Supergravity: Finite after all?

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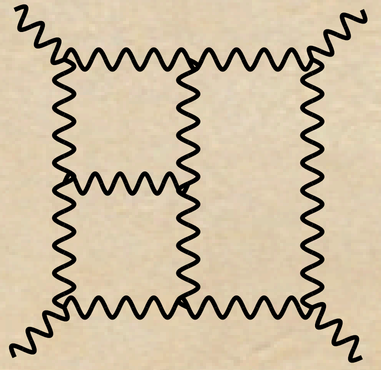
Ultraviolet Divergences in Gravity

- Simple power counting in gravity and supergravity theories leads to a naïve degree of divergence

$$\Delta = (D-2)L + 2$$

in D spacetime dimensions. So, for D=4, L=3, one expects $\Delta = 8$. In dimensional regularization, only logarithmic divergences are seen ($\frac{1}{\varepsilon}$ poles, $\varepsilon = D - 4$), so 8 powers of momentum would have to come out onto the external lines of such a diagram.



- Local supersymmetry implies that the pure curvature part of such a D=4, 3-loop divergent structure must be built from the square of the Bel-Robinson tensor Deser, Kay & K.S.S

$$\int \sqrt{-g}\, T_{\mu\nu\rho\sigma} T^{\mu\nu\rho\sigma}\,, \qquad T_{\mu\nu\rho\sigma} = R_{\mu}{}^{\alpha}{}_{\nu}{}^{\beta} R_{\rho\alpha\sigma\beta} + {}^{*}R_{\mu}{}^{\alpha}{}_{\nu}{}^{\beta}\, {}^{*}R_{\rho\alpha\sigma\beta}$$

- This is directly related to the α'^3 corrections in the superstring effective action, except that in the string context such contributions occur with finite coefficients. The question remains whether such string theory contributions develop poles in $(\alpha')^{-1}$ as one takes the zero-slope limit $\alpha' \to 0$ and how this bears on the ultraviolet properties of the corresponding field theory.

- The consequences of supersymmetry for the ultraviolet structure are not restricted, however, simply to the requirement that counterterms be supersymmetric invariants.

- There exist more powerful "non-renormalization theorems," the most famous of which excludes infinite renormalization within D=4, N=1 supersymmetry of chiral invariants, given in N=1 superspace by integrals over half the superspace:

$$\int d^2\theta W(\phi(x,\theta,\bar{\theta})) , \quad \bar{D}\phi = 0$$

- The strength of a given supersymmetric non-renormalization theorem depends on the extent of linearly realizable, or "off-shell" supersymmetry. This is the extent of supersymmetry for which the algebra can close without use of the equations of motion.
- Knowing the extent of this off-shell supersymmetry is tricky, and may involve formulations (*e.g.* harmonic superspace) with infinite numbers of auxiliary fields.
- For maximal N=4 Super Yang-Mills and maximal N=8 supergravity, the linearly realizable supersymmetry has been known since the 80's to be at least half the full supersymmetry of the theory.

- The key point about the non-renormalization theorems is that allowed counterterms have to be written as full $\int d^{4M}\theta$ superspace integrals for the linearly realized M-extended supersymmety, where the integrands must be written using a clearly defined set of basic objects, and where the integrated counterterms have to satisfy all applicable gauge symmetries and also must be locally constructed (*i.e.* written without such operators as $\Box^{-1}$).

- So, in D=4, N=1 supersymmetry, full superspace integrals like $\int d^4x d^4\theta f(\phi,\bar{\phi})$ (or "D terms") are allowed, but chiral integrals like $\int d^4x d^2\theta\, g(\phi)$ (or "F terms") are not.

- The full extent of a theory's supersymmetry, even though it may be non-linear, also restricts the infinities since the *leading* counterterms have to be invariant under the original unrenormalized supersymmetry transformations.

Howe, K.S.S & Townsend

- Assuming that 1/2 supersymmetry is linearly realizable and requiring gauge and supersymmetry invariances, one derives predictions for the first divergent loop orders in maximal (N=4 ↔ 16 supercharge) SYM and (N=8 ↔ 32 sc.) SUGRA:

Max. SYM first divergences, assuming half SUSY off-shell (8 supercharges)

Dimension D	10	8	7	6	5	4
Loop order L	1	1	2	3	4	∞
Gen. form	$\partial^2 F^4$	F^4	$\partial^2 F^4$	$\partial^2 F^4$	F^4	finite

Max. SUGRA first divergences, assuming half SUSY off-shell (16 supercharges)

Dimension D	11	10	8	7	6	5	4
Loop order L	2	1	1	2	2	2	3
Gen. form	$\partial^6 R^4$	$\partial^2 R^4$	R^4	$\partial^6 R^4$	$\partial^2 R^4$	R^4	R^4

Unitarity-based calculations

Bern, Dixon, Dunbar, Kosower, Perelstein, Rozowsky et al.

