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Holography and the dynamics of strongly coupled gauge theories

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Introduction

♠ An analytic quantitative approach to strongly coupled gauge theories is one of the holy grails of modern theoretical physics.

The reasons are:

♠ QCD a very successful theory for the strong interactions is such a strongly coupled gauge theory.

♠ New strongly-coupled gauge theories are one of the expected ingredients at the TeV scale. They may appear in various forms:

- Technicolor-like theories or little Higgs versions.
- Warped higher dimensions theories in the Randall-Sundrum family that are qualitatively similar and in many cases dual to strongly-coupled 4d gauge theories.
- Hidden sector (or "hidden valley") theories, that are necessary for supersymmetry breaking, or omnipresent and generically required for the consistency of string theory SM-like vacua.

Remarkably, we do not have analytical control over most of the energy regime. Even numerically (lattice), many aspects of the theory are still beyond reach

.

♠ Despite, analytical weak coupling tools,numerical (lattice) calculations, and (semi)-phenomenological approaches (chiral perturbation theory, traditional large-N techniques, resummations, bag models, Lund and fragmentation models etc) we cannot reliably calculate in QCD several observables of interest:

- Glueball spectra for higher glueballs, mesons and baryons. Decay widths for essentially all particles.

- There are at least two weak matrix elements that cannot be computed so far reliably enough by lattice computations: The $\Delta I = \frac{1}{2}$ matrix elements of type $\langle K|\mathcal{O}_{\Delta I=1/2,3/2}|\pi\pi\rangle$, and the $B_K \sim \langle K|\mathcal{O}_{\Delta S=2}|\bar{K}\rangle$.

- Data associated to the chiral symmetry breaking (like the quark condensate), or its restauration at higher temperatures.

- In general matrix elements with at least two particle final states.

- Real time finite temperature correlation functions (associated to QGP dynamics) and badly needed for comparison with current data from RHIC and future data from ALICE

- Finite temperature physics at finite baryon density (potentially relevant for astrophysical purposes).

AdS/CFT and holography

♠ 't Hooft had indicated in 1974 that pure SU(N_c) YM theory has an extra "parameter": N_c. In the limit

$$N_c \to \infty \quad , \quad \lambda \equiv g_{YM}^2 N_c \quad \to \quad \text{fixed}$$

the perturbative series in $\frac{1}{N_c}$ resembles that of a string theory (with string coupling constant $\sim \frac{1}{N_c}$) and as dominant diagrams the "planar ones" (corresponding to "classical" string diagrams).

♠ Assuming confinement, the observable fields are finite mass color singlets (glueballs, mesons) with negligible interactions. Baryons are a more complicated though.

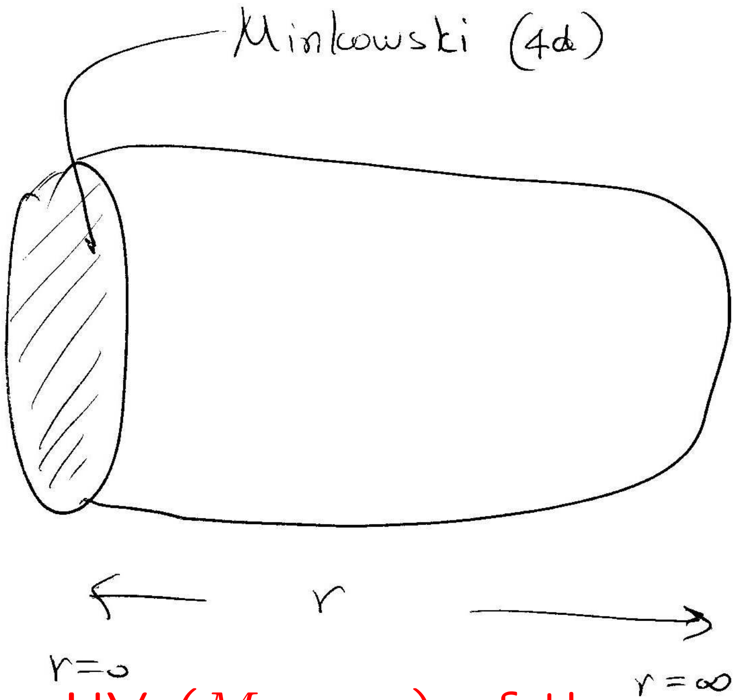
• This has spurned the quest for a low-energy weakly-coupled string description of hadron physics.

♠ The surprise in this quest emergent in 1997 when it was realized in a different 4d gauge theory (a strongly-coupled conformal theory with 4 supersymmetries) that, the relevant string theory lives in 10 rather than in the expected 4 dimensions.

Maldacena

The spacetime (string) background geometry is that of $AdS_5 \times S^5$

$$AdS_5 \quad \to \quad ds^2 = \frac{\ell^2_{\rm AdS}}{r^2}(dr^2 + \eta_{\mu\nu}dx^\mu dx^\nu) \quad , \quad R = -\frac{6}{\ell^2_{\rm AdS}}$$



• The extra coordinate r is the "holographic coordinate". There is Poincaré invariance in the 4d coordinates x^μ.

• The space is non-compact with a boundary at r=0 (isomorphic to Minkowski space).

• The holographic coordinate can be interpreted as a RG scale M.

• The boundary at $r = 0$ corresponds to the UV ($M = \infty$) of the gauge theory. $r = \infty$ is the IR ($M = 0$).

• There is a 1-1 correspondence between UV divergences in the gauge theory and IR divergences (near the AdS boundary) of the gravity (string) theory. Both theories need "renormalization" as usual.

The correspondence

gauge theory $\leftrightarrow$ string theory

• To every gauge-invariant gauge theory operator $\leftrightarrow$ a particle (on-shell) state of the string theory.

• Matching parameters:

$$g^2_{YM} = 4\pi g_s \quad , \quad \lambda \equiv g^2_{YM} N_c = \frac{\ell^4_{\text{AdS}}}{\ell^4_{\text{string}}}$$

• As $N_c \to \infty$, $\quad \lambda \to$ fixed, $\quad g_s \sim \frac{\lambda}{N_c} \to 0$ string loops are suppressed.

• If $\lambda >> 1$, the $\ell_{\text{AdS}} >> \ell_{\text{string}}$ the geometry is weakly curved $\to$ effectively $\ell_s \to 0$ $\quad \to \quad$ the string is "stiff" $\to$ we can approximate it with its zero modes (and drop the oscillation modes).

The effective theory

The generic string "zero modes" are:

- The graviton $g_{\mu\nu} \quad \rightarrow \quad T_{\mu\nu} \sim Tr[F^2_{\mu\nu} - \frac{1}{4}\eta_{\mu\nu}F^2]$
- The dilaton scalar $\phi \quad \rightarrow \quad Tr[F^2]$
- The RR (pseudoscalar) axion $a \quad \rightarrow \quad Tr[F \wedge F]$

with effective string theory action

$$S_{\text{string}} \sim M_P^3 \int d^5x\sqrt{g}\ \left[e^{-2\phi}\left(R - \frac{4}{3}(\partial\phi)^2 + \cdots\right) + (\partial a)^2 + \cdots\right]$$

$$\lambda \sim N_c\ e^{\phi} \quad , \quad \theta \sim a$$

- The (Lorentz invariant in 4d) classical solution for $g_{\mu\nu}$, ϕ, a etc corresponds to the "vacuum" of the gauge theory.

- Fluctuations around the vacuum solution represent the color-singlet propagating particles (glueballs here) of the gauge theory. This is an eigenvalue (Schrondinger-like) problem that gives a discrete spectrum of masses (and a mass gap) in confining gauge theories.

- Fluctuations of $g_{\mu\nu}$ gives a tower of bound states with spin 2 (2^{++} glueballs). The dilaton gives the tower of 0^{++} glueballs. The axion gives the tower of 0^{+-} glueballs, etc.

The thermal gauge theory

• Putting the gauge theory at finite temperature T amounts to compactifying Euclidean time to a circle of radius $\beta = \frac{1}{T}$

• In the dual string theory, we must consider solutions that near the AdS boundary look like $S^1_\beta \times R^3$

Unlike the $T = 0$ case now the "vacuum solution" is not unique. There are two generic kinds:

♠ The "thermal vacuum solution". This is the same as the vacuum solution but with the Euclidean time circle compactified with radius β.

$$ds^2_{TV} = e^{2A(r)} \left[dr^2 + dt^2 + d\vec{x} \cdot d\vec{x}\right]$$

In confining theories it describes the low-T confining phase.

♠ The "black hole solution"

$$ds^2_{BH} = e^{2A(r)} \left[\frac{dr^2}{f(r)} + f(r)\ dt^2 + d\vec{x} \cdot d\vec{x}\right]$$

In confining theories it describes the high-T deconfined phase (Quark-Gluon-Plasma phase).

The deconfining transition and QGP

• Both the TV and BH solutions are large-N_c saddle points (semiclassical minima)

• Which one dominates and is the true vacuum can be decided by comparing their free energies:

$$F_{TV} = N_c^2\ S(g_{TV}) \quad , \quad F_{BH} = N_c^2\ S(g_{BH})$$

• The two are equal at $T = T_c$. Below T_c, the true vacuum is the Thermal Vacuum (confinement). Above T_c it is the BH that dominates (QGP phase)

• The entropy of the gauge theory is O(1) in the confined phase and $\mathcal{O}(N_c^2)$ in the QGP phase. It coincides with the Bekenstein-Hawking entropy of the BH

• The low-energy dynamics of strongly coupled gauge theory QG plasma is described by the gravitational fluid dynamics of the black hole.

• Black holes have universal low-energy features that translate into a universality of the non-abelian plasmas.

Can we control the gauge theory?

• String duals of gauge theories involve RR backgrounds, and so far we do not know how to solve the associated string theories.

• If the string background is weakly curved, then we can solve the theory in the "zero mode" (classical gravity) approximation.

• There are several strongly-coupled gauge theories which are weakly curved. They are all 10d. We can engineer pure YM in the IR, but we cannot separate other (higher d) dynamics so far.

• QCD and several other gauge theories (a) live in 5d only (b) in the UV they are weakly coupled (asymptotic freedom). Therefore the dual string is "soft" (and the gravity approximation breaks down).

• To study them holographic (semi)-phenomenological models are used.

AdS/QCD

♠ The crudest model: use a slice of AdS_5, with a UV cutoff, and an IR cutoff.
Polchinski+Strassler, also Randall-Sundrum I

♠ It successfully exhibits confinement (trivially via IR cutoff), and power-like behavior in hard scattering amplitudes

♠ It may be equipped with $U(N_f)_L \times U(N_f)_R$, gauge fields and a bifundamental scalar, T, to describe mesons.
Erlich+Katz+Son+Stepanov, DaRold+Pomarol

♠ Chiral symmetry is broken by hand, via IR boundary conditions. The low-lying meson spectrum looks partly "reasonable".

♠ Shortcomings:

- The glueball spectrum does not fit very well the lattice calculations.In particular it has the wrong behavior $m_n^2 \sim n^2$ at large n.
- Magnetic quarks are confined instead of screened.
- Chiral symmetry breaking is input by hand.
- The meson spectrum has also the wrong UV asymptotics $m_n^2 \sim n^2$.

Improved Holographic QCD

- The effective action

$$S_{\text{string}} = M^3 N_c^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3}\frac{(\partial\lambda)^2}{\lambda^2} + V(\lambda) \right] \quad , \quad \lambda = N_c \, e^{\phi}$$

with a dilaton potential $V(\lambda)$.

- Find a classical solution: $\lambda(r)$ and $ds^2 = e^{2A(r)}(dr^2 + dx^\mu dx_\mu)$

- In the UV $\lambda(r) \to 0$ (asymptotic freedom) and the metric becomes AdS_5.

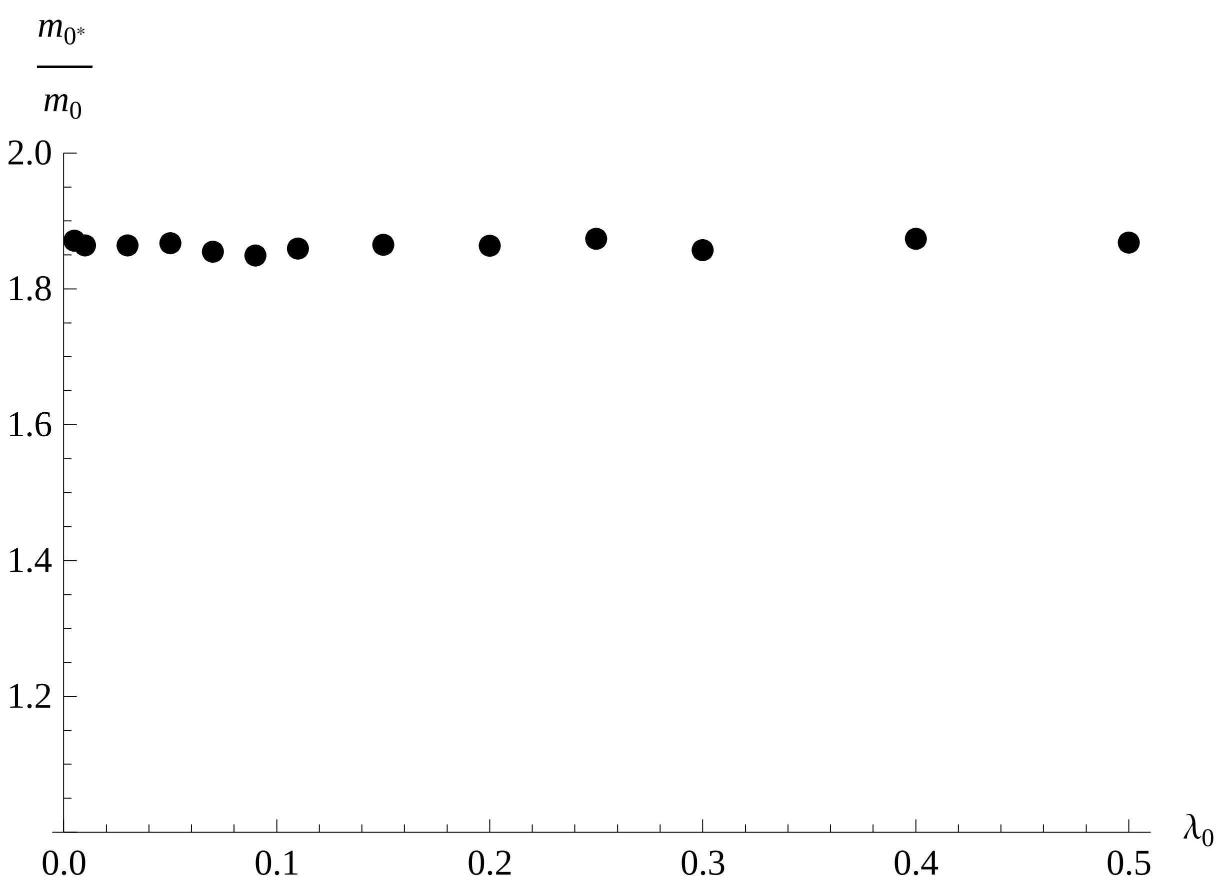
- There is a 1-1 correspondence between the QCD β-function, $\beta(\lambda)$ and the dilaton potential $V(\lambda)$

