Transition Radiation by Neutrinos at an interface

Ara N. Ioannisian

YerPhI, Armenia

Rencontres de Moriond,
EW section, March 2011

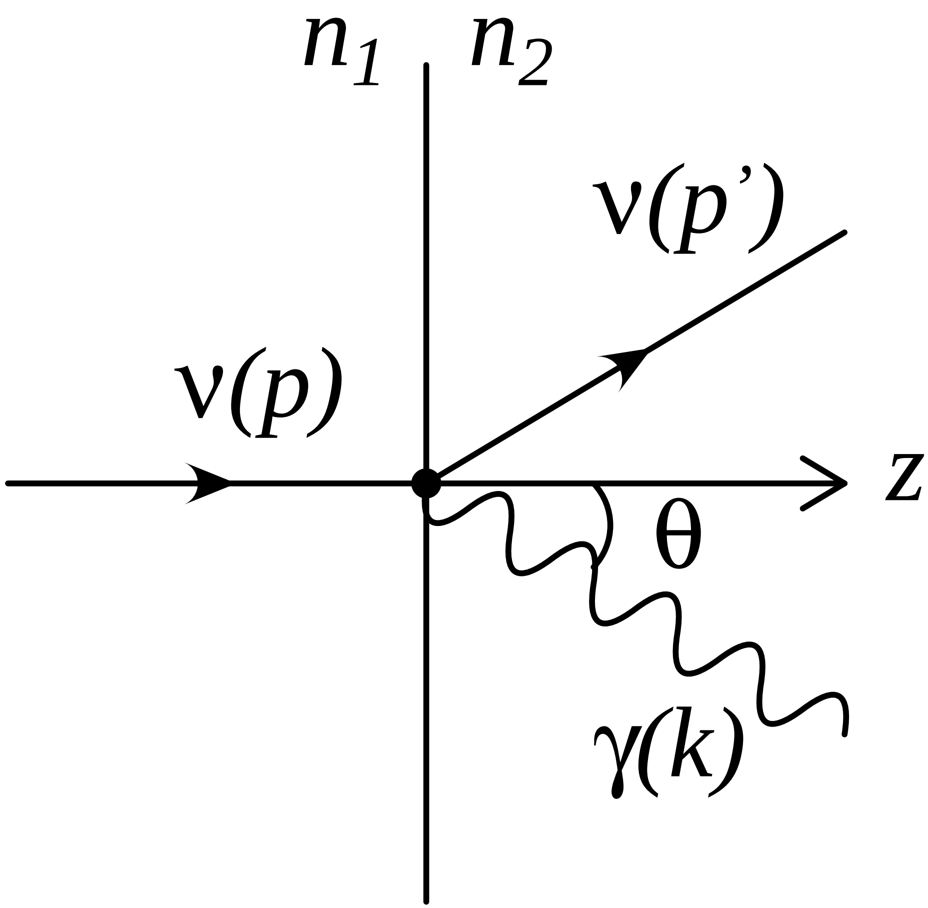

Figure 1: Transition radiation by neutrino at an interface of two media with refracive indexes n_1 and n_2.

The conservation of momentum is a consequence of translational invariance.

$p \neq p'_z + k_z$ Kinematically allowed.

