

XLIII Rencontres de Moriond

Electroweak Session
La Thuile 7 - III - 2008

Quantum Boltzmann equations in resonant leptogenesis

Andrea De Simone

Massachusetts Institute of Technology

Based on: JCAP **08**(2007) 002 and JCAP **08**(2007) 013 with *A. Riotto*,
and on JCAP **01**(2008)004 with *V. Cirigliano, G. Isidori, I. Masina* and *A. Riotto*

OUTLINE

- Introduction
- Quantum Boltzmann Equations for leptogenesis
- Applications to Resonant and MLFV leptogenesis

BARYOGENESIS VIA LEPTOGENESIS

- Attractive and successful scenario to explain the observed matter-antimatter asymmetry:

$$\left. \frac{n_B - n_{\bar{B}}}{s} \right|_{\text{today}} = (8.66 \pm 0.35) \times 10^{-11}$$

- It naturally arises when invoking the *see-saw* mechanism to account for the small neutrino masses.
- The same see-saw framework provides explanation for two seemingly unrelated problems: neutrino masses and BAU.
- “Minimal amount” of physics BSM: RH neutrinos.

$$\mathcal{L}_{\text{int}} = \lambda_{\alpha i} N_{\alpha} \ell_i H + h_i \bar{e}_i \ell_i H^c + \frac{1}{2} M_{\alpha} N_{\alpha}^2 + \text{h.c.}$$

- **The mechanism** (roughly speaking): in the early Universe at $T \sim M_N$, RH neutrinos decay out of equilibrium in an L - and CP -violating way, thus generating a L asymmetry.
Non-perturbative SM processes (“sphalerons”) play the role of converting L into a B asymmetry, which arrives to us.
- The three famous Sakharov conditions are fulfilled:
 - 1) L is violated by the Majorana mass of N . Sphalerons convert it into a **B violation**.
 - 2) **CP violation** in N decays $N \rightarrow \ell H$ (complex λ needed)
 - 3) If N -decays are slow compared to the expansion $\Gamma_D < H(T \sim M_N)$ the N 's decay **out of equilibrium**.
- Leptogenesis works in principle. Whether or not it is able to explain the observed BAU needs a *quantitative* analysis.






- Getting quantitative answers requires to keep track of the evolution of the abundances of all the particles involved.
- A lot of effort in recent years to refine the basic picture and include corrections: thermal effects (2003), spectator processes (2005), flavour structure in the lepton sector (2006) ...
- The dynamics of leptogenesis is usually analysed by means of semi-classical Boltzmann equations. The CP asymmetry

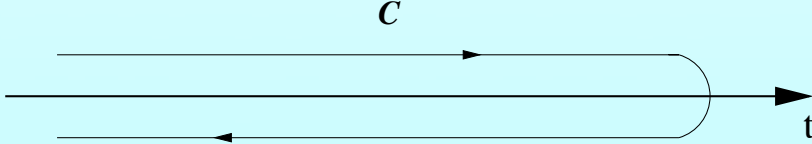
$$\epsilon_N = \frac{\Gamma(N \rightarrow \ell H) - \Gamma(N \rightarrow \bar{\ell} \bar{H})}{\Gamma(N \rightarrow \ell H) + \Gamma(N \rightarrow \bar{\ell} \bar{H})}$$


is assumed to be a constant.

- We tried to go beyond and look for a fully *quantum* approach, which led us to a set of quantum Boltzmann equations.
- **Main result:** *the CP asymmetry shows a time dependence.*
Depending on the model, this effect may or may not be significant.

