The Flavour Puzzle :
Why Neutrinos are Different ?

Fu-Sin Ling (ULB)

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Work in collaboration with

Jean-Marie Frère (ULB),
Maxim Libanov, Emin Nugaev, Sergei Troitsky (INR)

The flavour puzzle in a nutshell

- **Why three families in the SM ?**
  - Hierarchical masses + small mixing angles
- **Why massive neutrinos ?**
  - Tiny masses + two large mixing angles
- **Why very suppressed FCNC ?**
  - Strong limits on a TeV scale extension of the SM

THE FLAVOUR PUZZLE IN A NUTSHELL

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Proposed solution :

A model of family replication in 6D

3 FAMILIES IN 4D FROM 1 FAMILY IN 6D

- **Vortex in 6D**

  $U(1)_g$ gauge field A + background scalar field Φ

- **Family replication**

  One single fermion coupled to vortex leads to several (three ?) chiral zero-modes (index theorem)

- **New quantum number**

  Family number in 4D corresponds to winding number in extradimensions

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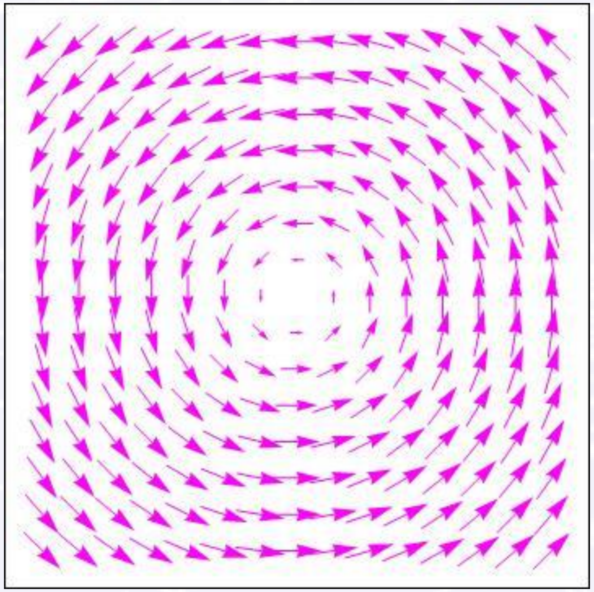
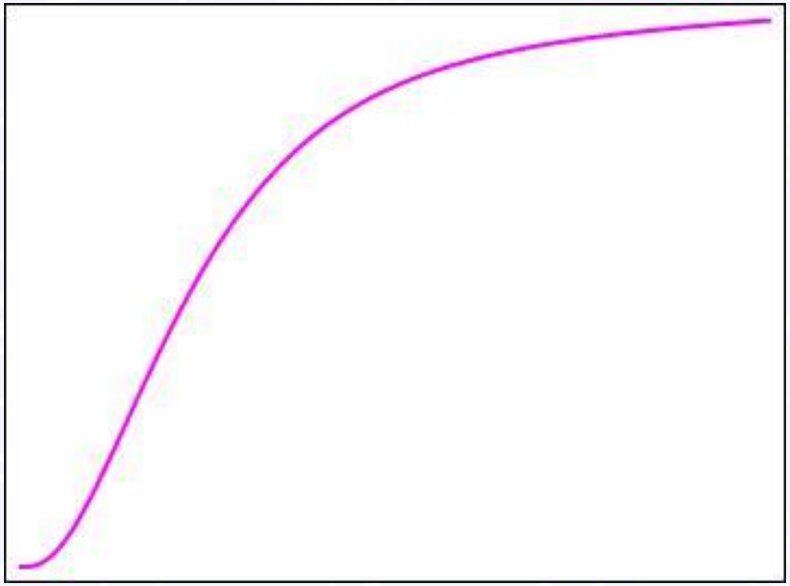
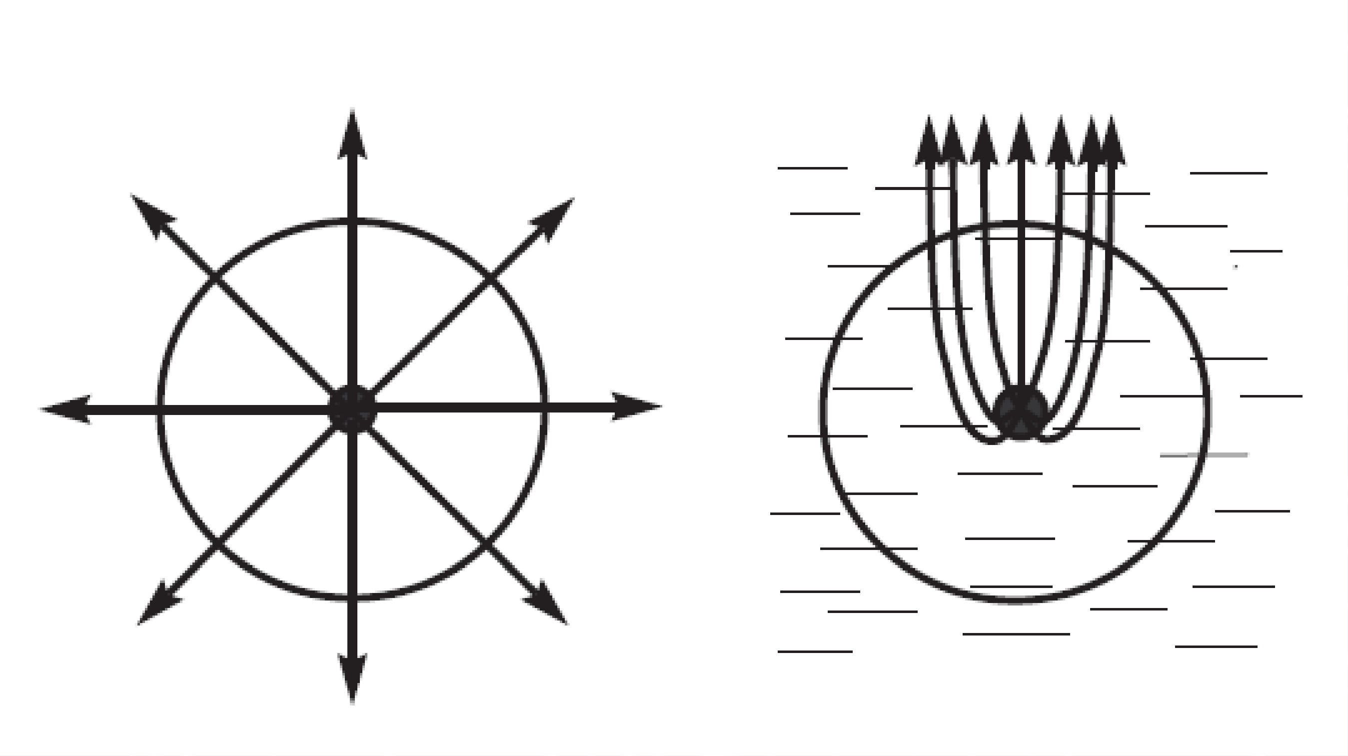
$U(1)_g$ gauge field A + background scalar field Φ





