NLO Event Simulation for Chargino Production at the ILC

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1. Introduction and Motivation
   - Charginos and Neutralinos in the MSSM
   - Experimental accuracy and NLO results
2. Inclusion of NLO results in WHIZARD
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   - Photons: fixed order vs resummation
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3. Summary and Outlook

Chargino and Neutralino sector: Reconstruction of SUSY parameters

- Charginos $\widetilde{\chi}_i^{\pm}$ and Neutralinos $\widetilde{\chi}_i^0$:
  superpositions of gauge and Higgs boson superpartners
- Chargino/ Neutralino sector:

  $$\tan\beta,\ \mu \text{ (Higgs sector)},\ M_1,\ M_2 \text{(soft breaking terms)}$$

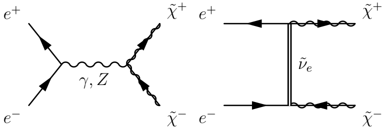
  can be reconstructed from

  $$\text{masses of } \widetilde{\chi}_1^{\pm},\ \widetilde{\chi}_2^{\pm},\ \widetilde{\chi}_1^0,\ 2\ \sigma \text{ in the } \widetilde{\chi}^{\pm} \text{ sector}$$

  (Choi ea 98, 00, 01)
- low-scale parameters + evolution to high scales (RGEs):
  $\Rightarrow$ hint at SUSY breaking mechanism (Blair ea, 02)
- requires high precision in ew-scale parameter determination

Chargino production at the ILC

- **ILC**: future e^+e^- collider, $\sqrt{s} = 500\,\mathrm{GeV}\,(1\,\mathrm{TeV})$
  "clean" environment, low backgrounds ⇒ high precision
- Charginos: (typically) light in the MSSM
  ⇒ easily accessible at colliders (ILC/ LHC) ⇐
- LO production at the ILC:



- decays: typically long decay chains

$$\text{e.g. } e^+\,e^- \;\rightarrow\; \widetilde{\chi}_1^+\,\widetilde{\chi}_1^- \;\rightarrow\; \tilde{\tau}_1^+\,\tilde{\tau}_1^-\,\nu_\tau\,\bar{\nu_\tau}\,(\rightarrow\; \tau^+\,\tau^-\,\nu_\tau\,\bar{\nu_\tau}\,\widetilde{\chi}_1^0\,\widetilde{\chi}_1^0)$$

Experimental accuracy and theoretical next-to-leading-order (NLO) corrections

- experimental errors: obtained from simulation studies (LHC/ ILC study, Weiglein ea, 04)
- generate "experimental data" with known SUSY input parameters
- errors: combination of statistical and systematic errors

combined **LHC + ILC:** ‰

same $\mathcal{O}$ errors from fitting routines determining SUSY parameters

- **Theory**:
  Full NLO SUSY corrections for $\sigma(ee \to \widetilde{\chi}\,\widetilde{\chi})$ at ILC:
  in the % regime (Fritzsche ea 04, Öller ea 04, 05)

$\Rightarrow$ include complete NLO contributions in analyses$\Leftarrow$

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From σ_{tot} to Monte Carlo event generators

MC event generators: Generate event samples
(same form as experimental outcome)

- experiments: see final decay products
- need to compare with simulated event samples
- also: important irreducible background effects,
(e.g. Hagiwara ea, 05)

$\Rightarrow$ include NLO results in Monte Carlo Generators $\Leftarrow$

- MC Generator WHIZARD (Kilian ea, arXiv:0708.4233 [hep-ph]):
- so far: LO Monte Carlo Event Generator for $2 \rightarrow n$ particle processes
- includes various physical models (SM, MSSM, non-commutative geometry, little Higgs models), initial state radiation,...

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NLO cross section contributions

σ_{tot} contributions and dependencies:

- σ_{born}
- virtual $\mathcal{O}(\alpha)$ corrections: $\sigma_{\text{virt}}(\lambda)$
- emission of soft/ hard collinear/ hard non-collinear photons:

  $\sigma_{\text{soft}}(\Delta E_\gamma, \lambda) + \sigma_{\text{hc}}(\Delta E_\gamma,\, \Delta\theta_\gamma) + \sigma_{2\to 3}(\Delta E_\gamma, \Delta\theta_\gamma)$
- higher order initial state radiation: $\sigma_{\text{ISR}} - \sigma_{\text{ISR}}^{\mathcal{O}(\alpha)}(Q)$