- In the IR, $\lambda \to \infty$ and

$$V(\lambda) \simeq \sqrt{\log \lambda} \; \lambda^{\frac{4}{3}} + \cdots \quad , \quad \beta(\lambda) \simeq -\frac{3}{2}\lambda \left[1 + \frac{3}{8}\frac{1}{\log \lambda} + \mathcal{O}\left(\frac{1}{\log^2 \lambda} \right) \right]$$

for confinement and asymptotically-linear Regge trajectories

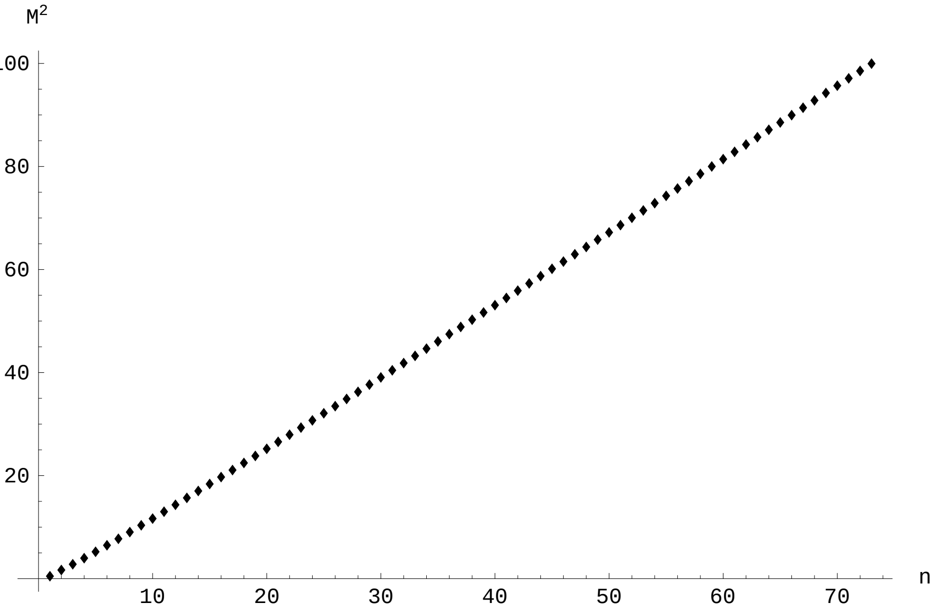
Dependence of mass ratios on λ_0



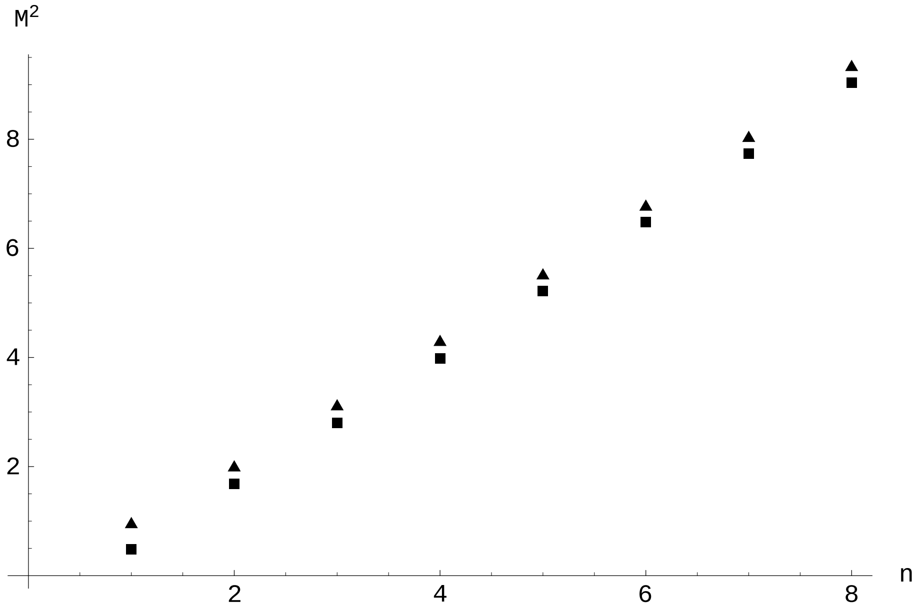
The mass ratios R_{20}

$$R_{20} = \frac{m_{2++}}{m_{0++}}.$$

Linearity of the glueball spectrum



(a)

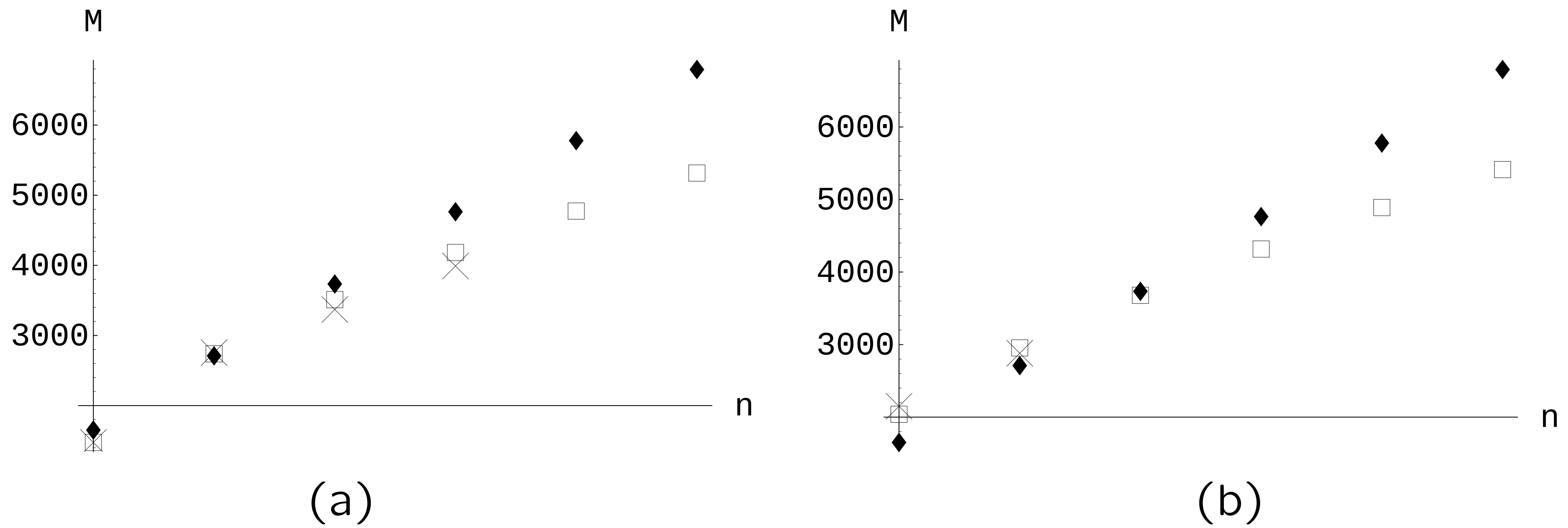


(b)

(a) Linear pattern in the spectrum for the first 40 0^{++} glueball states. M^2 is shown units of $0.015\ell^{-2}$.

(b) The first 8 0^{++} (squares) and the 2^{++} (triangles) glueballs. These spectra are obtained in the background I with $b_0 = 4.2, \lambda_0 = 0.05$.

Comparison with lattice data: Ref I



Comparison of glueball spectra from our model with $b_0 = 4.2, \lambda_0 = 0.05$ (boxes), with the lattice QCD data from Ref. I (crosses) and the AdS/QCD computation (diamonds), for (a) 0^{++} glueballs; (b) 2^{++} glueballs. The masses are in MeV, and the scale is normalized to match the lowest 0^{++} state from Ref. I.

$$\ell_{eff}^2 = 6.88\ \ell_{\text{AdS}}^2$$

and "predict"

$$\alpha_s(1.2GeV) = 0.34,$$

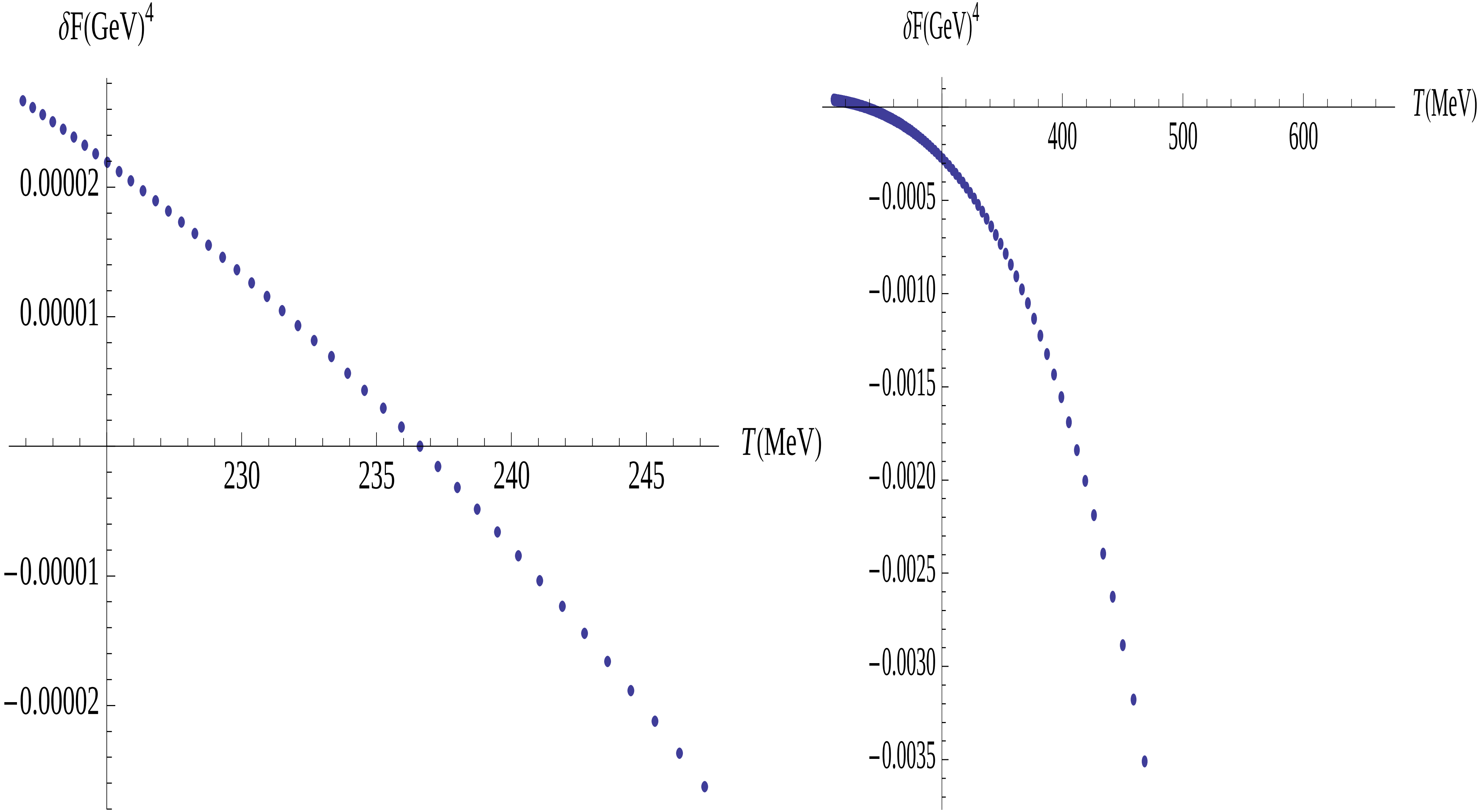
which is within the error of the quoted experimental value $\alpha_s^{(exp)}(1.2GeV) = 0.35 \pm 0.01$

