

Fitting New Physics

Michael Rauch | November 4, 2010

INSTITUTE FOR THEORETICAL PHYSICS



- Fitting techniques
- “LHC SUSY Weather Forecast”
- Low luminosity prospects
- High luminosity and ILC data
- Higgs couplings

Parameters in the Lagrangian

$m_0, \mu, \tan(\beta), M_{\{1,2,3\}}, \dots$

Feynman diagrams,
RG evolution, ...

Observables:

- Masses
- Kinematic endpoints
- Cross sections
- Branching ratios
- ...

Observables

$m_{h^0}, \Delta m_{\tilde{g}\chi_1^0}, \text{three-particle edge}(\chi_4^0, \tilde{e}_L, \chi_1^0), \text{BR}, \dots$

?

Lagrangian parameters

M_1	<input type="checkbox"/> ± <input type="checkbox"/> GeV
M_2	<input type="checkbox"/> ± <input type="checkbox"/> GeV
M_3	<input type="checkbox"/> ± <input type="checkbox"/> GeV
μ	<input type="checkbox"/> ± <input type="checkbox"/> GeV
$\tan \beta$	<input type="checkbox"/> ± <input type="checkbox"/>
...	...

⇒ Tools to reconstruct SUSY parameters

- SFitter:
Lafaye, Plehn, MR, Zerwas
- Fittino:
Bechtle, Desch, Wienemann
- MasterCode:
Buchmüller, Cavanaugh, De Roeck, Ellis, Flücher, Hahn,
Heinemeyer, Isidori, Olive, Rogerson, Ronga, Weiglein
- Roszkowski, Ruiz de Austra, Trotta
- Allanach, Cranmer, Lester, Weber
- AbdusSalam, Allanach, Dolan, Quevedo, Feroz, Hobson
- Brummer, Fichet, Kraml, Singh
- Dreiner, Krämer, Lindert, O'Leary
- ...

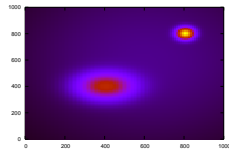
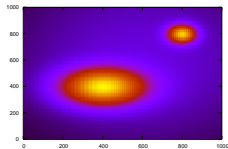


- MSSM parameter space is high-dimensional:
 - SM: 3+ parameters ($m_t, \alpha_s, \alpha, \dots$)
 - mSUGRA: 5 parameters ($m_0, m_{1/2}, A_0, \tan(\beta), \text{sgn}(\mu)$)
 - General MSSM: 105 parameters
- On loop-level observables depend on every parameter
Simple inversion of the relations not possible
⇒ Parameter scans
- Error estimates on parameters in the minimum

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Find best points (best χ^2) using different fitting techniques:

- Gradient search (Minuit) (+ Reasonably fast
- Limited convergence, only best fit)



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- Gradient search (Minuit) $\left(\begin{array}{l} + \text{ Reasonably fast} \\ - \text{ Limited convergence, only best fit} \end{array} \right)$
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- Markov Chains
- Simulated Annealing

Markov Chain (MC):

- Sequence of points, chosen by an algorithm (Metropolis-Hastings), only depending on its direct predecessor
 - Picks a set of "average" points according to a potential V (e.g. inverse log-likelihood, $1/\chi^2$)
 - Point density resembles the value of V (i.e. more points in region with high V)
 - Scans high dimensional parameter spaces efficiently [Baltz, Gondolo 2004]
- + Fast scans of high-dimensional spaces $\mathcal{O}(N)$
- + Does not rely on shape of χ^2 (no derivatives used)
- + Can find secondary distinct solutions
- Exact minimum difficult to find \Rightarrow Additional gradient fit
- Bad choice of proposal function for next point leads to bad coverage of the space

Simulated Annealing:

- Acceptance probability dependent on "temperature"
- Temperature gradually decreased during MC run
- Gradual changeover between parameter-space scanning and finding optimum

■ Bayesian

“How likely is a given parameter value given the data?”

