

**Precision predictions for  $Z'$  production at the LHC:  
QCD matrix elements, parton showers, and joint  
resummation**

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# Outline

- ▶ Introduction
- ▶ MC@NLO
- ▶  $Z'$  in MC@NLO
- ▶ Joint resummation for  $Z'$
- ▶ Numerical results
- ▶ Conclusions

# Introduction

- ▶ Many various Standard Model Extensions predict additional neutral vector bosons  $Z'$ : Left-Right Symmetric model, Little Higgs model, Universal extra dimension model,  $E_6$  GUT ...
- ▶  $E_6$  GUT as an example:
  - ▶ In 1984, Green and Schwarz showed that 10 dimensional string theories with  $E_8 \times E_8$  or  $SO(32)$  gauge symmetry are anomaly-free and thus potentially finite. Only the former one contains chiral fermions as they exist in the SM.
  - ▶ After Compactification,  $E_6$  symmetry appears as an effective GUT group, which is broken further into:

$$E_6 \rightarrow SO(10) \times U(1)_\psi \rightarrow SU(5) \times U(1)_\chi \times U(1)_\psi$$

- ▶ We will consider only the lower scale  $Z'_\chi$ , and not the mixing between  $Z'_\chi$  and  $Z'_\psi, Z'_\chi$  and  $Z_{SM}$

- ▶  $Z' f \bar{f}$  interaction Lagrangian (in Pythia's convention) :

$$\mathcal{L} = \frac{g}{4 \cos \theta_W} \bar{f} \gamma^\mu (v_f - a_f \gamma_5) f Z'_\mu$$

$v_d$	$a_d$	$v_u$	$a_u$	$v_l$	$a_l$	$v_\nu$	$a_\nu$
$\frac{2\sqrt{6}s_W}{3}$	$-\frac{\sqrt{6}s_W}{3}$	0	$\frac{\sqrt{6}s_W}{3}$	$-\frac{2\sqrt{6}s_W}{3}$	$-\frac{\sqrt{6}s_W}{3}$	$-\frac{\sqrt{6}s_W}{2}$	$-\frac{\sqrt{6}s_W}{2}$

- ▶ Experiment search for  $Z'$ :

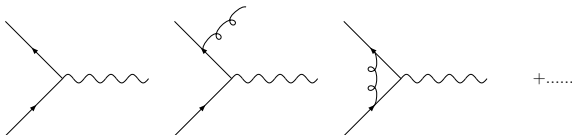
CDF collaboration has searched the Tevatron Run II data for  $Z'$  in the  $e^+e^-$  decay channel, using the di-electron invariant mass and angular distributions and setting lower mass limits of 650 to 900 GeV for a large variety of models.

Within the ATLAS collaboration, the discovery reach in  $Z' \rightarrow e^+e^-$  decays has recently been analysed for assumed  $Z'$  mass 1.5 and 4 TeV. The CMS collaboration has claimed a discovery reach of masses between 3.4 and 4.3 TeV for the  $Z' \rightarrow \mu^+\mu^-$  decay channel and an integrated luminosity of  $100 \text{ fb}^{-1}$ .

- ▶ The currently available simulations for the LHC experiments rely completely on the PYTHIA Monte Carlo generator, which is based on LO QCD matrix elements, parton showers, and the Lund string hadronization model.
  - ▶ The PT distribution can be improved by matching parton shower with hard emission of extra partons.
  - ▶ Problems:
    - ▶ The total cross section accuracy is still at LO.  
(Can not be resolved by adding K factors event by event.)
    - ▶ Parton showers not right for wide separated emission.
    - ▶ Double counting.
    - ▶ Extra dependence on matching scale.
- ⇒ CKKW, MC@NLO...
- ▶ Here we will implement  $Z'$  into MC@NLO and study

$$pp \rightarrow (\gamma, Z, Z') \rightarrow e^+e^-$$

# MC@NLO



- ▶ MC@NLO is a full event generator: hard emission are treated with NLO ME; soft/collinear emissions by Parton Shower.
- ▶ Compared with Matrix Element Corrections:  
no merging dependence, total cross sections more reliable;  
less hard emissions, technique complicated.

