

# Learning from WIMPs

*Manuel Drees*

*Bonn University*

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# Introduction: WIMPs as Dark Matter

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- Models of structure formation,  $X$  ray temperature of clusters of galaxies, ...
- **Cosmic Microwave Background anisotropies (WMAP)**  
imply  $\Omega_{\text{DM}}h^2 = 0.105^{+0.007}_{-0.013}$  Spergel et al., astro-ph/0603449

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- In standard cosmology, roughly weak cross section automatically gives roughly right relic density for thermal WIMPs! (On logarithmic scale)
- Roughly weak interactions may allow both *direct* and *indirect* detection of WIMPs

# WIMP production

Let  $\chi$  be a generic DM particle,  $n_\chi$  its number density (unit:  $\text{GeV}^3$ ). Assume  $\chi = \bar{\chi}$ , i.e.  $\chi\chi \leftrightarrow \text{SM particles}$  is possible, but single production of  $\chi$  is forbidden by some symmetry.

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Evolution of  $n_\chi$  determined by **Boltzmann equation**:

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle\sigma_{\text{ann}}v\rangle (n_\chi^2 - n_{\chi,\text{eq}}^2)$$

$H = \dot{R}/R$  : Hubble parameter

$\langle \dots \rangle$  : Thermal averaging

$\sigma_{\text{ann}} = \sigma(\chi\chi \rightarrow \text{SM particles})$

$v$  : relative velocity between  $\chi$ 's in their cms

$n_{\chi,\text{eq}}$  :  $\chi$  density in full equilibrium



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Gives

$$\Omega_\chi h^2 \propto \frac{1}{\langle v \sigma_{\text{ann}} \rangle} \sim 0.1 \text{ for } \sigma_{\text{ann}} \sim \text{pb}$$

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Can we test these assumptions, if  $\Omega_\chi$  and “all” particle physics properties of  $\chi$  are known?

# Low temperature scenario

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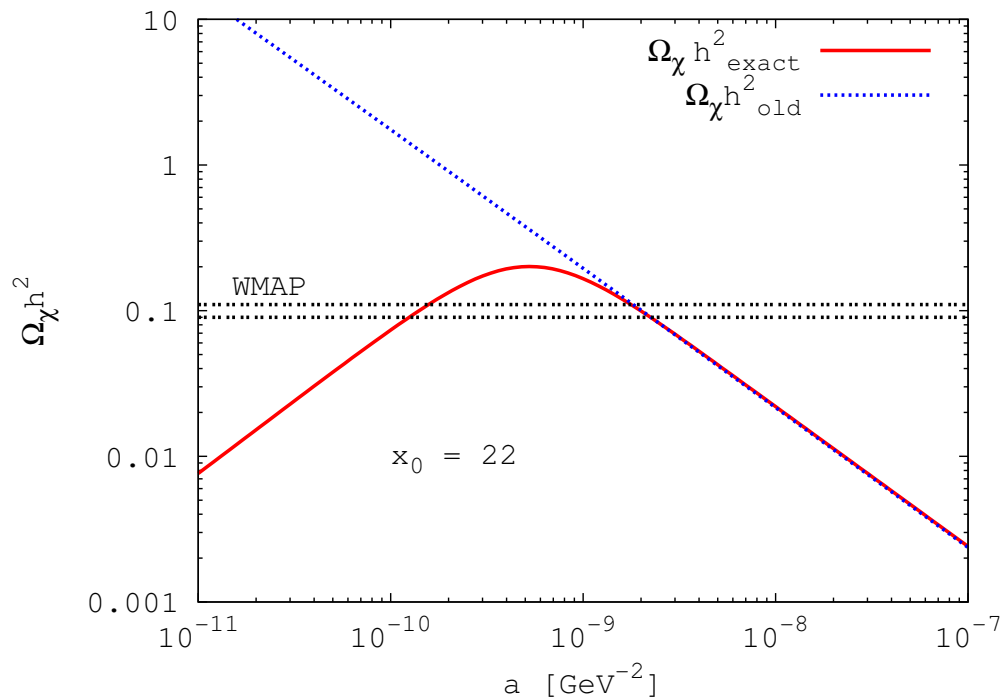
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## Low temperature scenario (cont.'d)

Using explicit form of  $H$ ,  $Y_{\chi,\text{eq}}$ , Boltzmann eq. becomes

$$\frac{dY_{\chi}}{dx} = -f \left( a + \frac{6b}{x} \right) x^{-2} \left( Y_{\chi}^2 - cx^3 e^{-2x} \right) .$$

$$f = 1.32 m_{\chi} M_{\text{Pl}} \sqrt{g_*}, \quad c = 0.0210 g_{\chi}^2 / g_*^2$$



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For  $T_0 \ll T_F$ : Annihilation term  $\propto Y_{\chi}^2$  negligible: defines 0-th order solution  $Y_0(x)$ , with

$$Y_0(x \rightarrow \infty) = fc \left[ \frac{a}{2} x_R e^{-2x_R} + \left( \frac{a}{4} + 3b \right) e^{-2x_R} \right] .$$

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For intermediate temperatures,  $T_0 \lesssim T_F$ : Define 1st-order solution

$$Y_1 = Y_0 + \delta.$$

$\delta < 0$  describes pure annihilation:

$$\frac{d\delta}{dx} = -f \left( a + \frac{6b}{x} \right) \frac{Y_0(x)^2}{x^2}.$$

$\delta(x)$  can be calculated analytically:  $\delta \propto \sigma_{\text{ann}}^3$

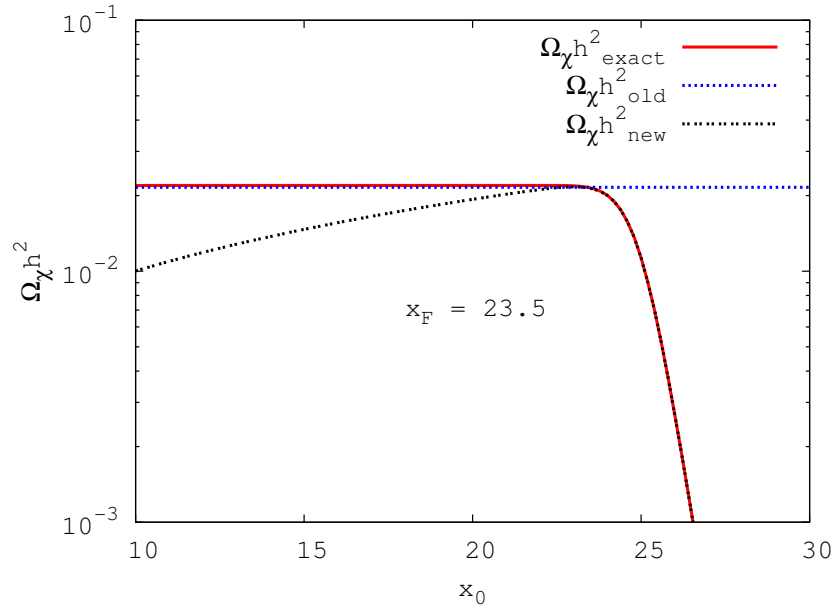
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Get good results for  $\Omega_\chi h^2$  for all  $T_0 \leq T_F$  through “resummation”:

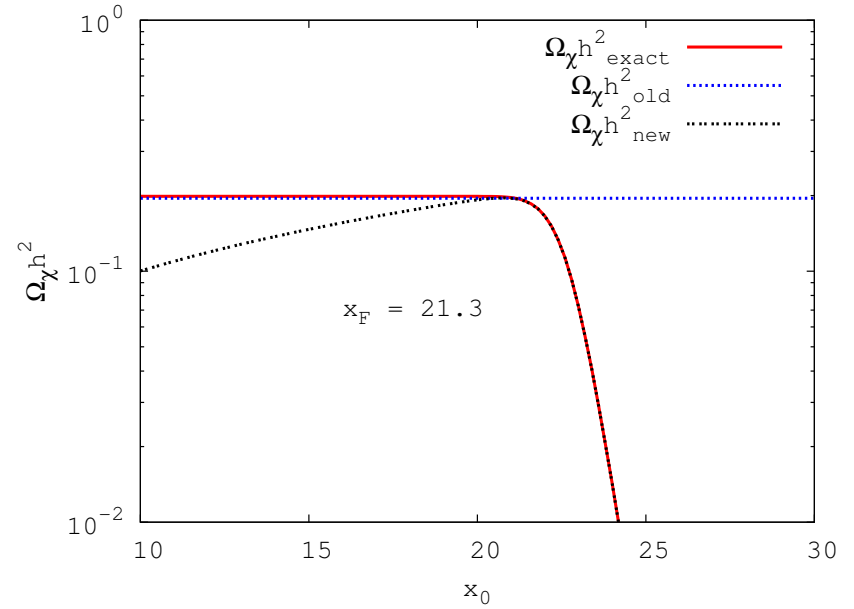
$$Y_1 = Y_0 \left( 1 + \frac{\delta}{Y_0} \right) \simeq \frac{Y_0}{1 - \delta/Y_0} \equiv Y_{1,r}$$

$Y_{1,r} \propto 1/\sigma_{\text{ann}}$  for  $|\delta| \gg Y_0$  MD, Imminniyaz, Kakizaki, hep-ph/0603165

