# CP-violation from Non-Unitary Leptonic Mixing

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# Motivations

- Neutrino masses and mixing → evidence of Physics Beyond the SM
- Typical explanations involve New Physics at higher energies
- This NP often induces deviations from unitarity of the PMNS at low energy

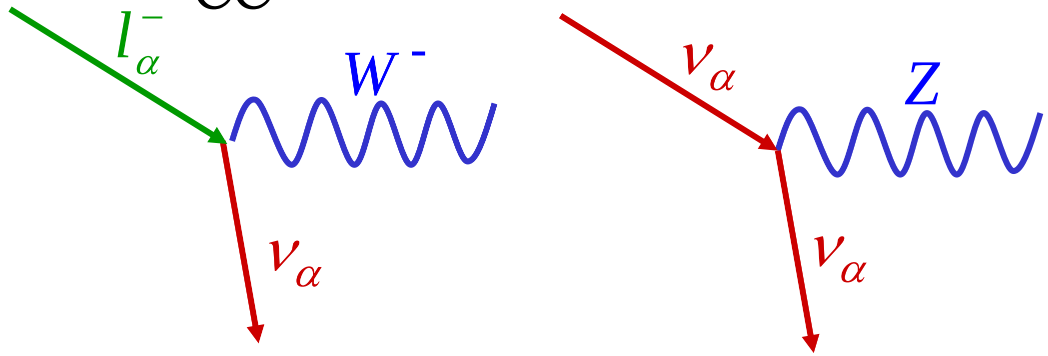
↓

We will analyze the present constraints on the mixing matrix
*without* assuming unitarity

Study of CP violation in the context of non-unitarity

# Usual Analysis

$$\mathrm{L} = \frac{1}{2}\left(i\bar{\nu}_\alpha \partial \nu_\alpha - \bar{\nu}_\alpha M_{\alpha\beta}\nu_\beta + h.c.\right) - \underbrace{\frac{g}{\sqrt{2}}\left(W^+_\mu \bar{l}_\alpha \gamma^\mu P_L \nu_\alpha + h.c.\right)}_{\text{CC}} - \underbrace{\frac{g}{\cos\theta_W}\left(Z_\mu \bar{\nu}_\alpha \gamma^\mu P_L \nu_\alpha + h.c.\right)}_{\text{NC}} + \ldots$$



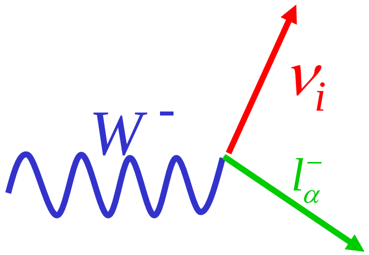

Diagonalizing

$$L = \frac{1}{2}\left(i\bar{\nu}_i \partial \nu_i - \bar{\nu}_i m_i \nu_i\right) - \frac{g}{\sqrt{2}}\left(W^+_\mu \bar{l}_\alpha \gamma^\mu P_L \underbrace{U_{\alpha i}\nu_i}\right) - \frac{g}{\cos\theta_W}\left(Z_\mu \bar{\nu}_i \gamma^\mu P_L \nu_i\right) + h.c. + \ldots$$

$$\nu_\alpha = U_{\alpha i}\nu_i$$

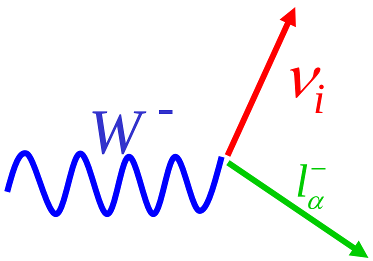
The mixing matrix $U$ is unitary in the usual analysis with 3 light $\nu$

# The general idea




$$\sim\sim g\gamma^{\mu}\ P_L U_{\alpha i}$$

$$U=\begin{pmatrix} c_{13}c_{12} & s_{12}c_{13} & s_{13}e^{i\delta} \\ -s_{12}c_{23}-s_{23}s_{13}c_{12}e^{-i\delta} & c_{12}c_{23}-s_{23}s_{13}s_{12}e^{-i\delta} & s_{23}c_{13} \\ s_{23}s_{12}-c_{23}s_{13}c_{12}e^{-i\delta} & -s_{23}c_{12}-c_{23}s_{13}s_{12}e^{-i\delta} & c_{23}c_{13} \end{pmatrix}\begin{pmatrix} e^{i\alpha} & & \\ & e^{i\beta} & \\ & & 1 \end{pmatrix}$$




$$\sim\sim g\gamma^{\mu}\ P_L N_{\alpha i}$$

$$N=\begin{pmatrix} N_{e1} & N_{e2} & N_{e3} \\ N_{\mu 1} & N_{\mu 2} & N_{\mu 3} \\ N_{\tau 1} & N_{\tau 2} & N_{\tau 3} \end{pmatrix}$$

# Effective Lagrangian allowing non-unitarity

- 3 light $\nu$
- Assume NP at $\Lambda >> \Lambda_{EW}$. In the flavor basis,

$$L = \frac{1}{2}\left(i\bar{\nu}_\alpha \partial K_{\alpha\beta}\nu_\beta - \bar{\nu}^c_\alpha M_{\alpha\beta}\nu_\beta + h.c.\right) - \frac{g}{\sqrt{2}}\left(W^+_\mu \bar{l}_\alpha \gamma^\mu P_L \nu_\alpha + h.c.\right) - \frac{g}{\cos\theta_W}\left(Z_\mu \bar{\nu}_\alpha \gamma^\mu P_L \nu_\alpha + h.c.\right) + \dots$$

$$\nu_\alpha = N_{\alpha i}\nu_i$$

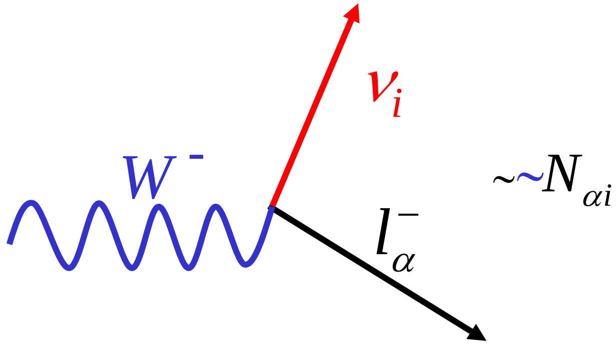
$$L = \frac{1}{2}\left(i\bar{\nu}_i \partial \nu_i - \bar{\nu}_i m_{ii}\nu_i\right) - \frac{g}{\sqrt{2}}\left(W^+_\mu \bar{l}_\alpha \gamma^\mu P_L N_{\alpha i}\nu_i\right) - \frac{g}{\cos\theta_W}\left(Z_\mu \bar{\nu}_i \gamma^\mu P_L (N^\dagger N)_{ij}\nu_j\right) + h.c. + \dots$$

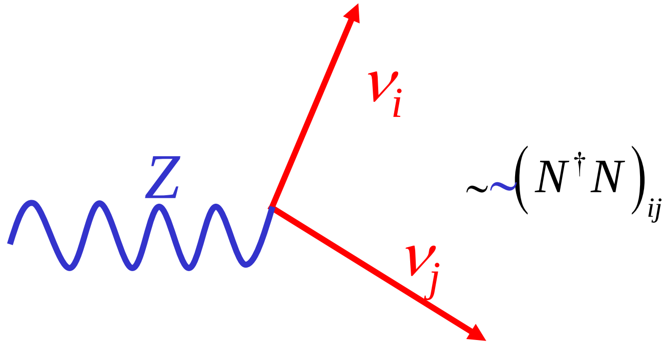
unchanged

$N$ non-unitary

$$\langle \nu_i | \nu_j \rangle = \delta_{ij}$$

# The effects of non-unitarity appear in the interactions




We can bound unitarity with weak decays

$$|\nu_\alpha\rangle = \frac{1}{\sqrt{\sum_i |N_{\alpha i}|^2}} \sum_i N^*_{\alpha i} |\nu_i\rangle \quad \Rightarrow \quad \langle \nu_\beta | \nu_\alpha \rangle \neq \delta_{\alpha\beta}$$

$$P_{\alpha\beta} = \left|\langle \nu_\beta | \nu_\alpha(L)\rangle\right|^2 = \frac{\left|\sum_i N_{\beta i} N^*_{\alpha i} e^{-i\frac{m_i^2}{2E}L}\right|^2}{\sum_i |N_{\alpha i}|^2 \sum_i |N_{\beta i}|^2}$$

$$\left|\langle \nu_\beta | \nu_\alpha(0)\rangle\right|^2 = \frac{\left|\sum_i N_{\beta i} N^*_{\alpha i}\right|^2}{\sum_i |N_{\alpha i}|^2 \sum_i |N_{\beta i}|^2}$$

Zero Distance Effect

# Constraints on Unitarity from weak decays

$W \to l_\alpha \nu$ universality tests

$Z \to \text{invisible}$ rare lepton decays

$$|NN^\dagger| \approx \begin{pmatrix} 0.994 \pm 0.005 & < 7.1 \cdot 10^{-5} & < 1.6 \cdot 10^{-2} \\ < 7.1 \cdot 10^{-5} & 0.995 \pm 0.005 & < 1.0 \cdot 10^{-2} \\ < 1.6 \cdot 10^{-2} & < 1.0 \cdot 10^{-2} & 0.995 \pm 0.005 \end{pmatrix}$$

90% C.L.