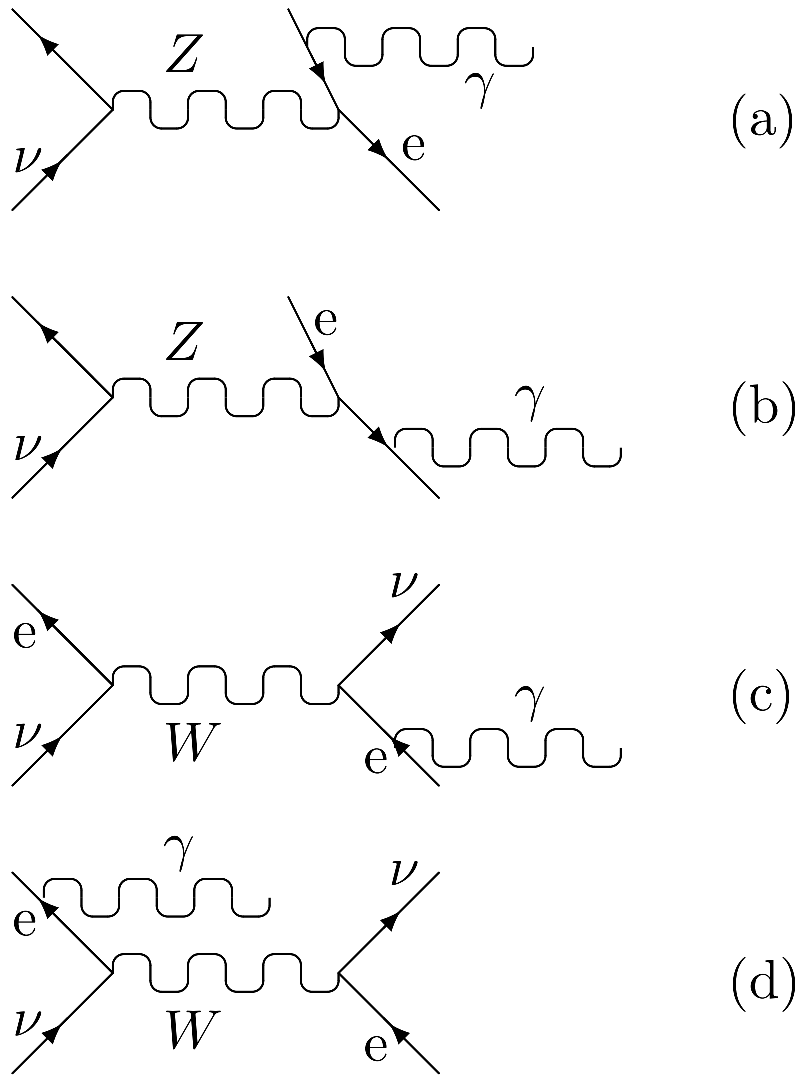

Figure 2: Neutrino-photon coupling in electrons background. (a,b) Z-γ-mixing. (c,d) "Penguin" diagram (only for ν_e).

Grigoriy Garibyan -armenian physicist, discoverer and developer of the theory of X-ray transition radiation. He used both - classical electrodynamics and quantum electrodynamics approaches to describe the phenomena.

$m_\nu \ll E_\nu, \omega, \omega_p$

Let us consider a neutrino crossing the interface between two media with refraction indices n_1 and n_2 (see fig. 1). In terms of the matrix element $\mathcal{M}$ the transition radiation probability of the process $\nu \to \nu\gamma$ is

$$W_{\nu\to\nu\gamma} = \frac{1}{(2\pi)^3}\frac{1}{2E\beta_z}\int \frac{d^3\mathbf{p}'}{2E'}\frac{d^3\mathbf{k}}{2\omega} \cdot$$
$$\delta(E - E' - \omega)\delta(p'_x + k_x)\delta(p'_y + k_y)\sum_{pols}\left|\int_{-\infty}^{\infty} dz e^{i(p_z - p'_z - k_z)z}\mathcal{M}_{\nu\to\nu\gamma}\right|^2$$

$p = (E, \mathbf{p})$, $p' = (E', \mathbf{p}')$, $k = (\omega, \mathbf{k})$, $\beta_z = p_z/E$. The sum is over photon polarizations.