# Numerical comparison: $b = 0$

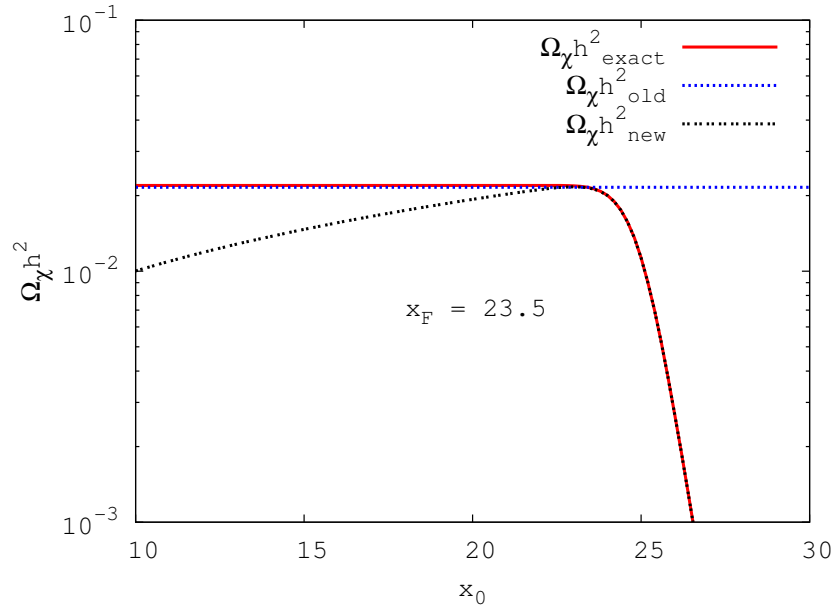


$$a = 10^{-8} \text{ GeV}^{-2}$$

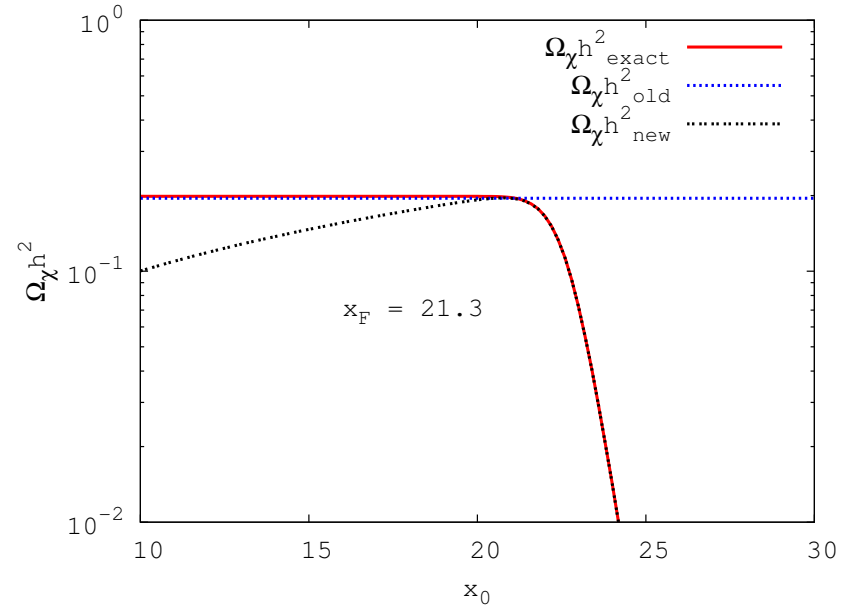


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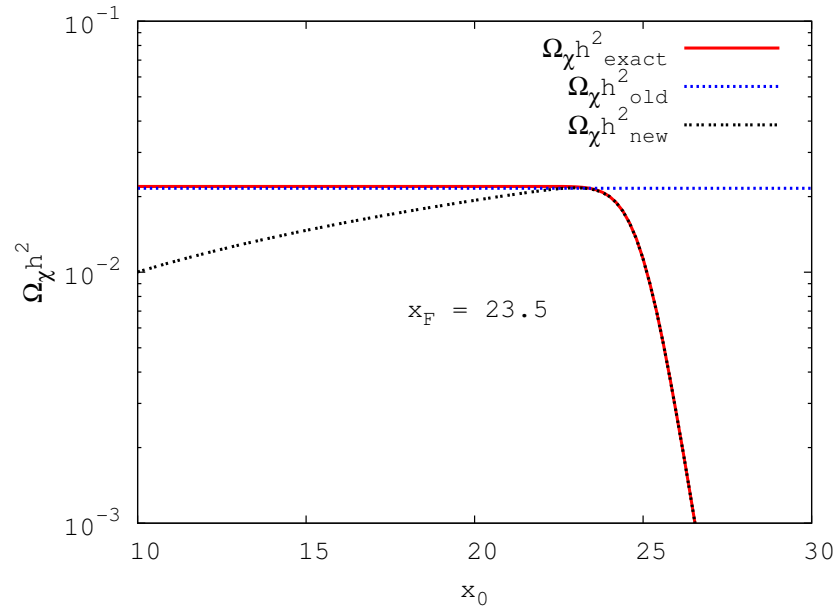
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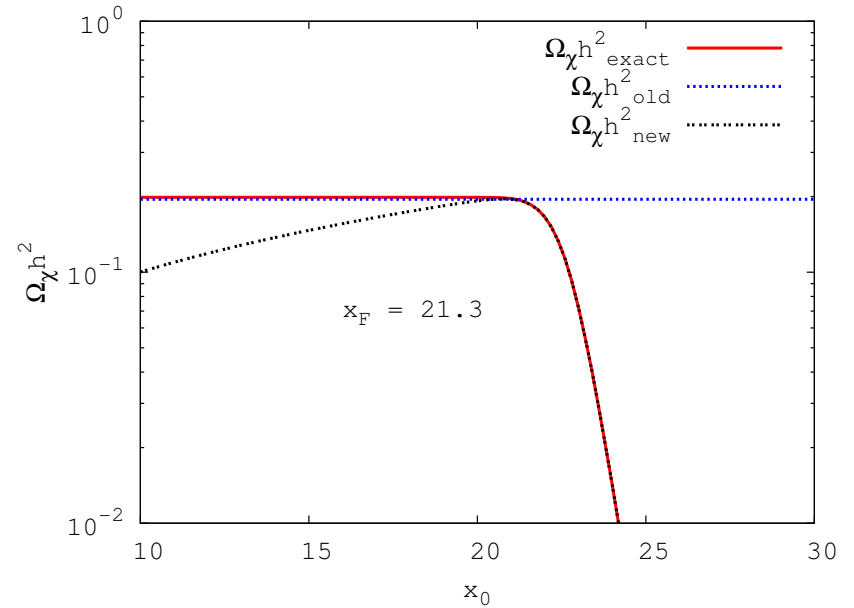
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Can extend validity of new solution to all  $T$ , including  $T \gg T_0$ , by using  $\Omega_\chi(T_{\text{max}})$  if  $T_0 > T_{\text{max}} \simeq T_F$

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Note:  $\Omega_\chi(T_0) \leq \Omega_\chi(T_0 \gg T_F)$

# Application: lower bound on $T_0$ for thermal WIMP

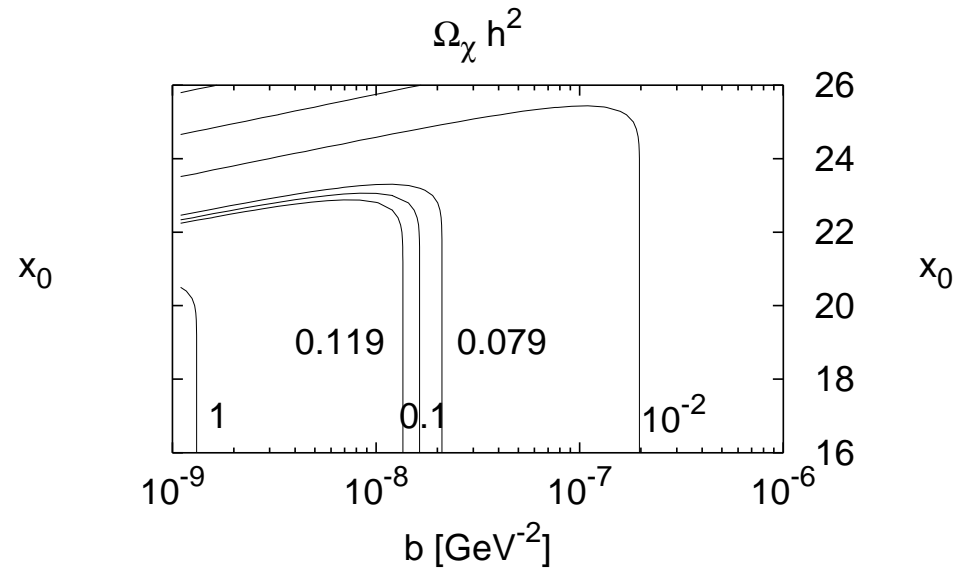
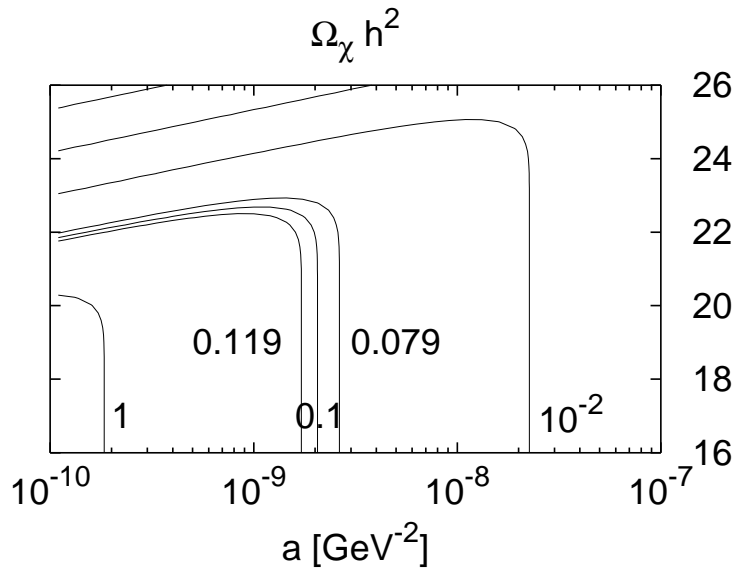
MD, Imminniyaz, Kakizaki, arXiv:0704.1590 [hep-ph]

