The NMSSM in SPheno: Two-loop RGEs and one-loop mass spectrum

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The Next-to-Minimal Supersymmetric Standard Model

Solution to the "μ-problem" of the MSSM by introducing an additional gauge singlet

$$W_{\text{MSSM}} \supset \mu \hat{H}_u \hat{H}_d \longrightarrow \mu \sim Q_{\text{EWSB}} \quad ???$$

$$W_{\text{NMSSM}} \supset \lambda \hat{H}_u \hat{H}_d \hat{S} + \frac{1}{3}\kappa \hat{S}^3 \longrightarrow \mu_{\text{eff}} = \frac{1}{\sqrt{2}}\lambda v_s \sim Q_{\text{EWSB}}$$

The singlet receives VEV after supersymmetry breaking and generates effective μ-term

$$S = \frac{1}{\sqrt{2}}\left(\phi_s + i\sigma_s + v_s\right)$$

New soft-breaking terms to be included in the scalar potential

$$V_{\text{NMSSM}} \supset m_s^2 S^2 + T_\lambda H_u H_d S + \frac{1}{3} T_\kappa S^3$$

New phenonological aspects: Five physical neutral Higgs-bosons and five neutralinos...

$$h_1, h_2, h_3, \qquad a_1, a_2, \qquad \tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0, \tilde{\chi}_5^0$$

Higgs sector

Higgs soft-breaking parameters fixed by minimum conditions of scalar potential (tadpole equations)

$$\boxed{T_i \; = \; \frac{\partial V}{\partial v_i}\Big|_{\phi_i=0} \; = \; 0} \qquad \boxed{T_i + \delta T_i \; = \; 0} \qquad (i = u, d, s)$$

Mass matrix obtained from scalar potential (tree-level) and self-energy (one-loop)

$$\boxed{\left(m^2_{h,0}\right)_{ij} \; = \; \frac{\partial^2 V}{\partial \phi_i \partial \phi_j}\Big|_{\phi_k=0,\sigma_k=0}} \qquad \boxed{m^2_{h,1}(p^2) \; = \; m^2_{h,0}(p^2) - \Pi_{hh}(p^2)}$$

One-loop masses from propagator poles and rotation matrix from diagonalization

$$\boxed{\mathrm{Det}\left[p_i^2 - m^2_{h,1}(p^2)\right] \; = \; 0} \qquad \boxed{Z_h m_h^2 Z_h^\dagger = \mathrm{diag}\left(m^2_{h_1}, m^2_{h_2}, m^2_{h_3}\right)}$$

Similar expressions for pseudo-scalar and charged Higgs-bosons

All calculations performed in ‘t Hooft gauge using the diagrammatic approach
using the Mathematica package SARAH [Staub 2008-2010]

Dominant two-loop contributions from (s)top and (s)bottom included [Degrassi & Slavich 2009]

Neutralino sector

Symmetric 5x5 mass matrix at tree-level in basis $\{\tilde{B}^0, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0, \tilde{S}\}$

$$\mathcal{M}_{\tilde{\chi}^0}^{(0)} = \begin{pmatrix} M_1 & 0 & -\frac{1}{2}g_1 v_d & \frac{1}{2}g_1 v_u & 0 \\ 0 & M_2 & \frac{1}{2}g_2 v_d & -\frac{1}{2}g_2 v_u & 0 \\ -\frac{1}{2}g_1 v_d & \frac{1}{2}g_2 v_d & 0 & -\frac{1}{\sqrt{2}}v_s\lambda & -\frac{1}{\sqrt{2}}v_u\lambda \\ \frac{1}{2}g_1 v_u & -\frac{1}{2}g_2 v_u & -\frac{1}{\sqrt{2}}v_s\lambda & 0 & -\frac{1}{\sqrt{2}}v_u\lambda \\ 0 & 0 & -\frac{1}{\sqrt{2}}v_u\lambda & -\frac{1}{\sqrt{2}}v_d\lambda & \sqrt{2}v_s\kappa \end{pmatrix}$$

One-loop mass matrix includes neutralino self-energy diagrams

$$\boxed{\mathcal{M}_{\tilde{\chi}^0}^{(1)} = \mathcal{M}_{\tilde{\chi}^0}^{(0)} - \frac{1}{2}\left[\Sigma_S(p_i^2) + \Sigma_R(p_i^2)\mathcal{M}_{\tilde{\chi}^0}^{(0)} + \mathcal{M}_{\tilde{\chi}^0}^{(0)}\Sigma_L(p_i^2)\right]}$$

Diagonalization trough 5x5 rotation matrix

$$\boxed{\mathcal{N}^*\mathcal{M}_{\tilde{\chi}^0}\mathcal{N}^\dagger = \mathrm{diag}(m_{\tilde{\chi}_1^0}, \ldots, m_{\tilde{\chi}_1^0})}$$

Slepton sector

General 6x6 slepton mass matrices at the tree-level

$$\mathcal{M}^2_{(0)} = \begin{pmatrix} \mathcal{M}^2_{\mathrm{LL}} & \mathcal{M}^2_{\mathrm{LR}} \\ \mathcal{M}^2_{\mathrm{RL}} & \mathcal{M}^2_{\mathrm{RR}} \end{pmatrix}$$

$$\boxed{\begin{aligned} \mathcal{M}^2_{\mathrm{LL}} &= M^2_{\tilde{L}} + \frac{1}{2} v_d^2 (Y_\ell)^* (Y_\ell)^T + \frac{1}{8}(g_1^2 - g_2^2)(v_d^2 - v_u^2)\mathbf{1}_3 \\ \mathcal{M}^2_{\mathrm{RR}} &= M^2_{\tilde{E}} + \frac{1}{2} v_d^2 (Y_\ell)^* (Y_\ell)^T + \frac{1}{4} g_1^2 (v_d^2 - v_u^2)\mathbf{1}_3 \\ \mathcal{M}^2_{\mathrm{LR}} &= -\frac{1}{2} v_s v_u \lambda^* (Y_\ell)^T + \frac{1}{\sqrt{2}} v_d (T_\ell)^T \end{aligned}}$$

One-loop mass matrix obtained by including self-energies

$$\boxed{\mathcal{M}^2_{(1)}(p_i^2) = \mathcal{M}^2_{(0)} - \Pi_{\tilde{\ell}\tilde{\ell}}(p_i^2)}$$

Diagonalization leads to mass eigenvalues and rotation matrix

$$\boxed{\mathcal{R}_{\tilde{\ell}} \mathcal{M}^2_{\tilde{\ell}} \mathcal{R}^\dagger_{\tilde{\ell}} = \mathrm{diag}(m^2_{\tilde{\ell}_1}, \ldots, m_{\tilde{\ell}_6})}$$

SARAH - A model builder's tool

Mathematica package to get all properties of a SUSY model from minimal amount of information

[F. Staub, arXiv:0806.0538; F. Staub, arXiv:0909.2863, F. Staub, arXiv:1002.0840]

Input: Gauge groups, particle content, superpotential → Lagrangian for gauge eigenstates

Input: Symmetry breaking(s) and rotations → Final Lagrangian, mass matrices, tadpole eqs.

