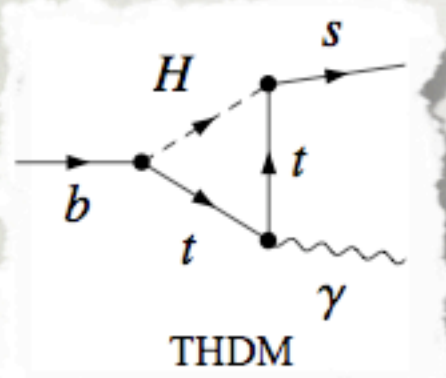
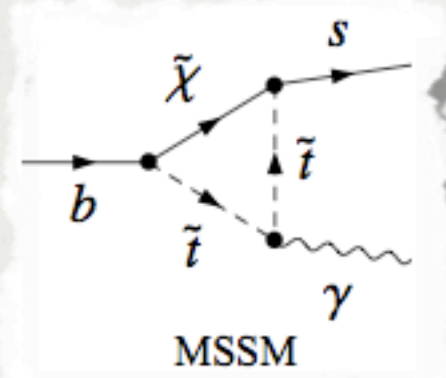
# $B \to X_s\gamma$: SM and Beyond

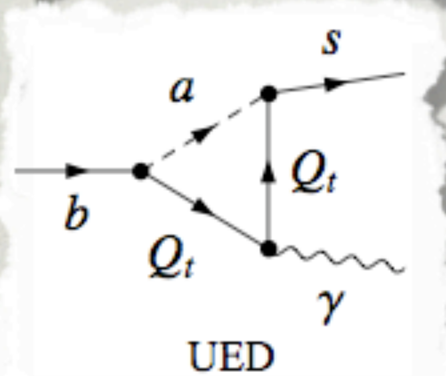
Ulrich Haisch
University of Mainz (THEP)

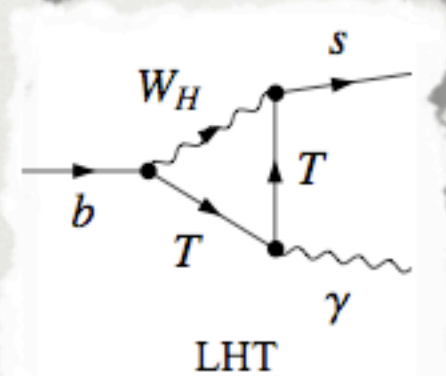
XLIIIrd Rencontres de Moriond,
Electroweak Session, La Thuile, March 1–8, 2008

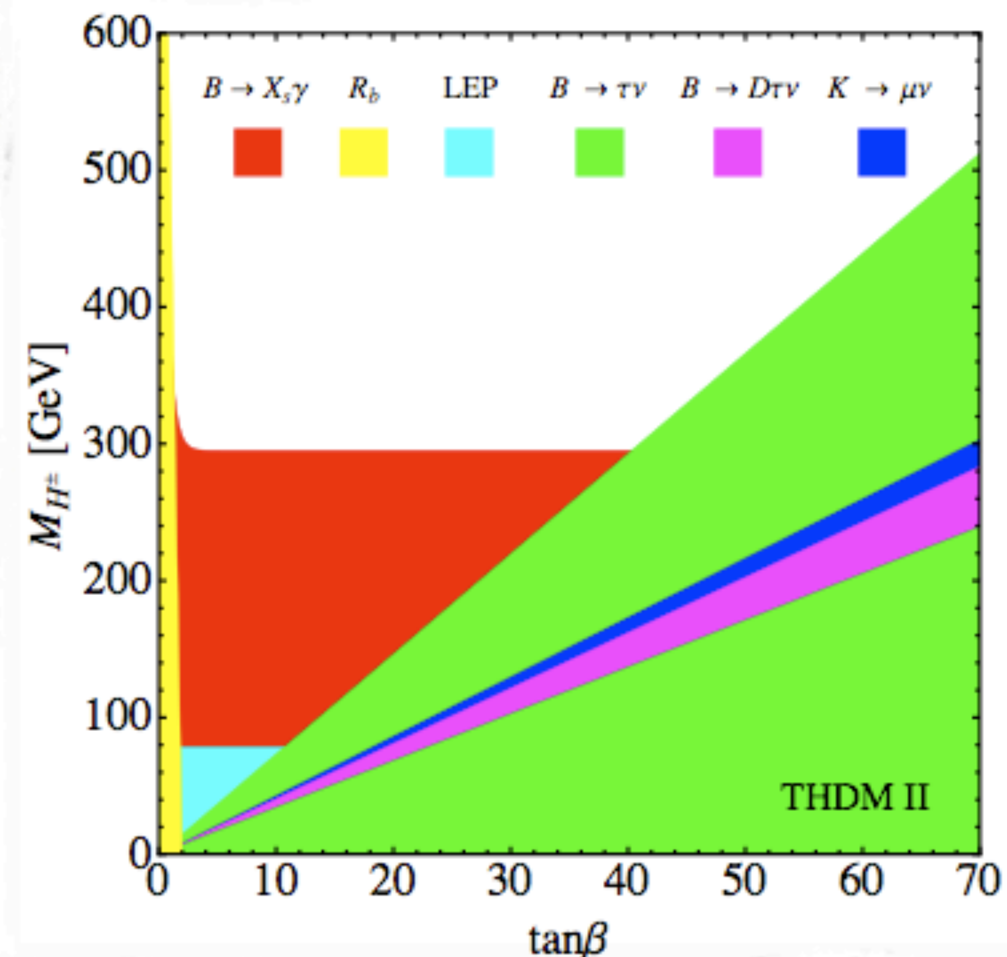
# Motivation



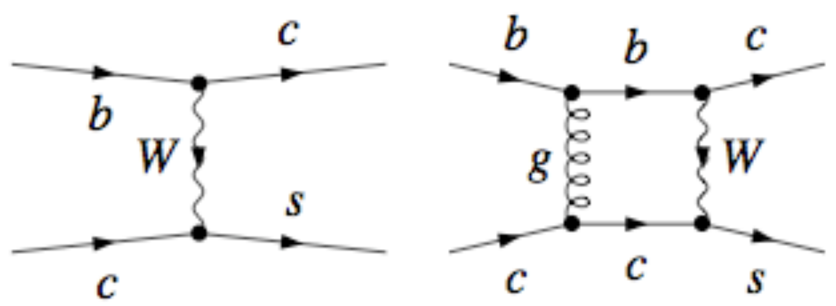



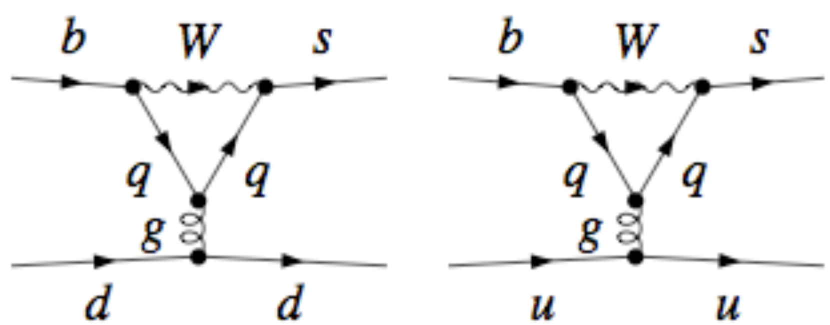

- Inclusive radiative $B$-meson decay provides stringent constraints on various NP scenarios at EW scale, since it is accurately measured and its theoretical determination is rather precise