  λ: photon mass , ΔE_γ: soft cut , $\Delta\theta_\gamma$: collinear angle

Including FormCalc $\mathcal{O}(\alpha)$ results in WHIZARD

- use FeynArts / FormCalc generated code for

$$\begin{array}{rcl} \mathcal{M}_{\text{virt}}(\lambda) & : & \text{virtual corrections} \\ f_s(\Delta E_\gamma, \lambda) & : & \text{soft photon factor} \\ (\mathcal{M}_{\text{born}} & : & \text{born contribution}) \end{array}$$

- fixed order: integrate over effective matrix element:

$$|\mathcal{M}_{\text{eff}}|^2(\Delta E_\gamma) = (1 + f_s(\Delta E_\gamma, \lambda))\,|\mathcal{M}_{\text{born}}|^2 + 2\,Re(\mathcal{M}_{\text{born}}\,\mathcal{M}^*_{\text{virt}}(\lambda))$$

$\Delta\, E_\gamma$: soft photon cut, λ: photon mass

- in practice: create library from FormCalc code, link this to WHIZARD

(1): Fixed $\mathcal{O}(\alpha)$ contributions

- integrate $|\mathcal{M}_{\text{eff}}|^2$ (born/ virtual/ soft photonic part)
- hard collinear photons: collinear approximation ($\mathcal{M}_{\text{born}}$)
- hard non-collinear photons: explicit $e\,e \longrightarrow \widetilde{\chi}\,\widetilde{\chi}\,\gamma$ process ($\mathcal{M}^{2\rightarrow 3}_{\text{born}}$)
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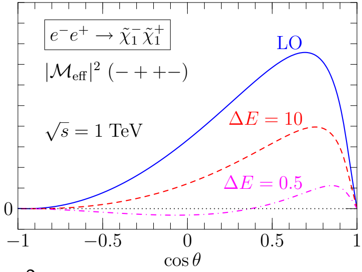
problem: too low energy cuts: $|\mathcal{M}_{\text{eff}}|^2 < 0$ $\Rightarrow$ use negative weights or set $\mathcal{M}_{\text{eff}} = 0$

event generator specific problem ($\sigma_{\text{tot}} \geq 0$)



$\mathcal{M}^2$ behaviour, different cuts [GeV]

(2): Resumming leading logs to all orders

- idea: subtract $\mathcal{O}(\alpha)$ soft + virtual collinear contributions in $\mathcal{M}_{\text{eff}}$:

$$|\widetilde{\mathcal{M}}_{\text{eff}}|^2 = (1 + f_s(\Delta E_\gamma))\,|\mathcal{M}_{\text{born}}|^2 + 2\,Re(\mathcal{M}_{\text{born}}\,\mathcal{M}^*_{\text{virt}}) - 2\,f_s^{ISR,\mathcal{O}(\alpha)}(\Delta E_\gamma)\,|\mathcal{M}_{\text{born}}|^2$$

- fold this with ISR structure function:

$$\int d\Gamma \int_0^1 dx_1 \int_0^1 dx_2\, f^{\text{ISR}}(x_1)\, f^{\text{ISR}}(x_2)\, |\widetilde{\mathcal{M}}_{\text{eff}}|^2(s, x_i))$$

- $f^{\text{ISR}}(x)$: Initial state radiation (Jadach, Skrzypek, Z.Phys. 1991)

$\Rightarrow$ describes collinear (real + virtual) photons in leading log accuracy $\Leftarrow$

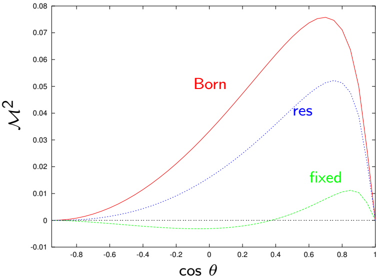
- $f_s^{\text{ISR},\mathcal{O}(\alpha)}$: soft integrated $\mathcal{O}(\alpha)$ contribution

Resumming: What do we get ??

- $\mathcal{O}(\alpha)$: equivalent to fixed order method

$\Rightarrow$ got rid of $|\mathcal{M}|^2 < 0$ effects !!

no negative weights



(-++-),

$\Delta E_\gamma = 0.5$ GeV

- higher orders:
  higher order ISR for $|\mathcal{M}_{\text{born}}|^2$ as well as Re $(\mathcal{M}_{\text{born}}\,\mathcal{M}^*_{\text{virt}})$!!!

