The initial stages of high energy hadronic or nuclear collisions

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François Gelis
IPhT, CEA/Saclay

Outline

1. **Gluon saturation**
2. **Collisions of two saturated projectiles**
3. **Correlations in the final state**
4. **Emergence of hydrodynamical flow**

Collaborators :

K. Dusling, R. Venugopalan	*(BNL)*
T. Epelbaum	*(IPhT)*
T. Lappi	*(Jyvaskyla University)*
A. Dumitru, J. Jalilian – Marian	*(Baruch College)*

1 **Gluon saturation**

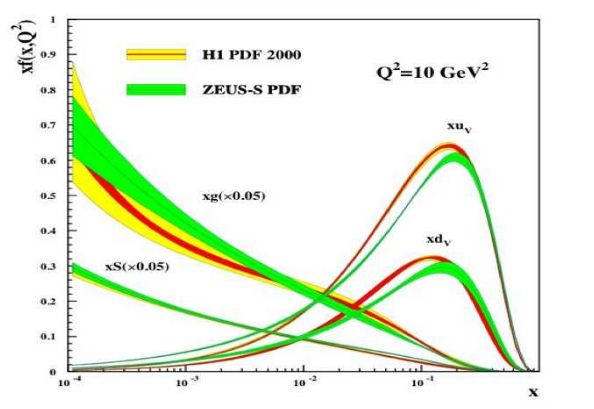
Why small-x gluons matter

Gluon evolution

Color Glass Condensate

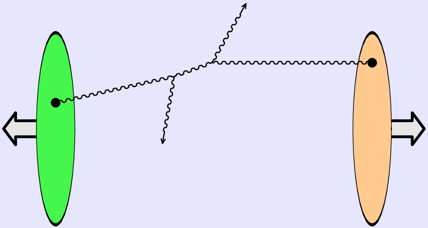
Growth of the gluon distribution at small x

Gluon distribution at small x



- Note: gluons have been divided by 20
- Gluons dominate at any $x \lesssim 10^{-1}$

Longitudinal momentum fraction in AA collisions



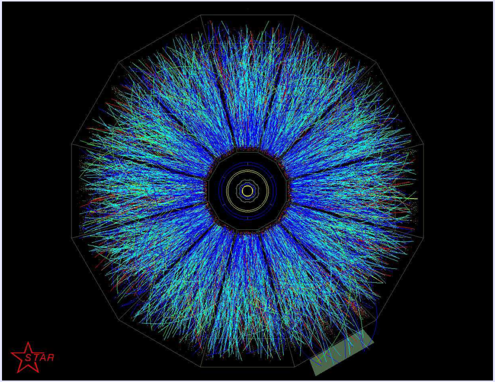
- The partons that are relevant for the process under consideration carry the longitudinal momentum fractions:

$$x_{1,2} = \frac{M_\perp}{\sqrt{s}}\, e^{\pm Y}$$

  - $M_\perp$: transverse mass
  - Y : rapidity
  - $\sqrt{s}$: collision energy

Longitudinal momentum fraction in AA collisions

Nucleus-Nucleus collision



- 99% of the multiplicity below $p_\perp \sim 2$ GeV
- $x \sim 10^{-2}$ at RHIC ($\sqrt{s} = 200$ GeV)
- $x \sim 4.10^{-4}$ at the LHC ($\sqrt{s} = 5.5$ TeV)

  ▷ gluons at small x are the most important

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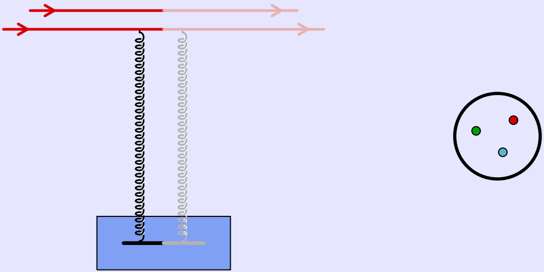
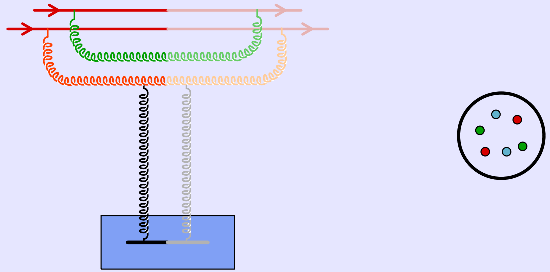


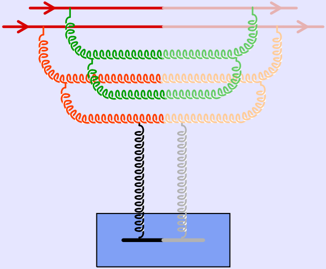
- consider a hadron or nucleus probed via gluon exchange
- at low energy, only valence quarks are present in the hadron wave function

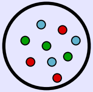
Gluon saturation



- when energy increases, new gluons are emitted
- the emission probability is $\alpha_s \int \frac{dx}{x} \sim \alpha_s \ln(\frac{1}{x})$, with x the longitudinal momentum fraction of the gluon
- at small-x (i.e. high energy), these logs need to be resummed

Gluon saturation

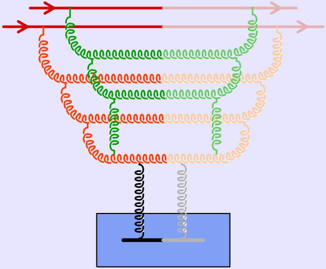


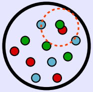


- as long as the gluon density remains small, the evolution is linear: the number of gluons produced at a given step is proportional to the number of gluons at the previous step

Balitsky, Fadin, Kuraev, Lipatov (1977)

Gluon saturation





- eventually, the gluons overlap in phase-space
- gluon recombination becomes likely
- after this point, the evolution is non-linear

Balitsky (1996), Kovchegov (2000)
Jalilian-Marian, Kovner, Leonidov, Weigert (1999)
Iancu, Leonidov, McLerran (2001)

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Criterion for gluon recombination

Gribov, Levin, Ryskin (1983)

Number of gluons per unit area :

$$\rho \sim \frac{xG_A(x, Q^2)}{\pi R_A^2}$$

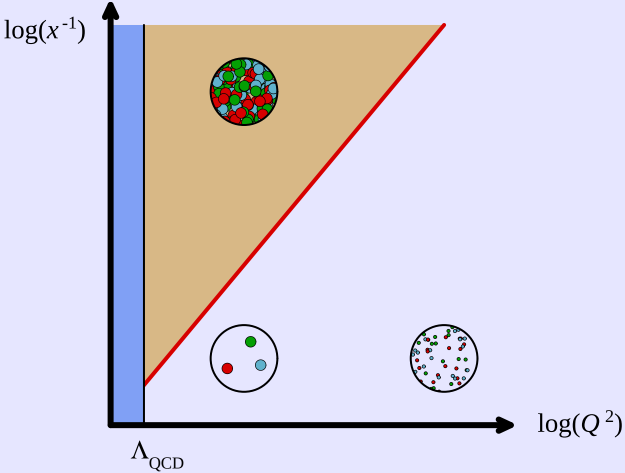
Recombination cross-section :

$$\sigma_{gg\to g} \sim \frac{\alpha_s}{Q^2}$$

Recombination happens if $\rho\sigma_{gg\to g} \gtrsim 1$, i.e. $Q^2 \lesssim Q_s^2$, with :

$$Q_s^2 \sim \frac{\alpha_s xG_A(x, Q_s^2)}{\pi R_A^2} \sim A^{1/3}\frac{1}{x^{0.3}}$$

Saturation domain



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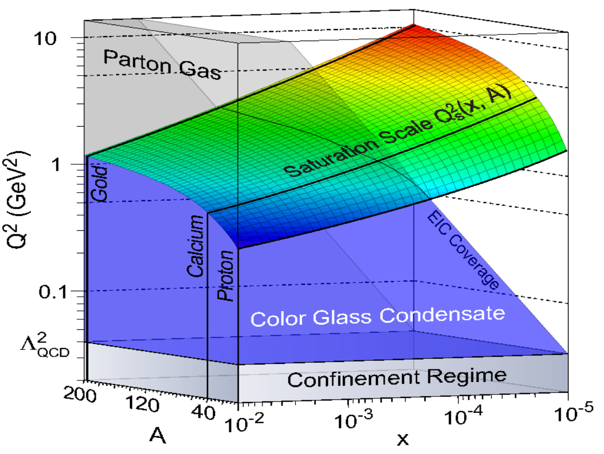
Saturation momentum: constraints from data

Q_s as a function of x and A



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Requirements

- Main difficulty: How to treat collisions involving a large number of partons?

