Search for Regions with Unification of GUT Scale Yukawa Couplings and WMAP Compatible Dark Matter Relic Density in SO(10) SUSY GUT Models Using the Markov Chain Monte Carlo Technique…

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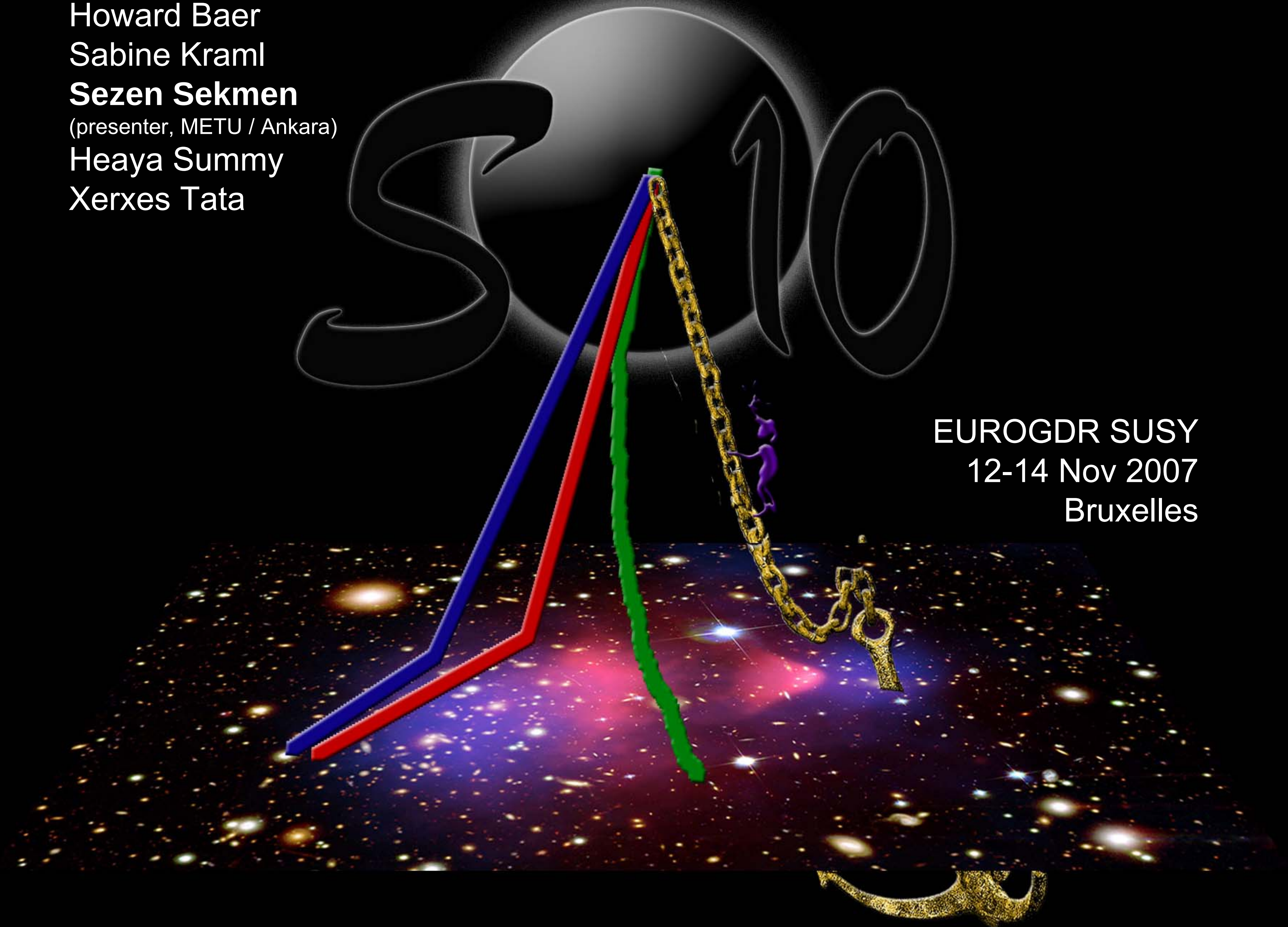

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- The Markov Chain Monte Carlo technique
- Our analysis
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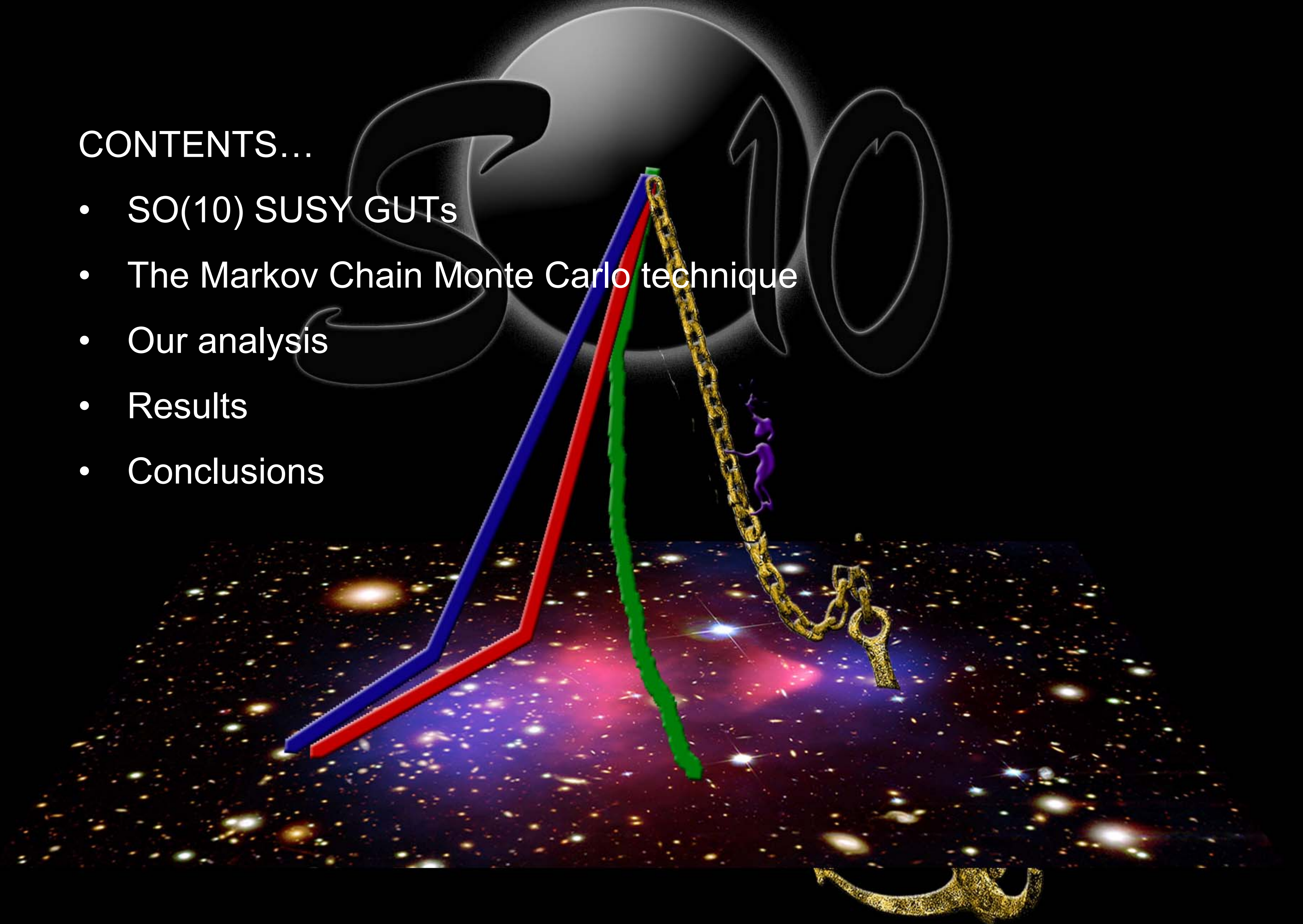
- Conclusions



Introducing SO(10) SUSY GUTs:

MSSM predicts gauge coupling unification at $M_{GUT} \sim 2 \times 10^{16}$ GeV, which is an indirect hint for SUSY GUTs.

Unification based on SO(10) Lie group is a highly motivated possibility.

In SO(10) SUSY GUTs:

- All matter in one generation reside in a single, irreducible 16-dimensional representation
- Two Higgs doublets necessary within the MSSM reside in a single irreducible 10-dmensional representation.

J.C. Pati and A. Salam, Phys. Rev. D10, 275 (1974); H. Georgi and S.L. Glashow, Phys. Rev. Lett. 32, 438 (1974). H. Georgi, H.R. Quinn and S. Weinberg, Phys. Rev. Lett. 33, 451 (1974); S. Weinberg, Phys. Lett. B91, 51 (1980)., etc.

Reviews:
R. Mohapatra, hep-ph/9911272 (1999) and S. Raby, in Phys. Rev. D66, 010001 (2002).

Motivation for SO(10) SUSY GUTs

- Fitting matter fields and Higgs fields in **irreducible representations is elegant**
- 16 dim representation naturally contains a **gauge singlet right-handed neutrino** state with $M_N \sim 10^{15}$ GeV. This is necessary for seesaw mechanism which generates non-zero left-handed neutrino masses. (in accordance with current measurements)
- Structure of the SO(10) neutrino sector leads to a **successful theory of baryogenesis** via intermediate scale leptogenesis
- The gauge group SO(10) is **left-right symmetric**, so it can solve the strong CP problem and naturally induce R parity conservation.
- …

Breaking the SO(10)

SO(10) could be **broken below the GUT scale** through various mechanisms such as:

$$SO(10) \to SM$$

$$SO(10) \to SU(5) \to SM$$

$$SO(10) \to SU(2)_L \times SU(2)_R \times SU(4)_C \times Z_2 \to SM$$

…or breaking via compactification in 5D or 6D …

BUT: **Rank of SO(10) = Rank of MSSM gauge group + 1**

→ So there must be an **extra $U(1)_X$ factor** broken at some high scale M_X.

→ This extra symmetry has its effects on sfermion/Higgs masses.

→ **"m_D^2"**, the magnitude of the D terms in U(1) scalar potential contributes to the definition of GUT scale **sfermion and higgs mass parameters**. It is a *free parameter*.