- ▶ Hard Part at NLO in Subtraction scheme (assume  $2 \rightarrow 2$  process at LO)

$$\frac{d\sigma}{dO} = \sum_{ab} \int dx_1 dx_2 d\phi_3 f_a(x_1) f_b(x_2) [O^{(2 \rightarrow 3)} M_{ab}^h(x_1, x_2, \phi_3) + O^{(2 \rightarrow 2)} (M_{ab}^{b,v,c}(x_1, x_2, \phi_2) - M_{ab}^{c.t.}(x_1, x_2, \phi_3))] ]$$

- ▶ Normalization factors omitted above.
- ▶  $O^{(2 \rightarrow 2,3)} = \delta(O - O(2 \rightarrow 2, 3))$ ,
- ▶  $M_{ab}^h$  is the NLO real emission contribution,
- ▶  $M_{ab}^{b,v,c}$  are the born, virtual, and collinear counter-term (finite) parts.
- ▶  $M_{ab}^{c.t.}$  is the counter-term that cancel the divergences of  $M_{ab}^h$ .

- ▶ Toy model: a quick look

$$\begin{aligned}\frac{d\sigma^B}{dx} &= B\delta(x), \\ \frac{d\sigma^V}{dx} &= a\left(\frac{B}{2\epsilon} + V\right)\delta(x), \\ \frac{d\sigma^R}{dx} &= a\frac{R(x)}{x}, \quad \lim_{x \rightarrow 0} R(x) = B,\end{aligned}$$

with  $0 < x < x_s < 1$ , and  $a$  is the coupling constant.

$$\begin{aligned}\frac{d\sigma^R}{dO} &= \int_0^1 dx x^{-2\epsilon} O(x) \frac{d\sigma^R}{dx} \\ &= aBO(0) \int_0^1 dx x^{-1-2\epsilon} + a \int_0^1 dx x^{-1-2\epsilon} (O(x)R(x) - BO(0)) \\ &= -aO(0)\frac{B}{2\epsilon} + a \int_0^1 dx (O(x)R(x)/x - BO(0)/x).\end{aligned}$$



- ▶ Now feed hard Part at NLO to MC

$$\mathcal{F}_{MC} = \sum_{ab} \int dx_1 dx_2 d\phi_3 f_a(x_1) f_b(x_2) \left[ \mathcal{F}_{MC}^{(2 \rightarrow 3)} M_{ab}^h(x_1, x_2, \phi_3) + \mathcal{F}_{MC}^{(2 \rightarrow 2)} (M_{ab}^{b,v,c}(x_1, x_2, \phi_2) - M_{ab}^{c.t.}(x_1, x_2, \phi_3)) \right]$$

- ▶  $\mathcal{F}_{MC}^{(2 \rightarrow 2)}$  and  $\mathcal{F}_{MC}^{(2 \rightarrow 3)}$  are MC generating functionals starting from 2 to 2 and 2 to 3 processes. The generated events attached with the relevant coefficients as weight factor.
- ▶ However the coefficients have divergences (although after summing up 2 to 2 and 2 to 3 results, the divergences cancel), and double counting exists ( $\mathcal{F}_{MC}^{(2 \rightarrow 3)}$  will generate non-branching events).  
 ⇒ MC counter-terms.

$$\mathcal{F}_{MC} = \sum_{ab} \int dx_1 dx_2 d\phi_3 f_a(x_1) f_b(x_2) \times$$

$$\left[ \mathcal{F}_{MC}^{(2 \rightarrow 3)} (M_{ab}^h - M_{ab}^{mc}) + \right.$$

$$\left. \mathcal{F}_{MC}^{(2 \rightarrow 2)} (M_{ab}^{b,v,c} - M_{ab}^{c,t.} + M_{ab}^{mc}) \right]$$

- ▶ Roughly speaking,  $M_{ab}^{mc}$  corresponds to the  $\mathcal{O}(\alpha_s)$  term in the Sudakov factor expansion (S. Frixione and B R. Webber, JHEP05 (2004) 056). It depends on the MC implementation details.
- ▶ Now the coefficients are finite respectively.
- ▶ There are negative weight events, and the sign should be kept.