ABIKOSOV-NIELSEN-OLESEN VORTEX

- A vortex on a sphere is in fact like a magnetic monopole configuration in 3D

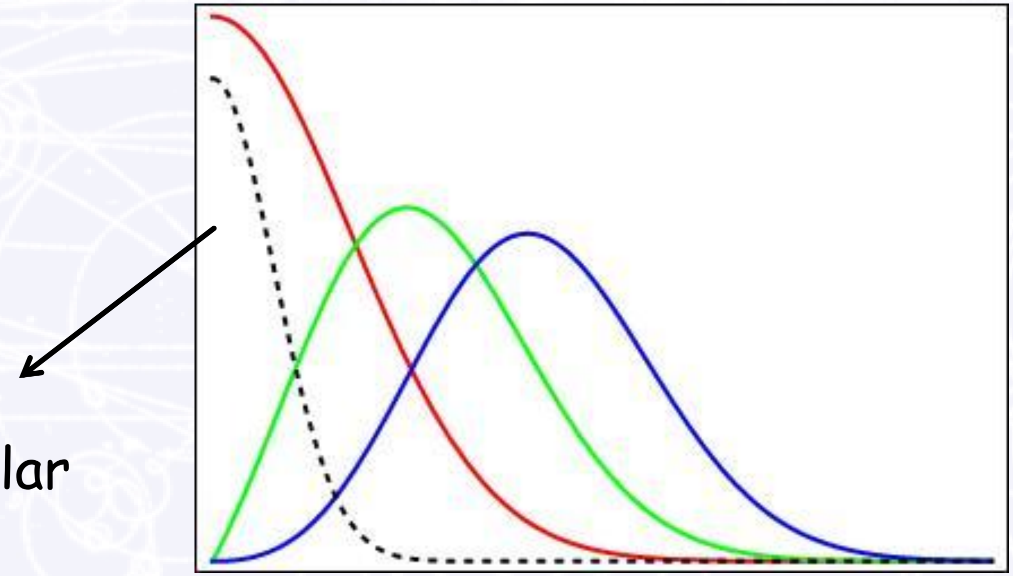


3 FAMILIES IN 4D FROM 1 FAMILY IN 6D

- **Fermion zero-modes**

  Different profile and different winding around the vortex

Narrow
B-E-H scalar



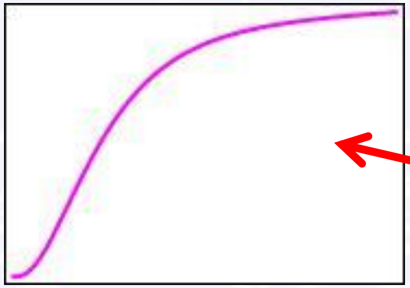
$e^{i0\phi}$

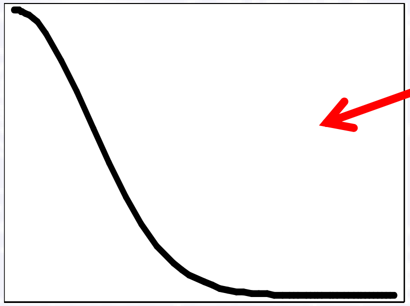
$e^{i1\phi}$

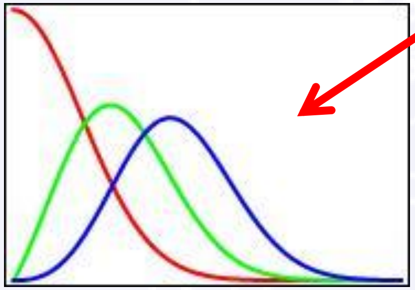
$e^{i2\phi}$

Note that the profiles are determined by a Dirac equation in the vortex background

Field content of the model







fields		charges		representations	
		$U(1)_g$	$U(1)_Y$	$SU(2)_W$	$SU(3)_C$
scalar	Φ	$+1$	0	$\mathbf{1}$	$\mathbf{1}$
scalar	X	$+1$	0	$\mathbf{1}$	$\mathbf{1}$
scalar	H	-1	$+1/2$	$\mathbf{2}$	$\mathbf{1}$
fermion	L_+, L_-	$(3,0)$	$-1/2$	$\mathbf{2}$	$\mathbf{1}$
fermion	E_+, E_-	$(0,3)$	-1	$\mathbf{1}$	$\mathbf{1}$
fermion	N	0	0	$\mathbf{1}$	$\mathbf{1}$

Hierarchical Dirac masses

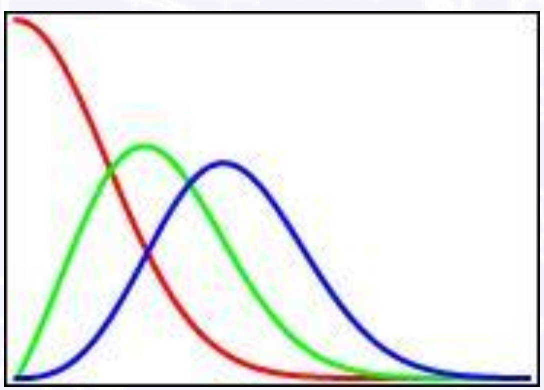
$$\Psi = \begin{pmatrix} \psi_{+R} \\ \psi_{+L} \\ \psi_{-L} \\ \psi_{-R} \end{pmatrix}$$

$$Y_X H X \bar{L} \frac{1-\Gamma_7}{2} E + Y_\Phi H \Phi \bar{L} \frac{1-\Gamma_7}{2} E$$