  $\Rightarrow$ new higher order effects $\Leftarrow$
- additional possibility: also fold 2 $\rightarrow$ 3 process with ISR ("res+")

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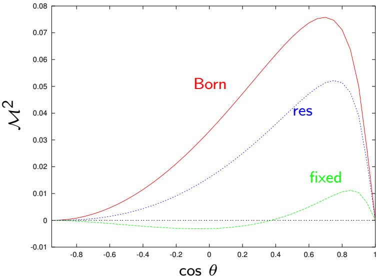

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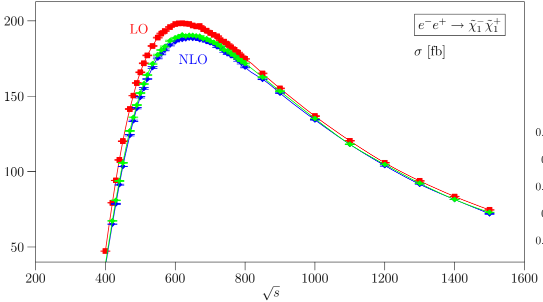
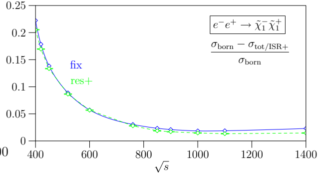
Results: cross sections



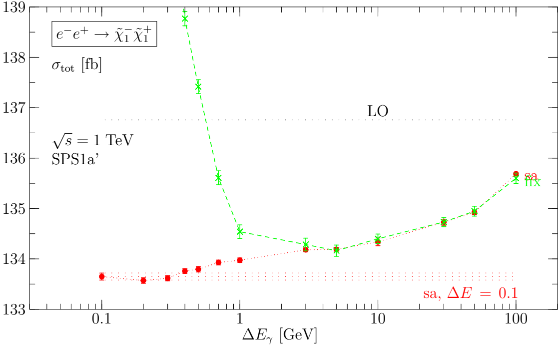
$\sigma(\sqrt{s})$



relative corrections

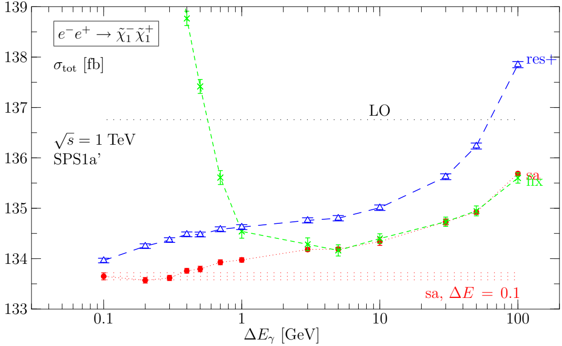
agrees with results in the literature (Fritzsche ea, Öller ea)

A closer look: ΔE_γ dependence of σ_{tot}



- semianalytic (FormCalc): tests soft approximation, shifts : 2 - 5 ‰ ($\Delta E_\gamma \leq 10\,\mathrm{GeV}$)
- fixed order result (WHIZARD): same as 'sa' for $\Delta E_\gamma \geq 3\,\mathrm{GeV}$, smaller values: $|\mathcal{M}_{\text{eff}}|^2 \leq 0$ effects

ΔE_γ dependence: resummation



$\sigma_{tot}(\Delta E_\gamma)$:
resummation includes higher order effects
5‰ difference to 'sa'
for $\Delta E_\gamma \leq 10\,\mathrm{GeV}$

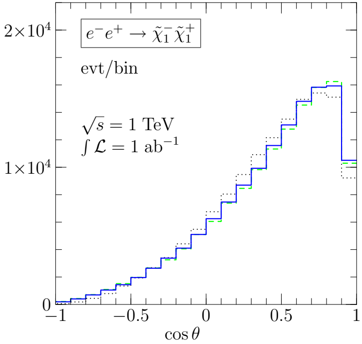
In summary:

shift in ΔE_γ leads to ‰ effects, match ILC accuracy

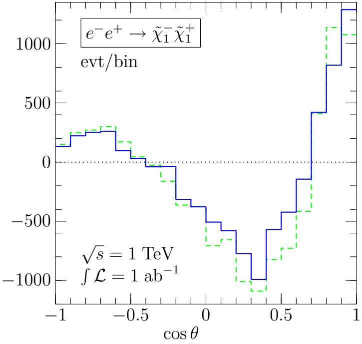
$\Rightarrow$ careful choice of ΔE_γ, method important