The fit to Ref I

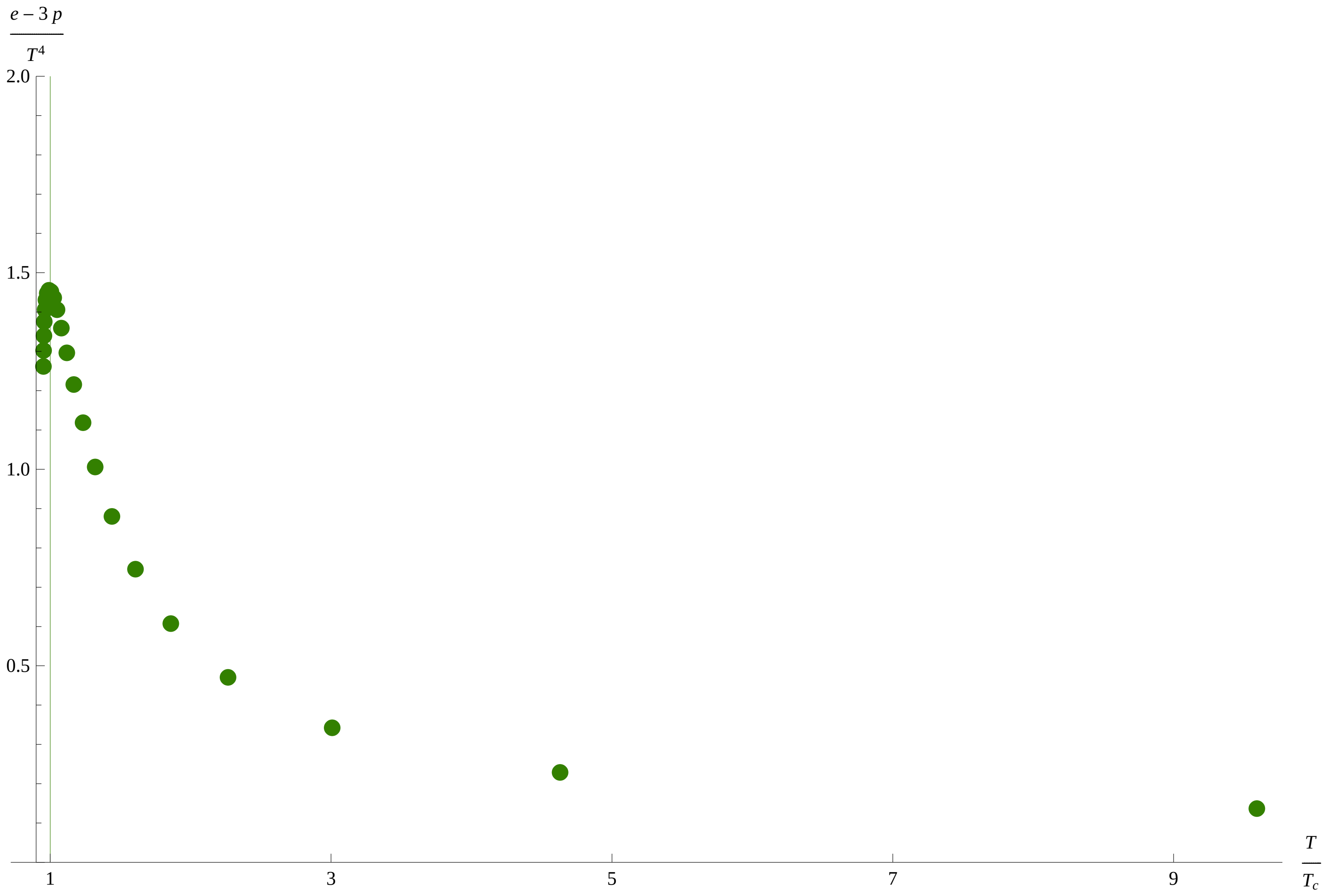
J^{PC}	Ref I (MeV)	Our model (MeV)	Mismatch	$N_c \to \infty$ [?]	Mismatch
0^{++}	**1475 (4%)**	**1475**	0	**1475**	0
2^{++}	2150 (5%)	2055	4%	2153 (10%)	5%
0^{-+}	**2250 (4%)**	**2243**	0		
0^{++*}	**2755 (4%)**	**2753**	0	2814 (12%)	2%
2^{++*}	2880 (5%)	2991	4%		
0^{-+*}	3370 (4%)	3288	2%		
0^{++**}	3370 (4%)	3561	5%		
0^{++***}	3990 (5%)	4253	6%		

Comparison between the glueball spectra in Ref. I and in our model. The states we use as input in our fit are marked in red. The parenthesis in the lattice data indicate the percent accuracy.

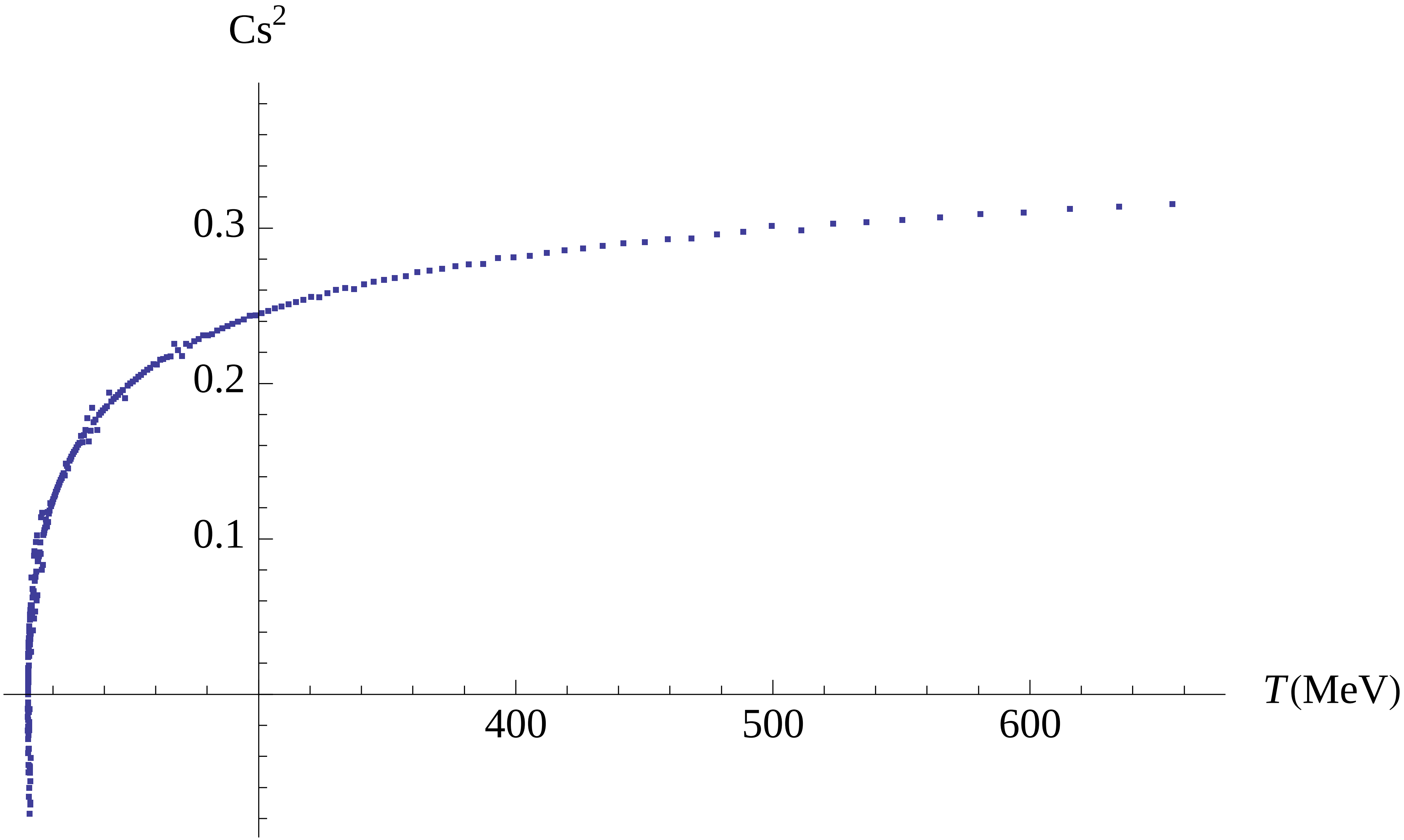
The transition in the free energy



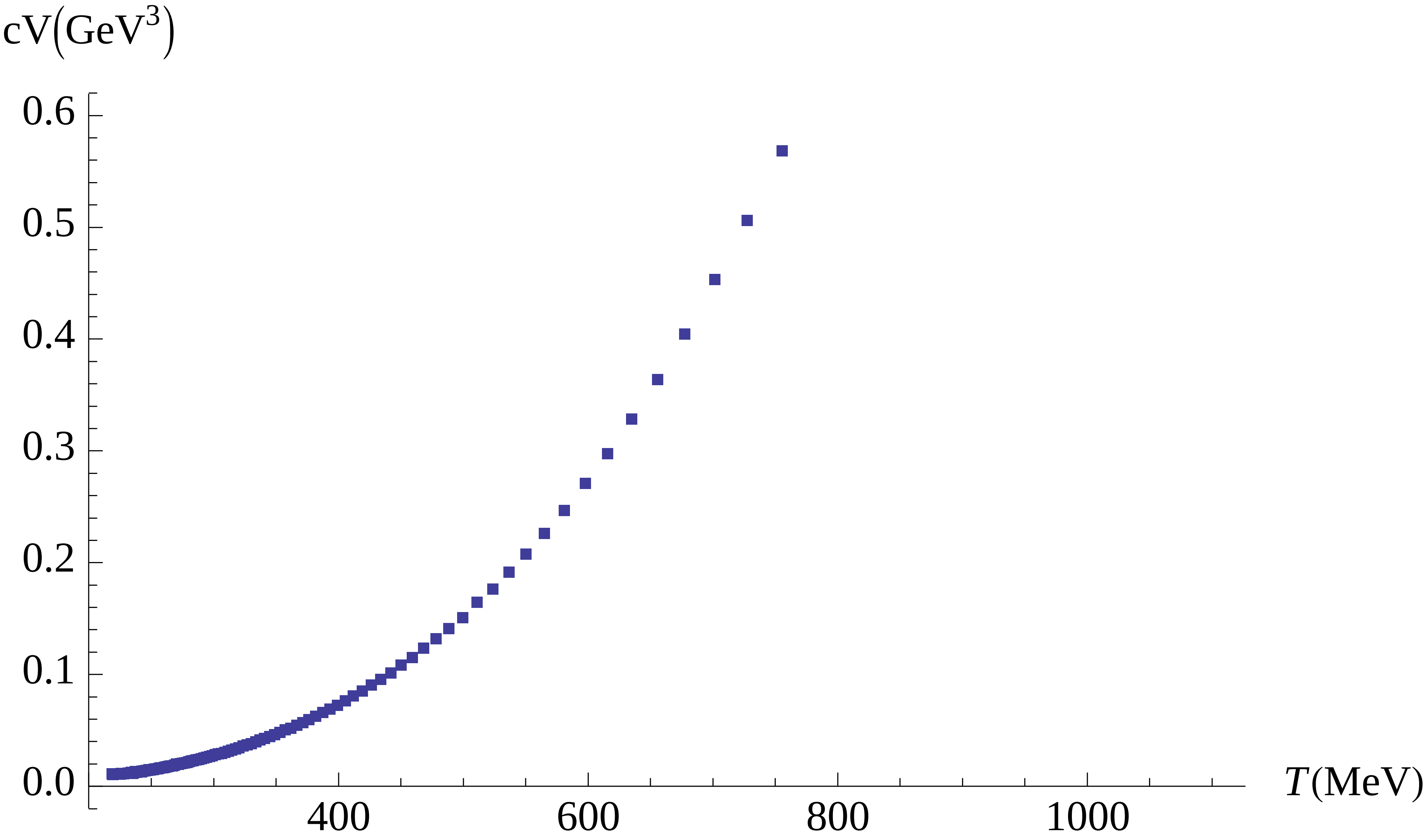
Equation of state



The speed of sound (bulk viscosity)



The specific heat



Many open ends

• This approach towards an improved holographic QCD model is preliminary but seems promising

• Several immediate directions:

♠ Calculate the meson spectrum and compare with data.

♠ Explore the baryon spectrum

♠ Diagonalize the $\eta' - 0^{+-}$ system and compare with data.

♠ Recalculate the dipole moment of the neutron in connection with the strong CP problem.

♠ Calculate RHIC/LHC finite T observables (like jet quenching)

♠ Analyze different strongly coupled theories in particular N=1 super YM.

Bibliography

- The work on the Improved Holographic QCD model has appeared in

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- Improved Holographic QCD: Part I: **arXiv:0707.1324**
U. Gursoy, E. Kiritsis

- Improved Holographic QCD: Part II: **arXiv:0707.1349**
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- Thermal Improved Holographic QCD: **to appear soon**
U. Gursoy, E. Kiritsis, L. Mazzanti, F. Nitti

A preview of the results: pure glue

♠ The starting point of pure QCD: a two-derivative action in 5d involving

$$g_{\mu\nu} \leftrightarrow T_{\mu\nu} \quad , \quad \phi \leftrightarrow Tr[F^2] \quad , \quad a \leftrightarrow Tr[F \wedge F]$$

$$S_{\text{Einstein}} = M^3 N_c^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} - \frac{Z(\lambda)}{2N_c^2} (\partial a)^2 + V(\lambda) \right] \quad , \quad \lambda = N_c \, e^{\phi}$$

with

$$V(\lambda) = V_0 \left(1 + \sum_{n=1}^{\infty} V_n \lambda^n \right) = -\frac{4}{3} \lambda^2 \left(\frac{dW}{d\lambda} \right)^2 + \frac{64}{27} W^2.$$

• There is a 1-1 correspondence between the QCD β-function, $\beta(\lambda)$ and W:

$$\beta(\lambda) = -\frac{9}{4} \lambda^2 \, \frac{d \log W(\lambda)}{d\lambda}$$

• There is a similar statement between $Z(\lambda)$ and the (non-perturbative) β-function for the θ-angle.

• The space is asymptotically AdS$_5$ in the UV ($r \to 0$) modulo log corrections (in the Einstein frame):

$$ds^2 = e^{2A(r)}(dr^2 + \eta_{\mu\nu}dx^\mu dx^\mu) \quad , \quad E \equiv e^{A(r)}$$

• There are various extra α' corrections to the potential ($\sim \beta$-function). They only correct the non-universal terms. Moreover, α' corrections to the energy definition E can be set to zero in a special scheme (the "holographic" scheme).

• ALL confining backgrounds have an IR singularity at $r = r_0$. There are two classes: r_0=finite and $r_0 = \infty$. The singularity is always "good": all spectra are well defined without extra input.

• $\lambda \to \infty$ at the IR singularity.

• In the $r_0 = \infty$ class of backgrounds, the curvature (in the string frame) vanishes in the neighborhood of the IR singularity.

Classification of confining superpotentials $W(\lambda)$ as $\lambda \to \infty$ in IR:

$$W(\lambda) \sim (\log\lambda)^{\frac{P}{2}}\ \lambda^{Q} \quad , \quad \lambda \sim E^{-\frac{9}{4}Q}\left(\log\frac{1}{E}\right)^{\frac{P}{2Q}}, \qquad E \to 0.$$

• $Q > 2/3$ or $Q = 2/3$ and $P > 1$ leads to confinement and a singularity at finite $r = r_0$.

• $Q = 2/3$, and $0 \leq P < 1$ leads to confinement and a singularity at $r = \infty$ The scale factor e^A vanishes there exponentially in the r coordinate.

• For all potentials that confine, the spectrum of 0^{++} and 2^{++} glueballs has a mass gap and is purely discrete. For the 0^{+-} glueballs this is the case if

$$Z(\lambda) \sim \lambda^d \quad , \quad d > 2 \quad \text{as} \quad \lambda \to \infty.$$

We will later derive that $d = 4$.

• In all physically interesting confining backgrounds the magnetic color charges are screened. This is an improvement with respect to AdS/QCD models (magnetic quarks are also confined instead) .

