Low energy effects of neutrino masses

Thomas Hambye
Université Libre de Bruxelles

In collaboration with A. Abada, C. Biggio, F. Bonnet and B. Gavela:
arXiv:0707.0633 (hep-ph), JHEP '07

Neutrino masses: origin and effective theory

- neutrino masses require new fields beyond the Standard Model

e.g. heavy

- effects of these heavy fields for SM particle processes: low energy effective theory

in powers of (1/M)

dimension 5 operators: $\propto 1/M$

dimension 6 operators: $\propto 1/M^2$

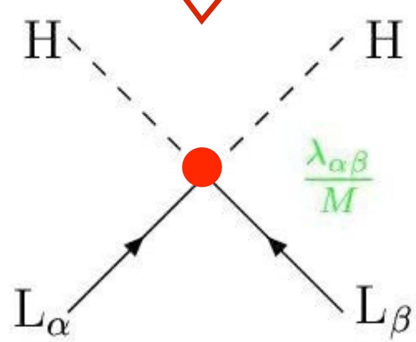
dimension 7 operators: $\propto 1/M^3$

....

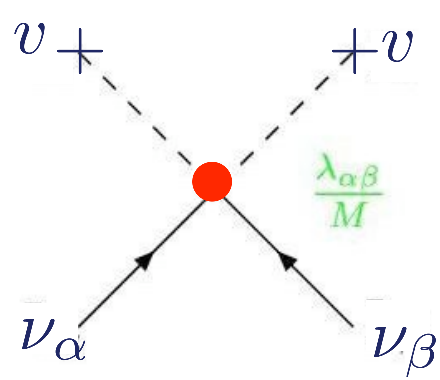
Dimension 5 operator

there is only one: $\mathcal{L} \ni \frac{\lambda}{M}(LLHH) = c_{d=5}(LLHH)$

$$\Downarrow$$



$$\Downarrow$$



$$\Longrightarrow \quad m_\nu = \frac{\lambda}{M} v^2$$

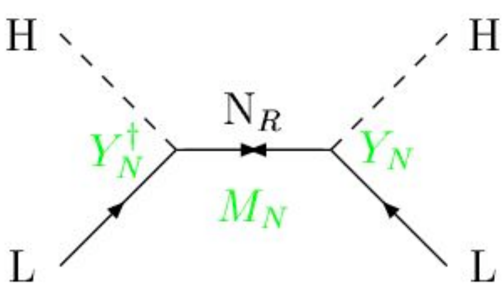
Majorana

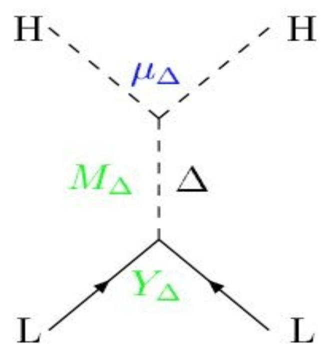
$\Longrightarrow$ dim 5 operator $\Longleftrightarrow$ neutrino masses

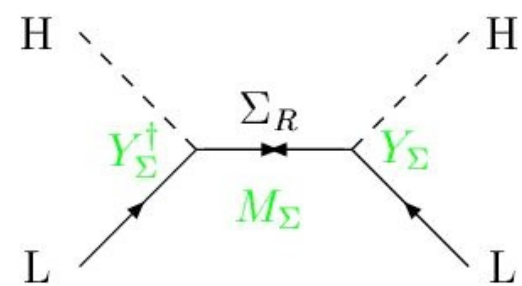
The 3 basic seesaw models

i.e. tree level ways to generate the dim 5 operator

Right-handed singlet: (type-I seesaw)	Scalar triplet: (type-II seesaw)	Fermion triplet: (type-III seesaw)







$$m_\nu = Y_N^T \frac{1}{M_N} Y_N v^2$$

$$m_\nu = Y_\Delta \frac{\mu_\Delta}{M_\Delta^2} v^2$$

$$m_\nu = Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma v^2$$

Minkowski; Gellman, Ramon, Slansky; Yanagida;Glashow; Mohapatra, Senjanovic

Magg, Wetterich; Lazarides, Shafi; Mohapatra, Senjanovic; Schechter, Valle

Foot, Lew, He, Joshi; Ma; Ma, Roy;T.H., Lin, Notari, Papucci, Strumia; Bajc, Nemevsek, Senjanovic; Dorsner, Fileviez-Perez;....