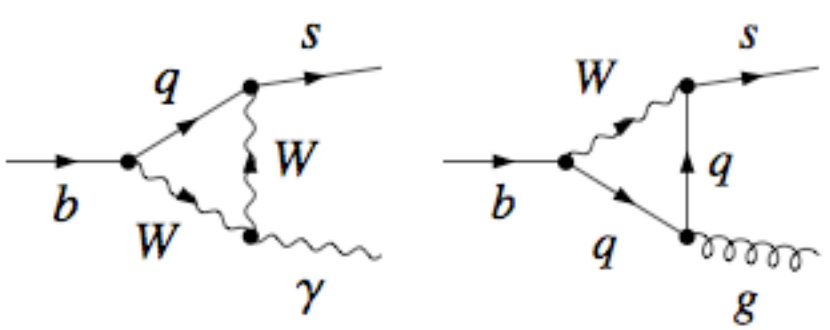
- As $\Delta BR_{SUSY} \sim (0.1\ \text{TeV}/\tilde{m})^2\ BR_{SM}$ and squark masses of at least a few hundred GeV are plausible, SUSY corrections to BR of only a few percent are likely, which calls for precise SM calculations of $b \to s\gamma$

*Misiak et al. '06; Antonelli et al. '08; Kamenik & Mescia '08

# Effective Theory




$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QED}\times\text{QCD}} + \frac{4G_F}{\sqrt{2}} V_{ts}^{*} V_{tb} \sum_{i=1}^{8} C_i(\mu) Q_i + \dots$$

$$Q_{1,2} = (\bar{s}\Gamma_i c)\,(\bar{c}\Gamma_i' b)$$

$$|C_{1,2}(m_b)| \sim 1$$

$$Q_{3\text{-}6} = (\bar{s}\Gamma_i b) \textstyle\sum_q (\bar{q}\Gamma_i' q)$$

$$|C_{3\text{-}6}(m_b)| < 0.07$$

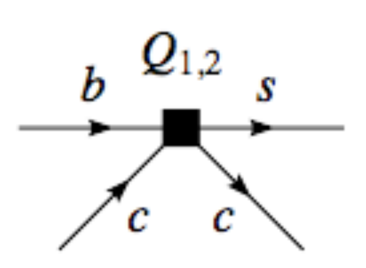
$$Q_7 = \frac{e m_b}{16\pi^2} (\bar{s}_L \sigma^{\mu\nu} b_R)\, F_{\mu\nu}$$

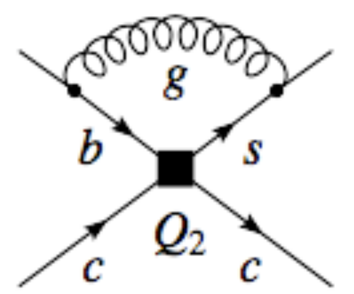
$$Q_8 = \frac{g m_b}{16\pi^2} (\bar{s}_L \sigma^{\mu\nu} T^a b_R)\, G^a_{\mu\nu}$$

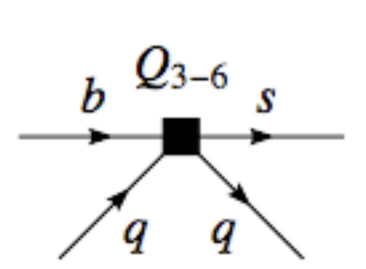
$$C_7(m_b) \sim -0.3$$

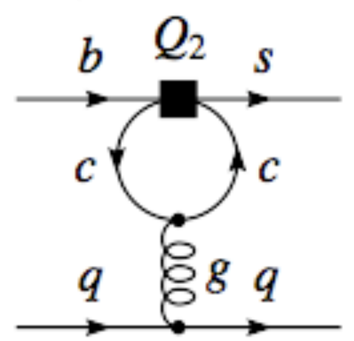
$$C_8(m_b) \sim -0.15$$

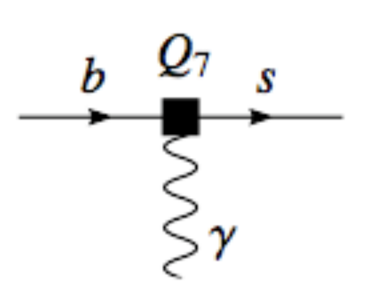
# Effective Theory

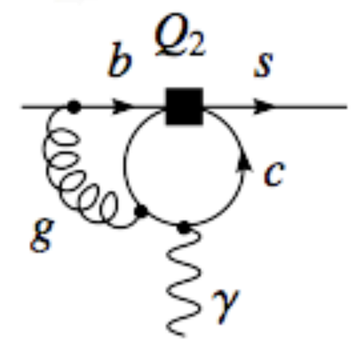



$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QED}\times\text{QCD}} + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i(\mu) Q_i + \ldots$$

$$Q_{1,2} = (\bar{s}\Gamma_i c)\,(\bar{c}\Gamma_i' b)$$

$$|C_{1,2}(m_b)| \sim 1$$

$$Q_{3\text{–}6} = (\bar{s}\Gamma_i b) \sum\nolimits_q (\bar{q}\Gamma_i' q)$$

$$|C_{3\text{–}6}(m_b)| < 0.07$$

$$Q_7 = \frac{e m_b}{16\pi^2} (\bar{s}_L \sigma^{\mu\nu} b_R)\, F_{\mu\nu}$$

$$Q_8 = \frac{g m_b}{16\pi^2} (\bar{s}_L \sigma^{\mu\nu} T^a b_R)\, G^a_{\mu\nu}$$

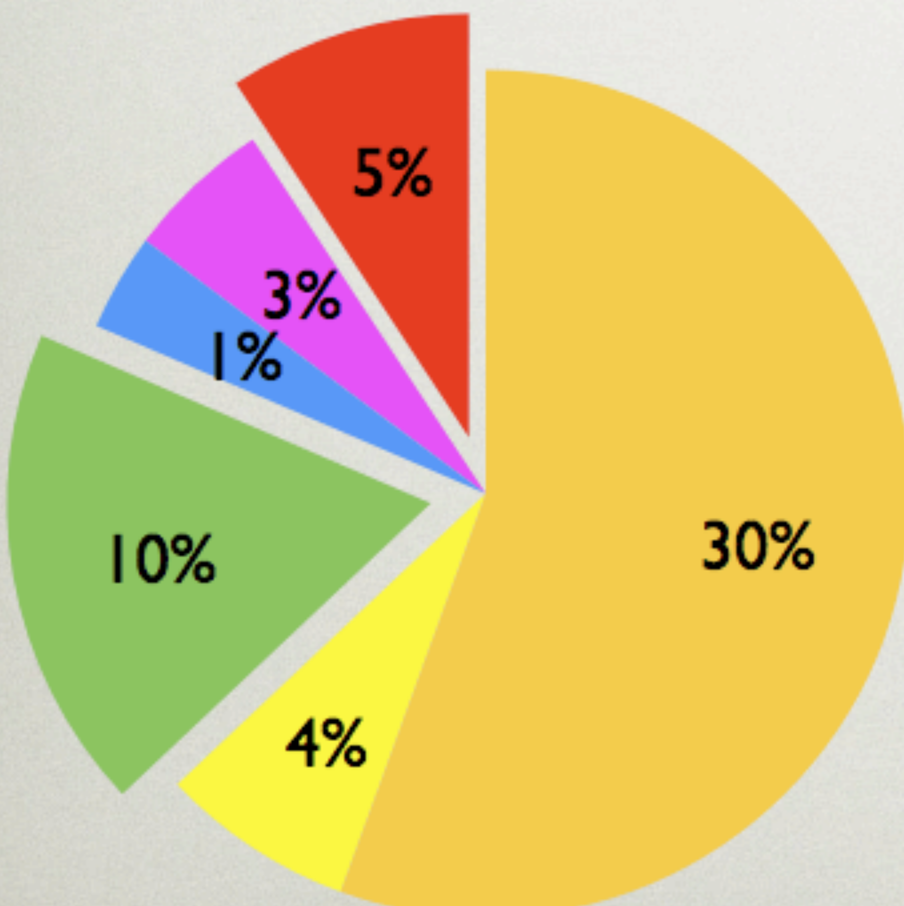
$$C_7(m_b) \sim -0.3$$

$$C_8(m_b) \sim -0.15$$

# General Structure

$$\mathrm{BR}(B \to X_s\gamma)_{\mathrm{SM}}^{E_\gamma > 1.6\ \mathrm{GeV}} = \mathrm{BR}(B \to X_c e\bar{\nu})_{\mathrm{exp}} \left[\frac{\Gamma(b \to s\gamma)}{\Gamma(b \to ce\bar{\nu})}\right]_{\mathrm{LO}}$$

$$\times \left\{1 + \mathcal{O}(\alpha_s) + \mathcal{O}(\alpha) + \mathcal{O}(\alpha_s^2) + \mathcal{O}\left(\frac{\Lambda^2}{m_b^2}\right) + \mathcal{O}\left(\frac{\Lambda^2}{m_c^2}\right) + \mathcal{O}\left(\alpha_s \frac{\Lambda}{m_b}\right)\right\}$$