• Of all the possible confining asymptotics, there is a unique one that guarantees "linear confinement" ($m_n^2 \sim n$) for all glueballs. It corresponds to the case $Q = 2/3, P = 1/2$, i.e.

$$W(\lambda) \sim (\log \lambda)^{\frac{1}{4}} \lambda^{\frac{2}{3}} \quad , \quad \beta(\lambda) = -\frac{3}{2}\lambda \left[1 + \frac{3}{8 \log \lambda} + \cdots\right] \quad , \quad \lambda \sim E^{-\frac{3}{2}} \left(\log \frac{1}{E}\right)^{\frac{3}{8}}$$

This choice also seems to be preferred from considerations of the meson sector as discussed below.

• Numerical calculation of the 0^{++} and 2^{++} glueball spectra and comparison with lattice data gives a clear preference for the $r_0 = \infty$ asymptotics.

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- We can find the background solution for the axion:

$$a(r) = (\theta_{UV} + 2\pi k) \int_r^{r_0} \frac{dr}{e^{3A} Z(\lambda)} \Big/ \int_0^{r_0} \frac{dr}{e^{3A} Z(\lambda)}$$

written in terms of the axion coupling function $Z(\lambda)$ and the scale factor e^A. This provides the "running" of the effective QCD θ angle.

- A direct holographic calculation of the θ-dependent vacuum energy gives

$$E(\theta_{UV}) \sim \text{Min}_k (\theta_{UV} + 2\pi k)^2$$

.

- Note that always $a(E=0) = 0$. This suggests that the θ angle is screened in the IR.

Preview: quarks ($N_f \ll N_c$) and mesons

• Flavor is introduced by N_f $D_4 + \bar{D}_4$ branes pairs inside the bulk background. Their back-reaction on the bulk geometry is suppressed by N_f/N_c.

• The important world-volume fields are

$$T_{ij} \leftrightarrow \bar{q}^i_a \frac{1+\gamma^5}{2} q^j_a \quad , \quad A_\mu^{ij\,L,R} \leftrightarrow \bar{q}^i_a \frac{1\pm\gamma^5}{2} \gamma^\mu q^j_a$$

Generating the $U(N_f)_L \times U(N_f)_R$ chiral symmetry.

• The UV mass matrix m_{ij} corresponds to the source term of the Tachyon field. It breaks the chiral (gauge) symmetry. The normalizable mode corresponds to the vev $\langle \bar{q}^i_a \frac{1+\gamma^5}{2} q^j_a \rangle$.

• We show that the expectation value of the tachyon is non-zero and $T \sim \mathbf{1}$, breaking chiral symmetry $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$. The anomaly plays an important role in this (holographic Coleman-Witten)

• The fact that the tachyon diverges in the IR (fusing D with $\bar{D}$) constraints the UV asymptotics and determines the quark condensate $\langle\bar{q}q\rangle$ in terms of m_q. A GOR relation is satisfied (for an asymptotic AdS$_5$ space)

$$m_\pi^2 = -2\frac{m_q}{f_\pi^2}\ \langle\bar{q}q\rangle \quad , \quad m_q \to 0$$

• We can derive formulae for the anomalous divergences of flavor currents, when they are coupled to an external source.

• When $m_q = 0$, the meson spectrum contains N_f^2 massless pseudoscalars, the $U(N_f)_A$ Goldstone bosons.

• The WZ part of the flavor brane action gives the Adler-Bell-Jackiw $U(1)_A$ axial anomaly and an associated Stuckelberg mechanism gives an $O\left(\frac{N_f}{N_c}\right)$ mass to the would-be Goldstone boson η', in accordance with the Veneziano-Witten formula.

• Studying the spectrum of highly excited mesons, we find the expected property of linear confinement: $m_n^2 \sim n$.

Motivating the effective action

- Spectrum in 5d: NSNS ($g_{\mu\nu}, B_{\mu\nu}, \phi$) and RR ($C_0 \leftrightarrow C_3$, $C_1 \leftrightarrow C_2$ and C_4).

- The basic string motivated action for the 5d theory is

$$S_5 = M^3 \int d^5x\sqrt{g}\left[e^{-2\phi}\left(R + 4(\partial\phi)^2 + \frac{\delta c}{\ell_s^2}\right) - \frac{1}{2\cdot 5!}F_5^2 - \frac{1}{2}(da)^2\right]$$

$F_5 = dC_4$ seeds the D_3 branes that generate the $U(N_c)$ group.

- The C_4 equation of motion gives ${}^*F_5 = N_c$ and the dual action in the Einstein frame $g_E = e^{\frac{4}{3}\phi}\ g_s$

$$S_E = M^3 \int d^5x\sqrt{g}\left[R - \frac{4}{3}(\partial\phi)^2 - \frac{e^{2\phi}}{2}(\partial a)^2 + V_s(\phi)\right] \quad , \quad V_s(\phi) = \frac{e^{\frac{4}{3}\phi}}{\ell_s^2}\left[\delta c - \frac{N_c^2}{2}e^{2\phi}\right]$$

- Higher derivative corrections involving the F_5 upon dualization provide further terms in the dilaton potential

$$V_s(\phi) = \frac{e^{\frac{4}{3}\phi}}{\ell_s^2}\left[\delta c + \sum_{n=1}^{\infty} a_n\ (N_c e^{\phi})^{2n}\right]$$

MORE INFO

♠ This potential is very good for the IR behavior but in the UV it vanishes with λ and this is not the correct behavior.

♠ We need a potential that in the Einstein frame asymptotes to a constant $V_0 = \frac{12}{\ell^2}$ as $\lambda \to 0$.

♠ This is generated by higher-derivative corrections in the curvature. Here we postulate it.

♠ The five form will then generate a series of (perturbative) terms in λ:

$$V(\lambda) = V_0 \left(1 + \sum_{n=1}^{\infty} a_n \lambda^{a\ n} \right)$$

we will take $a = 1$ for simplicity (by adjusting the kinetic term).

♠ This matches the weak coupling expansion of perturbative QCD and will give the perturbative β-function expansion.

♠ We will ignore other effects of higher-derivative terms associated with R and $(\partial\Phi)^2$. Motivated partly by the success of SVZ sum rules

♠ The "resumed" two-derivative action reads

$$S_{\text{Einstein}} = M^3 N_c^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V(\lambda) \right] \quad , \quad \lambda = N_c\, e^{\phi}$$

after redefining the kinetic terms.

• We must choose the holographic energy: the natural choice is $E = e^{A_E}$ frame as it is monotonic and end at zero in the IR singularity.

• We may now solve the equations perturbatively in λ around $\lambda = 0$ and $r = 0$ (this is a weak coupling expansion) to find

$$\frac{d\lambda}{d \log E} \equiv \beta(\lambda) = -b_0\lambda^2 + b_1\lambda^3 + b_2\lambda^4 + \cdots$$

with

$$\frac{1}{\lambda} = L - \frac{b_1}{b_0} \log L + \mathcal{O}\left(\frac{\log L}{L}\right) \quad , \quad L \equiv -b_0 \log(r\Lambda)$$

$$e^{2A} = \frac{\ell^2}{r^2} \left[1 + \frac{8}{3^2 \log r\Lambda} + \cdots \right]$$

$$V = \frac{12}{\ell^2} \left[1 + \frac{8}{9}(b_0\lambda) + \frac{23 - 36\frac{b_1}{b_0^2}}{3^4}(b_0\lambda)^2 + \cdots \right]$$

♠ One-to-one correspondence with the perturbative β-function, and the perturbative potential.

Organizing the vacuum solutions

A useful variable is the phase variable

$$X \equiv \frac{\Phi'}{3A'} = \frac{\beta(\lambda)}{3\lambda} \quad , \quad e^{\Phi} \equiv \lambda$$

and a superpotential

$$W^2 - \left(\frac{3}{4}\right)^2 \left(\frac{\partial W}{\partial \Phi}\right)^2 = \left(\frac{3}{4}\right)^3 V(\Phi).$$

with

$$A' = -\frac{4}{9} W \quad , \quad \Phi' = \frac{dW}{d\Phi}$$

$$X = -\frac{3}{4}\frac{d\log W}{d\log\lambda} \quad , \quad \beta(\lambda) = -\frac{9}{4}\lambda\frac{d\log W}{d\log\lambda}$$

♠ The equations have three integration constants: (two for Φ and one for A) One corresponds to the "gluon condensate" in the UV. It must be set to zero otherwise the IR behavior is unacceptable. The other is Λ. The third one is a gauge artifact (corresponds to overall translation in the radial coordinate).

The IR regime

For any asymptotically AdS$_5$ solution ($e^A \sim \frac{\ell}{r}$):

• The scale factor $e^{A(r)}$ is monotonically decreasing

Girardelo+Petrini+Porrati+Zaffaroni
Freedman+Gubser+Pilch+Warner

• Moreover, there are only three possible, mutually exclusive IR asymptotics:

♠ *there is another asymptotic AdS_5 region, at $r \to \infty$, where* $\exp A(r) \sim \ell'/r$, *and* $\ell' \leq \ell$ (equality holds if and only if the space is exactly AdS_5 everywhere);

♠ there is a curvature singularity at some finite value of the radial coordinate, $r = r_0$;

♠ **there is a curvature singularity at $r \to \infty$, where the scale factor vanishes and the space-time shrinks to zero size.**

Wilson-Loops and confinement

• Calculation of the static quark potential using the vev of the Wilson loop calculated via an F-string worldsheet.

Rey+Yee, Maldacena

$$T\ E(L) = S_{minimal}(X)$$

We calculate

$$L = 2\int_0^{r_0} dr \frac{1}{\sqrt{e^{4A_S(r)-4A_S(r_0)}-1}}.$$

It diverges when e^{A_s} has a minimum (at $r = r_*$). Then

$$E(L) \sim T_f\ e^{2A_S(r_*)}\ L$$

• Confinement $\to$ $A_s(r_*)$ is finite. This is a more general condition than was considered before as A_S `is not monotonic in general`.

• Effective string tension

$$T_{\rm string} = T_f\ e^{2A_S(r_*)}$$

General criterion for confinement

• **the geometric version:**
A geometry that shrinks to zero size in the IR is dual to a confining 4D theory if and only if the Einstein metric in conformal coordinates vanishes as (or faster than) e^{-Cr} as $r \to \infty$, for some $C > 0$.

• It is understood here that a metric vanishing at finite $r = r_0$ also satisfies the above condition.

♠ **the superpotential**
A 5D background is dual to a confining theory if the superpotential grows as (or faster than)

$$W \sim (\log \lambda)^{P/2} \lambda^{2/3} \quad \text{as} \quad \lambda \to \infty \quad , \quad P \geq 0$$

♠ **the β-function** A 5D background is dual to a confining theory if and only if

$$\lim_{\lambda \to \infty} \left(\frac{\beta(\lambda)}{3\lambda} + \frac{1}{2} \right) \log \lambda = K, \qquad -\infty \leq K \leq 0$$

(No explicit reference to any coordinate system) Linear trajectories correspond to $K = -\frac{3}{16}$

Comments on confining backgrounds

• For all confining backgrounds with $r_0 = \infty$, although the space-time is singular in the Einstein frame, the string frame geometry is asymptotically flat for large r. Therefore only λ grows indefinitely.

• String world-sheets do not probe the strong coupling region, at least classically. The string stays away from the strong coupling region.

• Therefore: singular confining backgrounds have generically the property that the singularity is *repulsive*, i.e. only highly excited states can probe it. This will also be reflected in the analysis of the particle spectrum (to be presented later)

• The confining backgrounds must also screen magnetic color charges. This can be checked by calculating 't Hooft loops using D_1 probes:

♠ All confining backgrounds with $r_0 = \infty$ and most at finite r_0 screen properly

♠ In particular "hard-wall" AdS/QCD confines also the magnetic quarks.

• scalar glueballs

$$B(r) = \frac{3}{2}A(r) + \frac{1}{2}\log\frac{\beta(\lambda)^2}{9\lambda^2}$$

• tensor glueballs

$$B(r) = \frac{3}{2}A(r)$$

• pseudo-scalar glueballs

$$B(r) = \frac{3}{2}A(r) + \frac{1}{2}\log Z(\lambda)$$

• Universality of asymptotics

$$\frac{m^2_{n\to\infty}(0^{++})}{m^2_{n\to\infty}(2^{++})} \to 1 \quad , \quad \frac{m^2_{n\to\infty}(0^{+-})}{m^2_{n\to\infty}(0^{++})} = \frac{1}{4}(d-2)^2$$

predicts $d = 4$ via

$$\frac{m^2}{2\pi\sigma_a} = 2n + J + c,$$

The axion background

- The kinetic term of the axion is suppressed by $1/N_c^2$. (it is an angle in the gauge theory, it is RR in string theory)

$$\ddot{a} + \left(3\dot{A} + \frac{\dot{Z}(\lambda)}{Z(\lambda)}\right)\dot{a} = 0 \quad \to \quad \dot{a} = \frac{C\ e^{-3A}}{Z(\lambda)}$$

It can be interpreted as the flow equation of the effective θ-angle.

