QED IS endangered by the proton's charge rms radius
QED is NOT endangered by the proton's running radii.
The leading proton finite size contribution

\[ m_r \equiv \frac{m_\ell \, m_p}{m_\ell + m_p} \]

\[ r_p^2 \equiv \int d^3r \, \rho(r) r^2 \]

\[ F(q^2) = \int d^3r \, \rho(r) e^{-iq \cdot r} \simeq Z (1 - \frac{q^2}{6} r_p^2 + \cdots) \]

\[ \Delta V(r) = V(r) - \left( -\frac{Z \alpha}{r} \right) \]

\[ \Delta V(q) = \frac{4\pi Z \alpha}{q^2} (1 - F(q)) \simeq \frac{2\pi (Z \alpha)}{3} r_p^2 \]

\[ \Delta E_{FS} = \frac{2\pi (Z \alpha)}{3} r_p^2 |\Psi_n(0)|^2 \]

[Not Quite Right]

[\[ m_r \equiv \quad \frac{m_\ell \, m_p}{m_\ell + m_p} \quad \text{rms: root mean square} \]

Saturday, 19 March, 2011

A. Antognini, CERN 10.08.2010 – p.5
Not Defensible

Not Quite There
Not Defensible

Not Quite There

Pretty amazing

CODATA-06

e-p scattering

H₂O calib.
# Contributions to the $\mu p$ Lamb shift

<table>
<thead>
<tr>
<th>#</th>
<th>Contribution</th>
<th>Value</th>
<th>Unc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Relativistic one loop VP</td>
<td>205.0282</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>NR two-loop electron VP</td>
<td>1.5081</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Polarization insertion in two Coulomb lines</td>
<td>0.1509</td>
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</tr>
<tr>
<td>6</td>
<td>NR three-loop electron VP</td>
<td>0.00529</td>
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</tr>
<tr>
<td>7</td>
<td>Polarisation insertion in two and three Coulomb lines (corrected)</td>
<td>0.00223</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Three-loop VP (total, uncorrected)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Wichmann-Kroll</td>
<td>-0.00103</td>
<td>0.00135</td>
</tr>
<tr>
<td>10</td>
<td>Light by light electron loop ((Virtual Delbrück)</td>
<td>0.00135</td>
<td>0.00135</td>
</tr>
<tr>
<td>11</td>
<td>Radiative photon and electron polarization in the Coulomb line $\alpha^2(Z\alpha)^4$</td>
<td>-0.00500</td>
<td>0.0010</td>
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<tr>
<td>12</td>
<td>Electron loop in the radiative photon of order $\alpha^2(Z\alpha)^4$</td>
<td>-0.00150</td>
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<tr>
<td>13</td>
<td>Mixed electron and muon loops</td>
<td>0.00007</td>
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<tr>
<td>14</td>
<td>Hadronic polarization $\alpha(Z\alpha)^4 m_T$</td>
<td>0.01077</td>
<td>0.00038</td>
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<tr>
<td>15</td>
<td>Hadronic polarization $\alpha(Z\alpha)^5 m_T$</td>
<td>0.000047</td>
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<tr>
<td>16</td>
<td>Hadronic polarization in the radiative photon $\alpha^2(Z\alpha)^4 m_T$</td>
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<tr>
<td>17</td>
<td>Recoil contribution</td>
<td>0.05750</td>
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</tr>
<tr>
<td>18</td>
<td>Recoil finite size</td>
<td>0.01300</td>
<td>0.001</td>
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<tr>
<td>19</td>
<td>Recoil correction to VP</td>
<td>-0.00410</td>
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<tr>
<td>20</td>
<td>Radiative corrections of order $\alpha^n (Z\alpha)^k m_T$</td>
<td>-0.66770</td>
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<tr>
<td>21</td>
<td>Muon Lamb shift 4th order</td>
<td>-0.00169</td>
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<tr>
<td>22</td>
<td>Recoil corrections of order $\alpha(Z\alpha)^5 \frac{m_T}{M} m_T$</td>
<td>-0.04497</td>
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<tr>
<td>23</td>
<td>Recoil of order $\alpha^6$</td>
<td>0.00030</td>
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<td>24</td>
<td>Radiative recoil corrections of order $\alpha(Z\alpha)^5 \frac{m_T}{M} m_T$</td>
<td>-0.00960</td>
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<tr>
<td>25</td>
<td>Nuclear structure correction of order $(Z\alpha)^5$ (Proton polarizability)</td>
<td>0.015</td>
<td>0.004</td>
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<td>26</td>
<td>Polarization operator induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_T$</td>
<td>0.00019</td>
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<td>27</td>
<td>Radiative photon induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_T$</td>
<td>-0.00001</td>
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</tbody>
</table>

Sum: 206.0573, 0.0045
Lamb shift prediction

radius dependent contributions

<table>
<thead>
<tr>
<th>Contribution</th>
<th>Value</th>
<th>(&lt; r_p^2 &gt;)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leading nuclear size contribution</td>
<td>-5.19745</td>
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<tr>
<td>Radiative corrections to nuclear finite size effect</td>
<td>-0.0275</td>
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<tr>
<td>Nuclear size correction of order ((Z\alpha)^6)</td>
<td>-0.001243</td>
<td></td>
</tr>
<tr>
<td>Total (&lt; r_p^2 &gt;) contribution</td>
<td>-5.22619</td>
<td></td>
</tr>
<tr>
<td>Nuclear size correction of order ((Z\alpha)^5)</td>
<td>0.0347</td>
<td>(&lt; r_p^3 &gt;)</td>
</tr>
<tr>
<td>Nuclear size correction of order ((Z\alpha)^6)</td>
<td>-0.000043</td>
<td>(&lt; r_p^4 &gt;)^2</td>
</tr>
</tbody>
</table>