SO(10) parameters

At the GUT scale…

m_{16} :	common SSB scalar mass
$m_{1/2}$:	common SSB gaugino mass
NEW! m_{10} :	common SSB Higgs mass
NEW! m_D :	$U(1)_X$ D term magnitude
A_0,	Trilinear gauge coupling
tanß:	Ratios of VEVs

1. D-term (DT) model

$$m^2_{Q,E,U} = m^2_{16} + M^2_D$$

$$m^2_{D,L} = m^2_{16} - 3M^2_D$$

$$m^2_{H_{u,d}} = m^2_{10} \mp 2M^2_D$$

$$m^2_N = m^2_{16} + 5M^2_D$$

2. Higgs splitting (HS) model

$$m^2_{Q,E,U,D,L,N} = m^2_{16}$$

$$m^2_{H_{u,d}} = m^2_{10} \mp 2M^2_D$$

3. Arbitrary Higgs splitting

$$m^2_{Q,E,U,D,L,N} = m^2_{16}$$

$$m^2_{H_{u,d}} = m^2_{10}(1 \mp \Delta m^2_H)$$

Yukawa unification in SO(10)

The **superpotential** of SO(10) models contain the following term:

$$\hat{f} \supset y\psi(16)_3\phi(10)_H\psi(16)_3 + \ldots$$

At tree level, the Yukawa couplings y are unified at the GUT scale:

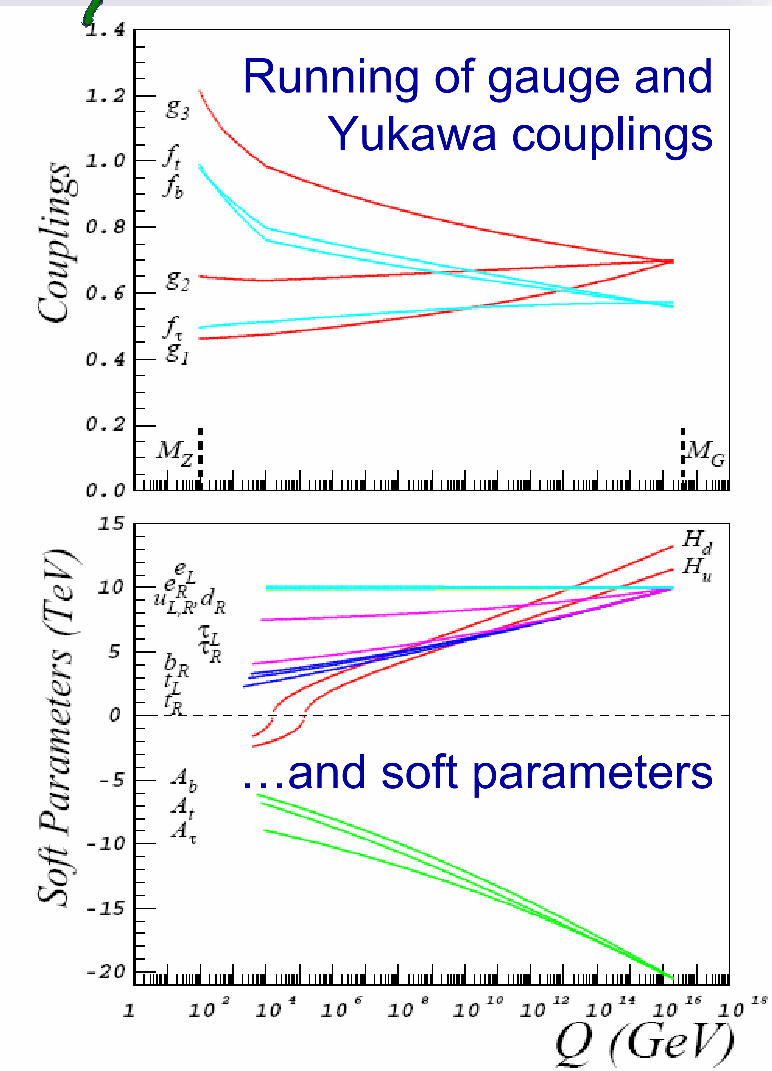
$$y_t = y_b = y_\tau = y_{\nu_\tau} \equiv y$$

However at **1-loop level** there are **~several % corrections** to this unification.

Hence GUT scale Yukawa unification is an important signature of SO(10) models.

B.D. Wright, unpublished, hep-ph/9404217 (1994).

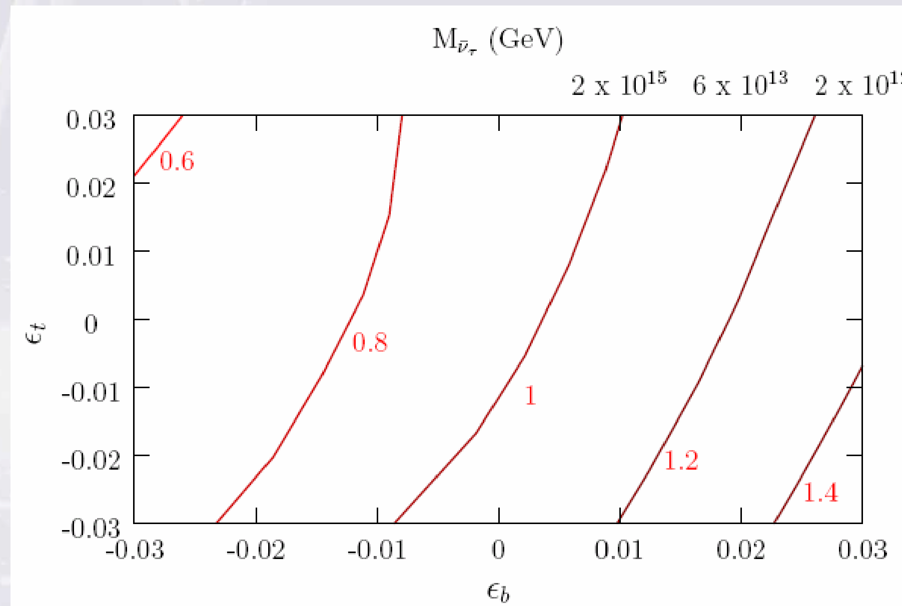
Yukawa unification in SO(10) models



D. Auto, H. Baer, C. Balazs, A. Belyaev, J. Ferrandis and X. Tata, J. High Energy Phys.0306 (2003) 023.

Variation of χ^2 (defined in terms of low energy observables) with respect to GUT scale threshold corrections to y_b and y_t, where $y_i = y(1 + \varepsilon_i)$.

$m_{1/2}$ = 300GeV, μ = 150GeV, m_{16} = 2TeV, rest is varied to minimize χ^2. (Arbitrary Higgs splitting)



T. Blazek, R. Dermisek and S. Raby, Phys. Rev. Lett. 88, 111804 (2002) and Phys. Rev. D65, 115004 (2002).

DM in SO(10)

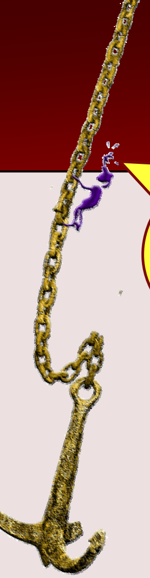
WMAP limits: $0.094 < \Omega h^2 < 0.136$

- In general SO(10) models predict high relic densities since mass spectrum challenges efficient eutralino annihilation
- But still there are WMAP compatible solutions (more later…)
- In any case, WMAP compatible relic densities could be reconciled via:
  - Lowering GUT scale mass value of first and second generation scalars
  - Relaxing gaugino mass universality

D. Auto, H. Baer, A. Belyaev, T. Krupovnickas, JHEP 0410:066 (2004), hep-ph/0407164

R. Dermisek, S. Raby, L. Roszkowski, R. Ruiz de Austri, JHEP 0509:029 (2005), hep-ph/0507233

Markov Chain Monte Carlo



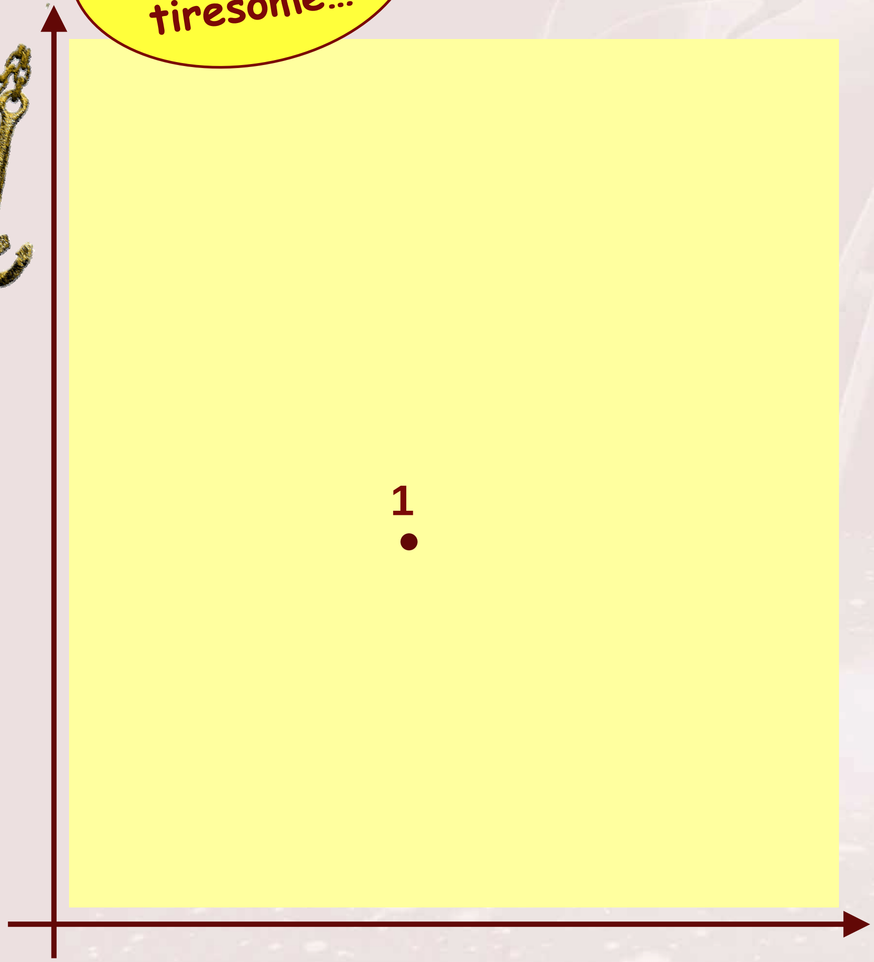
A Markov Chain is a **discrete-time, stochastic (radom)** process where the next step only depends on the present one – not on any of the previous states.

A Markov Chain Monte Carlo (MCMC) is an algorithm that constructs a Markov Chain by **sampling from a probability distribution.**

It aims to **converge to a stationary distribution** within an acceptable error from an **arbitrary position** in a parameter space, using as few steps as possible.