- NLO QCD
- NLO EW
- NNLO QCD

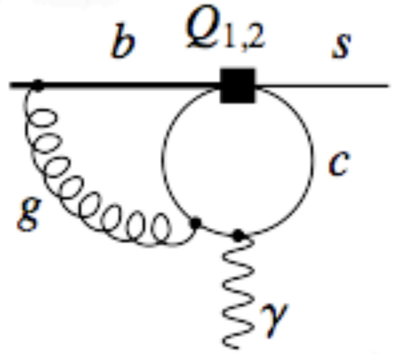
perturbative

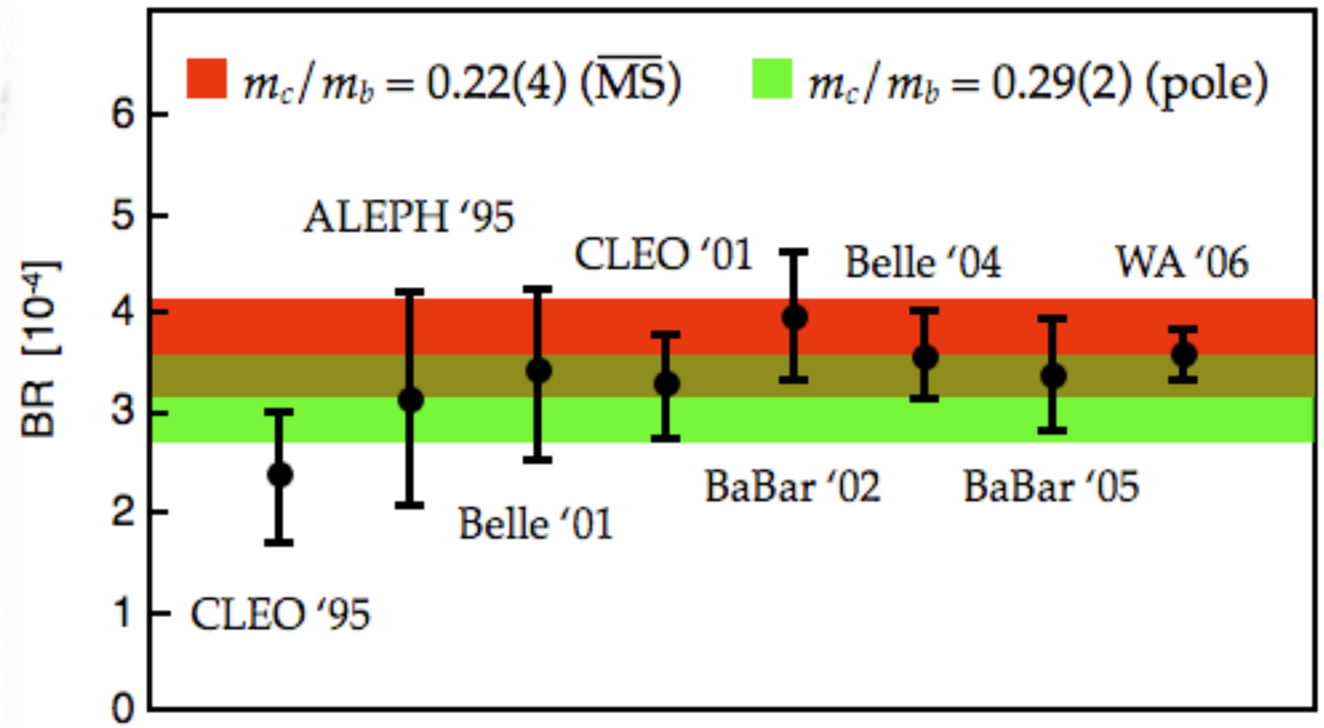
- LO QCD + NLO $m_b$
- LO QCD + NLO $m_c$
- NLO QCD + LO $m_b$

non-perturbative

relative size compared to LO

# Role of Charm Quark



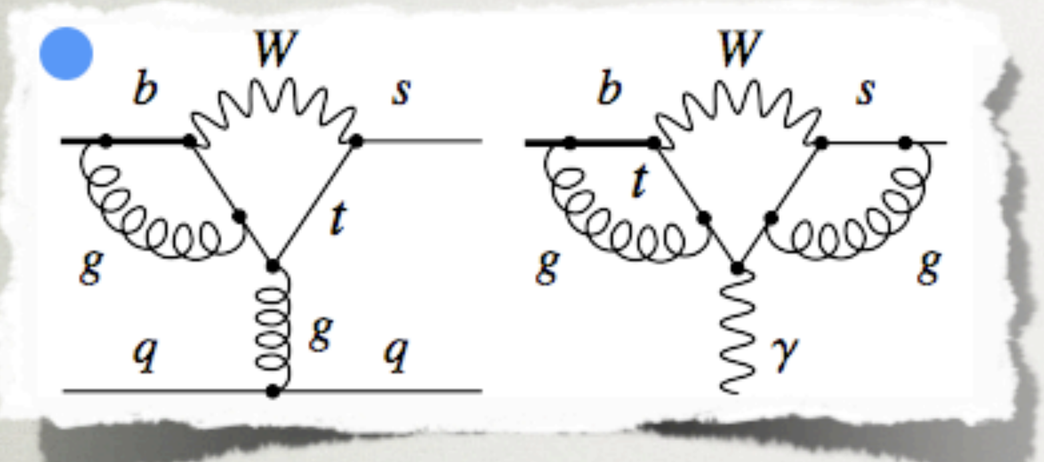

- Changing pole scheme value $m_c/m_b = 0.29(2)$ to $\overline{\rm MS}$ value 0.22(4) leads to a shift of SM BR of more than 10% at NLO level

- Charm quark mass dependence of $\mathrm{BR}(B \to X_s\gamma)$ so pronounced because charm quark mass first enters through $b \to s\gamma$ matrix elements of $Q_{1,2}$ at NLO level

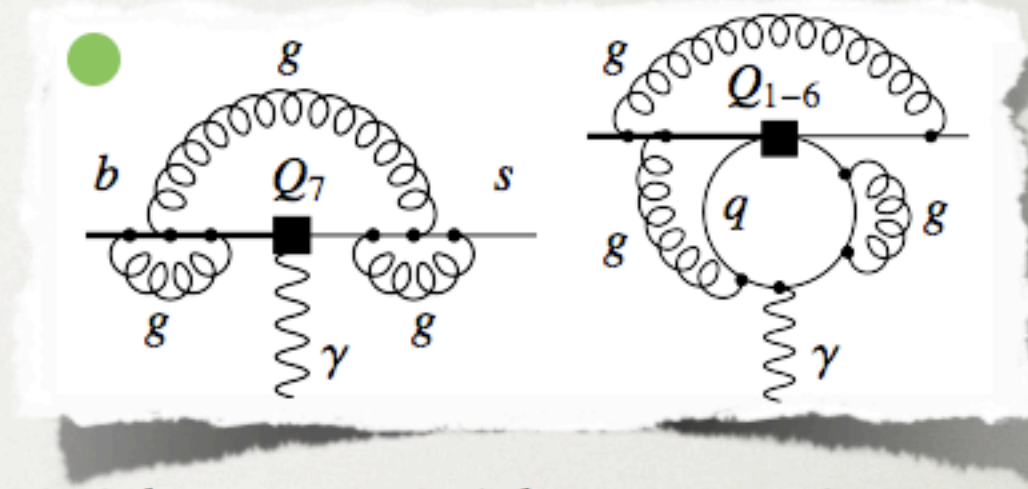
- Natural scale at which charm quark mass should be normalized can thus only be determined by a dedicated calculation of charm quark mass dependence of BR at NNLO