## Z' in MC@NLO

In MC@NLO, the implementation of SM Z-boson interactions with fermions  $f$  is based on the Lagrangian:

$$\frac{g}{\cos \theta_W} \bar{f} \gamma^\mu (a_f + b_f \gamma_5) f Z_\mu.$$

The averaged squared matrix elements for di-lepton production :

$$\begin{aligned} \overline{|\mathcal{M}_i|^2}(q\bar{q} \text{ or } qg \rightarrow \gamma, Z \rightarrow e^- e^+ + X) &= \frac{1}{4} e^4 C_i \left\{ \frac{e_q^2}{M^4} T_i|_{1,0}^{1,0} \right. \\ &+ \frac{1}{\sin^4 \theta_W \cos^4 \theta_W} \frac{1}{(M^2 - m_Z^2)^2 + (\Gamma_Z m_Z)^2} T_i|_{A_q, B_q}^{A_l, B_l} \\ &\left. - \frac{2e_q}{M^2} \frac{1}{\sin^2 \theta_W \cos^2 \theta_W} \frac{M^2 - m_Z^2}{(M^2 - m_Z^2)^2 + (\Gamma_Z m_Z)^2} T_i|_{a_q, b_q}^{a_l, b_l} \right\}, \end{aligned}$$

with  $i = \text{DY, A, C}$  corresponding to DY, Annihilation and Compton processes' results,  $A_f = a_f^2 + b_f^2$  and  $B_f = 2a_f b_f$ . For the LO DY process, the colour factor is  $C_{\text{DY}} = N_C/N_C^2 = 1/3$  and

$$T_{\text{DY}}|_{A_q, B_q}^{A_l, B_l} = 8 [A_l A_q (t_{\text{DY}}^2 + u_{\text{DY}}^2) - B_l B_q (t_{\text{DY}}^2 - u_{\text{DY}}^2)].$$

To implement  $Z'$  in MC@NLO, We use the same convention.

$$\begin{aligned}
 |\overline{\mathcal{M}}_i|^2 (q\bar{q} \text{ or } qg \rightarrow \gamma, Z, Z' \rightarrow e^- e^+ + X) &= \frac{1}{4} e^4 C_i \left\{ \frac{e_q^2}{M^4} T_i|_{1,0}^{1,0} \right. \\
 &+ \frac{1}{\sin^4 \theta_W \cos^4 \theta_W} \frac{1}{(M^2 - m_Z^2)^2 + (\Gamma_Z m_Z)^2} T_i|_{A_q, B_q}^{A_l, B_l} \\
 &+ \frac{1}{\sin^4 \theta_W \cos^4 \theta_W} \frac{1}{(M^2 - m_{Z'}^2)^2 + (\Gamma_{Z'} m_{Z'})^2} T_i|_{A'_q, B'_q}^{A'_l, B'_l} \\
 &- \frac{2e_q}{M^2} \frac{1}{\sin^2 \theta_W \cos^2 \theta_W} \frac{M^2 - m_Z^2}{(M^2 - m_Z^2)^2 + (\Gamma_Z m_Z)^2} T_i|_{a_q, b_q}^{a_l, b_l} \\
 &- \frac{2e_q}{M^2} \frac{1}{\sin^2 \theta_W \cos^2 \theta_W} \frac{M^2 - m_{Z'}^2}{(M^2 - m_{Z'}^2)^2 + (\Gamma_{Z'} m_{Z'})^2} T_i|_{a'_q, b'_q}^{a'_l, b'_l} \\
 &+ 2 \frac{1}{\sin^4 \theta_W \cos^4 \theta_W} \frac{(M^2 - m_Z^2)(M^2 - m_{Z'}^2) + \Gamma_Z m_Z \Gamma_{Z'} m_{Z'}}{[(M^2 - m_Z^2)^2 + (\Gamma_Z m_Z)^2] \times [(M^2 - m_{Z'}^2)^2 + (\Gamma_{Z'} m_{Z'})^2]} \\
 &\times \left. T_i|_{a_q a'_q + b_q b'_q, a_q b'_q + a'_q b_q}^{a_l a'_l + b_l b'_l, a_l b'_l + a'_l b_l} \right\}.
 \end{aligned}$$