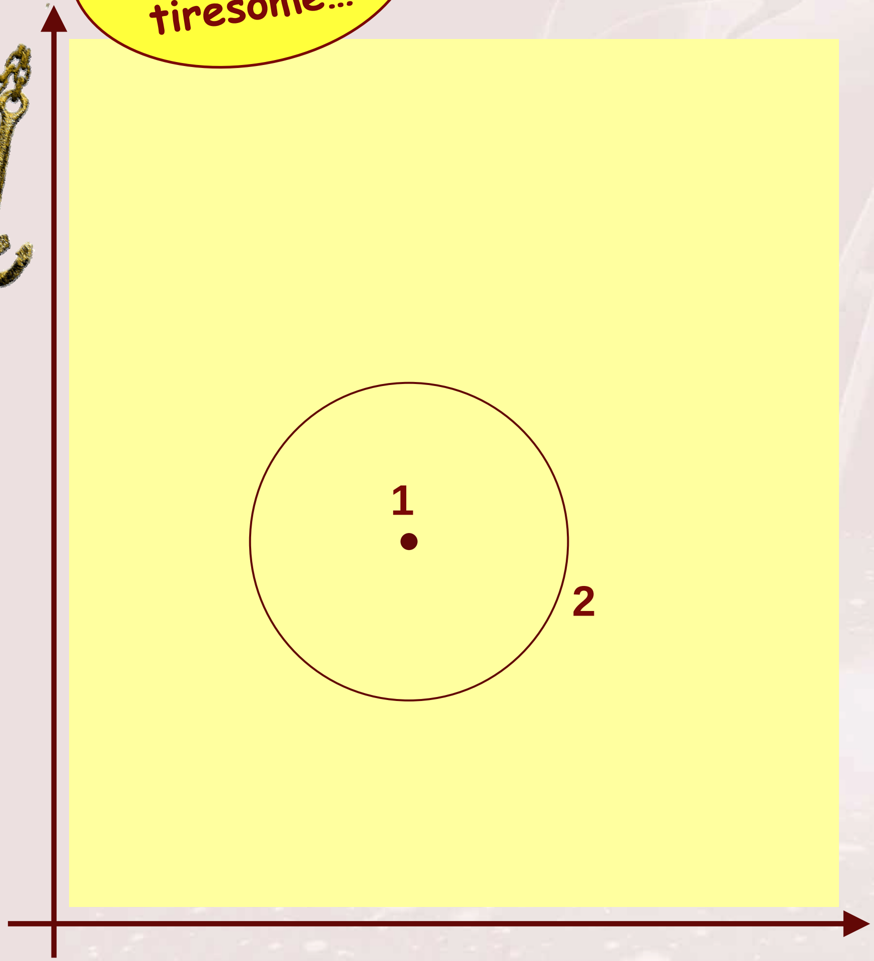
A. A. Markov, Izvestiya Fiziko-matematicheskogo obschestva pri Kazanskom universitete, 2-ya seriya, tom 15, pp 135-156, 1906.

MCMC at work: Metropolis-Hastings Algorithm



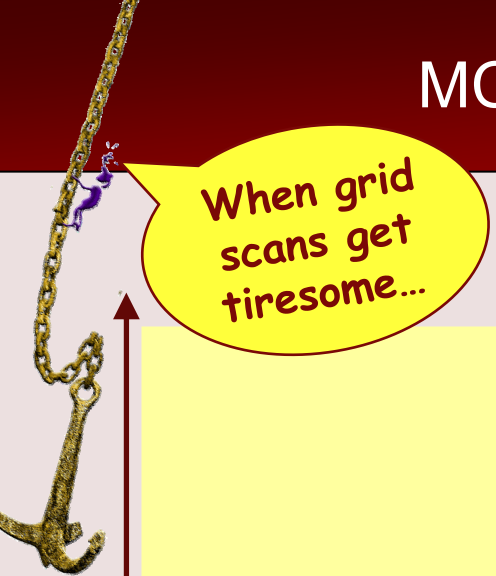
1. Pick an arbitrary point xi and set $x^i = x^t$.

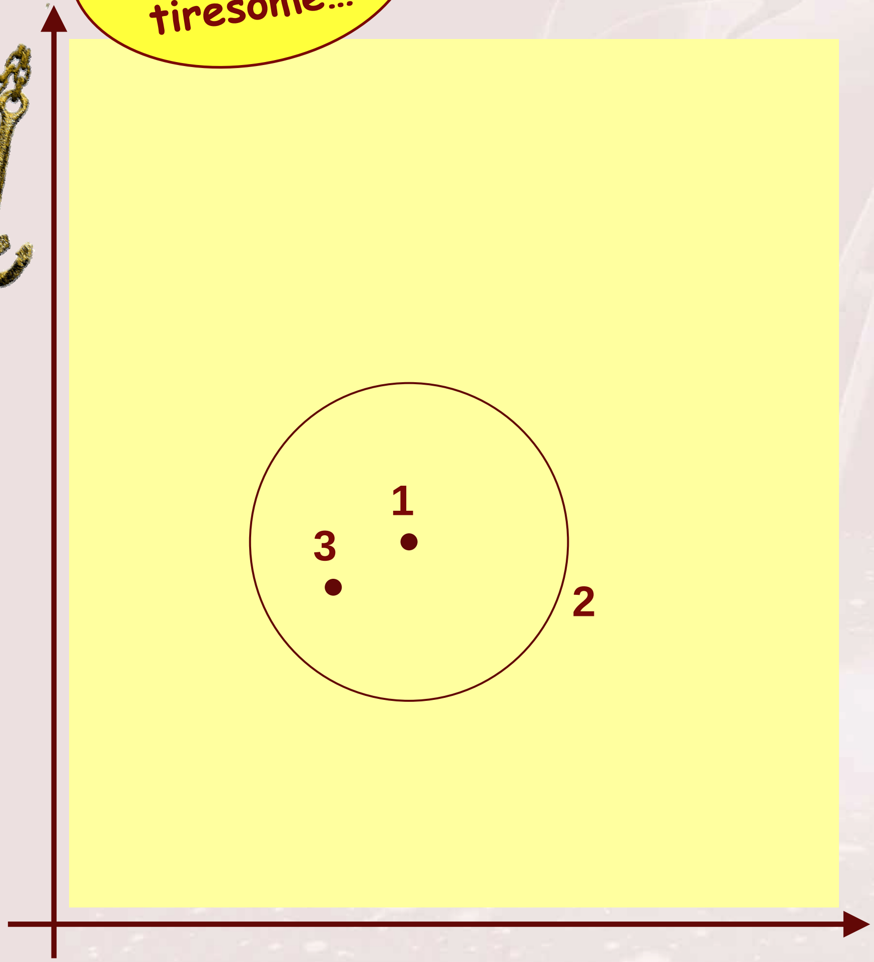
MCMC at work: Metropolis-Hastings Algorithm



1. Pick an arbitrary point xi and set $x^i = x^t$.
2. Propose a probability density Q (Generally a Gaussian with width w centered aroud x^t)

MCMC at work: Metropolis-Hastings Algorithm





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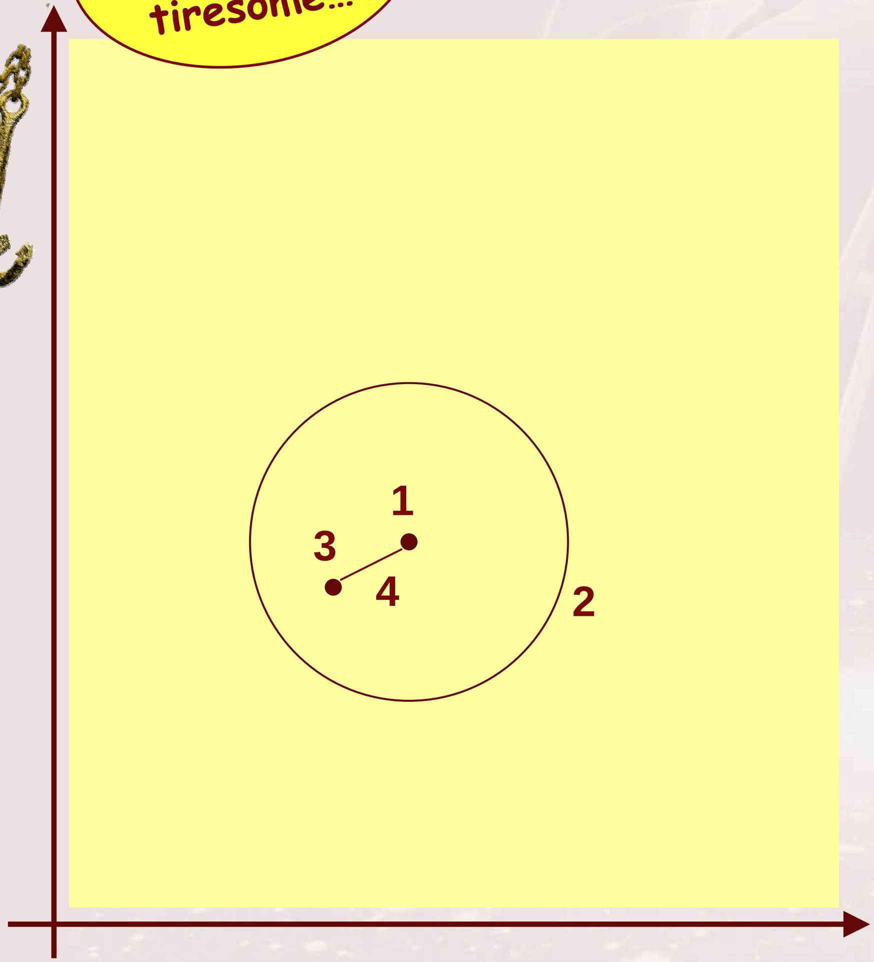
3. Pick a random point x^{i+1} from Q