"best" choice: fully resummed version with low energy cut

simulation results: angular distributions



N_{tot}

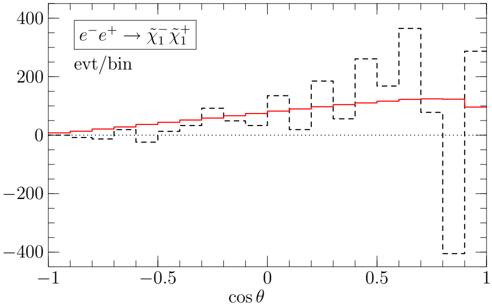


$N_i - N_{\text{Born}}$

Born, fixed order, resummation

!! more than 1 σ deviation !! $\sqrt{n_{\max}} \approx \mathcal{O}(10^2)$; nbins = 20

Angular distributions: higher orders

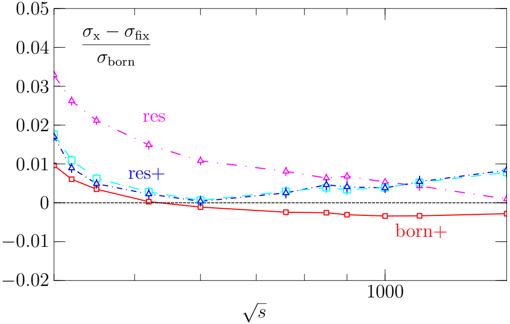


$N_{\text{res},+} - N_{\text{fix}}$
red: 1 standard dev from Born result

N^+_{res}: resummation, additionaly $2 \to 3$ folded w ISR; most complete

also higher order contributions statistically significant

$\sqrt{s}$ dependence of different higher order contributions



relative difference:

$$\frac{\sigma_x - \sigma_{fix}}{\sigma_{Born}}$$

Born+: only Born folded w ISR (standard way in the literature),
fully resummed result: subtraction, also fold $2 \to 3$ part with ISR
difference between Born+ and fully resummed result: multiple photon emission from interaction term

Summary and Outlook

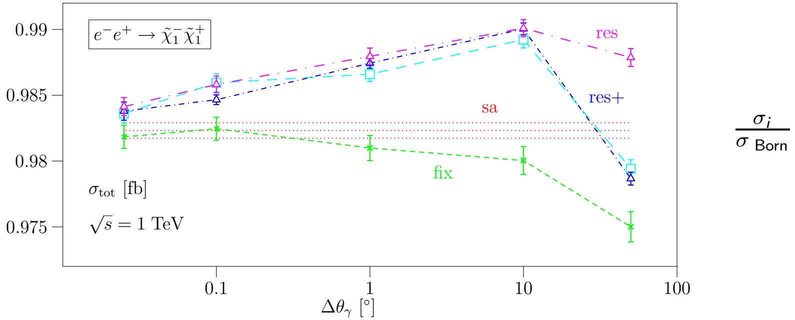
- Chargino/ neutralino sector of MSSM: high precision in SUSY paramater analysis at EW scale (‰ at ILC)
- same size/ larger NLO corrections

$\Rightarrow$ include NLO results in Monte Carlo Event generators

- resummation method for photons allows lower soft cuts/ inclusion of higher order contributions
- NLO as well as higher order contributions significant !!
- next steps: include NLO corrections to $\tilde{\chi}$ decays, non-factorizing contributions (start with photonic corrections in the double-pole approximation)
- general interface to FormCalc generated matrix elements: extendable to other processes...

cut dependencies: $\Delta\theta_\gamma$

tests: collinear photon approximation



σ_{tot} again larger for resummation method
for higher angles: second order ISR effects between 0.05^o and 0.1^o $(\mathcal{O}(‰))$

η, f_s, hard collinear approximation, $ISR^{\mathcal{O}(\alpha)}$

- $\eta = \frac{2\alpha}{\pi}\left(\log\left(\frac{Q^2}{m_e^2}\right) - 1\right)$ (Q = scale of process)
- $$f_s = -\frac{\alpha}{2\pi} \sum_{i,j = e^{\pm}} \int_{|\mathbf{k}| \leq \mathbf{\Delta E}} \frac{d^3k}{2\omega_k} \frac{(\pm)\, p_i\, p_j\, Q_i\, Q_j}{p_i\, k\, p_j\, k},$$

  (Denner 1992)

  $\omega_k = \sqrt{\mathbf{k^2} + \lambda^2}$, p_i initial/ final state momenta, k: γ momentum
- hard collinear factor ($\pm$ helicity conserving/ flipping):

  $$f^{+}(x) = \frac{\alpha}{2\,\pi} \frac{1+x^2}{(1-x)} \left(\ln\left(\frac{s\,(\Delta\theta)^2}{4\,m^2}\right) - 1\right), f^{-}(x) = \frac{\alpha}{2\,\pi} x.$$