A1 collaboration at MAMI, Mainz has started the reevaluation of the various proton moments:
\(< r_p^2 >, R_{\text{Zemach}}, < r_p^4 > \ldots \)

New evaluations of structure leads to a shift < 10% of the measured discrepancy.

\[ E(2S_{1/2}^F = 1 - 2P_{3/2}^F = 2) = 209.9779(49) - 5.2262 \ r_p^2 + 0.0347 \ r_p^3 \ \text{meV} \] (HFS+FS included)
Ladies & Gentlemen
Ladies & Gentlemen

THE
THIRD
ZEMACH
MOMENT
\[ L^{th} \left[ \langle r_p^2 \rangle, \langle r_p^3 \rangle(2) \right] = 209.9779(49) - 5.2262 \langle r_p^2 \rangle + 0.00913 \langle r_p^3 \rangle(2) \] (in meV, with r in fermi)

\[ \Delta E_3(n) = \frac{\alpha^5}{3 n^3} m_r^4 \delta l_0 \langle r_p^3 \rangle(2) \]

\[ \langle r_p^3 \rangle(2) \equiv \int d^3r_1 d^3r_2 \rho(r_1)\rho(r_2) |r_1 - r_2|^3 \]

For a dipole form factor

\[ \left[ \langle r_p^3 \rangle(2) \right]^2 = \left( \frac{3675}{256} \right) \left[ \langle r^2 \rangle \right]^3 \quad \text{with} \quad r_p \equiv \sqrt{\langle r_p^2 \rangle} \]

\[ L_{th}(r_p) = 209.9779(49) - 5.22619 r_p^2 + 0.0347 r_p^3 \]
It is intrepid to use a model of the proton

\[ G_E(Q^2) = \frac{1}{\left[1 + \frac{|Q^2|}{0.71 \text{ GeV}^2}\right]^2} \]

... to challenge QED
THE
RUNNING
RMS P-RADIUS

\langle r^2 \rangle_p
\[ \frac{2\pi\alpha}{3} \langle r_p^2 \rangle |\Psi_{2,0}(0)|^2 \]

The slope of FF at \( q^2 = 0 \)

\[ \int_0^\infty e^{i\vec{q}\cdot\vec{r}} d^3q \quad (\text{all } q) \]

Rephrase “atomic” result:

\[ \langle r_p^2 \rangle |(0,\infty) \simeq \frac{\langle r_p^2 \rangle |(\alpha m_r,m)\rangle}{1 - 4\alpha m_r \sqrt{\langle r_p^2 \rangle |(\alpha m_r,m)\rangle}/6} \]

\( q = \mathcal{O}(\alpha m_r) \) up to \( q \sim m/4 \)

Momentum range is which the proton is “probed” in the atom

Correction is form-factor dependent

Saturday, 19 March, 2011
Extracting

\[ \langle r^2 \rangle \]

from electron-proton scattering data
5-parameter continued-fraction fit
\[ f(z, n) = \frac{2^{-n/2} e^{-z/2} \left( \frac{n}{2} \right) - 1}{\Gamma \left[ \frac{n}{2} \right]} \]

\[ p(\chi^2, n_{\text{dof}}) = \int_{\chi^2}^{\infty} f(z, n_{\text{dof}}) \, dz = \frac{\Gamma(n_{\text{dof}}/2, \chi^2/2)}{\Gamma(n_{\text{dof}}/2)} \]

The data are tickled to agree with H

H and the data do not agree

Saturday, 19 March, 2011
\[ f(z, n) = \frac{2^{-n/2} e^{-z/2} z^{\frac{n}{2} - 1}}{\Gamma \left[ \frac{n}{2} \right]} \]

\[ p(\chi^2, n_{\text{dof}}) = \int_{\chi^2}^{\infty} f(z, n_{\text{dof}}) \, dz = \frac{\Gamma(n_{\text{dof}}/2, \chi^2/2)}{\Gamma(n_{\text{dof}}/2)} \]

The data are tickled to agree with H

H and the data do not agree
\( n = 310 \)

\[ p(512, 310) \approx 3.92 \times 10^{-12} \]
what to say ??????
what to say ??????

% & !! §° ≠ ≠ ℓ

¢™¢¶∞§**

!!!!!!!!!!!!!!!!!!!!!!
Stephens, Atkins, and Kingston of Keele University, for confirming the widely held belief that swearing relieves pain.

\[ q = 1 \text{ f}^{-1} \quad q = 2 \text{ f}^{-1} \]

\[ p(\text{most flexible}) = 1.9 \times 10^{-4} \]
\[ \sqrt{\langle r_p^2 \rangle} \text{(polynom)} = 0.883(5)_{\text{stat}}(5)_{\text{syst}}(3)_{\text{model}} \text{ fm} \]

\[ \sqrt{\langle r_p^2 \rangle} \text{(spline)} = 0.875(5)_{\text{stat}}(4)_{\text{syst}}(2)_{\text{model}} \text{ fm} \]

THEOREM
Proton form factors at low $Q^2$

Parton-Distribution Functions at low $x$

Study form-factor with the same neural-network techniques only instances ??
Extracting
\[
\langle r^3_{\nu p} \rangle (2)
\]
from (the same) electron-proton scattering data


Pluchino, Rapisarda, and Garofalo of the Univ. of Catania, for demonstrating mathematically that organizations would become more efficient if they promoted people at random.

BASTA