Flavour Permutation Symmetry and Fermion Mixing [a]

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We discuss our recently proposed $S3_{\downarrow} \times S3_{\uparrow}$ flavour-permutation-symmetric mixing observables, giving expressions for them in terms of (moduli-squared) of the mixing matrix elements. We outline their successful use in providing flavour-symmetric descriptions of (non-flavour-symmetric) lepton mixing schemes. We develop our partially unified flavour-symmetric description of both quark and lepton mixings, providing testable predictions for CP-violating phases in both B decays and neutrino oscillations.

1 Introduction

Flavour observables, namely quark and lepton masses and mixings are neither predicted nor predictable in the Standard Model. Neither are they correlated with each other in any way. However, their experimentally determined values display striking structure: viewed on a logarithmic scale, the fermion masses of any given non-zero charge are approximately equi-spaced; the spectrum of quark mixing angles is described by the Wolfenstein form, [1] suggestive of correlations between mixing angles and quark masses, and the lepton mixing matrix is well-approximated by the tri-bimaximal form. [2] These striking patterns are the modern-day equivalents of the regularities observed around a century ago in hydrogen emission spectra, which were mathematically well-described by the Rydberg formula, but nevertheless had no theoretical basis before the advent of quantum mechanics. While consistent with the Standard Model, they lie completely outside its predictive scope, and are surely evidence for some new physics beyond it.

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[b] Speaker.

In this talk, we report on our recent attempts [3] to find a new description of fermion mixing which builds on the Standard Model and allows constraints on the mixing observables which make no reference to individual flavours, while describing mixing structures which are manifestly not flavour-symmetric, as observed experimentally. This approach does not in itself constitute a complete theory of flavour mixing beyond the Standard Model, but we hope that it might help stimulate new developments in that direction.

2 The Jarlskogian and Plaquette Invariance

Jarlskog's celebrated CP-violating invariant, [4] J, is important in the phenomenology of both quarks and leptons. As well as parameterising the violation of a specific symmetry, it has two other properties which set it apart from most other mixing observables. First, its value (up to its sign) is independent of any flavour labels.[c] Mixing observables are in general dependent on flavour labels, eg. the moduli-squared of mixing matrix elements, $|U_{\alpha i}|^2$, certainly depend on α and i. Indeed, J itself is often calculated in terms of a subset of four mixing matrix elements, namely those forming a given plaquette [5] (whose elements are defined by deleting the γ-row and the k-column [d] to leave a rectangle of four elements):

$$J = \text{Im}(\Pi_{\gamma k}) = \text{Im}(U_{\alpha i} U^*_{\alpha j} U^*_{\beta i} U_{\beta j}). \quad (1)$$

However, it is well-known [4] that the value of J does not depend on the choice of plaquette (ie. on its flavour labels, γ and k above) - it is "plaquette-invariant". This special feature originates in the fact that J is *flavour-symmetric*, carrying information sampled evenly across the whole mixing matrix. We recently pointed-out [3] that in fact, *any* observable function of the mixing matrix elements, flavour-symmetrised (eg. by summing over both rows and columns), and written in terms of the elements of a single plaquette (eg. using unitarity constraints), will be similarly plaquette-invariant. Both its expression in terms of mixing matrix elements, as well as its value, will be independent of the particular choice of plaquette.

The second exceptional property of J is that it may be particularly simply related to the fermion mass (or Yukawa) matrices:

$$J = -i\,\frac{\text{Det}[L, N]}{2 L_\Delta N_\Delta} \quad (2)$$

where for leptons, L and N are the charged-lepton and neutrino mass matrices respectively [e] (in an arbitrary weak basis) and $L_\Delta = (m_e - m_\mu)(m_\mu - m_\tau)(m_\tau - m_e)$ (with an analogous definition for N_Δ in terms of neutrino masses and likewise for the quarks). This is useful, as, despite J being defined purely in terms of mixing observables via Eq. (1), by contrast, Eq. (2) relates it to the mass matrices, which appear in the Standard Model Lagrangian.

We will discuss our recently proposed [3] plaquette-invariant (ie. flavour-symmetric mixing) observables, which, in common with J, are independent of flavour labels and can be simply related to the mass matrices. Again like J, we find that our observables parameterise the violation of certain phenomenological symmetries which have already been considered significant [6] [7] [8] [9] in leptonic mixing. In the next section, we define more precisely what we mean by flavour symmetry.

