TRANSITION RADIATION BY STANDARD MODEL NEUTRINOS AT AN INTERFACE

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We discuss the transition radiation process $\nu \to \nu\gamma$ at an interface of two media. The medium fulfills the dual purpose of inducing an effective neutrino-photon vertex and of modifying the photon dispersion relation. The transition radiation occurs when at least one of those quantities have different values in different media. We present a result for the probability of the transition radiation which is both accurate and analytic. For $E_\nu = 1$MeV neutrino crossing polyethylene-vacuum interface the transition radiation probability is about 10^{-39} and the energy intensity (deposition) is about 10^{-34}eV. At the surface of the neutron stars the transition radiation probability may be $\sim 10^{-20}$. Our result on three orders of magnitude is larger than the results of previous calculations.

1 Introduction

In many astrophysical environments the absorption, emission, or scattering of neutrinos occurs in media, in the presence of magnetic fields [1] or at the interface of two media.

In the presence of a media, neutrinos acquire an effective coupling to photons by virtue of intermediate charged particles. The violation of the translational invariance at the direction from one media into another leads to the non conservation of the momentum at the same direction so that transition radiation becomes kinematically allowed.

The theory of the transition radiation by charged particle has been developed in [2 3]. It those articles authors used classical theory of electrodynamics. In [4] the quantum field theory was used for describing the phenomenon. The neutrinos have very tiny masses. Therefore one has to use the quantum field theory approach in order to study transition radiation by neutrinos.

The presence of a magnetic field induces an effective ν-γ-coupling. The Cherenkov decay in a magnetic field was calculated in [5].

At the interface of two media with different refractive indices the transition radiation $\nu \to \nu\gamma$ was studied in [6] with an assumption of existence of large (neutrino) magnetic dipole moment.

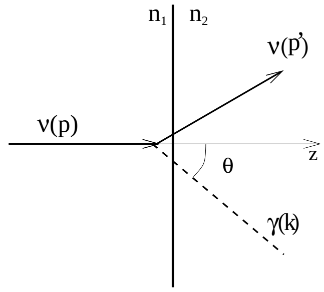

Figure 1: Transition radiation by neutrino at an interface of two media with refractive indexes n_1 and n_2.

We presently extend previous studies of the transition radiation to neutrinos with only standard-model couplings. The media changes the photon dispersion relation. In addition, the media causes an effective ν-γ-vertex by standard-model neutrino couplings to the background electrons. We neglect neutrino masses and medium-induced modifications of their dispersion relation due to their negligible role. Therefore, we study the transition radiation entirely within the particle-physics standard model.

A detailed literature search reveals that neutrino transition radiation has been studied earlier in [7]. They used vacuum induced ν-γ vertex ("neutrino toroid dipole moment") for the $\nu \to \nu\gamma$ matrix element. We do not agree with their treatment of the process. The media itself induces ν-γ vertex. The vacuum induced vertex can be treated as a radiation correction to the medium induced one. We found that the result of [7] for the transition radiation rate is more than three orders of magnitude, $(\frac{8\alpha}{\pi})^2$, smaller than our result.

2 Transition Radiation

Let us consider a neutrino crossing the interface of two media with refraction indices n_1 and n_2 (see Fig. 1). In terms of the matrix element $\mathcal{M}$ the transition radiation probability of the process $\nu \to \nu\gamma$ is

$$W = \frac{1}{(2\pi)^3}\frac{1}{2E\beta_z}\int\frac{d^3\mathbf{p}'}{2E'}\frac{d^3\mathbf{k}}{2\omega}\sum_{pols}\left|\int_{-\infty}^{\infty}dz e^{i(p_z-p_z'-k_z)z}\mathcal{M}\right|^2 \delta(E-E'-\omega)\delta(p_x'+k_x)\delta(p_y'+k_y)\ . \quad (1)$$

Here, $p=(E,\mathbf{p})$, $p'=(E',\mathbf{p}')$, and $k=(\omega,\mathbf{k})$ are the four momenta of the incoming neutrino, outgoing neutrino, and photon, respectively and $\beta_z = p_z/E$. The sum is over photon polarizations.