- based on Bayes theorem: $p(m|d) = p(d|m) \frac{p(m)}{p(d)}$
 - $p(m|d)$: probability of the model, given the data
 - $p(d|m)$: probability of the data, given the model (“ χ^2 ”)
 - $p(d)$: probability of the data, typically $p(d) = 1$
 - $p(m)$: probability of the model = prior
- prior $p(m)$: subjective quantity of initial knowledge/ignorance about parameters introduces measure in parameter space
- typical choices:
 - linear: $p(m) \propto \text{const.}$, flat in m
 - logarithmic: $p(m) \propto \frac{1}{m}$, flat in $\log(m)$
 - Gaussian: $p(m) \propto \exp^{-(m-\bar{m})^2/2\sigma^2}$
- Nuisance Parameters:
 - parameters which are “uninteresting” (e.g. SM parameters)
 - Marginalization (integrating over them)

■ Frequentist

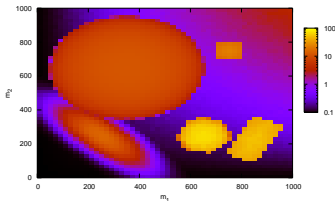
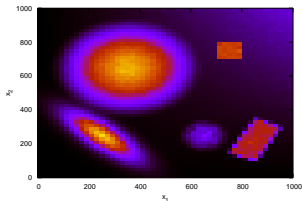
“How probable is the data, given certain parameters?”

- Bayesian *“How likely is a given parameter value given the data?”*
- Frequentist *“How probable is the data, given certain parameters?”*
 - Frequency interpretation of probability
Infinite repetition of the same experiment, statistically independent results
Outcome distributed around true value
(Gaussian, Poisson for counting experiments, . . .)
 - $x\%$ Confidence level: in $x\%$ of the cases outcome at least as extreme
 - Defined only for single points in parameter space
 - Profile likelihood:
Maximum likelihood over unseen parameters

Toy example (5-dim):

■ Bayesian (left)

“How likely is a given parameter value given the data?”



■ Frequentist (right)

“How probable is the data, given certain parameters?”

Objects:

- Small Hypersphere $r = 100$, $V_{\max} = 75$ @ $(650, 250, 350, 350, 350)$
- Cuboid $d = (173, 120, 200, 200, 200)$, $V_{\max} = 60$ @ $(850, 225, 650, 650, 650)$
- Cube $d = (100, 100, 300, 300, 300)$, $V_{\max} = 25$ @ $(750, 750, 450, 450, 450)$
- Gaussian $\sigma = (50, 150, 150, 150, 150)$, $V_{\max} = 16$ @ $(250, 250, 550, 550, 550)$
- Big Hypersphere $r = 300$, $V_{\max} = 12$ @ $(350, 650, 650, 650, 650)$
- Background $V = 0.1 + 4 \cdot 10^{-30} \cdot x_1^2 x_2^2 x_3^2 x_4^2 x_5^2$

If TeV-scale supersymmetry is out there, what can we tell today? (without LHC data)
→ Fits of current data to supersymmetry (mSUGRA)

[Allanach, Cranmer, Lester, Weber]

[Roszkowski, Ruiz de Austa, Trotta]

[MasterCode, Fittino]

[AbdusSalam, Allanach, Quevedo, Feroz, Hobson]

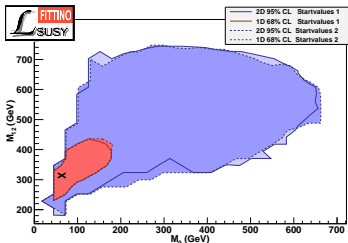
(pMSSM)

Observables:

- Dark Matter $\Omega_{\text{DM}} h^2$
- $(g - 2)_\mu$
- M_W
- $\sin^2 \theta_W$
- $\text{BR}(b \rightarrow s\gamma)$
- $\text{BR}(B_s \rightarrow \mu^+ \mu^-)$
- ...

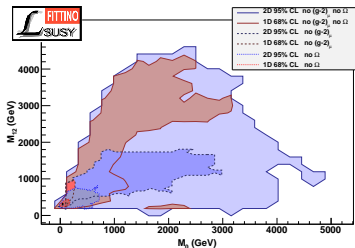
⇒ Predictions for SUSY mass spectrum

Fit Results (mSUGRA)



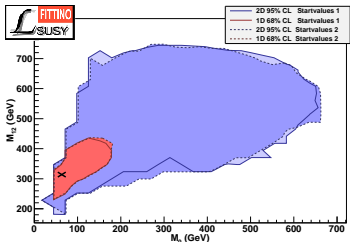
Prediction for m_0 vs. $m_{1/2}$
including all measurements

red: 68% CL
blue: 95% CL



dashed: $(g - 2)_\mu$ constraint removed
dotted: Ωh^2 constraint removed
solid: both $(g - 2)_\mu$ and Ωh^2 constraint removed