*Gambino & Misiak '01

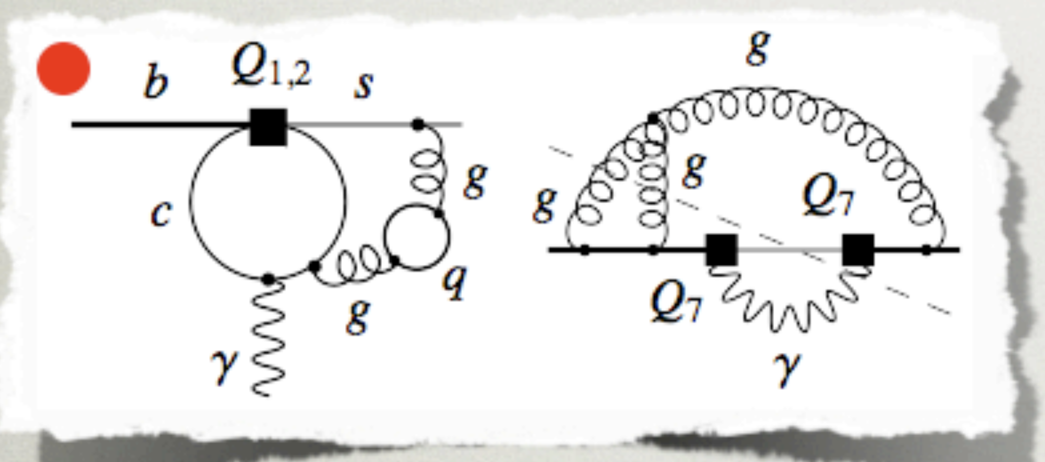
# Flavor of NNLO Calculation




Bobeth, Misiak & Urban '00; Misiak & Steinhauser '04




Gorbahn & UH '04; Gorbahn, UH & Misiak '05;
Czakon, UH & Misiak '06



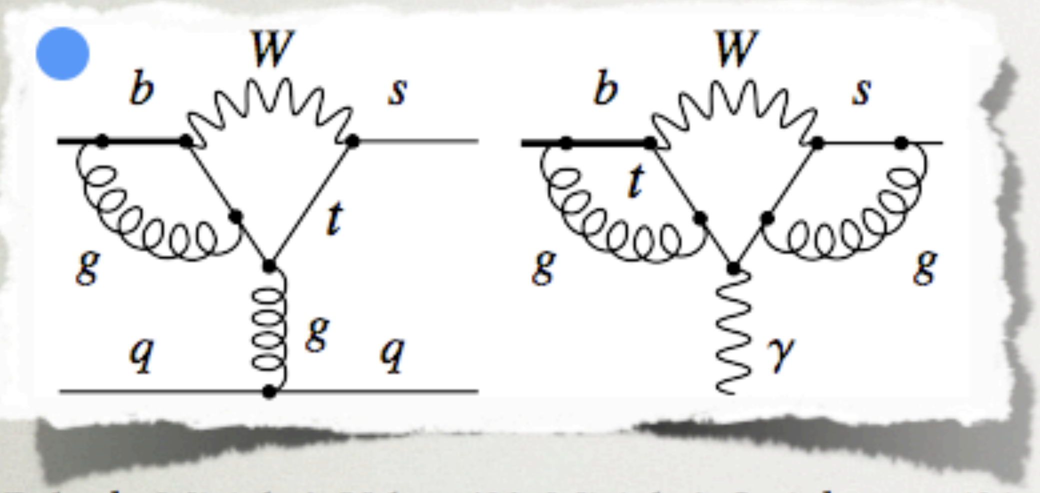

Bieri, Greub & Steinhauser '03; Blokland et al. '05;
Melnikov & Mitov '05; Asatrian et al. '05, '06;
Misiak & Steinhauser '06

- Matching at high scale $\sim M_W$: three- for $C_{7,8}$ and two-loop for $C_{1-6}$
- Running between $M_W$ and $m_b$: four- for $b \to s\gamma(g)$ while three-loop in remaining cases

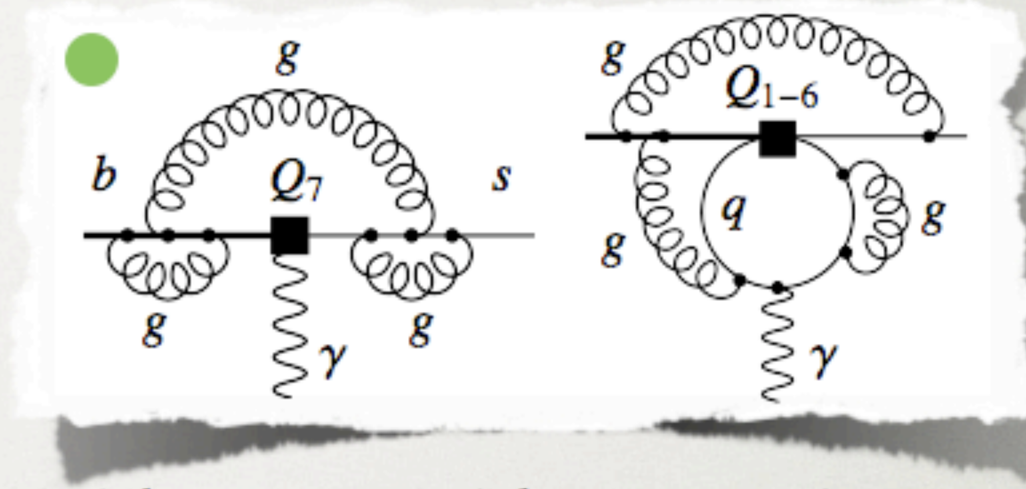
---

all Wilson coefficients $C_{1-8}(m_b)$ known at NNLO

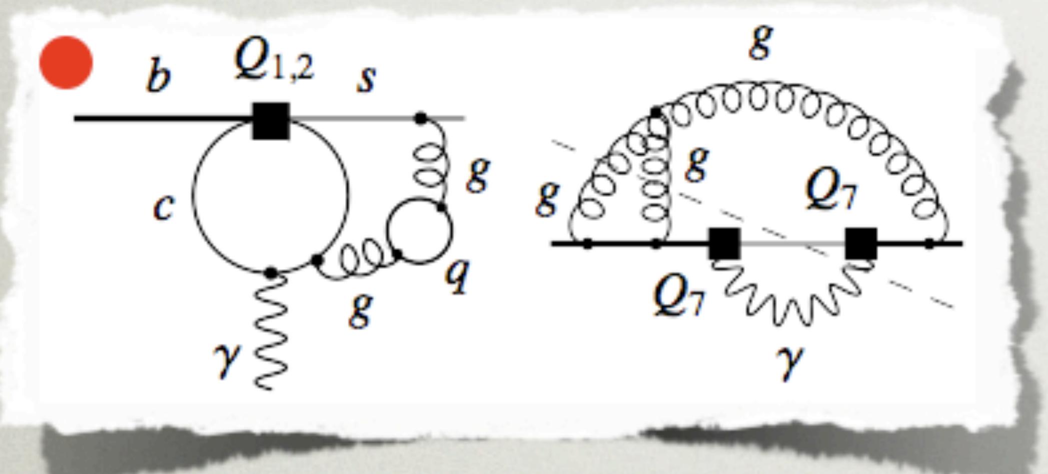
# Flavor of NNLO Calculation




Bobeth, Misiak & Urban '00; Misiak & Steinhauser '04




Gorbahn & UH '04; Gorbahn, UH & Misiak '05; Czakon, UH & Misiak '06




Bieri, Greub & Steinhauser '03; Blokland et al. '05; Melnikov & Mitov '05; Asatrian et al. '05, '06; Misiak & Steinhauser '06