We shall neglect the neutrino masses and the deformation of its dispersion relations due to the forward scattering. Thus we assume that the neutrino dispersion relation is precisely light-like so that $p^2=0$ and $E=|\mathbf{p}|$.

The formation zone length of the medium is

$$|p_z - p_z' - k_z|^{-1}. \quad (2)$$

The integral over z in eq. (1) oscillates beyond the length of the formation zone. Therefore the contributions to the process from the depths over the formation zone length may be neglected.

The z momentum $(p_z - p'_z - k_z)$ transfers to the media from the neutrino. Since photons propagation in the media suffers from the attenuation(absorption) the formation zone length must be limited by the attenuation length of the photons in the media when the later is shorter than the formation zone length.

After integration of (1) over $\mathbf{p}'$ and z we find

$$W = \frac{1}{(2\pi)^3}\frac{1}{8E\beta_z}\int \frac{|\mathbf{k}|^2 d|\mathbf{k}|}{\omega E'\beta'_z}\ \sin\theta\ d\theta\ d\varphi \sum_{pols}\left|\frac{\mathcal{M}^{(1)}}{p_z - p_z'^{(1)} - k_z^{(1)}} - \frac{\mathcal{M}^{(2)}}{p_z - p_z'^{(2)} - k_z^{(2)}}\right|^2, \quad (3)$$

where $\beta'_z = p'_z/E'$, θ is the angle between the emitted photon and incoming neutrino. $\mathcal{M}^{(1,2)}$ are matrix elements of the $\nu \to \nu\gamma$ in each media. $k_z^{(i)}$ and $p_z'^{(i)}$ are z components of momenta of the photon and of the outgoing neutrino in each media.

As it will be shown below main contribution to the process comes from large formation zone lengths and ,thus, small angle θ. Therefore the rate of the process does not depend on the angle between the momenta of the incoming neutrino and the boundary surface of two media (if that angle is not close to zero). The integration over φ drops out and we may replace $d\varphi \to 2\pi$. $k_z^{(i)}$ and $p_z'^{(i)}$ have the forms

$$k_z^{(i)} = n^{(i)}\omega\cos\theta, \;\; p_z'^{(i)} = \sqrt{(E-\omega)^2 - {n^{(i)}}^2\omega^2\sin^2\theta}\ , \quad (4)$$

here we have used $n^{(1,2)} = |\mathbf{k}|^{(1,2)}/\omega$.

If the medium is isotropic and homogeneous the polarization tensor, $\pi^{\mu\nu}$, is uniquely characterized by a pair of two polarization functions which are often chosen to be the longitudinal and transverse polarization functions. They can be projected from the full polarization matrix. In this paper we are interested in transverse photons, since they may propagate in the vacuum as well. The transverse polarization function is

$$\pi_t = \frac{1}{2}T_{\mu\nu}\pi^{\mu\nu}, \;\; T^{\mu\nu} = -g^{\mu i}(\delta_{ij} - \frac{\mathbf{k}_i\mathbf{k}_j}{\mathbf{k}^2})g^{j\nu}. \quad (5)$$

The dispersion relation for the photon in the media is the location of its pole in the effective propagator (which is gauge independent)

$$\frac{1}{\omega^2 - \mathbf{k}^2 - \pi_t} \quad (6)$$

3 Neutrino-photon vertex

In a media, photons couple to neutrinos via interactions to electrons by the amplitudes shown in Fig 2. One may take into account similar graphs with nuclei as well, but their contribution are usually negligible. When photon energy is below weak scale $(E \ll M_W)$ one may use four-fermion interactions and the matrix element for the ν-γ vertex can be written in the form

$$\begin{aligned} M &= -\frac{G_F}{\sqrt{2}e}Z\epsilon_\mu\bar{u}(p')\gamma_\nu(1-\gamma_5)u(p)\ (g_V\pi_t^{\mu\nu} - g_A\pi_5^{\mu\nu}) & (7)\\ &= \frac{G_F}{\sqrt{2}e}Z\epsilon_\mu\bar{u}(p')\gamma_\nu(1-\gamma_5)u(p)\ g^{\mu i}\left(g_V\pi_t(\delta_{ij} - \frac{\mathbf{k}_i\mathbf{k}_j}{\mathbf{k}^2}) - ig_A\pi_5\epsilon_{ijl}\frac{\mathbf{k}_l}{|\mathbf{k}|}\right)g^{j\nu}, & (8)\end{aligned}$$