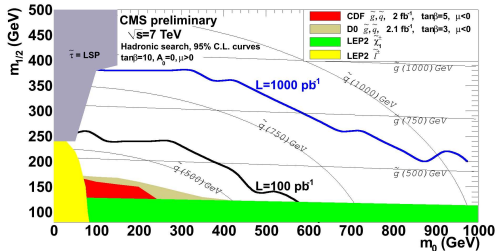
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Prediction for m_0 vs. $m_{1/2}$ including all measurements

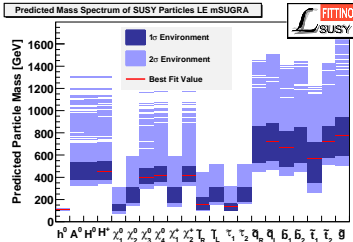
red: 68% CL

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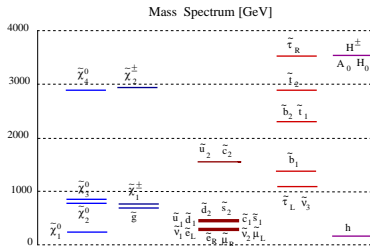


CMS physics reach for the LHC ($\sqrt{s} = 7$ TeV)

Mass Spectrum



mSUGRA fit



pMSSM fit
 (20 MSSM parameters +
 5 SM parameters)
 → much heavier mass spectrum

[Allanach et al.]

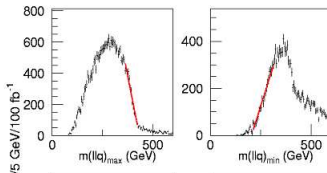
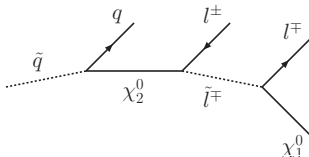
mSUGRA constraints ↔ Bayesian preference for decoupling limit (?)

Experimental Input (Edges)

mSUGRA SPS1a as a benchmark point:

$m_0 = 100 \text{ GeV}$, $m_{1/2} = 250 \text{ GeV}$, $A_0 = -100 \text{ GeV}$, $\tan \beta = 10$, $\text{sgn } \mu = +1$,
 $m_t = 171.4 \text{ GeV}$

LHC “experimental” data from cascade decays (best precision obtainable)



Measurement	Value (GeV)	Errors (GeV)			
		(stat)	(LES)	(JES)	(theo)
(m_{llq}^{\max}) :Edge($\tilde{q}_L, \chi_2^0, \chi_1^0$)	449.08	1.4		4.3	5.1
(m_{llq}^{\min}) :Thres($\tilde{q}_L, \chi_2^0, \tilde{l}_R, \chi_1^0$)	216.00	2.3		2.0	3.3
(m_{ll}^{\max}) :Edge($\chi_2^0, \tilde{l}_R, \chi_1^0$)	80.852	0.042	0.08		1.2
m_h	108.7	0.01	0.25		2.0
...	

Treatment of errors:

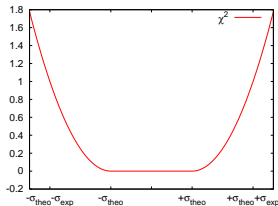
- All experimental errors are Gaussian

$$\sigma_{\text{exp}}^2 = \sigma_{\text{stat}}^2 + \sigma_{\text{syst}(j)}^2 + \sigma_{\text{syst}(l)}^2$$

- Systematic errors from jet ($\sigma_{\text{syst}(j)}$) and lepton energy scale ($\sigma_{\text{syst}(l)}$) correlated

- Theory errors (missing higher-order corrections):

- Gaussian
- Box-shaped (RFit scheme [Hoecker, Lacker, Laplace, Lediberder])