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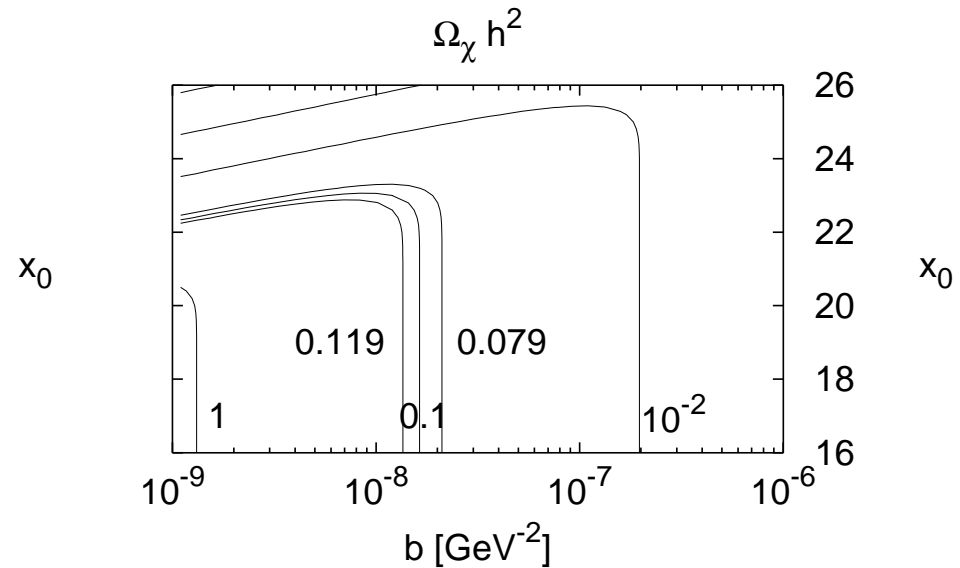
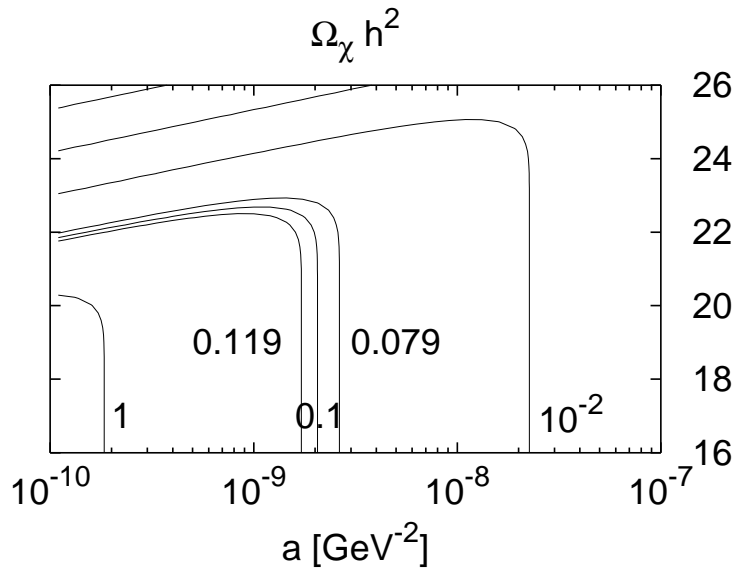




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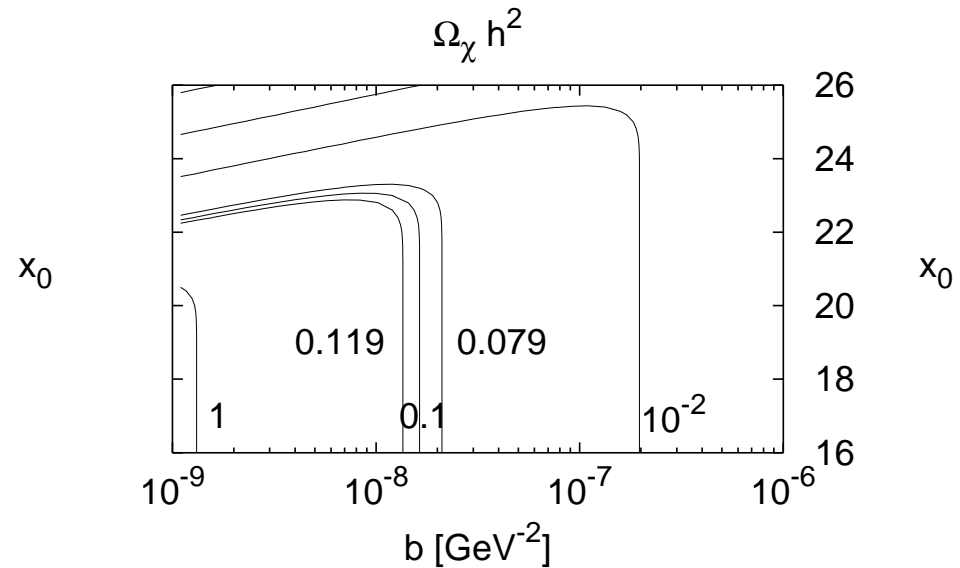
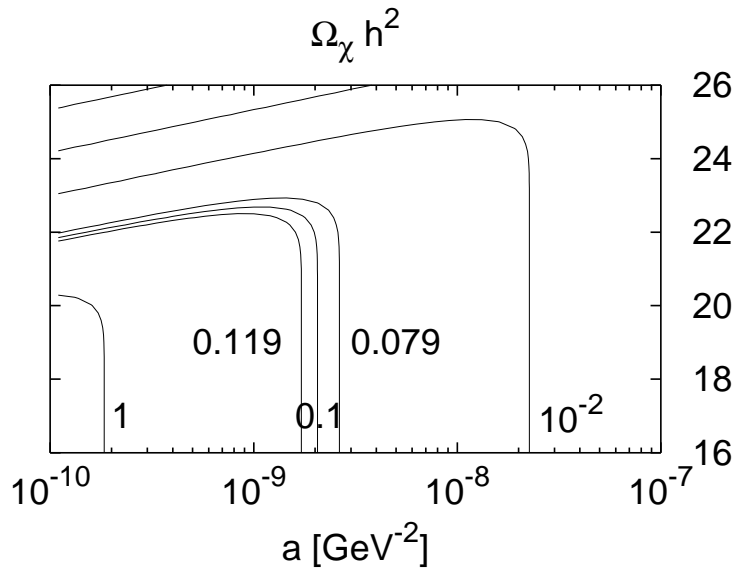


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If  $T_0 \simeq m_\chi/22$ : Get right  $\Omega_\chi h^2$  for wide range of cross sections!

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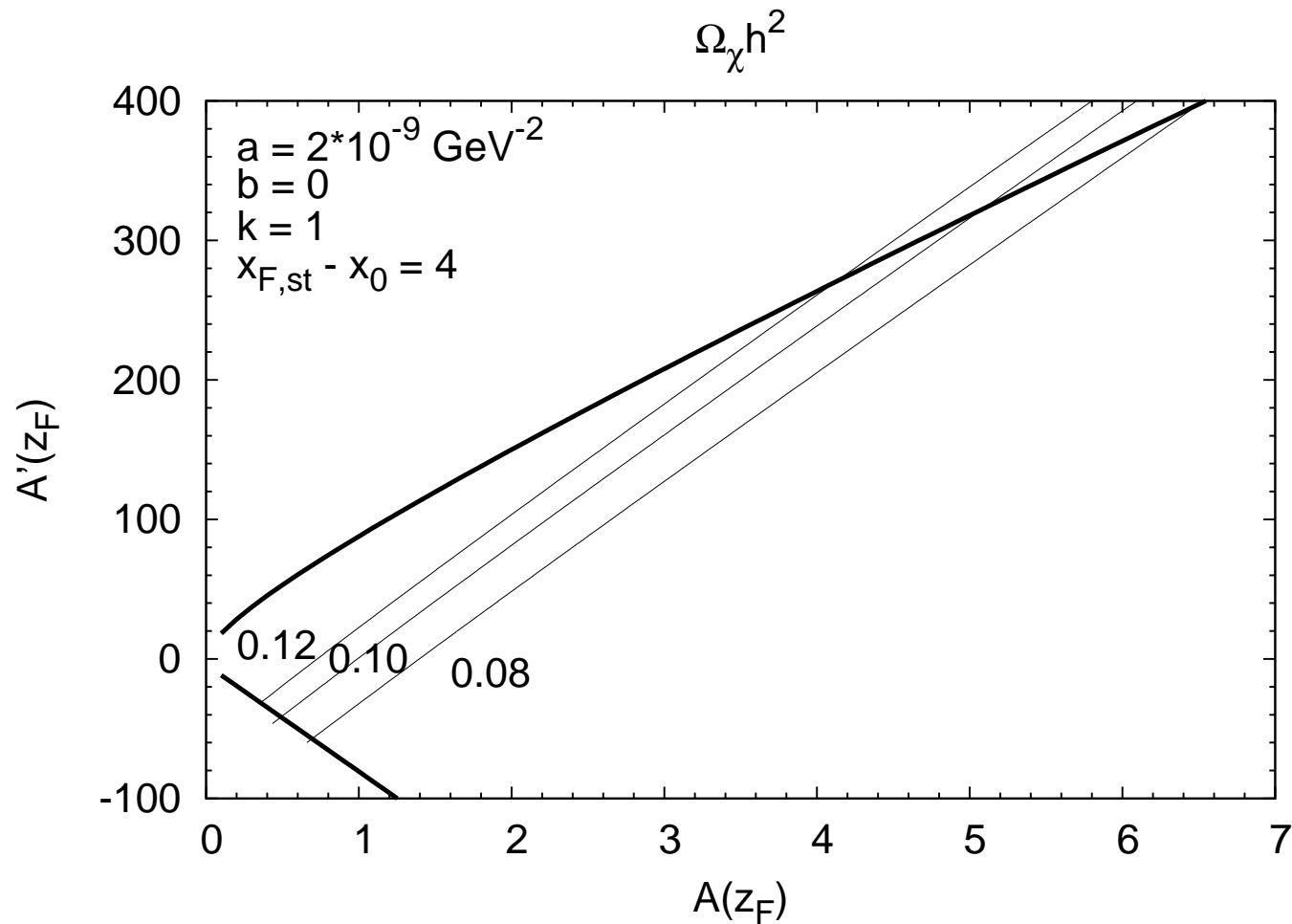
- Successful BBN  $\implies k \equiv A(z \rightarrow 0) = 1.0 \pm 0.2$