How to distinguish experimentally the 3 seesaw models??

not from the dim-5 operator (i.e. not from neutrino oscillations experim., $0\nu 2\beta$,...)

$\Downarrow$

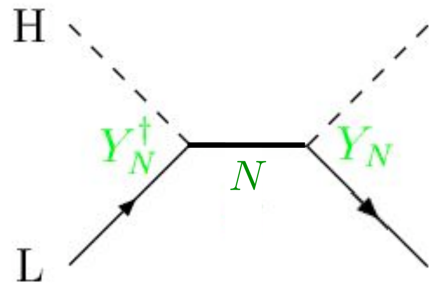
same unique operator for all 3 models

we need : - either to be able to produce the heavy states

- and/or to distinguigsh them from dim-6 operators

Dimension 6 operators in seesaw models

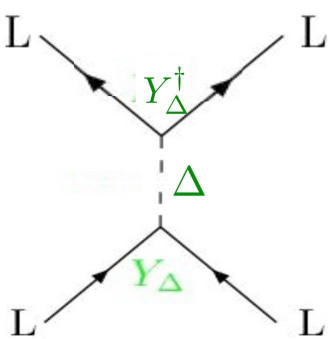
- **Type-I:** $\mathcal{L}^{d=6} = Y_N^\dagger \frac{1}{M_N^2} Y_N \, (\bar{L}H) \, \partial\!\!\!/ (HL)$



Broncano, Gavela, Jenkins '02

- **Type-III:** $\mathcal{L}^{d=6} = Y_\Sigma^\dagger \frac{1}{M_\Sigma^2} Y_\Sigma \, (\bar{L}H) \, D\!\!\!\!/ \, (HL)$

- **Type-II:** $$\mathcal{L}^{d=6} = Y_\Delta^\dagger \frac{1}{M_\Delta^2} Y_\Delta \, (\bar{L}L)(\bar{L}L) + \frac{\mu_\Delta^2}{M_\Delta^4}(H^\dagger H)^3 + \lambda_i \frac{\mu_\Delta^2}{M_\Delta^4}(H\tau H) D_\mu D^\mu (H\tau H)$$



Dim 5 and dim 6 operator summary

Model	Effective Lagrangian $\mathcal{L}_{eff} = c_i \mathcal{O}_i$		
	$c^{d=5}$	$c_i^{d=6}$	$\mathcal{O}_i^{d=6}$
Fermionic Singlet	$Y_N^T \frac{1}{M_N} Y_N$	$Y_N^\dagger \frac{1}{\lvert M_N\rvert^2} Y_N$	$\left(\overline{L}\widetilde{H}\right) i\partial\!\!\!/ \left(\widetilde{H}^\dagger L\right)$
Fermionic Triplet	$Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma$	$Y_\Sigma^\dagger \frac{1}{\lvert M_\Sigma\rvert^2} Y_\Sigma$	$\left(\overline{L}\vec{\tau}\widetilde{H}\right) i D\!\!\!\!/ \left(\widetilde{H}^\dagger \vec{\tau} L\right)$
Scalar Triplet	$4Y_\Delta \frac{\mu_\Delta}{\lvert M_\Delta\rvert^2}$	$Y_\Delta^\dagger \frac{1}{2\lvert M_\Delta\rvert^2} Y_\Delta$	$\left(\overline{\widetilde{L}}\vec{\tau}L\right)\left(\overline{L}\vec{\tau}\widetilde{L}\right)$
		$\frac{\lvert\mu_\Delta\rvert^2}{\lvert M_\Delta\rvert^4}$	$\left(H^\dagger\vec{\tau}\widetilde{H}\right)\left(\overleftarrow{D}_\mu \overrightarrow{D}^\mu\right)\left(\widetilde{H}^\dagger\vec{\tau}H\right)$
		$-2\left(\lambda_3+\lambda_5\right)\frac{\lvert\mu_\Delta\rvert^2}{\lvert M_\Delta\rvert^4}$	$\left(H^\dagger H\right)^3$