→ *N* is unitary at % level

# In the future…

## TESTS OF UNITARITY

### RARE LEPTON DECAYS

- $\mu \rightarrow e\gamma$ $\quad \left(NN^{\dagger}\right)_{e\mu} < 10^{-6}$ (MEG)
- $\tau \rightarrow e\gamma$ $\quad \left(NN^{\dagger}\right)_{e\tau} < 1.6 \cdot 10^{-2}$ (BABAR)
- $\tau \rightarrow \mu\gamma$ $\quad \left(NN^{\dagger}\right)_{\mu\tau} < 1.0 \cdot 10^{-2}$ (Belle)

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ZERO-DISTANCE EFFECT @ NF

- $\nu_e \rightarrow \nu_\mu$ $\quad (NN^\dagger)_{e\mu} < 2.3 \cdot 10^{-4}$
- $\nu_e \rightarrow \nu_\tau$ $\quad (NN^\dagger)_{e\tau} < 2.9 \cdot 10^{-3}$
- $\nu_\mu \rightarrow \nu_\tau$ $\quad (NN^\dagger)_{\mu\tau} < 2.6 \cdot 10^{-3}$

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## PHASES

- phases → *appearance* experiments: NUFACTs, *b*-beams

# Can we measure the phases of N ?

E. Fdez-Martinez, M.B. Gavela, J. López-Pavón and O.Yasuda hep-ph/0703098

If we parametrize $N = (1+\eta)\cdot U$ where $U \approx U_{PMNS}$ and $\eta = \eta^\dagger$

$$\eta = \begin{pmatrix} \eta_{ee} & \eta_{e\mu} & \eta_{e\tau} \\ \eta^*_{e\mu} & \eta_{\mu\mu} & \eta_{\mu\tau} \\ \eta^*_{e\tau} & \eta^*_{\mu\tau} & \eta_{\tau\tau} \end{pmatrix}$$

If $\frac{\Delta m^2 L}{2E} \equiv \Delta << 1$ ➡ $\langle \nu_\alpha | \nu_\beta(L) \rangle = A^{SM}_{\alpha\beta} + 2\eta^*_{\alpha\beta} + O(\eta^2)$

$P_{\alpha\beta}$ only depends on $\eta_{\alpha\beta}$

# Can we measure the phases of N ?

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For instance, for two families:

$$P_{\alpha\beta} = \sin^2(2\theta)\sin^2\left(\frac{\Delta}{2}\right) - 4\,|\eta_{\alpha\beta}|\sin(\delta_{\alpha\beta})\sin(2\theta)\sin\Delta + 4\left|\eta_{\alpha\beta}\right|^2$$

SM

CP violating interference

Zero distance effect

$$\eta_{\alpha\beta} \equiv |\eta_{\alpha\beta}|\, e^{-\delta_{\alpha\beta}}$$

$$\Delta_{ij} = \Delta m_{ij}^2 L / 2E \ll 1$$

New CP-violation signals

# Which is the best channel?

$\nu_e \leftrightarrow \nu_\mu$ ?

$|\eta_{e\mu}| < 3.5 \cdot 10^{-5}$

$\nu_e \leftrightarrow \nu_\tau$ ?

$|\eta_{e\tau}| < 8.0 \cdot 10^{-3}$

$\nu_\mu \leftrightarrow \nu_\tau$ ?

$|\eta_{\mu\tau}| < 5.1 \cdot 10^{-3}$

$$P_{\mu\tau} \approx \sin^2 2\theta_{23} \sin^2\left(\frac{\Delta_{31}}{2}\right) - 4|\eta_{\mu\tau}| \sin\delta_{\mu\tau} \sin 2\theta_{23} \sin(\Delta_{31}) + 4|\eta_{\mu\tau}|^2$$

This is the best channel to measure new CP phases:

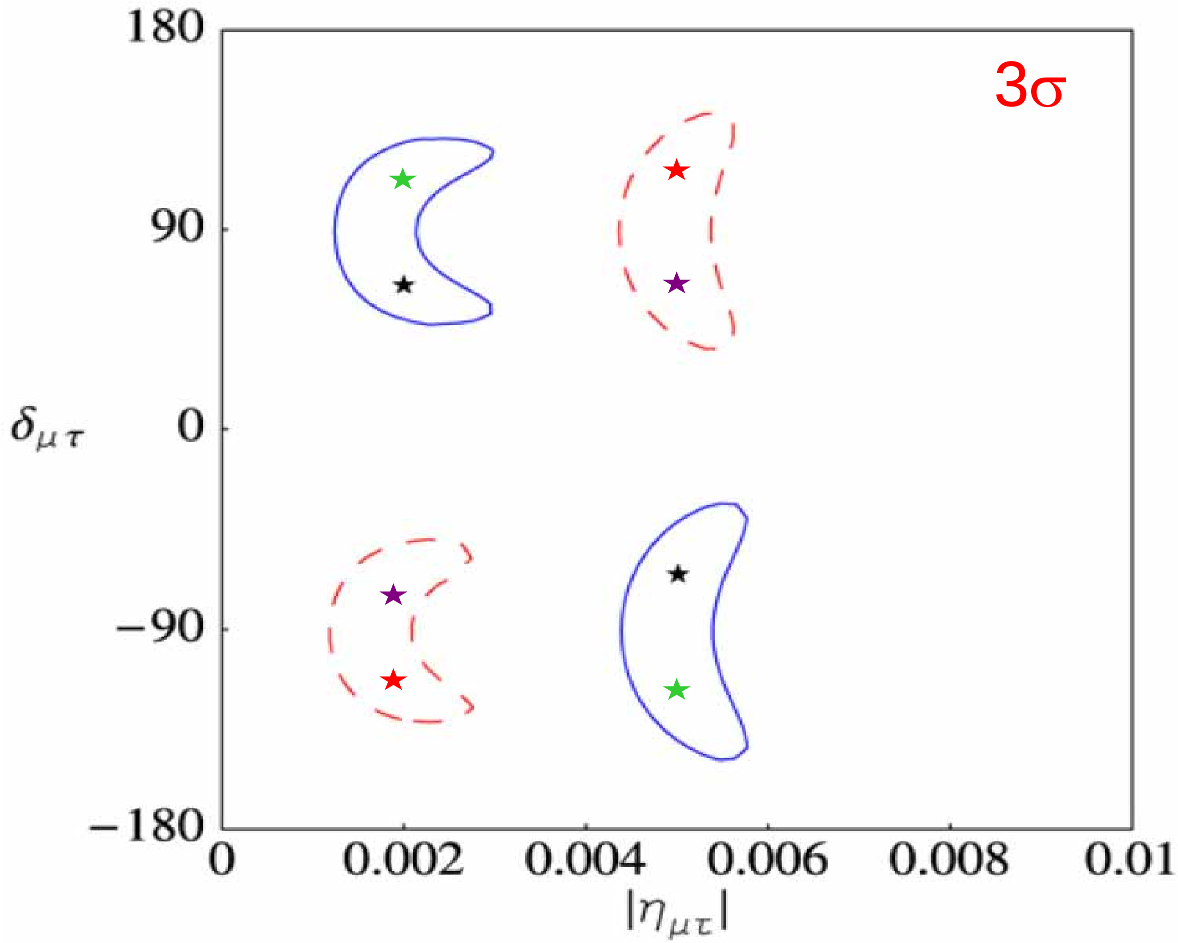
$$P_{\mu\tau} - P_{\overline{\mu}\overline{\tau}} \approx 8|\eta_{\mu\tau}| \sin\delta_{\mu\tau} \sin(2\theta_{23}) \sin(\Delta_{13})$$

## $\nu_\mu \leftrightarrow \nu_\tau$ The CP phase $\delta_{\mu\tau}$ can be probed

$$N_{\mu\tau} - N_{\overline{\mu}\overline{\tau}} \propto 8\,|\eta_{\mu\tau}|\sin\delta_{\mu\tau}\,\sin(2\theta_{23})\sin\left(\Delta_{13}\right)$$

