CALCULATING THE RELIC DENSITY AT ONE-LOOP IN SUSY

A FEW EXAMPLES

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LAPTH

EURO-GDR
Dark Matter Session

Relic density of Dark Matter

$0.092 < \Omega_{DM}h^2 < 0.122$ Precision 10%(WMAP) $\rightarrow$ 2%!(PLANCK)

Era of precision measurement

Cosmology (radiation domination) + Particle physics

$$\Omega_{DM}h^2 \approx \frac{3 \times 10^{-27} cm^3 s^{-1}}{< \sigma(\chi^0\chi^0 \rightarrow SM)v >}$$

Precision

Need to know σ precisely

⇒ Computation of relic density

⇒ Parameters reconstruction at the LHC/LC

⇒ Check the underlying cosmological scenario

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Minimal Supersymmetric Standard Model

A lot of parameters (~100 without CP violation)

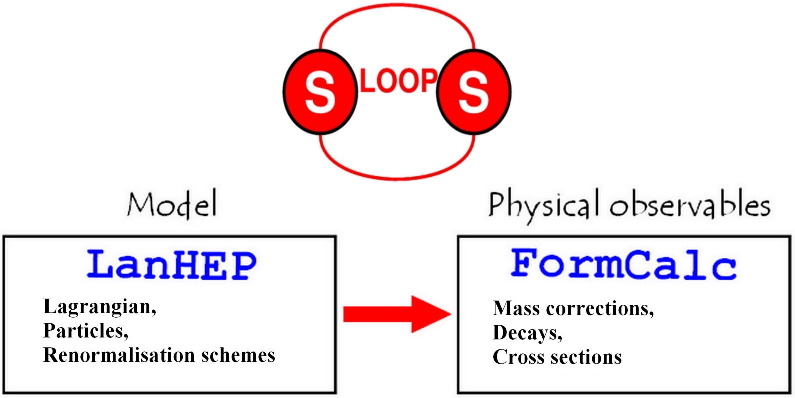
Sectors

Fermion	f
Gauge	$\gamma, Z^0, W^{\pm}$
Higgs	$H^0, h^0, A^0, H^{\pm}$
Sfermion	$\tilde{f}$
Chargino/Neutralino	$\chi_i^{\pm}, \chi_i^0$

A lot of vertices (~ 5000)

... and a lot of counter-terms!

SLOOPS (AUTOMATIC TOOL)



A code for the calculation of loops diagrams in the MSSM with application to collider physics, astrophysics and cosmology

Features of the code

- Complete and coherent renormalisation of the MSSM (On Shell scheme)
- Flexibility (between renormalisation schemes)
- Non linear gauge fixing

$$
\begin{aligned}
\mathcal{L}^{GF} &= -\frac{1}{\xi_W}|(\partial_\mu - ie\tilde{\alpha}A_\mu - igc_W\tilde{\beta}Z_\mu)W^{\mu+} \\
&+ i\xi_W\frac{g}{2}(v + \tilde{\delta}h^0 + \tilde{\omega}H^0 + i\tilde{\kappa}G^0 + i\tilde{\rho}A^0)G^+|^2 \\
&- \frac{1}{2\xi_Z}(\partial_\mu Z^\mu + \xi_Z\frac{g}{2c_W}(v + \tilde{\varepsilon}h^0 + \tilde{\gamma}H^0)G^0)^2 - \frac{1}{2\xi_A}(\partial_\mu A^\mu)^2
\end{aligned}
$$

Checks

- Gauge independent
- Finite

INPUT PARAMETERS

The MSSM contains 8 × 3 SUSY breaking parameters for sfermions, 3 × 3 fermion masses and 12 parameters for gauge couplings, scalar potential and the SUSY breaking gaugino masses:

$$\underbrace{g, g', g_s}_{\text{gauge}}, \underbrace{v_1, v_2}_{\text{v.e.v.}}, \underbrace{m_1, m_2, m_{12}}_{\text{scalar potential}}, \mu, \underbrace{M_1, M_2, M_3}_{\textit{breaking}}, \underbrace{M_L, M_R, A_f}_{\textit{sfermion}}$$