What about the size of dim 6 effects??

expected e.g. very suppressed:

$$c_{d=6} \sim Y_N^\dagger \frac{1}{M_N^2} Y_N$$

$$c_{d=5} \sim Y_N^T \frac{1}{M_N} Y_N$$

$$\Rightarrow \quad c_{d=6} \sim \frac{c_{d=5}}{M_N} \sim \underbrace{\frac{m_\nu}{M_N}}_{} \frac{1}{v^2}$$

$$\sim 10^{-13} \quad (M_N \sim \mathrm{TeV})$$

$$Y_N \sim 10^{-6}$$

but not necessarily so:

- $c_{d=5}$ and $c_{d=6}$: not same Yukawa combination

- $c_{d=5}$ breaks lepton number but $c_{d=6}$ do not!

$\Rightarrow$ there is no symmetry reasons why if $c_{d=5}$ is suppressed $c_{d=6}$ should also be

“Direct Lepton number Violation,,

assume a L conserving setup with not too large $M_{N,\Delta,\Sigma}$ and large Yukawas

$M_{N,\Delta,\Sigma} \sim 100\,\mathrm{GeV} - 100\,\mathrm{TeV}$

$$\Downarrow$$

$$c_{d=6} \sim \frac{Y^2}{M^2} \quad \longleftarrow \quad \text{large}$$

$$c_{d=5} = m_\nu = 0 \quad \longleftarrow \quad \text{no L violation}$$

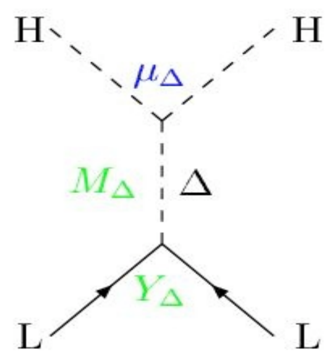
assume L is broken by a small perturbation μ

$$\Downarrow$$

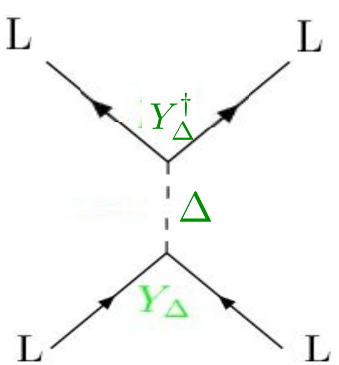
$$m_\nu = f(Y)\,\frac{\mu}{M^2}\,v^2 \quad \longleftarrow$$

neutrino masses directly proport. to a small source of L violation μ rather than inversely proport. to a large mass M

Direct Lepton number Violation in type-II model



$$\Longrightarrow \quad c_{d=5} = \frac{Y_\Delta \mu_\Delta}{M_\Delta^2} \quad \leftarrow \quad \left(= \frac{m_\nu}{v^2} \right)$$



$$\Longrightarrow \quad c_{d=6} = \frac{Y_\Delta^\dagger Y_\Delta}{M_\Delta^2}$$

if Y_Δ large, μ_Δ small $\Longrightarrow$ small enough neutrino masses with large dim 6 effects

if $\mu_\Delta = 0$ no L violation

DLV in type-I (and type-III) model

example with one light neutrino and 2 N:

$$\begin{array}{cc} & \begin{array}{ccc} \nu_L & N_1 & N_2 \end{array} \\ \begin{array}{c} \nu_L \\ N_1 \\ N_2 \end{array} & \begin{pmatrix} 0 & Y_N \frac{v}{\sqrt{2}} & 0 \\ Y_N \frac{v}{\sqrt{2}} & 0 & M_N \\ 0 & M_N & 0 \end{pmatrix} \end{array}$$