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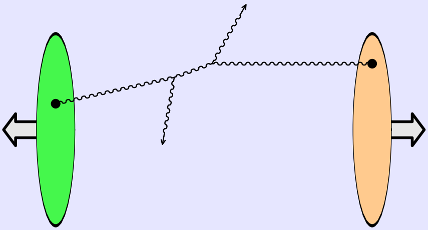
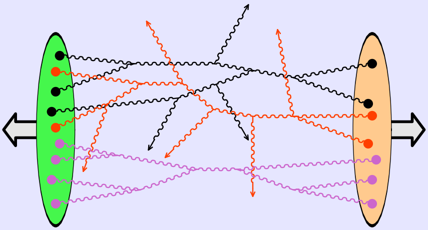


- Main difficulty: How to treat collisions involving a large number of partons?
- Dilute regime : one parton in each projectile interact (what the standard perturbative techniques are made for)

Requirements



- Main difficulty: How to treat collisions involving a large number of partons?

- Dense regime : multiparton processes become crucial
  ▷ new techniques are required
  ▷ multi-parton distributions are needed

CGC: Degrees of freedom

CGC = effective theory of small x gluons

- The fast partons ($k^+ > \Lambda^+$) are frozen by time dilation
  ▷ described as static color sources on the light-cone :

$$J^\mu = \delta^{\mu+}\rho(x^-, \vec{\boldsymbol{x}}_\perp) \qquad (0 < x^- < 1/\Lambda^+)$$

- The color sources ρ are random, and described by a probability distribution $W_{\Lambda^+}[\rho]$

- Slow partons ($k^+ < \Lambda^+$) cannot be considered static over the time-scales of the collision process
  ▷ must be treated as standard gauge fields
  ▷ eikonal coupling to the current J^μ : $A_\mu J^\mu$

CGC: renormalization group evolution

Independence w.r.t Λ^+ $\rightarrow$ evolution equation (JIMWLK) :

$$\frac{\partial W_{\Lambda^+}}{\partial \ln(\Lambda^+)} = \mathcal{H} \;\; W_{\Lambda^+}$$

$$\mathcal{H} = \frac{1}{2} \int_{\vec{\boldsymbol{x}}_\perp, \vec{\boldsymbol{y}}_\perp} \frac{\delta}{\delta\alpha(\vec{\boldsymbol{y}}_\perp)} \eta(\vec{\boldsymbol{x}}_\perp, \vec{\boldsymbol{y}}_\perp) \frac{\delta}{\delta\alpha(\vec{\boldsymbol{x}}_\perp)}$$

where $-\boldsymbol{\partial}_\perp^2 \, \alpha(\vec{\boldsymbol{x}}_\perp) = \rho(1/\Lambda^+, \vec{\boldsymbol{x}}_\perp)$

- $\eta(\vec{\boldsymbol{x}}_\perp, \vec{\boldsymbol{y}}_\perp)$ is a non-linear functional of ρ
- Resums all the powers of $\alpha_s \ln(1/x)$ and of $Q_s/p_\perp$ that arise in loop corrections
- Simplifies into the BFKL equation when the source ρ is small (expand η in powers of ρ)

2 Collisions of two saturated projectiles

Power counting

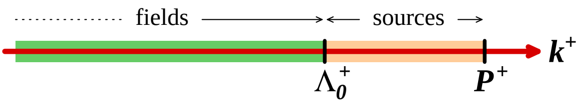
Leading Order

Next to Leading Order

Initial state factorization

Power counting

- CGC effective theory with cutoff at the scale Λ_0^+ :



$$\mathcal{S} = \underbrace{-\frac{1}{4}\int F_{\mu\nu}F^{\mu\nu}}_{\mathcal{S}_{\rm YM}} + \int \underbrace{(J_1^\mu + J_2^\mu)}_{\text{fast partons}} A_\mu$$

- Expansion in g^2 in the saturated regime:

$$\frac{dN_1}{dy d^2\vec{\boldsymbol{p}}_\perp} \sim \frac{1}{g^2}\Big[c_0 + c_1\, g^2 + c_2\, g^4 + \cdots\Big]$$

❷ Collisions of two saturated projectiles

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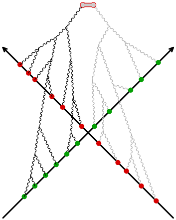
Leading Order

$$\left.\frac{dN_1}{dyd^2\vec{\boldsymbol{p}}_\perp}\right|_{\rm LO} = \sum_{\rm trees}$$



Inclusive gluon spectrum at LO :

$$\left.\frac{dN_1}{dyd^2\vec{\boldsymbol{p}}_\perp}\right|_{\rm LO} \propto \int d^4x d^4y\; e^{ip\cdot(x-y)} \cdots \mathcal{A}^\mu(x)\mathcal{A}^\nu(y)$$

$$\underbrace{\left[\mathcal{D}_\mu, \mathcal{F}^{\mu\nu}\right] = J_1^\nu + J_2^\nu}_{\rm Yang-Mills\;\; equation} \quad , \quad \lim_{t\to-\infty} \mathcal{A}^\mu(t,\vec{\boldsymbol{x}}) = 0$$

(at LO, everything comes from classical fields)

❷ Collisions of two saturated projectiles

Power counting

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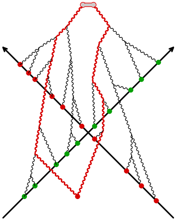
Next to Leading Order [FG, Venugopalan (2006)]

$$\left.\frac{dN_1}{dyd^2\vec{p}_\perp}\right|_{\rm NLO} = \sum_{\rm trees}$$



Inclusive gluon spectrum at NLO :

$$\left.\frac{dN_1}{dyd^2\vec{p}_\perp}\right|_{\rm NLO} = \left[\frac{1}{2}\int_{\vec{u},\vec{v}\in\Sigma} \mathcal{G}(\vec{u},\vec{v})\,\mathbb{T}_{\boldsymbol{u}}\mathbb{T}_{\boldsymbol{v}} + \int_{\vec{u}\in\Sigma} \beta(\vec{u})\,\mathbb{T}_{\boldsymbol{u}}\right] \left.\frac{dN_1}{dyd^2\vec{p}_\perp}\right|_{\rm LO}$$

$\Sigma =$ initial Cauchy surface , $\mathbb{T}_{\boldsymbol{u}} \sim \delta/\delta\mathcal{A}_{\rm init}(\boldsymbol{u})$

(for certain choices of Σ, the functions $\mathcal{G}(\vec{u},\vec{v})$ and $\beta(\vec{u})$ are calculable analytically)

❷ Collisions of two saturated projectiles

Power counting

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Leading Logs [FG, Lappi, Venugopalan (2008)]