[c] We focus first on the leptons, although many of our considerations may be applied equally well to the quarks. In the leptonic case, neutrino mass eigenstate labels $i = 1...3$ take the analogous role to the charge $-\frac{1}{3}$ quark flavour labels in the quark case. In this sense, we will often use the term "flavour" to include neutrino mass eigenstate labels, as well as charged lepton flavour labels.

[d] We use a cyclic labelling convention such that $\beta = \alpha + 1$, $\gamma = \beta + 1$, $j = i + 1$, $k = j + 1$, all indices evaluated mod 3.

[e] Throughout this paper, L and N are taken to be Hermitian, either by appropriate choice of the flavour basis for the right-handed fields, or as the Hermitian squares, $MM^\dagger$, of the relevant mass or Yukawa coupling matrices. The symbols m_α, m_i generically refer to their eigenvalues in either case.

3 The $S3_\downarrow \times S3_\uparrow$ Flavour Permutation Group

The $S3_\downarrow$ group is the group of the six possible permutations of the charged lepton flavours and/or of the charge $-\frac{1}{3}$ quark flavours, while the $S3_\uparrow$ group is the group of the six possible permutations of the neutrino flavours (ie. mass eigenstates) or of the charge $\frac{2}{3}$ quark flavours (the arrow subscript corresponds to the direction of the z-component of weak isospin of the corresponding left-handed fields). We consider all possible such permutations, which together constitute the direct product $S3_\downarrow \times S3_\uparrow$ flavour permutation group (FPG) [3] with 36 elements.

We next consider the P matrix (for "probability") [10] of moduli-squared of the mixing matrix elements, eg. for leptons:

$$P = \begin{pmatrix} |U_{e1}|^2 & |U_{e2}|^2 & |U_{e3}|^2 \\ |U_{\mu 1}|^2 & |U_{\mu 2}|^2 & |U_{\mu 3}|^2 \\ |U_{\tau 1}|^2 & |U_{\tau 2}|^2 & |U_{\tau 3}|^2 \end{pmatrix}. \tag{3}$$

It should be familiar: for quarks, semileptonic weak decay rates of hadrons are proportional to its elements, while for leptons, the magnitudes of neutrino oscillation probabilities may be written in terms of its elements. [10] Moreover, the P matrix may easily be related to the fermion mass matrices, as we will see in Section 5 below. The P matrix manifestly transforms as the natural representation of $S3_\downarrow \times S3_\uparrow$, the transformations being effected by pre- and/or post-multiplying by 3×3 real permutation matrices.[f]

Jarlskog's invariant J is a pseudoscalar under the FPG: under even permutations, it is invariant, while under odd permutations (eg. single swaps of rows or columns of the mixing matrix, or odd numbers of them), it simply changes sign. This is our prototype Flavour Symmetric Mixing Observable (FSMO). As we commented in the previous section, it is easy to find other similar such quantities, which, surprisingly had not appeared in the literature until recently. [3] There are two types of singlets under the S3 group: even ($\mathbf{1}$) which remain invariant under all permutations, and odd ($\overline{\mathbf{1}}$) which flip sign under odd permutations. So, under the FPG, there are four types of singlet: $\mathbf{1}\times\mathbf{1}$, $\overline{\mathbf{1}}\times\overline{\mathbf{1}}$ (like J), $\mathbf{1}\times\overline{\mathbf{1}}$ and $\overline{\mathbf{1}}\times\mathbf{1}$. By Flavour Symmetric Observables (FSOs), we mean observables with any of these transformation properties under the FPG. They may be functions of mixing matrix elements alone (FSMOs), or functions of mass eigenvalues alone, or functions of both.

Starting with elements of P and combining and (anti-)symmetrising them over flavour labels in various ways, we find that, apart from their (trivial) overall normalisation, and possibly scalar offsets, there are a finite number of independent FSMOs at any given order in P. Enumerating them, we found that there are no non-trivial ones linear in P, while at 2nd order in P, there is only one each of $\mathbf{1}\times\mathbf{1}$, $\overline{\mathbf{1}}\times\overline{\mathbf{1}}$. At third order, there is exactly one each of the four types of singlet, while at higher orders in P, there are multiple instances of each. Recognising that we need only four independent variables to specify the mixing, it is clearly enough to stop at third order, up to which, the singlets are essentially uniquely defined by their order in P and their transformation property under the FPG.