## Constraining $H(T)$ (cont.d)

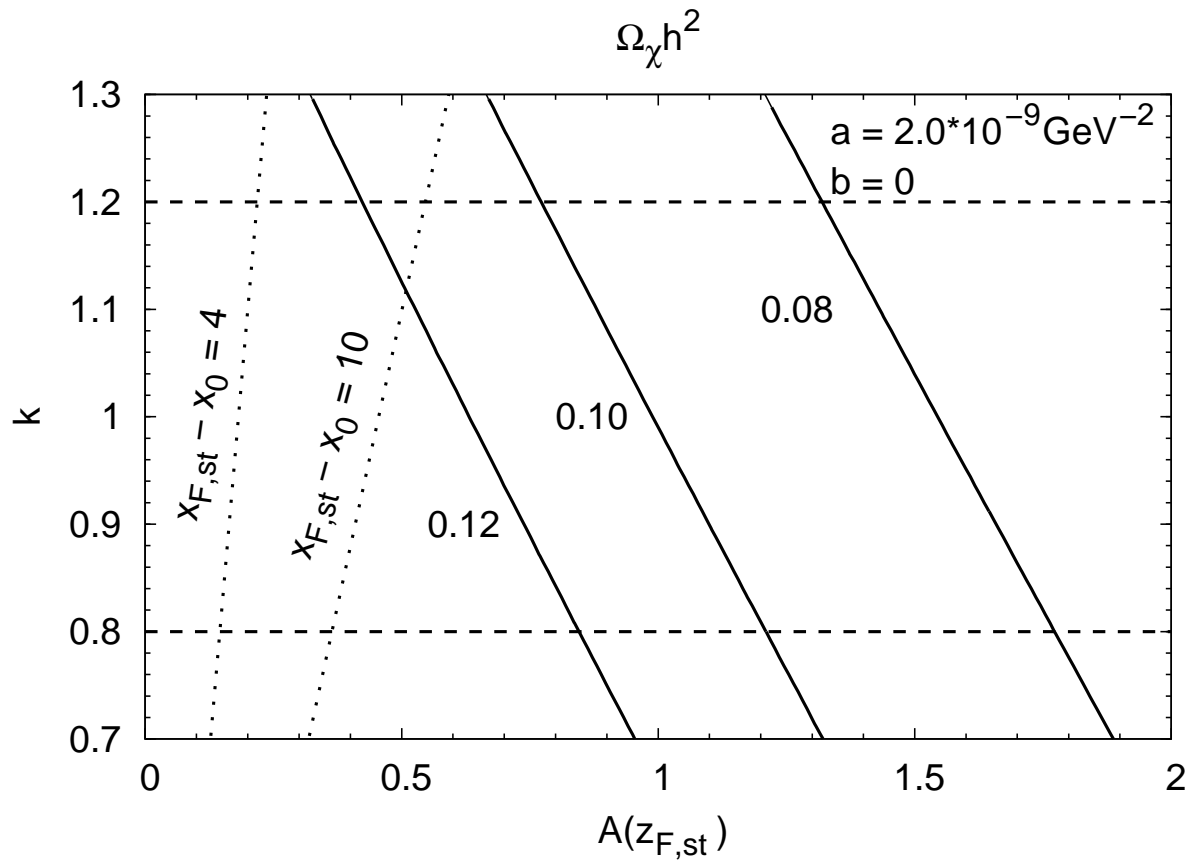
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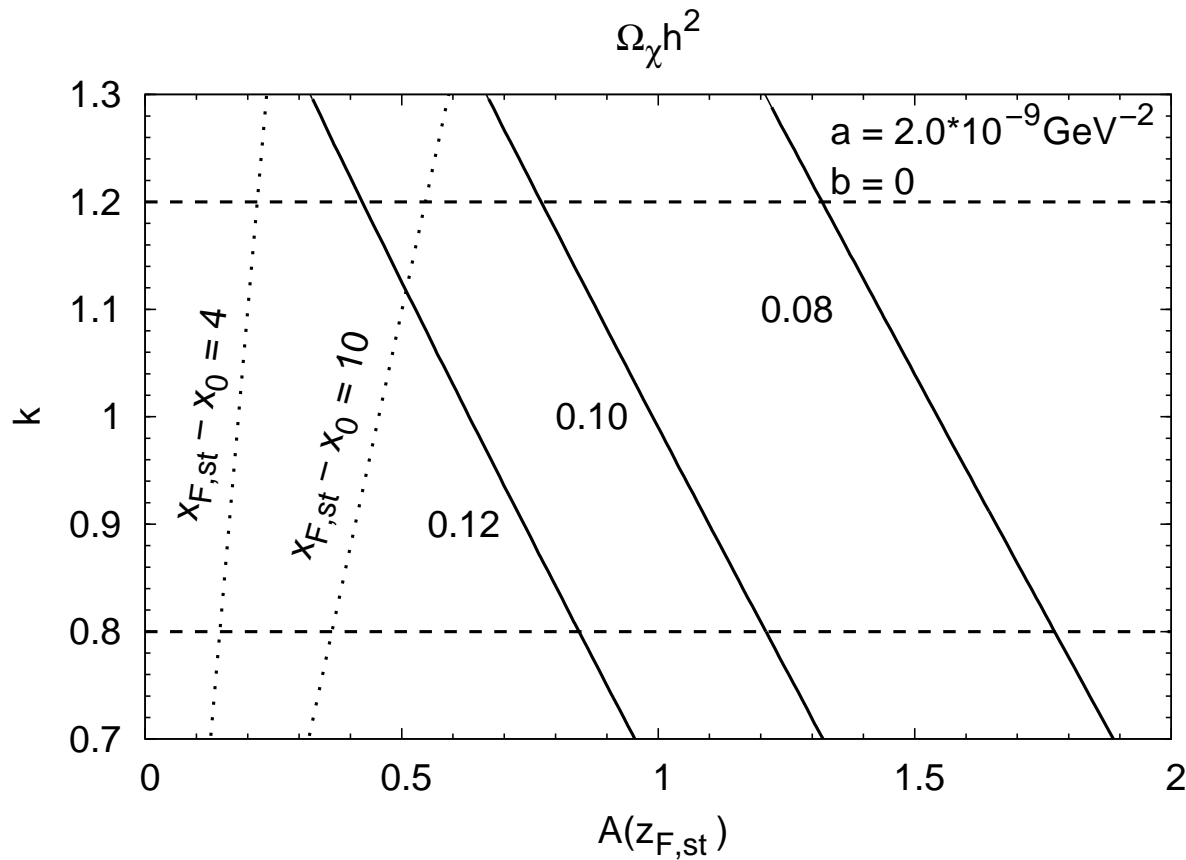
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Relative constraint on  $A(z_{F,st})$  weaker than that on  $\Omega_\chi h^2$ .

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- Is being pursued vigorously around the world!

# Direct WIMP detection: theory

Counting rate given by

$$\frac{dR}{dQ} = AF^2(Q) \int_{v_{\min}}^{v_{\text{esc}}} \frac{f_1(v)}{v} dv$$

$Q$ : recoil energy

$A = \rho\sigma_0 / (2m_\chi m_r) = \text{const.}$ : encodes particle physics

$F(Q)$ : nuclear form factor

$v$ : WIMP velocity in lab frame

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In principle, can invert this relation to measure  $f_1(v)$ !

# Direct reconstruction of $f_1$

MD & C.L. Shan, astro-ph/0703651

$$f_1(v) = \mathcal{N} \left\{ -2Q \frac{d}{dQ} \left[ \frac{1}{F^2(Q)} \frac{dR}{dQ} \right] \right\}_{Q=2m_r^2 v^2 / m_N}$$

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Need to know  $m_\chi$ , but do *not* need  $\sigma_0, \rho$ .

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$$f_1(v) = \mathcal{N} \left\{ -2Q \frac{d}{dQ} \left[ \frac{1}{F^2(Q)} \frac{dR}{dQ} \right] \right\}_{Q=2m_r^2 v^2 / m_N}$$

$\mathcal{N}$ : Normalization ( $\int_0^\infty f_1(v) dv = 1$ ).

Need to know form factor  $\implies$  stick to spin-independent scattering.

Need to know  $m_\chi$ , but do *not* need  $\sigma_0, \rho$ .

Need to know *slope* of recoil spectrum!



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$dR/dQ$  is approximately exponential: better work with logarithmic slope

# Determining the logarithmic slope of $dR/dQ$

- Good local observable: Average energy transfer  $\langle Q \rangle_i$  in  $i$ -th bin