Logs of Λ^+ and Λ^-

$$\frac{1}{2}\int\limits_{\vec{\boldsymbol{u}},\vec{\boldsymbol{v}}\in\Sigma} \mathcal{G}(\vec{\boldsymbol{u}},\vec{\boldsymbol{v}})\,\mathbb{T}_{\boldsymbol{u}}\mathbb{T}_{\boldsymbol{v}} + \int\limits_{\vec{\boldsymbol{u}}\in\Sigma} \beta(\vec{\boldsymbol{u}})\,\mathbb{T}_{\boldsymbol{u}} =$$

$$= \ln\left(\Lambda^+\right)\,\mathcal{H}_1 + \ln\left(\Lambda^-\right)\,\mathcal{H}_2 + \text{terms w/o logs}$$

$\mathcal{H}_{1,2}$ = JIMWLK Hamiltonian

$\triangleright$ ensures the factorizability of these logs into JIMWLK-evolved distributions $W[\rho_{1,2}]$

Factorization of the Leading Logs of $1/x$

- One can factorize all the powers of $\alpha_s \log(1/x_{1,2})$

Single inclusive gluon spectrum at Leading Log accuracy

$$\left\langle \frac{dN_1}{dyd^2\vec{\boldsymbol{p}}_\perp} \right\rangle_{\text{LLog}} = \int \left[D\rho_1\, D\rho_2\right]\, W_1\left[\rho_1\right]\, W_2\left[\rho_2\right]\, \underbrace{\frac{dN_1[\rho_{1,2}]}{dyd^2\vec{\boldsymbol{p}}_\perp}}_{\text{for fixed } \rho_{1,2}}$$

- The factor $dN_1/dyd^2\vec{\boldsymbol{p}}_\perp$ under the integral does not depend on y: the rapidity dependence comes entirely from the distributions $W_{1,2}$

- This factorization establishes a link to other reactions (such as DIS) in the saturated regime

Multi-gluon correlations at Leading Log

- The previous factorization can be extended to multi-particle inclusive spectra :

$$\left\langle \frac{dN_n}{dy_1 d^2\vec{\boldsymbol{p}}_{1\perp} \cdots dy_n d^2\vec{\boldsymbol{p}}_{n\perp}} \right\rangle_{\text{LLog}} =$$

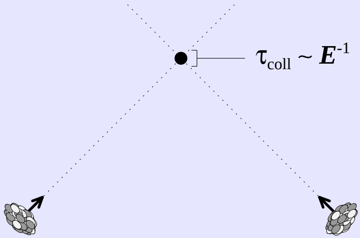
$$= \int \big[D\rho_1\, D\rho_2\big]\; W_1\big[\rho_1\big]\; W_2\big[\rho_2\big]\; \frac{dN_1[\rho_{1,2}]}{dy_1 d^2\vec{\boldsymbol{p}}_{1\perp}} \cdots \frac{dN_1[\rho_{1,2}]}{dy_n d^2\vec{\boldsymbol{p}}_{n\perp}}$$

- Note: at Leading Log accuracy, all the rapidity correlations come from the evolution of the distributions $W[\rho_{1,2}]$

  ▷ they are a property of the pre-collision initial state

- This formula predicts long range ($\Delta y \sim \alpha_s^{-1}$) correlations in rapidity

Why factorization works: causality



- The duration of the collision is very short: $\tau_{\text{coll}} \sim E^{-1}$

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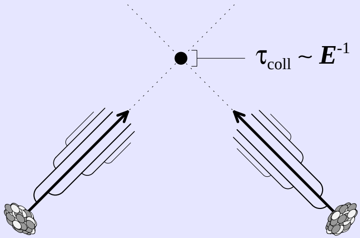

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- The logarithms we want to resum arise from the radiation of soft gluons, which takes a long time
  $\triangleright$ it must happen (long) before the collision

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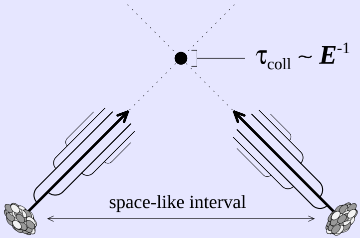


- The duration of the collision is very short: $\tau_{\text{coll}} \sim E^{-1}$
- The logarithms we want to resum arise from the radiation of soft gluons, which takes a long time
  ▷ it must happen (long) before the collision
- The projectiles are not in causal contact before the impact
  ▷ the logarithms are intrinsic properties of the projectiles, independent of the measured observable

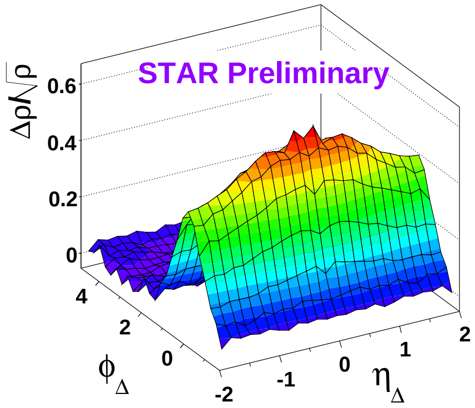
3 Correlations in the final state

2-hadron correlations in data

Ridge in Au-Au collisions

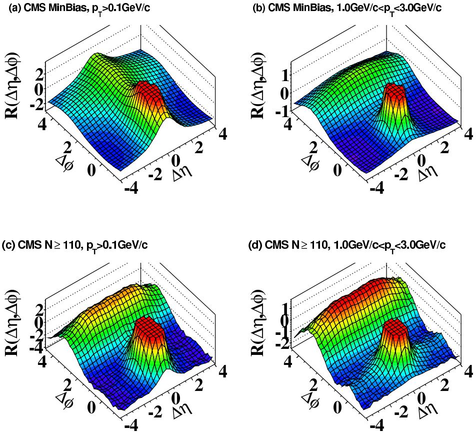
Ridge in p-p collisions

Nucleus-nucleus collisions at RHIC



- Long range correlation in $\Delta\eta$ (rapidity)
- Narrow correlation in $\Delta\varphi$ (azimuthal angle)

Proton-proton collisions at the LHC

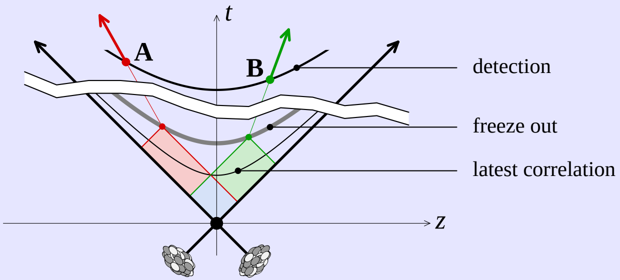


- Similar effect visible for high multiplicity p-p collisions, in an intermediate $p_\perp$ window
- Much weaker than in AA collisions

Importance of the initial stages

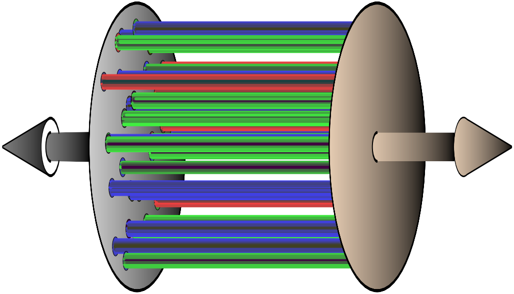
Long range rapidity correlations probe early times

$$t_{\text{correlation}} \le t_{\text{freeze out}} \;\; e^{-\frac{1}{2}|\eta_A - \eta_B|}$$



Initial classical color fields [Lappi, McLerran (2006)]

- At $\tau = 0^+$, the chromo-$\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{B}}$ fields form longitudinal "flux tubes" extending between the projectiles:



- Correlation length in the transverse plane: $\Delta r_\perp \sim Q_s^{-1}$
- Correlation length in rapidity: $\Delta\eta \sim \alpha_s^{-1}$

3 Correlations in the final state

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Ridge in Au-Au collisions

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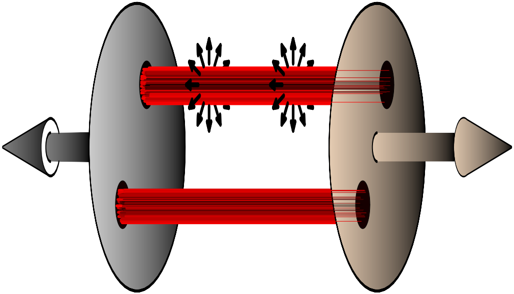
2-hadron correlations at RHIC

Dumitru, FG, McLerran, Venugopalan (2008)

- η-independent fields lead to long range correlations :



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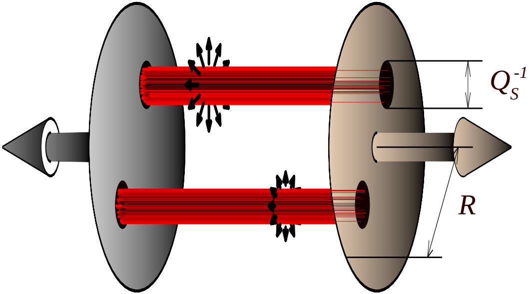

- Particles emitted by different flux tubes are not correlated
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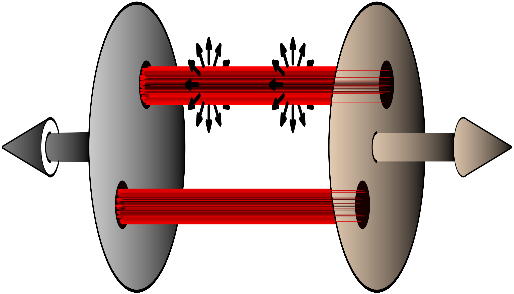

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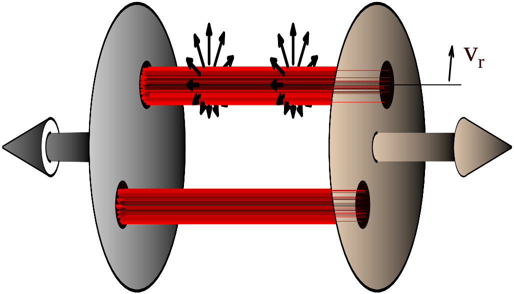
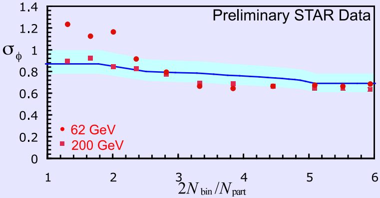


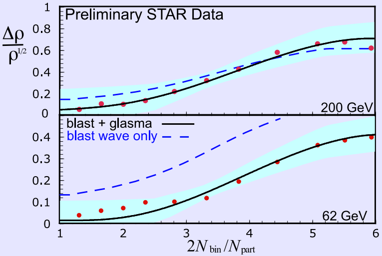
- Particles emitted by different flux tubes are not correlated
  $\triangleright$ $(RQ_s)^{-2}$ sets the strength of the correlation
- At early times, the correlation is flat in $\Delta\varphi$
  A collimation in $\Delta\varphi$ is produced later by radial flow

Ridge: centrality dependence

Gavin, McLerran, Moschelli (2008)

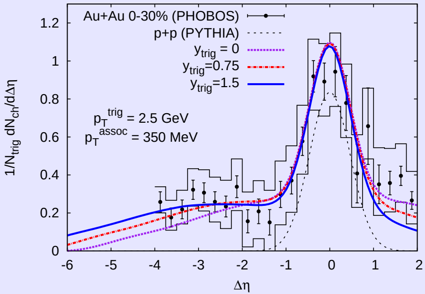
Radial flow modelled by a blast wave





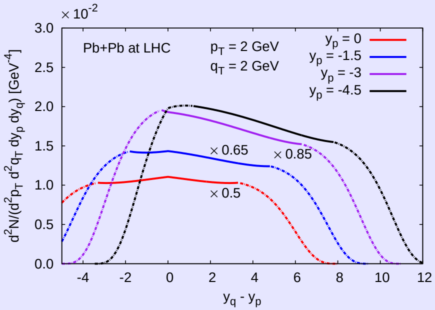
Ridge: rapidity dependence

Dusling, FG, Lappi, Venugopalan (2009)



- Note: the central peak is not a CGC prediction. It is good old jet fragmentation, taken from PYTHIA in this plot

Ridge: rapidity dependence



- The shape of the $y_q - y_p$ dependence depends on the rapidity of the trigger particle

  the CGC provides testable predictions that may distinguish it from other models that also have "boost invariance"

❸ Correlations in the final state

2-hadron correlations in data

Ridge in Au-Au collisions

Ridge in p-p collisions

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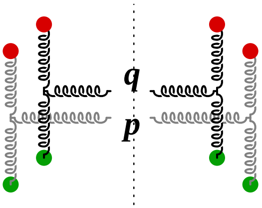
Origin of the angular correlation

Dumitru, Dusling, FG, Jalilian-Marian, Lappi, Venugopalan (2010)

- The long range rapidity correlations invoked in A-A collisions are also present in p-p collisions
- Whether there is a sufficient amount of radial flow to induce the azimuthal collimation is unknown
  - less particles are produced
  - the system freezes out much earlier
- There is however an “intrinsic” angular correlation, that exists in the absence of flow (it was there in A-A collisions as well, but neglected because it is a small effect)

Intrinsic angular correlations

- 2-gluon inclusive spectrum before the average over $\rho_{1,2}$:

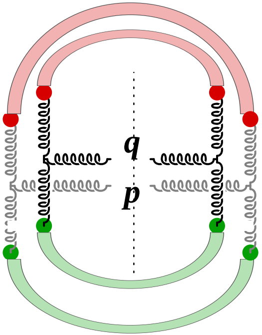


$\triangleright$ this contribution dominates the 2-gluon spectrum in the regime where the parton densities are large

$\triangleright$ the average over $\rho_{1,2}$ amounts to connecting the red and green lines in all the possible ways (pairwise if the sources have Gaussian distributions)

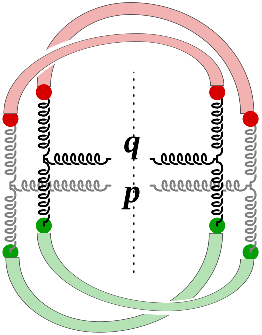
Intrinsic angular correlations

- Trivial connection (no correlation):



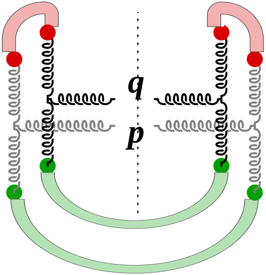
Intrinsic angular correlations

- Non-trivial connection (1):



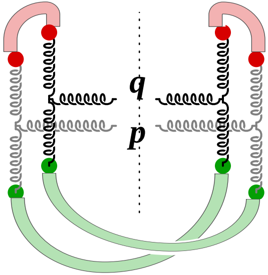
Intrinsic angular correlations

- Non-trivial connection (2):



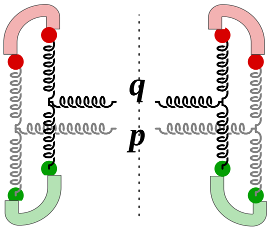
Intrinsic angular correlations

- Non-trivial connection (3):



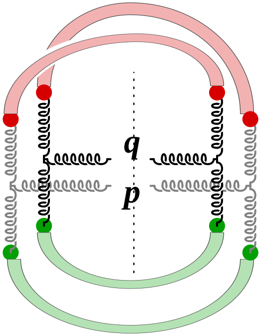
Intrinsic angular correlations

- Non-trivial connection (4):



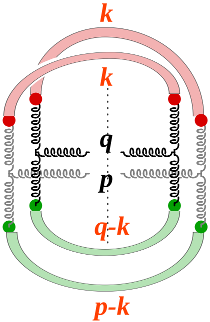
Intrinsic angular correlations

- Non-trivial connection (5):



Intrinsic angular correlations

- Diagrams (1-4) have only back-to-back correlations
- (5) is the interesting one!