here

$$g_V = \left\{ \begin{array}{l} 2\sin^2\theta_W + \frac{1}{2} \text{ for } \nu_e \\ 2\sin^2\theta_W - \frac{1}{2} \text{ for } \nu_\mu, \nu_\tau \end{array}\right. , \quad g_A = \left\{ \begin{array}{l} +\frac{1}{2} \text{ for } \nu_e \\ -\frac{1}{2} \text{ for } \nu_\mu, \nu_\tau \end{array}\right. . \quad (9)$$

$\pi_t^{\mu\nu}$ is the polarization tensor for transverse photons, while $\pi_5^{\mu\nu}$ is the axialvecor-vector tensor [8].

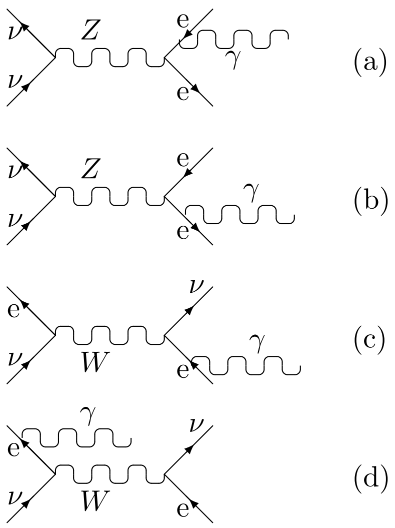

Figure 2: Neutrino-photon coupling in electrons background. (a,b) Z-γ-mixing. (c,d) "Penguin" diagrams (only for ν_e).

4 Transition radiation probability

Armed with these results we may now turn to an evaluation of the $\nu \to \nu\gamma$ rate at the interface of the media and the vacuum. We find that the transition probability is

$$W = \frac{G_F^2}{16\pi^3\alpha}\int \frac{\omega\ d\omega\ \sin\theta d\theta}{(E - p'_z - n\omega\cos\theta)^2} \tag{10}$$

$$\times\Big[(g_V^2\pi_t^2 + g_A^2\pi_5^2)(1 - \frac{\cos^2\theta}{E-\omega}(p'_z - n\omega\frac{\sin^2\theta}{\cos\theta})) - 2g_V g_A \pi_t\pi_5\cos\theta(1 - \frac{p'_z - n\omega\frac{\sin^2\theta}{\cos\theta}}{E-\omega})\Big], \tag{11}$$

here

$$p'_z = \sqrt{(E-\omega)^2 - n^2\omega^2\sin^2\theta}\ . \tag{12}$$

Now we expand underintegral expressions on small angle, since only in that case the denominator is small (and the formation zone length is large). Thus we write the transition probability in the form

$$W \simeq \frac{G_F^2}{16\pi^3\alpha}\int\frac{d\omega\ \theta^2 d\theta^2}{\omega\ [\ \theta^2 + (1-n^2)(1-\frac{\omega}{E})\ ]^2}\Big[(g_V^2\pi_t^2 + g_A^2\pi_5^2)(2 - 2\frac{\omega}{E} + \frac{\omega^2}{E^2}) - 2g_V g_A\pi_t\pi_5\frac{\omega}{E}(2 - \frac{\omega}{E})\Big]\ . \tag{13}$$

Eq.(13) tells us that the radiation is forward peaked within an angle of order $\theta \sim \sqrt{1-n^2}$.

After integration over angle θ we get

$$W \simeq \frac{G_F^2}{16\pi^3\alpha}\int\frac{d\omega}{\omega}\Big[-\ln[(1-n^2)(1-\frac{\omega}{E})] + \ln[(1-n^2)(1-\frac{\omega}{E}) + \theta_{max}^2] - 1\Big] \tag{14}$$

$$\times\Big[(g_V^2\pi_t^2 + g_A^2\pi_5^2)(2 - 2\frac{\omega}{E} + \frac{\omega^2}{E^2}) - 2g_V g_A\pi_t\pi_5\frac{\omega}{E}(2 - \frac{\omega}{E})\Big]. \tag{15}$$

Numerically eq. (15) does not depend much on θ_{max}.