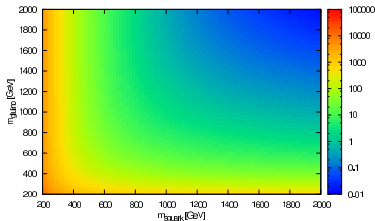
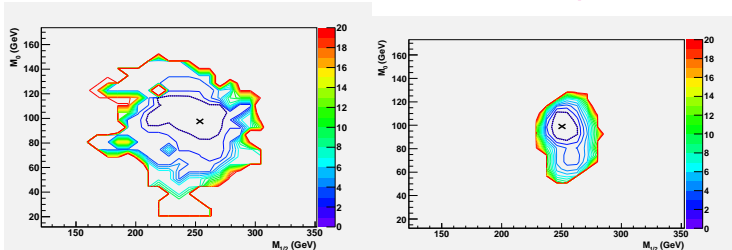
$$\Rightarrow -2 \log L \equiv \chi^2 = \sum_{\text{measurements}} \begin{cases} 0 & \text{for } |x_{\text{data}} - x_{\text{pred}}| < \sigma_{\text{theo}} \\ \left(\frac{|x_{\text{data}} - x_{\text{pred}}| - \sigma_{\text{theo}}}{\sigma_{\text{exp}}} \right)^2 & \text{for } |x_{\text{data}} - x_{\text{pred}}| \geq \sigma_{\text{theo}} \end{cases}$$

- All values within 1 unit equally likely
- No preference for vanishing corrections
- No huge corrections possible
($> \mathcal{O}(1)$ break-down of perturbation theory)

Cross sections

At early running cross sections yield additional information

[Dreiner, Krämer, Lindert, O'Leary]



- Top left: only early-running kinematic edges ($7 \text{ TeV}, 1 \text{ fb}^{-1}$)
- Top right: including rate information
- Left: integrated cross section for squark/gluino production
- significant improvement

mSUGRA with LHC measurements (SPS1a kinematic edges):
 pick one set of "measurements", randomly smeared from the true values

[SFitter]

Free parameters:

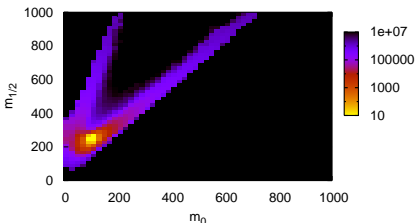
m_0 , $m_{1/2}$, $\tan(\beta)$, A_0 , $\text{sgn}(\mu)$, m_t

SFitter output 1:

Fully-dimensional exclusive likelihood map

(colour:

minimum χ^2 over all unseen parameters)



SFitter output 2:

Ranked list of minima:

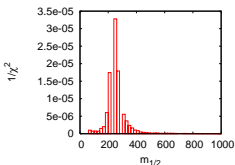
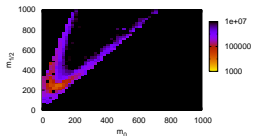
	χ^2	m_0	$m_{1/2}$	$\tan(\beta)$	A_0	μ	m_t
SPS1a		100.0	250.0	10.0	-100.0	+	171.4
1)	0.09	102.0	254.0	11.5	-95.2	+	172.4
2)	1.50	104.8	242.1	12.9	-174.4	-	172.3
3)	73.2	108.1	266.4	14.6	742.4	+	173.7
4)	139.5	112.1	261.0	18.0	632.6	-	173.0

Bayesian or Frequentist?

Fully-dimensional log-likelihood map

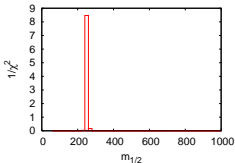
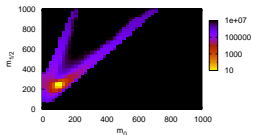
→ "project" onto plotable 1- or 2-dimensional spaces

Bayesian:



Marginalisation of χ^2 in all other directions

Frequentist:



Profile likelihood: Value of bin is value of smallest χ^2 occurring in this bin

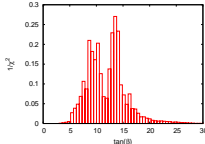
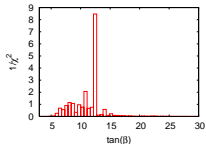
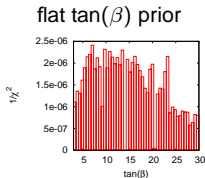
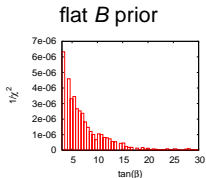
Purely high-scale model

$m_0, m_{1/2}, A_0$ defined at the GUT-scale $\Leftrightarrow \tan(\beta)$ defined at the weak scale

\Rightarrow Replace $\tan(\beta)$ with high-scale quantity B

\Rightarrow Flat prior in B yields prior $\propto \frac{1}{\tan(\beta)^2}$

SPS1a with LHC kinematic edges ($\tan(\beta)$ vs. $1/\chi^2$):



Bayesian:

Large influence of choice of prior

Choosing flat B prior strongly favours low values of $\tan(\beta)$.