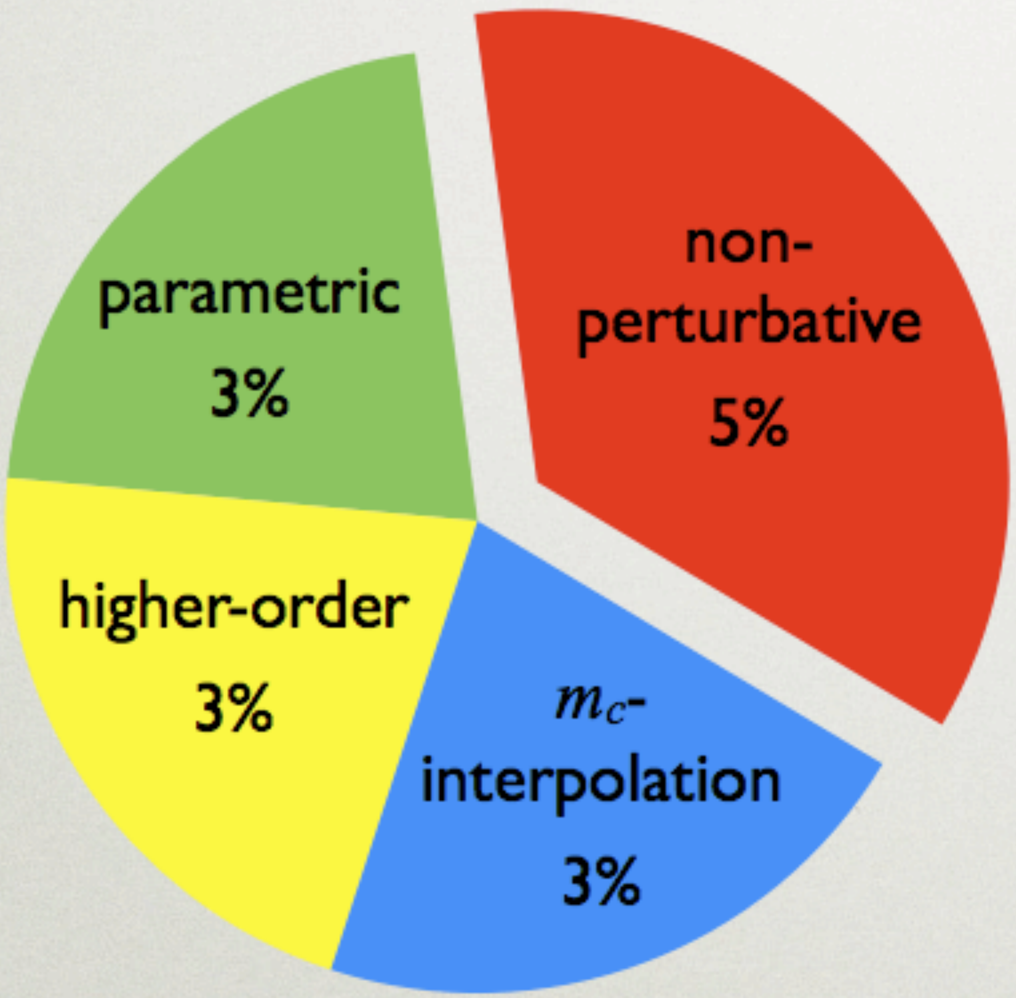
- Matrix elements at low scale $\sim m_b$: complete for $(Q_7, Q_7)$, virtual large-$\beta_0$ corrections for $Q_{1,2,7,8}$ and asymptotic form of all matrix elements in heavy charm quark mass limit $m_c \ll m_b/2$

---

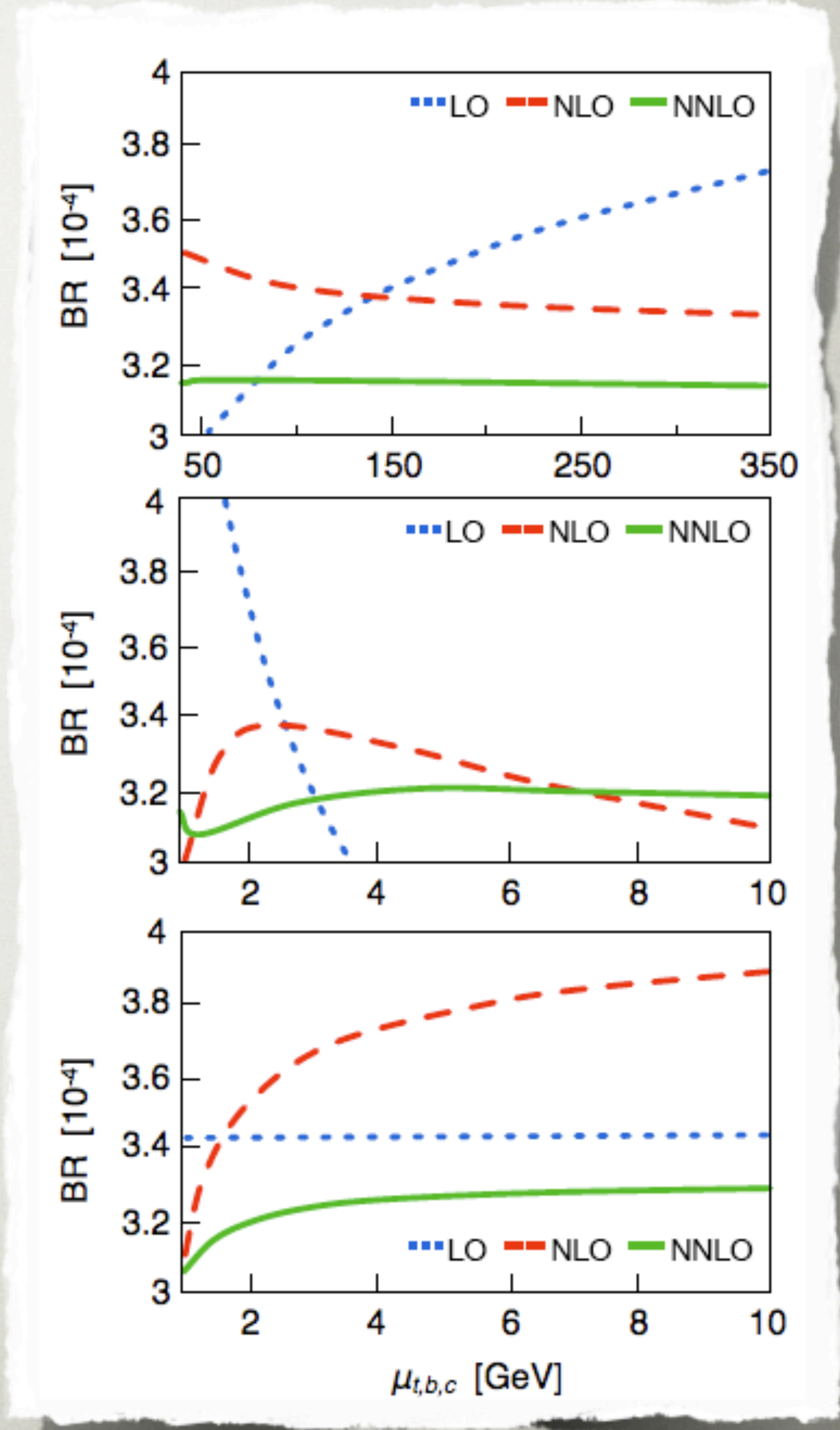
interpolation to $m_c \sim 0.25 m_b$ assuming that large-$\beta_0$ part describes full result well for $m_c = 0$

# First NNLO Estimate

$$\mathrm{BR}(B \to X_s \gamma)_{\mathrm{SM}}^{E_\gamma > 1.6\ \mathrm{GeV}} = (3.15 \pm 0.23) \times 10^{-4}$$