We assume $p^2 = 0$ and $E = |\mathbf{p}|$. After integration over $d\mathbf{p}'$ and z

$$W_{\nu\to\nu\gamma} = \frac{1}{(2\pi)^3}\frac{1}{8E\beta_z}\int\frac{|\mathbf{k}|^2 d|\mathbf{k}|}{\omega E'\beta'_z}\ \sin\theta\ d\theta\ d\varphi$$
$$\sum_{pols}\left|\frac{\mathcal{M}^{(1)}_{\nu\to\nu\gamma}}{p_z - p_z'^{(1)} - k_z^{(1)}} - \frac{\mathcal{M}^{(2)}_{\nu\to\nu\gamma}}{p_z - p_z'^{(2)} - k_z^{(2)}}\right|^2$$

$\beta'_z = p'_z/E'$, θ is the angle between the emitted photon and incoming neutrino. $\mathcal{M}^{(1,2)}_{\nu\to\nu\gamma}$ are matrix elements in each media.

$$p_z'^{(i)} = \sqrt{(E-\omega)^2 - {n^{(i)}}^2\omega^2 + {k_z^{(i)}}^2}\ .$$

Formation zone length in each media

$$l^{(i)} = (p_z - p_z'^{(i)} - k^{(i)})^{-1}$$

$$max(l^{(i)}) = |\omega(1-n)|^{-1} \approx \frac{2\omega}{\omega_p^2} \approx 1\ cm\ \frac{\omega}{MeV}(\frac{20\ eV}{\omega_p})^2$$

In a media the photon refractive index is $n = \sqrt{\epsilon\mu}$.

$$\pi^{\mu\nu} = 2e^2 \int \frac{d^3\mathbf{q}}{\varepsilon(2\pi)^3}(f(\varepsilon) + \bar{f}(\varepsilon))\frac{qk(q^\mu k^\nu + q^\nu k^\mu) - k^2 q^\mu q^\nu - (qk)^2 g^{\mu\nu}}{(qk)^2 - k^4/4}$$

$$f(\varepsilon) = \frac{1}{e^{\frac{\varepsilon-\mu}{T}} + 1}, \quad \bar{f}(\varepsilon) = \frac{1}{e^{\frac{\varepsilon+\mu}{T}} + 1}, \quad \varepsilon = \sqrt{\mathbf{q}^2 + m_e^2}$$

$$\pi^{\mu\nu} = \pi_t T^{\mu\nu} + \pi_L L^{\mu\nu}$$

$$k_\mu \pi^{\mu\nu} = k_\mu \pi_5^{\mu\nu} = k_\mu T^{\mu\nu} = k_\mu L^{\mu\nu} = 0$$

$$\pi_t = \frac{1}{2} T_{\mu\nu}\pi^{\mu\nu}$$

$$\pi_t = (1 - n_t^2)\omega^2$$

$$T^{\mu\nu} = g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} - \frac{\upsilon^\mu \upsilon^\nu}{\upsilon^2} = -g^{\mu i}(\delta_{ij} - \frac{\mathbf{k}_i \mathbf{k}_j}{\mathbf{k}^2})g^{j\nu}, \quad \sum_{trans} \epsilon_\mu \epsilon_\nu = -T_{\mu\nu}$$

$$\upsilon^\mu = (g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2})\mathrm{v}^\nu, \qquad \mathrm{v} = (1, 0, 0, 0)$$

$$M_{\nu\to\nu\gamma} = -\frac{G_F}{\sqrt{2}e} Z\epsilon_\mu \bar{u}(p')\gamma_\nu(1-\gamma_5)u(p)\ (g_V\pi^{\mu\nu} - g_A\pi_5^{\mu\nu})$$

$$= \frac{G_F}{\sqrt{2}e} Z\epsilon_\mu \bar{u}(p')\gamma_\nu(1-\gamma_5)u(p)\ g^{\mu i}\left(g_V\pi_t(\delta_{ij} - \frac{\mathbf{k}_i\mathbf{k}_j}{\mathbf{k}^2}) - ig_A\pi_5\epsilon_{ijl}\frac{\mathbf{k}_l}{|\mathbf{k}|}\right)g^{j\nu}$$

