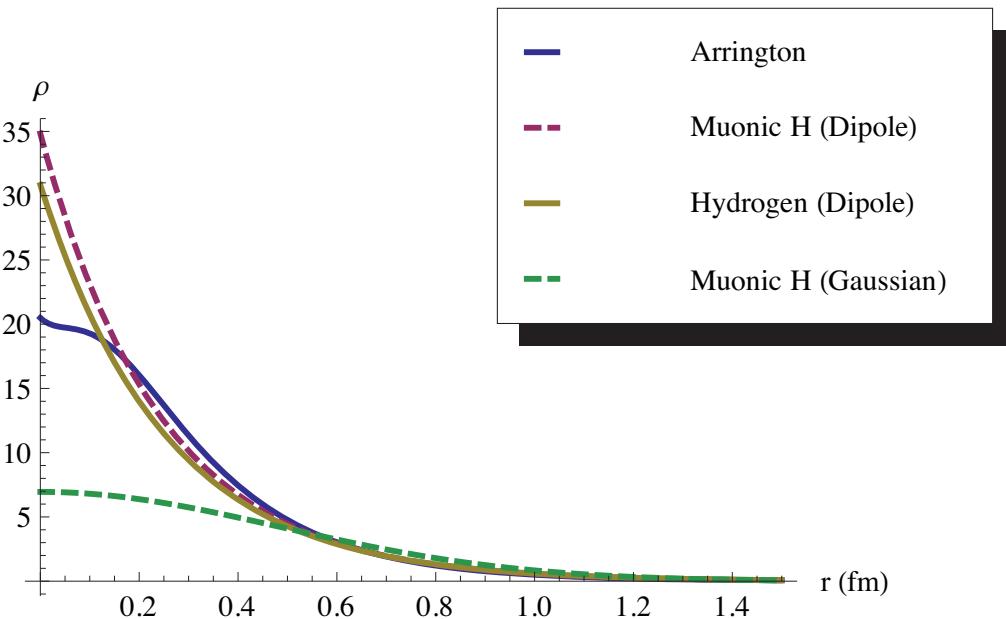
Table 1. Proton charge radii. For the result corresponding to Ref. [17] see the analysis in Sec.2.1

	SC approach	Explicit pQCD app.	Ref. [23]	Recent determ.
r_E^p (fm)	0.844 (0.840 ... 0.852)	0.830 (0.822 ... 0.835)	0.848	0.886(15) [72–74]
r_M^p (fm)	0.854 (0.849 ... 0.859)	0.850 (0.843 ... 0.852)	0.857	0.855(35) [73,75]
$(r_E^n)^2$ (fm ²)	-0.117 (-0.11 ... -0.128)	-0.119 (-0.108 ... -0.13)	-0.12	-0.115(4) [52]
r_M^n (fm)	0.862 (0.854 ... 0.871)	0.863 (0.859 ... 0.871)	0.879	0.873(11) [76]

Resonance	Mass (GeV)	a_1 (GeV ²)	a_2 (GeV ²)	Γ (GeV)
ω	0.782	0.755960	0.370592	—
ϕ	1.019	-0.776537	-2.913229	—
s_1	1.124860	0.902379	2.484859	—
s_2	2.019536	0.022798	-0.130622	5.158635
v_1	1.062128	-0.127290	-2.162533	—
v_2	1.300946	-1.243412	3.704233	—
v_3	1.493630	4.191380	-7.091021	—
v_4	1.668522	-3.176013	3.723858	—
v_5	2.915451	0.048987	0.075965	19.088297

Dispersion analysis of the nucleon form factors including meson continua, M.A. Belushkin, H.W. Hammer and U.-G. Meißner. Phys. Rev. C **75**, 035202 (2007).