Vertices, loop corrections, renormalization group equations

Model files for FeynArts and CalcHEP, LaTeX output

Large variety of supported models (with N=1 supersymmetry)

Full control over parameters (real or complex, allow/forbid off-diagonal entries, relations between parameters....)

Automatic check for gauge anomalies and charge conservation

Automatic calculation of soft-breaking terms and ghost interactions

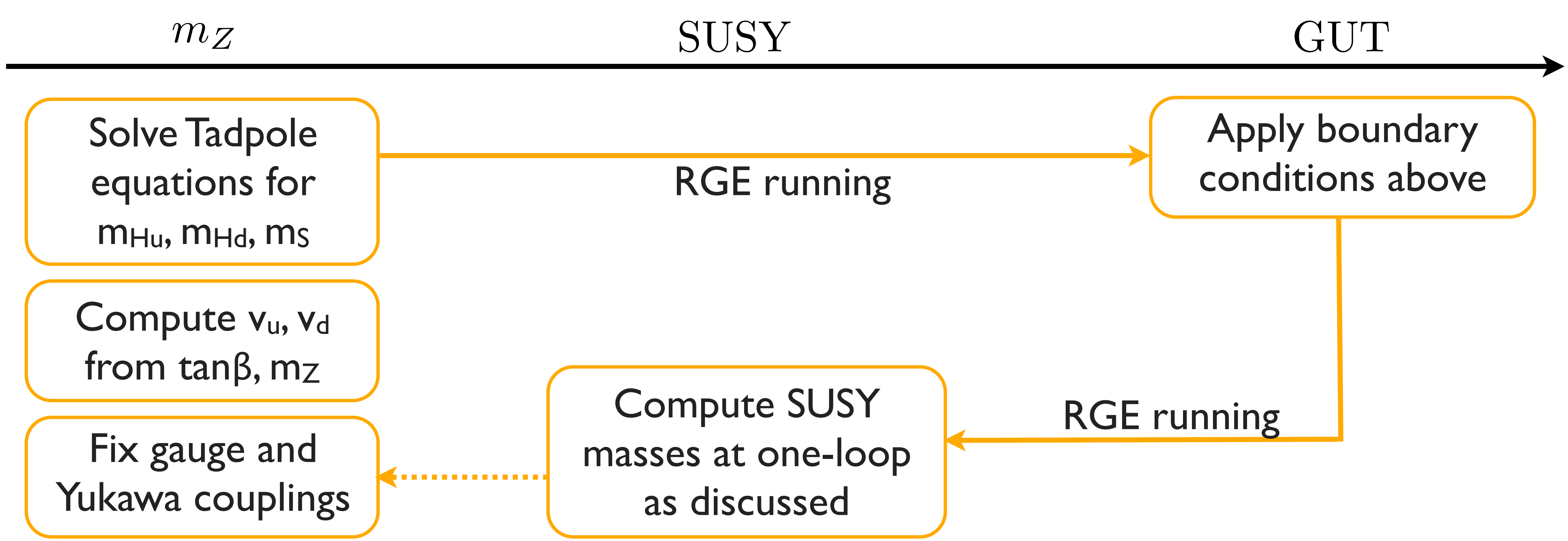
All analytical expressions for the NMSSM were calculated using SARAH
and exported to Fortran code to be included in SPheno...

The constrained NMSSM in SPheno

Consider a nine-parameter "mSUGRA-inspired" realization with unification at GUT-scale

$$\boxed{m_0, \quad M_{1/2}, \quad A_0, \quad \lambda, \quad \kappa, \quad A_\lambda, \quad A_\kappa, \quad v_s, \quad \tan\beta}$$

Parameters at various scales connected through renormalization group equations (RGEs)



Iterative solution until the physical masses converge with relative precision of 10^{-5}

An example spectrum

Particle	m_T [GeV]	m_{1L} [GeV]	Δ [%]	m_{2L} [GeV]	Δ [%]
h_1	86.7	113.3	23.5	119.6	5.2
h_2	863.1	934.2	7.6	937.3	0.3
h_3	2073.9	2073.9	< 0.1	2073.9	< 0.1
A_1^0	76.4	69.3	10.2	69.5	0.3
A_2^0	865.2	937.2	7.7	940.4	0.3
$\tilde{\chi}_1^0$	211.6	207.6	1.9	-	-
$\tilde{\chi}_2^0$	388.2	398.4	2.6	-	-
$\tilde{\chi}_3^0$	987.9	980.5	0.7	-	-
$\tilde{\chi}_4^0$	993.0	985.1	0.8	-	-
$\tilde{\chi}_5^0$	2074.8	2074.9	< 0.1	-	-
$\tilde{\chi}_1^+$	388.2	398.6	2.6	-	-
$\tilde{\chi}_2^+$	993.3	985.9	0.7	-	-
$\tilde{\tau}_1$	191.1	193.3	1.2	-	-
$\tilde{\tau}_2$	388.1	393.1	1.1	-	-
$\tilde{t}_1$	506.9	541.8	6.4	-	-
$\tilde{t}_2$	914.4	949.3	3.7	-	-
$\tilde{b}_1$	845.3	880.4	3.9	-	-
$\tilde{b}_2$	961.9	1008.5	4.6	-	-
$\tilde{g}$	1107.2	1154.2	4.1	-	-

Large correction for light Higgs-boson, which is main motivation for including the two-loop contribution

Input parameters:

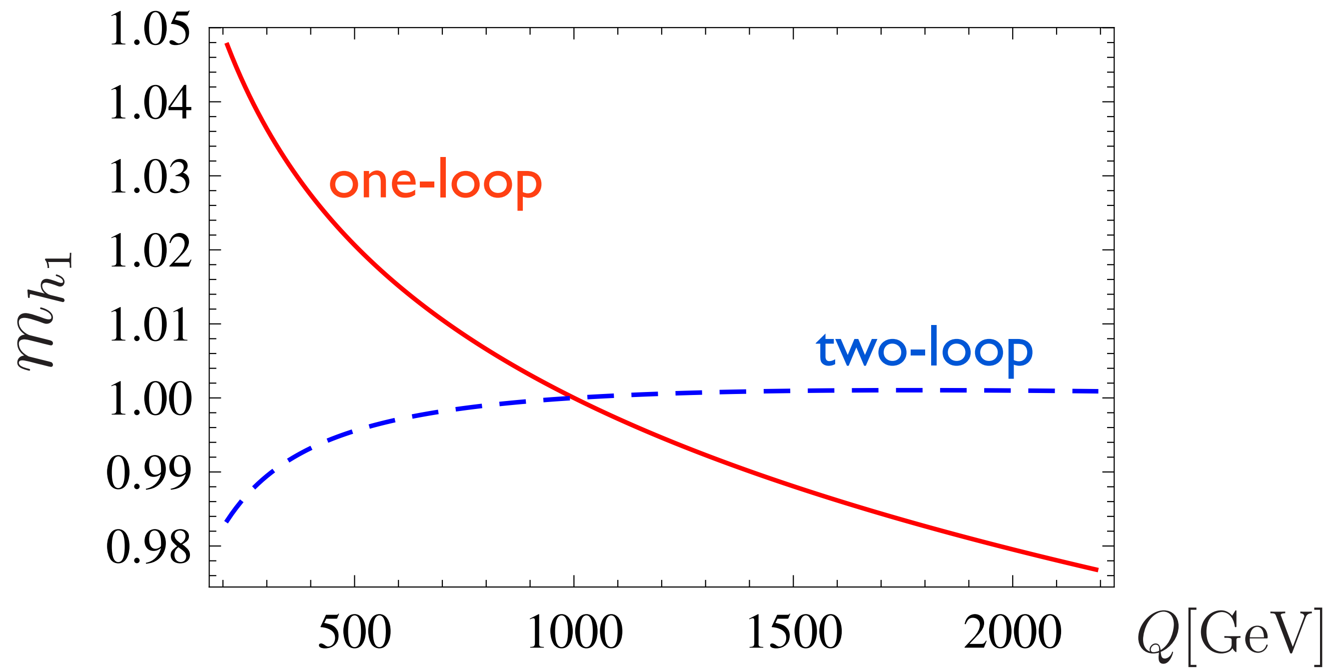
$$\begin{aligned} m_0 &= 180\ \text{GeV} \\ M_{1/2} &= 500\ \text{GeV} \\ A_{0,\lambda} &= -1500\ \text{GeV} \\ A_\kappa &= -36\ \text{GeV} \\ \tan\beta &= 10 \\ \lambda &= 0.1 \\ \kappa &= 0.11 \\ v_s &= 13689\ \text{GeV} \end{aligned}$$

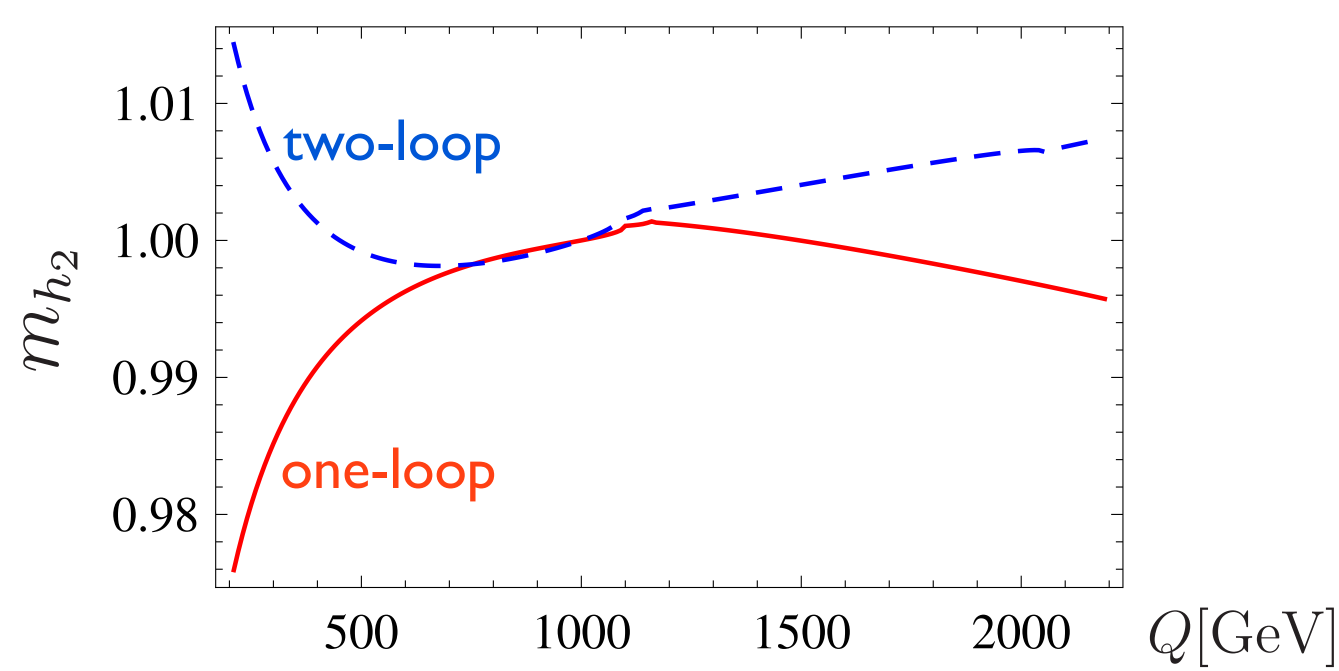
[Djouadi *et al.* 2008]