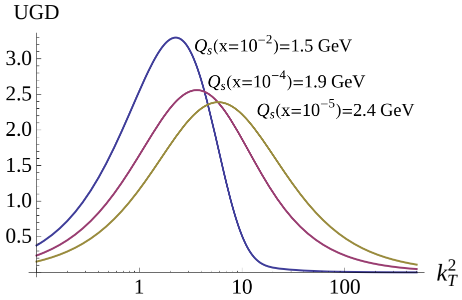


▷ Momentum assignment of the unintegrated gluon distributions:

$$[\phi_1(\boldsymbol{k}_\perp)]^2\;\phi_2(|\boldsymbol{p}_\perp-\boldsymbol{k}_\perp|)\;\phi_2(|\boldsymbol{q}_\perp-\boldsymbol{k}_\perp|)$$

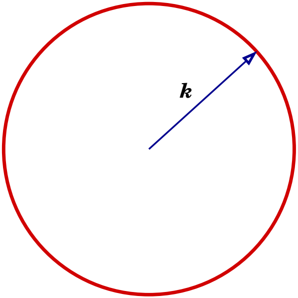
Intrinsic angular correlations

- In the saturation regime, unintegrated gluon distributions are peaked near Q_s:



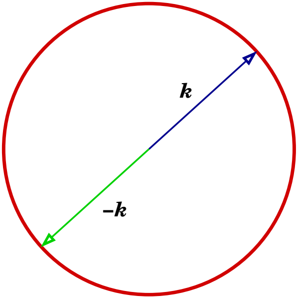
- The presence of this peak is what correlates the directions of $\vec{p}_\perp$ and $\vec{q}_\perp$ around $\Delta\phi = 0$ when we perform the integration over $\vec{k}_\perp$

Intrinsic angular correlations



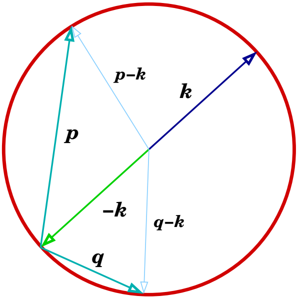
- $|\vec{k}_\perp| \sim Q_s$

Intrinsic angular correlations



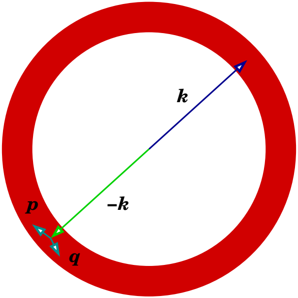
- $|\vec{k}_\perp| \sim Q_s$

Intrinsic angular correlations



- $|\vec{\boldsymbol{k}}_\perp| \sim Q_s$
- $|\vec{\boldsymbol{p}}_\perp - \vec{\boldsymbol{k}}_\perp| \sim |\vec{\boldsymbol{q}}_\perp - \vec{\boldsymbol{k}}_\perp| \sim Q_s$

Intrinsic angular correlations

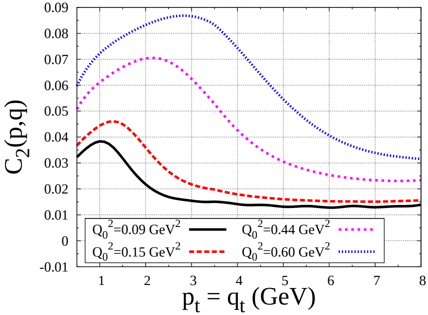


- $|\vec{\boldsymbol{k}}_\perp| \sim Q_s$
- $|\vec{\boldsymbol{p}}_\perp - \vec{\boldsymbol{k}}_\perp| \sim |\vec{\boldsymbol{q}}_\perp - \vec{\boldsymbol{k}}_\perp| \sim Q_s$
- If the momenta are smaller than the width of the distributions, there is no significant angular correlation

  Similarly, for large momenta there is no correlation because the main contribution does not come from the peak of the distributions anymore

Intrinsic angular correlations

- The effect is maximal for intermediate $p_\perp, q_\perp \sim Q_s$:



François Gelis

4 **Emergence of hydrodynamical flow**

Hydrodynamics in AA collisions

Toy scalar model

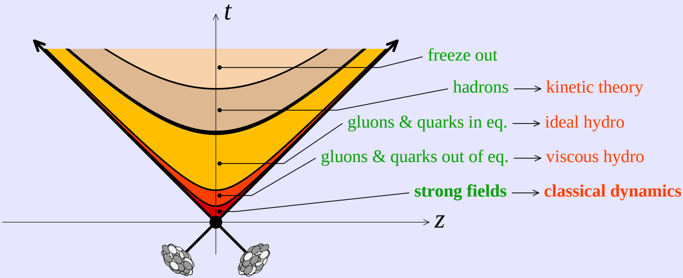
Pressure at LO and NLO

Unstable modes and resummation

Zero mode fluctuations only

Complete fluctuation spectrum

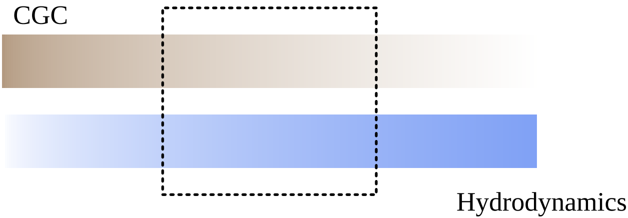
Stages of a nucleus-nucleus collision



- The Color Glass Condensate provides a framework to describe nucleus-nucleus collisions up to a time $\tau \sim Q_s^{-1}$
- The subsequent stages are well described by nearly ideal hydrodynamics

Matching to hydrodynamics

- If a smooth matching from the CGC to Hydro is possible, there should be an overlap between their respective domain of applicability:



▷ one should be able to recover the fluid behavior by starting from the CGC. However, this is problematic at the moment

Reminder on hydrodynamics

Equations of hydrodynamics :

$$\partial_\mu T^{\mu\nu} = 0$$

Additional inputs :

EoS : $p = f(\epsilon)$, Transport coefficients : $\eta, \zeta, \cdots$

- Required initial conditions : $T^{\mu\nu}(\tau = \tau_0, \eta, \vec{x}_\perp)$

❹ Emergence of hydrodynamical flow

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ϕ^4 field theory with a strong external source

Dusling, Epelbaum, FG, Venugopalan (2010)

Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\alpha \phi)^2 - V(\phi) + J\phi$$

$$V(\phi) = \frac{g^2}{4!}\phi^4 \qquad J = \theta(-x^0)\, e^{aQx^0}\, \frac{Q^3}{g}$$

- In 3+1-dim, g is dimensionless, and the only scale in the problem is Q, provided by the external source
- Q mimics the saturation scale
- The source is active only at $x^0 < 0$, and is switched off adiabatically when $x^0 \to -\infty$