$$L_n \sim \begin{pmatrix} 0 \\ e^{-i\phi(n-7/2)} f_2(n,\theta)\, l_n(x^\mu) \\ e^{-i\phi(n-1/2)} f_3(n,\theta)\, l_n(x^\mu) \\ 0 \end{pmatrix} \qquad E_m \sim \begin{pmatrix} e^{-i\phi(m-1/2)} f_3(m,\theta)\, \bar{e}_m(x^\mu) \\ 0 \\ 0 \\ e^{-i\phi(m-7/2)} f_2(m,\theta)\, \bar{e}_m(x^\mu) \end{pmatrix}$$

$$f_2 \sim \theta^{3-n} \quad f_3 \sim \theta^{n-1} \quad n = 1, 2, 3$$

Integration over ϕ → $\delta(n-m)$



$$M_l \sim \begin{pmatrix} \delta^4 & \delta^3 & \\ & \delta^2 & \delta \\ & & 1 \end{pmatrix}$$

NEUTRINOS MASSES

$$\Psi = \begin{pmatrix} \psi_{+R} \\ \psi_{+L} \\ \psi_{-L} \\ \psi_{-R} \end{pmatrix}$$

- Why is it different ?

See-saw mechanism → $\bar{L}^c L \;\; + \text{h.c.}$

$$L_n \sim \begin{pmatrix} 0 \\ \mathrm{e}^{-i\phi(n-7/2)} f_2(n,\theta)\, l_n(x^\mu) \\ \mathrm{e}^{-i\phi(n-1/2)} f_3(n,\theta)\, l_n(x^\mu) \\ 0 \end{pmatrix}$$

$$f_2 \sim \theta^{3-n} \quad f_3 \sim \theta^{n-1} \quad n = 1,2,3$$

Integration over ϕ → $\delta(4-n-m)$

$$M_\nu \sim \begin{pmatrix} \cdot & \cdot & \times \\ \cdot & \cdot & \cdot \\ \times & \cdot & \cdot \end{pmatrix}$$

Neutrinos masses

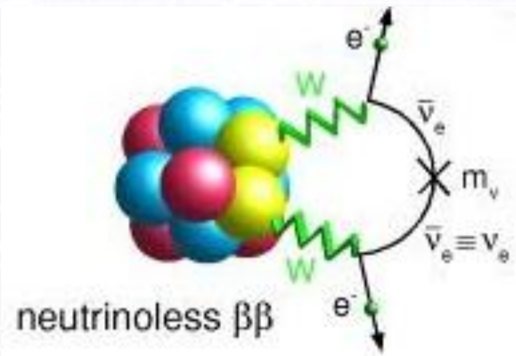
$$M_\nu \sim \begin{pmatrix} \cdot & \cdot & \times \\ \cdot & \cdot & \cdot \\ \times & \cdot & \cdot \end{pmatrix}$$

- Consequences of this structure
  - Inverted hierarchy with a pseudo-Dirac pair $m_1 \simeq -m_2 \gg m_3$
  - Solar angle automatically large
  - Small reactor angle $U_{e3} \sim \delta$
  - Correct prediction for Δm^2 ratio $\sim \delta^2$