Usually the axialvector polarization function is much less than the vector one. For instance in nonrelativistic and nondegenerate plasma these functions are [8]

$$\pi_t = \omega_p^2 \ \ \text{and} \ \ \pi_5 = \frac{|\mathbf{k}|}{2m_e}\frac{\omega_p^4}{\omega^2}, \tag{16}$$

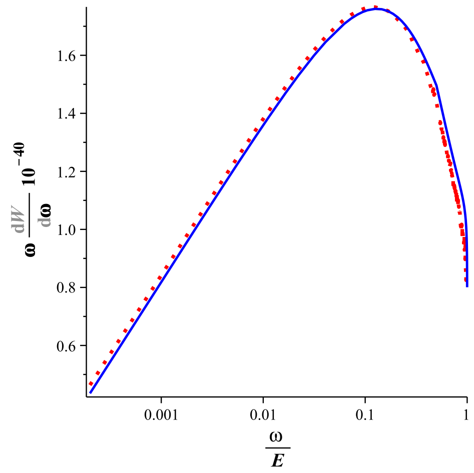

Figure 3: Energy spectrum of the transition radiation by electron neutrinos at an interface of media with plasma frequency $\omega_p = 20$eV and vacuum. The energy of the incoming neutrino is $E = 1$ MeV. The dot line is numerical and the solid line is semi-analytical integration over angle between photon and incoming neutrino momenta.

where $\omega_p^2 = \frac{4\pi\alpha N_e}{m_e}$ is the plasma frequency. Therefore we may ignore the term proportional to π_5.

Since we are interested in the forward radiation in the gamma ray region, we assume that the index of refraction of the photon is

$$n^2 = 1 - \frac{\omega_p^2}{\omega^2} \tag{17}$$

and the photons from the medium to the vacuum propagate without any reflection or/and refraction.

In Fig.3 we plot the energy spectrum of the photons from the transition radiation by electron neutrinos with energy $E = 1$MeV.

After integration over photon energy we find the neutrino transition radiation probability as

$$W = \int_{\omega_{min}}^{E} dW \simeq \frac{g_V^2 G_F^2 \omega_p^4}{16\pi^3\alpha} \left(2\ \ln^2 \frac{E}{\omega_p} - 5 \ln \frac{E}{\omega_p} + \delta\right) \tag{18}$$

here $\delta \simeq 5$ for $\omega_{min} = \omega_p$, $\delta \simeq -1$ for $\omega_{min} = 10\omega_p$.

The energy deposition of the neutrino in the media due to the transition radiation

$$\int_{\omega_p}^{E} \omega\ dW_{\nu\to\nu\gamma} \simeq \frac{g_V^2 G_F^2 \omega_p^4}{16\pi^3\alpha} E\Big[\frac{8}{3} \ln \frac{E}{\omega_p} - 4.9 + 9\frac{\omega_p}{E} + O(\frac{\omega_p^2}{E^2})\Big] \tag{19}$$

The eqs.(15),(18) and (19) are main results.

For MeV electronic neutrinos the transition radiation probability is about $W \sim 10^{-39}$ and the energy deposit is about $1.4 \cdot 10^{-34}$ eV when they cross the interface of the media with $\omega_p = 20$ eV to vacuum.

Unfortunately the transition radiation probability is extremely small and cannot be observed at the Earth.

On the other hand at the surface of the neutron stars electron layer may exist with density $\sim m_e^3$, due to the fact that the electrons not being bound by the strong interactions are displaced to the outside of the neutron star [9]. Therefore MeV energy neutrinos emited by the neutron stars during its cooling processes will have transition radiation with the probability of $\sim 10^{-20}$ and energy spectrum given in Eq. (15).

5 Summary and Conclusion

We have calculated the neutrino transition radiation at the interface of two media. The charged particles of the media provide an effective ν-γ vertex, and they modify the photon dispersion relation. We got analytical expressions for the energy spectrum of the transition radiation, its probability and the energy deposition at the process. The radiation is forward peaked within an angle of order $\frac{\omega_p}{\omega}$. The photons energy spectrum is falling almost linearly over photon energy.

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