CTP FORMALISM

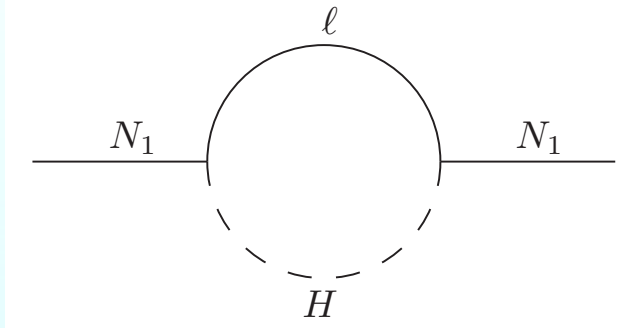
-  We need the time evolution of quantum correlators with definite initial conditions and not the transition amplitude of particle reactions. So, the ordinary equilibrium QFT at finite temperature is not the appropriate tool.
-  The most appropriate extension of QFT to deal with non-equilibrium phenomena amounts to generalizing the time contour of integration to a closed time-path  **Closed Time-Path (CTP)** formalism (a.k.a. Schwinger-Keldysh formalism).
-  It is a powerful Green's function formulation for describing out-of-equilibrium phenomena in field theory (widely used in condensed matter and nuclear theory).
-  All time integrations are performed along a deformed time contour



 and time-ordering operators are considered along the path.
-  It allows to follow the particle abundances at a given time as functions of the previous dynamical history of the system (“*memory effects*”).

QBE for the RH NEUTRINOS

The BE's are obtained starting from the Dyson's equations for propagators. We need to compute the self-energy functions explicitly for our case.



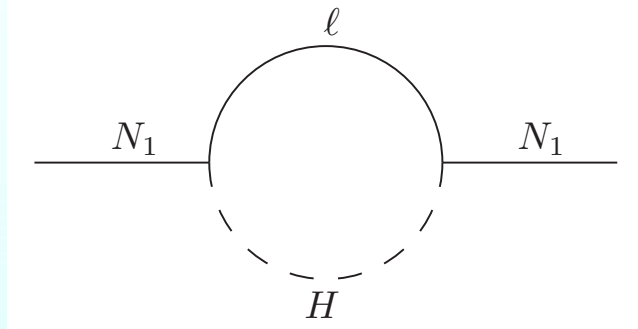
Assign to the interaction points a + or - sign in all possible manners and sum all the possible diagrams.

$$\Sigma_{N_1}^{>,<}(x,y) = i G_H^{>,<}(x,y) G_\ell^{>,<}(x,y).$$

$$\frac{\partial n_{N_1}}{\partial t} = -\langle \Gamma_{N_1}(t) \rangle n_{N_1} + \langle \tilde{\Gamma}_{N_1}(t) \rangle n_{N_1}^{\text{eq}}.$$

QBE for the RH NEUTRINOS

The BE's are obtained starting from the Dyson's equations for propagators. We need to compute the self-energy functions explicitly for our case.



Assign to the interaction points a + or - sign in all possible manners and sum all the possible diagrams.

$$\Sigma_{N_1}^{>,<}(x, y) = i G_H^{>,<}(x, y) G_\ell^{>,<}(x, y).$$

$$\frac{\partial n_{N_1}}{\partial t} = -\underbrace{\langle \Gamma_{N_1}(t) \rangle}_{\downarrow} n_{N_1} + \underbrace{\langle \tilde{\Gamma}_{N_1}(t) \rangle}_{\downarrow} n_{N_1}^{\text{eq}}.$$

$$\propto \int_0^t dt_z \cos[(E_{N_1} - E_\ell - E_H)(t - t_z)]$$

$$\downarrow \quad t \rightarrow \infty$$

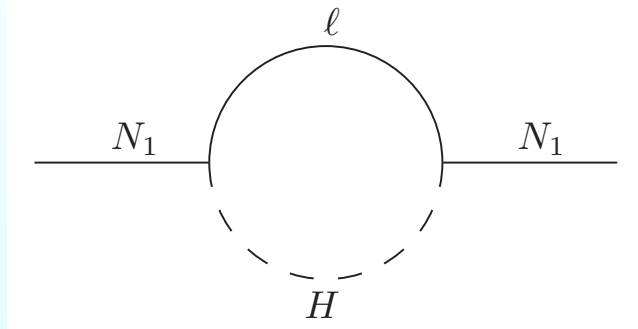
$$\pi \delta(E_{N_1} - E_\ell - E_H)$$

Energy-conserving
delta function

Only on-shell processes contribute

QBE for the RH NEUTRINOS

The BE's are obtained starting from the Dyson's equations for propagators. We need to compute the self-energy functions explicitly for our case.