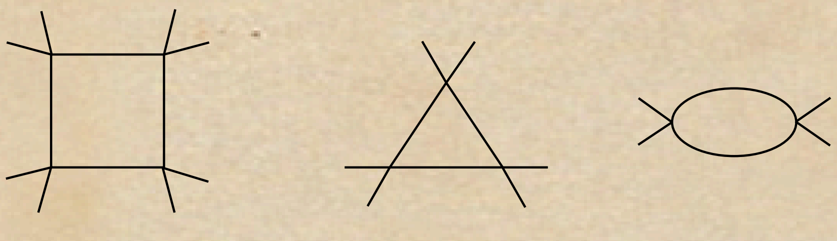
- Within the last decade, there have been significant advances in the computation of loop corrections in quantum field theory.
- These developments include the organization of amplitudes into a new kind of perturbation theory starting with maximal helicity violating amplitudes (MHV), then next-to-MHV (NMHV), *etc.*
- They also incorporate a specific use of dimensional regularization together with a clever use of unitarity cutting rules.

- Normally, one thinks of unitarity relations such as the optical theorem as giving information only about the imaginary parts of amplitudes. However, if one keeps all orders in an expansion in $\varepsilon = 4 - D$ then loop integrals like $\int d^{(4-\varepsilon)}p$ require integrands to have an additional momentum dependence $f(s) \to f(s)s^{-\varepsilon/2}$, where s is a momentum invariant. Then, since $s^{-\varepsilon/2} = 1 - (\varepsilon/2)\ln(s) + \ldots$ and $\ln(s) = \ln(|s|) + i\pi\Theta(s)$, one can learn about the real parts of an amplitude by retaining imaginary terms at order ε .

- This gives rise to a procedure for the *cut construction* of higher-loop diagrams.

- Another key element in the unitarity-based analysis of amplitudes is the Passarino-Veltman procedure for the reduction of Feynman-diagram propagators, replacing numerator factors like $2k \cdot p$ where $p^2 = 0$ by $(k+p)^2 - k^2$ and then canceling corresponding denominators.

- This procedure can yield a variety of resulting irreducible configurations in the reduced diagram, including boxes, triangles and bubbles.



- Important simplifications occur if one can show there are ultimately no bubbles or triangles in the reduced amplitude.

- For maximal supergravity amplitudes, another specific relation allowing amplitudes to be evaluated is the Kawai-Lewellen-Tai relation between open- and closed-string amplitudes. This gives rise to tree-level relations between max. SUGRA and max. SYM field-theory amplitudes, *e.g.*

$$M_4^{\rm tree}(1,2,3,4) = -is_{12}A_4^{\rm tree}(1,2,3,4)\,A_4^{\rm tree}(1,2,4,3)$$

- Combining this with unitarity-based calculations, in which all amplitudes are ultimately reduced to integrals of products of tree amplitudes, one has a way to obtain higher-loop supergravity amplitudes from SYM amplitudes.

- In this way, a different set of anticipated first loop orders for ultraviolet divergences has arisen from the unitarity-based approach:

Max. SYM first divergences, unitarity-based predictions

Dimension D	10	8	7	6	5	4
Loop order L	1	1	2	3	6	∞
Gen. form	$\partial^2 F^4$	F^4	$\partial^2 F^4$	$\partial^2 F^4$	$\partial^2 F^4$	finite

