FLAVOUR IN WARPED EXTRA DIMENSIONS

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Models in warped extra dimensions are very attractive because they can easily explain the hierarchy between the Planck and Electroweak scale, and generate the hierarchies in the fermion spectrum. The bounds on flavour are naturally less severe then in 4D extensions of the Standard Model, however they are still more severe than the electroweak precision tests, therefore worsening the fine tuning or little hierarchy problem. We review some recent attempts to soften such bounds either by means of flavour symmetries in the bulk or of a 5D minimal flavour violation paradigm.

1 Introduction

The Higgs sector is the only part of the Standard Model (SM) that has not been unveiled by experimental searches yet. What puzzles theorists is not only the lack of direct evidence of a Higgs boson sofar, but also a theoretical prejudice against a light fundamental scalar particle. In fact, quantum corrections would like to push the Higgs mass and the electroweak scale near the cutoff, that can be as high as the Planck scale. Recent efforts, however, have focused on a less severe problem that has more impact on the LHC experiments: the little hierarchy problem. The stability of the electroweak scale would require the presence of new particles below or around a TeV, however precision electroweak measurements generically push such scale above 5-10 TeV. This bound is severely worsened if flavour is also taken into account: measurements in the Kaon and B systems push the scale of new physics up to 10^4 TeV, thus requiring a fine tuning of several orders of magnitude unless a protection mechanism is summoned.

In the early '90, is has been realized that extra space dimensions are a rich playground for models of new physics. L. Randall and R. Sundrum [1] proposed an interesting metric in 5 dimensions that may account for large hierarchies in a natural way: such metric can be written as

$$ds^2 = \left(\frac{R}{z}\right)^2 \left(dx_\mu dx^\mu - dz^2\right) . \tag{1}$$

In this parametrization the 5D metric is explicitly invariant if we riscale the 4D coordinates x_μ and z by the same amount: this means that moving along the coordinate z is equivalent, from the 4D point of view, to a rescaling of lenghts and energies. The space is compactified by placing two branes at the boundaries. The brane at small z (UV-brane) will feel a large fundamental scale and therefore acts as

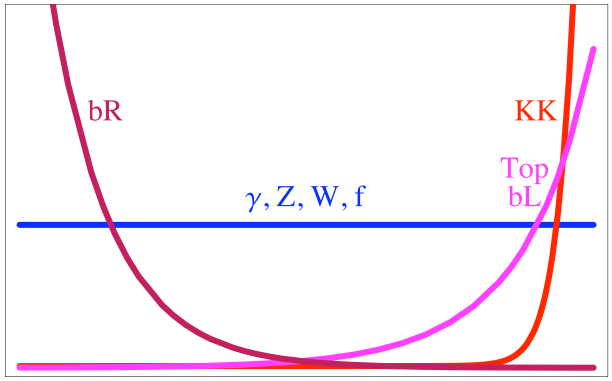

Figure 1: Portrait of a generic model of EWSB in warped space.

a fundamental cutoff of the theory, while the brane at large z will feel a smaller scale which cound be identified with the electroweak scale. In this setup a large hierarchy is rephrased in terms of order one parameters thanks to the exponential nature of the metric: this is more evident if one uses the coordinate $z = R \exp \frac{y}{R}$.

This idea has sprouted many interesting models. Among them, one can identify the Higgs as the 5th polarization of a gauge boson from a broken bulk symmetry [2]: gauge invariance itself will protect the Higgs potential and solve the little hierarchy problem, given that the Kaluza-Klein (KK) resonances that cut off the loop divergencies are light enough. However, precision electroweak tests (PEWTs) require the heavy bosons to be above 2 TeV. Flavour physics plays an important role in these models: in fact one can use warped geometry to generate the hierarchies in the fermion mass spectrum naturally [3]. Once the fermions propagate in the bulk of the extra dimension, there will be more sources of flavour than in the SM: schematically the relevant terms in the action can be written as

$$S = \int d^4x \int_{z_{UV}}^{z_{IR}} dz \left(\frac{R}{z}\right)^4 \left[\frac{c_{Q,u,d}}{z}\ \bar{\psi}_{Q,u,d}\psi_{Q,u,d} + Y_{u,d}\ \bar{\psi}_Q H \psi_{u,d}\ \delta(z - z_{IR})\right] + \dots\,, \tag{2}$$

where the dots represent eventual UV localized terms and higher order operators. The SM fermion masses are generated by the interactions with the Higgs which is localized on or near the IR brane: for instance in gauge-Higgs unification models the delta function is replaced by the Higgs profile, peaked at large z. The bulk masses c, matrices in flavour space, are not real masses: they control the fermion localization along the extra dimension, and therefore the overlap with the Higgs. The wave functions are in fact exponentially sensitive to the c's. Generically, this flavour-dependence of the wave functions will induce flavour non-universal couplings with the gauge KK modes, in particular the KK gluons, which will generate flavour changing neutral currents (FCNCs) at tree level. Flavour therefore may constrain the KK masses well above the TeV scale! Moreover, one needs to worry about new CP violating phases and higher order operators which may be suppressed by the IR scale.