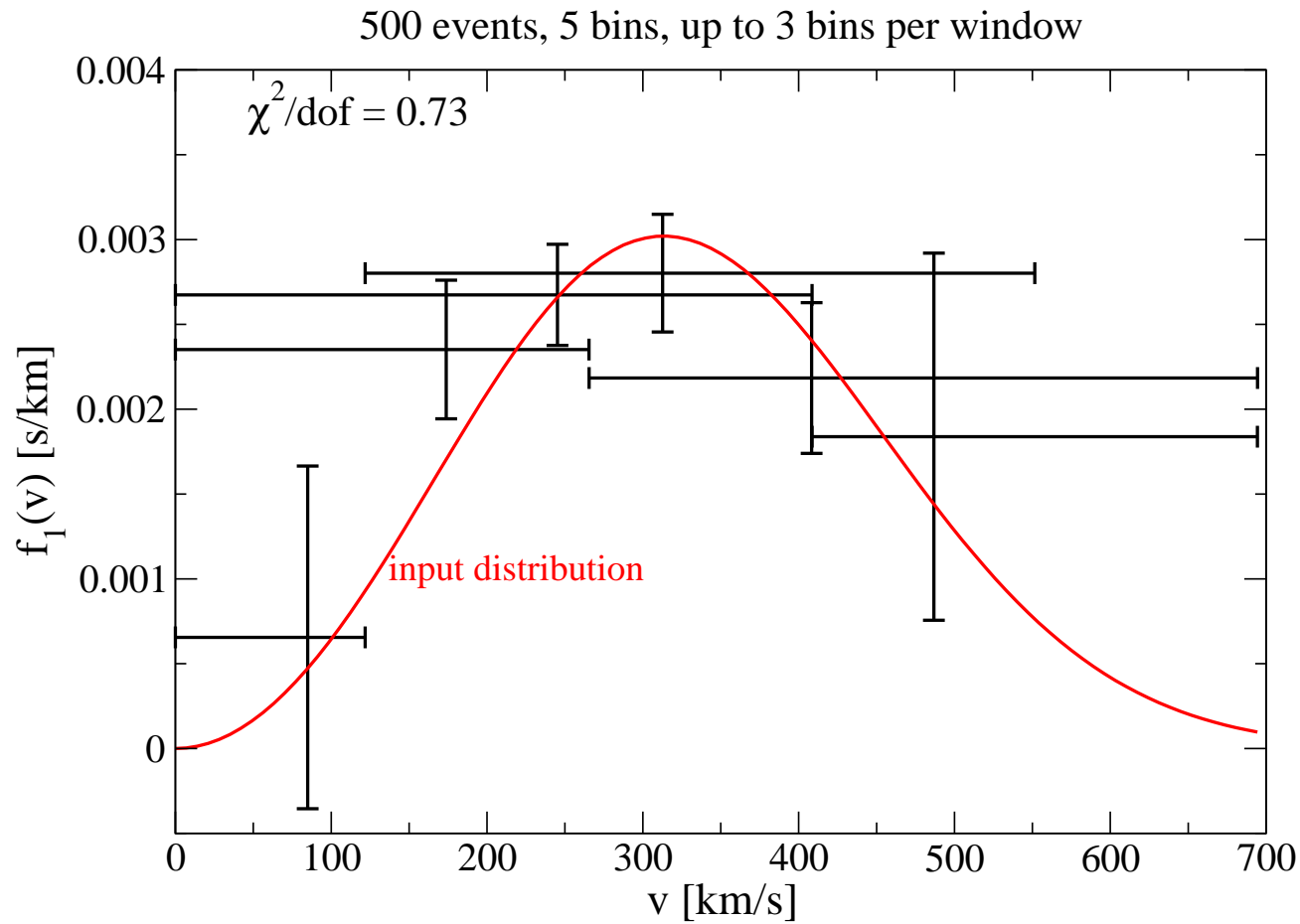
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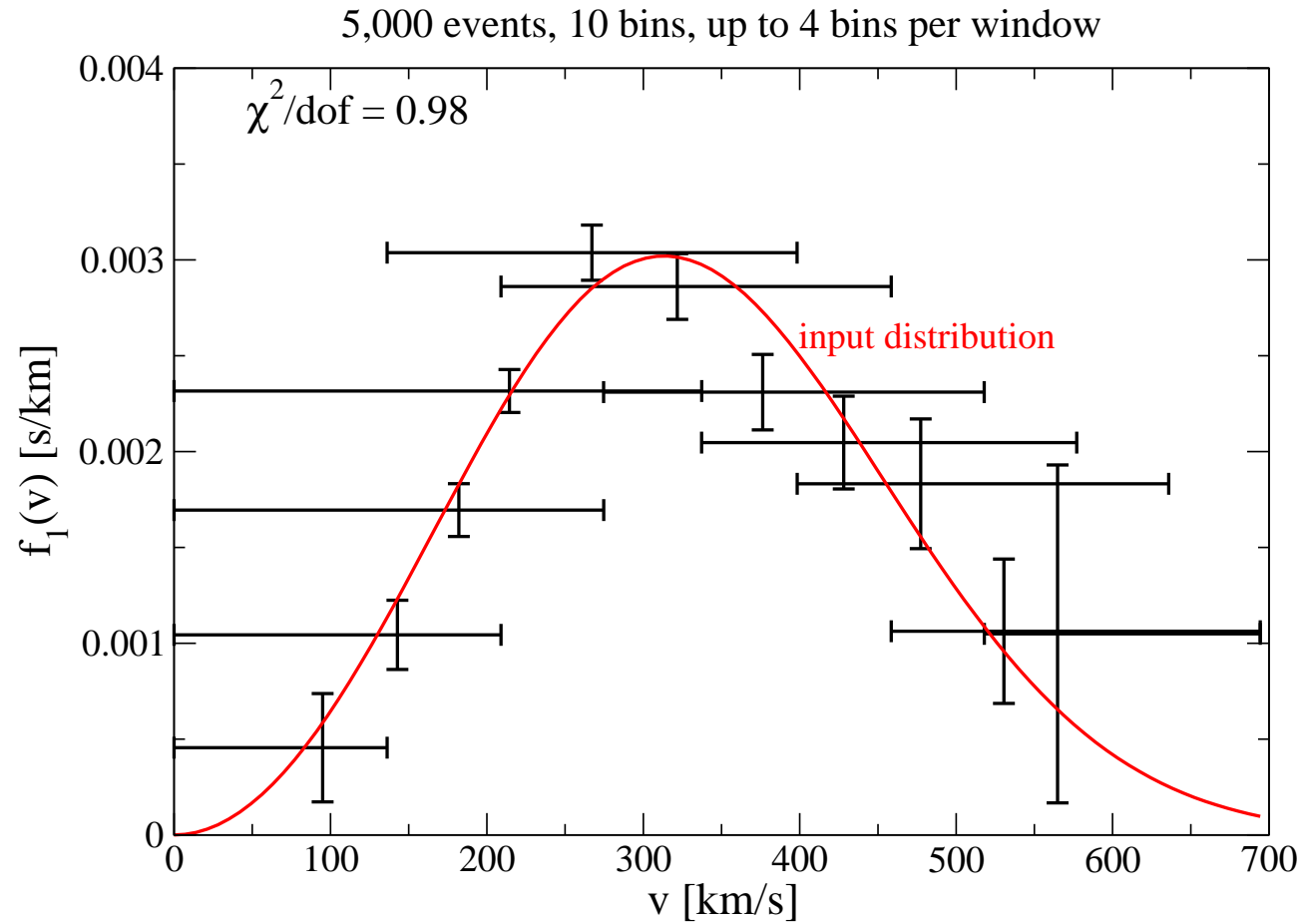
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- To maximize information: **use overlapping bins** (“windows”)

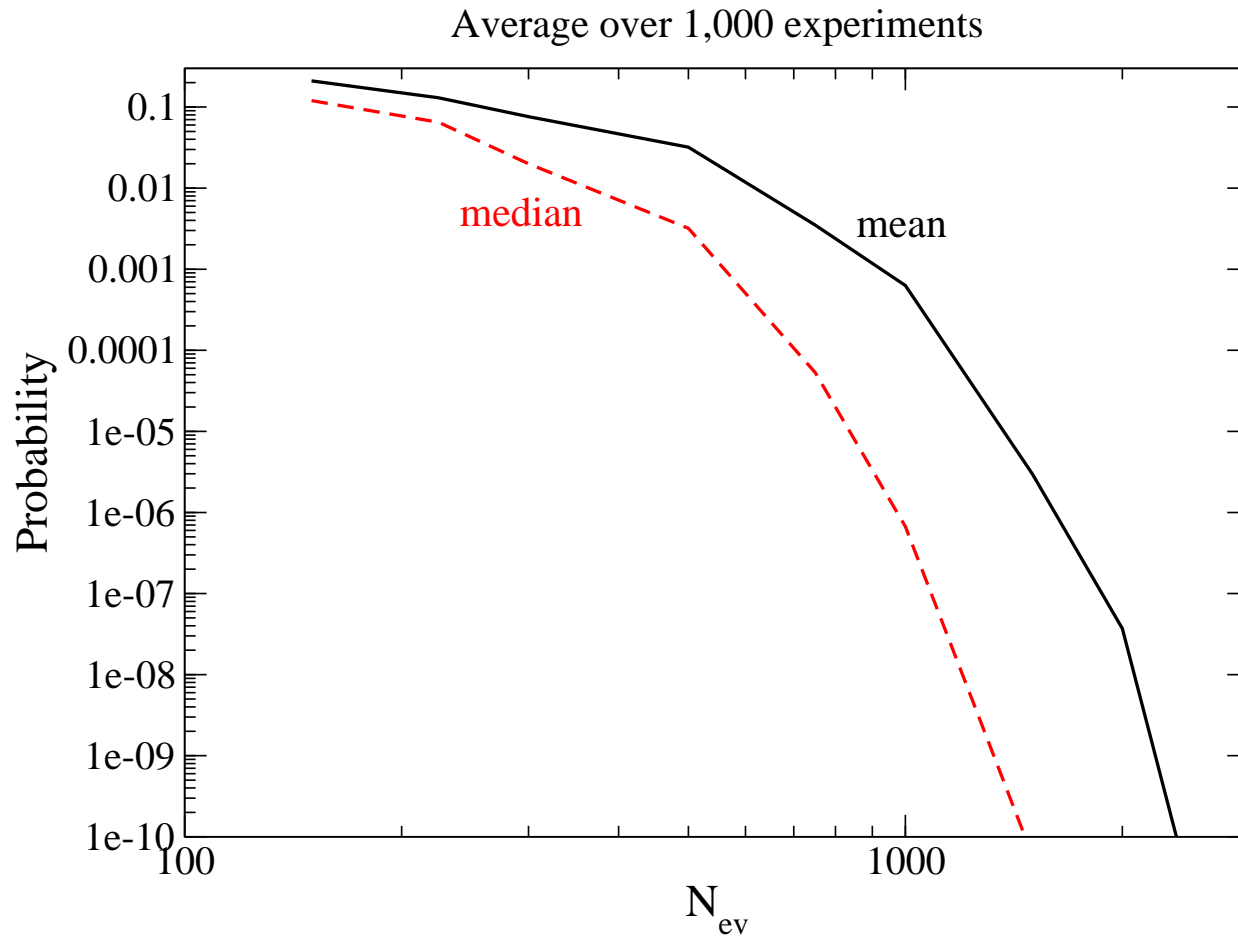
# Recoil spectrum: prediction and simulated measurement



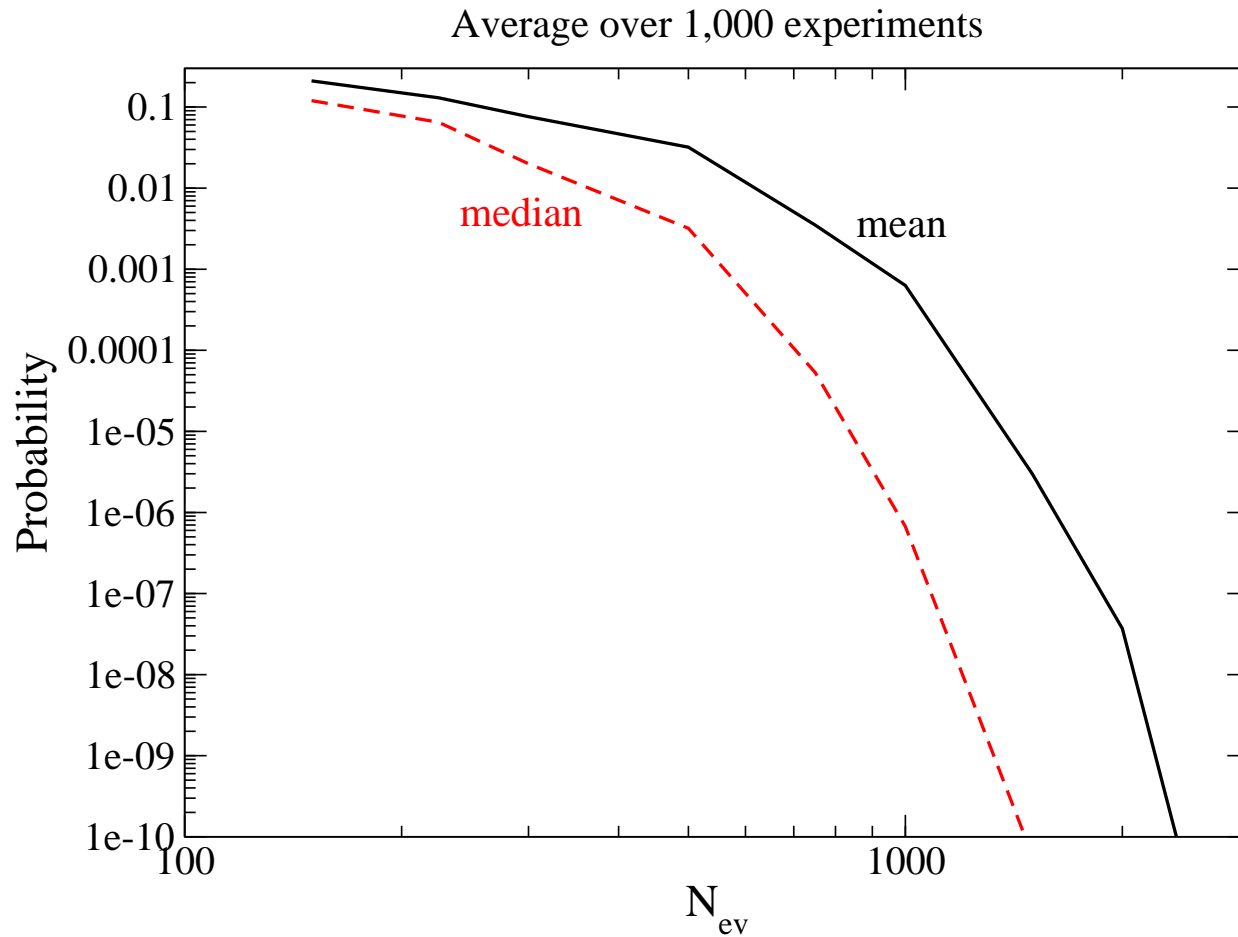
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Need several hundred events to begin direct reconstruction!