MCMC at work: Metropolis-Hastings Algorithm

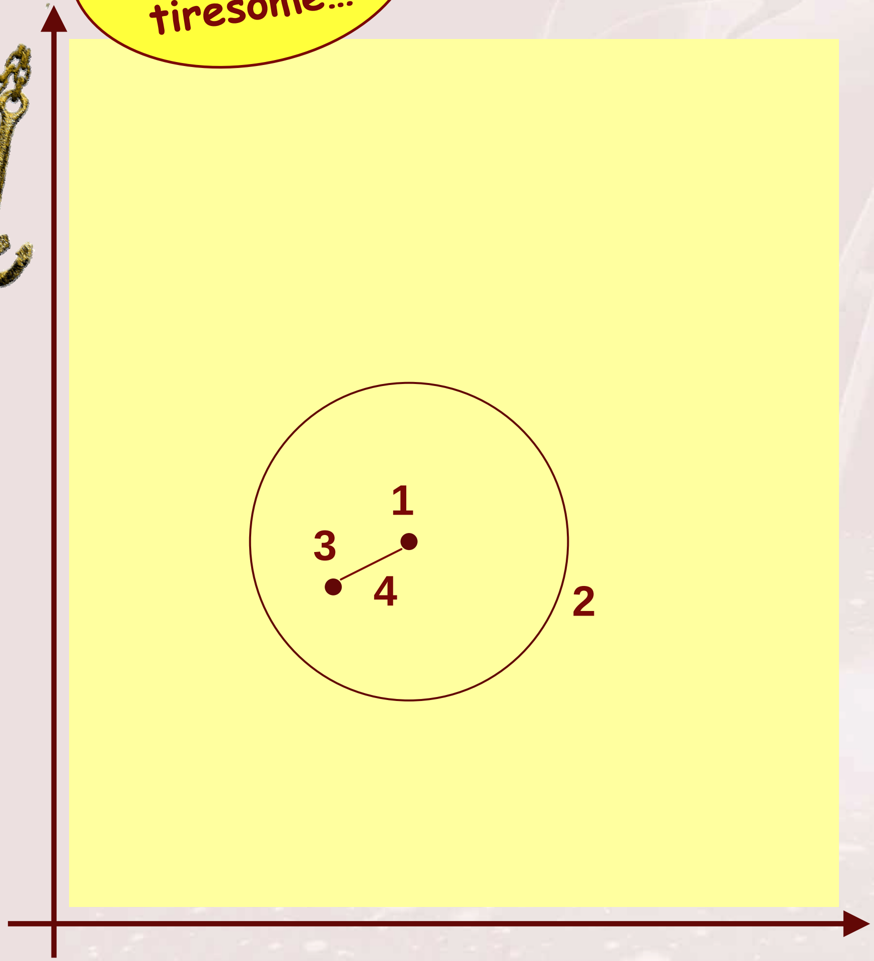


1. Pick an arbitrary point xi and set $x^i = x^t$.
2. Propose a probability density Q (Generally a Gaussian with width w centered aroud x^t)
3. Pick a random point x^{i+1} from Q
4. Calculate

$$p = \min\left(1, \frac{P(x^{i+1})Q(x^i; x^{i+1})}{P(x^i)Q(x^{i+1}; x^i)}\right)$$

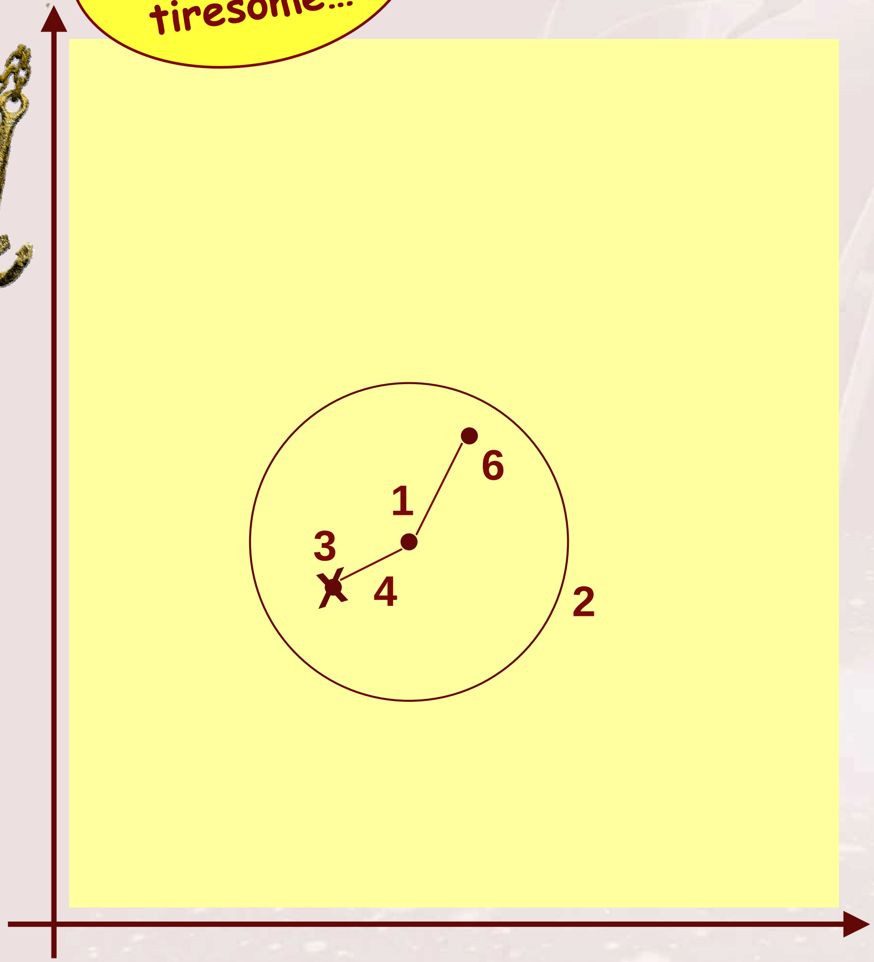
where P(x) is the probability

MCMC at work: Metropolis-Hastings Algorithm (MHA



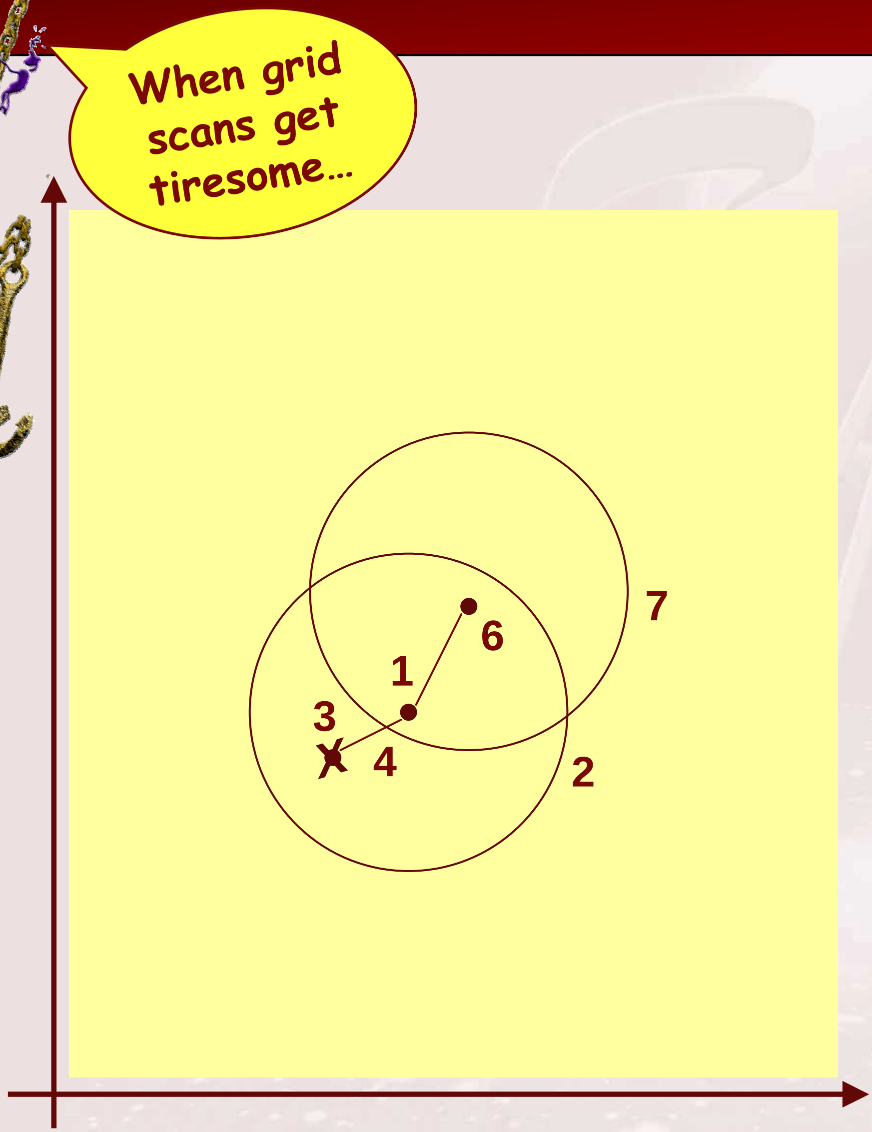
5. Generate a uniform random number a = [0,1]

MCMC at work: Metropolis-Hastings Algorithm (MHA



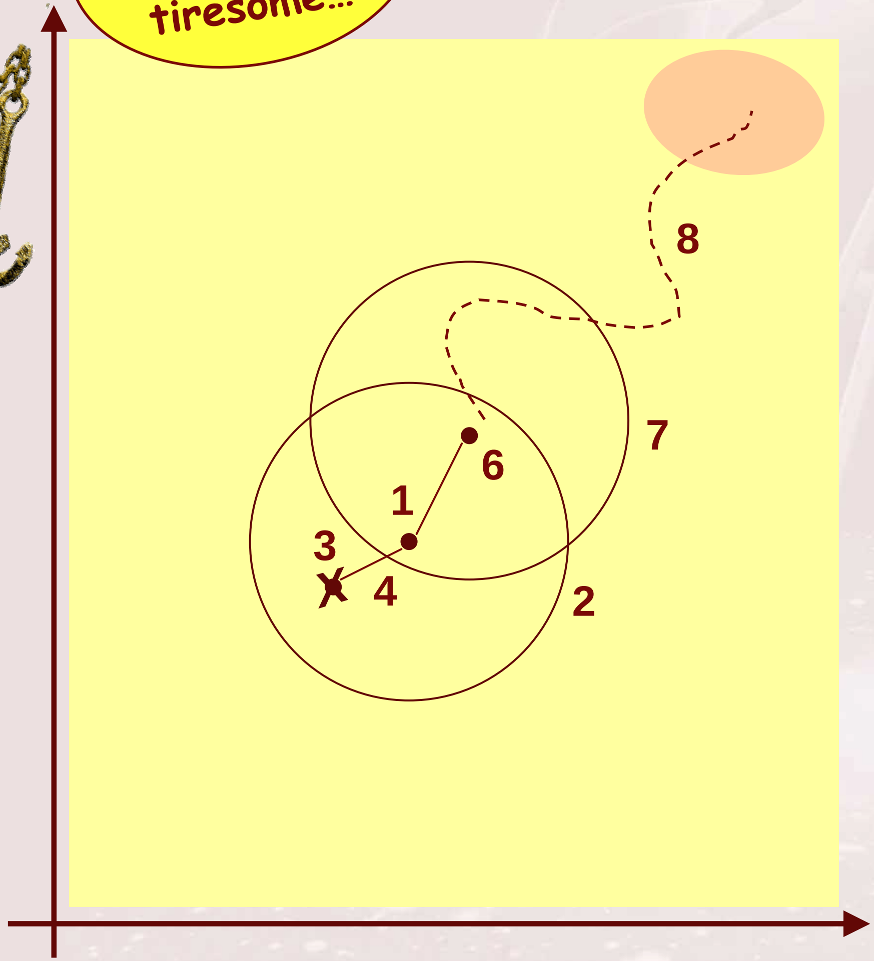
5. Generate a uniform random number a = [0,1]
6. If $p < a$, reject the point and pick another point from Q and recalculate p wrt x^t.