Frequentist:

Two plots should be identical (no prior in χ^2 calculation)

Indirect influence via Markov Chain proposal function

- No need to assume specific SUSY-breaking scenario
⇒ SUSY-breaking mechanism should be induced from data
- Use of Markov Chains makes scanning the **19-dimensional** parameter space feasible
- Lack of sensitivity on one parameter does not slow down the scan
(no need to fix parameters)

Full scan of 19D parameter space challenging

Four-step procedure yields better and faster results:

- Markov-Chain run with flat pdf over full parameter space
5 best points additionally minimised
(full scan, no bias on starting point)
- Markov Chain with flat pdf on Gaugino-Higgsino subspace:
 $M_1, M_2, M_3, \mu, \tan \beta, m_t$
Additional Minuit run with 15 best solutions

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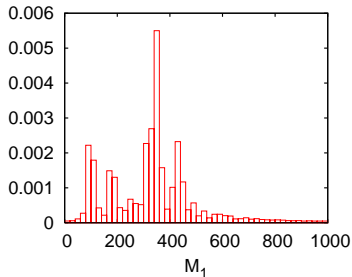
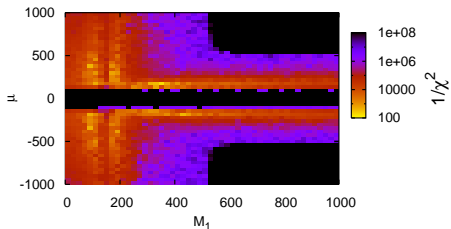
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Search Strategy (2) - results

- Only three neutralinos ($\chi_1^0, \chi_2^0, \chi_4^0$) with masses (97.2 GeV , 180.5 GeV , 375.6 GeV) and no charginos observable at the LHC in SPS1a
- \Rightarrow Mapping $(M_1, M_2, \mu) \rightarrow (\chi_1^0, \chi_2^0, \chi_4^0)$ not unique
- $\text{sgn } \mu$ basically undetermined by collider data
- \Rightarrow 8-fold solution



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- \Rightarrow 8-fold solution

	DS1	DS2	DS3	DS4	DS7 (SPS1a-like)	DS8	DS9	DS10
$\tan \beta$	12.3 ± 5.6	12.4 ± 5.0	14.9 ± 9.8	8.9 ± 5.9	13.8 ± 7.5	12.6 ± 7.9	19.2 ± 14.3	23.0 ± 15.6
M_1	102.7 ± 7.1	189.5 ± 6.2	107.2 ± 9.2	383.2 ± 9.1	105.0 ± 6.9	191.7 ± 6.6	116.3 ± 7.5	380.9 ± 9.3
M_2	185.5 ± 7.0	$96. \pm 6.4$	356.9 ± 8.7	114.2 ± 10.7	194.7 ± 7.3	105.5 ± 7.3	354.0 ± 8.2	137.2 ± 9.1
μ	-362.7 ± 7.8	-364.7 ± 6.8	-186.0 ± 8.5	-167.0 ± 9.6	353.0 ± 7.7	357.1 ± 8.3	188.9 ± 7.1	172.8 ± 8.7
$\Delta \chi_{\text{ILC}}^2$	73	22000	1700	25000	0.4	22000	2000	24000
ILC	$\tilde{\tau}_1$	χ_1^\pm	χ_3^0	χ_1^\pm	$\tilde{\tau}_1$	χ_1^\pm	χ_3^0	χ_1^\pm

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Additional Minuit run with 15 best solutions
- Markov Chain with Breit-Wigner-shaped pdf on remaining parameters for all solutions of previous step
Minimisation for best 5 points
- Minuit run for best points of last step keeping all parameters variable

- In total 19 parameters constrained by 22 measurements
- Measurements constructed from only 15 underlying masses
- \Rightarrow Complete determination of parameter set not possible
- Five parameters not well constrained
 - m_A
 - \leftarrow no heavy Higgses measurable
 - $M_{\tilde{t}_R}$
 - A_t
 - \leftarrow stop sector parameters do not enter edge measurements
 - $M_{\tilde{\tau}_L}$ or $M_{\tilde{\tau}_R}$
 - \leftarrow only the lighter stau measured
 - $\tan \beta$
 - \leftarrow change can always be accommodated by rotating $M_1, M_2, M_{\tilde{a}_3}, \dots$
- Single common link: m_{h^0}
- \Rightarrow 4-dimensional hyperplane in parameter space undetermined
- Can still assign errors to some of the badly determined parameters

How well can we determine the SM Higgs couplings?