  (Dittmaier 1993)
- $$f_s^{ISR,\mathcal{O}(\alpha)} = \left[\int_{x_0}^{1} f_{ISR}(x)\, dx\right]_{\mathcal{O}(\alpha)} = \frac{\eta}{4}\left(2\ln(1-x_0) + x_0 + \frac{1}{2}x_0^2\right)$$

ISR in its full beauty (Skrzypek ea, 91)

$$\Gamma_{ee}^{LL}(x,Q^2) = \frac{\exp(-\frac{1}{2}\eta\gamma_E + \frac{3}{8}\eta)}{\Gamma(1+\frac{\eta}{2})}\,\frac{\eta}{2}\,(1-x)^{(\frac{\eta}{2}-1)}$$

$$- \frac{\eta}{4}(1+x) + \frac{\eta^2}{16}\left(-2\,(1-x)\log(1-x) - \frac{2\log x}{1-x} + \frac{3}{2}(1+x)\log x - \frac{x}{2}\right.$$

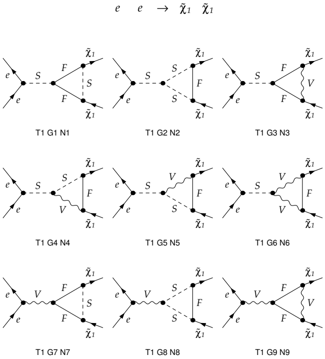
$$\left. - \frac{5}{2}\right) + \left(\frac{\eta}{2}\right)^3\left[-\frac{1}{2}(1+x)\left(\frac{9}{32} - \frac{\pi^2}{12} + \frac{3}{4}\log(1-x) + \frac{1}{2}\log^2(1-x)\right.\right.$$

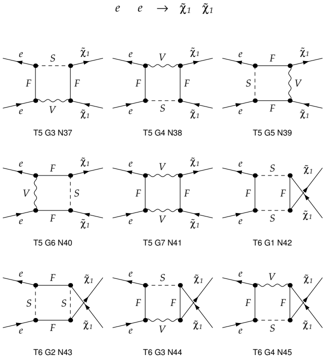
$$\left. - \frac{1}{4}\log x\log(1-x) + \frac{1}{16}\log^2 x - \frac{1}{4}\mathrm{Li}_2(1-x)\right)$$

$$+ \frac{1}{2}\,\frac{1+x^2}{1-x}\left(-\frac{3}{8}\log x + \frac{1}{12}\log^2 x - \frac{1}{2}\log x\log(1-x)\right)$$

$$\left. - \frac{1}{4}(1-x)\left(\log(1-x) + \frac{1}{4}\right) + \frac{1}{32}(5-3x)\log x\right] \;;\; \eta = \frac{2\,\alpha}{\pi}\left(\log\left(\frac{Q^2}{m_e^2}\right) - 1\right)$$

Some NLO matrix elements





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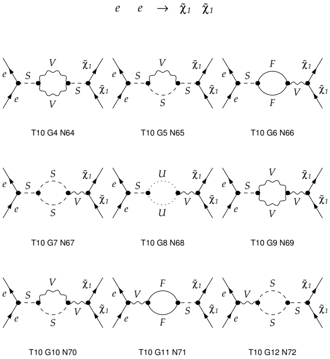
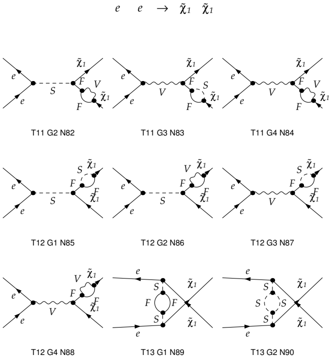
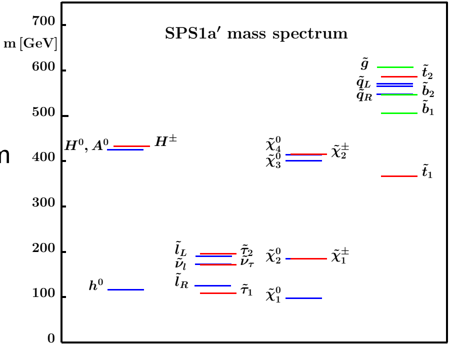




Point SPS1a'

- mSUGRA scenario
- according to Snowmass Points (Allanach ea, 02), in agreement with cosmology data/ WMAP ($\tilde{\chi}_1^0$ as DM candidate)

spectrum $\Longrightarrow$



light sleptons
heavy squarks
some light $\tilde{\chi}$s
all masses < 1 TeV