Assign to the interaction points a + or - sign in all possible manners and sum all the possible diagrams.

$$\Sigma_{N_1}^{>,<}(x,y) = i G_H^{>,<}(x,y) G_\ell^{>,<}(x,y).$$

$$\frac{\partial n_{N_1}}{\partial t} = -\underbrace{\langle \Gamma_{N_1}(t) \rangle}_{\downarrow} n_{N_1} + \underbrace{\langle \tilde{\Gamma}_{N_1}(t) \rangle}_{\downarrow} n_{N_1}^{\text{eq}},$$

$$\propto \int_0^t dt_z \cos[(E_{N_1} - E_\ell - E_H)(t - t_z)]$$

$$\downarrow t \rightarrow \infty$$

$$\pi \delta(E_{N_1} - E_\ell - E_H)$$

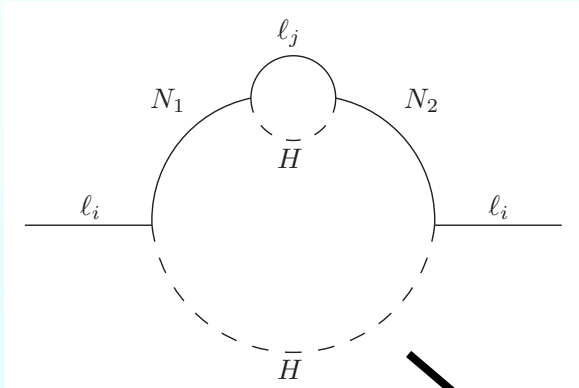
Energy-conserving
delta function

Only on-shell processes contribute

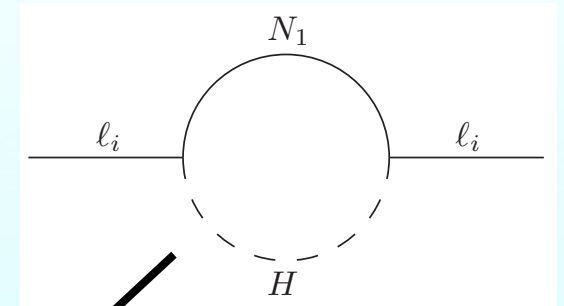
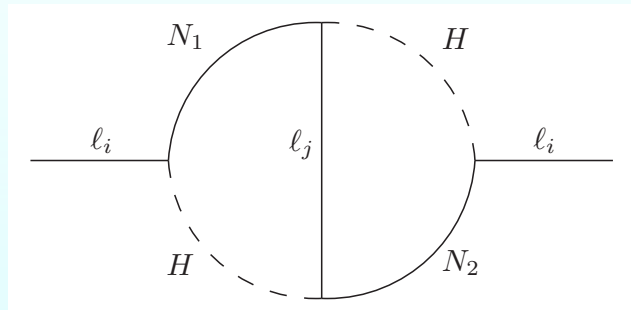
$$\xrightarrow{\text{orange arrow}} \frac{\partial n_{N_1}}{\partial t} = -\langle \Gamma_{N_1} \rangle (n_{N_1} - n_{N_1}^{\text{eq}}) \quad \text{Usual BE}$$