Comparison of charge densities

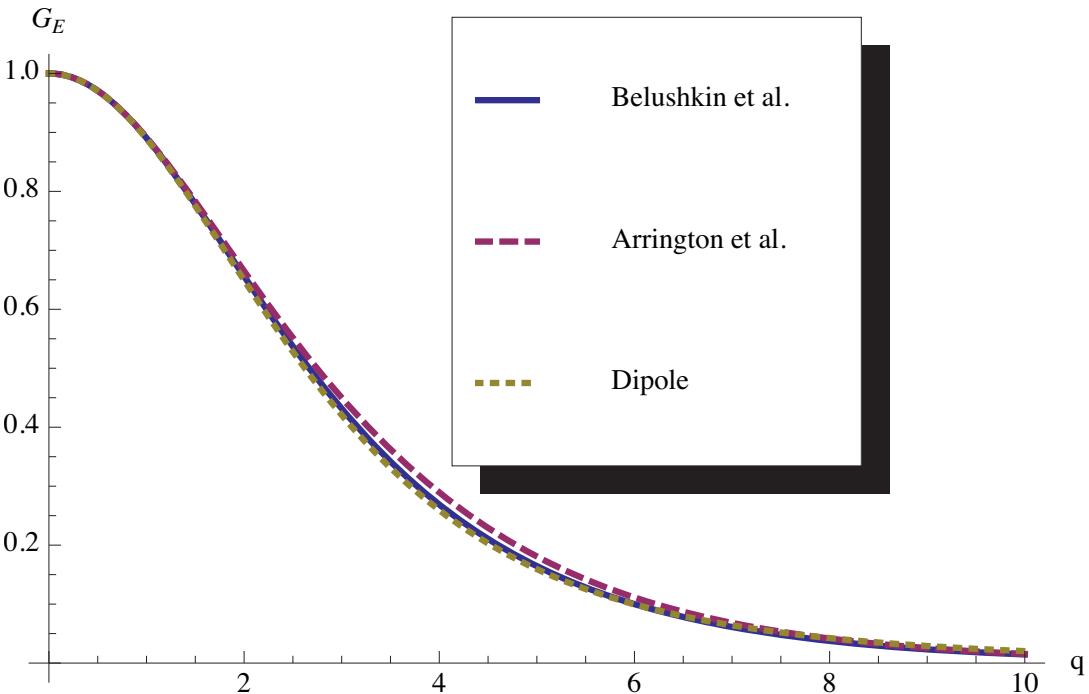


[1] QED is not endangered by the proton's size, A. De Rújula. Physics Letters B 693, 555-558 (2010).

Comparison of charge densities

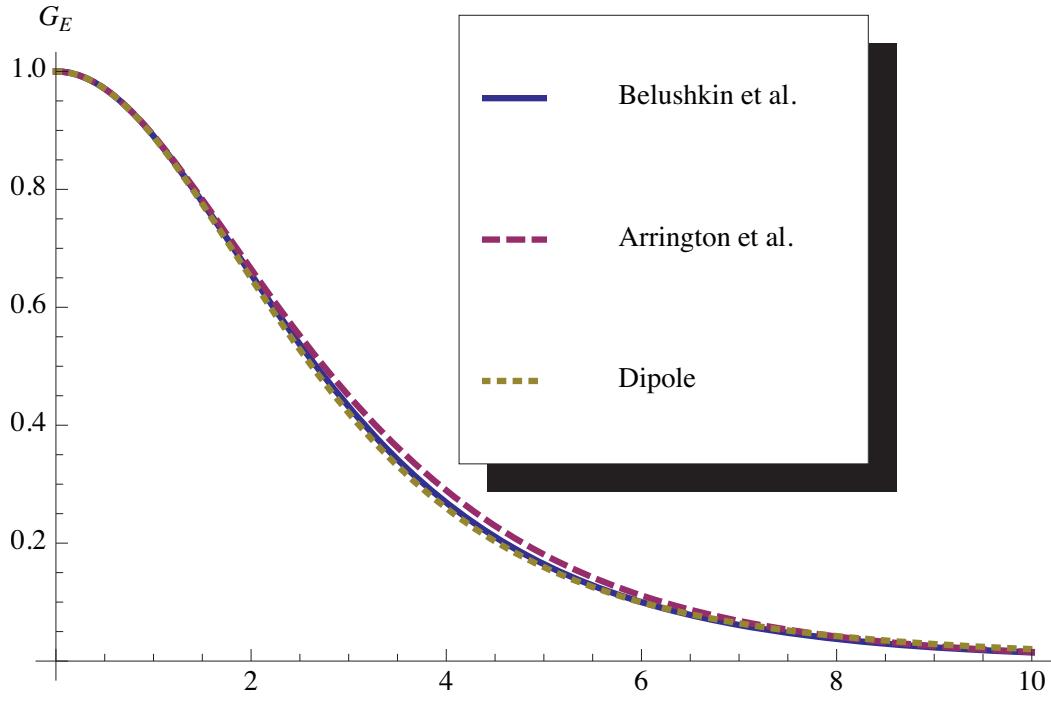
- [1] QED is not endangered by the proton's size, A. De Rújula. Physics Letters B 693, 555-558 (2010).