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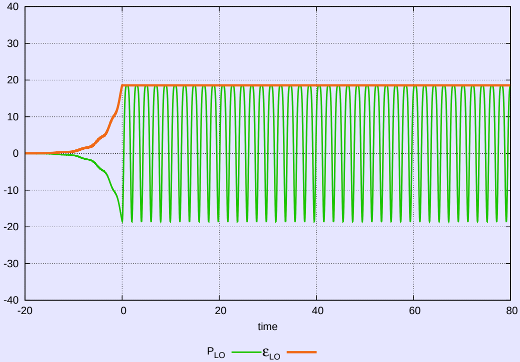
$T^{\mu\nu}$ at Leading Order

$$T^{\mu\nu}_{\rm LO} = \partial^\mu\varphi\partial^\nu\varphi - g^{\mu\nu}\mathcal{L}\;,\quad \square\varphi + V'(\varphi) = J\;,\quad \lim_{x^0\to-\infty}\varphi(x) = 0$$



▷ no single-valued relation between ϵ and p: no EoS at LO

$T^{\mu\nu}$ at Next to Leading Order

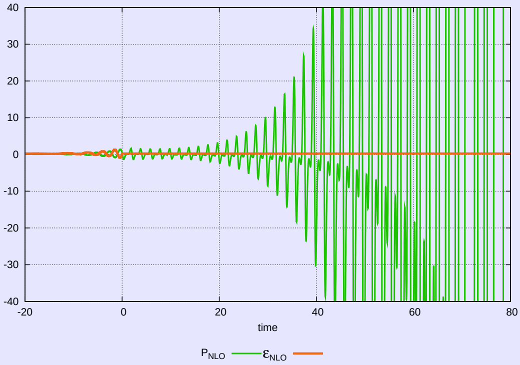
$$T^{\mu\nu}_{\rm NLO} = \partial^\mu\varphi\partial^\nu\beta + \partial^\mu\beta\partial^\nu\varphi - g^{\mu\nu}\Big[\partial_\alpha\beta\partial^\alpha\varphi - \beta V'(\varphi)\Big]$$
$$+\int_{\boldsymbol k}\Big[\partial^\mu a_{-\boldsymbol k}\partial^\nu a_{+\boldsymbol k} - \frac{g^{\mu\nu}}{2}\Big(\partial_\alpha a_{-\boldsymbol k}\partial^\alpha a_{+\boldsymbol k} - V''(\varphi)a_{-\boldsymbol k}a_{+\boldsymbol k}\Big)\Big]$$

$$\Big[\square + V''(\varphi)\Big]a_{\pm\boldsymbol k} = 0\,, \qquad \lim_{x^0\to-\infty} a_{\pm\boldsymbol k}(x) = e^{\pm ik\cdot x}$$

$$\Big[\square + V''(\varphi)\Big]\beta = -\tfrac{1}{2}V'''(\varphi)\int_{\boldsymbol k} a_{-\boldsymbol k}a_{+\boldsymbol k}\,, \qquad \lim_{x^0\to-\infty}\beta(x) = 0$$

$T^{\mu\nu}$ at Next to Leading Order: secular divergences

NLO corrections for $g = 1$



- $\epsilon_{\rm NLO}$ is a small correction at all times (it is protected from divergences by the conservation of $T^{\mu\nu}$)
- $p_{\rm NLO}$ is still small at $x^0 = 0$
- $p_{\rm NLO}$ diverges exponentially when $x^0 \to +\infty$

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Stability analysis

- Consider the (linearized) equation for fluctuations of momentum $\vec{\boldsymbol{k}}$ over the uniform background $\varphi_0(t)$:

$$\begin{aligned} \ddot{\varphi}_0(t) + V'(\varphi_0(t)) &= 0 \\ \ddot{a} + \Big[\boldsymbol{k}^2 + \underbrace{V''(\varphi_0(t))}_{m^2(t)}\Big]\, a &= 0 \end{aligned}$$

- It has two independent solutions a_1 and a_2

- If T is the period of $\varphi_0(t)$, one can relate $a_{1,2}(T)$ to $a_{1,2}(0)$ by a linear transformation :

$$M_{\boldsymbol{k}} \equiv \begin{pmatrix} a_1(T) & a_2(T) \\ \dot{a}_1(T) & \dot{a}_2(T) \end{pmatrix} \begin{pmatrix} a_1(0) & a_2(0) \\ \dot{a}_1(0) & \dot{a}_2(0) \end{pmatrix}^{-1}$$

- $\det(M_{\boldsymbol{k}}) = 1$ (from unitarity, since $\varphi_0(t) \in \mathbb{R}$)

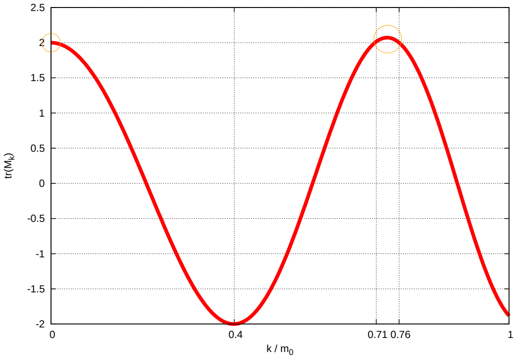
Stability analysis

(In)stability conditions

$\mathrm{tr}\,(M_{\boldsymbol{k}}) < 2$: a_1 and a_2 are stable

$\mathrm{tr}\,(M_{\boldsymbol{k}}) = 2$: a_1 is stable and a_2 diverges linearly

$\mathrm{tr}\,(M_{\boldsymbol{k}}) > 2$: a_1 is stable and a_2 diverges exponentially



Resummation [FG, Lappi, Venugopalan (2007)]

Reminder: energy-momentum tensor at NLO

$$T^{\mu\nu}_{\rm NLO} = \left[\frac{1}{2}\int\limits_{\vec{\boldsymbol{u}},\vec{\boldsymbol{v}}\in\Sigma} \mathcal{G}(\vec{\boldsymbol{u}},\vec{\boldsymbol{v}})\,\mathbb{T}_{\boldsymbol{u}}\mathbb{T}_{\boldsymbol{v}} + \int\limits_{\vec{\boldsymbol{u}}\in\Sigma} \beta(\vec{\boldsymbol{u}})\,\mathbb{T}_{\boldsymbol{u}}\right] T^{\mu\nu}_{\rm LO}$$

Resummation [FG, Lappi, Venugopalan (2007)]

Resummed energy-momentum tensor

$$T^{\mu\nu}_{\text{resummed}} = \exp\left[\frac{1}{2}\int_{\vec{u},\vec{v}\in\Sigma} \mathcal{G}(\vec{u},\vec{v})\,\mathbb{T}_{u}\mathbb{T}_{v} + \int_{\vec{u}\in\Sigma} \beta(\vec{u})\,\mathbb{T}_{u}\right] T^{\mu\nu}_{\text{LO}}$$

- contains the complete LO $+$ NLO
  + a subset of the higher orders :

$$T^{\mu\nu}_{\text{resummed}} = \frac{Q^4}{g^2}\left[\underbrace{c_0 + c_1\,g^2}_{\text{fully}} + \underbrace{c_2\,g^4 + \cdots}_{\text{partly}}\right]$$

- resums the leading divergent terms (i.e. the terms where each extra g^2 is accompanied by a growing fluctuation)

Why does it cure the secular divergences?