Can we distinguish a non-Standard-Model-like Higgs sector?

- Theory: Standard Model plus general Higgs sector
- For Higgs couplings present in the Standard Model $j = W, Z, t, b, \tau$ replace general couplings by

$$g_{jjH} \longrightarrow g_{jjH}^{\text{SM}} (1 + \Delta_{jjH}) \quad (\rightarrow \Delta = -2 \text{ means sign flip})$$

- For loop-induced Higgs couplings $j = \gamma, g$ replace by

$$g_{jjH} \longrightarrow g_{jjH}^{\text{SM}} (1 + \Delta_{jjH}^{\text{SM}} + \Delta_{jjH})$$

where g_{jjH}^{SM} : (loop-induced) coupling in the Standard Model

Δ_{jjH}^{SM} : contribution from modified tree-level couplings
to Standard-Model particles

Δ_{jjH} : additional (dimension-five) contribution

- Additional free parameters:
 - Higgs boson mass m_H
 - Top-quark mass m_t
 - Bottom-quark mass m_b

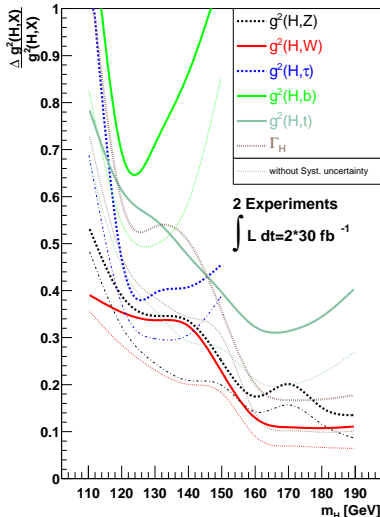
Higgs at the LHC

[Zeppenfeld, Kinnunen, Nikitenko, Richter-Was; Dührssen et al.]

production	decay
$gg \rightarrow H$	ZZ
qqH	ZZ
$gg \rightarrow H$	WW
qqH	WW
$t\bar{t}H$	$WW(3\ell)$
$t\bar{t}H$	$WW(2\ell)$
inclusive	$\gamma\gamma$
qqH	$\gamma\gamma$
$t\bar{t}H$	$\gamma\gamma$
WH	$\gamma\gamma$
ZH	$\gamma\gamma$
qqH	$\tau\tau(2\ell)$
qqH	$\tau\tau(1\ell)$
$t\bar{t}H$	$b\bar{b}$
WH/ZH	$b\bar{b}$ (subject)

Total width

- degeneracy $\sigma \cdot BR \propto g_p^2 \frac{g_d^2}{\Gamma_H} \quad (\Gamma_H \propto g^2)$
- additional contribution to total width can compensate increase in coupling constant
- Here: $\Gamma_H = \sum_{SM} \Gamma_i$



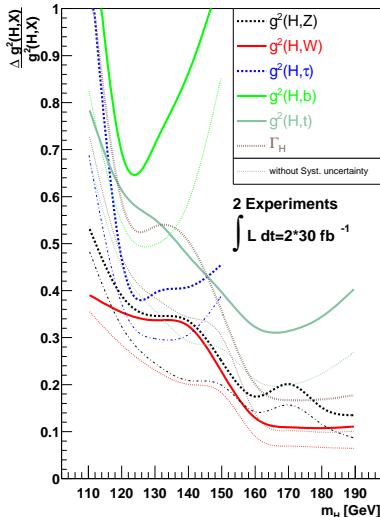
Higgs at the LHC

[Zeppenfeld, Kinnunen, Nikitenko, Richter-Was; Dührssen et al.]