"inverse seesaw" as in
Gonzalez-Garcia, Valle '89
Kersten, Smirnov '07
Abada, Biggio, Bonnet, Gavela, T.H. '07

if Y_N is large, M_N not too high:

$c_{d=6}$ large with $c_{d=5} = 0$ (L is conserved)

$$L(\nu) = 1,\ L(N_1) = -1,\ L(N_2) = 1$$

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Gonzalez-Garcia, Valle '89
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if Y_N is large, M_N not too high:

$c_{d=6}$ large with $c_{d=5} = \frac{m_\nu}{v^2} = Y_N^T \frac{\mu}{M_N^2} Y_N$

(L is approximately conserved)

Phenomenology of dim 6 operators

long list of effects depending on the seesaw model:

-rare lepton decays: $\mu \to e\gamma,\ \tau \to e\gamma, \tau \to \mu\gamma, \mu \to eee, \tau \to 3\,l$

Phenomenology of dim 6 operators

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-Z and W decays: $Z \to l\bar{l}$, $W \to l\nu$

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-Z and W decays: $Z \to l\bar{l},\ W \to l\nu$

-Z invisible width: $Z \to \nu\bar{\nu}$

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- -Z and W decays: $Z \to l\bar{l},\ W \to l\nu$
- -Z invisible width: $Z \to \nu\bar{\nu}$
- - ρ parameter
- - W mass
- - ....

Bounds on Yukawa couplings from dim 6 operator induced processes: type-II model

Process	Constraint on	Bound ($\times\left(\frac{M_\Delta}{1\,\mathrm{TeV}}\right)^2$)
M_W	$\lvert Y_{\Delta\mu e}\rvert^2$	$< 7.3\times 10^{-2}$
$\mu^-\to e^+e^-e^-$	$\lvert Y_{\Delta\mu e}\rvert\lvert Y_{\Delta ee}\rvert$	$< 1.2\times 10^{-5}$
$\tau^-\to e^+e^-e^-$	$\lvert Y_{\Delta\tau e}\rvert\lvert Y_{\Delta ee}\rvert$	$< 1.3\times 10^{-2}$
$\tau^-\to \mu^+\mu^-\mu^-$	$\lvert Y_{\Delta\tau\mu}\rvert\lvert Y_{\Delta\mu\mu}\rvert$	$< 1.2\times 10^{-2}$
$\tau^-\to \mu^+e^-e^-$	$\lvert Y_{\Delta\tau\mu}\rvert\lvert Y_{\Delta ee}\rvert$	$< 9.3\times 10^{-3}$
$\tau^-\to e^+\mu^-\mu^-$	$\lvert Y_{\Delta\tau e}\rvert\lvert Y_{\Delta\mu\mu}\rvert$	$< 1.0\times 10^{-2}$
$\tau^-\to \mu^+\mu^-e^-$	$\lvert Y_{\Delta\tau\mu}\rvert\lvert Y_{\Delta\mu e}\rvert$	$< 1.8\times 10^{-2}$
$\tau^-\to e^+e^-\mu^-$	$\lvert Y_{\Delta\tau e}\rvert\lvert Y_{\Delta\mu e}\rvert$	$< 1.7\times 10^{-2}$
$\mu\to e\gamma$	$\lvert\Sigma_{l=e,\mu,\tau} Y^\dagger_{\Delta l\mu} Y_{\Delta el}\rvert$	$< 4.7\times 10^{-3}$
$\tau\to e\gamma$	$\lvert\Sigma_{l=e,\mu,\tau} Y^\dagger_{\Delta l\tau} Y_{\Delta el}\rvert$	< 1.05
$\tau\to \mu\gamma$	$\lvert\Sigma_{l=e,\mu,\tau} Y^\dagger_{\Delta l\tau} Y_{\Delta \mu l}\rvert$	$< 8.4\times 10^{-1}$