4 Flavour-Symmetric Mixing Observables

We introduce four FSMOs, [3] uniquely defined as outlined above:

	$\underline{\mathbf{1}\times\mathbf{1}}$	$\underline{\overline{\mathbf{1}}\times\overline{\mathbf{1}}}$
<u>2nd Order in P :</u>	$\mathcal{G} = \frac{1}{2}\,[\sum_{\alpha i}(P_{\alpha i})^2 - 1\,]$	$\mathcal{F} = \text{Det}P$
<u>3rd Order in P :</u>	$\mathcal{C} = \frac{3}{2}\sum_{\alpha i}[\,(P_{\alpha i})^3 - (P_{\alpha i})^2\,] + 1$	$\mathcal{A} = \frac{1}{18}\sum_{\gamma k}(L_{\gamma k})^3$

(4)

[f] Less obviously, any given plaquette of P transforms as a 2-dimensional (real) irreducible representation of $S3_\downarrow \times S3_\uparrow$.

where $L_{\gamma k} = (P_{\alpha i} + P_{\beta j} - P_{\beta i} - P_{\alpha j})$. Alternative, but equivalent definitions in terms of the elements of a single plaquette are given elsewhere.[3] Note that $\mathcal{F}$ is only quadratic in P, because of the constraints of unitarity. We comment briefly on the normalisations and offsets we have given them. $\mathcal{F}$ and $\mathcal{A}$, being anti-symmetric, need no offset, as they are already centred on zero, which they reach for threefold maximal mixing[11] (uniquely defined by all 9 elements of the mixing matrix having magnitude $\frac{1}{\sqrt{3}}$). $\mathcal{G}$ and $\mathcal{C}$ are defined with offsets such that they likewise vanish for threefold maximal mixing. All four variables are normalised so that their maximum value is unity, which they attain for no mixing. In Ref.[3], we also give the $\overline{\mathbf{1}} \times \mathbf{1}$ and the $\mathbf{1} \times \overline{\mathbf{1}}$ FSMOs at 3rd order (called $\mathcal{B}$ and $\mathcal{D}$ respectively), but they will not concern us here.

The four FSMOs introduced in Eq. 4 are the simplest ones[g] in terms of P and are sufficient to completely specify the mixing, up to a number of discrete ambiguities associated with the built-in flavour symmetry. J is of course not independent, and is given by $18J^2 = 1/6 - \mathcal{G} + (4/3)\,\mathcal{C} - (1/2)\,\mathcal{F}^2$. In Table 1, we summarise their properties and values (estimated at 90% CL from compilations of current experimental results) for both quarks[12] and leptons.[13]

Table 1: Properties and values of flavour-symmetric mixing observables for quarks and leptons. The experimentally allowed ranges are estimated (90% CL) from compilations of current experimental results, neglecting any correlations between the input quantities.

Observable Name	Order in P	Symmetry: $S3_{\downarrow} \times S3_{\uparrow}$	Theoretical Range	Experimental Range for Leptons	Experimental Range for Quarks
$\mathcal{F}$	2	$\overline{\mathbf{1}}\times\overline{\mathbf{1}}$	$(-1, 1)$	$(-0.14, 0.12)$	$(0.893, 0.896)$
$\mathcal{G}$	2	$\mathbf{1}\times\mathbf{1}$	$(0, 1)$	$(0.15, 0.23)$	$(0.898, 0.901)$
$\mathcal{A}$	3	$\overline{\mathbf{1}}\times\overline{\mathbf{1}}$	$(-1, 1)$	$(-0.065, 0.052)$	$(0.848, 0.852)$
$\mathcal{C}$	3	$\mathbf{1}\times\mathbf{1}$	$(-\frac{1}{27}, 1)$	$(-0.005, 0.057)$	$(0.848, 0.852)$

5 Flavour-Symmetric Mixing Observables in Terms of Mass Matrices

Equation (2) gives J, our prototype FSMO, in terms of the fermion mass matrices, which in turn are proportional to the matrices of Yukawa couplings which appear in the Standard Model Lagrangian. In this section, we show how to write the FSMOs of Section 4 above also in terms of the mass matrices. It is useful to define a reduced P matrix:

$$\widetilde{P} = P - D \tag{5}$$

where D is the 3×3 democratic matrix with all 9 elements equal to $\frac{1}{3}$. We also define the reduced (ie. traceless) powers of the fermion mass matrices: $\widetilde{L^m} := L^m - \frac{1}{3}\mathrm{Tr}(L^m)$ (similarly for $\widetilde{N^m}$), in terms of which, we can define the 2×2 matrix of weak basis-invariants:

$$\widetilde{T}_{mn} := \mathrm{Tr}(\widetilde{L^m}\widetilde{N^n}), \quad m, n = 1, 2. \tag{6}$$

