# The non-linear evolution of CMB: non-Gaussianity and spectral distortions

Cyril Pitrou

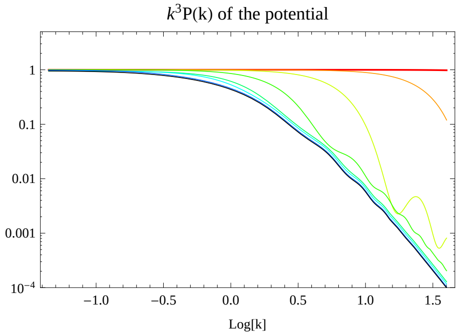
Institute of Cosmology and Gravitation, Portsmouth

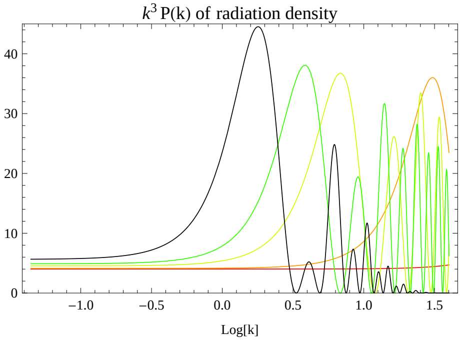
18 Novembre 2010

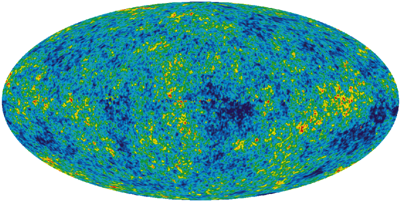
# Outline

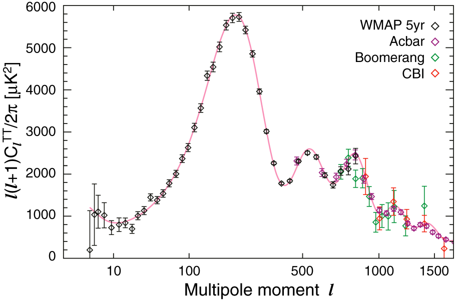
1. Motivations for non-Gaussianity search
2. Theory of perturbations
3. Spectral distortions
4. The flat-sky approximation
5. Numerical resolution and analytic insight

# From initial conditions to observations








## Harmonic analysis

Analysis in the space of $Y^{\ell m}$

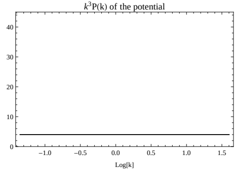
$\langle a_{\ell m} a_{\ell' m'} \rangle = \delta_{\ell\ell'} \delta_{mm'} C_\ell$

## Standard lore of perturbation theory

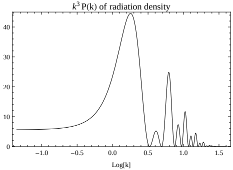
- *Initial conditions*: quantization of the free theory implies Gaussian initial conditions: $P(k)$
  $\langle \Phi(\mathbf{k})\Phi(\mathbf{k}')\rangle = \delta(\mathbf{k}+\mathbf{k}')P(k)$
- *Evolution*: linearisation of GR.

## Transfer scheme of perturbations

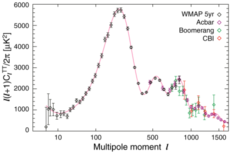
- Linear equations, modes $k$ are independent,
  $\Longrightarrow$ Gaussianity conserved.
- $P(k) \rightarrow \Theta(k,\eta) \rightarrow a_{\ell m} \rightarrow C_\ell$




$\longrightarrow$




$\longrightarrow$



## non-Gaussianity (NG)

- Initial conditions non-Gaussian?
  We want to test the models of inflation with other moments of the statistics.
- Non-linear dynamics is intrinsic to GR,

## Statistics of the primordial gravitational potential $\Phi = \Phi^{(1)} + \frac{1}{2}\Phi^{(2)}$

- Gaussian part $\Phi^{(1)}$ and non-Gaussian part $\Phi^{(2)}$:
- $\langle \Phi(\mathbf{k})\Phi(\mathbf{k}')\rangle = \delta(\mathbf{k}+\mathbf{k}')P(k)$
- $\langle \Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3)\rangle = \delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) f_{\rm NL} F(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$
- $F(\dots) =$ type of non-Gaussianity
- $f_{\rm NL} =$ its amplitude.