Combined bounds		
Process	Yukawa	Bound $\left(\times\left(\frac{M_\Delta}{1\,\mathrm{TeV}}\right)^4\right)$
$\mu\to e\gamma$	$\left\lvert Y^\dagger_{\Delta\mu\mu} Y_{\Delta\mu e} + Y^\dagger_{\Delta\tau\mu} Y_{\Delta\tau e}\right\rvert$	$< 4.7\times 10^{-3}$
$\tau\to e\gamma$	$\left\lvert Y^\dagger_{\Delta\tau\tau} Y_{\Delta\tau e}\right\rvert$	< 1.05
$\tau\to \mu\gamma$	$\left\lvert Y^\dagger_{\Delta\tau\tau} Y_{\Delta\tau\mu}\right\rvert$	$< 8.4\times 10^{-1}$

Abada, Biggio, Bonnet, Gavela, T.H. ‘07

Partly from: Barger et al ’82; Pal ’83; Bernabeu et al ’84, ‘86; Bilenky, Petcov’87; Gunion et al ’89, ‘06; Swartz ‘89; Mohapatra ’92

Bounds on Yukawa couplings from dim 6 operator induced processes: type-I model

Antusch, Biggio, Fernandez-Martinez, Lopez-Pavon, Gavela '06

$$\frac{v^2}{2}|c^{d=6}|_{\alpha\beta} = \frac{v^2}{2}|Y_N^\dagger \frac{1}{|M_N|^2} Y_N|_{\alpha\beta} \lesssim \begin{pmatrix} 10^{-2} & 7.2\cdot 10^{-5} & 1.6\cdot 10^{-2} \\ 7.2\cdot 10^{-5} & 10^{-2} & 1.1\cdot 10^{-2} \\ 1.6\cdot 10^{-2} & 1.1\cdot 10^{-2} & 10^{-2} \end{pmatrix}$$

Abada, Biggio, Bonnet, Gavela, T.H. '07

effects come mostly from the mixings between the ν and N which induce modifications of W couplings to leptons and Z couplings to ν (through non-unitarity effects)

-rare lepton decays: $\mu \to e\gamma,\ \tau \to e\gamma, \tau \to \mu\gamma, \mu \to eee, \tau \to 3\,l$

-universality tests: $W \to l\bar{\nu}$

-Z and W decays: $Z \to l\bar{l},\ W \to l\nu$

-Z invisible width: $Z \to \nu\bar{\nu}$

Bounds on Yukawa couplings from dim 6 operator induced processes: type-III model

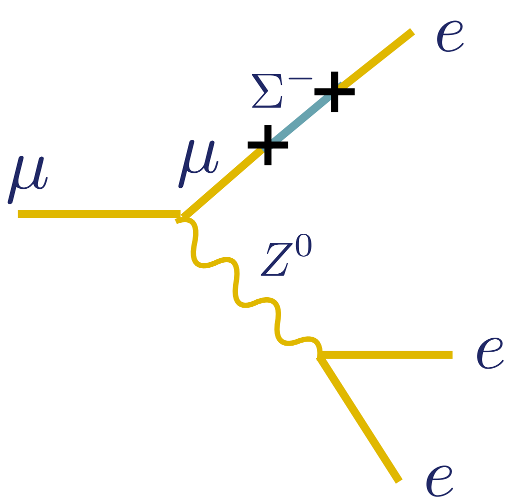
Abada, Biggio, Bonnet, Gavela, T.H. ‘07

$$\frac{v^2}{2}|c^{d=6}|_{\alpha\beta} = \frac{v^2}{2}\left|Y_\Sigma^\dagger \frac{1}{M_\Sigma^\dagger}\frac{1}{M_\Sigma} Y_\Sigma\right|_{\alpha\beta} \lesssim \begin{pmatrix} 3\cdot 10^{-3} & 1.1\cdot 10^{-6} & 1.2\cdot 10^{-3} \\ 1.1\cdot 10^{-6} & 4\cdot 10^{-3} & 1.2\cdot 10^{-3} \\ 1.2\cdot 10^{-3} & 1.2\cdot 10^{-3} & 4\cdot 10^{-3} \end{pmatrix}$$

