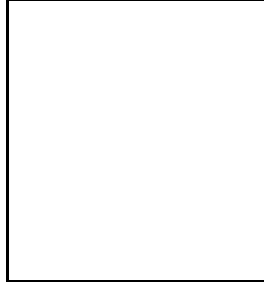


NEUTRINO SELF INTERACTIONS IN SUPERNOVAE

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Oscillations of neutrino emerging from a supernova core are studied. In this extremely high density region neutrino self interactions induce collective flavor transitions. When collective transitions are decoupled from matter oscillations, as for our chosen matter profile, an analytical interpretation of the collective effects is possible, by means of a mechanical analogy with a spherical pendulum. For inverted neutrino hierarchy the neutrino propagation can be divided in three regimes: synchronization, bipolar oscillations, and spectral split. Our simulation shows that averaging over neutrino trajectories does not alter the nature of these three regimes.

1 Introduction

Supernova neutrino oscillations are a very important tool to study astrophysical processes and to better understand neutrino properties¹. When neutrinos leave the surface of the neutrinosphere, they undergo vacuum and matter oscillations. Beside this, in the first few hundred kilometers neutrino-neutrino interactions induce collective flavor transitions, whose effect can be very important, depending on the neutrino mass hierarchy. Self-interaction effects are expected to be non negligible when $\mu(r) \sim \omega$, where $\mu(r)$ is the neutrino potential associated to the neutrino background ($\mu = \sqrt{2}G_F(N_\nu(r) + \bar{N}_\nu(r))$), analogously to the MSW potential $\lambda = \sqrt{2}G_F N_e(r)$ and ω is the vacuum oscillation frequency. We neglect the solar mass square difference $\delta m^2 = m_2^2 - m_1^2 \ll \Delta m^2 = |m_3^2 - m_{1,2}^2|$, and consider a two-neutrino mixing scenario where the oscillations are governed by the mixing angle θ_{13} . Since in the supernova context ν_μ and ν_τ cannot be distinguished we generically speak of $\nu_e \leftrightarrow \nu_x$ oscillations. In our work we assume $\Delta m^2 = 10^{-3} \text{ eV}^2$ and $\sin^2 \theta_{13} = 10^{-4}$. Figure 1 shows the radial profiles of the matter

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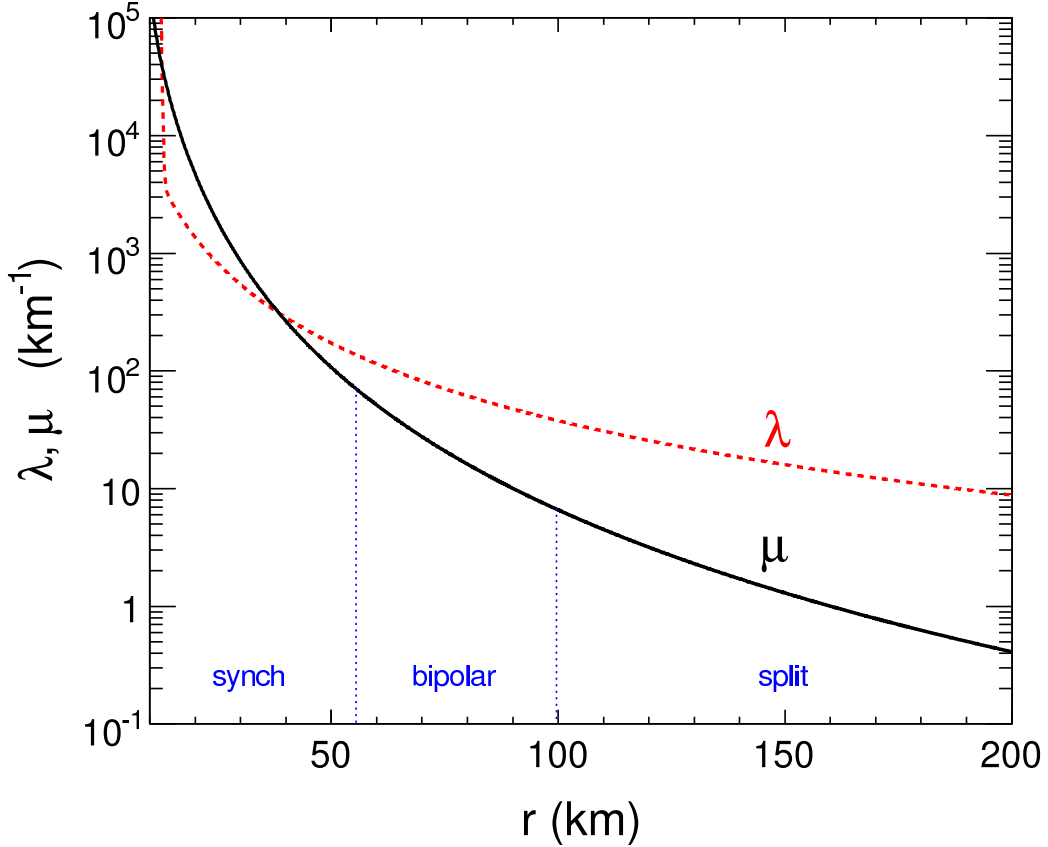