## The transfer to temperature fluctuations $\Theta_{\ell m}$

- In general $\Theta \equiv \mathcal{T}(\Phi)$
- Order 1 $\Theta^{(1)}_{\ell m} \equiv \mathcal{T}^{\ell m}_{\rm L}(\Phi^{(1)})$
- Order 2 $\Theta^{(2)\ell m} \equiv \mathcal{T}^{\ell m}_{\rm L}(\Phi^{(2)}) + \mathcal{T}^{\ell m}_{\rm NL}(\Phi^{(1)}\Phi^{(1)})$

## In Fourier space

- $\Theta^{(1)}_{\ell m}(\mathbf{k}) = \mathcal{T}^{\ell m}_{\rm L}(k)\Phi^{(1)}_{\mathbf{k}}$
- $\Theta^{(2)}_{\ell m}(\mathbf{k}) = \mathcal{T}^{\ell m}_{\rm L}(k)\Phi^{(2)}_{\mathbf{k}} + \int {\rm d}^3\mathbf{k}_1 {\rm d}^3\mathbf{k}_2 \delta^3(\mathbf{k} - \mathbf{k}_1 - \mathbf{k}_2)\mathcal{T}^{\ell m}_{\rm NL}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k})\Phi^{(1)}_{\mathbf{k}_1}\Phi^{(1)}_{\mathbf{k}_2}$

## $f_{\rm NL}$ ou $\mathcal{T}_{\rm NL}$ ?

$\langle \Theta_{\ell_1 m_1}\Theta_{\ell_2 m_2}\Theta_{\ell_3 m_3}\rangle \neq 0$ because of $\langle \Theta^{(1)}\Theta^{(1)}\Theta^{(2)}\rangle$.

# Description of perturbations

## General idea

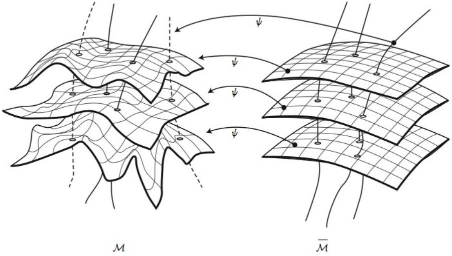
- We need to give a precise meaning to $\delta T(P) = T(P) - \bar{T}(P)$
- $T(P)$ ¨lives¨ in a perturbed space-time
- $\bar{T}(P)$ ¨lives¨ in a background space-time, homogeneous and isotropic

## Example: metric perturbations

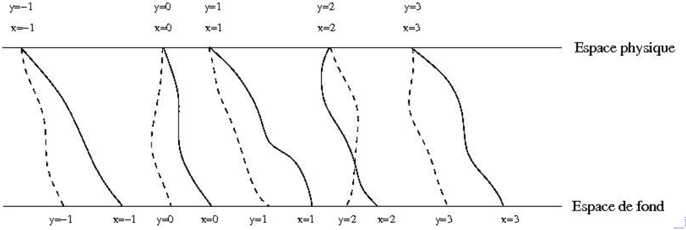
$$\mathrm{d}s^2 = a(\eta)^2\{-\mathrm{d}\eta^2 + \delta_{IJ}\mathrm{d}x^I\mathrm{d}x^J\},$$

$$\mathrm{d}s^2 = a(\eta)^2\{-e^{2\Phi}\mathrm{d}\eta^2 + 2B_I\mathrm{d}x^I\mathrm{d}\eta + [e^{-2\Psi}\delta_{IJ} + 2H_{IJ}]\mathrm{d}x^I\mathrm{d}x^J\},$$

# Correspondence between space-times




embedded into $4 + 1$ dimensions



# Characteristics of perturbations theory:

## Get rid of the gauge dependence

- Gauge-invariant variables
- A tensor equation is always expressed with such variables

## Structure of equations in orders of perturbations

- $\mathcal{E}[\delta^{(1)}g, \delta^{(1)}T] = 0$
- $\mathcal{E}[\delta^{(2)}g, \delta^{(2)}T] = \mathcal{S}[\delta^{(1)}g, \delta^{(1)}T]$

$\Longrightarrow$ Iterative resolution with gauge-invariant variables

# Describing the matter content

## The fluid approximation

- $T^{\mu\nu} = (P+\rho)u^{\mu}u^{\nu} + Pg^{\mu\nu} + \Pi^{\mu\nu}$
- Conservation Eq. $\nabla_{\mu} T^{\mu 0} = 0$
  $\Longrightarrow \rho' + \cdots = 0$
- Euler Eq. $\nabla_{\mu} T^{\mu i} = 0$
  $\Longrightarrow u'^{i} + \cdots + \partial_j \Pi^{ji} = 0$

## Problems

- Equation of state $P = w\rho$?
- Expression and evolution of the anisotropic stress tensor?
- Multifluid: $\nabla_{\mu} T^{\mu\nu} = F^{\nu} \neq 0$. Expression of forces ?