$\mu \to eee$ $\quad\mu \to e\gamma$ $\quad Z \to \mu e$
$\tau \to eee$ $\quad\tau \to e\gamma$ $\quad Z \to \tau e$
$\tau \to \mu ee$ $\quad\tau \to \mu\gamma$ $\quad Z \to \tau\mu$
$\tau \to \mu\mu e$ $\quad +$
$\tau \to \mu\mu\mu$ $\quad$ W decays
Invisible Z width
Universality tests

effects come from mixings between ν and neutral triplets and between charged leptons and charged triplets which induce modification of W couplings to leptons and Z couplings to both l and ν (through non-unitarity effects)

example: $\mu \to eee$:



at tree level (not at one-loop as in type-I)

General trend

$$\boxed{|Y| \lesssim 10^{-1}\frac{M}{1\,TeV}}$$

or stronger

Summary

- dimension 6 operator effects are crucial for distinguishing the seesaw models
- dimension 6 effects are e.g. suppressed but not necessarily

$\Downarrow$

Direct Lepton number Violation pattern

$\Uparrow$

same for all 3 seesaw models

- rich associated phenomenology

provides bounds on Yukawa coupling - mass combinations

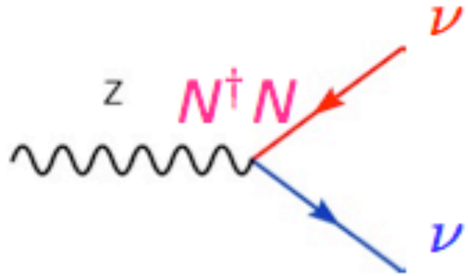
Backup

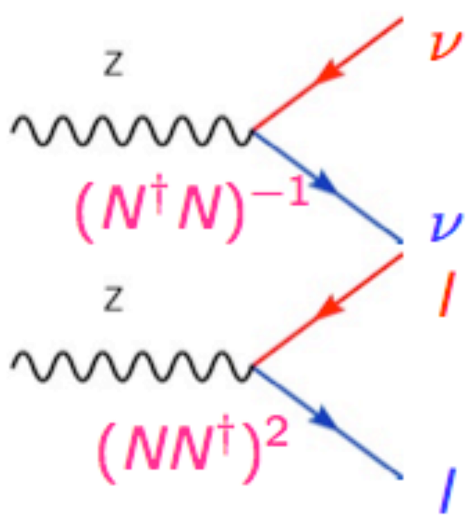
3 nus +3 N DFV case

$$\begin{array}{c}
(\nu_e, \nu_\mu, \nu_\tau, N_1, N_2, N_3) \\
\begin{pmatrix}
0 & 0 & 0 & c & 0 & 0 \\
0 & 0 & 0 & d & 0 & 0 \\
0 & 0 & 0 & e & 0 & 0 \\
c & d & e & 0 & 0 & a \\
0 & 0 & 0 & 0 & b & 0 \\
0 & 0 & 0 & a & 0 & 0
\end{pmatrix}
\end{array}$$