$$g_V = \{\begin{array}{l} 2\sin^2\theta_W + \frac{1}{2}\ \text{for}\ \nu_e \\ 2\sin^2\theta_W - \frac{1}{2}\ \text{for}\ \nu_\mu, \nu_\tau \end{array}\ ,\quad g_A = \{\begin{array}{l} +\frac{1}{2}\ \text{for}\ \nu_e \\ -\frac{1}{2}\ \text{for}\ \nu_\mu, \nu_\tau \end{array}$$

$$\pi_5^{\mu\nu} = -i\pi_5\frac{1}{\sqrt{(k\mathrm{v})^2 - k^2}}\epsilon^{\mu\nu\lambda\sigma}k_\lambda \mathrm{v}_\sigma = -i\pi_5\epsilon_{ijl}\frac{\mathbf{k}_l}{|\mathbf{k}|}g^{\mu i}g^{\nu j}$$

$$\pi_5 = e^2\frac{k^2}{|\mathbf{k}|}\int\frac{d^3\mathbf{q}}{\varepsilon(2\pi)^3}(f(\varepsilon) - \bar{f}(\varepsilon))\frac{(qk)\omega - k^2\varepsilon}{(qk)^2 - k^4/4}$$

$$\pi_5 \simeq \frac{|\mathbf{k}|}{2m_e}\omega^2(1-n_t^2)^2 = \frac{|\mathbf{k}|}{2m_e}\frac{\omega_p^4}{\omega^2}$$

$$\pi_t = \omega_p^2\ ,\ \omega_p^2 = \frac{4\pi\alpha N_e}{m_e}. \qquad \omega_p = 20eV$$

Let us consider a neutrino crossing the interface between media with refraction indices n ($n^2 = 1 - \frac{\omega_p^2}{\omega^2}$) and vacuum.

$$W_{\nu\to\nu\gamma} = \frac{G_F^2}{16\pi^3\alpha} \int \frac{\omega \ d\omega \ \sin\theta d\theta}{(E - p'_z - n\omega\cos\theta)^2} \cdot$$

$$\left[(g_V^2\pi_t^2 + g_A^2\pi_5^2)\left(1 - \frac{\cos\theta}{E-\omega}(p'_z\cos\theta - n\omega\sin^2\theta)\right)\right.$$

$$\left. -2g_V g_A\pi_t\pi_5\left(\cos\theta - \frac{1}{E-\omega}(p'_z\cos\theta - n\omega\sin^2\theta)\right)\right]$$

$$p'_z = \sqrt{(E-\omega)^2 - n^2\omega^2\sin^2\theta}$$

$$\omega\,\frac{dW_{\nu\to\nu\gamma}}{d\omega} \simeq \frac{g_V^2 G_F^2\omega_p^4}{16\pi^3\alpha}\,(2-2\frac{\omega}{E}+\frac{\omega^2}{E^2})\,(\,\ln\frac{\omega^2}{\omega_p^2}-\ln(1-\frac{\omega}{E})-1\,)\,,\ \omega < 0.3E_\nu$$

$$W_{\nu\to\nu\gamma} \simeq \frac{g_V^2 G_F^2\omega_p^4}{16\pi^3\alpha}\,2\,\ln^2\frac{\kappa E_\nu}{\omega_p} = 1.5\ 10^{-39}\,(\frac{\omega_p}{20\ eV})^4(1+\ln[\frac{E_\nu}{MeV}\frac{20\ eV}{\omega_p}])^2$$

Summary

We calculate the transition radiation process $\nu \to \nu\gamma$ at the border of two media.

The neutrinos are taken to be with only standard-model couplings and we ignore the neutrino masses due to their negligible contribution.

The medium fulfills the dual purpose of inducing an effective neutrino-photon vertex and of modifying the photon dispersion relation.