# Statistical description

Distribution function $f(x, p^a)$

## Tetrad in order to define locally a free-fall frame

$$\mathbf{e}_a.\mathbf{e}_b \equiv e_a^{\ \mu} e_b^{\ \nu} g_{\mu\nu} = \eta_{ab}$$
$$\mathbf{e}^a.\mathbf{e}^b \equiv e^a_{\ \mu} e^b_{\ \nu} g^{\mu\nu} = \eta^{ab}$$

## Momentum $\mathbf{p}$

Decomposed in energy and direction $\mathbf{p} = E(\mathbf{e}_o + \mathbf{n})$

## Link to the fluid description

$$T^{ab}(x) \equiv \int \delta_D^1(\mathbf{p}.\mathbf{p}) f(x, p^c) p^a p^b \frac{\mathrm{d}p^o \mathrm{d}^3 p^i}{(2\pi)^3}$$

# Evolution of the distribution function

## Boltzmann equation

$$L[f] = C[f]$$

- Liouville operator: Free-fall

$$L[f] = \frac{\mathrm{d}f}{\mathrm{d}s} = p^c \nabla_c f(x, p^a) + \frac{\partial f(x, p^a)}{\partial p^c} \frac{\mathrm{d}p^c}{\mathrm{d}s}$$

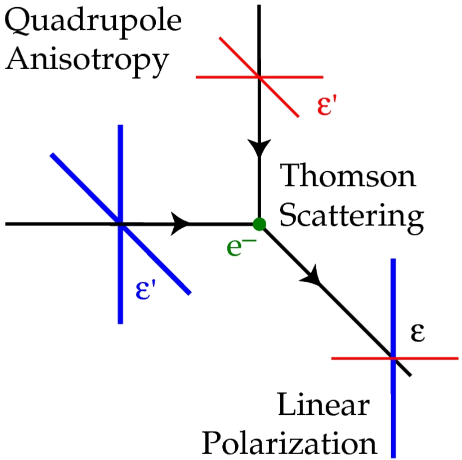
  Geodesic equation

$$p^b \nabla_b p_a = \frac{\mathrm{d}p_a}{\mathrm{d}s} + \omega_{bac} p^c p^b = 0$$

- Collision operator: Compton scattering on free electrons.

# Why do we also need to describe polarization?

Because if radiation has a quadrupole,
Compton scattering generates polarisation.



# Description of polarisation by the Stokes parameters

## Tensorial distribution function

If $n^i = (0,0,1)$: $f_{ab} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & I+Q & U+\mathrm{i}V & 0 \\ 0 & U-\mathrm{i}V & I-Q & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

## Covariant expression

$$f_{\mu\nu}(x,p^a) \equiv \frac{1}{2} I(x,p^a) S_{\mu\nu} + P_{\mu\nu}(x,p^a) + \frac{\mathrm{i}}{2} V(x,p^a) e_o^\rho \epsilon_{\rho\mu\nu\sigma} n^\sigma$$

# Covariant description of polarized radiation

## Tensor valued distribution function

- A photon is characterized by $p^\mu$ and $\varepsilon^\mu$ ($p_\mu \varepsilon^\mu = 0$)
- $F_{\mu\nu}(x, p^a) \equiv \frac{1}{2} f(x, p^a) \varepsilon_\mu \varepsilon_\nu$

## Screen projection

- Screen projector $S_{\mu\nu} = g_{\mu\nu} + e^o_\mu e^o_\nu - n_\mu n_\nu$
- $S^\nu_\mu \varepsilon_\nu$ is independent of the electromagnetic gauge choice for the polarization.
- We thus work with $f_{\mu\nu}(x, p^a) = S^\rho_\mu S^\sigma_\nu F_{\rho\sigma}(x, p^a)$

# Boltzmann equation with polarization

$$L[f_{ab}(x,p^h)] = C_{ab}\left(x,p^h\right)$$

- $L[f_{ab}(x,p^a)] = \frac{1}{2}L[I(x,p^d)]S_{ab} + L[P_{ab}(x,p^a)] + \frac{\mathrm{i}}{2}L[V(x,p^d)]n^c\epsilon_{ocab}$
- $C_{ab}\left(p^h\right) = n_{\mathrm{e}}\sigma_{\mathrm{T}}p^o\left[\frac{3}{2}\int\frac{\mathrm{d}^2\Omega'}{4\pi}S_a^cS_b^d f_{cd}\left(p'^h\right) - f_{ab}\left(p^h\right)\right]$