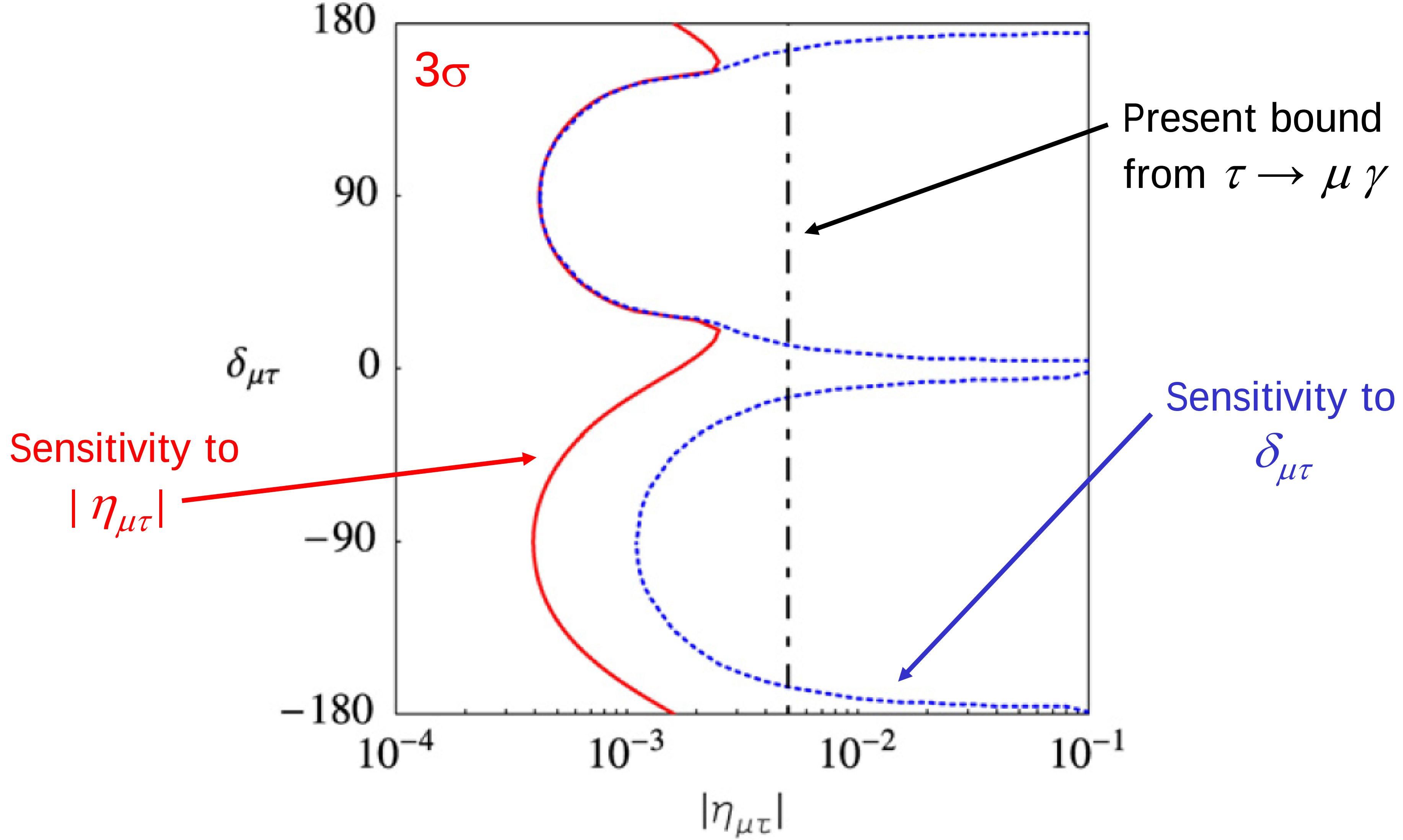
1. Confusion between
   $sign(\delta_{\mu\tau}) \leftrightarrow sign\left(\Delta_{13}\right)$

2. Degeneracy
   $180^o - \delta_{\mu\tau} \leftrightarrow \delta_{\mu\tau}$




@ a Neutrino Factory
with L = 130 Km

# $\nu_\mu \leftrightarrow \nu_\tau$ The CP phase $\delta_{\mu\tau}$ can be probed




For non-trivial $\delta_{\mu\tau}$, one order of magnitude improvement for $|\eta_{\mu\tau}|$

# Conclusions

•Analyze neutrino data without assuming unitarity. We started the first analyses and conclude that, at present:

-EW decays confirms unitarity at % level

-Present bounds on unitarity are strong enough to match the unitarity analysis

• $\nu_\mu - \nu_\tau$ CP-asymmetry is a clean probe of the new phases.

• Non-unitary effects in typical models are too small to be detected at present experiments. They could be sizable in extensions/others models with M~ TeV,...).

-> keep tracking them in the future.
They are excellent signals of new physics.

# Back-up slides

# Constraints on unitarity

$$|\eta| \approx \begin{pmatrix} |\eta_{ee}| < 5.5\cdot 10^{-3} & |\eta_{e\mu}| < 3.5\cdot 10^{-5} & |\eta_{e\tau}| < 8.0\cdot 10^{-3} \\ |\eta_{e\mu}| < 3.5\cdot 10^{-5} & |\eta_{\mu\mu}| < 5.0\cdot 10^{-3} & |\eta_{\mu\tau}| < 5.1\cdot 10^{-3} \\ |\eta_{e\tau}| < 8.0\cdot 10^{-3} & |\eta_{\mu\tau}| < 5.1\cdot 10^{-3} & |\eta_{\tau\tau}| < 5.0\cdot 10^{-3} \end{pmatrix}$$

# Leptonic Mixing Matrix Elements From Oscillations

$$N = \begin{pmatrix} N_{e1} & N_{e2} & N_{e3} \\ N_{\mu 1} & N_{\mu 2} & N_{\mu 3} \\ N_{\tau 1} & N_{\tau 2} & N_{\tau 3} \end{pmatrix}$$

# *N* elements from oscillations: *e*-row

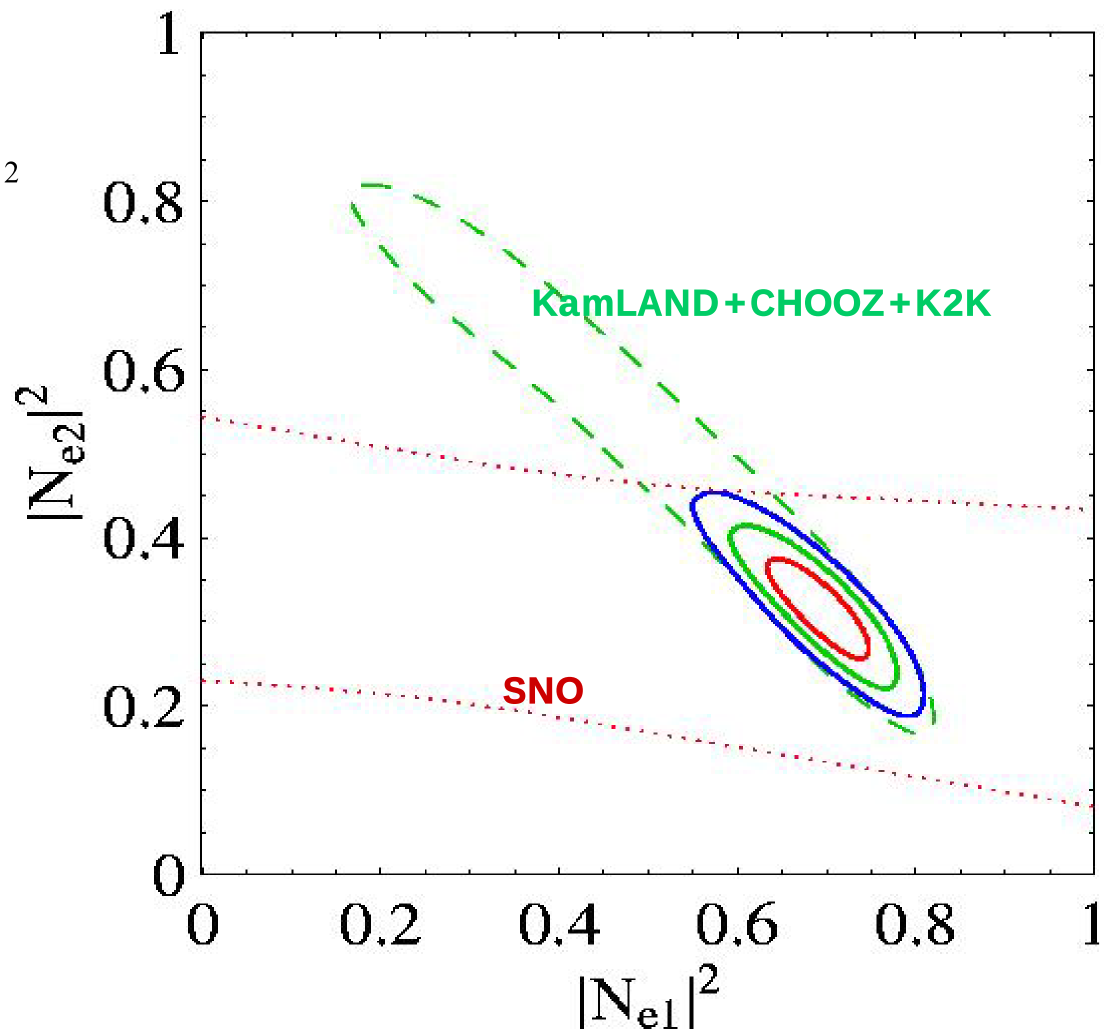
CHOOZ: $P(\bar{\nu}_e \to \bar{\nu}_e) \cong \left(|N_{e1}|^2 + |N_{e2}|^2\right)^2 + |N_{e3}|^4 + 2\left(|N_{e1}|^2 + |N_{e2}|^2\right)|N_{e3}|^2 \cos(\Delta_{13})$