Singlet and triplet Seesaws differ in the the pattern of the Z *couplings*

Singlet	Triplet
$J_\mu^{-\,CC} \equiv \overline{l_L}\,\gamma_\mu\, N\,\nu$	$J_\mu^{-\,CC} \equiv \overline{l_L}\,\gamma_\mu\, N\,\nu$
$J_\mu^{NC} \equiv \frac{1}{2}\overline{\nu}\,\gamma_\mu (N^\dagger\, N)\,\nu$	$J_\mu^{Z}(\text{neutrinos}) \equiv \frac{1}{2}\overline{\nu}\,\gamma_\mu (N^\dagger\, N)^{-1}\,\nu$
	$J_\mu^{3}(\text{leptons}) \equiv \frac{1}{2}\overline{l}\,\gamma_\mu (N N^\dagger)^2\, l$





l->l' gamma versus l->3l' ratios are predicted to fixed values in the type-III seesaw model

$$\begin{aligned} Br(\mu \to e\gamma) &= 1.3 \cdot 10^{-3} \cdot Br(\mu \to eee)\,, \\ Br(\tau \to \mu\gamma) &= 1.3 \cdot 10^{-3} \cdot Br(\tau \to \mu\mu\mu) = 2.1 \cdot 10^{-3} \cdot Br(\tau^- \to e^- e^+ \mu^-) \\ Br(\tau \to e\gamma) &= 1.3 \cdot 10^{-3} \cdot Br(\tau \to eee) \ = 2.1 \cdot 10^{-3} \cdot Br(\tau^- \to \mu^- \mu^+ e^-) \end{aligned}$$

l->l' gamma:

$$\begin{aligned} |\epsilon_{e\mu}| &= \frac{v^2}{2} |Y_\Sigma^\dagger \frac{1}{M_\Sigma^\dagger} \frac{1}{M_\Sigma} Y_\Sigma|_{\mu e} \lesssim 1.1 \cdot 10^{-4} \\ |\epsilon_{\mu\tau}| &= \frac{v^2}{2} |Y_\Sigma^\dagger \frac{1}{M_\Sigma^\dagger} \frac{1}{M_\Sigma} Y_\Sigma|_{\tau\mu} \lesssim 1.5 \cdot 10^{-2} \\ |\epsilon_{e\tau}| &= \frac{v^2}{2} |Y_\Sigma^\dagger \frac{1}{M_\Sigma^\dagger} \frac{1}{M_\Sigma} Y_\Sigma|_{\tau e} \lesssim 2.4 \cdot 10^{-2} \end{aligned}$$

l->3l':

Constraints on	Process	Bound
$\|(NN^\dagger)_{e\mu}\|$	$\mu^- \to e^+e^-e^-$	$< 1.1 \cdot 10^{-6}$
$\|(NN^\dagger)_{e\tau}\|$	$\tau^- \to e^+e^-e^-$	$< 1.2 \cdot 10^{-3}$
$\|(NN^\dagger)_{\mu\tau}\|$	$\tau^- \to \mu^+\mu^-\mu^-$	$< 1.2 \cdot 10^{-3}$
$\|(NN^\dagger)_{\tau e}\|$	$\tau^- \to \mu^+\mu^-e^-$	$< 1.6 \cdot 10^{-3}$
$\|(NN^\dagger)_{\tau\mu}\|\|(NN^\dagger)_{e\mu}\|$	$\tau^- \to e^+\mu^-\mu^-$	$< 3.1 \cdot 10^{-4}$
$\|(NN^\dagger)_{\tau\mu}\|$	$\tau^- \to e^+e^-\mu^-$	$< 1.5 \cdot 10^{-3}$
$\|(NN^\dagger)_{\tau e}\|\|(NN^\dagger)_{\mu e}\|$	$\tau^- \to \mu^+e^-e^-$	$< 2.9 \cdot 10^{-4}$
$\|(NN^\dagger)_{e\mu}\|$	$\mu \to e\gamma$	$2.8 \cdot 10^{-5}$
$\|(NN^\dagger)_{\mu\tau}\|$	$\tau \to \mu\gamma$	$5.2 \cdot 10^{-3}$
$\|(NN^\dagger)_{e\tau}\|$	$\tau \to e\gamma$	$6.6 \cdot 10^{-3}$