MAGNETIC AND MECHANICAL STUDY OF THE HORN PROTOTYPE

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OUTLINE

- Goal
  calculate magnetic flux distribution, surface currents, mechanical stress
- Model
  geometry, equations; boundary conditions
- Results
  magnetic flux ; currents; stress

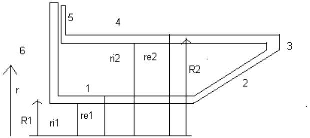


FIGURE: Horn geometry

- peak current $I_0 = 300$ kA; $T = 20$ms; $\tau = 100\mu s$
- $r_{i1} = 28$ mm; $r_{e1} = 34$ mm; $r_{i2} = 208$ mm; $r_{e2} = 214$ mm; $L_1 = 30$ cm ; $L_2 = 70$ cm

$$i_{rms}{}^2 = \frac{1}{T}\int_0^T i^2 dt = \frac{1}{T}\int_0^{\tau_0} i^2 dt = \frac{{I_0}^2}{T}\frac{\tau_0}{2} \quad (1)$$

$$i_{rms} = I_0\sqrt{\frac{\tau_0}{2T}} = 3 \times 10^5\sqrt{\frac{100 \times 10^{-6}}{2 \times 2 \times 10^{-3}}} = 15kA \quad (2)$$

At peak current $I_0 = 300$ kA the magnetic field is maximum for approximately $5\mu s$. Four domains are identified:

- $r \leq r_{i1}$, domain 1, $B = 0$: no current is flowing through the surface.
- $r_{i1} \leq r \leq r_{e1}$, domain 2, B(r): magnetic field inside the inner conductor.
- $r_{e1} \leq r \leq r_{i2}$, domain 3, B(r): magnetic field in the horn cavity between the 2 conductors.
- $r_{i2} \leq r \leq r_{e2}$, domain 4, B(r): magnetic field inside the outer conductor.

Assuming constant current density, the current is:

$$J(r) = \begin{cases} 0 & \text{if } 0 < r < r_{i1} \\ J_{01} & \text{if } r_{i1} < r < r_{e1} \\ 0 & \text{if } r_{e1} < r < r_{i2} \\ J_{02} & \text{if } r_{i2} < r < r_{e2} \end{cases}$$

and

$$J_{01}\pi(r_{e1}^2 - r_{i1}^2) = J_{02}\pi(r_{e2}^2 - r_{i2}^2) = I_0 \quad (3)$$

Using Ampere law; the magnetic flux is:

$$B(r) = \begin{cases} 0 & \text{if } 0 < r < r_{i1} \\ \frac{\mu I_0 (r^2 - r_i^2)}{2\pi r (r_{e1}^2 - r_{i1}^2)} & \text{if } r_{i1} < r < r_{e1} \\ \frac{\mu I_0}{2\pi r} & \text{if } r_{e1} < r < r_{i2} \\ \frac{\mu I_0}{2\pi r} - \frac{\mu I_0}{2\pi r} \frac{r^2 - r_{i2}^2}{r_{e2}^2 - r_{i2}^2} & \text{if } r_{i2} < r < r_{e2} \end{cases}$$

MAGNETIC PRESSURE AND FORCES

The magnetic pressure exerted on the horn conductors is:

$$p(r) \simeq \frac{\mu I^2}{8\pi^2 r^2} \tag{4}$$

The axial magnetic forces applied on the conical and end plates conductors are:

$$\begin{aligned} F_{conical} &= \frac{\mu I^2 \cos\beta}{4\pi} \ln\frac{r_2}{r_1} \\ F_{plates} &= \frac{\mu I^2}{4\pi} \ln\frac{r_2}{r_1} \end{aligned}$$

with β the half opening angle and r_1 and r_2 the entrance and exit radius of the cone

COMSOL MODEL

2D axisymmetric trapezoidal contour with currents in the r-z plan.

$$\begin{aligned} \nabla \times \mathbf{H} &= \sigma \mathbf{E} + \epsilon \frac{\partial \mathbf{E}}{\partial t} \\ \nabla \times \mathbf{H} &= (\sigma + j\omega\epsilon)\mathbf{E} \end{aligned} \tag{5}$$