For known lepton masses, $\widetilde{T}$ is completely equivalent to P. In fact, it is straightforward to show that $\widetilde{P}$ is a mass-moment transform of $\widetilde{T}$:

$$\widetilde{P} = \widetilde{M_\ell}^T \cdot \widetilde{T} \cdot \widetilde{M_\nu} \tag{7}$$

where

$$\widetilde{M_\ell} = \frac{1}{L_\Delta} \begin{pmatrix} m_\mu^2 - m_\tau^2 & m_\tau^2 - m_e^2 & m_e^2 - m_\mu^2 \\ m_\mu - m_\tau & m_\tau - m_e & m_e - m_\mu \end{pmatrix}, \tag{8}$$

[g] They also treat the two weak-isospin sectors symmetrically, though this is not an essential feature.

with an analogous definition for $\widetilde{M_\nu}$ (the inverse transform is easily obtained).

Starting from Eq. (4) and substituting for P from Eqs. (5) and (7), we find that:

$$\mathcal{F} \equiv \text{Det } P = 3\frac{\text{Det } \widetilde{T}}{L_\Delta N_\Delta}; \qquad \left[\text{cf. Eq. (2)} : J = -i\,\frac{\text{Det}[L,N]}{2L_\Delta N_\Delta}\right] \tag{9}$$

$$\mathcal{G} = \frac{\widetilde{T}_{mn}\,\widetilde{T}_{pq}\,\mathcal{L}^{mp}\,\mathcal{N}^{nq}}{(L_\Delta N_\Delta)^2}; \qquad \mathcal{C},\mathcal{A} = \frac{\widetilde{T}_{mn}\,\widetilde{T}_{pq}\,\widetilde{T}_{rs}\,\mathcal{L}^{(mpr)}_{\mathcal{C},\mathcal{A}}\,\mathcal{N}^{(nqs)}_{\mathcal{C},\mathcal{A}}}{(L_\Delta N_\Delta)^{n_{\mathcal{C},\mathcal{A}}}}, \tag{10}$$

where the $\mathcal{L}$ ($\mathcal{N}$) are simple functions of traces of $\widetilde{L^m}$ ($\widetilde{N^m}$), given in Ref.[3], and $n_\mathcal{C}$ ($n_\mathcal{A}$) = 2(3).

6 Application 1: Flavour-Symmetric Descriptions of Leptonic Mixing

The tribimaximal mixing[2] ansatz for the MNS lepton mixing matrix:

$$U \simeq \begin{pmatrix} -2/\sqrt{6} & 1/\sqrt{3} & 0 \\ 1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \end{pmatrix} \tag{11}$$

is compatible with all confirmed leptonic mixing measurements from neutrino oscillation experiments, and may be considered a useful leading-order approximation to the data. It is defined by three phenomenological symmetries:[6] CP symmetry, μ-τ-reflection symmetry and Democracy, which may each be expressed (flavour-symmetrically) in terms of our FSMOs. For example, as is well known, the zero in the U_{e3} position, if exact, ensures that no CP violation can arise from the mixing matrix. CP symmetry is thus represented simply by $J = 0$ (which is a necessary, but not sufficient condition for a single zero in the mixing matrix, see Section 7 below). μ-τ-reflection symmetry[7] means that corresponding elements in the μ and τ rows have equal moduli: $|U_{\mu i}| = |U_{\tau i}|,\ \forall i$, and this implies the two flavour-symmetric constraints:

$$\mathcal{F} = \mathcal{A} = 0 \tag{12}$$

(flavour symmetry means that although these two constraints imply just such a set of equalities, they do not define *which* pair of rows or columns are constrained). Democracy[8 9] ensures that one row or column is trimaximally mixed, ie. has the form $\frac{1}{\sqrt{3}}(1,1,1)^{(T)}$, as is the case for the ν_2 column in tribimaximal mixing. Democracy is ensured flavour-symmetrically by the two constraints:

$$\mathcal{F} = \mathcal{C} = 0. \tag{13}$$

Taking all three symmetries, tribimaximal mixing (or one of its trivial permutations) is ensured by the complete set of constraints $\mathcal{F} = \mathcal{C} = \mathcal{A} = J = 0$, which may be written as the single flavour-symmetric condition:

$$\mathcal{F}^2 + \mathcal{C}^2 + \mathcal{A}^2 + J^2 = 0. \tag{14}$$

Tribimaximal mixing is manifestly not flavour symmetric. The flavour-symmetry of our constraint, Eq. (14), is spontaneously broken by its tribimaximal solutions. The symmetry is manifested by the existence of a complete set of solutions of the generalised tribimaximal form, each related to the other by a member of the flavour permutation group.