Neutrinos masses

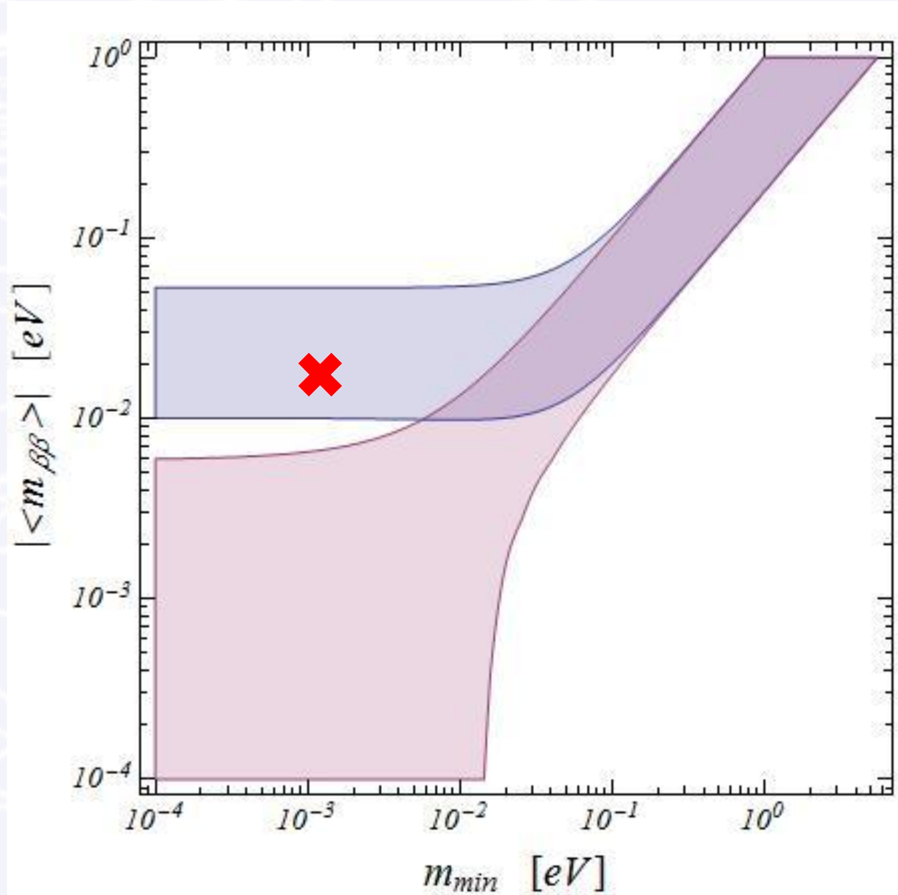
$$M_\nu \sim \begin{pmatrix} \cdot & \cdot & \times \\ \cdot & \cdot & \cdot \\ \times & \cdot & \cdot \end{pmatrix}$$

- Consequences of this structure
  - $0\nu\beta\beta$ decay



partial suppression

$$|\langle m_{\beta\beta}\rangle| \simeq \frac{1}{3}\sqrt{\Delta m^2_{\oplus}}$$



Numerical example

- With a good selection of Yukawa operators, we can get

$$M_\nu \sim \begin{pmatrix} \cdot & \times & \times \\ \times & \cdot & \cdot \\ \times & \cdot & \cdot \end{pmatrix}$$

→ ***Possibility to have a bimaximal mixing***

$$S_+ = \boxed{\Phi^*,\ X^*,}\ X^{*2}\Phi,\ \ldots$$
$$S_- = X^2,\ \boxed{X\Phi,}\ \Phi^2,\ \ldots$$

$$\tilde{Y}_\nu^+ = y_\nu\,\{1, 1.7\} \qquad y_\nu = 2.8 \cdot 10^{-2}$$
$$\tilde{Y}_\nu^- = y_\nu \qquad M = 1/R = 70\ \text{TeV}$$

Numerical example

$$M_\nu = \begin{pmatrix} 0 & 3.62 \cdot 10^{-2} & 3.50 \cdot 10^{-2} \\ 3.62 \cdot 10^{-2} & 1.46 \cdot 10^{-3} & 0 \\ 3.50 \cdot 10^{-2} & 0 & 0 \end{pmatrix} \quad [\mathrm{eV}]$$

$$\Delta m^2_{21} = 7.63 \times 10^{-5}\ \mathrm{eV}^2$$
$$\Delta m^2_{13} = 2.50 \times 10^{-3}\ \mathrm{eV}^2$$

$\rightarrow$ $\Delta m^2_{21}/\Delta m^2_{13} = 3.05\%$

$$M_l = \begin{pmatrix} 4.21 \cdot 10^{-4} & 1.08 \cdot 10^{-3} & 0 \\ 0 & 4.19 \cdot 10^{-3} & 5.98 \cdot 10^{-2} \\ 0 & 0 & 1.71 \end{pmatrix} \quad [\mathrm{GeV}]$$

$$U_l^\dagger M_l V_l = D_l = \mathrm{diag}\{4.07 \cdot 10^{-4}, 4.33 \cdot 10^{-3}, 1.71\} \quad [\mathrm{GeV}]$$

Numerical example

$$U_{MNS} = \begin{pmatrix} 0.808 & 0.555 & 0.196 \\ -0.286 & 0.662 & -0.693 \\ -0.514 & 0.504 & 0.694 \end{pmatrix}$$