Using Faraday law's, we can obtain an equation for the magnetic field:

$$\mu j\omega \mathbf{H} + \nabla \times \mathbf{E} = 0 \tag{6}$$

$$\mu j\omega \mathbf{H} + (\sigma + j\omega\epsilon)^{-1} \nabla \times [\nabla \times \mathbf{H}] = 0 \tag{7}$$

From axi symmetry, the current density vector is in the r-z plane $\mathbf{J} = J_r\mathbf{e}_r + J_z e_z$ and the magnetic field has only an azimuthal component, $\mathbf{H} = H_\phi \mathbf{e}_\phi$. Using ampere law with the quasi static assumption ($\lambda >> L$), the current density is calculated from the magnetic field components:

$$
\begin{aligned}
\mathbf{J} &= \nabla \times \mathbf{H} \qquad (8)\\
&= -\frac{\partial H_\phi}{\partial z}\mathbf{e}_r + \left(\frac{H_\phi}{r} + \frac{\partial H_\phi}{\partial r}\right)\mathbf{e}_z
\end{aligned}
$$

The time average resistive heating is calculated as follow:

$$
Q_{av} = \frac{1}{2\sigma}\mathbf{J}\mathbf{J}^* \qquad (9)
$$

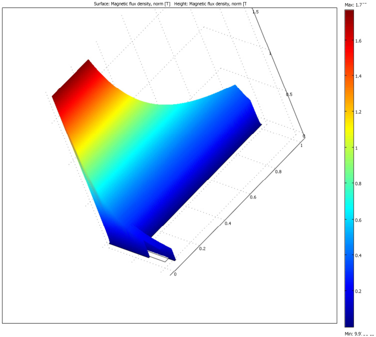
For time harmonic fields, the time average of the product of two vectors is:

$$
\begin{aligned}
\overline{\vec{A}(\mathbf{r},t)\cdot\vec{B}(\mathbf{r},t)} &= \frac{1}{2}Re(\mathbf{A}\cdot\mathbf{B}^*) \qquad (10)\\
\vec{A}(\mathbf{r},t) &= Re(\mathbf{A}e^{j\omega t}) \qquad (11)
\end{aligned}
$$

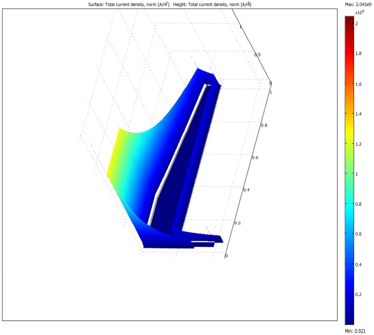
with $\mathbf{A}^*$ is the complex conjugate phasor boundary conditions: used ampere law and applied on the input(+) and output(-) end plates

$$H_\phi = +/- \frac{I_0}{2\pi r} \tag{12}$$

for the contour of the geometry; magnetic insulation: $\mathbf{n} \times \mathbf{E} = \mathbf{H}_{plan} = 0$
Material is Aluminium with electrical conductivity $\sigma = 2.08 \times 10^7$S/m



a)



b)

FIGURE: Magnetic flux [T] distribution in the horn, dimension based on Cern horn prototype, peak current 300 kA and total current distribution