It includes now the squared  $Z'$ -boson exchange as well as its interferences with the photon and SM  $Z$ -boson exchanges.

```
# renormalization scale factor
FREN=1.0
# factorization scale factor
FFACT=1.0
# Z mass
ZMASS=91.188
# Z width
ZWIDTH=2.49520

#Z' mass
ZPMAS=1000.0
# If want the width to be calculated (-1) ; if want to set it explicitly (1)
gammap=1.0
#Z' width
ZPWID=12.04
# LO (0) or NLO (1)
LONLO=1.0

# Z' coupling set by user if ZPCOU=-1; Z' coupling set by default U(1)_chi case, if
ZPCOU=1,
# ZPUA,ZPUV,ZPDA,ZPDV,ZPLA,ZPLV,ZPNUA,ZPNUV, axial and vector couplings to upper,down
quarks, charged lepton and neutrino,
# defined according to af and vf in pythia convention
ZPCOU=1.0
ZPUA=1.0
ZPUV=1.0
ZPDA=1.0
ZPDV=1.0
ZPLA=1.0
ZPLV=1.0
ZPNUA=1.0
ZPNUV=1.0
```

## Joint resummation for $Z'$

- ▶ When the  $Z'$ -boson is produced close to the partonic threshold, i.e.  $z = M^2/s \rightarrow 1$ , or when its transverse momentum is small, i.e.  $p_T \rightarrow 0$ , large logarithmical terms appear in NLO QCD corrections, which should be resummed to all orders.
- ▶ In Mellin ( $N$ ) and impact parameter ( $b$ ) space, the resummed cross section is:

$$\frac{d^2\sigma^{(\text{res})}}{dM^2 dp_T^2}(N, b) = \sum_{a,b,c} f_{a/h_a}(N+1; \mu_F) f_{b/h_b}(N+1; \mu_F) \hat{\sigma}_{c\bar{c}}^{(0)} \\ \times \exp[\mathcal{G}_c(N, b; \alpha_s, \mu_R)] \times \left[ \delta_{ca}\delta_{\bar{c}b} + \sum_{n=1}^{\infty} \left( \frac{\alpha_s(\mu_R)}{\pi} \right)^n \mathcal{H}_{ab \rightarrow c\bar{c}}^{(n)}(N; \mu_R, \mu_F) \right]$$

The perturbatively calculable eikonal factor

$$\mathcal{G}_c(N, b; \alpha_s, \mu_R) = g_c^{(1)}(\lambda) \ln \chi + g_c^{(2)}(\lambda; \mu_R),$$

which depends through the functions

$$g_c^{(1)}(\lambda) = \frac{A_c^{(1)}}{\beta_0} \frac{2\lambda + \ln(1 - 2\lambda)}{\lambda} \quad \text{and}$$
$$g_c^{(2)}(\lambda; \mu_R) = \frac{A_c^{(1)} \beta_1}{\beta_0^3} \left[ \frac{1}{2} \ln^2(1 - 2\lambda) + \frac{2\lambda + \ln(1 - 2\lambda)}{1 - 2\lambda} \right]$$
$$+ \left[ \frac{A_c^{(1)}}{\beta_0} \ln \frac{M^2}{\mu_R^2} - \frac{A_c^{(2)}}{\beta_0^2} \right] \left[ \frac{2\lambda}{1 - 2\lambda} + \ln(1 - 2\lambda) \right] + \frac{B_c^{(1)}(N)}{\beta_0} \ln(1 - 2\lambda)$$

on the logarithm  $\lambda = \beta_0 / \pi \alpha_s(\mu_R) \ln \chi$ , and

$$\chi(\bar{b}, \bar{N}) = \bar{b} + \frac{\bar{N}}{1 + \eta \bar{b} / \bar{N}} \quad \text{with} \quad \bar{b} \equiv b M e^{\gamma_E} / 2, \quad \bar{N} \equiv N e^{\gamma_E}.$$