MCMC at work: Metropolis-Hastings Algorithm (MHA



5. Generate a uniform random number a = [0,1]
6. If $p < a$, reject the point and pick another point from Q and recalculate p wrt x^t.
7. If $p \geq a$, accept the point and set $x^t = x^{i+1}$, and define a new Q around x^t.

MCMC at work: Metropolis-Hastings Algorithm (MHA



5. Generate a uniform random number a = [0,1]
6. If $p < a$, reject the point and pick another point from Q and recalculate p wrt x^t.
7. If $p \geq a$, accept the point and set $x^t = x^{i+1}$, and define a new Q around x^t.
8. Iterate until the chain converges to a region with maximal probability…

One can pick many starting points and run several MCMCs in order to guarantee finding the global minimum.

MCMCs in HEP phenomenology

M. Rauch, R. Lafaye, T. Plehn, D. Zerwas, "SFitter: Reconstructing the MSSM Lagrangian from LHC data", arXiv:0710.2822

S.Hennestad “Global neutrino parameter estimation using Markov Chain Monte Carlo”, arXiv:0710.1952

R. Lafaye, T. Plehn, M. Rauch, D. Zerwas “Measuring Supersymmetry”, arXiv:0709.3986

L. Roszkovski, R. Ruiz de Austri, R. Trotta, “Implications for the Constrained MSSM from a new prediction for b to s gamma”, arXiv:0705.2012; L. Roszkovski, R. Ruiz de Austri, R. Trotta, “On the detectability of the CMSSM light Higgs boson at the Tevatron”, arXiv:0611173; L. Roszkovski, R. Ruiz de Austri, R. Trotta, “A Markov Chain Monte Carlo Analysis of the CMSSM”, JHEP 0605 (2006) 002

B.C. Allanach, C.G. Lester, A.M. Weber, “Natural Priors, CMSSM Fits and LHC Weather Forecasts”, arXiv: 0705.0487; B.C. Allanach, C.G. Lester, A.M. Weber, “The Dark Side of mSUGRA”, JHEP 0612 (2006) 065; B.C. Allanach, “Naturalness Priors and Fits to the Constrained Minimal Supersymmetric Standard Model”, Phys.Lett. B635 (2006) 123-130; B.C. Allanach, C.G. Lester, Multi-Dimensional mSUGRA Likelihood Maps”, Phys.Rev. D73 (2006) 015013

C.G. Lester, M.A. Parker, M.J. White, “Determining SUSY model parameters and masses at the LHC using cross-sections, kinematic edges and other observables”, JHEP 0601 (2006) 080

E.A. Baltz, P.Gondolo, “Markov Chain Monte Carlo Exploration of Minimal Supergravity with Implications for Dark Matter”, JHEP 0410 (2004) 052

Constructing the MHA for SO(10) studies

- With the help of MHA, we search the SO(10)-motivated parameter spaces starting from following sets of parameters:
  - **HS:** m_{16}, m_{10}, m_D, $m_{1/2}$, A_0, tanß
  - **Low μ motivated arbitrary HS:** m_{16}, $m_{1/2}$, A_0, tanß, μ, mA^0
- We look for regions having
  - GUT scale Yukawa unification
    Here, Yukawa unification is parametrized by R:
    $$R = \max(y_t, y_b, y_\tau) / \min(y_t, y_b, y_\tau)$$
  - WMAP-compatible DM relic density
  - compliance with LEP sparticle and Higgs mass limits.
- We use **ISAJET 7.75** for spectrum calculations and **micrOMEGAs 2.0.7** for DM relic density computations.
- We run **~10 chains for each case study**, to find various compatible regions. **Starting points are random**.

Constructing the MHA for SO(10) studies

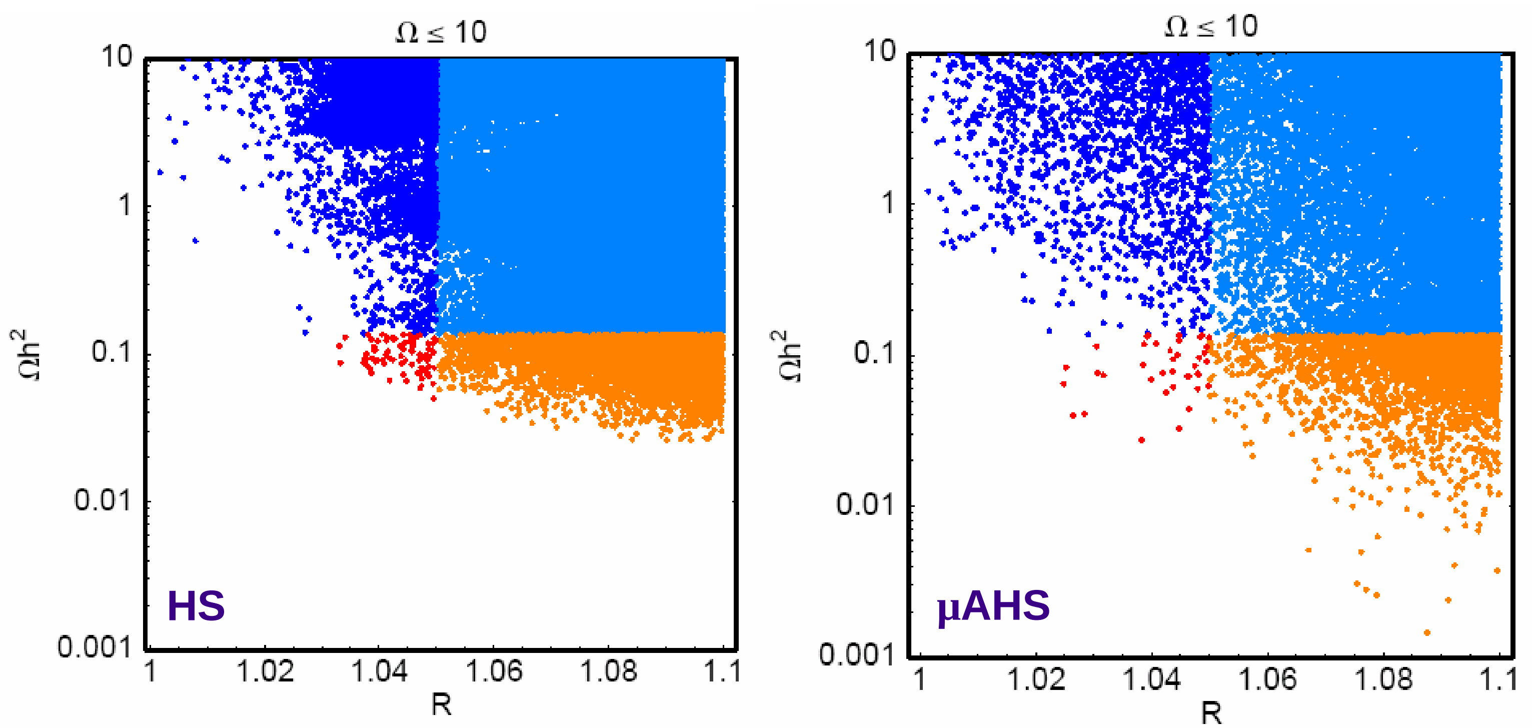
Probability: $P_{R,\Omega}(x) = e^{-\chi^2_{R,\Omega}(x)}$

$$\chi^2_R(x) = \left(\frac{R(x)-1}{\Delta R}\right)^2$$

$$\chi^2_\Omega(x) = \begin{cases} 1, & for\ 0.094 \le \Omega \le 0.136 \\ \left(\frac{\Omega(x)-\Omega_{central}}{\Delta\Omega}\right)^2, & for\ \Omega < 0.094,\ \Omega > 0.136 \end{cases}$$

- We assume that
  - Minimal unification must be %10: $R < 1.1$ and $\Delta R = 0.1$
  - $\Omega_{central} = 0.115$, $\Delta\Omega = \sigma_\Omega = 0.0105$
- We consider two cases
  - MHA for only R: $p_R \geq a$
  - MHA for R and Ω : $p_R \geq a$ and $p_\Omega \geq a$
- Points must satisfy LEP sparticle mass limits to be accepted