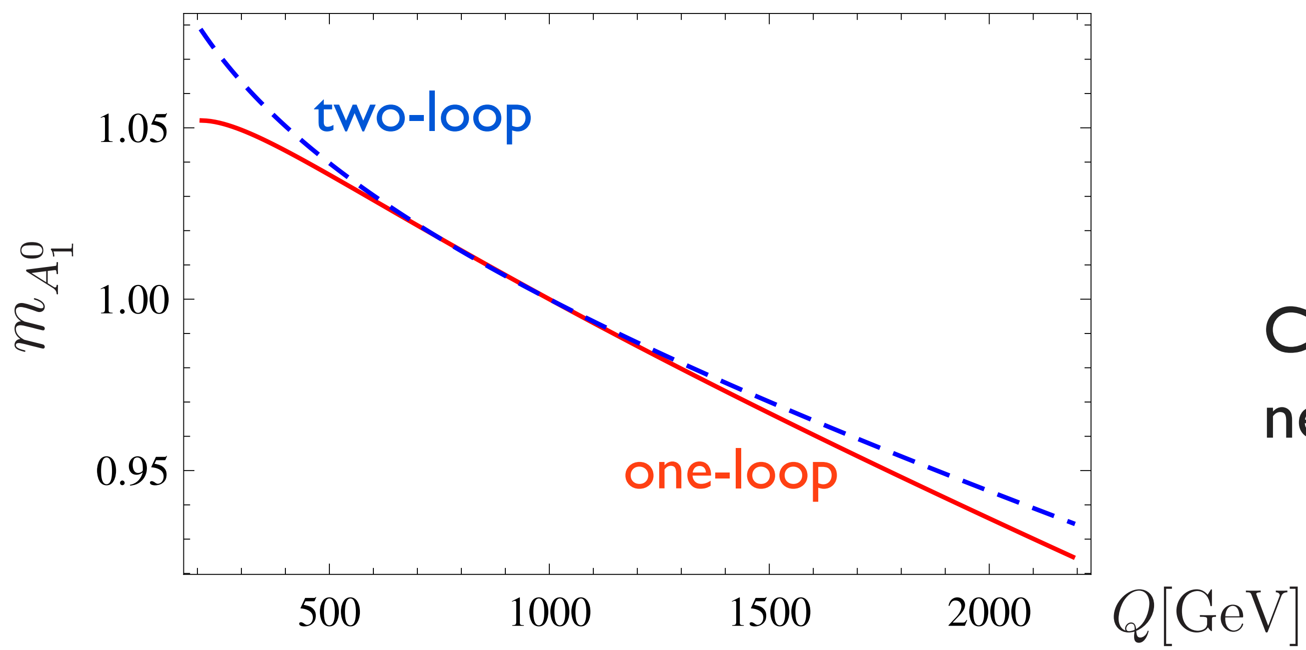
Higgs masses: Theoretical error estimate

Dependence on renormalization scale improves significantly at two-loop level for scalars

No improvement for lightest pseudo-scalar due to only strong contributions in the two-loop part







Contributions from Yukawa couplings needed for further improvement...

Comparison with NMSSM-Tools

Only spectrum generator for NMSSM to date: NMSSM-Tools 2.3.1 [Ellwanger & Hugonie 2006-2010]

SPheno NMSSM

- Masses are computed at SUSY scale
- Full momentum-dependence in Higgs-sector
- Full one-loop calculation for scalars and pseudoscalars plus two-loop contributions [Degrassi & Slavich 2009]
- Complete one-loop corrections to neutralino/chargino and slepton masses

NMSSM-Tools

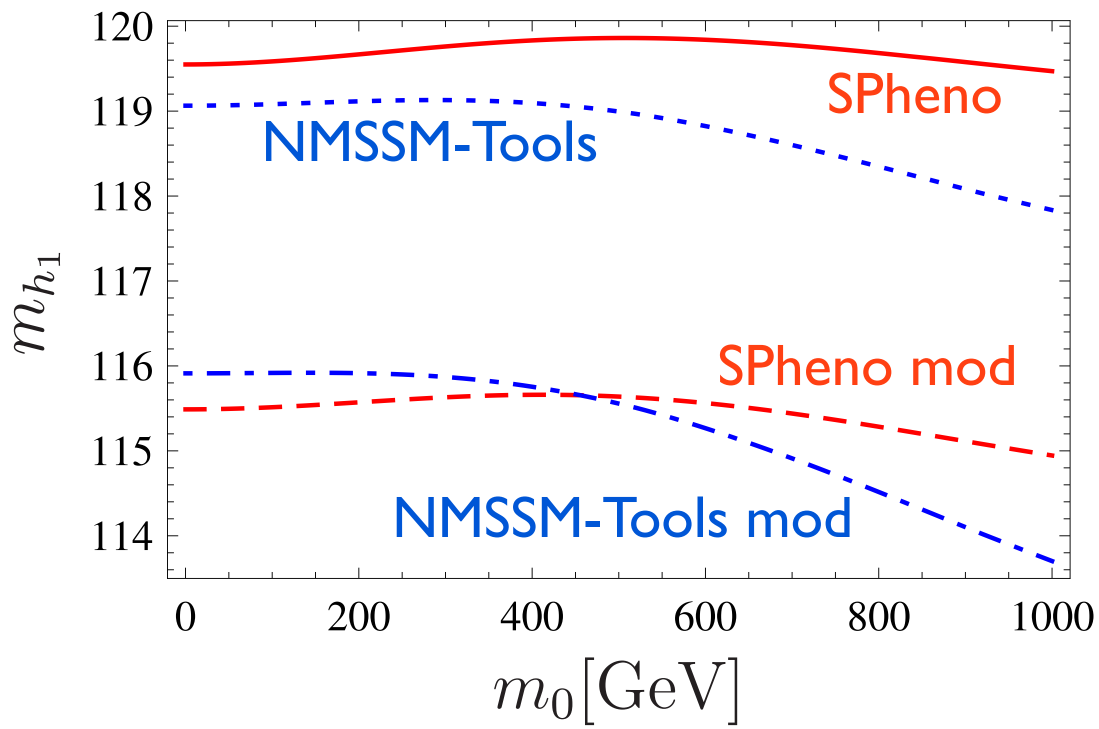
- Different scale for mass computation
- Effective potential approach in Higgs sector (external momenta zero and momentum-dependant contributions from top/bottom)
- For pseudo-scalars, only dominant one-loop corrections from (s)tops/(s)bottoms
- Only corrections to M_1, M_2, and μ_{eff} for neutralinos/charginos
- No one-loop corrections for sleptons