QBE for the LEPTON ASYMMETRY

Two-loop self-energy



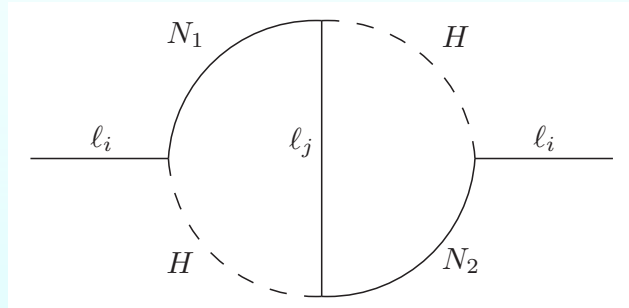
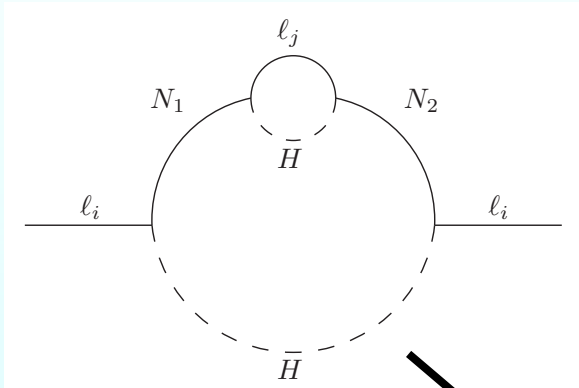
One-loop self-energy



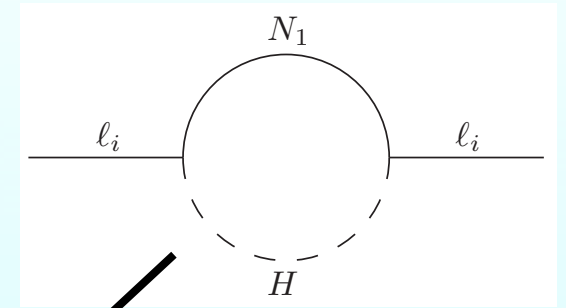
$$\frac{\partial n_{\mathcal{L}_i}}{\partial t} = \epsilon_{N_1}^i(t) \langle \Gamma_{N_1} \rangle (n_{N_1} - n_{N_1}^{\text{eq}}) - \langle \Gamma_{N_1}^{\text{ID}}(t) \rangle \frac{n_{N_1}^{\text{eq}}}{2 n_{\mathcal{L}_i}^{\text{eq}}} n_{\mathcal{L}_i}$$

QBE for the LEPTON ASYMMETRY

Two-loop self-energy



One-loop self-energy



$$\frac{\partial n_{\mathcal{L}_i}}{\partial t} = \underbrace{\epsilon_{N_1}^i(t)}_{\text{Time-dependent}} \langle \Gamma_{N_1} \rangle (n_{N_1} - n_{N_1}^{\text{eq}}) - \langle \Gamma_{N_1}^{\text{ID}}(t) \rangle \frac{n_{N_1}^{\text{eq}}}{2 n_{\mathcal{L}_i}^{\text{eq}}} n_{\mathcal{L}_i}$$

$$\epsilon_{N_1}^i \left[2 \sin^2 \left(\frac{1}{2} \Delta M t \right) - \frac{\Gamma}{\Delta M} \sin(\Delta M t) \right] \quad \text{Time-dependent}$$



It averages to $\epsilon_{N_1}^i$ as $t \rightarrow \infty$

$$\Delta M \equiv M_{N_2} - M_{N_1}$$

The usual BE is recovered in the $t \rightarrow \infty$ limit

The main effect of the quantum approach is the time-dependence of the CP asymmetry.

When is this new effect important?

- The typical time-scale to build up coherently the CP asymmetry is $\sim (\Delta M)^{-1}$
- If the time-scale associated with the other processes relevant for leptogenesis (e.g. $(\Gamma_N)^{-1}$) is much larger than $(\Delta M)^{-1}$, the CP asymmetry will average to a constant  no significant effect.
- If the interactions are faster than $(\Delta M)^{-1}$  the effect is expected to be relevant.
- This condition is attained in Resonant (and MLFV) leptogenesis