Figure 1: Radial profiles of the neutrino self-interaction parameter $\mu(r) = \sqrt{2} G_F (N + \bar{N})$ and of the matter-interaction parameter $\lambda(r) = \sqrt{2} G_F N_{e-}$ adopted in this work, in the range $r \in [10, 200]$ km.

potential $\lambda(r)$ and of the neutrino potential $\mu(r)$, and the approximate ranges where collective flavor transitions of different type occur: synchronization, bipolar oscillation and spectral split. The nonlinearity of the self interactions induce neutrino oscillations very different from the ordinary MSW effect. When undergoing collective flavor transition neutrinos and antineutrinos of any energy behave similarly, as we will see in the following. This kind of transitions occurs for small r , well before the ordinary MSW resonance, allowing for a clear interpretation of the numerical simulations. For matter profiles different from our own, the MSW resonance condition can occur in the same region of the collective transitions: shallow electron density profiles² can trigger MSW effects around $O(100)$ km. In that case it is much more difficult to disentangle collective from MSW effects in the results of the simulations.

2 Reference model and pendulum analogy

In our work, we use normalized thermal spectra with $\langle E_e \rangle = 10$ MeV, $\langle \bar{E}_e \rangle = 15$ MeV, and $\langle E_x \rangle = \langle \bar{E}_x \rangle = 24$ MeV for ν_e , $\bar{\nu}_e$, ν_x and $\bar{\nu}_x$, respectively. The geometry of the model, the so called “bulb model”², has a spherical symmetry, since we assume that neutrinos are half-isotropically emitted from the neutrinosphere. Along any radial trajectory there is, therefore, a cylindrical symmetry. By virtue of that, we need only two independent variables to describe the neutrino propagation and interaction: the distance from the supernova center r , and the angle ϑ between two interacting neutrinos. If the dependence on ϑ is integrated out, we speak of “single-angle” approximation, while the general situation of variable ϑ is dubbed “multi-angle” case. The numerical simulation in the multi-angle case is extremely challenging, since it requires the solution of a large system (size of order 10^5) of coupled non-linear equations. The propagation

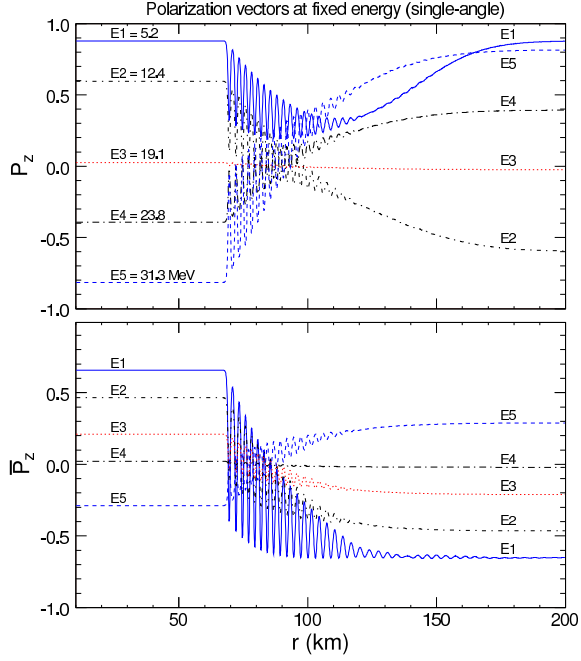


Figure 2: Single-angle simulation in inverted hierarchy: P_z (neutrinos) and \bar{P}_z (antineutrinos) as a function of radius, for five energy values.