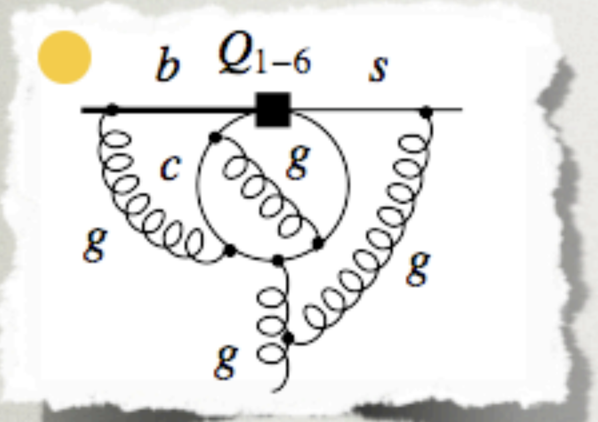



individual errors on BR

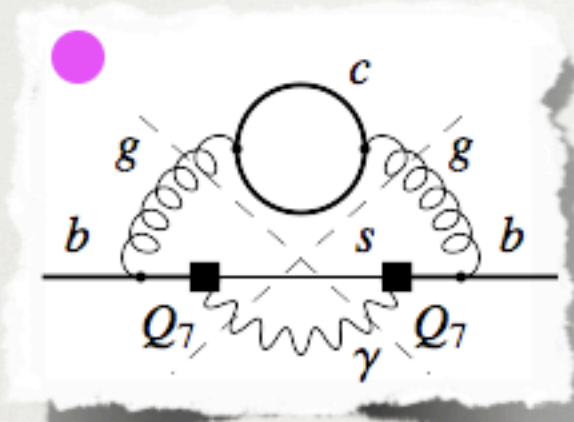



*Misiak et al. '06; Misiak & Steinhauser '06

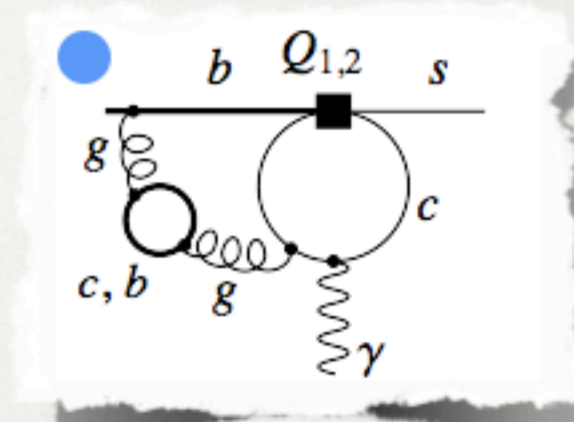
# Further Progress



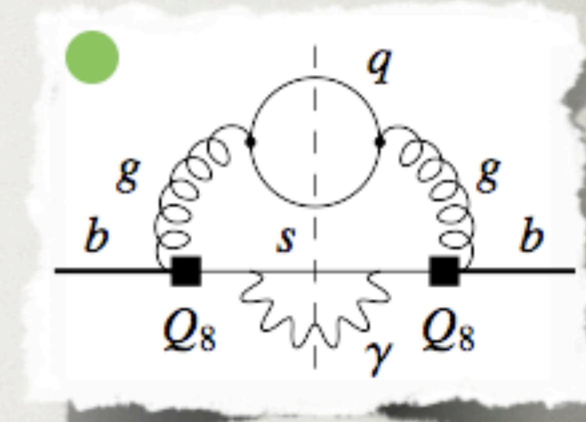

Czakon, UH & Misiak '06



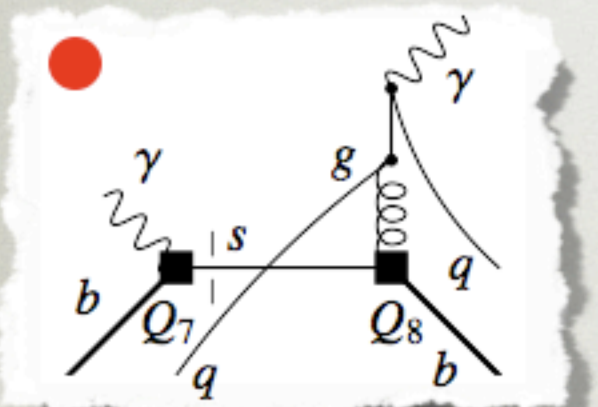

Asatrian et al. '06; Czarnecki & Pak, unpublished




Boughezal, Czakon & Schutzmeier '07




Ligeti et al. '99; Ferroglia, Gambino & UH, unpublished



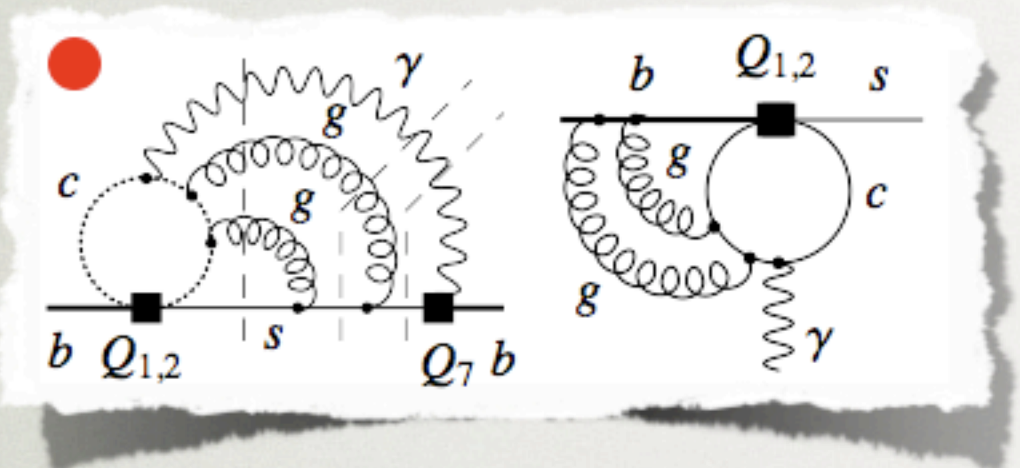

Lee, Neubert & Paz '06

- $\alpha_s^2$ anomalous dimensions $Q_{1\text{–}6}$ into $Q_8$: −0.3%
- $m_c$ effects due to gluon lines of ($Q_7$, $Q_7$): +0.3%
- $m_{c,b}$ effects due to gluon lines of ($Q_{1,2}$, $Q_7$): +1.1%
- dominant large-$\beta_0$ bremsstrahlungs effects: +2.0%
- $\alpha_s \Lambda / m_b$ non-perturbative effects of ($Q_7$, $Q_8$): −1.5%