- The full solution is

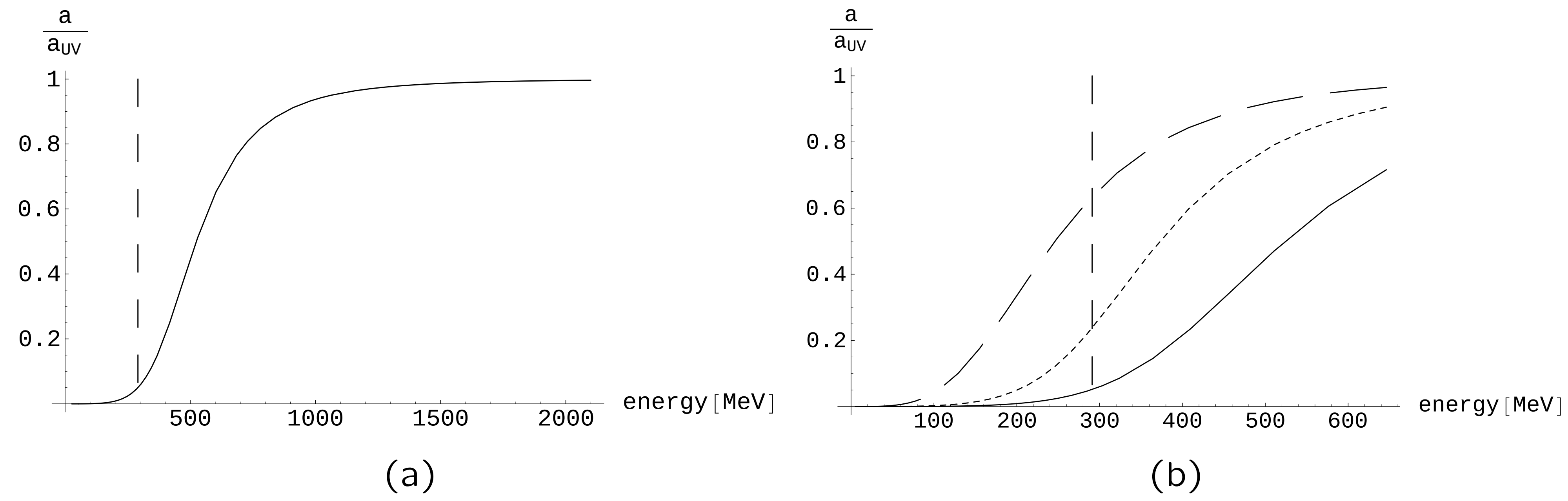
$$a(r) = \theta_{UV} + 2\pi k + C\ \int_0^r r\frac{e^{-3A}}{Z(\lambda)} \quad , \quad C = \langle Tr[F \wedge F]\rangle$$

- The vacuum energy is

$$E(\theta_{UV}) = \frac{M^3}{2N_c^2}\int d^5x\sqrt{g}Z(\lambda)(\partial a)^2 = \frac{M^3}{2N_c^2}Ca(r)\ \Big|_{r=0}^{r=r_0}$$

- Consistency requires to impose that $a(r_0) = 0$. This determines C and

$$E(\theta_{UV}) = -\frac{M^3}{2}\text{Min}_k\ \frac{(\theta_{UV} + 2\pi k)^2}{\int_0^{r_0}\frac{dr}{e^{3A}Z(\lambda)}} \quad , \quad \frac{a(r)}{\theta_{UV} + 2\pi k} = \frac{\int_r^{r_0}\frac{dr}{e^{3A}Z(\lambda)}}{\int_0^{r_0}\frac{dr}{e^{3A}Z(\lambda)}}$$



(a) An example of the axion profile (normalized to one in the UV) as a function of energy, in one of the explicit cases we treat numerically. The energy scale is in MeV, and it is normalized to match the mass of the lowest scalar glueball from lattice data, $m_0 = 1475 MeV$. The axion kinetic function is taken as $Z(\lambda) = Z_a(1 + c_a\lambda^4)$, with $c_a = 100$ (the masses do not depend on the value of Z_a). The vertical dashed line corresponds to $\Lambda_p \equiv \frac{1}{\ell}\frac{\exp\left[A(\lambda_0)-\frac{1}{b_0\lambda_0}\right]}{(b_0\lambda_0)^{b1/b_0^2}}$. In this particular case $\Lambda = 290 MeV$.

(b)A detail showing the different axion profiles for different values of c_a. The values are $c_a = 0.1$ (dashed line), $c_a = 10$ (dotted line) and $c_a = 100$ (solid line).

QCD at finite temperature

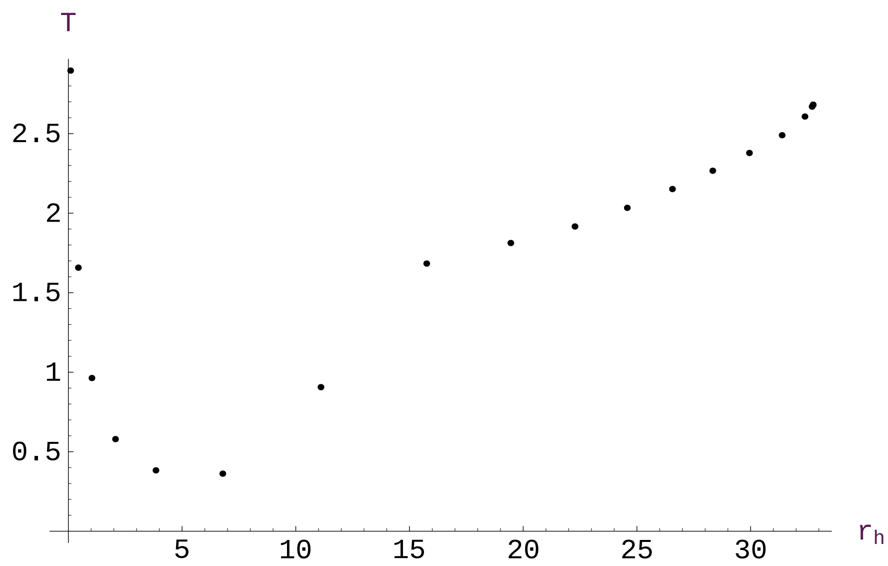
The thermal vacuum can be described by
(1) The "thermal vacuum solution". This is the zero temperature solution we desribed so far with time periodically identified with period β.

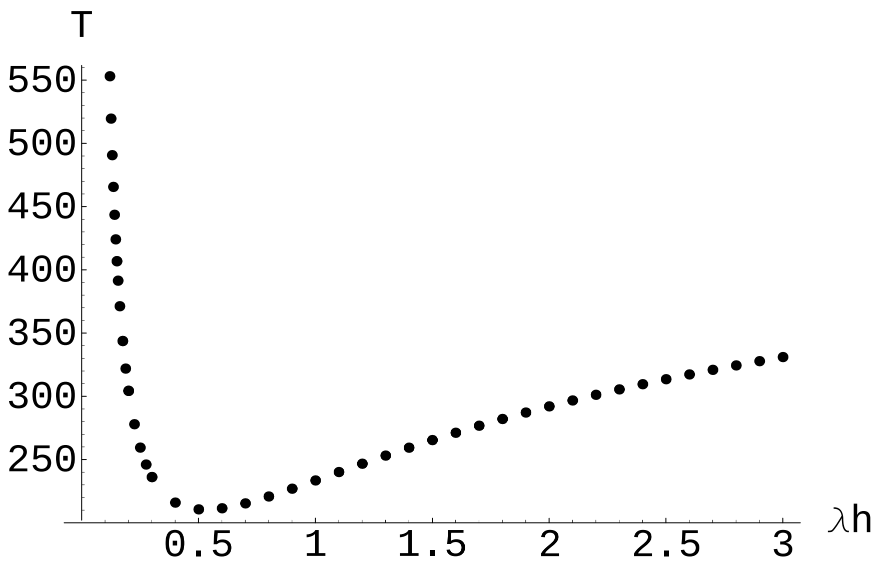
(2) The "black-hole solution"

$$ds^2 = b(r)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^i dx^i\right], \qquad \Phi = \Phi(r)$$

We can show the following:

• For $T > T_{\rm min}$ there are two black-hole solutions with the same temperature but different horizon positions. One is a "large" BH the other is "small".





• When $T < T_{\rm min}$ only the "thermal vacuum solution" exists: it describes the confined phase at finite temperature.

• When $T > T_{\rm min}$ three competing solutions exist. The large BH has the lowest free energy. It describes the deconfined QGP phase.

• The minimum temperature for the black-holes is $T_{\rm min} \simeq 210$ MeV with $\lambda_h = 0.34$. The critical temperature is

$$T_c \simeq 240 \quad \text{MeV} \quad , \quad \lambda_h = 0.54$$

• The specific heat for the QGP solution is positive as it should:

$$\frac{dE}{dT} = \frac{E}{T + \frac{3}{4\pi}\frac{\partial \log b}{\partial r_h}}$$

• In the QGP phase, the $q\bar{q}$ potential is screened. This is better than lattice results.

Critical string theory holography

♠ Several "successful" holographic models of non-trivial gauge dynamics

- The non-supersymmetric D_4 solution,due to Witten, dual to $\mathcal{N} = 4_5$ sYM on a circle, whose supersymmetry is broken by the boundary conditions of the fermions. It exhibits confinement in the IR.

- Flavor has been successfully incorporated by Sakai+Sugimoto by adding D_7 (dipole) branes.

- The Chamseddine-Volkov solution interpreted by Maldacena and Nuñes as the dual of a confining compactified gauge theory (emerging by wrapping NS_5 branes on a two-cycle).

- The Klebanov-Strassler solution corresponding to a cascade of quiver gauge theories, that confine in the IR.

.

♠ In all of the above, confinement related quantities (string tension, glue-ball, masses etc, finite temperature effects) can be calculated analytically.

♠ The same applies to the Sakai-Sugimoto model for flavor, except two major drawbacks:
The absence of bare quark masses and the chiral-symmetry-breaking condensate.

♠ In all the above solutions, the scale of KK excitations is of the same order as Λ of the confining gauge theory.

♠ None so far has managed to overcome this obstacle in critical string theory models.

Non-Critical holography

♠ Non-critical string theories have been explored in order to avoid the KK problem.

Kuperstein+Sonnenschein, Klebanov+Maldacena, Bigazzi+Casero+Cotrone+Kiritsis+Paredes

♠ They are expected to involve large curvatures (due to the δc term) and the supergravity approximation seems problematic.

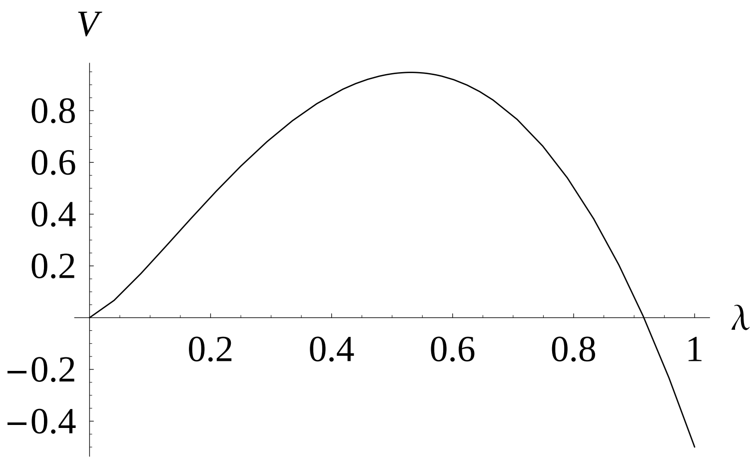
♠ They may provide reliable information on some quantities despite the strong curvature (cf. WZW CFTs).

♠ Recent progress in solving exactly for probe D-branes in non-critical backgrounds has provided important insights for non-critical holography.

Fotopoulos+Niarchos+Prezas, Ashok+Murthy+Troost

♠ It is fair to say that non-critical holography is so far largely unexplored.

Fluctuations around the AdS$_5$ extremum



- In QCD we expect that

$$\frac{1}{\lambda} = \frac{1}{N_c e^{\phi}} \sim \frac{1}{\log r} \quad , \quad ds^2 \sim \frac{1}{r^2}(dr^2 + dx_\mu dx^\mu) \qquad \text{as} \qquad r \to 0$$

- Any potential with $V(\lambda) \sim \lambda^a$ when $\lambda \ll 1$ gives a power different that of AdS$_5$

- There is an AdS$_5$ minimum at a finite value λ_*. This cannot be the UV of QCD as dimensions do not match.

Near an AdS extremum

$$V=\frac{12}{\ell^2}-\frac{16\xi}{3\ell^2}\phi^2+\mathcal{O}(\phi^3)\quad,\quad \frac{18}{\ell}\delta A'=\delta\phi'^2-\frac{4}{\ell^2}\phi^2=\mathcal{O}(\delta\phi^2)\quad,\quad \delta\phi''-\frac{4}{\ell}\delta\phi'-\frac{4\xi}{\ell^2}\delta\phi=0$$

where $\phi << 1$. The general solution of the second equation is

$$\delta\phi=C_+e^{\frac{(2+2\sqrt{1+\xi})u}{\ell}}+C_-e^{\frac{(2-2\sqrt{1+\xi})u}{\ell}}$$

For the potential in question

$$V(\phi)=\frac{e^{\frac{4}{3}\phi}}{\ell_s^2}\left[5-\frac{N_c^2}{2}e^{2\phi}-N_f\ e^{\phi}\right]\quad,\quad \lambda_0\equiv N_c e^{\phi_0}=\frac{-7x+\sqrt{49x^2+400}}{10}\quad,\quad x\equiv\frac{N_f}{N_c}$$

$$\xi=\frac{5}{4}\left[\frac{400+49x^2-7x\sqrt{49x^2+400}}{100+7x^2-x\sqrt{49x^2+400}}\right]\quad,\quad \frac{\ell_s^2}{\ell^2}=e^{\frac{4}{3}\phi_0}\left[\frac{100+7x^2-x\sqrt{49x^2+400}}{400}\right]$$

The associated dimension is $\Delta=2+2\sqrt{1+\xi}$ and satisfies

$$2+3\sqrt{2}<\Delta<2+2\sqrt{6}\qquad \text{or equivalently}\qquad 6.24<\Delta<6.90$$

It corresponds to an irrelevant operator. It is most probably relevant for the Banks-Zaks fixed points.

Bigazzi+Casero+Cotrone+Kiritsis+Paredes

RETURN

Further α' corrections

There are further dilaton terms generated by the 5-form in:

• The kinetic terms of the graviton and the dilaton $\sim \lambda^{2n}$.

• The kinetic terms on probe D_3 branes that affect the identification of the gauge-coupling constant, $\sim \lambda^{2n+1}$. There is also a multiplicative factor relating g_{YM^2} to e^{ϕ}, (not known). Can be traded for b_0.

• Corrections to the identification of the energy. At $r = 0$, $E = 1/r$. There can be log corrections to our identification $E = e^A$, and these are a power series in $\sim \lambda^{2n}$.

• It is a remarkable fact that all such corrections affect the higher that the first two terms in the β-function (or equivalently the potential), that are known to be non-universal!

the metric is also insensitive to the change of b_0 by changing Λ.

Holographic meson dynamics: the models

- Flavor is obtained by adding $N_f << N_C$ D+$\bar{D}$ pairs

- There are several working models of flavor:

♠ Non-supersymmetric backgrounds with abelian D_7flavor brane.