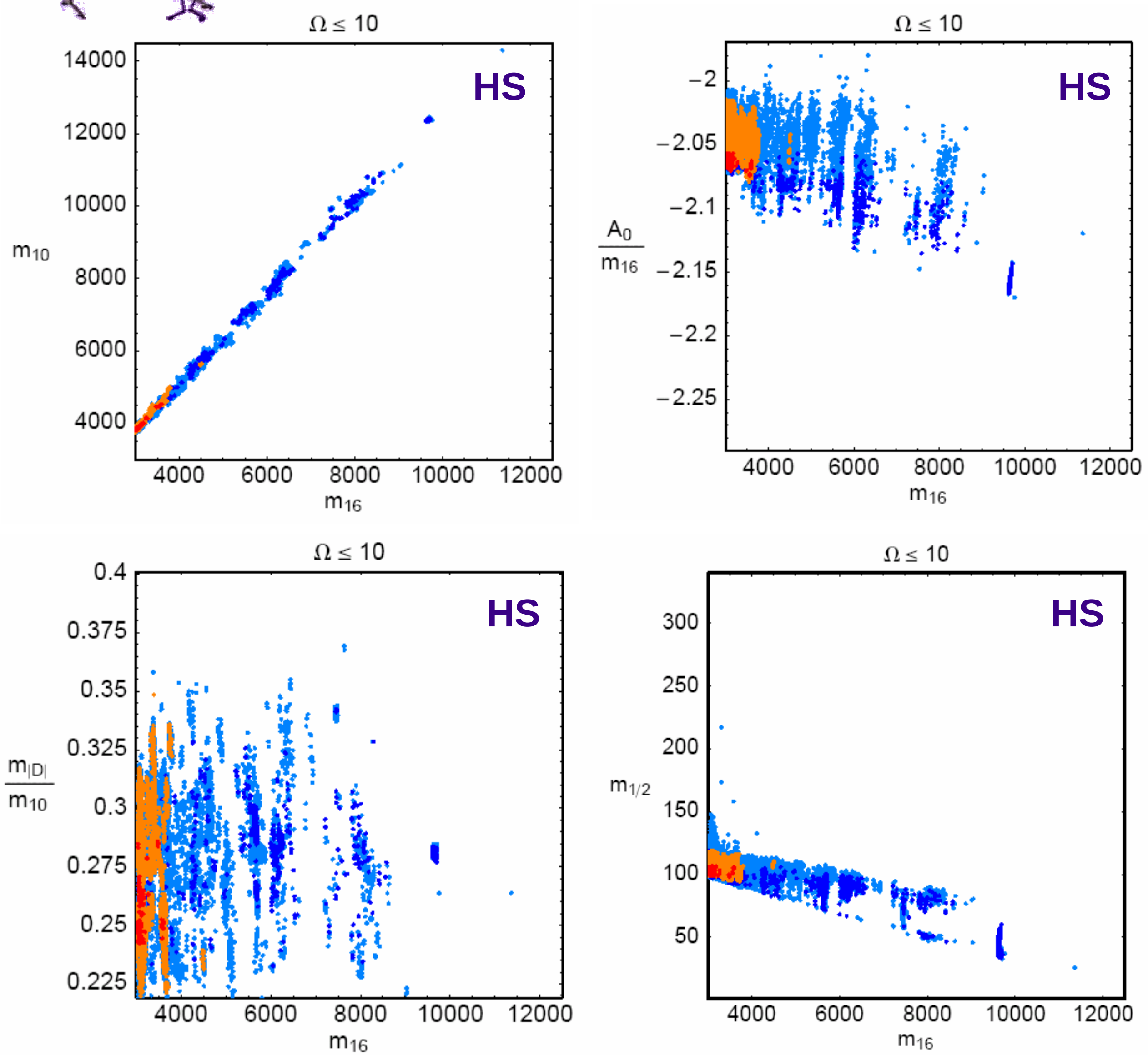
Results: Ω vs. R



$R \leq 1.10$, $R \leq 1.05$, $R \leq 1.10$ & $\Omega \leq 0.136$, $R \leq 1.05$ & $\Omega \leq 0.136$

Plots contain a collection of data from different MHA runs with different starting points

DM in SO(10)



$R \leq 1.10$, $R \leq 1.05$, $R \leq 1.10$ & $\Omega \leq 0.136$, $R \leq 1.05$ & $\Omega \leq 0.136$

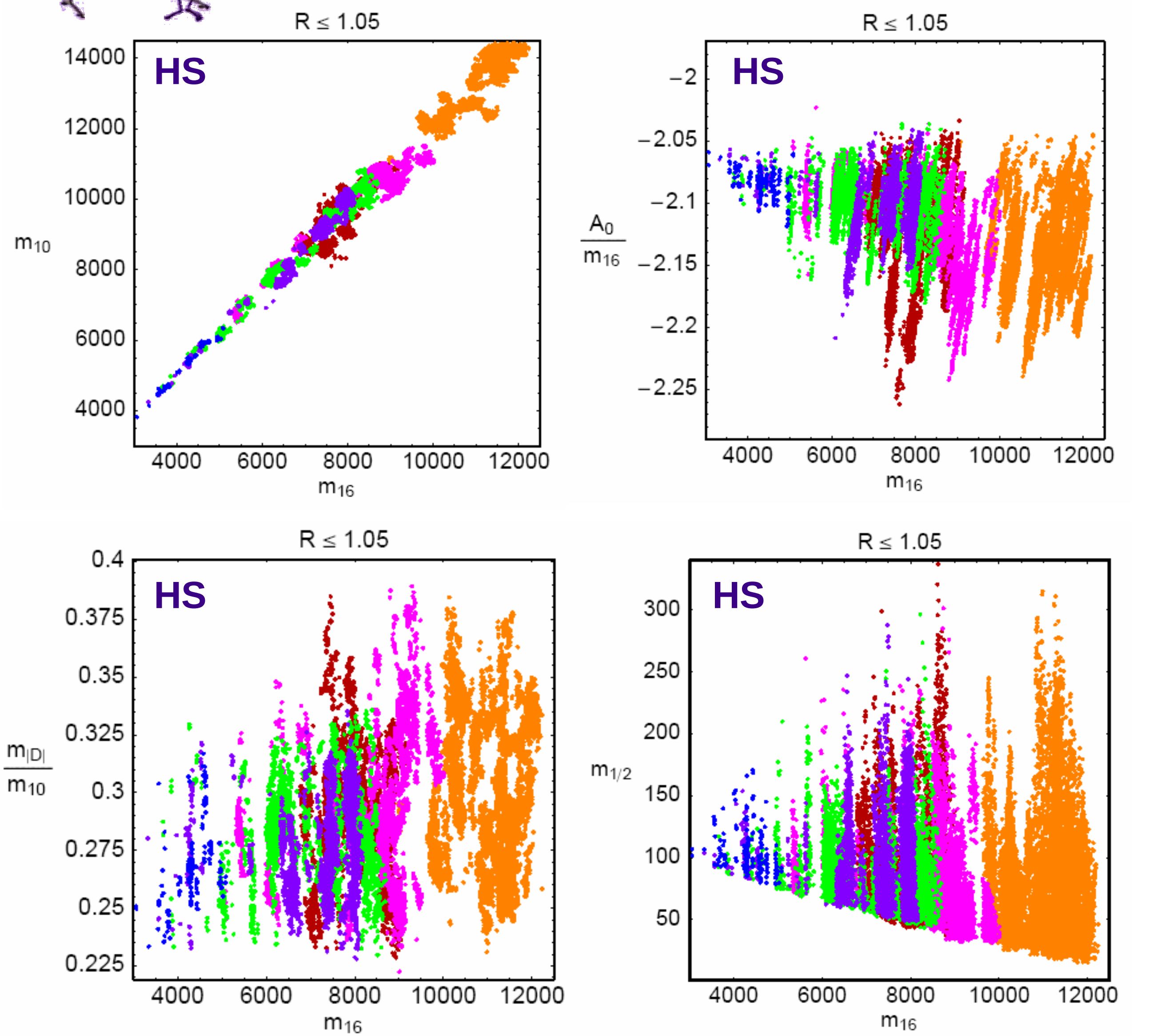
$$m_{10} \simeq \sqrt{2} m_{16}$$

$$A_0 \simeq -2 m_{16}$$

These relations agree with the theoretical predictions made İn the context of **radiatively driven inverted scalar mass hierarchy models**.

J. Feng, C. Kolda and N. Polonsky, Nucl. Phys. B546, 3 (1999); J.Bagger, J. Feng and N.Polonsky, Nucl. Phys. B563, 3 (1999); J. Bagger, J. Feng, N. Polonsky and R. Zhang, Phys.Lett. B473, 264 (2000).

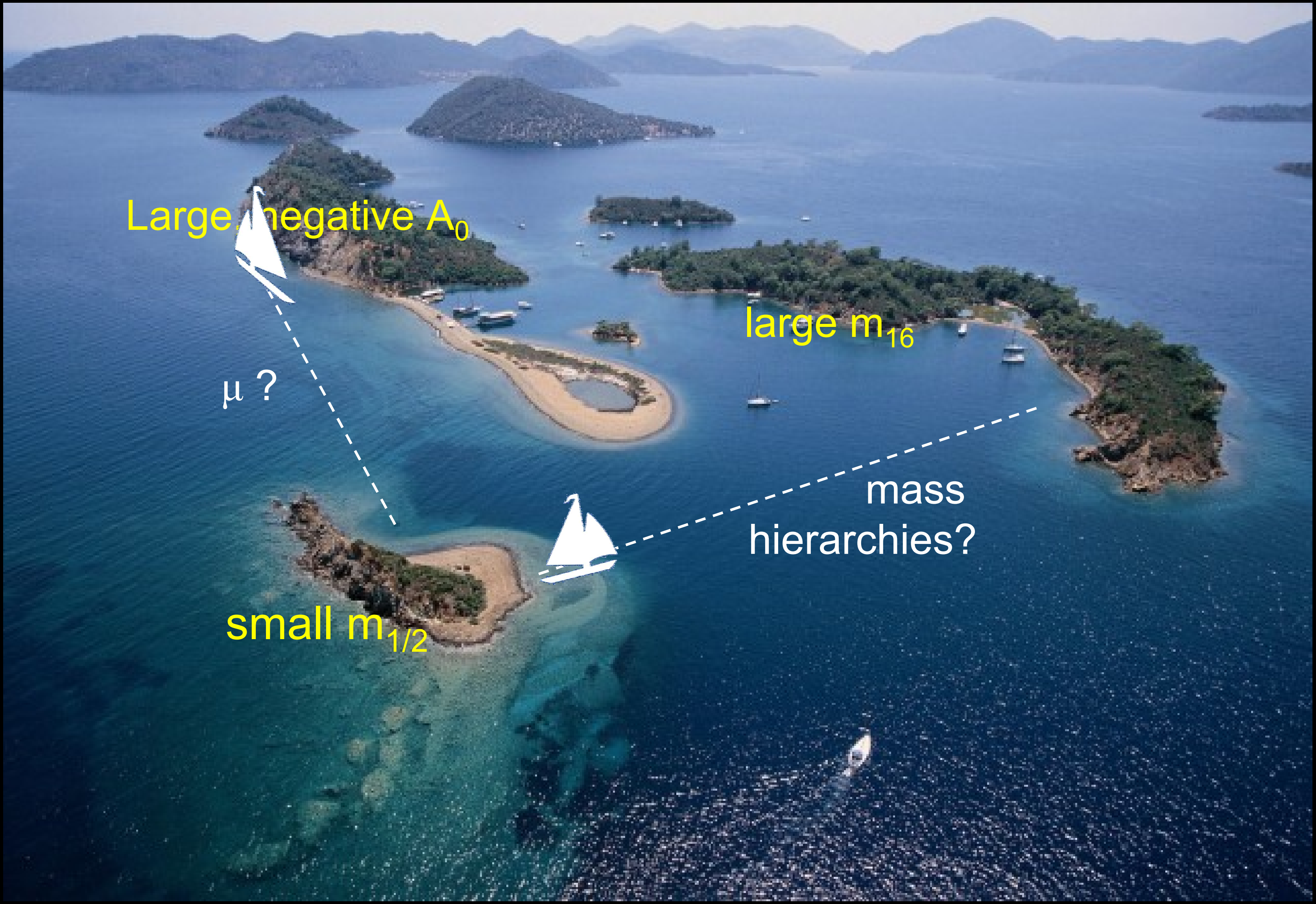
DM in SO(10)