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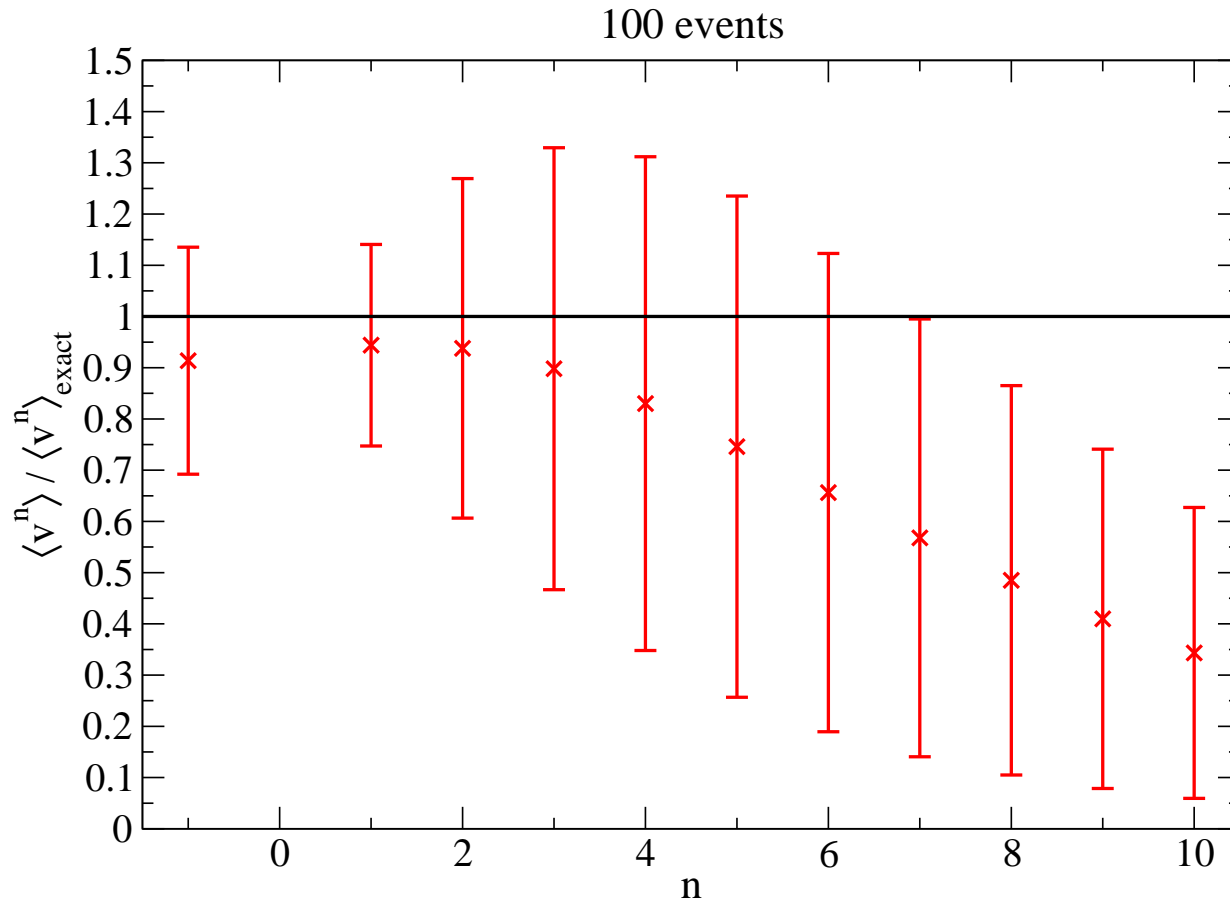
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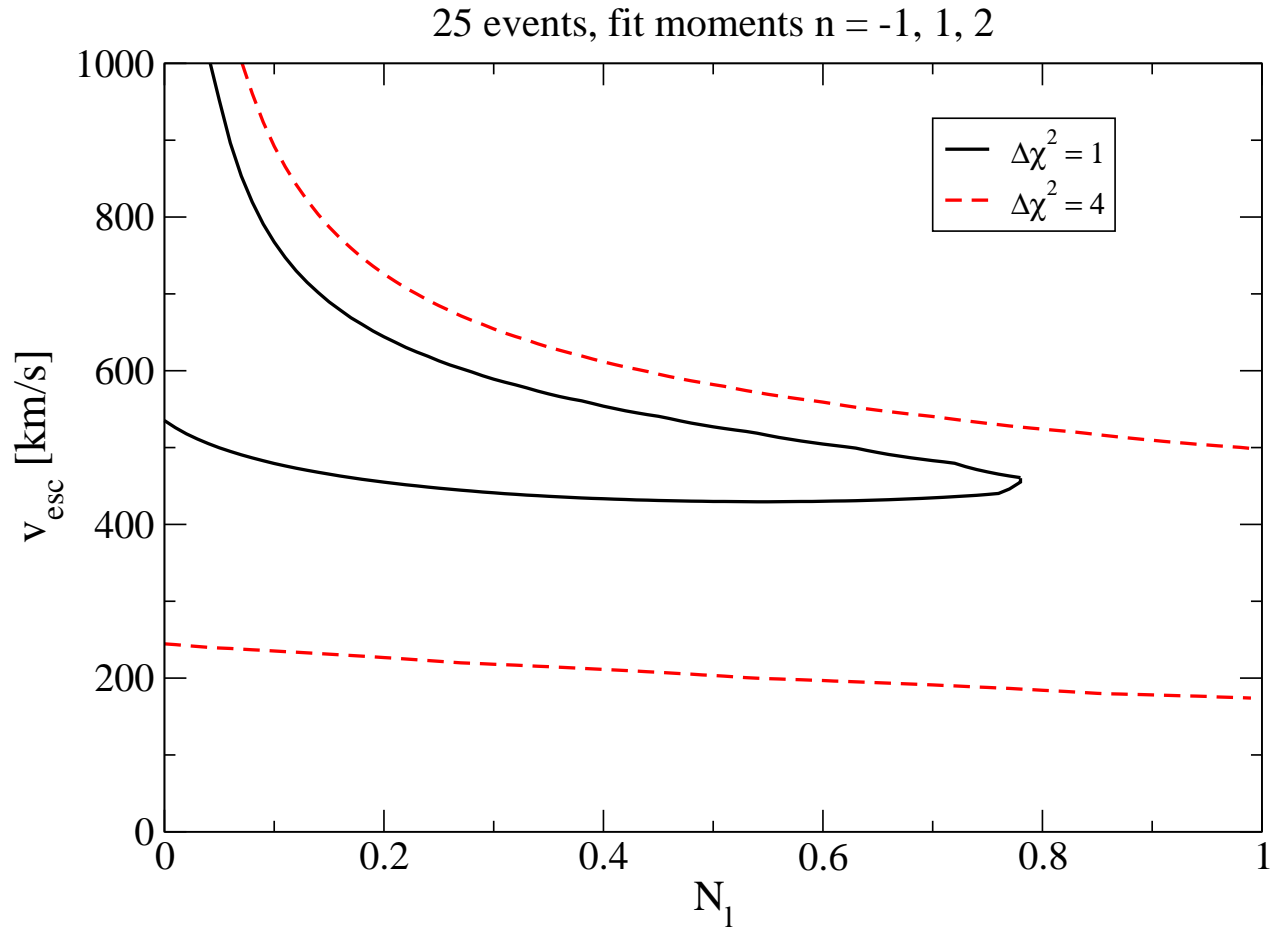
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High moments, and their errors, are underestimated in “typical” experiment: get large contribution from large  $Q$