The process is kinematikally allowed. The violation of the translational invariance at the direction from one media into another leads to the momentum non conservation at the same direction.

We find that the probability of transition radiation is larger by three orders of magnitude (medium induced neutrino-photon vertex) than previous calculations (vacuum induced vertex **PL B435 (1998) 134**).

The process may be important for the SuperNovae neutrinos.

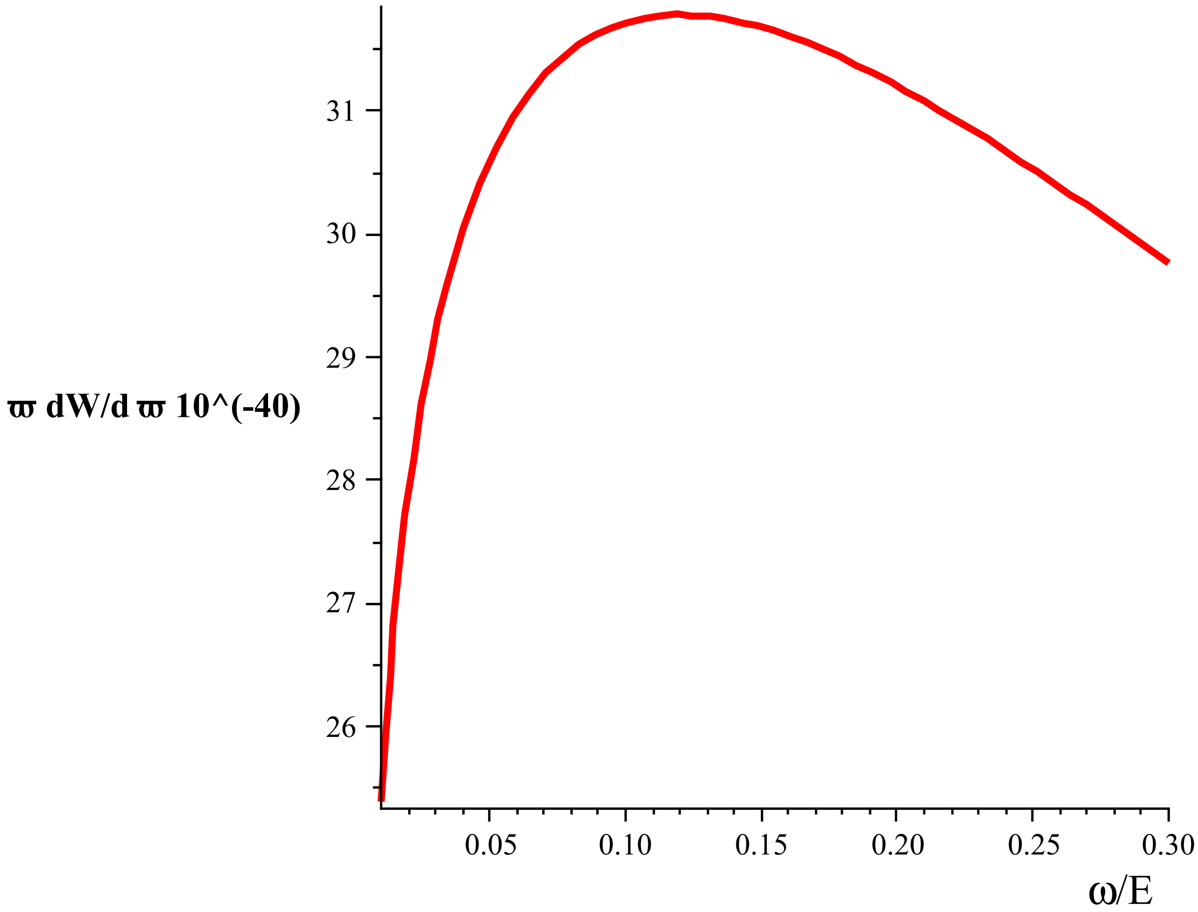

Figure 3: Energy spectrum of the NTR. $E_\nu = 1$ MeV and the plasma frequencie- $\omega_p = 20$ eV.