production	decay
$gg \rightarrow H$	ZZ
qqH	ZZ
$gg \rightarrow H$	WW
qqH	WW
$t\bar{t}H$	$WW(3\ell)$
$t\bar{t}H$	$WW(2\ell)$
inclusive	$\gamma\gamma$
qqH	$\gamma\gamma$
$t\bar{t}H$	$\gamma\gamma$
WH	$\gamma\gamma$
ZH	$\gamma\gamma$
qqH	$\tau\tau(2\ell)$
qqH	$\tau\tau(1\ell)$
$t\bar{t}H$	$b\bar{b}$
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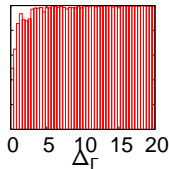
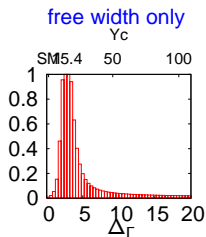
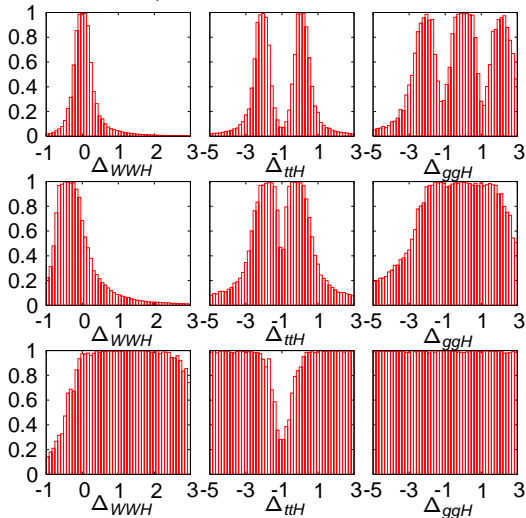
- Invisible Higgs decays actually observable

- Vector-Boson Fusion: tagging jets plus \cancel{E}_T [Eboli, Zeppenfeld]
- WH/ZH : recoil against nothing [Choudhury, Roy; Godbole, Guchait, Mazumdar, Moretti, Roy]

- Unobservable decays into particles with large backgrounds (like $H \rightarrow$ jets)
e.g. increased ccH coupling (corresponding to 15.4 GeV Yukawa coupling)

Invisible vs. Unobserved

- Unobservable decays into particles with large backgrounds (like $H \rightarrow \text{jets}$)
 e.g. increased ccH coupling (corresponding to 15.4 GeV Yukawa coupling)
 $\mathcal{L} = 30 \text{ fb}^{-1}$, SM data / increased ccH / increased ccH plus free width



- Fitting parameters of new-physics models requires dedicated tools
- Techniques: Markov Chain Monte Carlo plus Minuit
Bayesian vs Frequentist
Theory errors: Gaussian vs box-shaped (RFit)
- Fits of current data predict low-scale SUSY, accessible to 7-TeV LHC
↔ parts of 95% CL region out of reach for next year's LHC run
↔ mSUGRA vs. pMSSM results
- Kinematic edges as main input, cross sections helpful for early running
- Fitting a (18-parameter) pMSSM:
 - Reconstruction of Lagrangian parameters works fine
 - Discrete degenerate solutions for $M_1, M_2, |\mu|$
 - Undetermined parameters due to unseen sectors
- New-physics effects also in other sectors, e.g. Higgs sector
→ SM with general Higgs couplings

see also:

- S. Fichtel, On SUSY GUTs with a degenerate Higgs mass matrix
- J.-L. Kneur, Measuring unification
- R. Lafaye, Measuring Hidden Higgs and Strongly-Interacting Higgs Scenarios