KamLAND: $P(\bar{\nu}_e \to \bar{\nu}_e) \cong |N_{e1}|^4 + |N_{e2}|^4 + |N_{e3}|^4 + 2|N_{e1}|^2 |N_{e2}|^2 \cos(\Delta_{12})$

$\Delta_{13}$ from K2K

SNO:

$P(\nu_e \to \nu_e) \cong 0.1|N_{e1}|^2 + 0.9|N_{e2}|^2$




$\Delta_{ij} = \Delta m_{ij}^2 L / 2E$

$\rightarrow$ all $|N_{ei}|^2$ determined

# $N$ elements from oscillations: $\mu$-row

Atmospheric + K2K: $\Delta_{12} \approx 0$ $\Delta_{ij} = \Delta m_{ij}^2 L / 2E$

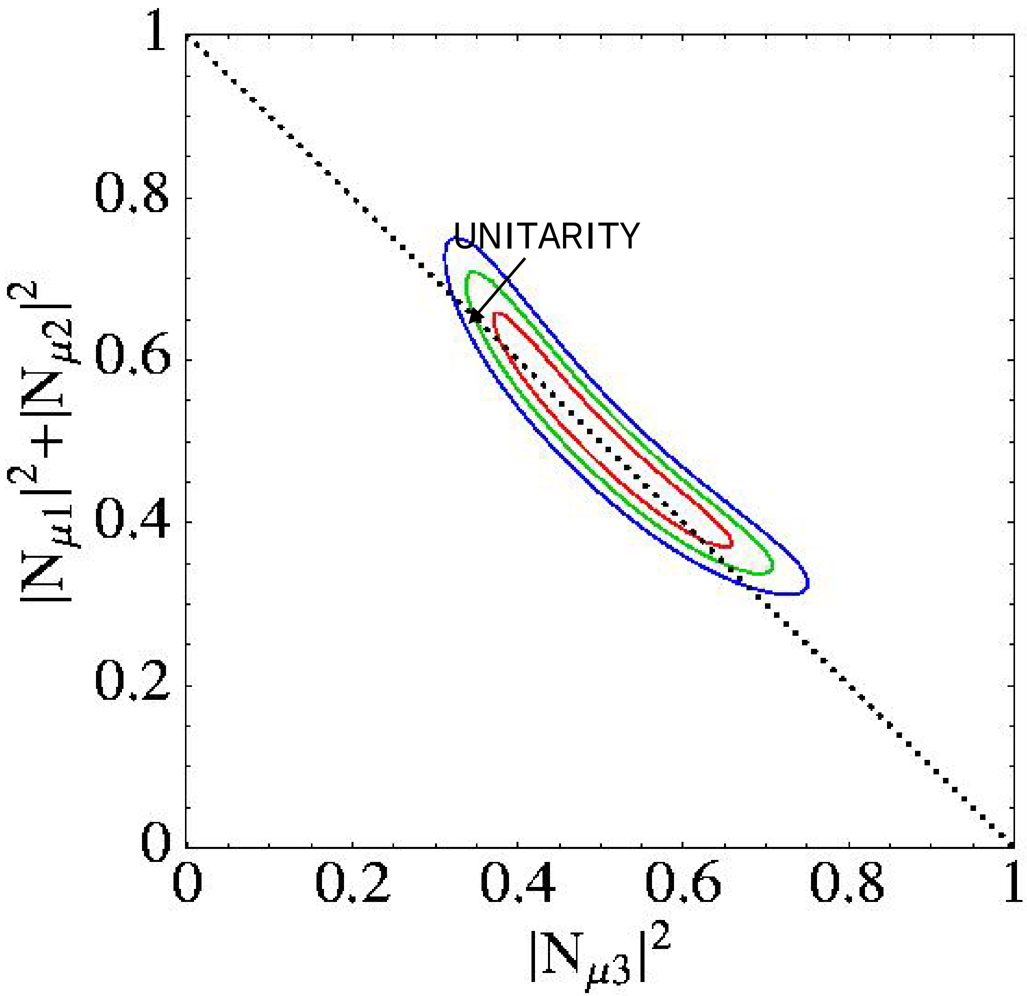
$$\hat{P}(\nu_\mu \to \nu_\mu) \cong \left(|N_{\mu 1}|^2 + |N_{\mu 2}|^2\right)^2 + |N_{\mu 3}|^4 + 2\left(|N_{\mu 1}|^2 + |N_{\mu 2}|^2\right)|N_{\mu 3}|^2 \cos(\Delta_{23})$$

1. Degeneracy

   $|N_{\mu 1}|^2 + |N_{\mu 2}|^2 \leftrightarrow |N_{\mu 3}|^2$

2. $|N_{\mu 1}|^2$ , $|N_{\mu 2}|^2$

   cannot be disentangled



# $N$ elements from oscillations only

without unitarity
OSCILLATIONS

$$|N| = \begin{pmatrix} 0.75-0.89 & 0.45-0.66 & <0.34 \\ \left[\left(|N_{\mu1}|^2+|N_{\mu2}|^2\right)^{1/2} = 0.57-0.86\right] & & 0.57-0.86 \\ ? & ? & ? \end{pmatrix}$$

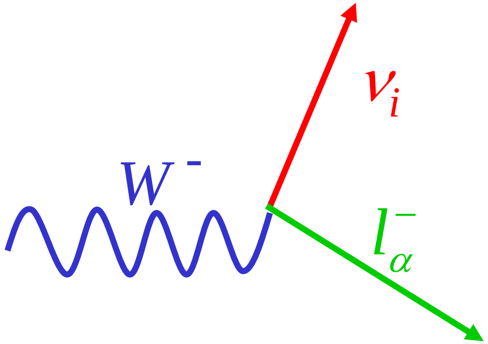
3σ

with unitarity
OSCILLATIONS

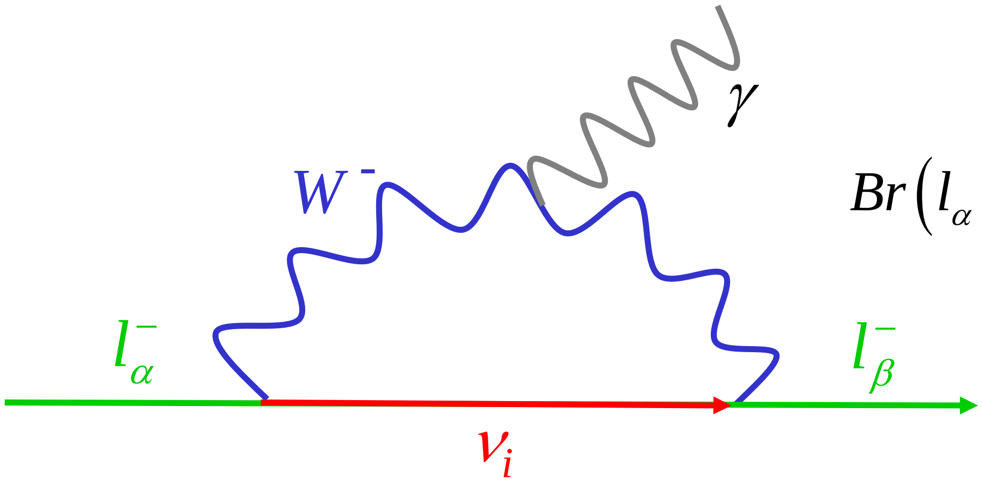
$$|U| = \begin{pmatrix} 0.79-0.86 & 0.50-0.61 & <0.20 \\ 0.25-0.53 & 0.47-0.73 & 0.56-0.79 \\ 0.21-0.51 & 0.42-0.69 & 0.61-0.83 \end{pmatrix}$$