Set of parameters directly connected to the physical quantities (On Shell inputs):

$$\underbrace{\alpha(0), m_W, m_Z}_{\text{EW}}, \underbrace{"t_\beta = v_2/v_1", m_A, T_1, T_2}_{\textit{Higgs}}, \underbrace{m_{\chi_1^+}, m_{\chi_2^+}}_{\text{Chargino}}, \underbrace{m_{\chi_1^0}}_{\text{Neutralino}}, \underbrace{m_{\tilde{f}_1}, m_{\tilde{f}_2}, A_f}_{\textit{sfermion}}, \underbrace{\alpha_s, m_{\tilde{g}}}_{\text{QCD}}$$

How to define $\tan(\beta)$?/Scheme dependence

t_β doesn't represent a physical/measurable quantity

We have many different ways/schemes to define it:

$\overline{DR}$

δt_β is a pure divergence (Heinemeyer, Hollik and Weiglein *hep-ph/0412214*)

(non gauge invariant $\rightarrow$ we use the linear gauge)

MH

δt_β is defined from the mass m_H

(we loose a correction but the definition is physical)

$A^0\tau\tau$

δt_β is defined from the decay $A^0 \rightarrow \tau\tau$ (vertex $\propto m_\tau t_\beta$)

- At Tree Level: Comparisons with `Grace` and `CompHEP`
- At One-Loop (without renormalisation): Comparisons with `PLATON` and `DarkSUSY` (Boudjema, Semenov and Temes *hep-ph/0507127*)
  $\chi_1^0\chi_1^0 \to \gamma\gamma$, $\chi_1^0\chi_1^0 \to gg$, $\chi_1^0\chi_1^0 \to Z^0\gamma$
- Mass corrections:
  $H^\pm$, h^0 (Freitas and Stöckinger *hep-ph/0205281*),
  $\tilde{b}_i$ (Hollik and Rzehak *hep-ph/0305328*),
  χ_2^0, χ_3^0, χ_4^0 (Fritzsche and Hollik *hep-ph/0203159*)
- Decoupling (SUSY loops into SM processes)
- Collaboration with the `GRACE` group

Application to Dark Matter

$$\chi_1^0 = N_{11}\tilde{B} + N_{12}\tilde{W}^3 + N_{13}\tilde{H}_1^0 + N_{14}\tilde{H}_2^0$$

Annihilation & Coannihilation

$\chi\chi \to X_{SM}Y_{SM} \quad \chi\tilde{f} \to X_{SM}Y_{SM}$

- A large number of diagrams: UV finite, gauge independent
- Calculation of Hard/Soft brems.: k_c stability, IR finite (photon mass regulator)

Average

$$<\sigma v>= \frac{\Sigma_{ij} g_i g_j \int_{(m_1+m_2)^2} ds\, \sqrt{s} K_1(\sqrt{s}/T) p_{ij}^2 \sigma_{ij}(s)}{2T(\Sigma_i g_i m_i^2 K_2(m_i/T))^2}$$

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Average

For a preliminary study, we compute the approximation:

$$< \sigma v > \simeq a + b < v^2 > \text{ with } v = \text{relative velocity} \simeq 0.1 - 0.3$$

A FEW EXAMPLES

BINO CASE

$\chi\chi \rightarrow l^+ l^-$

COANNIHILATION CASE

$\chi\tilde{\tau}^+ \rightarrow \tau^+\gamma$
$\chi\tilde{\tau}^+ \rightarrow \tau^+ Z$
$\tilde{\tau}^+\tilde{\tau}^+ \rightarrow \tau^+\tau^+$

MIXED CASE

$\chi\chi \rightarrow W^+ W^-$
$\chi\chi \rightarrow ZZ$

BINO CASE

$(\times 10^{26} cm^3/s)$

$\chi\chi \to \tau^+\tau^-$ (36%)	Tree		$A\tau\tau$	$\overline{DR}$	MH
a	0.081		+38%	+35%	+15%
b	3.858		+18%	+18%	+18%
Ωh^2	0.166		0.138	0.138	0.141
$\frac{\delta \Omega h^2}{\Omega h^2}$			-17%	-17%	-15%