Of course, generalisations of the tribimaximal form[6] possessing subsets of its three symmetries may be similarly defined, and their corresponding flavour-symmetric constraints may be obtained by analogy to the above. These, and those of other special mixing forms[14 15] are tabulated in Ref.[3].

7 Application 2: A Partially Unified, Flavour-Symmetric Description of Quark and Lepton Mixings

A unified understanding of quark and lepton mixings is highly desirable. This is difficult because their mixing matrices have starkly different forms: the quark mixing matrix is characterised by small mixing angles, [12] while the lepton mixing matrix is characterised mostly by large ones. [13] Many authors have ascribed this difference to the effect of the heavy majorana mass matrix in the leptonic case, via the see-saw mechanism. [17] Notwithstanding the attractiveness of this explanation, it is clearly still worthwhile to ask if there are any features of the respective mixings which the quark and lepton sectors have in common.

Neutrino oscillation data [13] require that $|U_{e3}|^2 \lesssim 0.05$, significantly less than the other MNS matrix elements-squared. At least one *small* mixing element is hence a common feature of both quark and lepton mixing matrices. We are thus led first to ask the question: "what is the flavour-symmetric condition for at least one zero element in the mixing matrix?" We should perhaps anticipate two constraints, as the condition implies that both real and imaginary parts vanish. A zero mixing element implies CP conservation, so that $J = 0$. A clue to the second constraint is that with μ-τ-reflection symmetry, $J = 0$ ensures a zero somewhere in the ν_e row of the MNS matrix. However, μ-τ-reflection symmetry implies two more constraints, Eq. (12).

In order to find a single additional constraint we consider the K matrix [16] [10] with elements:

$$K_{\gamma k} = \text{Re}(U_{\alpha i} U^*_{\alpha j} U^*_{\beta i} U_{\beta j}), \tag{15}$$

which is the CP-conserving analogue of J (cf. the definition of J, Eq. (1)). K should be familiar: in the leptonic case, its elements are often used to write the magnitudes of the oscilliatory terms in neutrino appearance probabilities; [10] in the quark case, its elements are just the CKM factors of the CP-conserving parts of the interference terms in penguin-dominated decay rates. A single zero in the mixing matrix leads to four zeroes in a plaquette of K and this clearly implies:

$$\text{Det}\, K = 0, \tag{16}$$

which is our sufficient second condition, along with $J = 0$. [h] We note that Eq. (16) can easily be cast in terms of our complete set of FSMOs, since $54\,\text{Det}\, K \equiv 2\mathcal{A} + \mathcal{F}(\mathcal{F}^2 - 2\mathcal{C} - 1)$. Hence, μ-τ-reflection symmetry, Eq. (12), is a special case of Eq. (16).

Experimentally, there is no exactly zero element in the CKM matrix, so that $\text{Det}\, K = 0$ *and* $J = 0$ cannot *both* be exact for quarks. Moreover, for leptons, despite there being no experimental lower limit for $|U_{e3}|$, there is no reason to suppose that the MNS matrix has an exact zero either. In order to ensure a small, but non-zero element in the mixing matrices, we need to consider a modest relaxation of either condition, or of both. For quarks, we know from experiment that CP is slightly violated, with [12] $|J_q/J_{max}| \simeq 3 \times 10^{-4}$, while [i] for leptons, fits to oscillation data [13] imply a fairly loose upper bound on their CP violation: $|J_\ell/J_{max}| \lesssim 0.33$. Turning to $\text{Det}\, K$, we find that for quarks, $|\text{Det}\, K_q/(\text{Det}\, K)_{max}| \lesssim 3 \times 10^{-7}$, while for leptons, $|\text{Det}\, K_\ell/(\text{Det}\, K)_{max}| \lesssim 0.6$ (the precision of lepton mixing data does not yet allow a strong constraint). However, there is no experimental lower limit for $|\text{Det}\, K|$ for quarks or for leptons, each being compatible with zero, so that it is sufficient to relax only the condition on J.