- $\exp[\cdots \mathbb{T}_{\boldsymbol{u}}]$ shifts the initial condition of a classical field:

$$\exp\left[\int_{\vec{\boldsymbol{u}}\in\Sigma} \alpha(\boldsymbol{u})\,\mathbb{T}_{\boldsymbol{u}}\right] \mathcal{F}[\varphi_{\rm init}] = \mathcal{F}[\varphi_{\rm init} + \alpha]$$

Equivalent form of the resummation

$$\exp\left[\frac{1}{2}\int_{\vec{\boldsymbol{u}},\vec{\boldsymbol{v}}\in\Sigma} \mathcal{G}(\vec{\boldsymbol{u}},\vec{\boldsymbol{v}})\,\mathbb{T}_{\boldsymbol{u}}\mathbb{T}_{\boldsymbol{v}} + \int_{\vec{\boldsymbol{u}}\in\Sigma} \beta(\vec{\boldsymbol{u}})\,\mathbb{T}_{\boldsymbol{u}}\right] T^{\mu\nu}_{\rm LO}[\varphi_{\rm init}]$$

$$= \int [D\alpha]\,\exp\left[-\frac{1}{2}\int_{\vec{\boldsymbol{u}},\vec{\boldsymbol{v}}\in\Sigma} \alpha(\vec{\boldsymbol{u}})\mathcal{G}^{-1}(\vec{\boldsymbol{u}},\vec{\boldsymbol{v}})\alpha(\vec{\boldsymbol{v}})\right] T^{\mu\nu}_{\rm LO}[\varphi_{\rm init} + \alpha + \beta]$$

$\triangleright$ initial condition at $x^0 = 0$ shifted by a Gaussian-distributed fluctuation, but full non-linear evolution at $x^0 > 0$ $\triangleright$ no runaway contributions **[Son (1996), Khlebnikov, Tkachev (1996)]**

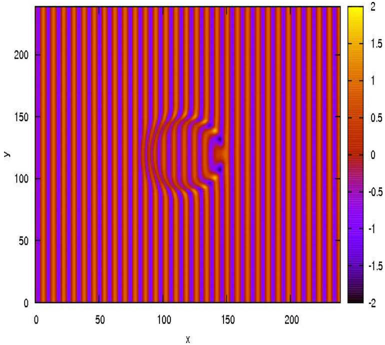
Spectrum of fluctuations

- The variance of the Gaussian fluctuations is:

$$\mathcal{G}(\vec{\boldsymbol{x}},\vec{\boldsymbol{y}}) = \int \frac{d^3\vec{\boldsymbol{k}}}{(2\pi)^3 2|\boldsymbol{k}|}\; a_{+\boldsymbol{k}}(0,\vec{\boldsymbol{x}})a_{-\boldsymbol{k}}(0,\vec{\boldsymbol{y}})$$

$$\left[\square + V''(\varphi_0(t))\right] a_{\pm\boldsymbol{k}} = 0 \qquad \lim_{x^0\to-\infty} a_{\pm\boldsymbol{k}}(x) = e^{\pm ik\cdot x}$$

- $a_{\pm\boldsymbol{k}}(t,\vec{\boldsymbol{x}}) =$ plane wave distorted by the background field :



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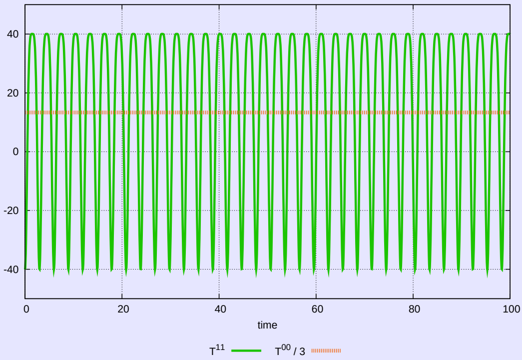
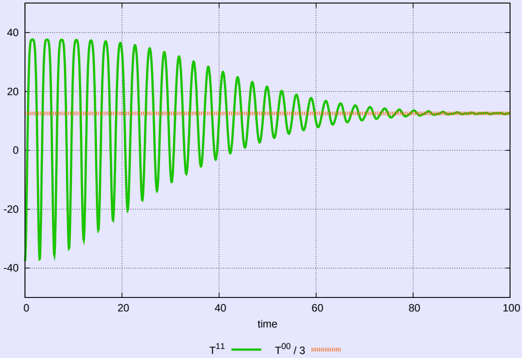
Time evolution of the pressure

LO result, i.e. no fluctuations :



Time evolution of the pressure

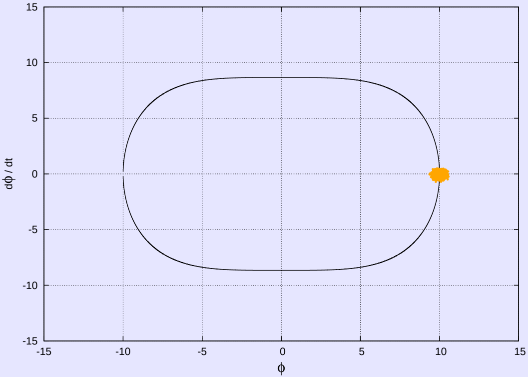
Resummed result (with k=0 fluctuations only) :



- No more divergence of the pressure when $x^0 \to +\infty$
- P relaxes to $\epsilon/3$ $\triangleright$ EoS of a 4-dim scale invariant system

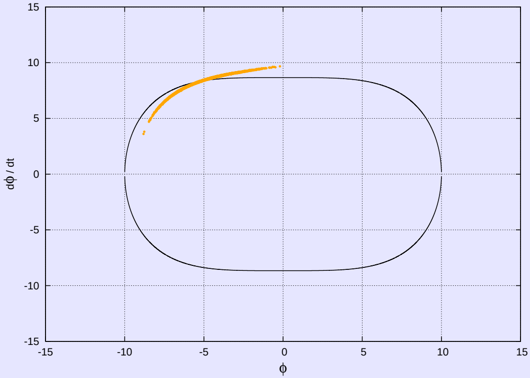
Time evolution of the phase-space density

$\rho(\phi, \dot{\phi})$ **at** $t = 0$ **:**



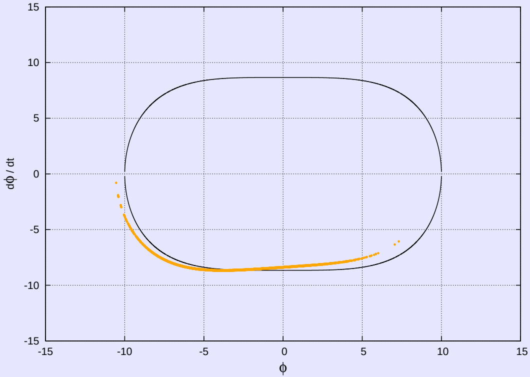
Time evolution of the phase-space density

$\rho(\phi, \dot{\phi})$ at $t = 10$:



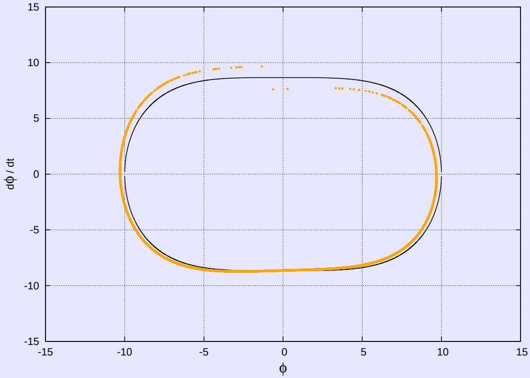
Time evolution of the phase-space density

$\rho(\phi, \dot{\phi})$ at $t = 20$:



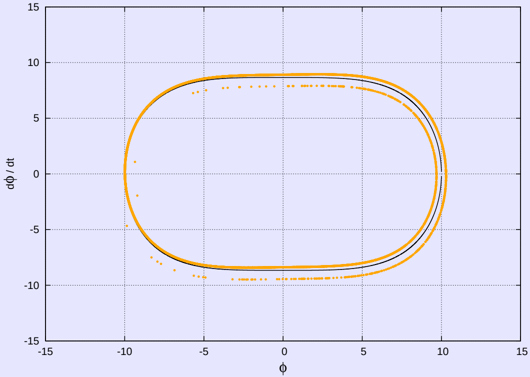
Time evolution of the phase-space density

$\rho(\phi, \dot{\phi})$ at $t = 50$:



Time evolution of the phase-space density

$\rho(\phi, \dot{\phi})$ **at** $t = 100$ **:**



- The phase-space density $\rho(\phi, \dot{\phi})$ spreads around a curve of fixed energy $\triangleright$ micro-canonical equilibrium

What makes $\rho(\phi, \dot{\phi})$ spread?