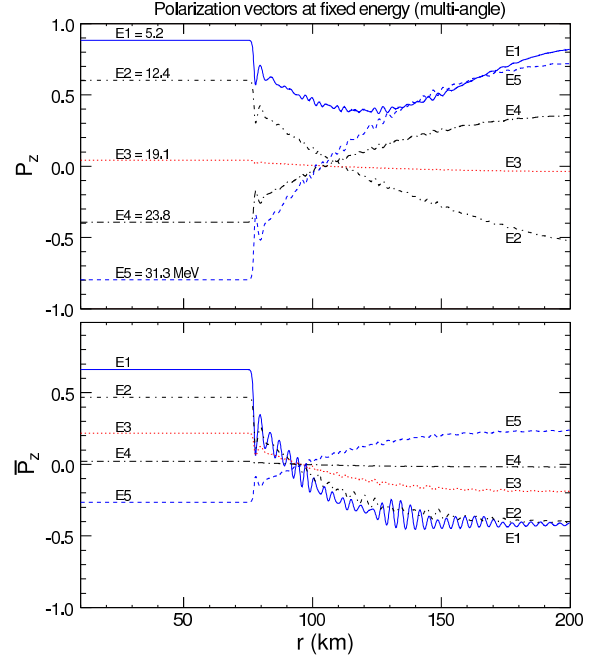


Figure 3: Multi-angle simulation in inverted hierarchy: P_z (neutrinos) and \bar{P}_z (antineutrinos) as a function of radius, for five energy values.

of neutrinos of given energy E is studied through the Liouville equation for the density matrix. By expanding the density matrix on the Pauli matrices and on the identity, the equations of motion can be expressed in terms of two polarization vectors, $\mathbf{P}(E)$ and $\bar{\mathbf{P}}(E)$, for neutrinos and antineutrinos, respectively. By introducing a vector \mathbf{B} that depends on the mixing angle θ_{13} , and a vector $\mathbf{D} = \mathbf{J} - \bar{\mathbf{J}}$ that is the difference between the integral over the energy of \mathbf{P} and $\bar{\mathbf{P}}$, the equations of motion can be written as

$$\dot{\mathbf{P}} = (+\omega\mathbf{B} + \lambda\mathbf{z} + \mu\mathbf{D}) \times \mathbf{P}, \quad (1)$$

$$\dot{\bar{\mathbf{P}}} = (-\omega\mathbf{B} + \lambda\mathbf{z} + \mu\mathbf{D}) \times \bar{\mathbf{P}}. \quad (2)$$

In the general case, the polarization vectors depend also on the neutrino emission angle θ_0 (the neutrino incidence angle ϑ can be expressed in terms of r and of the emission angle at the neutrinosphere θ_0). The electron neutrinos survival probability P_{ee} is a function of the polarization vector, $P_{ee} = 1/2(1 + P_f^z/P_z^i)$, where the i and f refer to the initial and final state respectively (analogously for antineutrinos). The equations of motion for $\mathbf{P}(E)$ and $\bar{\mathbf{P}}(E)$ can be reduced (under reasonable approximations³) to the equations of motion of a gyroscopic pendulum, a spherical pendulum of unit length in a constant gravity field, characterized by a point-like massive bob spinning around the pendulum axis with constant angular momentum. The pendulum inertia is inversely proportional to $\mu(r)$, while its total angular momentum depends on the difference of the integrated polarization vectors \mathbf{J} and $\bar{\mathbf{J}}$ ³. The motion of a spherical pendulum is, in general, a combination of a precession and a nutation^{4,5}. In the case of normal hierarchy of the neutrino mass spectrum the pendulum starts close to the stable, downward position and stays close to it, as μ slowly decreases and no collective effect is present. In the inverted hierarchy case, the pendulum starts close to the “unstable,” upward position. At the beginning, for small r , when μ is large (m is small), the bob spin dominates and the pendulum remains precessing in the upward position conserving angular momentum⁵, a situation named synchronization^{6,4}. Nevertheless, since μ decreases with r , at a certain point any $\theta_{13} \neq 0$ triggers the fall of the pendulum and its subsequent nutations, the so called bipolar oscillations. The increase of the