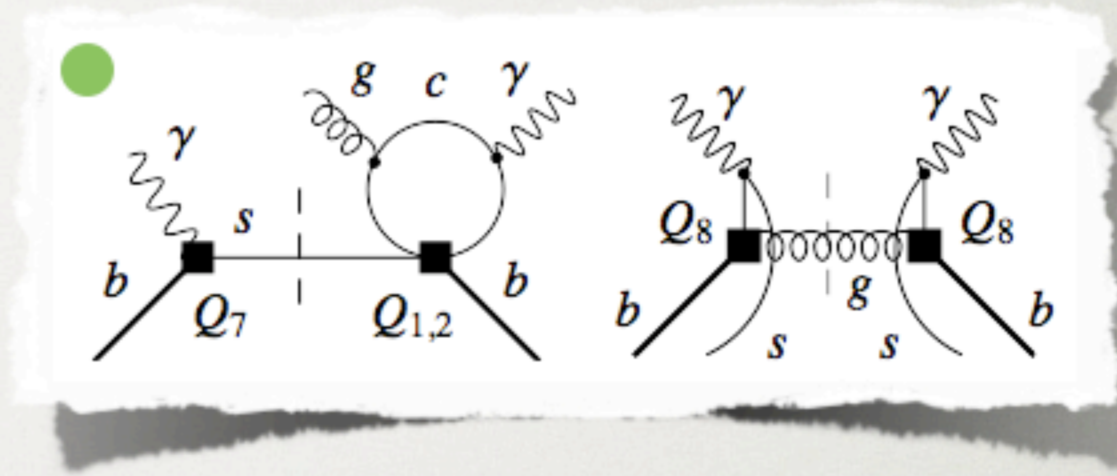
---

total correction to BR: +1.6%

# Ongoing Efforts




Boughezal, Czakon & Schutzmeier, in progress

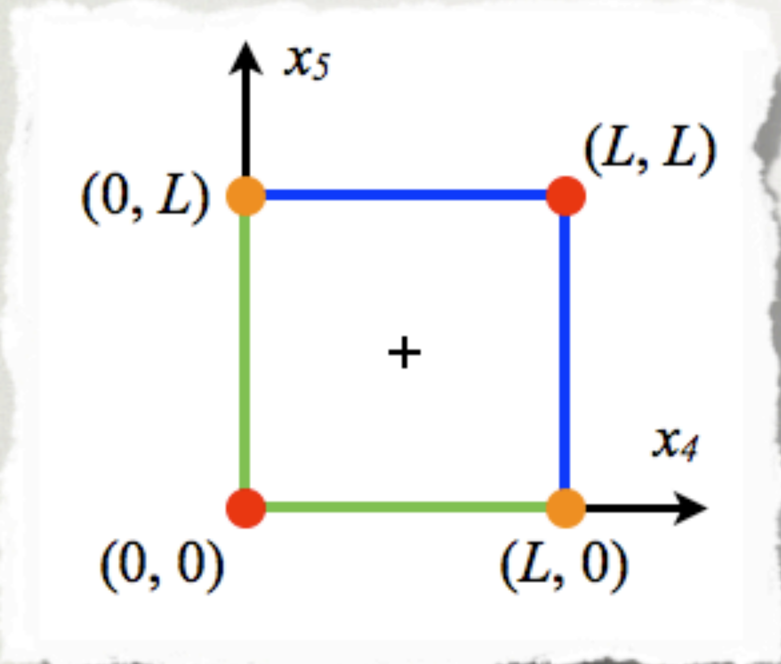


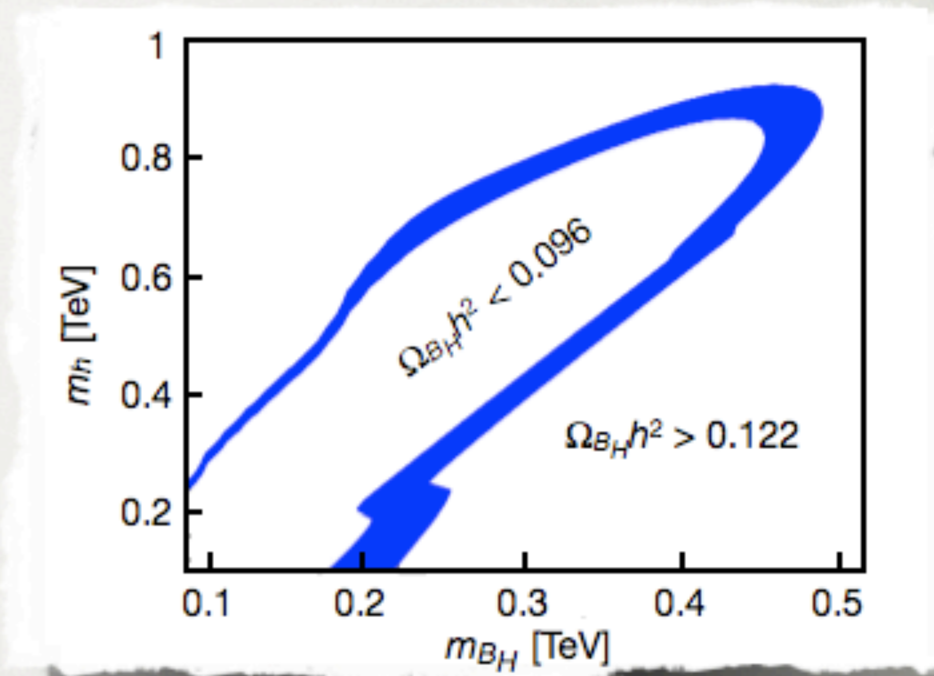

Lee, Neubert & Paz, in progress

- Calculation of $\alpha_s^2$ corrections to ($Q_{1,2}$, $Q_7$) for $m_c = 0$ and of virtual $\alpha_s^2$ corrections of $Q_{1,2}$ matrix elements for $m_c \sim 0.25 m_b$ with aim of significantly reducing or even removing theory uncertainty due to interpolation in charm quark mass

- Dedicated analysis of $\alpha_s \Lambda / m_b$ non-local power-corrections emerging from ($Q_{1,2}$, $Q_7$) and ($Q_8$, $Q_8$) terms may allow to set rough bound on size of non-perturbative effects, superseding current uncertainty guess-estimate of 5%