González-García, Maltoni 07

# Decays




$$\Gamma\left(W \to l_\alpha \nu\right) \propto \frac{\left(NN^\dagger\right)_{\alpha\alpha}}{\sqrt{\left(NN^\dagger\right)_{ee}\left(NN^\dagger\right)_{\mu\mu}}}$$




$$Br\left(l_\alpha \to l_\beta \gamma\right) \propto \frac{\left|\left(NN^\dagger\right)_{\beta\alpha}\right|^2}{\left(NN^\dagger\right)_{\alpha\alpha}\left(NN^\dagger\right)_{\beta\beta}} + \mathrm{O}\left(\frac{m_\nu^{\ 2}}{M_W^2}\right)$$

- $Z \to \text{invisible} \propto \dfrac{\sum_{i,j}\left(N^\dagger N\right)_{ij}}{\sqrt{\left(NN^\dagger\right)_{ee}\left(NN^\dagger\right)_{\mu\mu}}}$
- universality tests $\propto \dfrac{\left(NN^\dagger\right)_{\alpha\alpha}}{\left(NN^\dagger\right)_{\beta\beta}}$

# Can we measure the phases of N ?

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If we parametrize $N = (1+\eta)\cdot U$ where $U \approx U_{PMNS}$ and $\eta = \eta^{\dagger}$

$$\eta = \begin{pmatrix} \eta_{ee} & \eta_{e\mu} & \eta_{e\tau} \\ \eta^{*}_{e\mu} & \eta_{\mu\mu} & \eta_{\mu\tau} \\ \eta^{*}_{e\tau} & \eta^{*}_{\mu\tau} & \eta_{\tau\tau} \end{pmatrix}$$

Normalization factors

$$\langle \nu_\beta | \nu_\alpha(L) \rangle = A^{\rm SM}_{\alpha\beta}(L)(1 - \eta_{\alpha\alpha} - \eta_{\beta\beta}) + \sum_\gamma \left(\eta^*_{\alpha\gamma} A^{\rm SM}_{\gamma\beta}(L) + \eta_{\beta\gamma} A^{\rm SM}_{\alpha\gamma}(L)\right) + O(\eta^2)$$

Standard amplitude

If $\frac{\Delta m^2 L}{2E} \equiv \Delta << 1$ ➡ $\begin{cases} A^{\rm SM}_{\alpha\alpha} \approx 1 \\ A^{\rm SM}_{\alpha\beta} \sim |\eta_{\alpha\beta}| << 1 \end{cases}$

$$\langle \nu_\alpha | \nu_\beta(L) \rangle = A^{\rm SM}_{\alpha\beta} + 2\eta^*_{\alpha\beta} + O(\eta^2)$$

$P_{\alpha\beta}$ only depends on $\eta_{\alpha\beta}$

# Which is the best channel?

$\hookrightarrow$ ~~$\nu_e \leftrightarrow \nu_\mu$ ?~~ $\qquad \nu_e \leftrightarrow \nu_\tau$ ? $\qquad \nu_\mu \leftrightarrow \nu_\tau$ ?

Constraints on unitarity:

$$|\eta| \approx \begin{pmatrix} |\eta_{ee}| < 5.5\cdot 10^{-3} & |\eta_{e\mu}| < 3.5\cdot 10^{-5} & |\eta_{e\tau}| < 8.0\cdot 10^{-3} \\ |\eta_{e\mu}| < 3.5\cdot 10^{-5} & |\eta_{\mu\mu}| < 5.0\cdot 10^{-3} & |\eta_{\mu\tau}| < 5.1\cdot 10^{-3} \\ |\eta_{e\tau}| < 8.0\cdot 10^{-3} & |\eta_{\mu\tau}| < 5.1\cdot 10^{-3} & |\eta_{\tau\tau}| < 5.0\cdot 10^{-3} \end{pmatrix}$$

90% C.L.

$\hookrightarrow$ $|\eta_{e\mu}| << |\eta_{e\tau}|, |\eta_{\mu\tau}| \;\Rightarrow\;$ ~~$\nu_e \leftrightarrow \nu_\mu$~~

# Which is the best channel?

$\nu_e \leftrightarrow \nu_\mu$ ?    $\nu_e \leftrightarrow \nu_\tau$ ?    $\nu_\mu \leftrightarrow \nu_\tau$ ?

$$P_{e\tau} \approx P_{e\tau}^{SM} + $$
$$-2|\eta_{e\tau}| c_{23} \sin(\delta + \delta_{e\tau}) \sin 2\theta_{13} (\Delta_{31}) + 2|\eta_{e\tau}| s_{23} c_{13} \sin 2\theta_{12} \sin \delta_{e\tau} (\Delta_{21})$$
$$+4|\eta_{e\tau}|^2$$

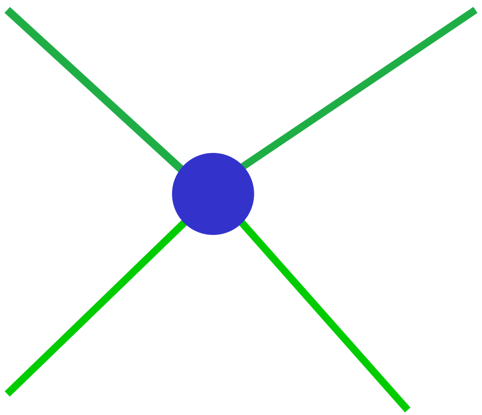
$$\Delta_{ij} = \Delta m_{ij}^2 L / 2E << 1$$

$\nu_e \leftrightarrow \nu_\tau$ is suppressed by the small parameters $\Delta_{21}$ and $\sin 2\theta_{13} \Delta_{31}$

# Our analysis will also apply to "non-standard" or "exotic" neutrino interactions.

(Grossman, Gonzalez-Garcia et al., Huber et al., Kitazawa et al., Davidson et al. Blennow et al...)

They add 4-fermion exotic operators which affect production
or detection
or propagation in matter



$$\bar{\Psi}\,\Psi\,\bar{\Psi}\,\Psi$$

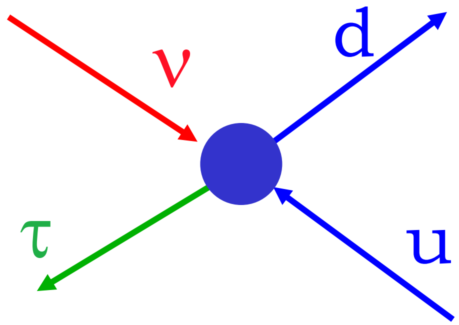
# “NSI”neutrino interactions vs NonUnitarity

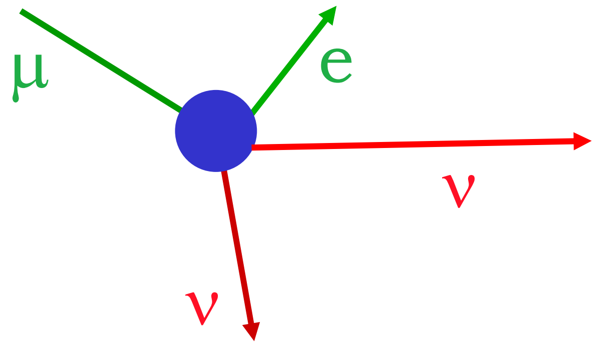
- SM

$$-\mathcal{L}^{\mathrm{CC}} = \frac{4G_F}{\sqrt{2}}\left[\underbrace{\sum_{\alpha\beta}(\bar{\nu}_\alpha^{\mathrm{SM}}\gamma_\mu P_L l_\alpha)(\bar{d}_\beta\gamma_\mu P_L u_\beta)} + \underbrace{(\bar{\nu}_\alpha^{\mathrm{SM}}\gamma_\mu P_L l_\alpha)(\bar{l}_\beta\gamma_\mu P_L \nu_\beta^{\mathrm{SM}})} + \mathrm{h.c.}\right]$$

Detection @ NF: $\bar{\nu}_\alpha d \to l_\alpha^- u$

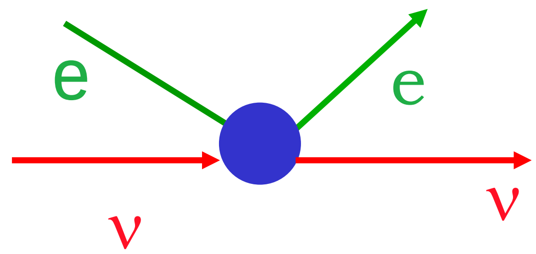
Production @ NF: $\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$