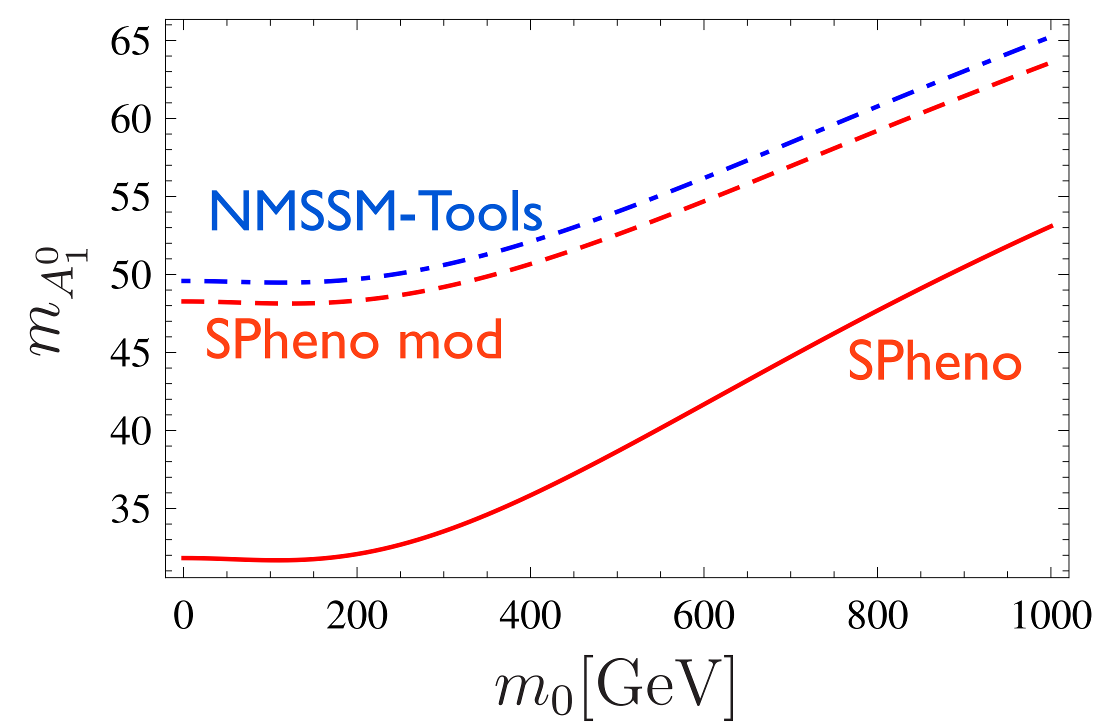
Modifications for numerical comparison:

- set scales $Q_{STSB}=Q_{SUSY}$ in NMSSM-Tools
- set external momenta to zero in SPheno
- switch off two-loop contributions in both codes
- keep in SPheno only same corrections to pseudoscalar masses as in NMSSM-Tools

Higgs masses: Comparison with existing results

Largest discrepancies for lighter scalar (up to 2.5%) and pseudo-scalar (up to 35%) Higgs bosons





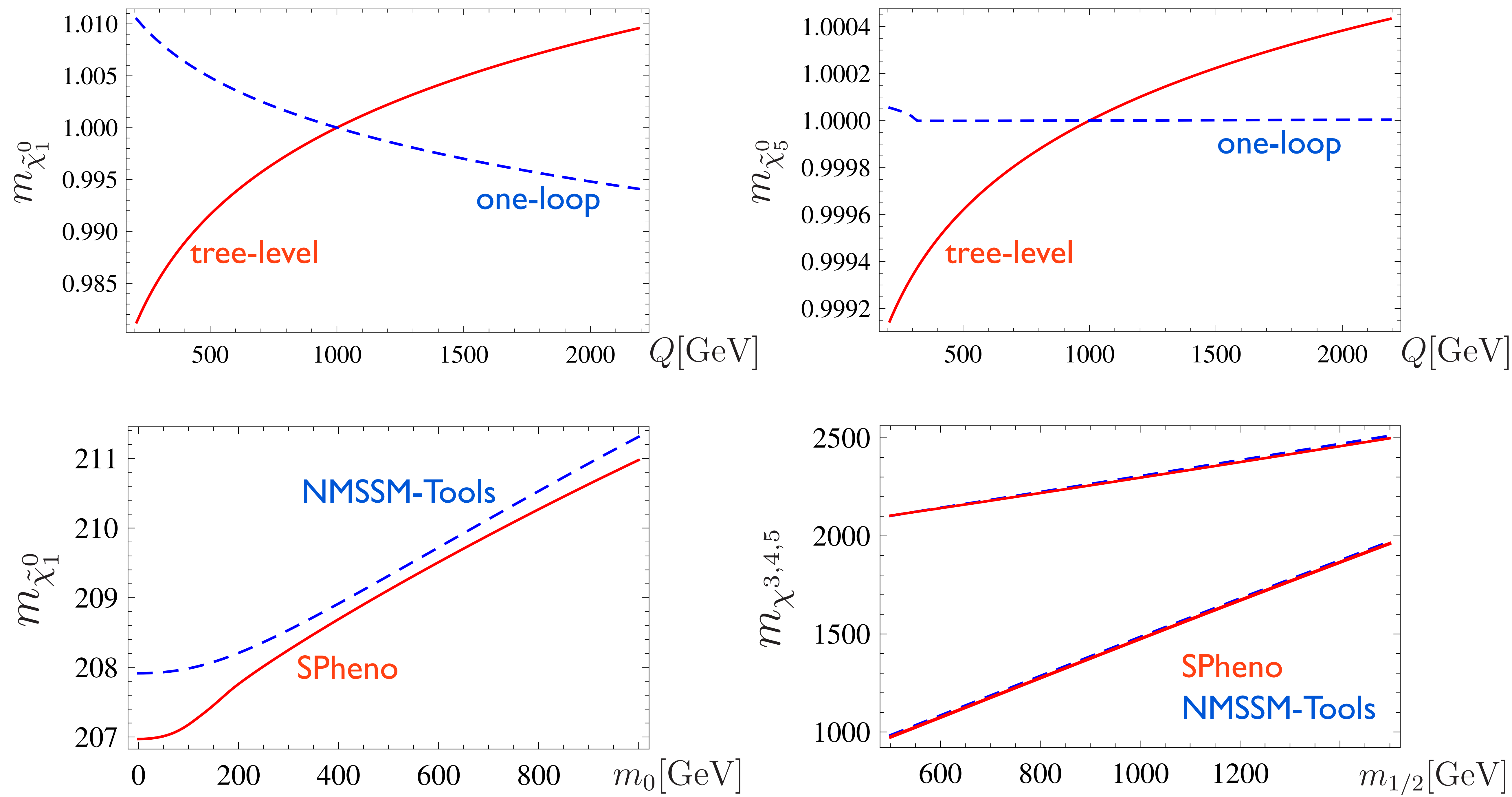
For lightest scalar: p^2-terms in loop functions and additional two-loop contributions
For pseudo-scalar: only contributions from 3rd sfermions in NMSSM-Tools while full one-loop result plus two-loop contributions in SPheno