Babington+Erdmenger+Evans+Guralnic+Kirsch

Kruczenski+Mateos+Myers+Winters

♠ Non-supersymmetric $D4+$ $D_8 + \bar{D}_8$

Sakai+Sugimoto

♠ Hard-wall AdS/QCD plus a scalar, plus $U(N_f)_L \times U(N_f)_R$ vectors

Erlich+Katz+son+Stephanov, DaRold+Pomarol

Classification of confining superpotentials

Classification of confining superpotentials $W(\lambda)$ as $\lambda \to \infty$ in IR:

$$W(\lambda) \sim (\log \lambda)^{\frac{P}{2}} \lambda^{Q} \quad , \quad \lambda \sim E^{-\frac{9}{4}Q} \left(\log \frac{1}{E} \right)^{\frac{P}{2Q}} , \qquad E \to 0.$$

• $Q > 2/3$ or $Q = 2/3$ and $P > 1$ leads to confinement and a singularity at finite $r = r_0$.

$$e^A(r) \sim \begin{cases} (r_0 - r)^{\frac{4}{9Q^2-4}} & Q > \frac{2}{3} \\ \exp\left[-\frac{C}{(r_0-r)^{1/(P-1)}} \right] & Q = \frac{2}{3} \end{cases}$$

• $Q = 2/3$, and $0 \le P < 1$ leads to confinement and a singularity at $r = \infty$ The scale factor e^A vanishes there as

$$e^A(r) \sim \exp[-Cr^{1/(1-P)}].$$

• $Q = 2/3, P = 1$ leads to confinement but the singularity may be at a finite or infinite value of r depending on subleading asymptotics of the superpotential.

♠ If $Q < 2\sqrt{2}/3$, no *ad hoc* boundary conditions are needed to determine the glueball spectrum $\to$ One-to-one correspondence with the β-function This is unlike standard AdS/QCD and other approaches.

• when $Q > 2\sqrt{2}/3$, the spectrum is not well defined without extra boundary conditions in the IR because both solutions to the mass eigenvalue equation are IR normalizable.

Confining β-functions

A 5D background is dual to a confining theory if and only if

$$\lim_{\lambda\to\infty}\left(\frac{\beta(\lambda)}{3\lambda}+\frac{1}{2}\right)\log\lambda = K, \qquad -\infty \le K \le 0$$

(No explicit reference to any coordinate system). Linear trajectories correspond to $K = -\frac{3}{16}$

- We can determine the geometry if we specify K:
- $K = -\infty$: the scale factor goes to zero at some finite r_0, not faster than a power-law.
- $-\infty < K < -3/8$: the scale factor goes to zero at some finite r_0 faster than any power-law.
- $-3/8 < K < 0$: the scale factor goes to zero as $r \to \infty$ faster than $e^{-Cr^{1+\epsilon}}$ for some $\epsilon > 0$.
- $K = 0$: the scale factor goes to zero as $r \to \infty$ as e^{-Cr} (or faster), but slower than $e^{-Cr^{1+\epsilon}}$ for any $\epsilon > 0$.

The borderline case, $K = -3/8$, is certainly confining (by continuity), but whether or not the singularity is at finite r depends on the subleading terms.

Calculating hadron Spectra

- A fluctuation equation (linearized) of a given string theory looks like :

$$\frac{d^2\xi}{dr^2}+2\frac{dB(r)}{dr}\frac{d\xi}{dr}+\Box_4\xi=0 \quad , \quad \Box_4\equiv\frac{\partial^2}{\partial x^\mu\partial x_\mu}$$

It is solved by separation of variables

$$, \quad \xi(r,x)=\xi(r)\xi^{(4)}(x), \qquad \Box\xi^{(4)}(x)=m^2\xi^{(4)}(x)$$

- The equation for the radial wavefunction $\xi(r)$ can be mapped to an effective Schrodinger problem

$$-\frac{d^2}{dr^2}\psi+V(r)\psi=m^2\psi \quad , \quad V(r)=\frac{d^2B}{dr^2}+\left(\frac{dB}{dr}\right)^2 \quad , \quad \xi(r)=e^{-B(r)}\psi(r)$$

- This is an eigenvalue problem with a discrete spectrum of masses (and a mass gap) in confining gauge theories.

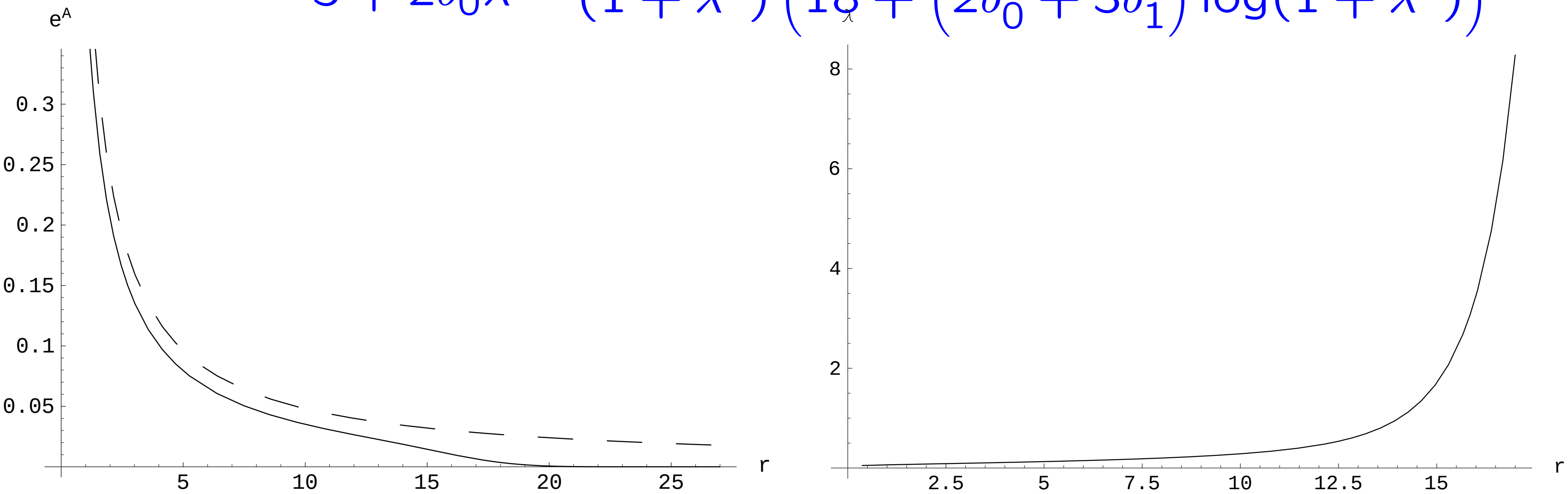
- The mass gap and discrete spectrum are visible from the asymptotics of the effective Schrodinger potential. Large n asymptotics of masses obtained from WKB

- Fluctuations of $g_{\mu\nu}$ gives a tower of bound states with spin 2 (2^{++} glueballs). The dilaton gives the tower of 0^{++} glueballs. The axion gives the tower of 0^{+-} glueballs, etc.

The concrete model

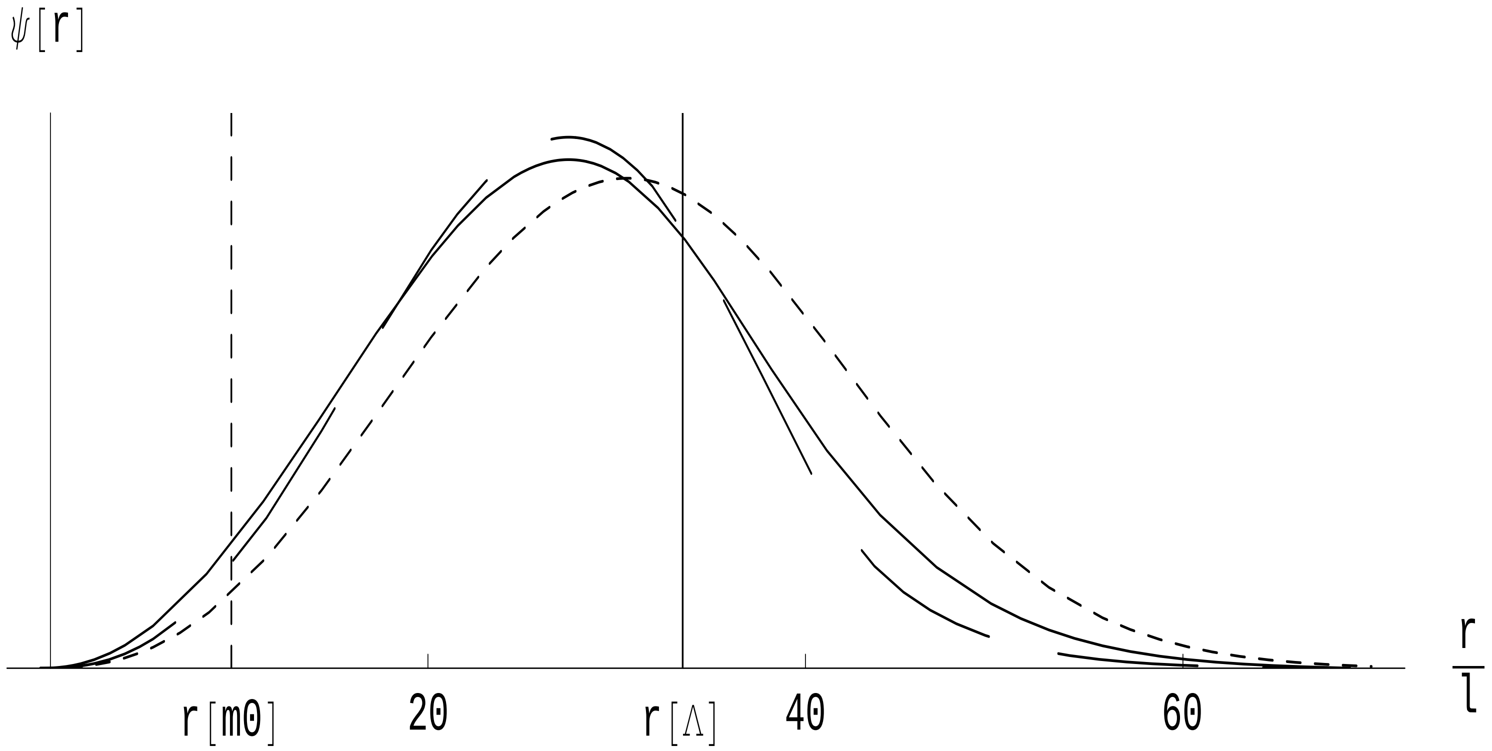
• Use a smooth interpolation between the one and two loop perturbative QCD β-function and the IR asymptotics.

$$\beta(\lambda) = -\frac{3b_0\lambda^2}{3+2b_0\lambda} - \frac{6(2b_0^2+3b_1^2)\lambda^3}{(1+\lambda^2)\left(18+\left(2b_0^2+3b_1^2\right)\log(1+\lambda^2)\right)}$$



The scale factor and 't Hooft coupling that follow from β. $b_0 = 4.2$, $\lambda_0 = 0.05$, $A_0 = 0$. The units are such that $\ell = 0.5$. The dashed line represents the scale factor for pure AdS.

The wave-functions of low-lying glueballs



Normalized wave-function profiles for the ground states of the 0^{++} (solid line),0^{-+} (dashed line), and 2^{++} (dotted line) towers, as a function of the radial conformal coordinate. The vertical lines represent the position corresponding to $E = m_{0^{++}}$ and $E = \Lambda_p$.

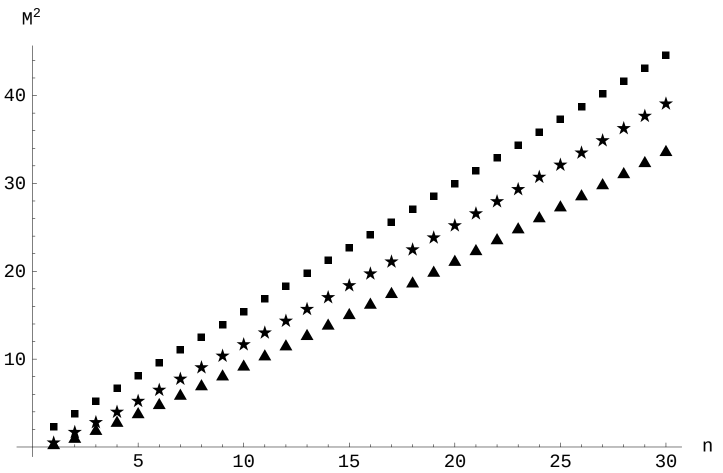
Estimating the importance of logarithmic scaling

We keep the IR asymptotics of background II,but change the UV to power asymptoting AdS_5, with a small λ_*.

$$e^A(r) = \frac{\ell}{r} e^{-(r/R)^2}, \qquad \Phi(r) = \Phi_0 + \frac{3}{2}\frac{r^2}{R^2}\sqrt{1+3\frac{R^2}{r^2}} + \frac{9}{4}\log\frac{2\frac{r}{R} + 2\sqrt{\frac{r^2}{R^2}+\frac{3}{2}}}{\sqrt{6}}.$$