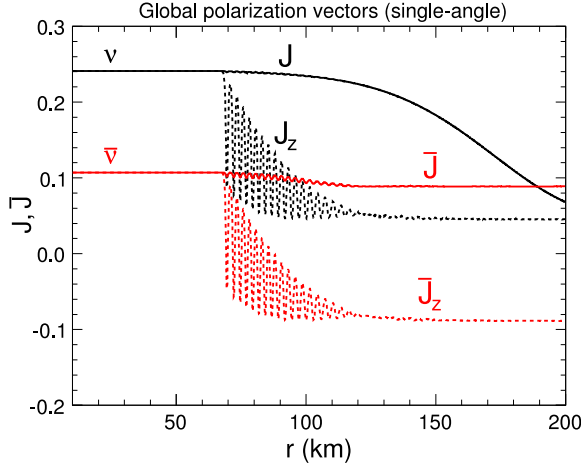


Figure 4: Single-angle simulation in inverted hierarchy: modulus and z -component of \mathbf{J} and $\bar{\mathbf{J}}$.

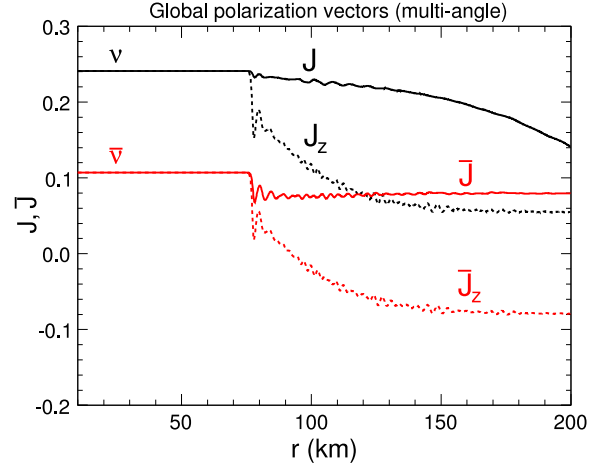


Figure 5: Multi-angle simulation in inverted hierarchy: modulus and z -component of \mathbf{J} and $\bar{\mathbf{J}}$.

pendulum inertia with r reduces the amplitude of the nutations, and bipolar oscillations are expected to vanish when self-interaction and vacuum effects are of the same size. At this point, at the end of the bipolar regime, self-interaction effects do not completely vanish and the spectral split builds up: a “stepwise swap” between the ν_e and ν_x energy spectra. The neutrino swapping can be explained by the conservation of the pendulum energy and of the lepton number⁷. The lepton number conservation is related to the constancy of $D_z = J_z - \bar{J}_z$, that is a direct consequence of the equation of motion. For a detailed description of the pendulum analogy and of our reference model the reader is referred to our previous work³ and references therein.

3 Simulations

Figures 2 and 3 show the third component of \mathbf{P} and $\bar{\mathbf{P}}$, as a function of the radius, for different energy values, for the single- and multi-angle simulations, respectively. Bipolar oscillations starts at the same r and their periods are equal for both ν and $\bar{\nu}$ at any energy, confirming the appearance of a self-induced collective behavior, in the single- and in the multi-angle case. The behavior of each P_z and \bar{P}_z depends on its energy. For neutrinos, Figure 2, the spectral split starts around the critical energy $E_c \simeq 7$ MeV: the curve relative to $E < E_c$ ends up at the same initial value ($P_{ee} = 1$), while the curves for $E > E_c$ show the P_z inversion ($P_{ee} = 0$). Neutrinos with an energy of ~ 19 MeV do not oscillate much, because this is roughly the energy for which the initial ν_e and ν_x fluxes are equal. For antineutrinos, all curves show almost complete polarization reversal, with the exception of small energies (of few MeV, not shown in Figure 3). Figures 4 and 5 show the evolution of J and J_z for neutrinos and antineutrinos, in the single- and multi-angle cases. The behavior of these vectors can be related to the gyroscopic pendulum motion. At the beginning, in the synchronized regime, all the polarization vectors are aligned so that $J = J_z$ and $\bar{J} = \bar{J}_z$: the pendulum just spins in the upward position without falling. Around ~ 70 km the pendulum falls for the first time and nutations appear. The nutation amplitude gradually decreases and bipolar oscillations eventually vanish for $r \sim 100$ km. At the same time, the spectral split builds up: antineutrinos tend to completely reverse their polarization, while this happens only partially for neutrinos. As said before, also for antineutrinos there is a partial swap of the spectra for $E \sim 4$ MeV. From Figure 5 it appears that bipolar oscillations of \mathbf{J} and $\bar{\mathbf{J}}$ are largely smeared out in the multi-angle case. The bipolar regime starts somewhat later with respect to the single-angle case, since neutrino-neutrino interaction angles can be larger than the (single-angle) average one, leading to stronger self-interaction effects, that force the system