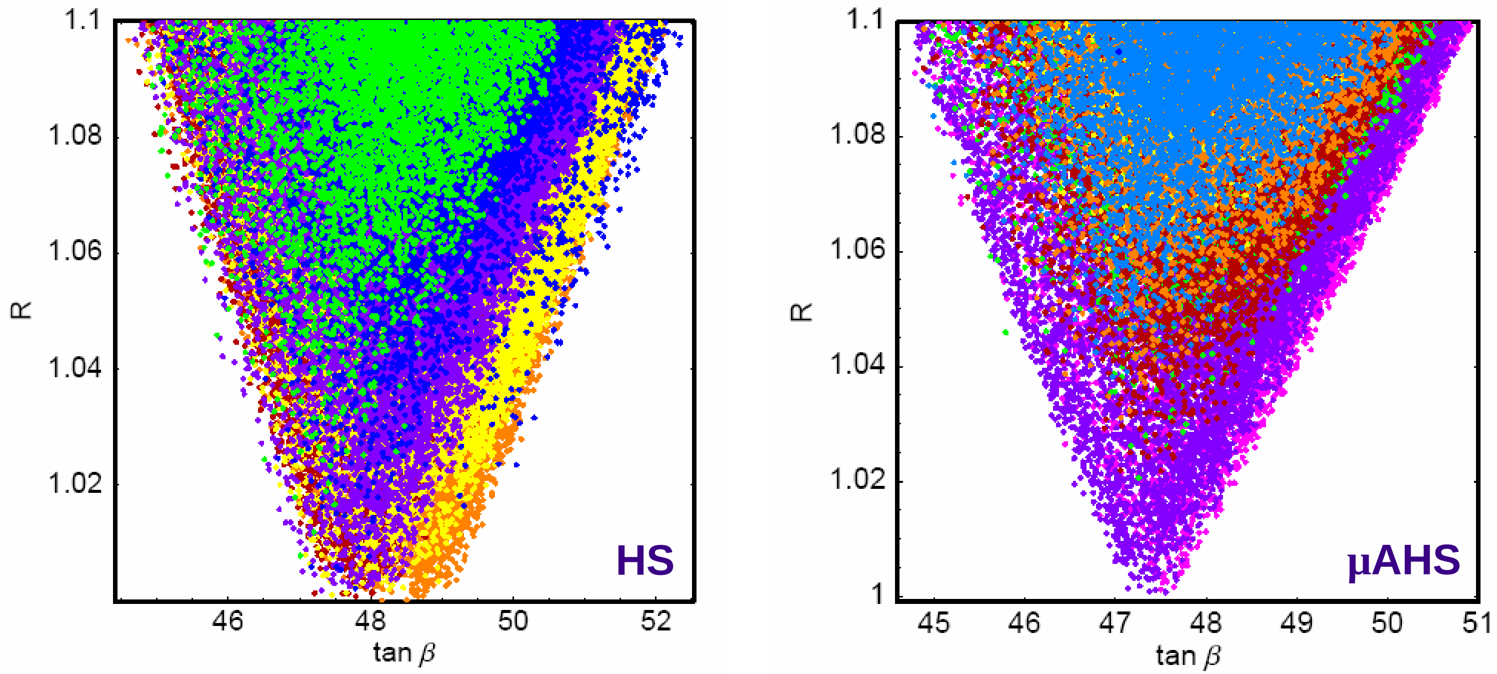
Points with different colors correspond to converged data from different MHA runs with different starting points.

Do these islands have something in common?

Do these islands have something in common?



Results: tanß



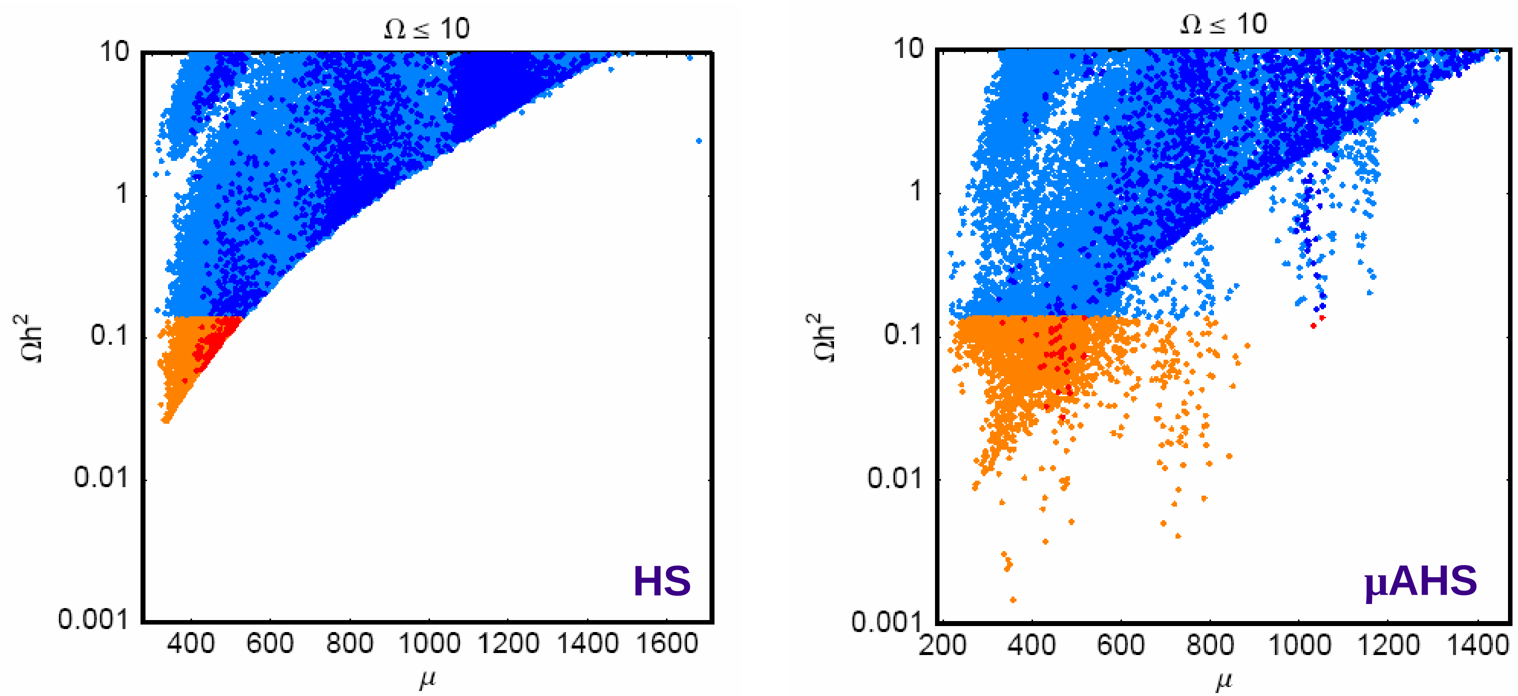
Different colors correspond to different starting points.

Yukawa unification favors high tanß, since.

$$\frac{m_t}{m_b} \sim \frac{v_u y_t}{v_d y_b} \sim \tan\beta \frac{y_t}{y_b}$$

but the effects from radiative corrections must also be considered.

Results: μ parameter

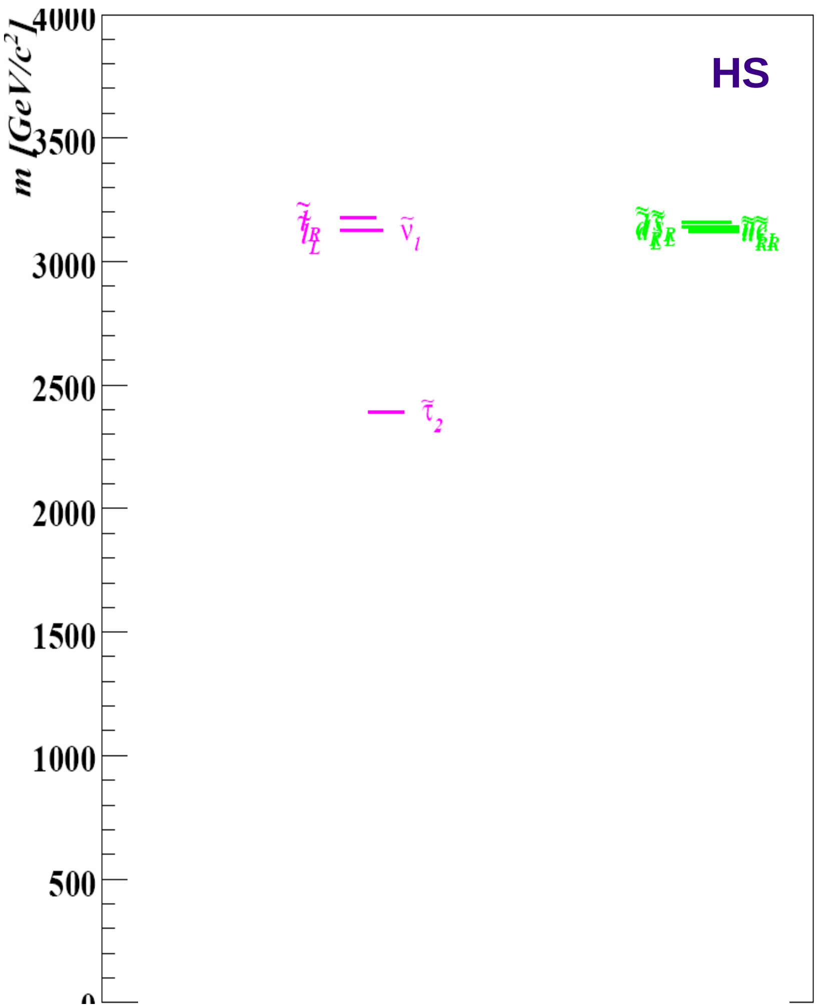


R ≤ 1.10, R ≤ 1.05, R ≤ 1.10 & Ω ≤ 0.136, R ≤ 1.05 & Ω ≤ 0.136

DM relic density at WMAP range favors a moderate μ parameter.