## We recover the case with no polarization

- $f_{ab} = \frac{1}{2}IS_{ab}$ or $I = S^{ab}f_{ab}$
- $S^{ab}C_{ab} \propto S'_{ab}S^{ab} = 1 + (\mathbf{n}.\mathbf{n}')^2 = 1 + \cos^2\theta$

# Multipolar expansion

## Multipoles for scalar functions ($I$ and $V$)

$$I(x, p^o, n^a) = \sum_{\ell=0}^{\infty} I_{\underline{a_\ell}}(x, p^o) n^{\underline{a_\ell}}$$

$$I_{\underline{a_\ell}}(x, p^o) = \Delta_\ell^{-1} \int I(x, p^o, n^a) n_{\langle \underline{a_\ell} \rangle} \mathrm{d}^2\Omega$$

## And for polarisation, E and B modes...

$$P_{ab}(x, p^a) = \sum_{\ell=2}^{\infty} \left[ E_{ab\underline{c_{\ell-2}}}(x, p^o) n^{\underline{c_{\ell-2}}} - n_c \epsilon^{cd}{}_{(a} B_{b)d\underline{c_{\ell-2}}}(x, p^o) n^{\underline{c_{\ell-2}}} \right]^{\mathrm{TT}}$$

$$E_{\underline{a_\ell}}(x, p^o) = M_\ell^2 \Delta_\ell^{-1} \int n_{\langle \underline{a_{\ell-2}}} P_{a_{\ell-1} a_\ell \rangle}(x, p^o, n^a) \mathrm{d}^2\Omega \,,$$

$$B_{\underline{a_\ell}}(x, p^o) = M_\ell^2 \Delta_\ell^{-1} \int n_b \epsilon^{bd}{}_{\langle a_\ell} n_{\underline{a_{\ell-2}}} P_{a_{\ell-1} \rangle d}(x, p^o, n^a) \mathrm{d}^2\Omega \,,$$

# Steps to follow

1. Perturb the metric $g_{\mu\nu} = \bar{g}_{\mu\nu} + g^{(1)}_{\mu\nu} + \frac{1}{2} g^{(2)}_{\mu\nu}$
2. Perturb the tetrad $e^{\mu}_{a} = \bar{e}^{\mu}_{a} + e^{(1)\mu}_{a} + \frac{1}{2} e^{(2)\mu}_{a}$
3. Perturb the connections $\omega_{abc} = \bar{\omega}_{abc} + \omega^{(1)}_{abc} + \frac{1}{2}\omega^{(2)}_{abc}$
4. Find the perturbed geodesic equations
5. Compute the perturbed Liouville operator
6. Compute the Thomson scattering for each electron
7. Sum over the electrons distribution to obtain the Collision tensor in full generalities
8. Expand it in perturbations
9. Take the multipoles $I_{\underline{a_\ell}}$ $E_{\underline{a_\ell}}$ and $B_{\underline{a_\ell}}$ of the Boltzmann equation
10. Solve it or integrate it numerically

## Evolution of brightness $\mathcal{I} = T^4$ along geodesics

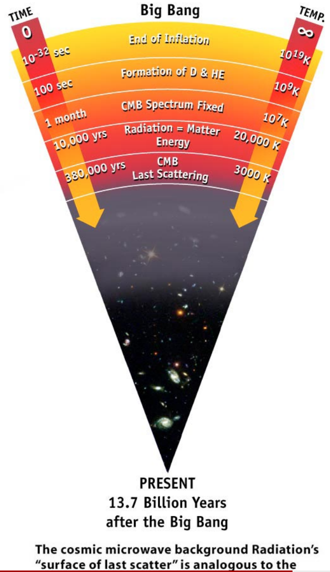
$$\frac{\mathrm{d}\left[e^{-\bar{\tau}}\mathcal{I}E^{-4}\right]}{\mathrm{d}\eta} = \bar{g}(\eta)E^{-4}\left[e^{\Phi}\mathcal{C}[\mathcal{I}] + \bar{\tau}'\mathcal{I}\right]$$

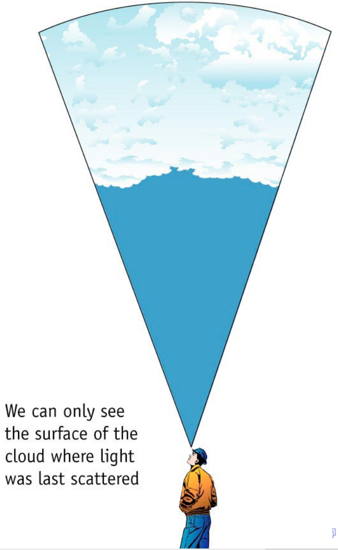
$$\frac{\mathrm{d}}{\mathrm{d}\eta} = \frac{\partial}{\partial\eta} + \frac{\mathrm{d}x^l}{\mathrm{d}\eta}\frac{\partial}{\partial x^l} + \frac{\mathrm{d}n^i}{\mathrm{d}\eta}\frac{\partial}{\partial n^i}\,, \qquad \frac{\mathrm{d}\ln E}{\mathrm{d}\eta} \simeq -\frac{\mathrm{d}\Phi}{\mathrm{d}\eta} + \Phi' + \Psi'$$