Modified programs differ by at most 2% due to different solutions of the tadpole equations

Perfect agreement with one-loop results from independent routines [Degrassi & Slavich, 2009]
(small difference of one per-mille due to Yukawa and trilinear couplings of 1st and 2nd generation)

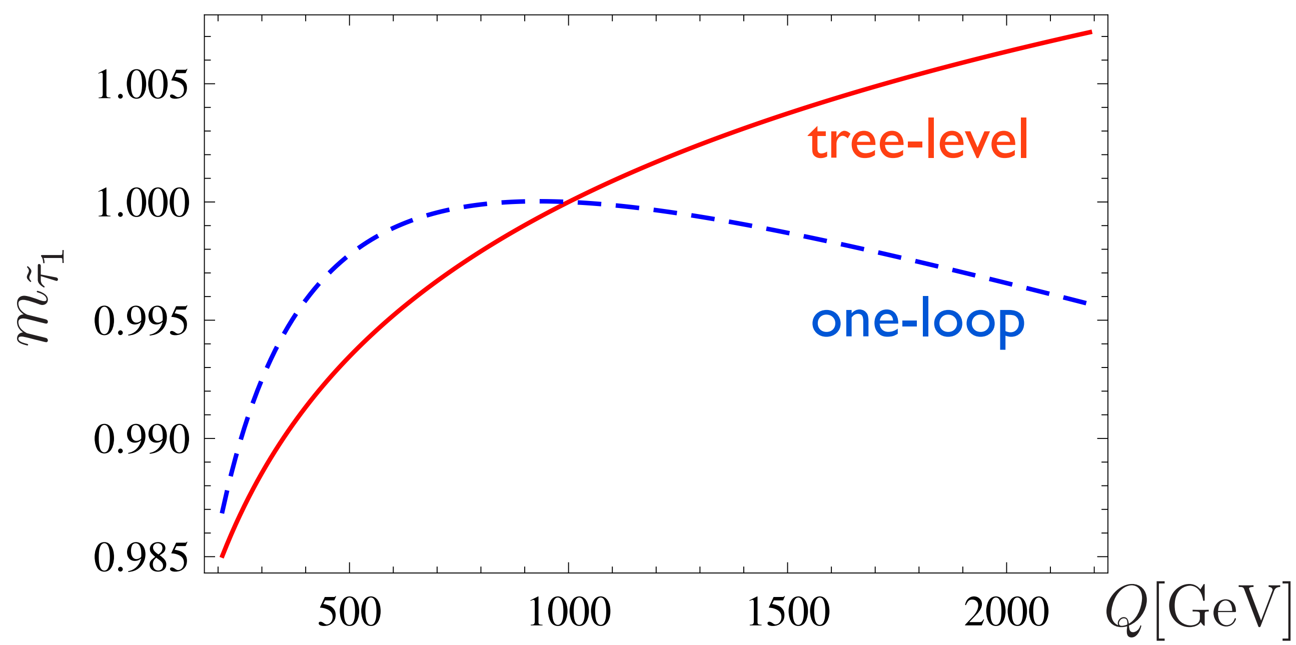
Neutralino masses

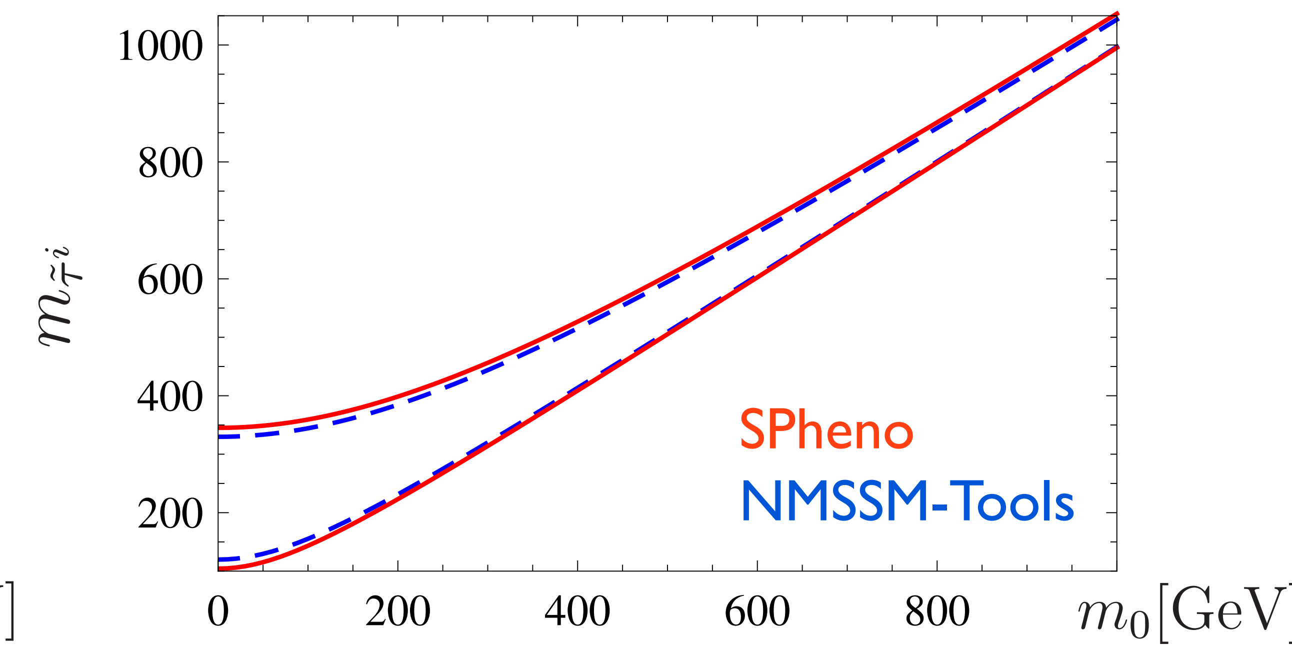
Reduced dependence on renormalization scale at the one-loop level,
good agreement with NMSSM-Tools for neutralino masses (at most 1%, generally slighly below 0.5%)