- In a non-harmonic potential, fields with different amplitudes oscillate at different frequencies

  ▷ their relative phase increases linearly in time

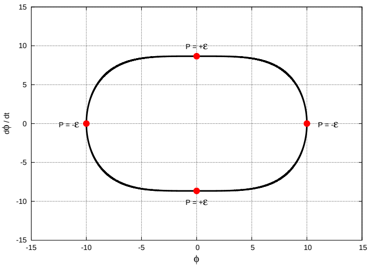
- They spread over the entire orbit in a time

$$t_{\rm relax} \approx \frac{2\pi}{\Delta\alpha \cdot \frac{d\omega}{d\phi}}$$

- In our toy model: $d\omega/d\phi \sim g$ and $\Delta\alpha \sim Q$

$$t_{\rm relax} \sim \frac{1}{gQ}$$

What makes the pressure relax?



$$\epsilon - 3P = -\frac{d(\phi\dot{\phi})}{dt}$$

- For a single field configuration, the average of $\epsilon - 3P$ over the orbit is zero
- If $\rho(\phi, \dot{\phi})$ spreads uniformly around the orbit, the ensemble average leads to the same result as the time average for one field configuration :

$$\lim_{t\to+\infty} \langle \epsilon - 3P \rangle_{\alpha,\dot{\alpha}} = \frac{1}{T}\int_0^T dt\ [\epsilon - 3P]_{\alpha,\dot{\alpha}=0} = 0\ .$$

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Gluon saturation
Why small-x gluons matter
Gluon evolution
Color Glass Condensate

High energy collisions
Power counting
Leading Order
Next to Leading Order
Factorization

Correlations
RHIC and LHC data
Ridge in Au-Au collisions
Ridge in p-p collisions

Emergence of flow
Hydro in AA collisions
Toy scalar model
Pressure at LO and NLO
Resummation
Zero mode fluctuations
Complete spectrum

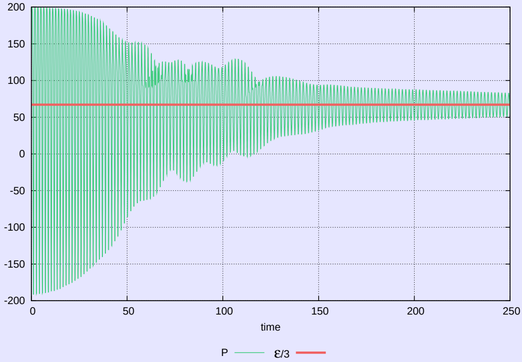
Summary

4 Emergence of hydrodynamical flow

- Hydrodynamics in AA collisions
- Toy scalar model
- Pressure at LO and NLO
- Unstable modes and resummation
- Zero mode fluctuations only
- Complete fluctuation spectrum

Relaxation of the pressure

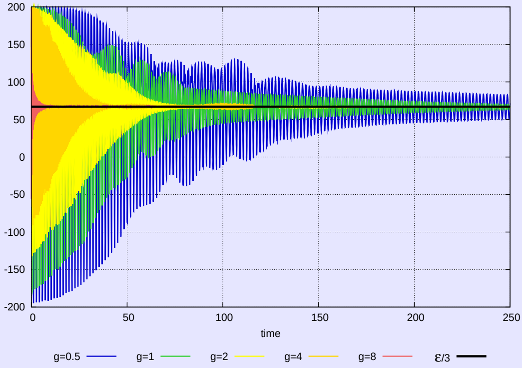
$g = 0.5$



- Note: $g = 0.5$ is a very small coupling in this model ($V(\phi)$ has a $1/4!$ prefactor)

Coupling dependence

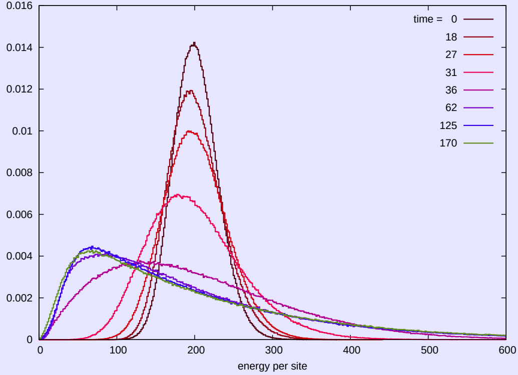
g dependence at fixed ϵ



- The relaxation time decreases rapidly with increasing g
- Note: $g = 8$ in this theory may be comparable to $g_s = 2$ in QCD, due to the $1/4!$ in the potential
- Caveat: our calculation becomes less reliable at large g

Energy distribution

Probability distribution $P(e)$ ($e =$ energy on one site, $g = 0.5$)



- At $t = 0$, narrow Gaussian fluctuations
- Very rapid change of shape around $t \approx 30$
- Shape closer to Bose-Einstein later on (but not quite)

Summary

- Collisions of hadrons/nuclei at high energy require some knowledge about the multigluon Fock states of the projectiles. An effective description of these states is provided by the color glass condensate

- Some new correlations among the final state particles emerge as a consequence of these high density states

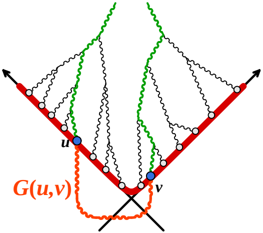
- The CGC computation of the energy-momentum tensor in heavy ion collisions has several problems :
    - No equation of state at LO
    - Secular divergences at NLO due to instabilities

- A resummation of higher order terms leads to :
    - Cancellation of all the secular divergences
    - Relaxation of the pressure towards an equation of state
    - More thermal-like energy density fluctuations

Power counting

François Gelis

- The instabilities are triggered by the 2-point function :



- Power counting : $\mathcal{G} \sim \mathcal{O}(1)$, $\bullet \sim \mathbb{T}\mathcal{A} \sim \mathcal{O}(g\, e^{\sqrt{\mu\tau}})$
- This 1-loop term is of order $g^2\, e^{2\sqrt{\mu\tau}}$ relative to the LO

Power counting

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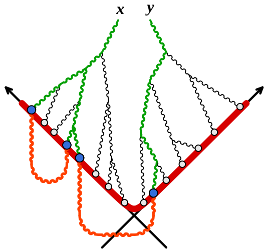
- At n-loop order, one should pick the terms that have the fastest growth in time

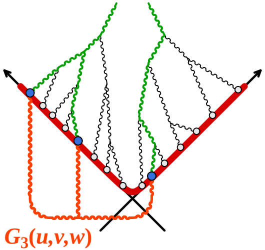
  $\triangleright$ maximize the number of locations where the initial field is perturbed on the light-cone, while minimizing the powers of α_s



- This 2-loop term is of order $g^4\, e^{4\sqrt{\mu\tau}}$ relative to the LO

Power counting

- Non-Gaussian correlations are suppressed :



- Power counting : $\mathcal{G}_3 \sim \mathcal{O}(g)$, $\bullet \sim \mathcal{O}(g\, e^{\sqrt{\mu\tau}})$
- This 2-loop term is of order $g^4\, e^{3\sqrt{\mu\tau}}$ relative to the LO
  - ▷ subleading