Example mass spectrum

m16 = 3147.52
m10 = 3973.51
mD = 1013.6
mhf = 101.908
A0 = -6482.75
tanß = 47.5693
R = 1.05
omg = 0.118



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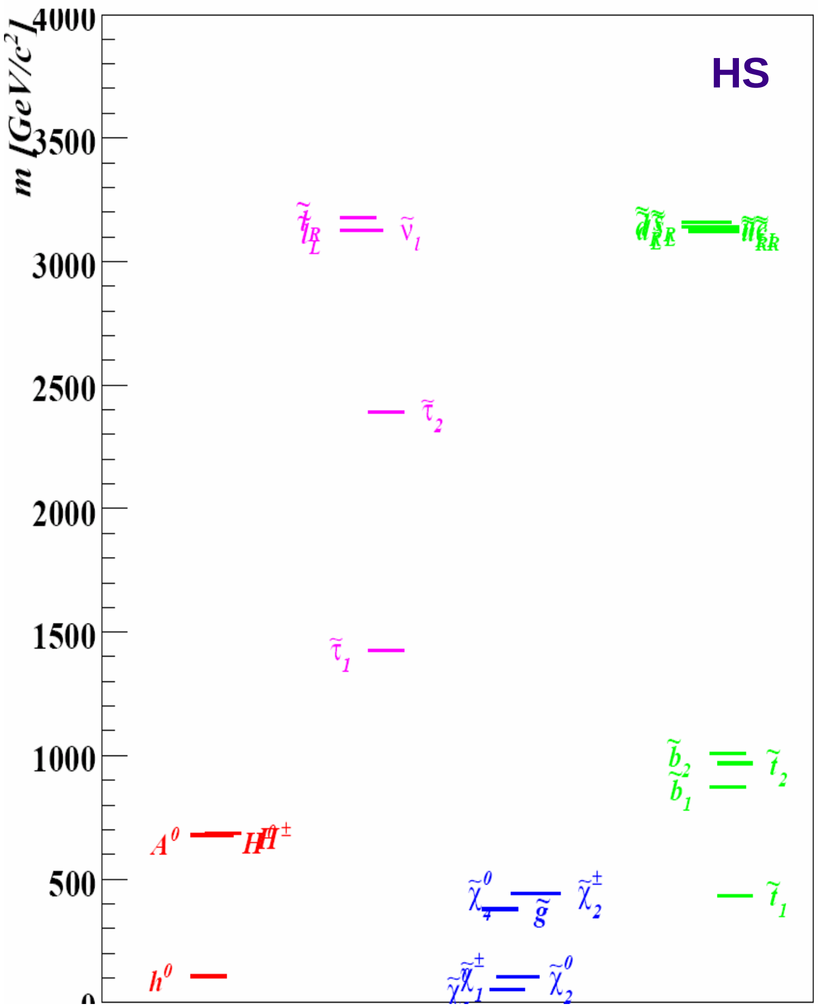
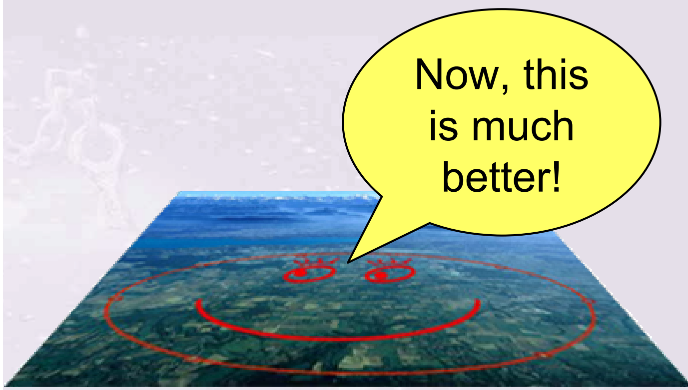
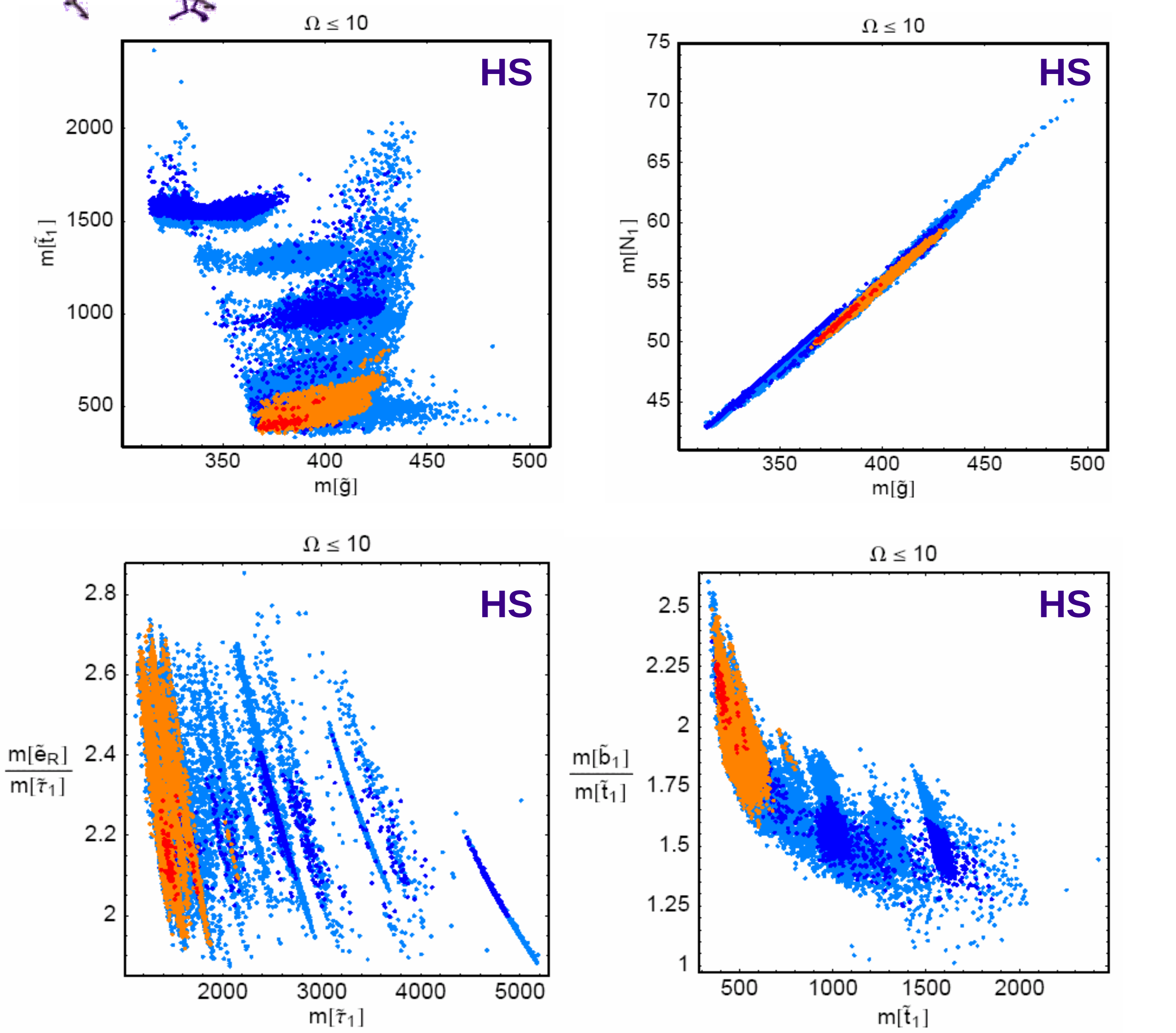




Mass relations

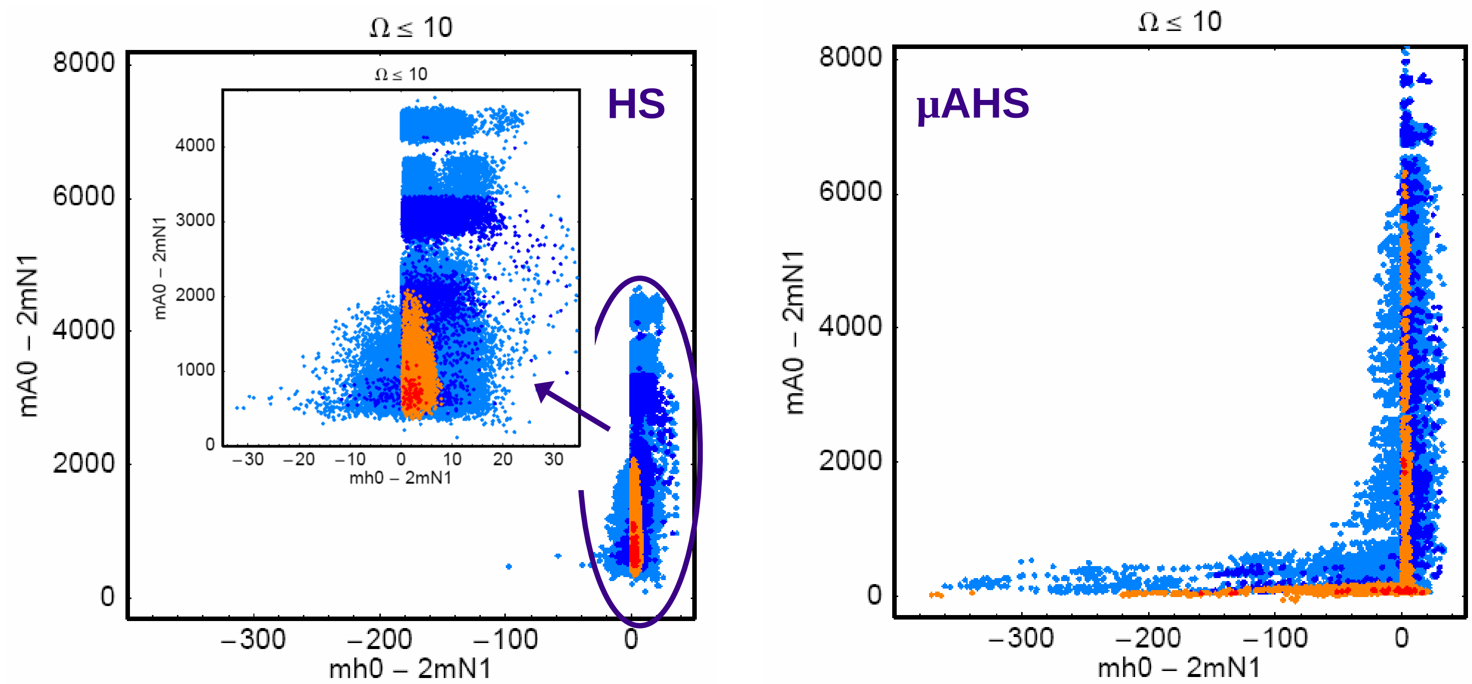


- 1st and 2nd generation sfermions are much heavier then 3rd family sfermions.
- $m(\tilde{g})$ ~400 GeV
- $m(\tilde{b}_1) > m(\tilde{t}_1)$ (important for LHC studies)

Besides these we have:

- $m(\tilde{\chi}_1^0)$ just above the LEP limits.
- $m(\tilde{\chi}_2^0) - m(\tilde{\chi}_1^0) < m(Z)$

Achieving a good Ω



R ≤ 1.10, R ≤ 1.05, R ≤ 1.10 & Ω ≤ 0.136, R ≤ 1.05 & Ω ≤ 0.136

Most effective neutralino annihilation mechanisms are:

- Annihilation through h^0 to bb or tt pairs
- Annihilation through A^0 to bb or tt pairs. Annihilation occurs even very far from the pole in μAHS case. We are investigating the the exact mechanism.

Conclusions:

- SO(10) SUSY GUTs are highly motivated models which predict Yukawa unification at GUT scale.

- We implemented the MCMC tool in order to search efficiently for the parameter space regions with both good Yukawa unification and WMAP-compatible DM relic density in two example SO(10) SUSY GUT scenarios.

- The regions we have discovered so far point out to distinguishable LHC signatures

- Further study is going on in order to understand the exact mechanism of achieving a WMAP-compatible DM relic density in SO(10) cases.