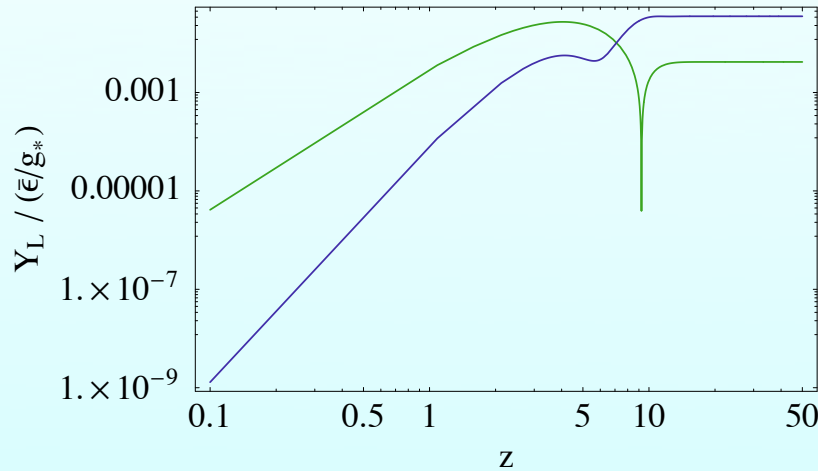
RESONANT LEPTOGENESIS and QUANTUM EFFECTS (1)

- The RH neutrinos are nearly mass-degenerate and $\Delta M \sim \Gamma_{N_1} \sim \Gamma_{N_2}$
- The CP asymmetry is resonantly enhanced, thus making leptogenesis viable at T as low as TeV (relaxed tension with “gravitino problem”).
- We found $\mathcal{O}(10) \div \mathcal{O}(100)$ **enhancements** wrt the usual case with no quantum effects (when at least one flavour is weakly coupled $K \lesssim 1$).
- The numerical results have been confirmed by analytical approximations.

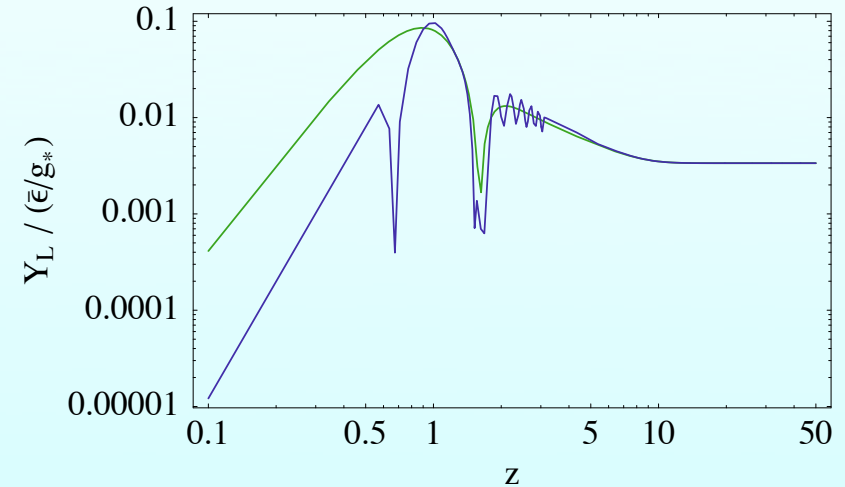
RESONANT LEPTOGENESIS and QUANTUM EFFECTS (2)

Some numerical simulations:

K=0.1 (weak washout)