$\tan^2\theta_{12} = 0.471$ $\tan^2\theta_{23} = 0.997$ $\sin^2\theta_{13} = 3.85 \cdot 10^{-2}$

- Consequence for $0\nu\beta\beta$ decay

$$\langle m_{\beta\beta}\rangle = \sum_i m_i U_{ei}^2$$

$$|\langle m_{\beta\beta}\rangle| = 17.0 \quad \text{meV}$$

→ ***Partially suppressed effective Majorana mass***

Flavour violation

Frère et al. hep-ph/0309014

- Like in the UED, vector bosons can travel in the bulk of space. From the 4D point of view :

1 massless vector boson in 6D =

1 massless vector boson (zero-mode)

+ KK tower of massive vector bosons

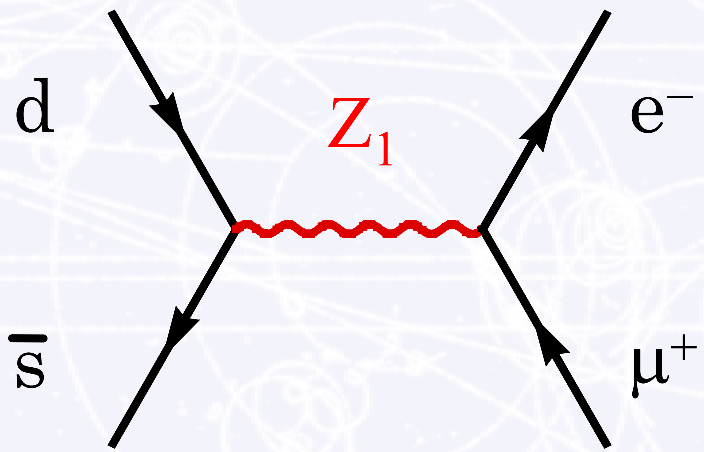
+ KK tower of massive scalar bosons in 4D

- KK scalar modes do not interact with fermion zero-modes

Flavour violation

Frère et al. hep-ph/0309014

- KK vector modes carry a family number = winding number. In the absence of fermion mixings, family number is an exactly conserved quantity
- Example: <u>FCNC with *ΔG=0*</u>



$K_L \to \mu^+ e^-$ or $\mu^- e^+$

→ ***Flavour violating***

→ ***Family conserving***

$$B.R. < 10^{-12} \quad \to \quad R^{-1} \geq \kappa \cdot 100 \text{ TeV}$$

FLAVOUR VIOLATION

Frère et al. hep-ph/0309014

- All processes with $\underline{\Delta G \neq 0}$ automatically suppressed by small fermion Cabibbo mixings

$\underline{\Delta G=1}$ $\mu^- \to e^- e^- e^+$

$\mu^- \to e^- \gamma$

$\mu^- \to e^-$ on nuclei

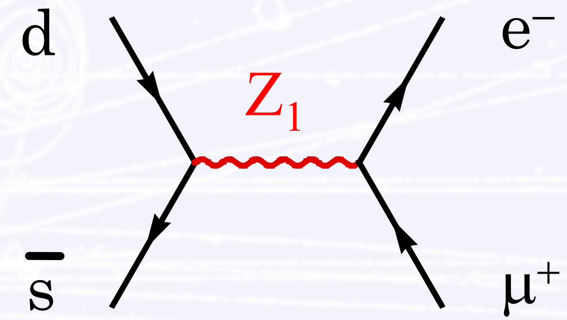
$\underline{\Delta G=2}$ $K_L - K_S$ mass difference and CP violation