We are thus led to conjecture that for both quarks and leptons:

$$\text{Det}\, K = 0; \quad |J/J_{max}| = \text{small} \tag{17}$$

(it is not implied that the small quantity necessarily has the same value in both sectors). Equation (17) is a unified and flavour-symmetric, partial description of both lepton and quark mixing

[h] The two conditions may even be expressed as one, noting that the product of all nine elements of P is given by $\frac{1}{144}\prod_{\alpha i} P_{\alpha i} = (\text{Det}\, K)^2 + J^2(2J^2 + \mathcal{R})^2$, which is zero iff $\text{Det}\, K = 0$ and $J = 0$ (as $\mathcal{R} > 0$, as long as $J \neq 0$).

[i] We note that $J_{max} = \frac{1}{6\sqrt{3}} \simeq 0.1$ and $(\text{Det}\, K)_{max} = \frac{2^6}{3^9} \simeq 0.0033$.

matrices, being associated with the existence of at least one small element in each mixing matrix, U_{e3} and V_{ub} respectively (it is partial in the sense that only two degrees of freedom are constrained for each matrix). However, in the case that J is not exactly zero, the condition $\mathrm{Det}\, K = 0$ also implies that in the limit, as $J \to 0$, there is at least one unitarity triangle angle which $\to 90°$. This is rather obvious in the μ-τ-symmetry case, but is less obvious more generally. While the flavour symmetry prevents an a priori prediction of *which* angle is $\simeq 90°$, we know from experiment [12] that for quarks, $\alpha \simeq 90°$. A detailed calculation shows that our conjecture, Eq. (17), predicts, in terms of Wolfenstein parameters: [1]

$$(90° - \alpha) = \overline{\eta}\lambda^2 = 1° \pm 0.2° \tag{18}$$

at leading order in small quantities, to be compared with its current experimental determination: [12]

$$(90° - \alpha) = 0°^{+3°}_{-7°}. \tag{19}$$

It will be interesting to test Eq. (18) more precisely in future experiments with B mesons, in particular, at LHCb and at a possible future Super Flavour Factory. For leptons, experiment tells us not only that it is the U_{e3} MNS matrix element which is small but also that only the unitarity triangle angles [j] $\phi_{\mu 1}$ or $\phi_{\tau 1}$ can be close to $90°$. Then Eq. (17) implies that:

$$|90° - \delta| = 2\sqrt{2}\, \sin\theta_{13}\, \sin(\theta_{23} - \frac{\pi}{4}) \lesssim 4° \tag{20}$$

at leading order in small quantities (we use the PDG convention here). It thus requires a large CP-violating phase in the MNS matrix, which is promising for the discovery of leptonic CP violation at eg. a future Neutrino Factory.

8 Discussion and Conclusions

Given that our flavour-symmetric variables are defined (essentially) uniquely by their flavour symmetry properties and by their order in P, it is remarkable that the leptonic data may be described simply by the constraints $\mathcal{F} = \mathcal{A} = \mathcal{C} = J = 0$. This is suggestive that these variables may be fundamental in some way. It is furthermore tantalising that the smallness of one element in each mixing matrix, the approximate μ-τ-symmetry in lepton mixing and the existence of a right unitarity triangle may all be related to each other, through our simple partially-unified constraint, Eq. (17). The precision of the resulting prediction, Eq. (18), motivates more sensitive tests at future B physics facilities, while the synergy with tests at a neutrino factory is manifest.

All elements of the Standard Model, apart from the Yukawa couplings of the fermions to the Higgs, treat each fermion of any given charge on an equal footing - they are already flavour-symmetric. The Yukawa couplings, on the other hand, depend on flavour in such a way that each flavour has unique mass and mixing matrix elements. Using our flavour-symmetric observables, or combinations of them appropriately chosen, we have shown how it is also possible to specify the flavour-dependent mixings in a flavour-independent way. [k] This recovers flavour symmetry at the level of the mixing description, the symmetry being broken only spontaneously by its solutions, which define and differentiate the flavours in terms of their mixings.

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[j] We use the nomenclature of unitarity triangle angles we defined in reference [46] of Ref. [9].

[k] We illustrated another variant of this in Ref. [18].