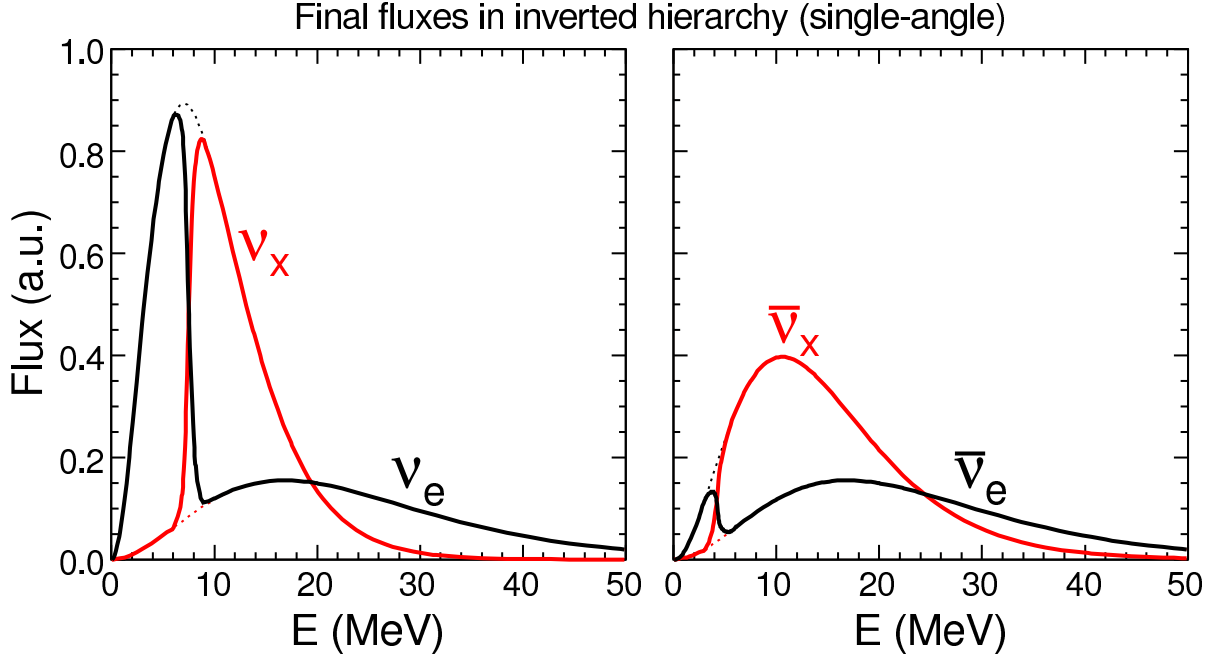


Figure 6: Single-angle simulation in inverted hierarchy: final fluxes (at $r = 200$ km, in arbitrary units) for different neutrino species as a function of energy. Initial fluxes are shown as dotted lines to guide the eye.

in synchronized mode slightly longer. However, just as in the single-angle case, the spectral split builds up, \bar{J}_z gets finally reversed, while the difference $D_z = J_z - \bar{J}_z$ remains constant. Figures 6 and 7 show the final neutrino and antineutrino fluxes, in the single- and multi-angle simulations. The neutrinos clearly show the spectral split effect and the corresponding sudden swap of ν_e and ν_x fluxes above $E_c \simeq 7$ MeV. In the right panel of Figure 6, the final antineutrino spectra are basically completely swapped with respect to the initial ones, except at very low energies, where there appears an “antineutrino” spectral split. This phenomenon can be related to the loss of \bar{J} and of $|\bar{J}_z|$ ³. Also in the multi-angle case of Figure 7, the neutrino spectral swap at $E > E_c \simeq 7$ MeV is rather evident, although less sharp with respect to the single-angle case, while the minor feature associated to the “antineutrino spectral split” is largely smeared out.