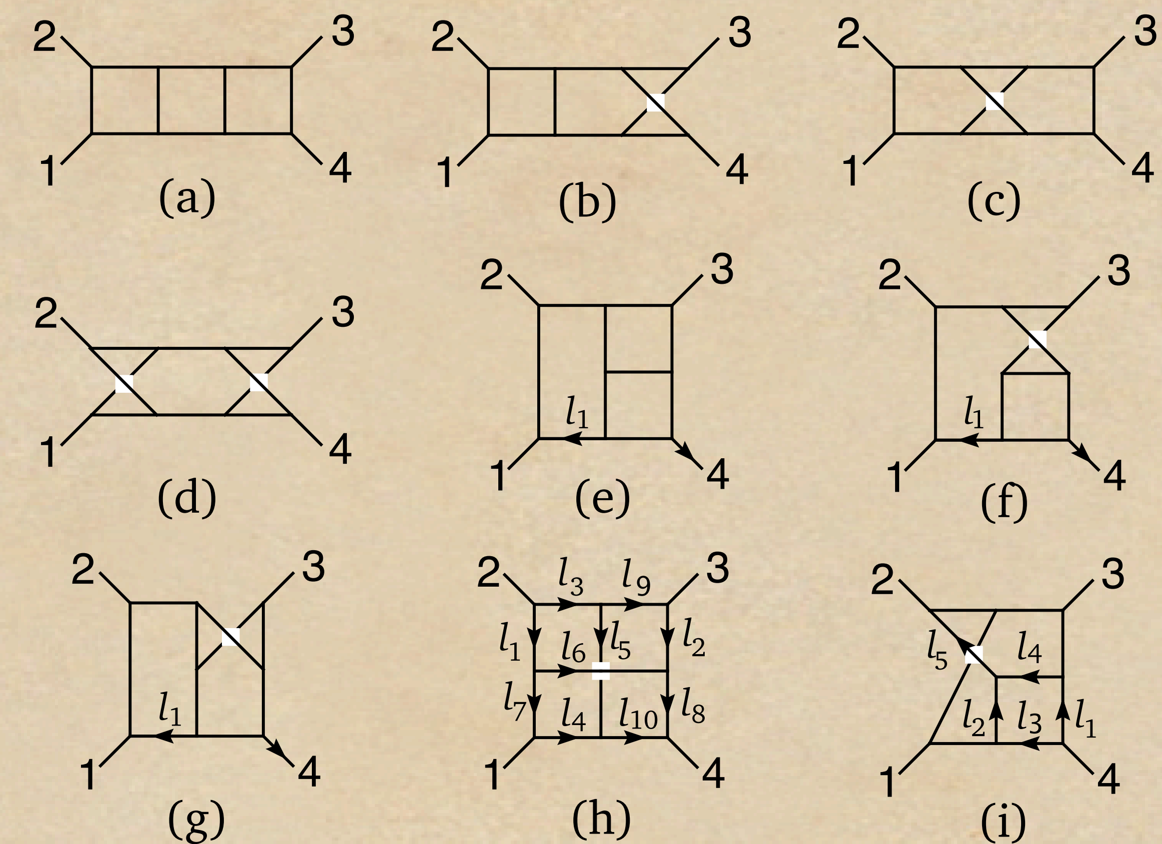
Max. SUGRA first divergences, unitarity-based predictions

Dimension D	11	10	8	7	6	5	4
Loop order L	2	1	1	2	3	4	5
Gen. form	$\partial^6 R^4$	$\partial^2 R^4$	R^4	$\partial^6 R^4$	$\partial^6 R^4$	$\partial^6 R^4$	$\partial^4 R^4$

- These anticipations are based on iterated 2-particle cuts, however. Full calculations can reveal different behavior.

- The main recent development is the completion of the 3-loop calculation:

Bern, Carrasco, Dixon, Johansson, Kosower & Roiban.



- Diagrams (a-g) can be evaluated using iterated two-particle cuts, but diagrams (h) & (i) cannot. The result is finite at L=3 in D=4, but the surprize is that the finite parts have an unexpected six powers of momentum that come out onto the external lines: a $\partial^6 R^4$ leading effective action correction.

Counterterm counterattack

- The 3-loop N=8 supergravity calculation is a remarkable *tour de force*, but does it indicate that there are "miracles" that cannot be understood from non-renormalization theorems?
- All known SYM divergences in the various dimensions D can be understood using non-renormalization theorems.
- Recently it has been realized that N=4 SYM can be quantized with 9=8+1 off-shell supersymmetries, at the price of manifest Lorentz invariance. Baulieu, Berkovits, Bossard & Martin
- A similar formulation for maximal supergravity exists with 17=16+1 off-shell supersymmetries in D=2. Preliminary indications are that a related construction is likely to rule out the L=3, D=4 counterterm. Bossard, Howe & K.S.S

Kallosh

- Moreover, the no-triangle property for the end result of the Passarino-Veltman graph reduction procedure has been shown to follow directly from N=8 supersymmetry at one loop for the *non-local* effective action. This argument follows closely the known *local* structure of supersymmetric ultraviolet counterterms. Kallosh; Howe, K.S.S. & Townsend

- This result can be read two ways: either as an indication of the validity of the no-triangle hypothesis, <u>or</u> as a warning that the simple box-only form of the reduced diagrams may follow from maximal supersymmetry only up to a limited loop order, similar to the ostensible finite reach of the non-renormalization theorems.

- To date, these questions remain unresolved. But, in a venerable tradition of marking points to be settled in physics, bets have been taken, for bottles of wine.

Which will be the payoff?

or