- $\text{Ampl}(v \to 0) \propto m_\tau$ thus $a \sim 0$
- $\alpha(0) \to \alpha(m_Z)$ implies a correction of -15%

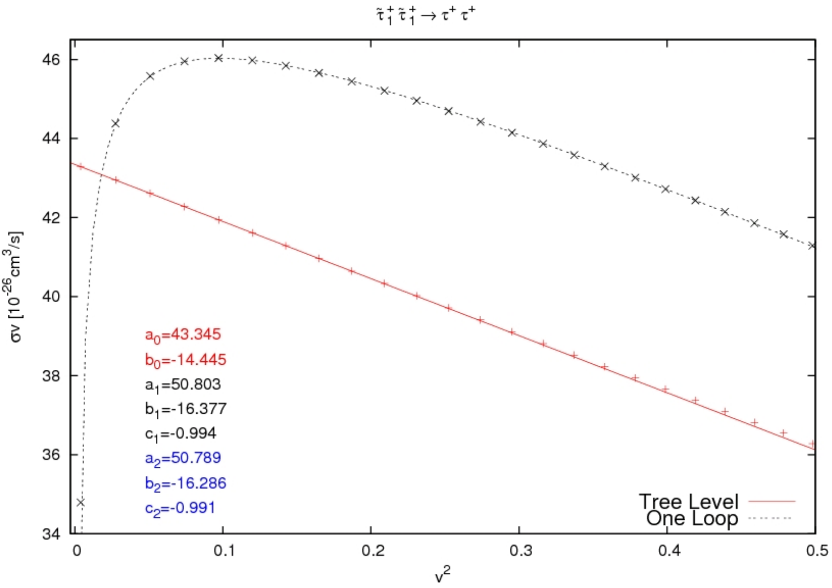
COANNIHILATION CASE

$(\times 10^{26} cm^3/s)$

$\chi\tilde{\tau}^+ \rightarrow \tau^+\gamma$ (37%)	Tree	$A\tau\tau$
a	4.342	+9%
b	-1.116	+9%
$\chi\tilde{\tau}^+ \rightarrow \tau^+ Z$ (10%)		
a	1.093	+21%
b	-0.214	+19%
$\tilde{\tau}^+\tilde{\tau}^+ \rightarrow \tau^+\tau^+$ (23%)		
a	43.345	+17%
b	-14.445	+13%
c	0	-0.994
Ωh^2	0.128	0.117
$\frac{\delta\Omega h^2}{\Omega h^2}$		-9%

- No t_β scheme dependence
- $\sigma_1 v = a_1 + b_1 v^2 + c_1/v$ with $c_1 = -\pi\alpha a_0$

Coannihilation case - Coulomb effect



MIXED CASE

$(\times 10^{26} cm^3/s)$

$\chi\chi \to W^+W^-$ (75%)	Tree		$A\tau\tau$	$\overline{DR}$	MH
a	3.099		-27%	-2%	+44%
b	5.961		-32%	-7%	+38%
$\chi\chi \to ZZ$ (5%)					
a	0.159		-22%	+3%	+50%
b	0.787		-30%	-6%	+39%
Ωh^2	0.053		0.068	0.054	0.039
$\frac{\delta\Omega h^2}{\Omega h^2}$			+28%	+2%	-26%

- Large t_β scheme dependence ($\delta\mu \supset 1/(\mu^2 - M_2^2)\delta t_\beta$)
- Large corrections

CONCLUSION

- Complete renormalisation of the MSSM
- Importance of radiative corrections in the relic density calculation
- Corrections seem to be small for the bino case either in the bulk throw coannihilation (after reabsorbing $\alpha(0) \to \alpha(m_Z)$) but still needed
- Large corrections for the mixed case
- New scenarii
- Interface with `MicrOMEGAs`