# Determination of first 10 moments

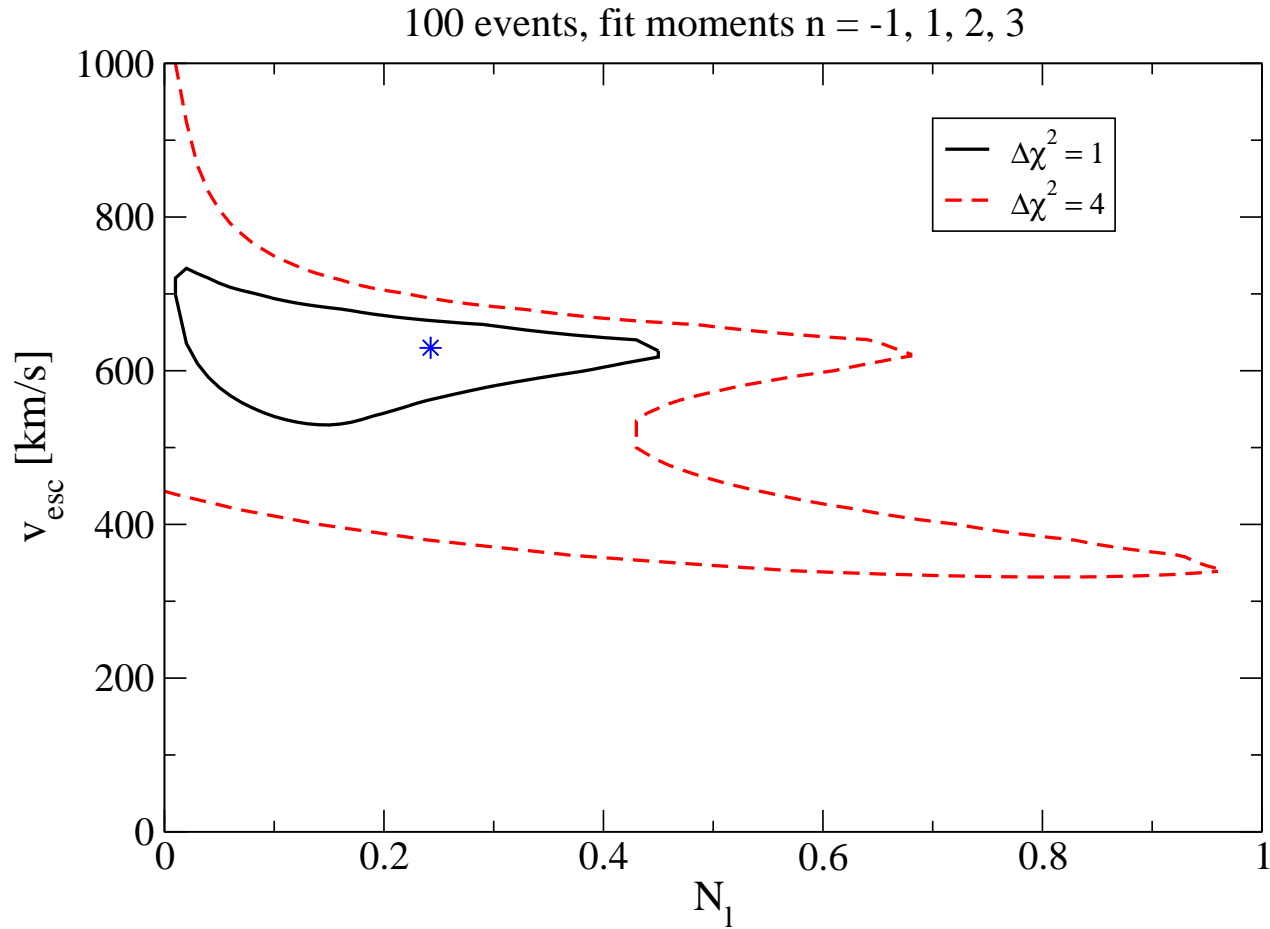


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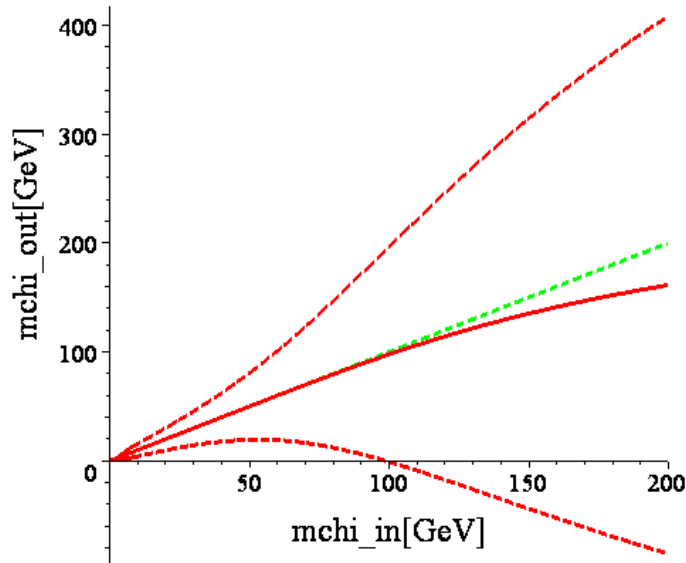


# Determining the WIMP mass

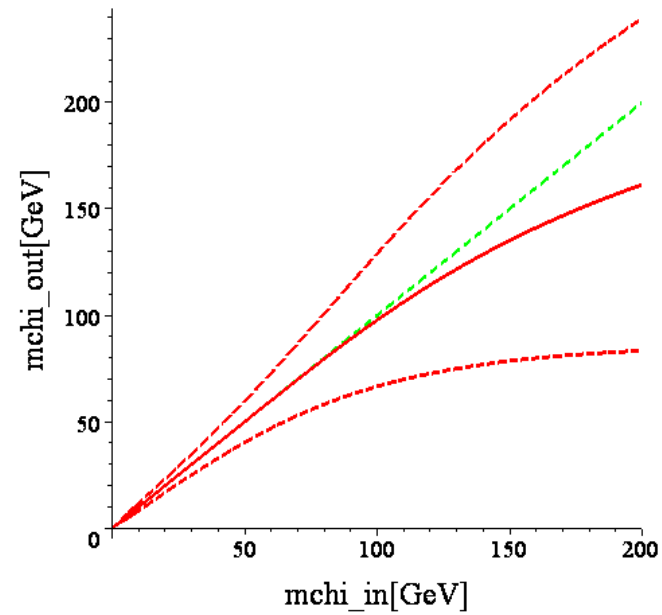
MD & C.L. Shan, in progress

Can determine  $m_\chi$  from requirement that different targets yield *same* moments of  $f_1$

$Q_{\max} = 200$  keV,  $Q_{\min} = 1$  keV,  $n = 1, 25 + 25$  events, Ge-76 + Si-28