# Two Universal Extra Dimensions

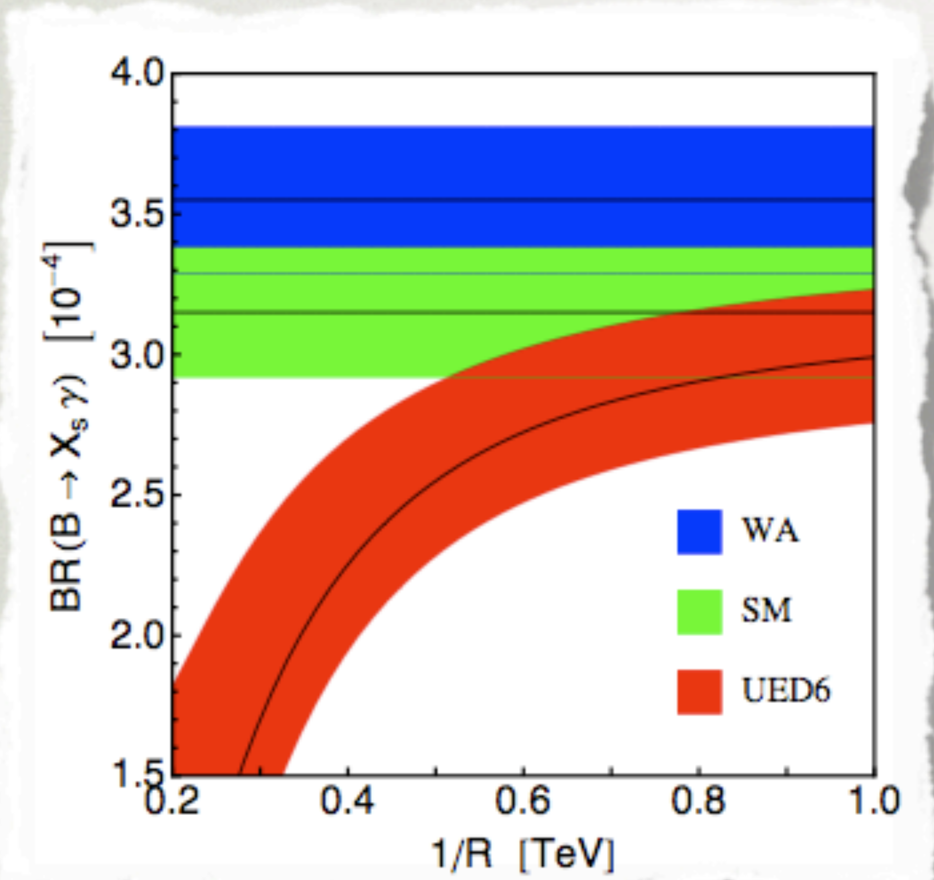


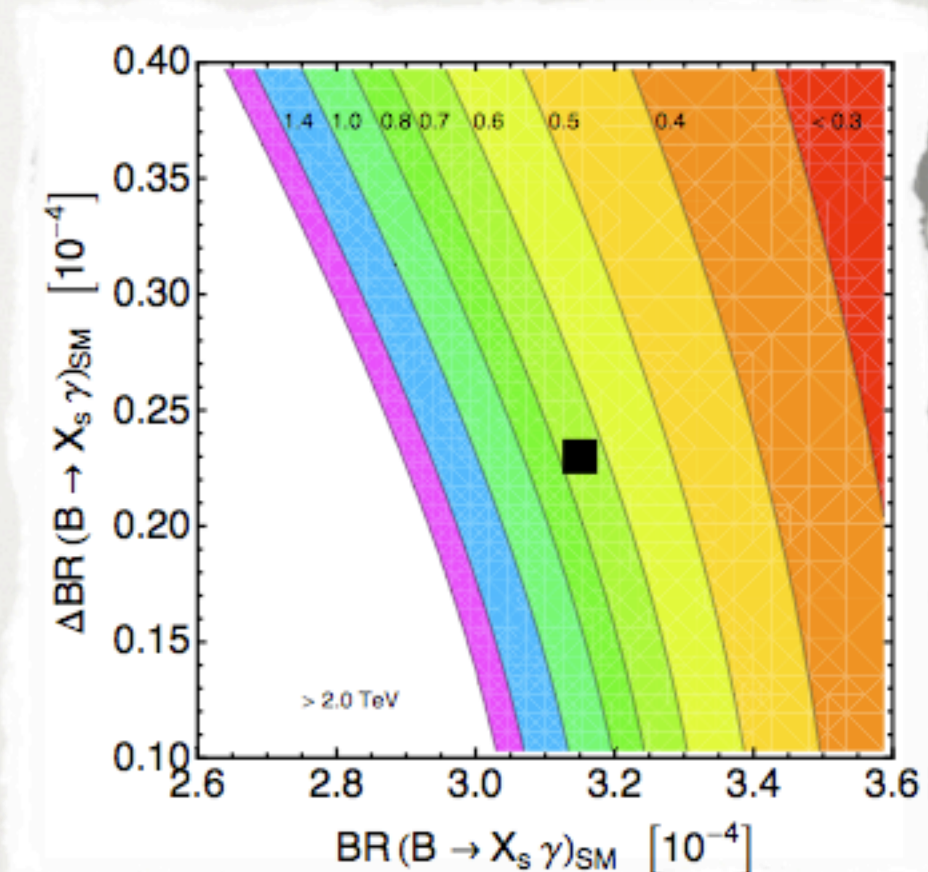




- All SM fields propagate in two flat extra dimensions, compactified on a square with side length $L = \pi R$ and adjacent sides being identified. This compactification leads to chiral fermion zero modes, while the higher fermionic KK modes are vector-like as usual
- Since geometry is invariant under rotations by $\pi$ about the center of square, the model respects an additional $Z_2$ symmetry. This implies that the lightest KK-odd particle is stable and could provide a viable dark matter candidate for a small KK scale $1/R \lesssim 600$ GeV

*Dobrescu & Ponton '04; Hashimoto & Hong '05; Burdman, Dobrescu & Ponton '06; Dobrescu et al. '07

# Bound on Compactification Scale

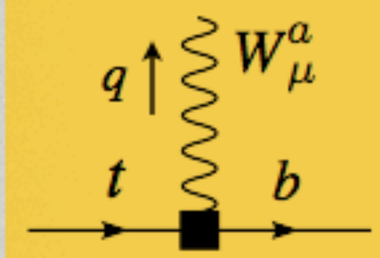


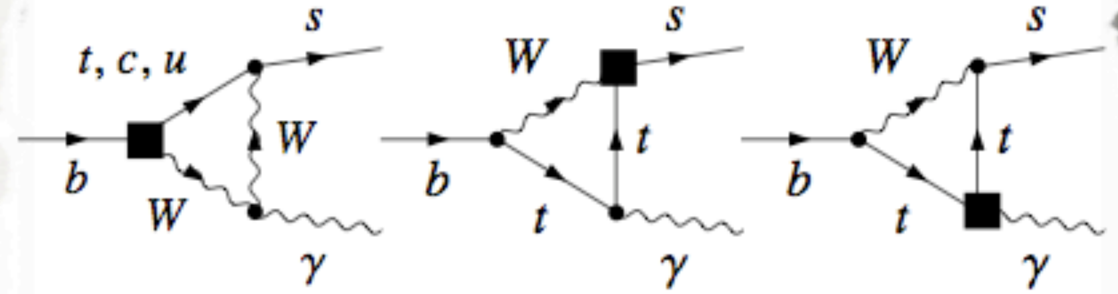

- Inclusion of virtual one-loop KK contributions and quadratic divergent mass corrections to KK scalar masses leads to strong suppression of BR in UED6 compared to SM prediction
- Although cut-off sensitivity weakens constraint on compactification scale, $B \to X_s\gamma$ leads to strongest direct bound of $1/R > 650$ GeV at 95% CL in variance with dark matter limit

*Freitas & UH '08

# Anomalous $Wtb$ Couplings




$$= -\frac{i g_W}{\sqrt{2}} \left[ \gamma_\mu (v_L P_L + v_R P_R) + \frac{i \sigma_{\mu\nu} q^\nu}{M_W} (g_L P_L + g_R P_R) \right]$$




| 95% CL bound | $\delta v_L$ | $v_R$ | $g_L$ | $g_R$ | $\mathrm{Re}\, C_7^{(p)}$ | $\mathrm{Re}\, C_8^{(p)}$ |
|---|---|---|---|---|---|---|
| upper | 0.03 | 0.0025 | 0.0004 | 0.57 | 0.04 | 0.15 |
| lower | −0.13 | −0.0007 | −0.0015 | −0.15 | −0.14 | −0.56 |

- ATLAS measurements should allow to put stronger bound of few $10^{-2}$ on $g_R$, while expected bounds on $v_R$ and $g_L$ are weaker than $B \to X_s\gamma$ limits by more than order of magnitude due to chiral $m_t/m_b$ enhancement in $b \to s\gamma$
- Single top production measurement at Tevatron imply $\delta v_L = 0.3 \pm 0.2$. Around order of magnitude better bounds are expected at LHC which would overcome current $B \to X_s\gamma$ constraint on BSM contribution to $v_L$

*Grzadkowski & Misiak '08; Aguilar-Saavreda et al. '07; Abazov et al. '07; O'Neil, Gonzalez-Pineiro & Lefebvre '02

# Conclusions

| Model | Accuracy | Effect | Bound |
|---|---|---|---|
| THDM II | NLO | $\Uparrow$ | $M_{H^\pm} > 295\,\text{GeV}$ (95% CL) |
| MFV MSSM | NLO | $\Updownarrow$ | — |
| LR | NLO | $\Updownarrow$ | — |
| general MSSM | LO | $\Updownarrow$ | $\lvert(\delta^d_{23})_{LL}\rvert \lesssim 4\times10^{-1}$, $\lvert(\delta^d_{23})_{RR}\rvert \lesssim 8\times10^{-1}$, $\lvert(\delta^d_{23})_{LR}\rvert \lesssim 6\times10^{-2}$, $\lvert(\delta^d_{23})_{RL}\rvert \lesssim 2\times10^{-2}$ |
| UED5 | LO | $\Downarrow$ | $1/R > 600\,\text{GeV}$ (95% CL) |
| UED6 | LO | $\Downarrow$ | $1/R > 650\,\text{GeV}$ (95% CL) |
| RS I | LO | $\Uparrow$ | $M_{\text{KK}} \gtrsim 2.4\,\text{TeV}$ |
| LH | LO | $\uparrow$ | — |
| LHT | LO | $\updownarrow$ | — |

- Inclusion of NNLO corrections has lead to significant reduction of renormalization scale and scheme dependences of $\text{BR}(B \to X_s\gamma)_{\text{SM}}$ that have been main source of theoretical uncertainty at NLO level
- As new SM prediction is lower than measurements by around $1\sigma$, BSM contributions should be preferably constructive, while NP that leads to suppression of $\text{BR}(B \to X_s\gamma)$ is more severely constrained than in past