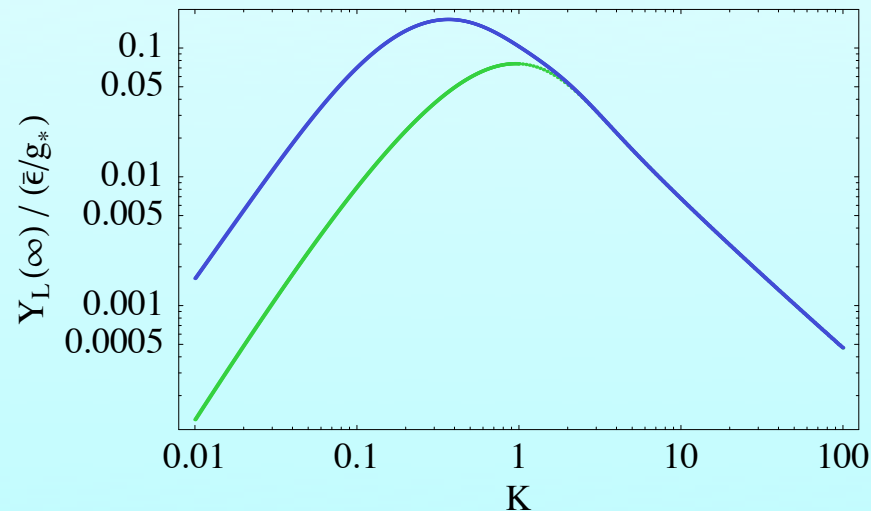


K=10 (strong washout)



$$K \equiv \left. \frac{\Gamma_D}{H} \right|_{z=1}$$

$$z = M_N/T$$



green: $\bar{\epsilon}$
blue: $\epsilon(z)$

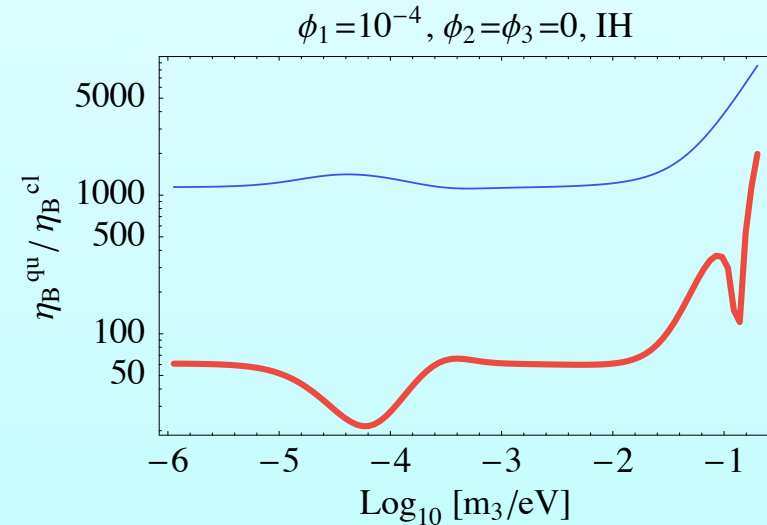
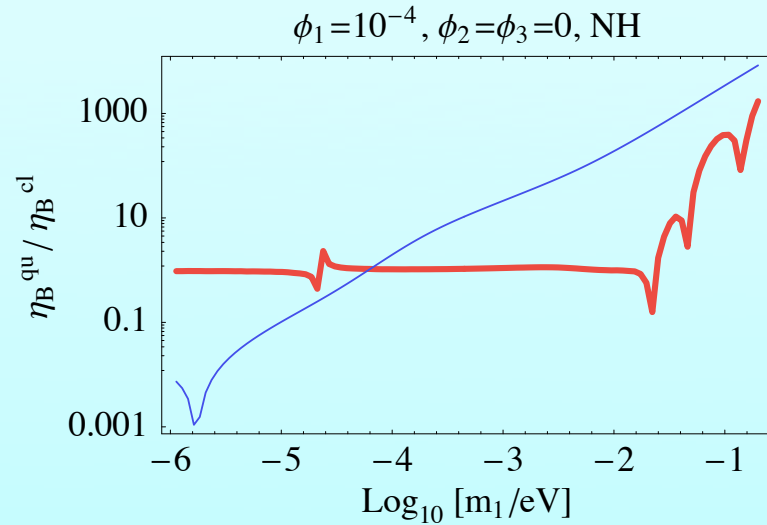
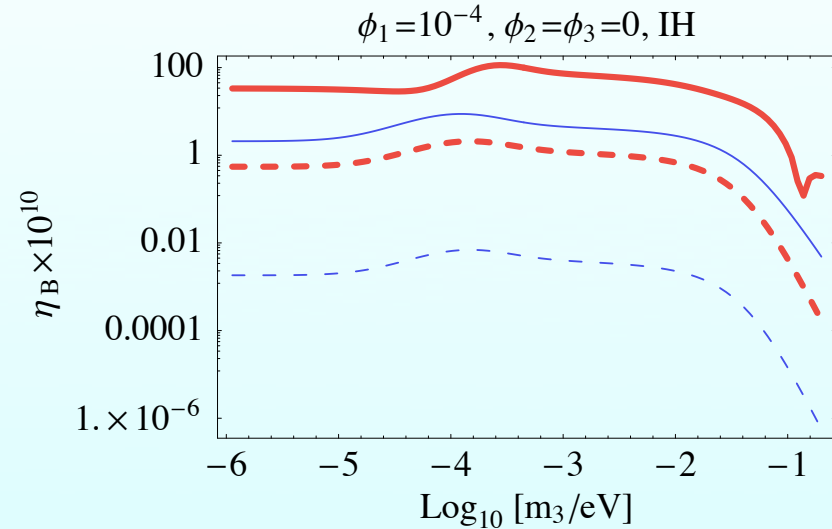
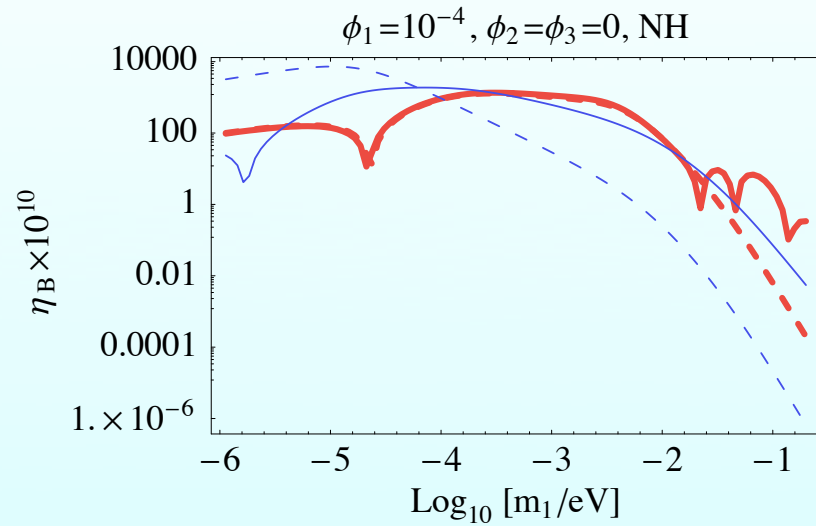
MLFV LEPTOGENESIS and QUANTUM EFFECTS (1)