References

1. L. Wolfenstein, Phys. Rev. Lett. 51 (1983) 1945.
2. P. F. Harrison, D. H. Perkins, and W. G. Scott, Phys. Lett. B530 (2002) 167, hep-ph/0202074; Phys. Lett. B458 (1999) 79, hep-ph/9904297; Z. Xing Phys. Lett. B533 (2002) 85, hep-ph/0204049; X. He and A. Zee, Phys. Lett. B560 (2003) 87, hep-ph/0301092; E. Ma, Phys. Rev. Lett. 90 (2003) 221802, hep-ph/0303126; Phys. Lett. B583 (2004) 157, hep-ph/0308282; Phys. Rev. D70 (2004) 031901, hep-ph/0404199; C. I. Low and R. R. Volkas, Phys. Rev. D68 (2003) 033007, hep-ph/0305243; G. Altarelli and F. Feruglio, Nucl. Phys. B 720 (2005) 64. hep-ph/0504165.
3. P.F. Harrison, D.R.J. Roythorne and W.G. Scott, Phys. Lett. B657 (2007) 210, arXiv:0709.1439.
4. C. Jarlskog, Phys. Rev. Lett. 55 (1985) 1039; Z. Phys. C29 (1985) 491; Phys. Rev. D35 (1987) 1685; Phys. Rev. Lett. 57 (1986) 2875; Phys. Rev. D36 (1987) 2128; Phys. Rev. D39 (1989) 988; Phys. Lett. B609 (2005) 323, hep-ph/0412288; Phys. Scripta, T127 (2006) 64, hep-ph/0606050; C. Jarlskog and A. Kleppe, Nucl. Phys. B286 (1987) 245.
5. J. D. Bjorken and I. Dunietz, Phys. Rev. D36 (1987) 2109.
6. P. F. Harrison and W. G. Scott, Phys. Lett. B535 (2002) 163, hep-ph/0203209.
7. C. S. Lam, Phys. Lett. B507 (2001) 214, hep-ph/0104116;
   M. Kitabayashi and M. Yasue, Phys. Lett. B524 (2002) 308, hep-ph/0110303; P. F. Harrison and W. G. Scott, Phys. Lett. B547 (2002) 219, hep-ph/0210197; J. F. Beacom et al. Phys. Rev. D69 (2004) 017303, hep-ph/0309267;
   W. Grimus and L. Lavoura, Phys. Lett. B579 (2004) 113, hep-ph/0305309.
8. P. F. Harrison and W. G. Scott, Phys. Lett. B557 (2003) 76, hep-ph/0302025; Proc. NOVE-II, Venice (2003) hep-ph/0402006; Phys. Lett. B594 (2004) 324, hep-ph/0403278; R. Friedberg and T. D. Lee, HEP & NP 30 (2006) 591, hep-ph/0606071; arXiv:0705.4156; C. S. Lam, Phys. Lett. B640 (2006) 260, hep-ph/0606220; Phys. Rev. D74 (2006), 113004, hep-ph/0611017.
9. J. D. Bjorken, P. F. Harrison and W. G. Scott, Phys. Rev. D74 (2006) 073012, hep-ph/0511201;
10. P. F. Harrison, W. G. Scott and T. J. Weiler, Phys. Lett. B641 (2006) 372, hep-ph/0607335.
11. P. F. Harrison and W. G. Scott, Phys. Lett. B349 (1995) 137, hep-ph/9503190.
12. CKMfitter Group (J. Charles et al.), Eur. Phys. J. C41 (2005) 1, hep-ph/0406184, updated results and plots: http://ckmfitter.in2p3.fr;
    M. Bona et al. (UTfit Collaboration), hep-ph/0701204.
13. A. Strumia and F. Vissani, (2007), hep-ph/0606054.
14. G. Altarelli and F. Feruglio, J. HEP 9811 (1998) 021, hep-ph/9809596.
15. V.D. Barger, S. Pakvasa, T.J. Weiler and K. Whisnant, Phys. Lett. B437 (1998) 107, hep-ph/9806387.
16. P.F. Harrison and W.G. Scott, Phys. Lett. B535 (2002) 229, hep-ph/0203021;
    P.F. Harrison, W.G. Scott and T.J. Weiler, Phys. Lett. B565 (2003) 159, hep-ph/0305175.
17. P. Minkowski Phys. Lett. B67 (1977) 421.
18. P. F. Harrison and W. G. Scott, Phys. Lett. B628 (2005) 93, hep-ph/0508012.