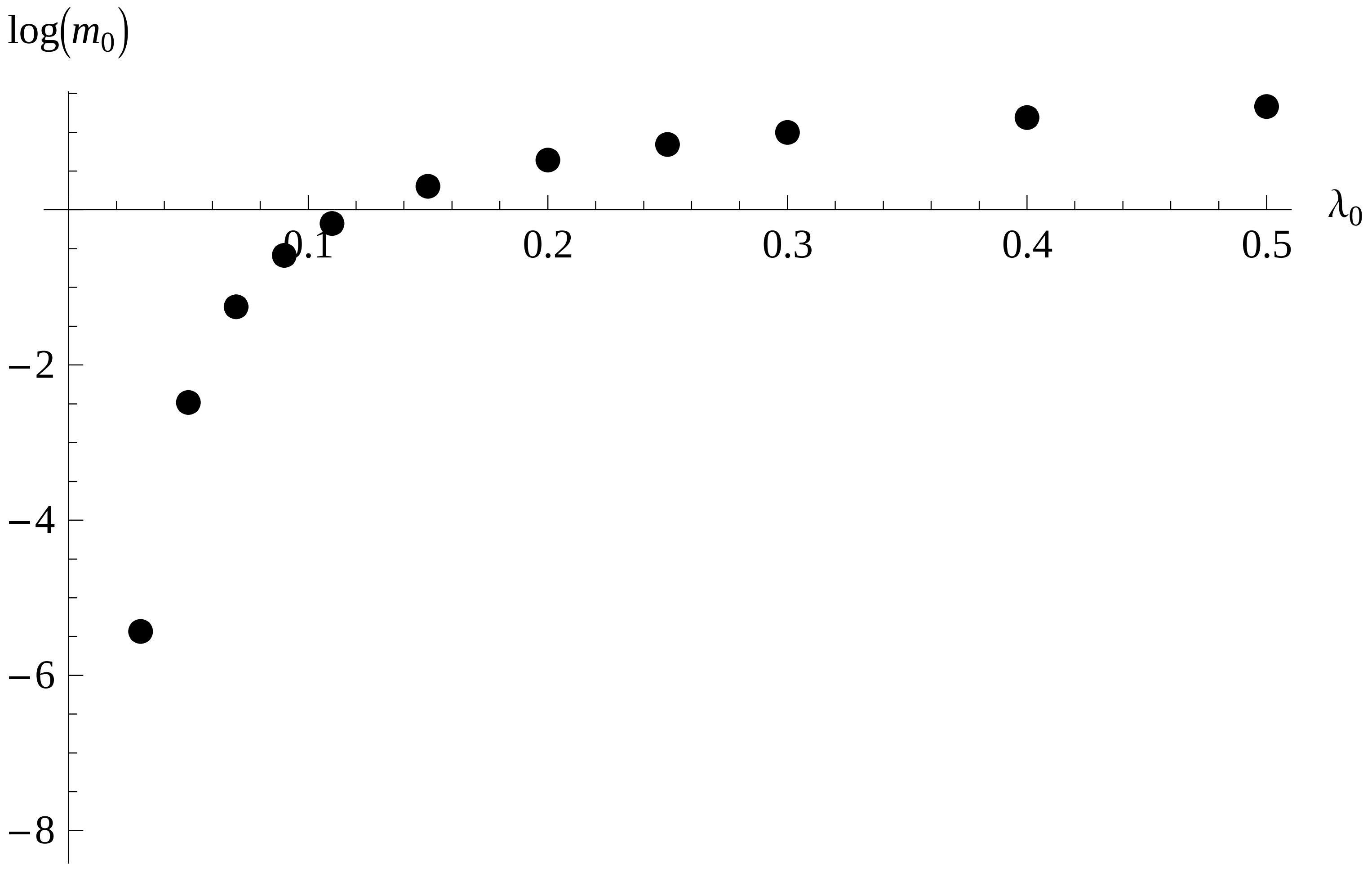
$$W_{conf} = W_0 \left(9 + 4b_0^2(\lambda-\lambda_*)^2)^{1/3}\right)\left(9a + (2b_0^2 + 3b_1)\log\left[1 + (\lambda-\lambda_*^2)\right]\right)^{2a/3}.$$

We fix parameters so that the physical QCD scale is the same (as determined from asymptotic slope of Regge trajectories.



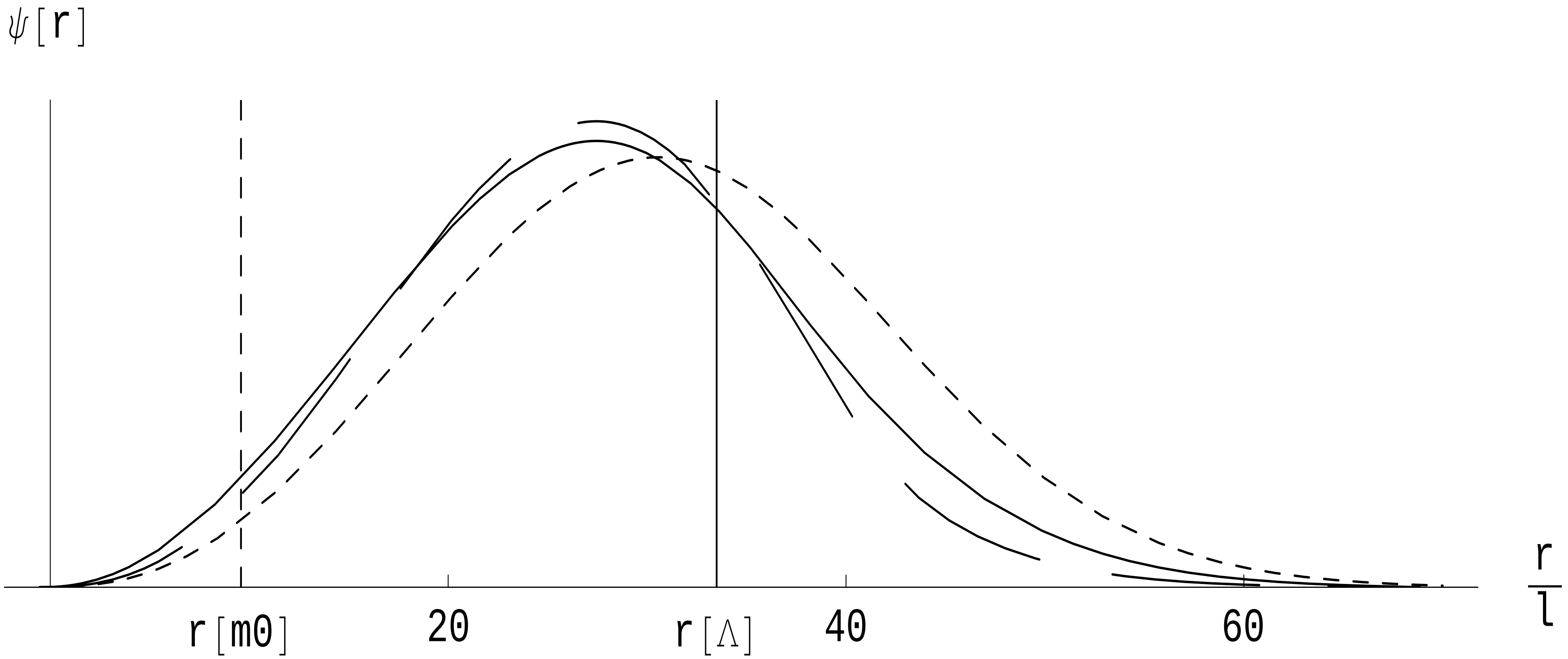
The stars correspond to the asymptotically free background I with $b_0 = 4.2$ and $\lambda_0 = 0.05$; the squares correspond the results obtained in the first background with $R = 11.4\ell$; the triangles denote the spectrum in the second background with $b0 = 4.2$, $li = 0.071$ and $l_* = 0.01$. These values are chosen so that the slopes coincide asymptotically for large n.

Dependence of absolute mass scale on λ_0



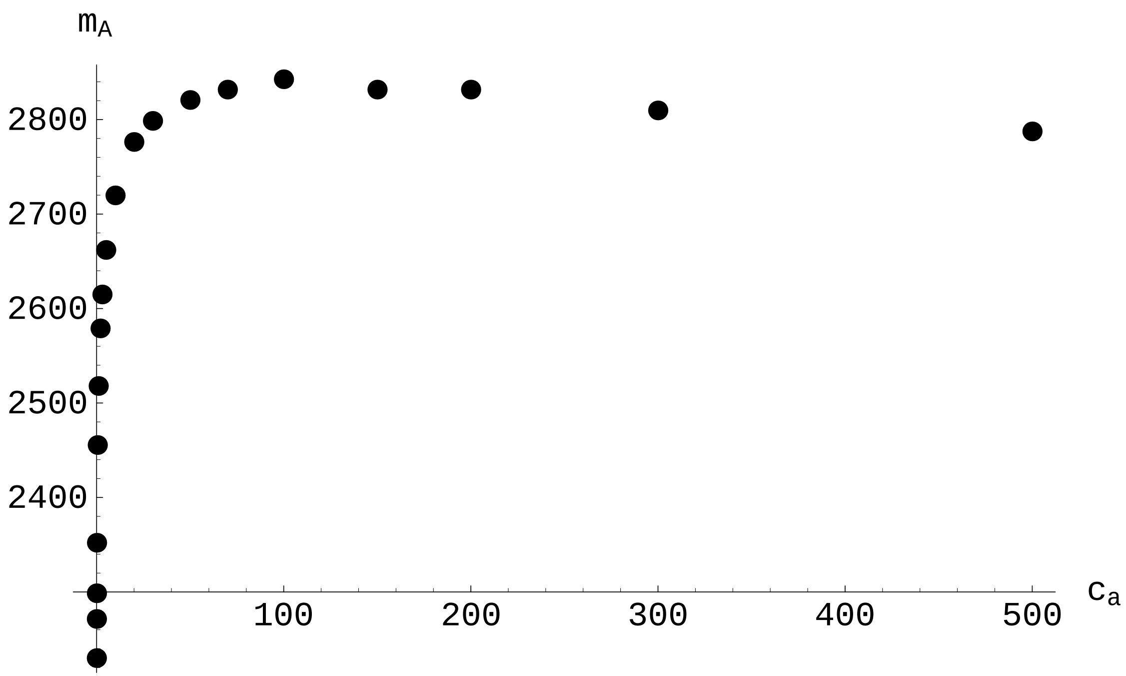
Dependence on initial condition λ_0 of the absolute scale of the lowest lying glueball (shown in Logarithmic scale)

The glueball wavefunctions



Normalized wave-function profiles for the ground states of the 0^{++} (solid line) ,0^{-+} (dashed line), and 2^{++} (dotted line) towers, as a function of the radial conformal coordinate. The vertical lines represent the position corresponding to $E = m_{0^{++}}$ and $E = \Lambda_p$.

Pseudoscalar glueballs



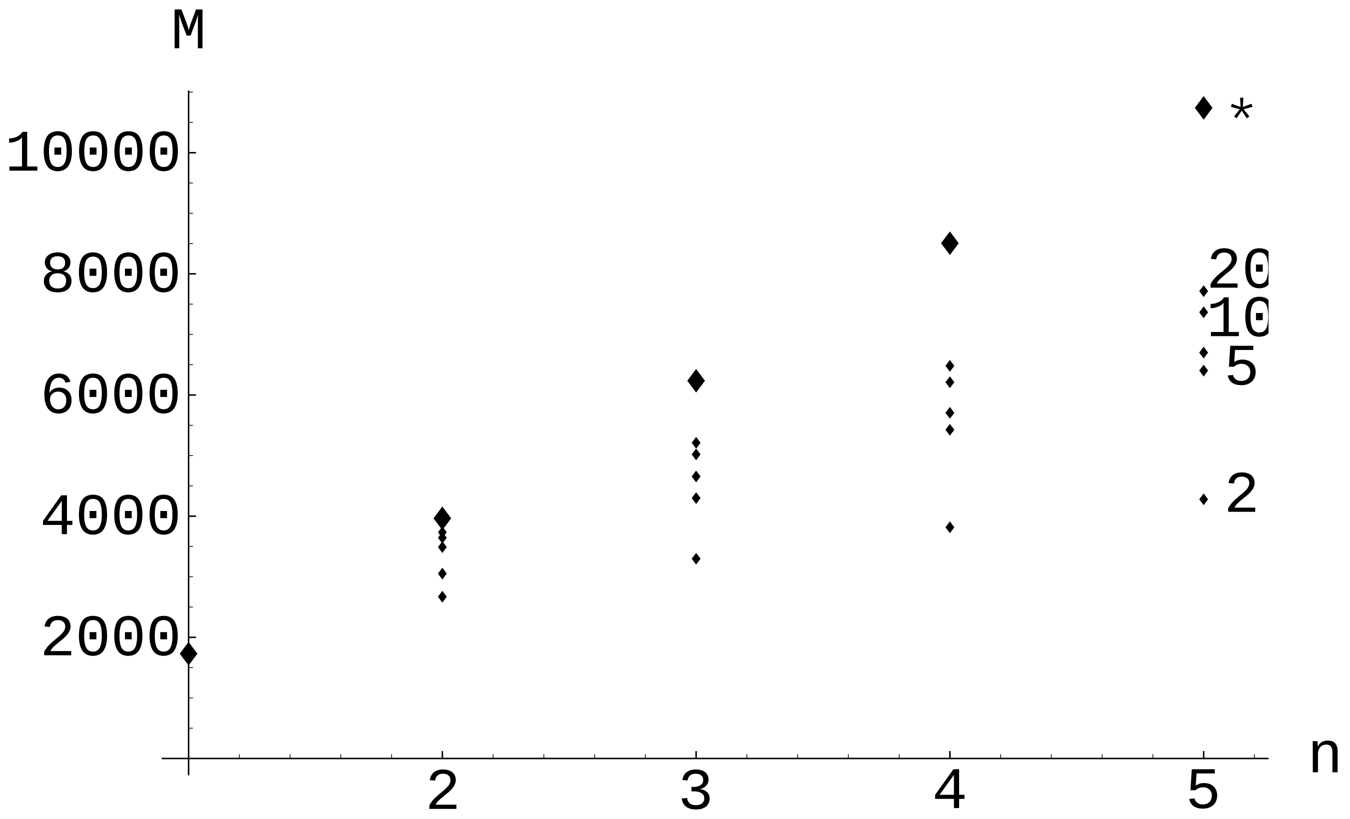
Lowest 0^{-+} glueball mass in MeV as a function of c_a in $Z(\lambda) = Z_a(1+c_a\lambda^4)$.

The lattice glueball data

J^{++}	Ref. I $(m/\sqrt{\sigma})$	Ref. I (MeV)	Ref. II (mr_0)	Ref. II (MeV)	$N_c \to \infty (m/\sqrt{\sigma})$
0	3.347(68)	1475(30)(65)	4.16(11)(4)	1710(50)(80)	3.37(15)
0^*	6.26(16)	2755(70)(120)	6.50(44)(7)	2670(180)(130)	6.43(50)
0^{**}	7.65(23)	3370(100)(150)	NA	NA	NA
0^{***}	9.06(49)	3990(210)(180)	NA	NA	NA
2	4.916(91)	2150(30)(100)	5.83(5)(6)	2390(30)(120)	4.93(30)
2^*	6.48(22)	2880(100)(130)	NA	NA	NA
R_{20}	1.46(5)	1.46(5)	1.40(5)	1.40(5)	1.46(11)
R_{00}	1.87(8)	1.87(8)	1.56(15)	1.56(15)	1.90(17)

Available lattice data for the scalar and the tensor glueballs. Ref. I =H. B. Meyer, [arXiv:hep-lat/0508002]. and Ref. II = C. J. Morningstar and M. J. Peardon, [arXiv:hep-lat/9901004] + Y. Chen *et al.*, [arXiv:hep-lat/0510074]. The first error corresponds to the statistical error from the the continuum extrapolation. The second error in Ref.I is due to the uncertainty in the string tension $\sqrt{\sigma}$. (Note that this does not affect the mass ratios). The second error in the Ref. II is the estimated uncertainty from the anisotropy. In the last column we present the available large N_c estimates according to B. Lucini and M. Teper, [arXiv:hep-lat/0103027]. The parenthesis in this column shows the total possible error followed by the estimations in the same reference.