Production @ conventional-beams
super-beams

Matter effects: α=β=e



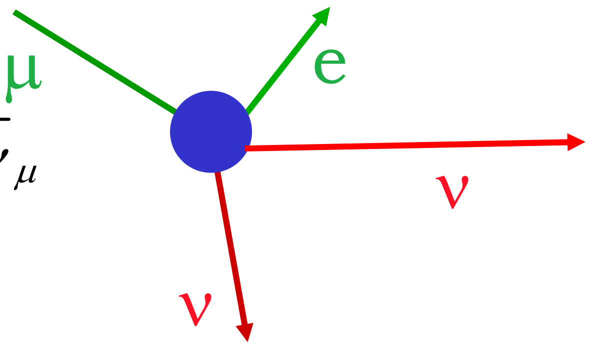
# “NSI”neutrino interactions vs NonUnitarity

- Exotic Interactions

Add new effective 4-fermion operators which affects to:

Production

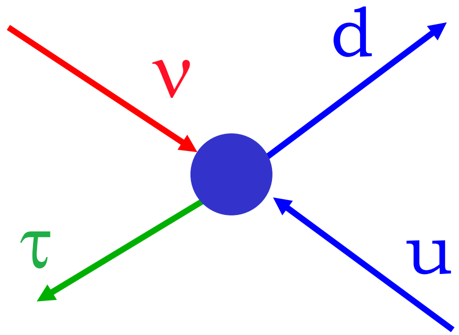
$$\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$$




$$\frac{-4}{\sqrt{2}} \sum_{\alpha \neq e} G^P_{e\alpha} (\bar{\mu} \gamma_\mu P_L \nu^{SM}_\mu)(\bar{\nu}^{SM}_\alpha \gamma_\mu P_L e) + \text{h.c.}$$

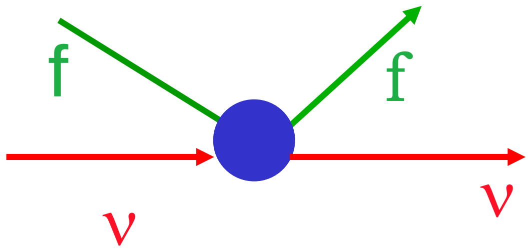
Detection

$$\bar{\nu}_\alpha d \to l^-_\alpha u$$




$$\frac{-4}{\sqrt{2}} \sum_{\beta \neq \mu} G^D_{\mu\beta} (\bar{\nu}^{SM}_\beta \gamma_\mu P_L \mu)(\bar{d} \gamma_\mu P_L u) + \text{h.c.}$$

Matter effects




$$\sim (\nu^{SM}_\alpha \gamma_\mu P_L \nu^{SM}_\beta)(\bar{f} \gamma_\mu f)$$

# “NSI”neutrino interactions vs NonUnitarity

- SM + Exotic Interactions

$$-\mathcal{L}^{\rm int} = \frac{4G_F}{\sqrt{2}} \sum_\alpha \Bigg[ \left( \delta_{\alpha\mu} + \frac{G^D_{\mu\alpha}}{G_F} \right) (\bar{\nu}^{\rm SM}_\alpha \gamma_\mu P_L \mu)(\bar{d}\gamma_\mu P_L u) + {\rm h.c.}$$

$$+ \left( \delta_{\alpha e} + \frac{G^P_{e\alpha}}{G_F} \right) (\bar{\nu}^{\rm SM}_\alpha \gamma_\mu P_L e)(\bar{\mu}\gamma_\mu P_L \nu^{\rm SM}_\mu) + {\rm h.c.} \Bigg]$$

$$\nu^{\rm SM}_\alpha \equiv U_{\alpha i} \nu_i$$

$$|\nu^p_e\rangle = (1+\epsilon^{*p})_{e\beta} U^*_{\beta i} |\nu_i\rangle,$$

$$|\nu^d_\mu\rangle = (1+\epsilon^{*d})_{\mu\beta} U^*_{\beta i} |\nu_i\rangle$$

Exotic Interactions

$$|\nu_\alpha\rangle = (1+\eta^*)_{\alpha\beta} U^*_{\beta i} |\nu_i\rangle$$

Non-Unitarity

# "NSI"neutrino interactions vs NonUnitarity

- Exotic Interactions

$$\left|\nu_e^p\right\rangle = \left(1 + \epsilon^{*p}\right)_{e\beta} U^*_{\beta i} \left|\nu_i\right\rangle,$$

$$\left|\nu_\mu^d\right\rangle = \left(1 + \epsilon^{*d}\right)_{\mu\beta} U^*_{\beta i} \left|\nu_i\right\rangle$$

No relationship between $\varepsilon^p_{\alpha\beta}$ and $\varepsilon^d_{\beta\alpha}$

- Non-Unitarity

$$|\nu_\alpha\rangle = (1 + \eta^*)_{\alpha\beta} U^*_{\beta i} |\nu_i\rangle$$

$$\eta_{\alpha\beta} = \eta^*_{\beta\alpha}$$

If $\varepsilon^p_{\alpha\beta} = \varepsilon^{d*}_{\beta\alpha}$ ➡ $P^{EXOTIC}_{\mu\tau} = P^{NON-UNITARITY}_{\mu\tau}$

Our constraints apply to Exotic Interactions !

If not ➡ Idem , barring extreme fine tuned cancelations

# Oscillation Probability in Matter

$$-\mathcal{L}_{eff}^{int} = \sqrt{2} G_F n_e \overline{\nu_e} \gamma^0 P_L \nu_e - \frac{1}{\sqrt{2}} G_F n_n \sum_\alpha \overline{\nu_\alpha} \gamma^0 P_L \nu_\alpha$$

$\sqrt{2} G_F n_e \rightarrow V_{CC}$ $\qquad$ $\frac{1}{\sqrt{2}} G_F n_n \rightarrow V_{NC}$

Effective Evolution

$$i \frac{d}{dt} \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \left[ \tilde{\tilde{N}}^* \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} \left(\tilde{\tilde{N}}^*\right)^{-1} + \right.$$

$$\left. + \begin{pmatrix} (V_{CC} - V_{NC})(NN^\dagger)_{ee} & -V_{NC}\sqrt{\frac{(NN^\dagger)_{\mu\mu}}{(NN^\dagger)_{ee}}}(NN^\dagger)_{\mu e} \\ (V_{cc} - V_{NC})\sqrt{\frac{(NN^\dagger)_{ee}}{(NN^\dagger)_{\mu\mu}}}(NN^\dagger)_{e\mu} & -V_{NC}(NN^\dagger)_{\mu\mu} \end{pmatrix} \right] \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix}$$

1. non-diagonal elements

2. NC effects do not disappear

# Matter effects in $\nu_\mu \leftrightarrow \nu_\tau$

Keeping only terms to second order in $\Delta_{21}$, $\Delta_{31}$, $AL$ and $\sin^2 2\theta_{13}$ and first in $|\eta_{\alpha\beta}|$ (setting $\eta_{e\mu} = 0$):

$$\begin{aligned} P_{\mu\tau} = & \sin^2 2\theta_{23}(\Delta_{31}L/2)^2 - 2|\eta_{\mu\tau}|\sin\delta_{\mu\tau}\sin 2\theta_{23}(\Delta_{31}L) + 4|\eta_{\mu\tau}|^2 \\ & - (1/2)c_{12}^2\sin^2 2\theta_{23}(\Delta_{31}L)(\Delta_{21}L) - |\eta_{\mu\tau}|\sin 2\theta_{23}\cos\delta_{\mu\tau}(AL)(\Delta_{31}L) \\ & + (1/4)c_{12}^4\sin^2 2\theta_{23}(\Delta_{21}L)^2 + 2|\eta_{\mu\tau}|c_{12}^2\sin\delta_{\mu\tau}\sin 2\theta_{23}(\Delta_{21}L) \\ & - (1/4)s_{13}\sin 4\theta_{23}\sin 2\theta_{12}\cos\delta(\Delta_{31}L)(\Delta_{21}L). \end{aligned}$$

$$\sin^2 2\theta_{13} \le 10^{-1} \quad \Delta_{31} \sim AL \sim 10^{-2} \quad \Delta_{21} \sim 10^{-3.5} \quad |\eta_{\alpha\beta}| \lesssim 10^{-2}$$

# Aproximate expression for $P_{e\tau}$

$$P_{e\tau} \approx c_{23}^2 \sin^2 2\theta_{13} \left(\frac{\Delta_{31}}{2}\right)^2 + s_{23}^2 \sin^2 2\theta_{12} \left(\frac{\Delta_{21}}{2}\right)^2 -$$