- MLFV hypothesis: the charged-lepton and neutrino Yukawa couplings are the *only* sources of lepton-flavour symmetry breaking (extension of MFV to the lepton sector)
[Cirigliano, Grinstein, Isidori, Wise 2005]
- In the limit of vanishing Yukawas, the $O(3)_N$ symmetry is exact and RH neutrinos are degenerate at a common scale M_N^0 .
- The degeneracy is lifted *only* by corrections induced by Yukawa couplings.

$$M_N = M_N^0 \left[1 + c^{(1)} \left(\lambda^0 \lambda^{0\dagger} + (\lambda^0 \lambda^{0\dagger})^T \right) + \dots \right]$$

- We end up with a constrained version of resonant leptogenesis, where we expect quantum effects to be important.

MLFV LEPTOGENESIS and QUANTUM EFFECTS (2)



$$c^{(1)} = 2 \times 10^{-5} \quad c^{(1)} = 6 \times 10^{-3}$$

CONCLUSIONS

- Leptogenesis is a viable and attractive candidate to account for the matter-antimatter asymmetry of the Universe.
- It requires a “minimal” amount of beyond-SM physics: heavy RH neutrinos (which may also explain the small neutrino masses).
- We extended the semi-classical treatments used so far, and derived a set of quantum Boltzmann equations.
- The quantum approach mainly leads to a time-dependent CP asymmetry, and the dynamics of the system manifests “memory”.
- In scenarios with nearly degenerate RH neutrinos (like resonant leptogenesis) the quantum effects play an important role and must be taken into account.