→ ***Less constraining !***

Search at LHC

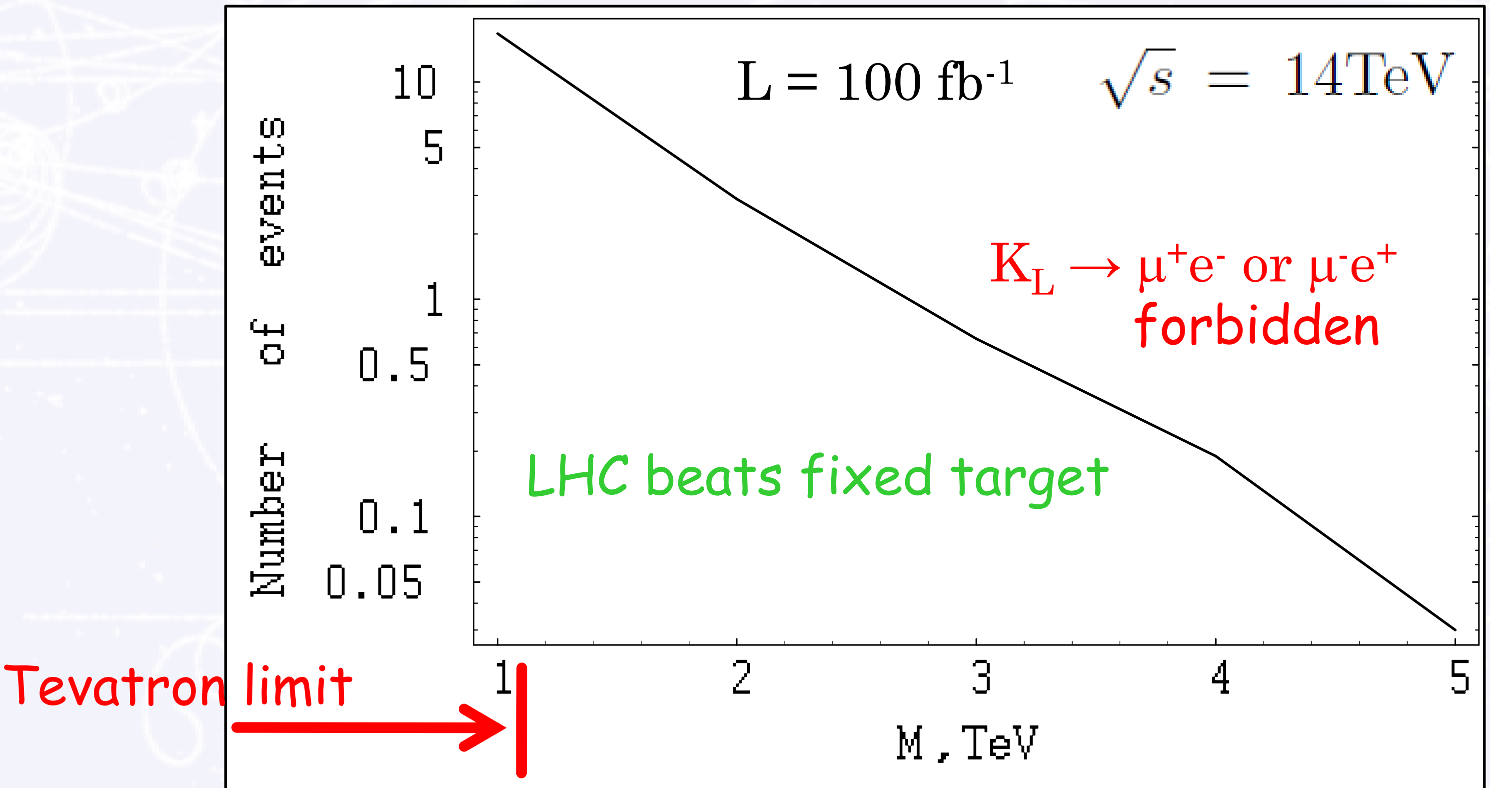
- Search for massive Z′

Frère et al. hep-ph/0404139



- Search for pp ⟶ $\mu^+ e^-$ + …

(pp ⟶ $\mu^- e^+$ + … lower by a factor 10 due to quark content of proton)



CONCLUSIONS

- Family replication model in 6D : elegant solution to the flavour puzzle
  - Hierarchical Dirac masses + small mixing angles
  - See-saw : can fit neutrino data
  - Universality of gauge structure like in SM
  - Family number violating FCNC suppressed by small fermion mixings
- Predictions for neutrinos
  - Inverted hierarchy
  - Reactor angle ~ 0.1
  - Partially suppressed neutrinoless $\beta\beta$ decay

Conclusions

- Testable at LHC
  - Massive gauge bosons can carry a family number
  - Search for massive gauge bosons with mass ~ TeV or higher
  - Search for $pp \rightarrow \mu^+ e^- + \ldots$ can beat fixed target