Up to next-to-leading logarithmic order, the coefficients needed in  $g_c^{(1,2)}$  are

$$A_q^{(1)} = C_F, \quad A_q^{(2)} = C_F \left[ C_A \left( \frac{67}{36} - \frac{\pi^2}{12} \right) - \frac{5}{9} T_R N_F \right],$$

$$\text{and } B_q^{(1)}(N) = -\frac{3}{2} C_F + 2\gamma_{q/q}^{(1)}(N).$$

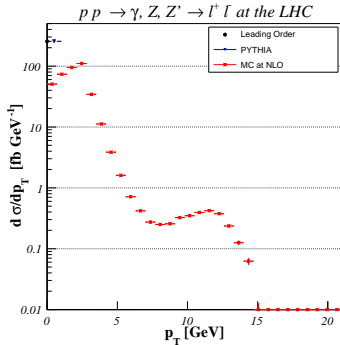
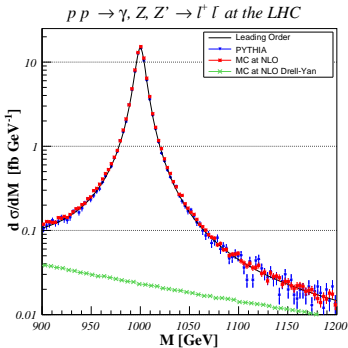
After matching the resummation results to fixed order results, we get

$$\begin{aligned} \frac{d^2\sigma}{dM^2 dp_T^2} &= \frac{d^2\sigma^{(\text{F.O.})}}{dM^2 dp_T^2} + \oint_C \frac{dN}{2\pi i} \tau^{-N} \int_0^\infty \frac{b db}{2} J_0(b p_T) \\ &\quad \times \left[ \frac{d^2\sigma^{(\text{res})}}{dM^2 dp_T^2}(N, b) - \frac{d^2\sigma^{(\text{exp})}}{dM^2 dp_T^2}(N, b) \right]. \end{aligned}$$

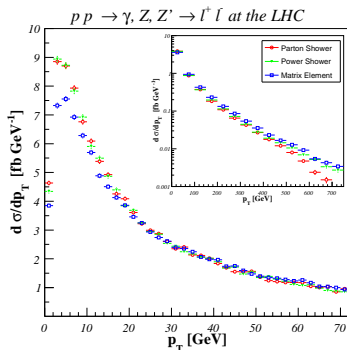
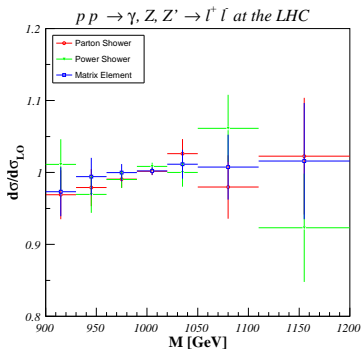