4 Conclusions

We have studied supernova neutrino oscillations in a model where the collective flavor transitions (synchronization, bipolar oscillations, and spectral split) are well separated from the MSW resonance. We have performed numerical simulations in both single- and multi-angle cases, using continuous energy spectra with significant ν - $\bar{\nu}$ and ν_e - ν_x asymmetry. The results of the single-angle simulation can be analytically understood to a large extent by means of a mechanical analogy with the spherical pendulum. The main observable effect is the swap of energy spectra, for inverted hierarchy, above a critical energy dictated by lepton number conservation. In the multi-angle simulation, the details of self-interaction effects change (e.g., the starting point of bipolar oscillations and their amplitude), but the spectral swap remains a robust, observable feature. In this sense, averaging over neutrino trajectories does not alter the main effect of the self interactions. The swapping of neutrino and antineutrino spectra could have an impact on r -process nucleosynthesis, on the energy transfer to the shock wave during the supernova explosion and on the propagation of the neutrinos through the shock wave. From the point of view of neutrino parameters, collective flavor oscillations in supernovae could be instrumental in identifying the inverse neutrino mass hierarchy, even for very small θ_{13} .⁸

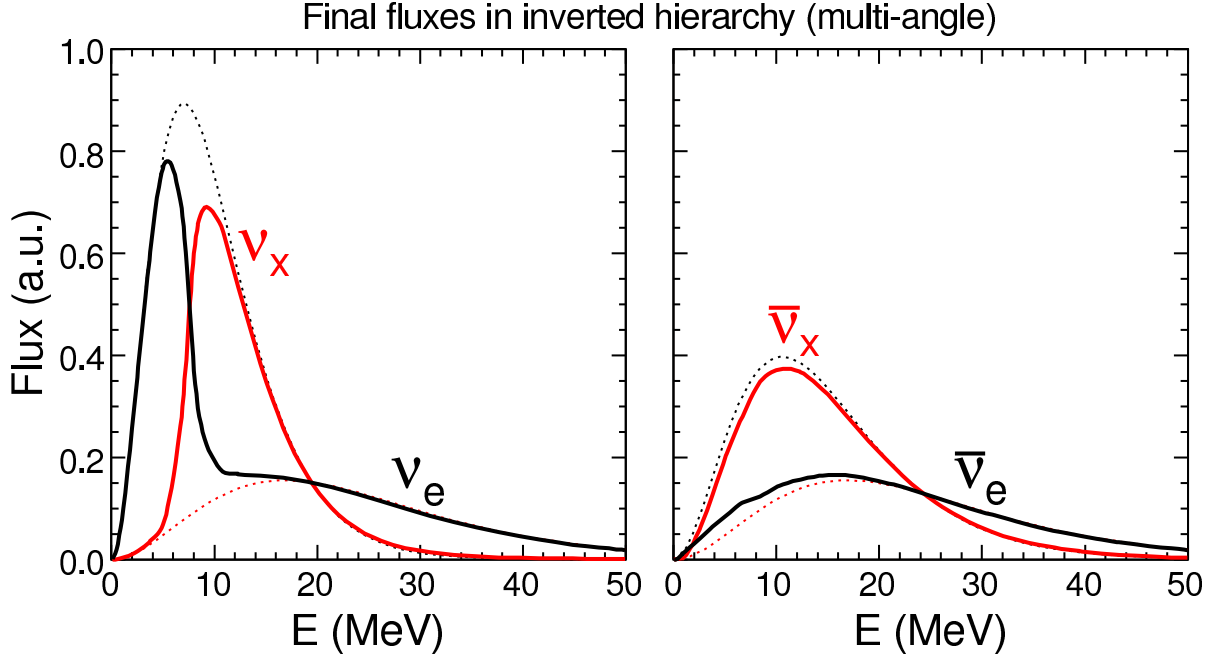


Figure 7: Multi-angle simulation in inverted hierarchy: final fluxes (at $r = 200$ km, in arbitrary units) for different neutrino species as a function of energy. Initial fluxes are shown as dotted lines to guide the eye.

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