If the bounds were as tight as in 4 dimensions, it would be the death of such models: however this is not the case. In order to understand this statement, we need to understand better the structure of a generic model of EWSB in warped geometry. The key is the localization of the wave functions: in fact it will determine both the spectrum via the boundary conditions, and the strength of their couplings via their overlap with other fields. Therefore, a generic model of EWSB can be portrayed in Fig. 1: the gauge boson wave functions are flat due to gauge invariance; the light fermions are localized towards the UV brane in order to suppress their coupling to the Higgs, or any other source of EWSB; on the other hand the top is necessarily localized toward the IR brane due to its heaviness. Finally the KK modes of all the bulk fields are localized towards the IR brane: as a generic consequence, they will couple more to the heavy SM particles than to the light ones. Assuming anarchic Yukawa couplings, the spectrum and mixings are both determined by the values of the fermion wave functions on the IR brane. The couplings of light fermions to the KK modes are small due to the localizations, and universal up to corrections of order $\mathcal{O}(m_f^2/m_{KK}^2)$ [4]: the light fermion are localized away from the IR brane, where KK wave functions are small and approximately constant. The flavour non-universal contribution comes from the values of the fermion wave functions on the IR brane, which are proportional to the fermion masses: this is the origin of the so-called Randall-Sundrum-GIM (Glashow Iliopoulos Maiani) mechanism. The situation is different for the top, which is localized on the IR brane. Therefore, all the FCNCs are induced by the third generation, and they are proportional to the mixing angles to the top. This mechanism allows to lower considerably the flavour bounds on KK masses from thousands to 8 TeV. However, bounds from flavour are still generally more severe that EWPTs, and reopen the little hierarchy problem and a fine tuning in the Higgs potential.

In recent years a lot of work has been dedicated to weaken this flavour bounds and push them below the EWPTs bounds [5,6,7]. In the rest of the paper we will review the two mechanisms involving flavour symmetries in the bulk [5] and minimal flavour violation in the bulk [6].

2 Flavour Symmetries in the Bulk

The easier way to avoid flavour bounds is to introduce flavour symmetries in the bulk. The simplest choice is to impose an $SU(3)_L\times$ $SU(3)_R$ in the bulk for both quarks and leptons, where we impose a single flavour symmetry $SU(3)_R$ for the right handed fermions due to an eventual custodial symmetry in the bulk [8] which will contain them in the same multiplet. The symmetry will be broken to the diagonal $SU(3)_D$ on the IR brane by the Yukawa couplings: in this way Yukawas, bulk masses and bulk operators are all flavour diagonal. The $SU(3)_R$ is broken on the UV brane where localized kinetic operators for the right-handed fermions will generate both the mass hierarchies and the mixings: therefore the number of flavour matrices in this model is the same as in the SM and no extra CP violating phases appear. Also, the symmetries forbid FCNCs: one can use two SU(3) rotations in the up and down sector to diagonalize the kinetic operators. The neutral sector of the gauge bosons will remain flavour universal, while flavour violation will only appear in the interactions with charged gage bosons like the W. Finally the only flavour violating higher order operators will be localized on the UV brane and will be suppressed by the large UV cutoff of the theory, therefore they can be safely neglected.

We can look more in detail to the main features of this scenario: the only flavour structure appears in the UV boundary conditions for the right-handed fields:

$$f_R(m, z_{UV})\, \bar{A}_{u,d} = m\, g_R(m, z_{UV})\, \mathcal{K}_{u,d} \cdot \bar{A}_{u,d}\,, \tag{3}$$

where f and g are generic flavour-blind wave functions, $\bar{A}$ is the normalization - a vector in flavour space, and $\mathcal{K}_{u,d}$ are the UV kinetic matrices. One can diagonalize the kinetic matrices

$$\mathcal{K}_{u,d} = U^\dagger_{u,d} \cdot \mathcal{K}^{\rm diag}_{u,d} \cdot U_{u,d}\,, \tag{4}$$

so the specrtum will be determined by the eigenvalues k_i while the mixing matrices will fix the normalization coefficients $\bar{A}$. Now, the couplings of neutral gauge bosons are diagonal, because they are proportional either to $U \cdot U^\dagger = 1$ or $U \cdot \mathcal{K} \cdot U^\dagger = \mathcal{K}^{\rm diag}$; on the other hand, the charged boson couplings will be proportional to $U_u^\dagger U_d$. Therefore

$$V_{CKM} = U_u^\dagger U_d + \mathcal{O}(m_i^2) \tag{5}$$

where the corrections are due to the mass dependence of the wave functions, and all the flavour violating contributions will be proportional to the Cabibbo-Kobayashi-Maskawa matrix.