Slepton masses

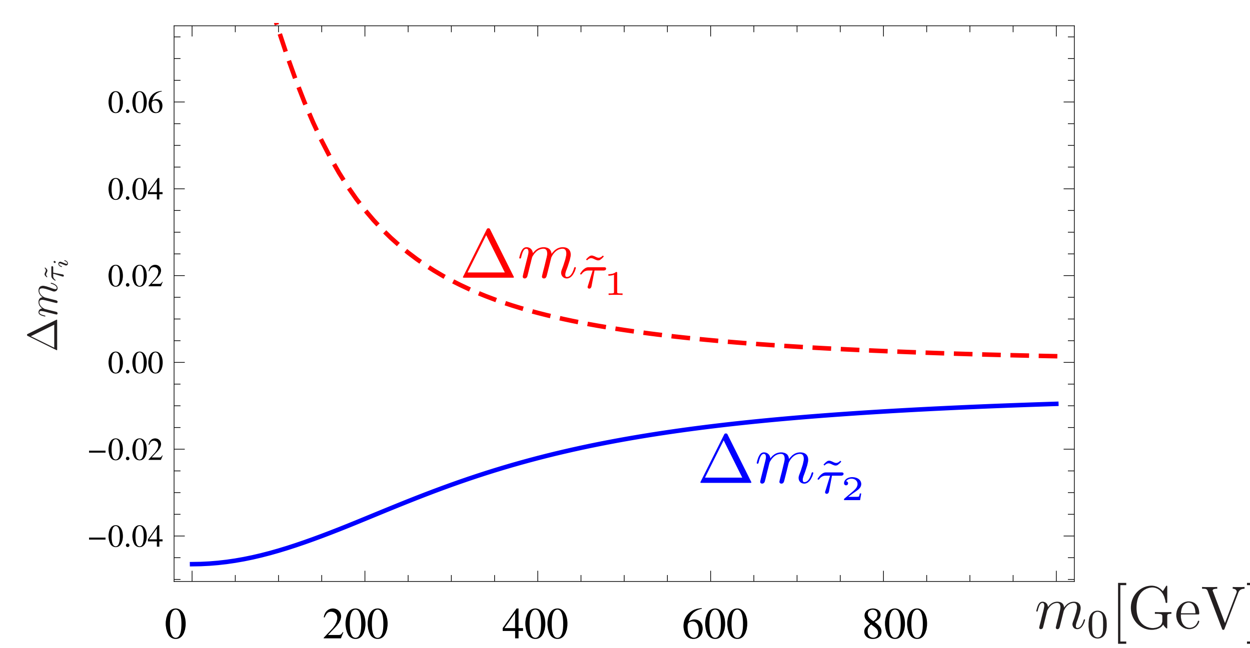
Lighter stau shows largest dependence on renormalization scale (about 1% at one-loop)





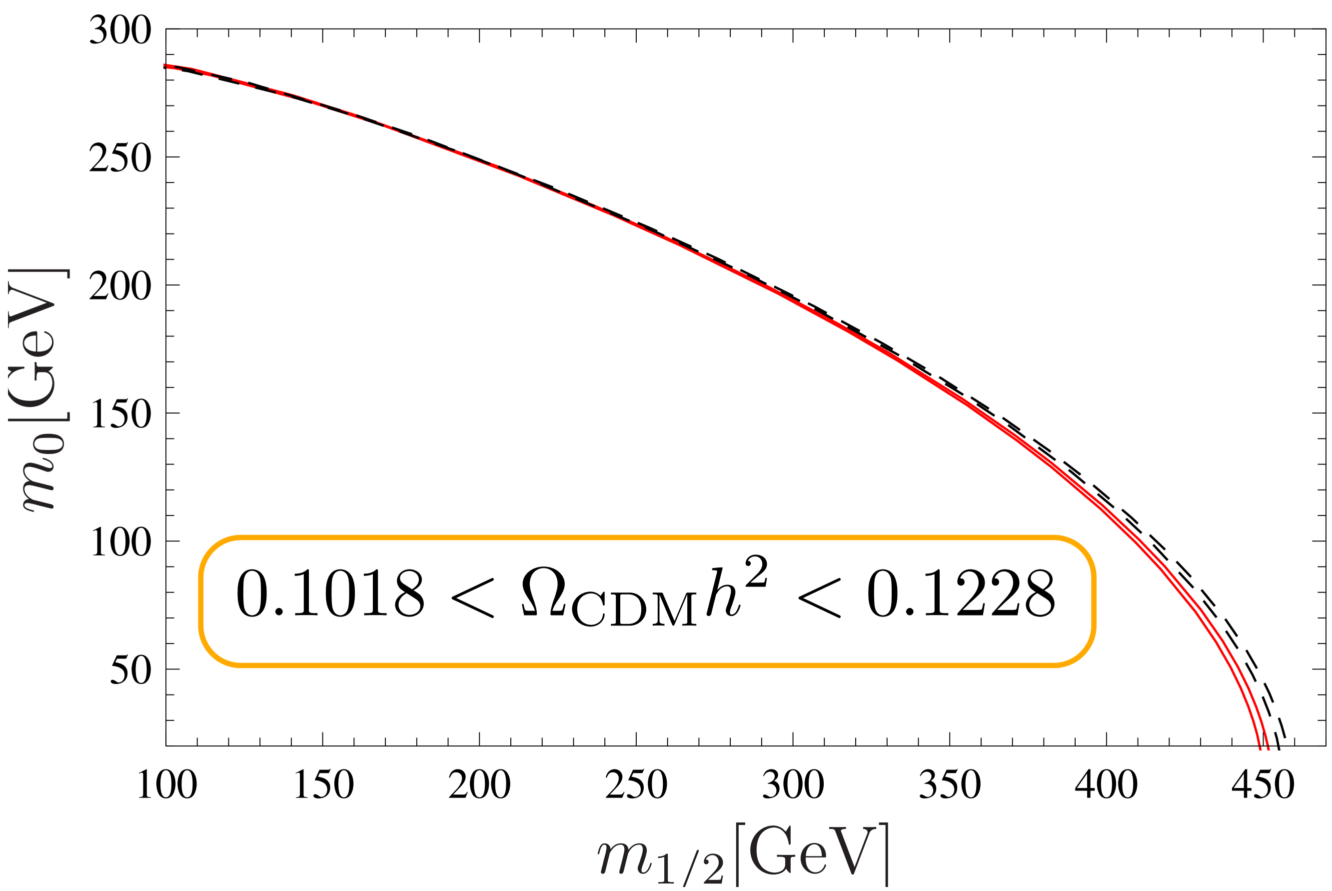
Differences in slepton masses more pronounced between the two codes due to the one-loop contributions included in SPheno