## Numerical results

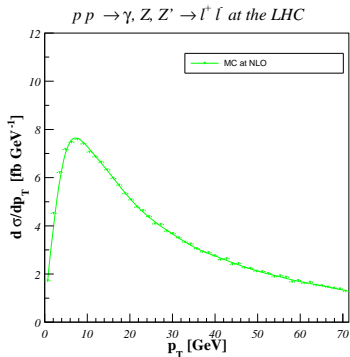
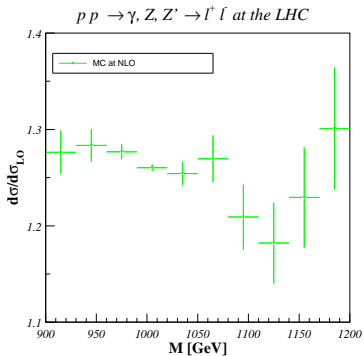
- ▶  $m_Z = 91.188$  GeV,  $\Gamma_Z = 2.4952$  GeV,  $\alpha = 1/137.04$ , and  $\sin^2 \theta_W = 0.23113$ , which are (still) used as default in the  $Z'$  analysis of the ATLAS collaboration;
- ▶  $m_{Z'} = 1$  TeV;  
 $\Gamma_{Z'} = 12.04$  GeV (in  $\chi$ -model): Got from Pythia, running the fine-structure constant to  $\alpha(1 \text{ TeV})=1/124.43$ , and including the NLO QCD correction factor  $1 + \alpha_s(\mu_F)/\pi$  for  $Z'$ -decays into quarks;
- ▶ CTEQ6L (LO) and CTEQ6M (NLO  $\overline{\text{MS}}$ ) for LO and NLO/NLL calculations;
- ▶  $900\text{GeV} < M_{e^+e^-} < 1200\text{GeV}$ ;
- ▶  $\alpha_s(\mu_R)$  is always computed with two-loop accuracy with  $\Lambda_{\overline{\text{MS}}}^{n_f=5} = 226$  MeV.  $\mu_r = \mu_f = M_{e^+e^-}$  unless specified;
- ▶ Running  $\alpha_{em}$  unless specified.



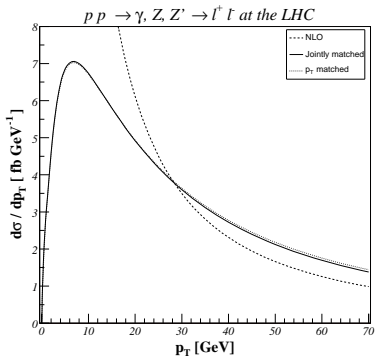
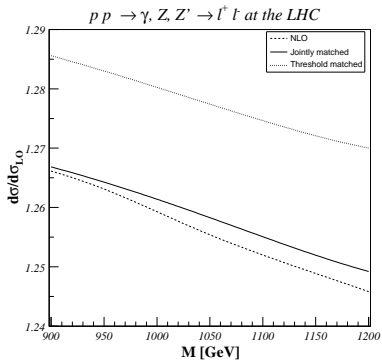
The LO mass- (left) and  $p_T$ -spectrum (right) for  $Z'$  production with fixed  $\alpha$  in PYTHIA (triangles), MC@NLO (stars) and at parton level (full line and circle), compared to the SM Drell-Yan background in MC@NLO (crosses).



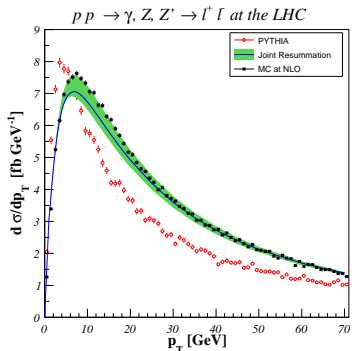
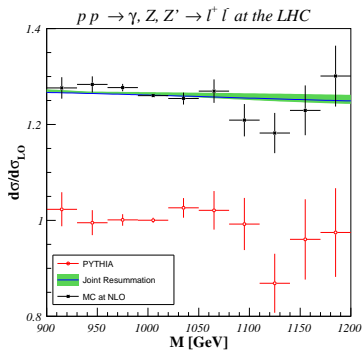
Three different ways of improving on the parton-level predictions that are implemented in PYTHIA. Mass (left) and transverse-momentum spectra (right) with PYTHIA with soft/collinear QCD parton showers (circles), QCD parton showers populating the full phase space (triangles), and after adding LO matrix element corrections (squares). The mass spectra have been normalized to the LO QCD prediction.