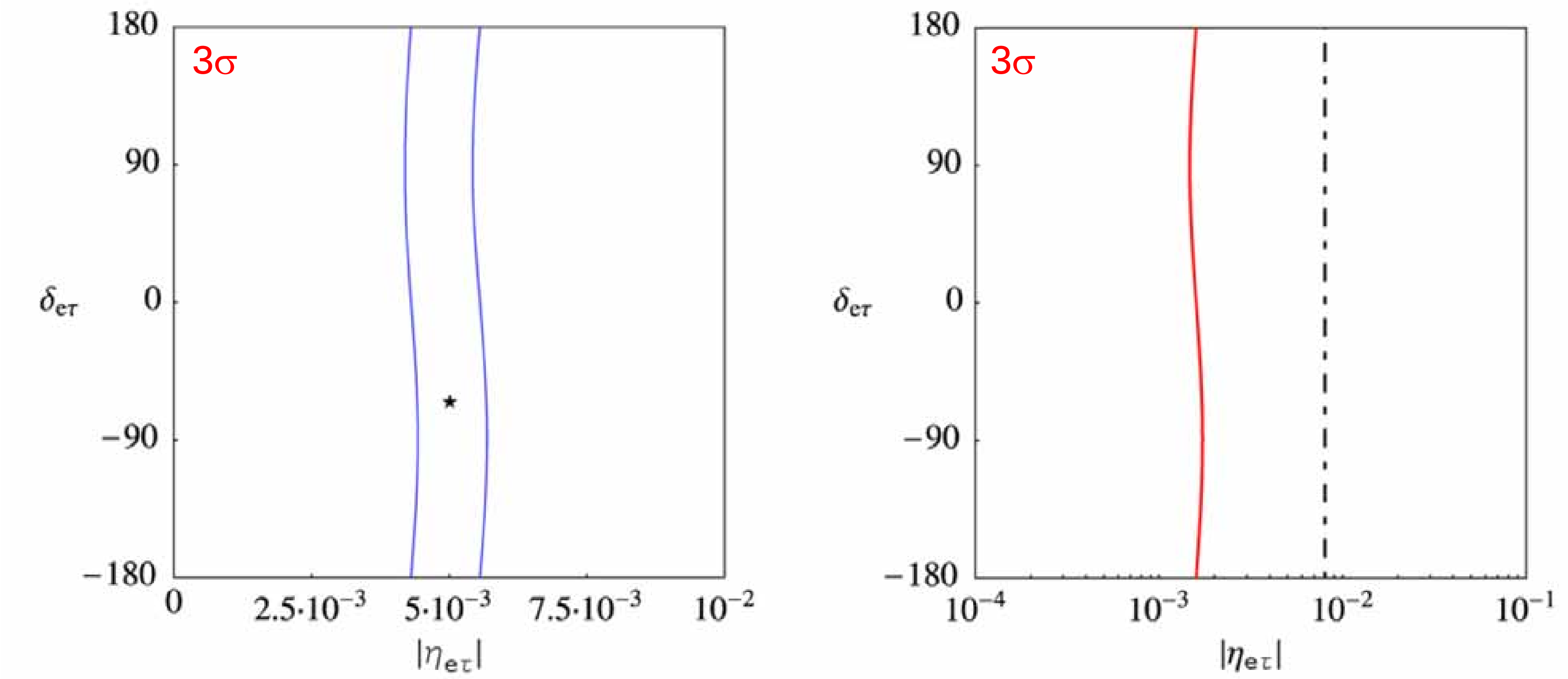
$$c_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\delta \left(\frac{\Delta_{21}}{2}\right)\left(\frac{\Delta_{31}}{2}\right)$$

$$-2\,|\eta_{e\tau}|\,c_{23} \sin 2\theta_{13} \sin\left(\delta + \delta_{e\tau}\right)\left(\Delta_{31}\right) + 2\,|\eta_{e\tau}|\,s_{23} c_{13} \sin 2\theta_{12} \sin\delta_{e\tau}\left(\Delta_{21}\right)$$

$$+4\,|\eta_{e\tau}|^2$$

$$P_{e\mu} = P_{e\tau}\left(s_{23} \to -c_{23}; c_{23} \to s_{23}; \eta_{e\tau} \to \eta_{e\mu}\right)$$

# Analysis of $\nu_e \leftrightarrow \nu_\tau$




The zero distance effect dominates over CP violating interference term

- No information on $\delta_{e\tau}$

- Sensitivities to $|\eta_{e\tau}|$ around $10^{-3}$

# Some details about the experiment $\nu_\mu \leftrightarrow \nu_\tau$ @ NF

We study a NF beam resulting from the decay of 50 GeV muons:

- Assuming $2 \times 10^{20}$ useful decays/year

- 5 years running with each polarity

- 5 Kt Opera-like detector

- L=130 km

- $\nu_\mu \leftrightarrow \nu_\tau$ sensitivities and backgrounds = $5 \times (\nu_e \leftrightarrow \nu_\tau$ sensitivities and backgrounds)

**hep-ph/0305185**

# $(NN^\dagger)$ and $(N^\dagger N)$ from decays

$$|NN^\dagger| \approx \begin{pmatrix} 1.002 \pm 0.005 & < 7.2\cdot 10^{-5} & < 1.6\cdot 10^{-2} \\ < 7.2\cdot 10^{-5} & 1.003\pm 0.005 & < 1.3\cdot 10^{-2} \\ < 1.6\cdot 10^{-2} & < 1.3\cdot 10^{-2} & 1.003 \pm 0.005 \end{pmatrix}$$

90% C.L.

$$N = HV \qquad NN^\dagger = H^2 = 1 + \varepsilon \quad \text{with } \varepsilon = \varepsilon^\dagger$$

$$N^\dagger N = 1 + V^\dagger \varepsilon V = 1 + \varepsilon' \qquad |\varepsilon'_{ij}| \le \sqrt{\sum_{\alpha\beta} |\varepsilon_{\alpha\beta}|^2} \approx 0.03$$

$$|N^\dagger N| \approx \begin{pmatrix} 1.00 \pm 0.032 & < 0.032 & < 0.032 \\ < 0.032 & 1.00\pm 0.032 & < 0.032 \\ < 0.032 & < 0.032 & 1.00 \pm 0.032 \end{pmatrix}$$

$\rightarrow$ $N$ is unitary at % level

# Low-energy theory

After EWSB:

$$L = \frac{1}{2}\left(i\,\overline{\nu_\alpha}\,\partial K_{\alpha\beta}\nu_\beta - \overline{\nu^c_\alpha} M_{\alpha\beta}\nu_\beta + h.c.\right) + \\ - \frac{g}{\sqrt{2}}\left(W^+_\mu \bar{l}_\alpha \gamma^\mu P_L \nu_\alpha + h.c.\right)$$

$M_{\alpha\beta} \rightarrow$ diagonalized $\rightarrow$ unitary transformation

$K_{\alpha\beta} \rightarrow$ diagonalized and normalized $\rightarrow$ unitary transf. + rescaling

$$L = \frac{1}{2}\left(i\,\overline{\nu_i}\,\partial \nu_i - \overline{\nu^c}_i m_{ii}\nu_i\right) - \frac{g}{\sqrt{2}}\left(W^+_\mu \bar{l}_\alpha \gamma^\mu P_L N_{\alpha i}\nu_i\right).$$

$N$ non-unitary

# Some numbers

- CHOOZ: systematic error
  hep-ex\0301017
  - Normalization flux ~ 2.7%
  - Energy ~ 1.1%
- SNO: $$\frac{n_{CC}}{n_{NC}} \simeq \underline{0.9}\,|N_{e1}|^2 + \underline{0.1}\,|N_{e2}|^2$$
  → we let it vary 2 %
- OPERA (NF): background: c-decays / charged current decays $\sim 10^{-6}$
  systematic error ~ 5%
  efficiency ~ 20%
  hep-ph\0305185
- K2K: Statistics
  hep-ex\0411038

**ZERO-DISTANCE EFFECT**

40Kt Iron calorimeter near NUFACT

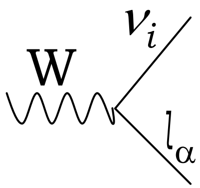
- $\nu_e \rightarrow \nu_\mu \qquad (NN^\dagger)_{e\mu} < 2.3 \cdot 10^{-4}$

4Kt OPERA-like near NUFACT

- $\nu_e \rightarrow \nu_\tau \qquad (NN^\dagger)_{e\tau} < 2.9 \cdot 10^{-3}$
- $\nu_\mu \rightarrow \nu_\tau \qquad (NN^\dagger)_{\mu\tau} < 2.6 \cdot 10^{-3}$