Comparison of charge densities



[1] QED is not endangered by the proton's size, A. De Rújula. Physics Letters B 693, 555-558 (2010).

Comparison of charge densities



- $\langle r^3 \rangle_{(2)} = 3.789 \langle r^2 \rangle^{3/2}$ Dipole
- $\langle r^3 \rangle_{(2)} = 1.960 \langle r^2 \rangle^{3/2}$ Gauss
- $\langle r^3 \rangle_{(2)} = 3.983 \langle r^2 \rangle^{3/2}$ Arrington et al.

- $\langle r^3 \rangle_{(2)} = 36.6 \pm 7.3 = 43 \langle r^2 \rangle^{3/2}$ De Rújula ?!

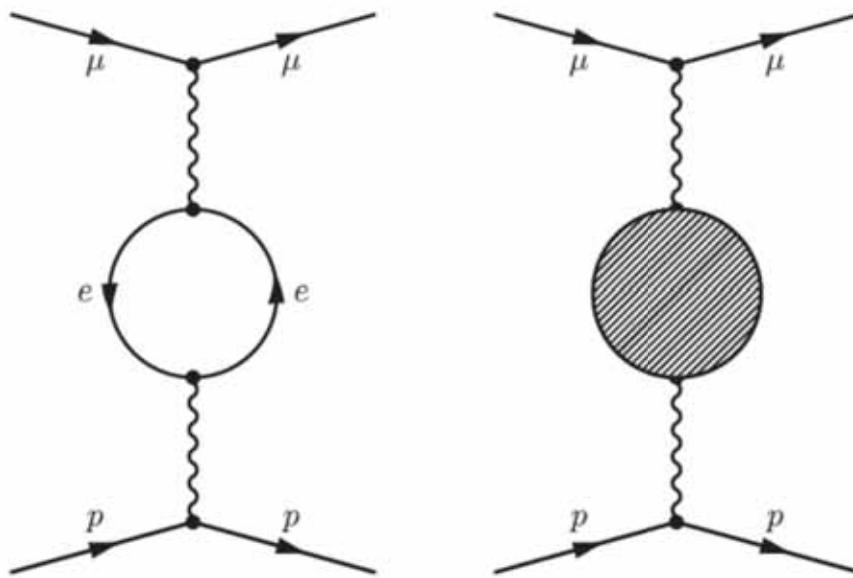
[1] QED is not endangered by the proton's size, A. De Rújula. Physics Letters B 693, 555-558 (2010).

- Extensions de type Stuekelberg du Modèle Standard, compatible avec les données du LEP [1]
- Charge $\varepsilon_e \ll 1$, candidats pour la matière noire
- Si dans la gamme de masse $m_e \leq m_M \leq m_\mu$ (sensibilité maximale) on prend la particule milichargée (la charge s'élimine) qui assure l'accord hydrogène-hydrogène muonique, alors on a un $g-2$ dans le muon qui est à 85 écarts standards de l'expérience [2]
- Si on prend $\varepsilon=0.0179$ et $m_M=0.221 m_e$ on a un bon accord, mais dans ce cas la constante de structure fine

[1] D. Feldman, Z. Liu, and P. Nath, Physical Review D **75**, 115001 (2007).

[2] U. D. Jentschura, Annals of Physics **326**, 516 (2011).

[3] J. Jaeckel and S. Roy, Physical Review D **82**, 125020 (2010).



$$\delta\rho(t) = \frac{1}{30} \Theta(t - 20m_e^2) \Theta(24m_e^2 - t).$$

- $m_e \leq m_\nu \leq m_\mu$ avec le choix précédent de ρ , on réduit à 2.5σ la différence sur l'hydrogène muonique et sur le moment magnétique anormal du muon
- Le moment magnétique anormal de l'électron ne change pas
- Mais le rayon du proton dans l'hydrogène bouge à 0.904

U. D. Jentschura, Annals of Physics **326**, 516 (2011).

- Tests de la loi de Coulomb à courtes distances pour des «photon cachés» supplémentaires de masse $m_{\gamma'}$
- Tests ~ke muonique

$$V(r) = -\frac{Z\alpha}{r}(1 + e^{-m_{\gamma'} r} \chi^2),$$

$$\Delta r_p^2 = -6\chi^2 \frac{a_o^4 m_{\gamma'}^2}{(1 + a_o m_{\gamma'})^4},$$

- Mais l'effet croît avec a_0 et donc est plus grand pour H que