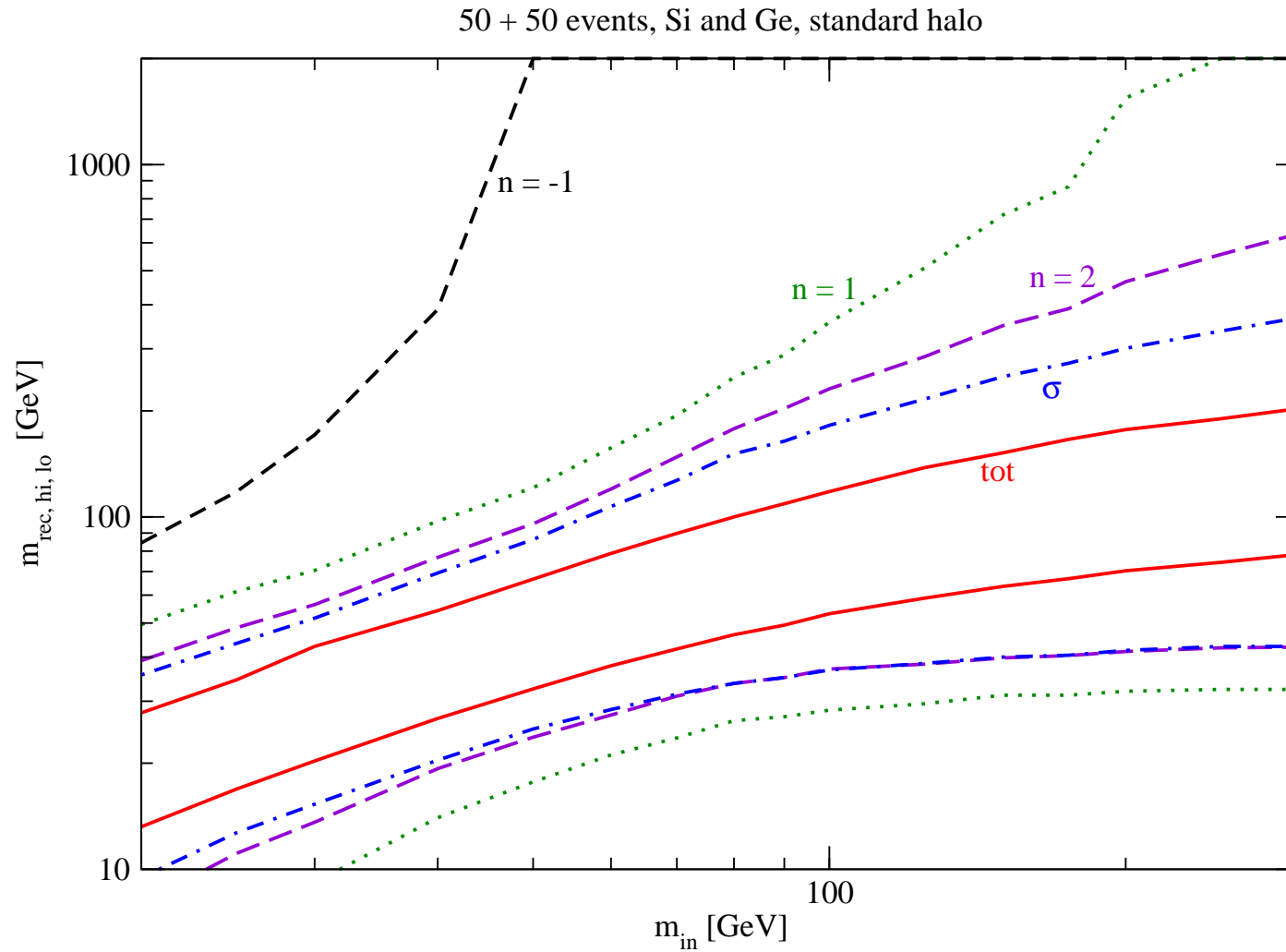


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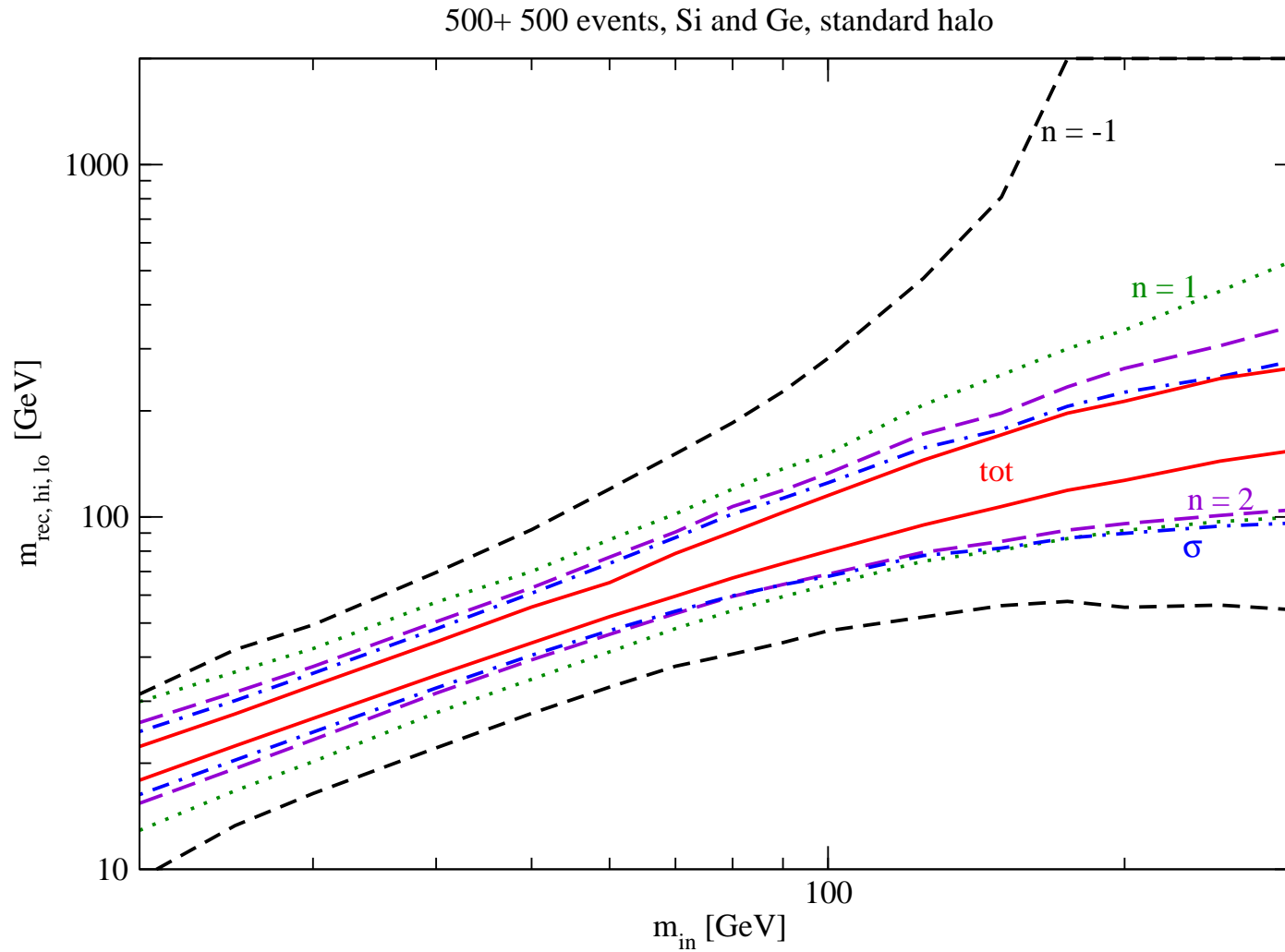
# Range of WIMP mass from simulation

Preliminary!



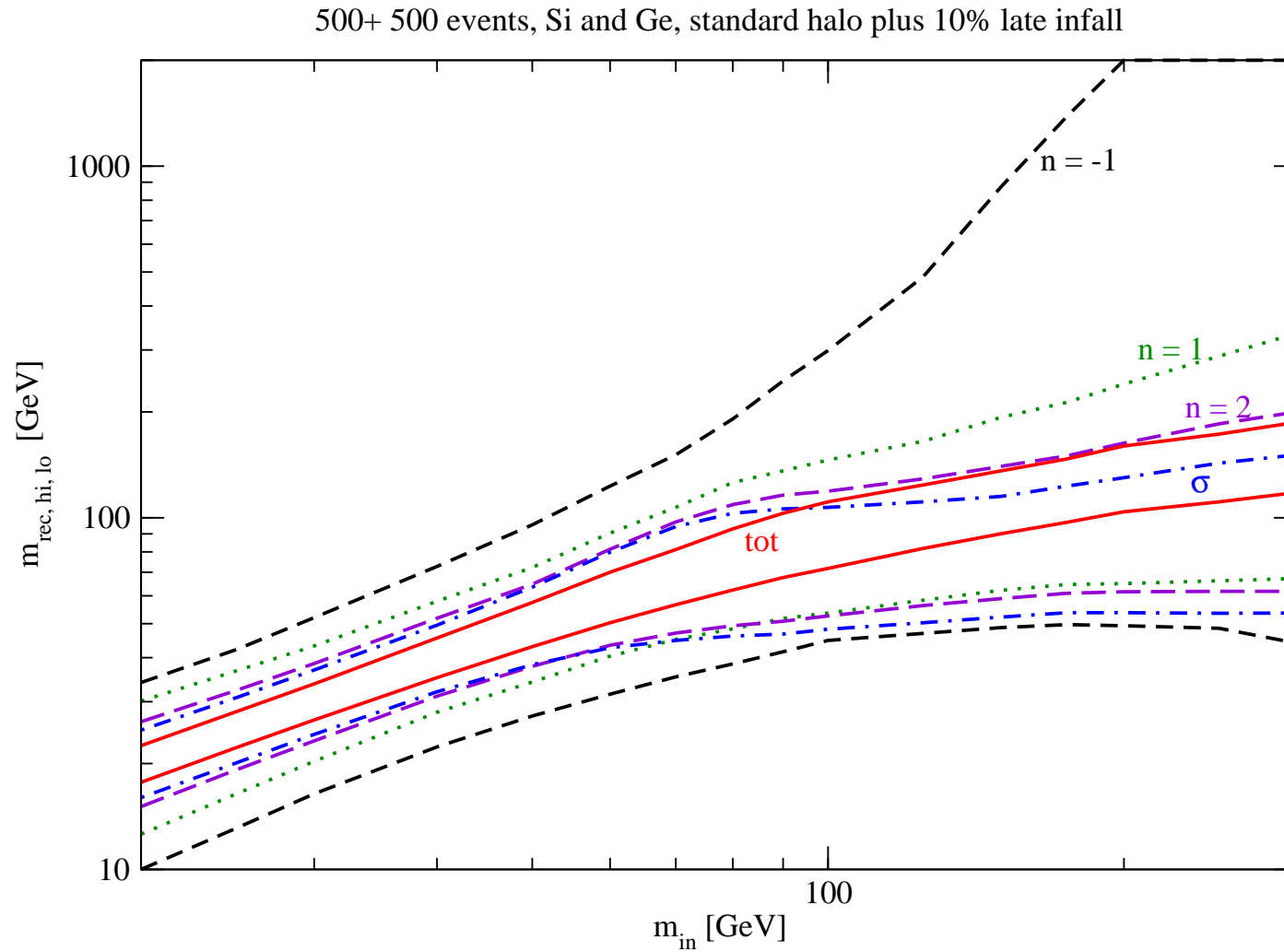
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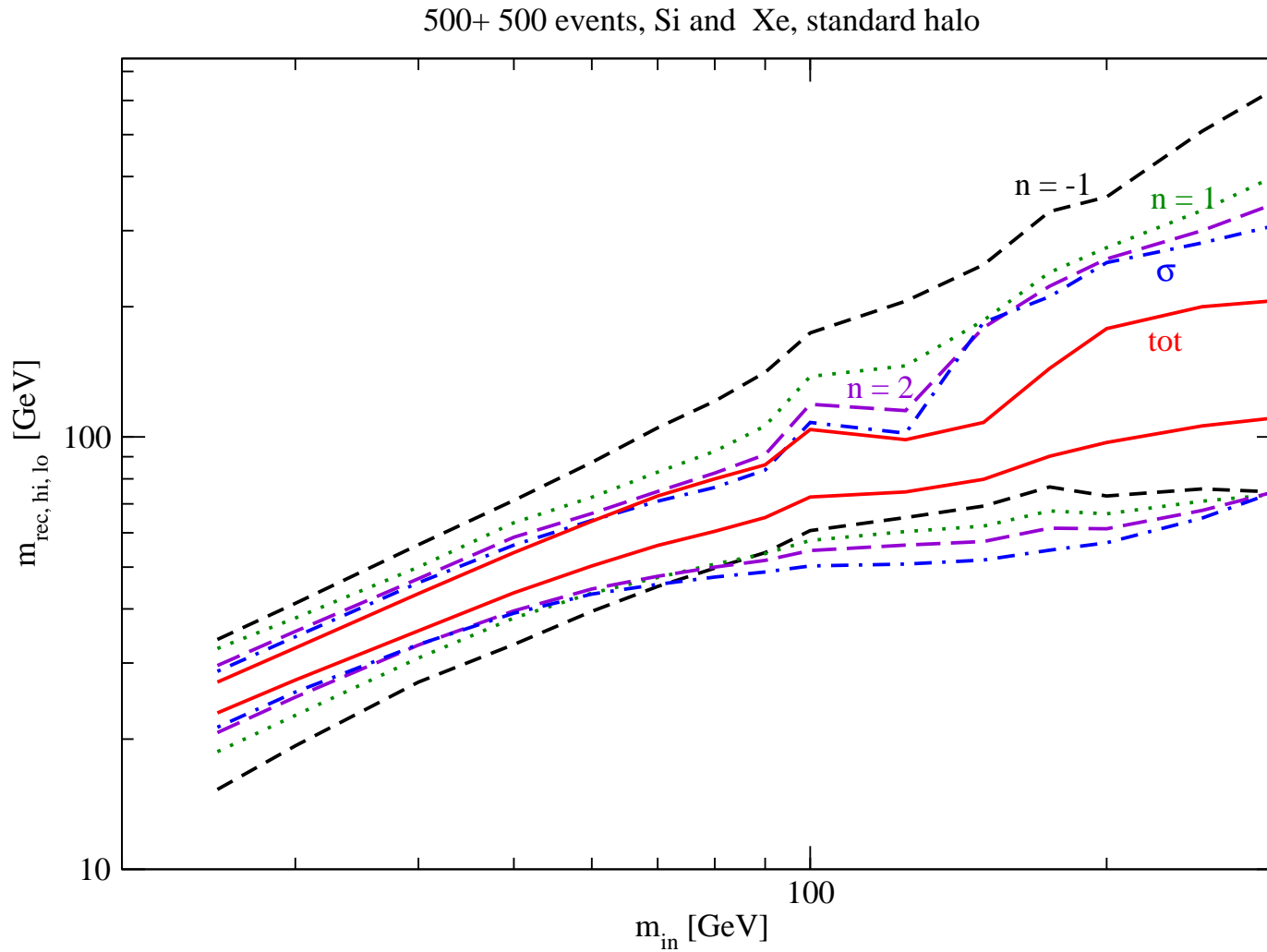
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- **Learning about WIMPs:** Can determine  $m_\chi$  from moments of  $f_1$  measured with two different targets.