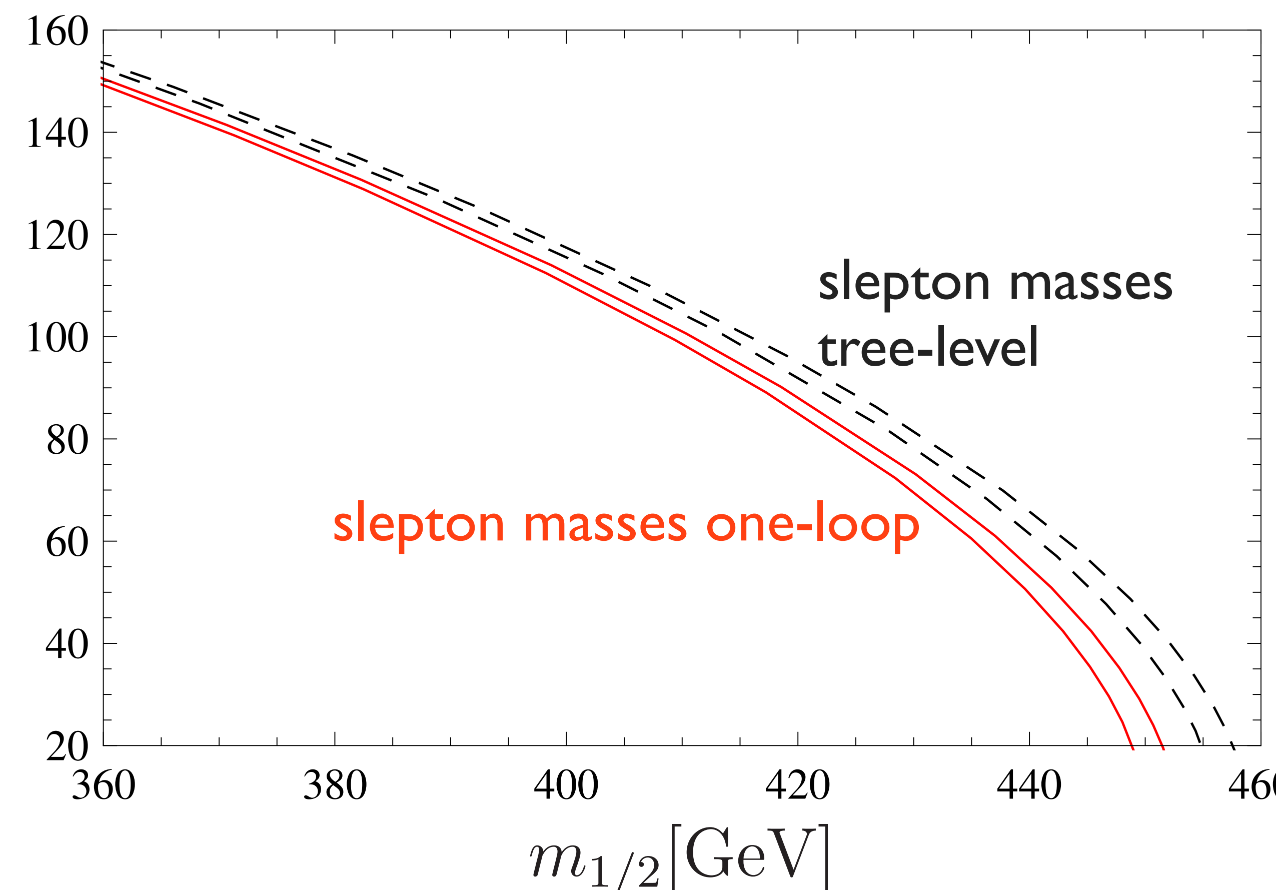
Similar uncertainties for LHC physics, but such precision necessary for linear collider and dark matter calculation...



Impact on dark matter relic density

Neutralino relic density can be very sensitive to mass configuration,
in particular for Higgs-funnels or co-annihilation regions (shown below)





Corrections to the stau mass shift preferred region in parameter space
Neutralino mass less important in this context

Relic density computed using micrOMEGAs with CalcHEP model files generated by SARAH and taking into account important QCD effects (as also in NMSSM-Tools)

Summary

The NMSSM is an attractive extension of the MSSM since it solves the "μ-problem"
This leads to interesting new phenomenological at present and future collider experiments
and can explain the amount of dark matter in our universe
However, more accurate theoretical prediction are needed, e.g. for constraints from WMAP data

Complete one-loop calculation of the electroweak sector (Higgs bosons, neutralinos/charginos, sleptons)
obtained with the help of the Mathematica package SARAH [Staub 2008-2010]
and implemented in spectrum calculator SPheno [Porod 2003-2010]

Numerical implementation of constrained NMSSM reproduces known results for the Higgs-sector
Corrections for other particles amount to a few percent
(below precision of LHC data, but clearly important in the context of WMAP data and a future linear collider)
[Staub, Porod, Herrmann 2010]

The special SPheno version can be obtained upon request and will become public in near future...
Further realizations of the NMSSM (other than 9-parameter cNMSSM) can easily be implemented...

Implementation and verification of Wilson coefficients for precision observables
($b \to s\gamma$, $b \to s\mu\mu$, Δm_{B_s},...) and particle decays in progress...