This model can be realized easily for the leptons, however it has problems when applied to quarks. The reason is that the top is very heavy and, due to the flavour symmetries, all the quarks share the same Yukawa coupling on the IR brane. The large Yukawa coupling will modify the fermion wave functions and generate universal corrections to the couplings. The flavour bound is therefore projected into EWPTs: the latter will push the KK masses above 10 TeV.

In order to solve this issue one needs to separate the top Yukawa from the light quarks. One can use different representations for the up and down right-handed quarks: using a singlet for the up quarks, including the top, can also help in lowering the bound from the coupling of the bottom with the Z boson [9]. Moreover, one can impose a looser $U(1)^3$ flavour symmetry for the right-handed up-type quarks and leave it unbroken. In this way the up type quarks Yukawas are all different:

$$Q \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} u + m_b Q \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} d \tag{6}$$

The down sector is as before, therefore all the flavour mixing is induced in the down sector. One can show that in this model FCNCs are still forbidden, and the strongest bound on the KK masses is again the 2 TeV from precision measurements [5].

3 5D MFV: 5 Dimensional Minimal Flavour Violation

Another interesting approach is to impose minimal flavour violation on the 5 dimensional model: flavour violating effects are not protected by a symmetry, but by the assumption that all the flavour structure can only be determined by the Yukawa matrices. In this case, the bulk masses in Eq. 2 are

$$c_{u,d} \sim Y^{\dagger}_{u,d} + \dots \qquad c_Q \sim rY^{\dagger}_u Y_u + Y^{\dagger}_d Y_d + \dots \tag{7}$$

The advantage of this approach is that one can still use different bulk masses to explain the hierarchies in the spectrum and the mixing angles, and at the same time gain a factor of ~ 3 suppression in the flavour bounds that makes them again as low as the precision tests. Assuming anarchic Yukawa matrices is still enough to generate the required hierarchies due to the exponential sensitivity to the c parameters. Moreover, in the limit when c_Q only depends on one Yukawa, for instance when $r \to 0$, one can diagonalize the down sector and eliminate all the flavour violating effects involving down type quarks. This means that the processes that violates flavour by 2 units, like for example the neural Kaon mixing which gives the strongest bounds, are suppressed by small r. Therefore a moderately small r can provide the required factor of 3 in the bound without any flavour symmetry. Those small values are also preferred by the fit of the masses and mixing angles. Moreover the CP problem is also removed, because there isn't any additional phase besides the SM one: for instance one can check that electric dipole moments only arise at two loops and they do not pose any additional bound [6].

4 Conclusion and Outlook

Flavour physics is an important component of models in warped extra dimension. In fact, flavour bounds generically apply to the KK masses of the gauge bosons which play an important role in the electroweak symmetry breaking sector and are required to be at or around a TeV in order for the model to be natural. The bounds are much lower than in a generic 4 dimensional model due to a Randall-Sundrum GIM mechanism, however they are still one order of magnitude tighter than bounds from precision electroweak tests. Moreover, the warped geometry offer the possibility to construct an elegant model of flavour where both the hierarchies in the masses and in the mixing angles are explained in terms of order one parameters. If we were not concerned by the two orders of magnitude still separating the scale of new physics and the electroweak scale, this would be one of the most appealing models of flavour.

However, trying to lower the bounds from flavour has inspired a dense activity in recent years. We reviewed two nice ideas. One involving the use of bulk flavour symmetry, and one proposal of a minimal flavour violation paradigm. In the former case, one can eliminate all the flavour changing neutral currents at the price of giving up the nice explanation of the hierarchies. The heaviness of the top quark still requires some massaging as the light quarks cannot share its large Yukawa, however it is still possible to construct models with a relaxed flavour symmetry where the flavour bound is as low as 2 TeV.

In the case of minimal flavour violation, no symmetry is needed and a relation between the Yukawa matrices and all the other sources of flavour violation is enough to solve the CP problem and to parametrically suppress the most dangerous flavour violating effects. It is important to notice that the required suppression is just a factor of 3, and that this suppression is also preferred by the fit of the fermion masses and mixing angles.

The precise bound from flavour physics is therefore very important as it can have severe consequences on the phenomenology and viability of such models. It can easily push the new physics above the reach of the LHC and the electroweak sector of the model un-natural. There cannot be a viable model unless its flavour structure is studied in detail.

Acknowledgments

I would like to thank C. Csaki, J. Galloway, G. Marandella, J. Terning and A. Weiler for their precious collaboration, and the organizers of the Rencontres of Moriond for the always inspiring environment offered during the meeting.

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