α-dependence of scalar spectrum



The 0^{++} spectra for varying values of α that are shown at the right end of the plot. The symbol $*$ denotes the AdS/QCD result.

Non-supersymmetric backgrounds with abelian flavor branes

• D_7 brane in deformed AdS_5.

• Only abelian axial symmetry $U(1)_A$ realized geometrically as an isometry.

• A quark mass can be introduced, and a quark condensate can be calculated.

• $U(1)_A$ is spontaneously broken du to the embedding.

• Correct GOR relation

• Qualitatively correct η' mass.

• No non-abelian flavor symmetry (due to N=2 Yukawas)

The Sakai-Sugimoto model

- D4 on non-susy S^1 plus $D8$ branes.

- The flavor symmetry is realized on world-volume

- Full $U(N_f)_L \times U(N_f)_R$ symmetry broken to $U(N_f)_V$.

- Chiral symmetry breaking as brane-antibrane recombination.

- Quark constituent mass

- Qualitatively correct η' mass

- No quark mass parameter, nor chiral condensate.

AdS/QCD

- Crude model: AdS_5 with a UV and IR cutoff.

- Addition of $U(N_f)_L \times U(N_f)_R$ vectors and a $(N_f, \bar{N}_f)$ scalar T.

- Chiral symmetry broken by hand via IR boundary conditions.

- Vector meson dominance and GOR relation incorporated.

- Chiral condensate not determined.

- Gluon sector problematic.

The meson sector ($N_f << N_c$)

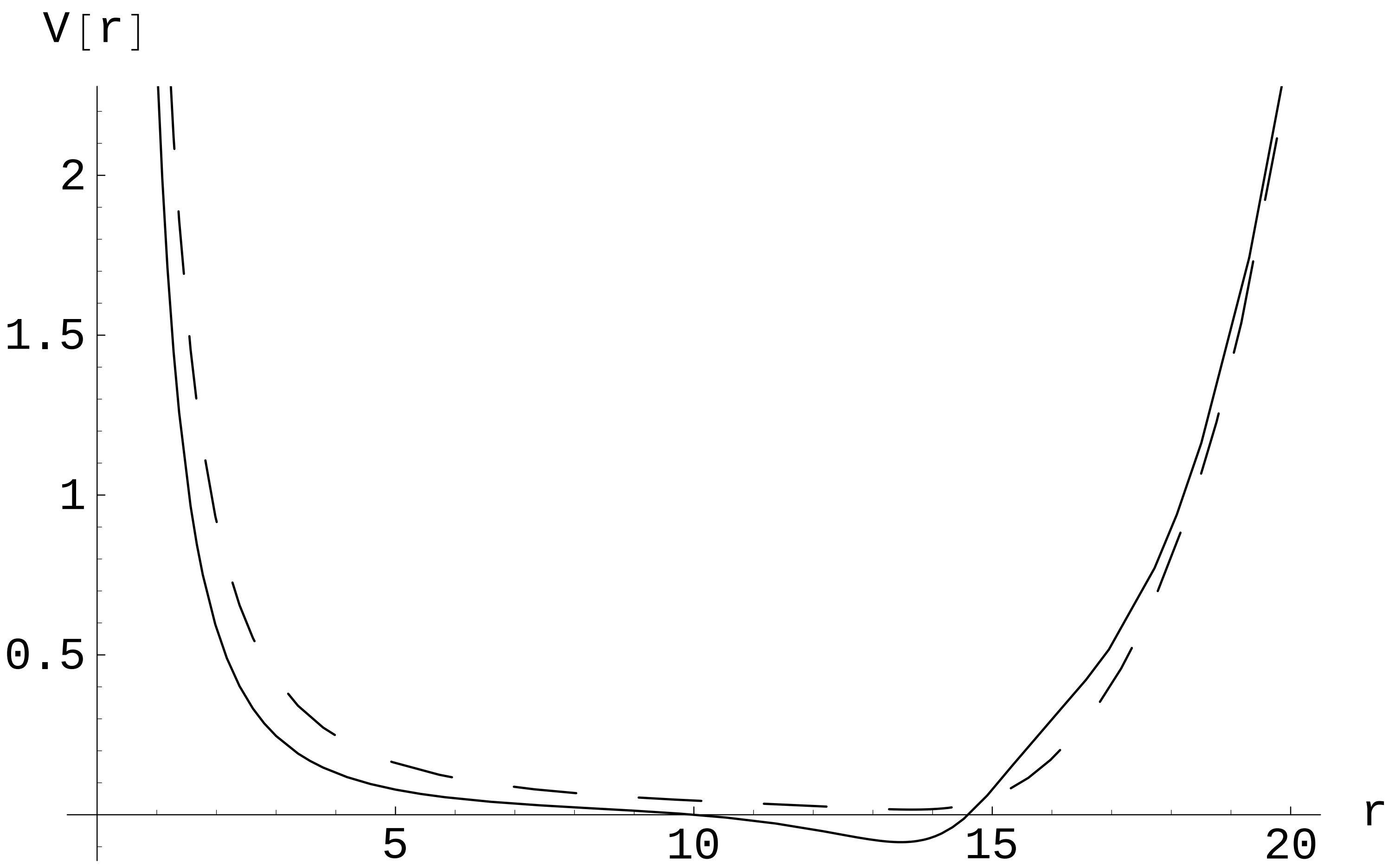
- Flavor is introduced via the introduction of N_f pairs of space filling $D_4+\bar{D}_4$ branes.

- The crucial world volume fields are the tachyon T_{ij} in $(N_f, \bar{N}_f)$ and the $U(N_f)_L \times U(N_f)_R$ vectors.

- The D-WZW sector depends nontrivial on T and realizes properly the P and C symmetries. It generates the appropriate gauge and global flavor anomalies.

- We can introduce explicitly mass matrices for the quarks, and we can dynamically determine the chiral condensate.

Comparison of scalar and tensor potential



Effective Schrödinger potentials for scalar (solid line) and tensor (dashed line) glueballs. The units are chosen such that $\ell = 0.5$.

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• We have naturally the χSB breaking order parameter T, and consistency with anomalies implies that it is non-zero and proportional to the identity (Holographic Coleman+Witten theorem).

• The pions appear as Goldstone bosons when $m_q = 0$.

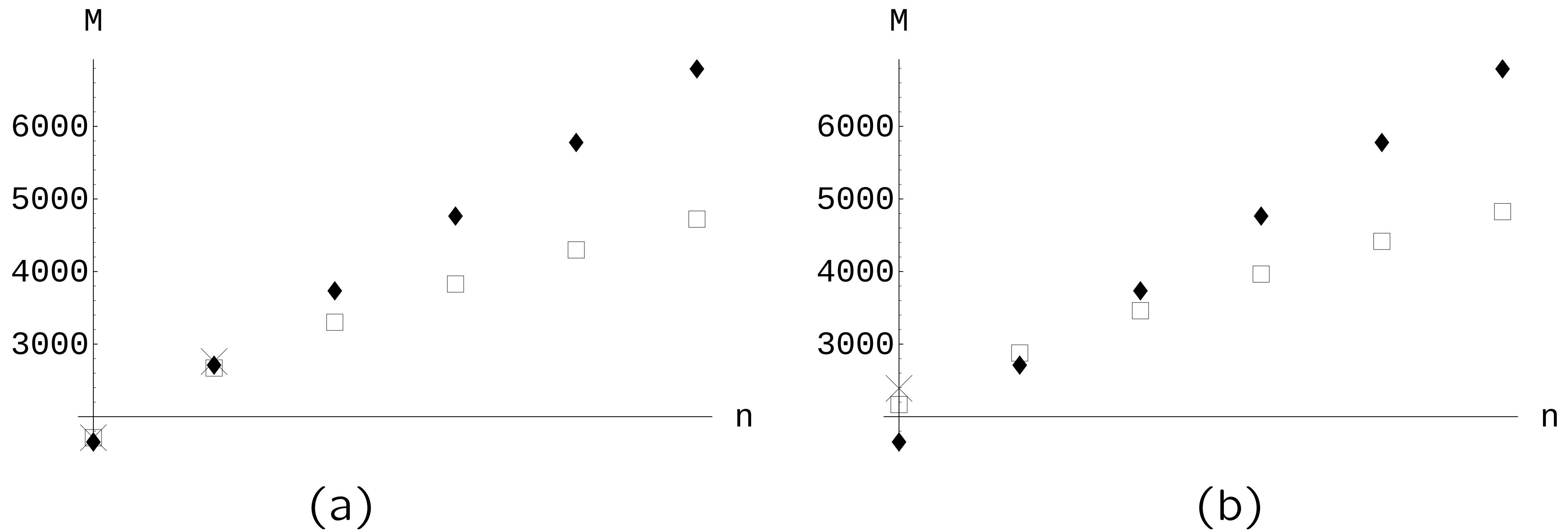
• The correct GOR relation is obtained.

$$m_\pi^2 = -2\frac{m_q}{f_\pi^2}\langle \bar{q}q\rangle \quad , \quad m_q \to 0$$

• There is linear confinement $(M_n^2 \sim n)$ associated with the vanishing of the tachyon potential at $T \to \infty$.

• We obtain the correct Stuckelberg coupling mixing with 0^{+-} and and mass for the η'.

Comparison with lattice data: Ref II



Comparison of glueball spectra from our model with $b_0 = 2.55, \lambda_0 = 0.05$ (boxes), with the lattice QCD data from Ref. II (crosses) and the AdS/QCD computation (diamonds), for (a) 0^{++} glueballs; (b) 2^{++} glueballs. The masses are in MeV, and the scale is normalized to match the lowest 0^{++} state from Ref. II.

Confining background II: r_0 =finite

- We choose a regular β-function with appropriate asymptotics:

$$\beta(\lambda) = -\frac{3b_0\lambda^2}{3+2b_0\lambda} - \frac{3\eta(2b_0^2+3b_1^2)\lambda^3}{9\eta+2\left(2b_0^2+3b_1^2\right)\lambda^2}, \qquad \eta \equiv \sqrt{1+\delta^{-1}}-1$$

- Confining backgrounds with r_0 =finite have a hard time to match the lattice results, even for the first few glueballs.

Tachyon dynamics

- In the vacuum the gauge fields vanish and $T \sim 1$. Only DBI survives

$$S[\tau] = T_{D_4} \int dr d^4x \, \frac{e^{4A_s(r)}}{\lambda} \, V(\tau) \, \sqrt{e^{2A_s(r)} + \dot{\tau}(r)^2} \quad , \quad V(\tau) = e^{-\frac{\mu^2}{2}\tau^2}$$

- We obtain the nonlinear field equation:

$$\ddot{\tau} + \left(3\dot{A}_S - \frac{\dot{\lambda}}{\lambda}\right)\dot{\tau} + e^{2A_S}\mu^2\tau + e^{-2A_S}\left[4\dot{A}_S - \frac{\dot{\lambda}}{\lambda}\right]\dot{\tau}^3 + \mu^2\tau \, \dot{\tau}^2 = 0.$$

- In the UV we expect

$$\tau = m_q \, r + \sigma \, r^3 + \cdots \quad , \quad \mu^2\ell^2 = 3$$

- We expect that the tachyon must diverge before or at $r = r_0$. We find that indeed it does `at the singularity`. For the $r_0 = \infty$ backgrounds

$$\tau \sim \exp\left[\frac{2}{a} \, \frac{R}{\ell^2} \, r\right] \qquad \text{as} \qquad r \to \infty$$

• Generically the solutions have spurious singularities: $\tau(r_*)$ stays finite but its derivatives diverges as:

$$\tau \sim \tau_* + \gamma\sqrt{r_* - r}.$$

The condition that they are absent determines σ as a function of m_q.

• The easiest spectrum to analyze is that of vector mesons. We find $(r_0 = \infty)$

$$\Lambda_{glueballs} = \frac{1}{R}, \qquad \Lambda_{mesons} = \frac{3}{\ell}\left(\frac{\alpha\ell^2}{2R^2}\right)^{(\alpha-1)/2} \propto \frac{1}{R}\left(\frac{\ell}{R}\right)^{\alpha-2}.$$

This suggests that $\alpha = 2$. preferred also from the glue sector.

Improved Holographic QCD

- The effective action

$$S_{\rm string} = M^3 N_c^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V(\lambda) \right] \quad , \quad \lambda = N_c \, e^{\phi}$$

with a dilaton potential $V(\lambda)$.

- In the UV $\lambda(r) \to 0$ (asymptotic freedom) and the metric becomes AdS_5.

- There is a 1-1 correspondence between the QCD β-function, $\beta(\lambda)$ and the dilaton potential:

- In the IR, $\lambda \to \infty$ and

$$V(\lambda) \simeq \sqrt{\log\lambda}\, \lambda^{\frac{4}{3}} + \cdots \quad , \quad \beta(\lambda) \simeq -\frac{3}{2}\lambda - \frac{9}{16} \frac{\lambda}{\log\lambda} + \cdots \quad , \quad \lambda(E) \sim E^{-\frac{3}{2}} (\log E)^{\frac{3}{8}}$$

Detailed plan of the presentation

- Title page 0 minutes
- Introduction 4 minutes
- AdS/CFT and holography 9 minutes
- The correspondence 11 minutes
- The effective action 13 minutes
- The thermal gauge theory 15 minutes
- The deconfining transition and QGP 17 minutes
- AdS/QCD 19 minutes
- Improved Holographic QCD 20 minutes

THE DATA

- Dependence of mass ratios on λ_0 21 minutes
- Linearity of the glueball spectrum 22 minutes
- Comparison with lattice data: Ref I 23 minutes
- The fit to Ref I 24 minutes
- The transition in the free energy 25 minutes
- Equation of state 26 minutes
- The speed of sound (bulk viscosity) 27 minutes
- The specific heat 28 minutes
- Open ends 29 minutes
- Bibliography 29 minutes

- A preview of the results: pure glue 43 minutes
- Preview: quarks ($N_f \ll N_c$) and mesons 48 minutes
- Motivating the effective action 58 minutes
- Organizing the vacuum solutions 60 minutes
- The IR regime 62 minutes
- Wilson loops and confinement 64 minutes
- General criterion for confinement 67 minutes
- Comments on confining backgrounds 69 minutes
- The axion background 72 minutes
- QCD at finite temperature 75 minutes
- Critical string theory holography 78 minutes
- Non-Critical holography 80 minutes
- Fluctuations around the AdS_5 extremum 83 minutes
- Further α' corrections 85 minutes
- Holographic meson dynamics: the models 87 minutes