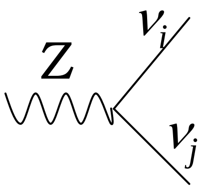
# $(NN^\dagger)$ from decays: $G_F$

- W decays




$$\to \frac{(NN^+)_{\alpha\alpha}}{\sqrt{(NN^+)_{ee}}\sqrt{(NN^+)_{\mu\mu}}}$$

- Invisible Z



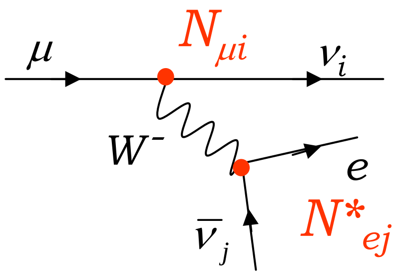

$$\to \frac{\sum_{ij}(N^+N)_{ij}}{\sqrt{(NN^+)_{ee}}\sqrt{(NN^+)_{\mu\mu}}}$$

- Universality tests

$$\to \frac{(NN^+)_{\alpha\alpha}}{(NN^+)_{\beta\beta}}$$

Info on $(NN^\dagger)_{aa}$

$G_F$ is measured in $\mu$-decay




$$\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3}\sum_i |N_{\mu i}|^2 \sum_j |N_{ej}|^2$$

$$G_F^2 = \frac{G_{F,\exp}^2}{\sum_i |N_{\mu i}|^2 \sum_j |N_{ej}|^2}$$

# Oscillation Probability in Vacuum

$$i\frac{d}{dt}|\nu_i\rangle = \hat{H}_{free}|\nu_i\rangle = \sum_j E_i\,\delta_{ij}\,|\nu_j\rangle \quad\leftrightarrow\quad \sum_i |\nu_i\rangle\langle\nu_i| = \mathrm{I}$$

Effective Evolution

$$i\frac{d}{dt}\begin{pmatrix}|\nu_e\rangle\\|\nu_\mu\rangle\\|\nu_\tau\rangle\end{pmatrix} = U^*\begin{pmatrix}E_1 & 0 & 0\\0 & E_2 & 0\\0 & 0 & E_3\end{pmatrix}U^{-1}\begin{pmatrix}|\nu_e\rangle\\|\nu_\mu\rangle\\|\nu_\tau\rangle\end{pmatrix} \leftrightarrow \sum_\alpha |\nu_\alpha\rangle\langle\nu_\alpha| = \mathrm{I}$$

$$i\frac{d}{dt}\begin{pmatrix}|\nu_e\rangle\\|\nu_\mu\rangle\\|\nu_\tau\rangle\end{pmatrix} = \tilde{N}^*\begin{pmatrix}E_1 & 0 & 0\\0 & E_2 & 0\\0 & 0 & E_3\end{pmatrix}\left(\tilde{N}^*\right)^{-1}\begin{pmatrix}|\nu_e\rangle\\|\nu_\mu\rangle\\|\nu_\tau\rangle\end{pmatrix} \leftrightarrow \sum_\alpha |\nu_\alpha\rangle\langle\nu_\alpha| \neq \mathrm{I}$$

$$P_{\alpha\beta} = \left|\langle\nu_\beta|\nu_\alpha(L)\rangle\right|^2 = \frac{\left|\sum_i N_{\beta i}N^*_{\alpha i}\, e^{-i\frac{m_i^2}{2E}L}\right|^2}{\sum_i |N_{\alpha i}|^2 \sum_i |N_{\beta i}|^2}$$

$$P_{\alpha\beta} = \left|\langle\nu_\beta|\nu_\alpha(0)\rangle\right|^2 = \frac{\left|\sum_i N_{\beta i}N^*_{\alpha i}\right|^2}{\sum_i |N_{\alpha i}|^2 \sum_i |N_{\beta i}|^2}$$

Zero Distance Effect

# Non-unitarity from see-saw

$$L = L_{SM} + i\, \bar{N}_R \not\partial N_R - Y_\nu\, \bar{L}\, H\, N_R - M\, N_R\, N_R$$

Integrate out $N_R$ $$L^{eff} = L_{SM} + \frac{1}{M} L^{d=5} + \frac{1}{M^2} L^{d=6} + \ldots$$

$YY^T/M\ (L\ L\ H\ H)$

$YY^{+}/M^2 (\bar{L}\ H) \not\partial (H\ L)$

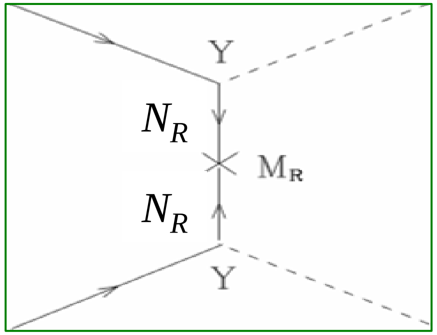
d=5 operator
it gives mass to $\nu$

d=6 operator
it renormalises kinetic energy

Broncano, Gavela, Jenkins 02

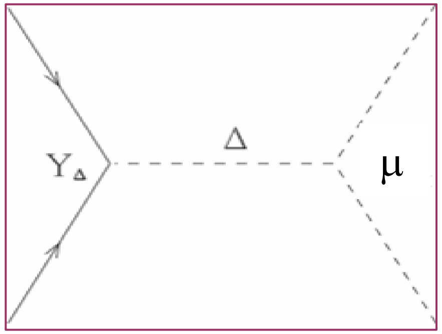
# ν masses beyond the SM

## ★ Tree-level realizations




Heavy fermion singlet $N_R$ (Seesaw I)

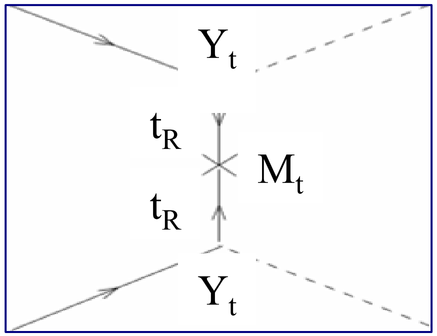
→ deviations from unitarity




Heavy scalar triplet $\Delta$

→ no deviations from unitarity

Abada, Antusch, Biggio, Bonnet, Hambye, M.B.G.




Heavy fermion triplet $t_R$

→ deviations from unitarity

# ...adding near detectors...

Test of zero-distance effect: $P_{\alpha\beta}(E,0) = \left|\left(NN^{\dagger}\right)_{\alpha\beta}\right|^2 \neq \delta_{\alpha\beta}$

- MINOS: $(NN^{\dagger})_{\mu\mu} = 1 \pm 0.05$
- BUGEY: $(NN^{\dagger})_{ee} = 1 \pm 0.04$
- NOMAD: $(NN^{\dagger})_{\mu\tau} < 0.09$ $(NN^{\dagger})_{e\tau} < 0.013$
- KARMEN: $(NN^{\dagger})_{\mu e} < 0.05$

$$|N| = \begin{pmatrix} 0.75-0.89 & 0.45-0.66 & <0.27 \\ 0.00-0.69 & 0.22-0.81 & 0.57-0.85 \\ ? & ? & ? \end{pmatrix}$$

$\rightarrow$ also all $|N_{\mu i}|^2$ determined

# Number of events

$$n_{ev} \sim \int dE \, \frac{d\Phi_\alpha(E)}{dE} P_{\alpha\beta}(E,L) \sigma_\beta(E) \varepsilon(E)$$

$\nu$ produced and detected in CC

$$\begin{cases} \dfrac{d\Phi_\alpha}{dE} \sim \dfrac{d\Phi_\alpha^{SM}}{dE} \left(NN^+\right)_{\alpha\alpha} \\ \sigma_\beta \sim \sigma_\beta^{SM} \left(NN^+\right)_{\beta\beta} \end{cases}$$

$$n_{ev} \sim \int dE \, \frac{d\Phi_\alpha^{SM}(E)}{dE} \left(NN^+\right)_{\alpha\alpha} P_{\alpha\beta}(E,L) \left(NN^+\right)_{\beta\beta} \sigma_\beta^{SM}(E) \varepsilon(E)$$

$$\hat{P}_{\alpha\beta}(E,L) = \left| \sum_i N_{\alpha i}^* e^{iP_i L} N_{\beta i} \right|^2$$

Exceptions:

- measured flux
- leptonic production mechanism
- detection via NC

# In the future…

<u>TESTS OF UNITARITY</u>

RARE LEPTON DECAYS

- $\mu \to e\gamma$ $\left(NN^\dagger\right)_{e\mu} < 7.1 \cdot 10^{-5}$ (MEGA)
- $\tau \to e\gamma$ $\left(NN^\dagger\right)_{e\tau} < 1.6 \cdot 10^{-2}$ (BABAR)
- $\tau \to \mu\gamma$ $\left(NN^\dagger\right)_{\mu\tau} < 1.0 \cdot 10^{-2}$ (Belle)