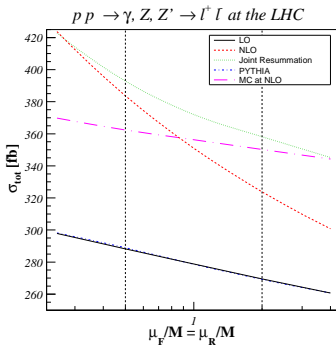
Mass (left) and transverse-momentum spectra (right) after matching the NLO QCD corrections to the HERWIG QCD parton shower (triangles). The mass spectra have been normalized to the LO QCD prediction.



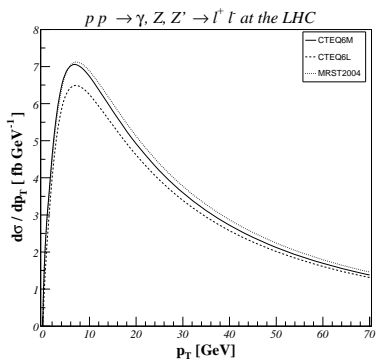
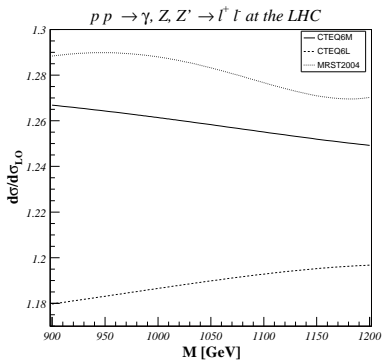
Mass (left) and transverse-momentum spectra (right) in NLO QCD (dashed) and after resumming threshold and  $p_T$  logarithms (dotted) or both at the same time (full line). The resummed cross sections have been matched to those at NLO, and the mass spectra have been normalized to the LO QCD prediction.



Mass (left) and transverse-momentum spectra (right) in PYTHIA with LO matrix elements matched to QCD parton showers (circles), in MC@NLO with NLO matrix elements matched to the HERWIG QCD parton shower (stars), and after matching the NLO QCD corrections to joint resummation (full line). The mass spectra have been normalized to the LO QCD prediction, and the renormalization and factorization scale uncertainties in the resummed predictions are indicated as shaded bands.



Dependence of the total  $Z'$ -boson production cross section at the LHC on the common factorization/renormalization scale  $\mu_{F,R}$  in LO QCD (full), NLO QCD (dashed), and after matching the NLO QCD corrections to joint resummation (dotted), LO matrix elements to the PYTHIA parton shower (dot-dashed), and NLO matrix elements to the HERWIG parton shower (long dot-dashed).



Mass (left) and transverse-momentum spectra (right) after matching the NLO QCD corrections to joint resummation with CTEQ6M (full), CTEQ6L (dashed), and MRST 2004 NLO (dotted) parton densities. The mass spectra have been normalized to the LO QCD prediction using CTEQ6L and MRST 2001 LO parton densities, respectively.



## Conclusions

- ▶ we have improved the theoretical predictions for the production of extra neutral gauge bosons at hadron colliders, which are currently based on the LO Monte Carlo generator PYTHIA, by implementing the  $Z'$  bosons in the MC@NLO generator and by computing their differential and total cross sections in joint  $p_T$  and threshold resummation.
- ▶ The two improved predictions were found to be in excellent agreement with each other for mass spectra,  $p_T$  spectra, and total cross sections, while the PYTHIA parton and “power” shower predictions show significant shortcomings both in normalization and shape.

- ▶ The theoretical uncertainties from scale and parton density variations were found to be 9 and 2%, respectively, and thus under good control.
- ▶ The implementation of our improved predictions in terms of the new MC@NLO generator or resummed  $K$  factors in the analysis chains of the Tevatron and LHC experiments should be straightforward and lead to more precise determinations or limits of the  $Z'$  boson masses and/or couplings.
- ▶ Joint resummation codes for  $Z'$  and Modified MC@NLO can be downloaded from LPSC (Grenoble, France) webpage:

<http://lpsc.in2p3.fr/klasen/software/>