	type of measurement	nominal value	stat.	LES error	JES	theo.
m_h		108.7	0.01	0.25		2.0
m_t		171.20	0.01		1.0	
$m_{\tilde{l}_L} - m_{\chi_1^0}$		102.38	2.3	0.1		1.1
$m_{\tilde{g}} - m_{\chi_1^0}$		511.38	2.3		6.0	6.1
$m_{\tilde{q}_R} - m_{\chi_1^0}$		446.39	10.0		4.3	5.5
$m_{\tilde{g}} - m_{\tilde{b}_1}$		89.01	1.5		1.0	8.0
$m_{\tilde{g}} - m_{\tilde{b}_2}$		62.93	2.5		0.7	8.2
$m_{\tilde{l}\tilde{l}}^{\max.}$	three-particle edge ($\chi_2^0, \tilde{l}_R, \chi_1^0$)	80.852	0.042	0.08		1.2
$m_{\tilde{l}\tilde{q}}^{\max.}$	three-particle edge ($\tilde{q}_L, \chi_2^0, \chi_1^0$)	449.08	1.4		4.3	5.1
$m_{\tilde{l}\tilde{q}}^{\text{low.}}$	three-particle edge ($\tilde{q}_L, \chi_2^0, \tilde{l}_R$)	326.32	1.3		3.0	5.2
$m_{\tilde{l}\tilde{l}}^{\max.}(\chi_4^0)$	three-particle edge ($\chi_4^0, \tilde{l}_L, \chi_1^0$)	277.36	3.3	0.3		2.0
$m_{\tilde{\tau}\tilde{\tau}}^{\max.}$	three-particle edge ($\chi_2^0, \tilde{\tau}_1, \chi_1^0$)	83.21	5.0		0.8	1.0
$m_{\tilde{l}\tilde{q}}^{\text{high.}}$	four-particle edge ($\tilde{q}_L, \chi_2^0, \tilde{l}_R, \chi_1^0$)	390.18	1.4		3.8	5.0
$m_{\tilde{l}\tilde{q}}^{\text{thres.}}$	threshold ($\tilde{q}_L, \chi_2^0, \tilde{l}_R, \chi_1^0$)	216.00	2.3		2.0	3.3
$m_{\tilde{l}\tilde{b}}^{\text{thres.}}$	threshold ($\tilde{b}_1, \chi_2^0, \tilde{l}_R, \chi_1^0$)	198.41	5.1		1.8	3.1

particle	m_{SPS1a}	value	± stat.err.	± theo.err.
h	108.7	± 0.05	± 2.0	
H	395.34	± 1.5	± 2.0	
A	394.9	± 1.5	± 2.0	
H^+	403.5	± 1.5	± 2.0	
χ_1^0	97.22	± 0.05	± 0.5	
χ_2^0	180.44	± 1.2	± 0.9	
χ_3^0	355.45	± 4.0	± 1.8	
χ_4^0	375.09	± 4.0	± 1.9	
$\chi_{1\pm}$	179.79	± 0.55	± 0.9	
$\chi_{2\pm}$	375.22	± 3.0	± 1.9	
\tilde{t}_1	398.93	± 2.0	± 4.0	
$\tilde{\theta}_L$	199.59	± 0.2	± 1.0	
$\tilde{\theta}_R$	142.68	± 0.05	± 0.7	
$\tilde{\mu}_L$	199.59	± 0.5	± 1.0	
$\tilde{\mu}_R$	142.68	± 0.2	± 0.7	
$\tilde{\tau}_1$	133.36	± 0.3	± 0.7	
$\tilde{\tau}_2$	203.62	± 1.1	± 1.0	
$\tilde{\nu}_e$	183.72	± 1.2	± 0.9	

	LHC		LHC+ILC		SPS1a
$\tan \beta$	13.8 ± 7.4		10.7 ± 3.1		10.0
M_1	105.0 ± 6.9		103.1 ± 0.7		103.1
M_2	194.7 ± 7.3		193.0 ± 1.6		192.9
M_3	568.3 ± 11.6		568.5 ± 7.8		567.7
$M_{\tilde{t}_L}$	321.4 ± 248		192.4 ± 4.7		193.5
$M_{\tilde{t}_R}$	164.3 ± 120		134.9 ± 5.7		133.4
$M_{\tilde{\mu}_L}$	196.3 ± 7.6		194.4 ± 1.2		194.3
$M_{\tilde{\mu}_R}$	138.0 ± 7.0		135.8 ± 0.6		135.8
$M_{\tilde{e}_L}$	196.4 ± 7.5		194.3 ± 0.8		194.3
$M_{\tilde{e}_R}$	137.9 ± 7.1		135.8 ± 0.6		135.8
$M_{\tilde{q}_3 L}$	491.4 ± 16.2		486.2 ± 11.1		481.1
$M_{\tilde{t}_R}$	483.4 ± 232		409.6 ± 17.1		409.4
$M_{\tilde{b}_R}$	502.6 ± 15.3		499.1 ± 13.1		502.7
$M_{\tilde{q}_L}$	529.6 ± 12.1		526.4 ± 5.3		526.4
$M_{\tilde{q}_R}$	508.9 ± 16.4		507.8 ± 14.4		506.8
A_τ	fixed 0		-102.9 ± 681		-249.3
A_t	-394.4 ± 353		-497.3 ± 74		-496.8
A_b	fixed 0		-274.2 ± 1830		-764.0
m_A	558.2 ± 271.2		394.9 ± 1.5		394.9
μ	353.1 ± 7.7		350.8 ± 2.5		351.0