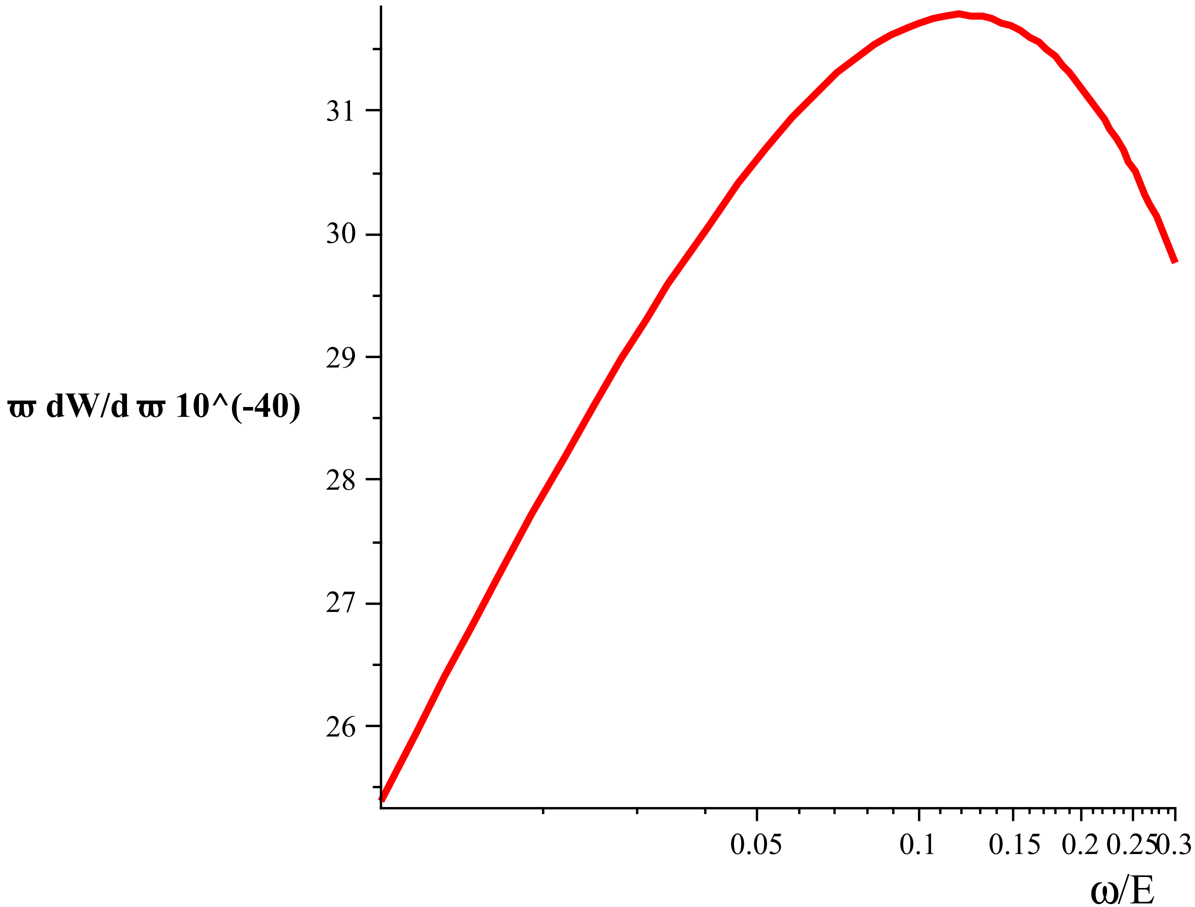

Figure 4: Energy spectrum of the NTR. $E_\nu = 1$ MeV and $\omega_p = 20$ eV. Logaritmical scale.