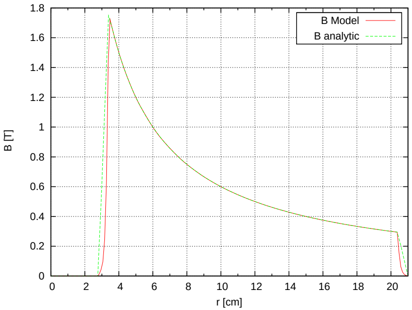


FIGURE: Magnetic flux, model in red and analytic in green

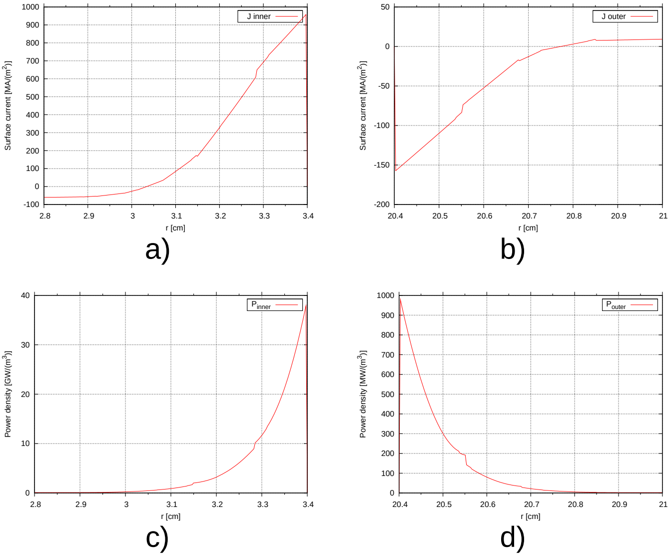


FIGURE: Current and power density in the inner conductor a), c) and outer conductor b), d) of the magnetic horn for a peak current of 300 kA at the frequency of 5000 Hz

For the inner conductor, the azimuthal stress for the mean radius is:

$$\begin{aligned} p(r_{i2}) &= \frac{\mu I^2}{8\pi^2 {r_{i2}}^2} \qquad (13) \\ &= \frac{4\pi \times 10^{-7} \times (3 \times 10^5)^2}{4\pi \times 2\pi \times (0.034)^2} = 1.24Mpa \\ \sigma_\phi &= \frac{pR}{e} = \frac{1.24 \times 0.031}{0.006} = 6.4MPa. \end{aligned}$$

MECHANICS

Assume no thermal stress! axial symmetry stress strain model, Comsol solve the following static equilibrium equations:

$$\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} + F_r = 0 \quad (14)$$

$$\frac{\partial \tau_{rz}}{\partial r} + \frac{\partial \sigma_z}{\partial z} + \frac{\tau_{rz}}{r} + F_z = 0 \quad (15)$$

linear elastic model with small deformation. The force are calculated using Lorenz equation. Elements of force are calculated from the current density

$$dF_r(t) = -Re(B_\phi) \times Re(J_z) \quad (16)$$

$$dF_z(t) = Re(J_r) \times Re(B_\phi) \quad (17)$$

Integrating the axial element of force F_z on each subdomain, the force on the bottom, conical and top segment are respectively $\{-16.1, 12.5, 3.7\}$ kN at the peak current and peak magnetic field.

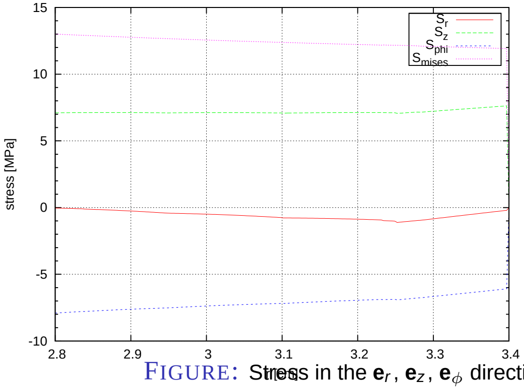


FIGURE: Stress in the $\mathbf{e}_r$, $\mathbf{e}_z$, $\mathbf{e}_\phi$ direction

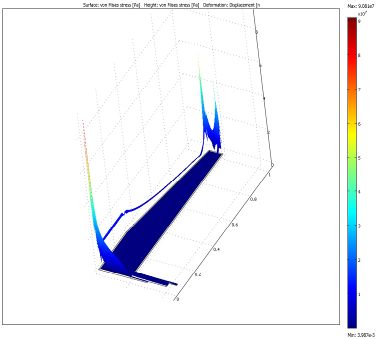


FIGURE: von mises stress

CONCLUSION-SUMMARY

- electromagnetic model is correct: predict the magnetic field distribution; currents , magnetic forces.
- stress on the horn structure from the magnetic pressure is acceptable $\sigma_{mises} \sim 10, 15$MPa.
- total stress: need to add magnetic and thermal stress.
- transient analysis; dynamics, fatigue.