## Classification of effects:

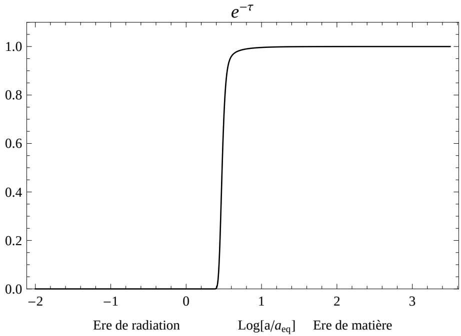
- 1) $\mathrm{d}E/\mathrm{d}\eta$: Evolution of the energy of photons:
  *Einstein effect* (potential $\Phi$) and *integrated effects*
- 2) $\bar{g}(\eta)\dots$ Collisions on the last scattering surface (LSS):
  *Intrinsic temperature*, and *Doppler effect*.
- 3) *Lensing effect* $\delta\left(\frac{\mathrm{d}n^i}{\mathrm{d}\eta}\right)$ **Order 2**
- 4) *Shapiro (or potential) time-delay* $\delta\left(\frac{\mathrm{d}x^l}{\mathrm{d}\eta}\right)$ **Order 2**

# Geometry of the background space-time



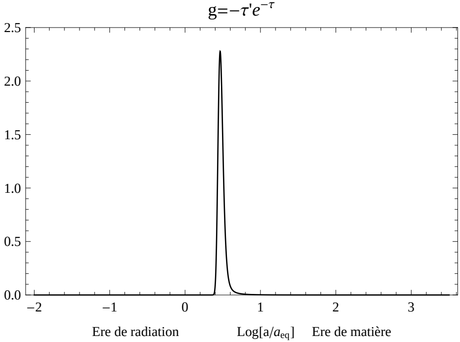



Integrated effects efficient since recombination




In the width of the LSS

Intrinsic Θ, Φ, Doppler



# But this is not enough to describe non-linear effects

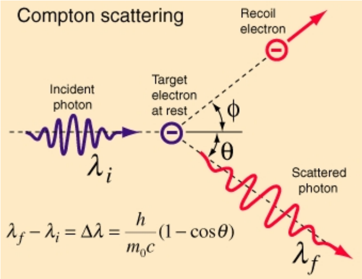
## Collisions distort the spectrum

In the collision term distorting effects can be classified as

- Thermal SZ effect (Kompanets collision term) $\rightarrow$ removes distortions of the $y$-type
- Kinetic SZ effect $\rightarrow$ creates distortions of the $y$-type

Important for reionization.

# Origin of distortions




## Expansion in $v_b$ to transform to the "lab" frame

- $\mathcal{O}(v_b)$: Doppler shift
  Can be described as a perturbed temperature
- $\mathcal{O}(v_b^2)$: Non-linear collisions: Spectral distortions.

### $\mathcal{O}(v_b)$, of the form $\frac{\partial f}{\partial E} E v_b$

Handled by defining

$$f(E) \equiv g(T, E) \simeq g(\bar{T}, E) - \frac{\partial g}{\partial E} E \frac{\delta T}{\bar{T}}$$

with $g(T, .)$ the BB spectrum of temperature $T$.

### $\mathcal{O}(v_b^2)$

Averaged over the distribution of electrons

$$\langle v_i v_j \rangle = v^2 \delta_{ij} + \frac{T_e}{m_e} \delta_{ij}$$

- $\bar{T}_e$ is responsible for background Comptonization: *Kompaneets* term
- $\delta T_e$ Spectral distortions from hot regions: *Thermal* SZ
- $v_b^2$ Spectral distortions from fast moving regions: *Kinetic* SZ

# Describing the distortion

## Parameterization (astro-ph/0703541)

$$f(E) = g(T, E) + yE^{-2}\partial_E \left[E^4 \partial_E g(T, E)\right]$$

- $T$ temperature. Takes the gravitational and $\mathcal{O}(v_b)$ effects into account
- $y$ takes the $\mathcal{O}(v_b^2)$ effects into account.

## Ambiguity of the temperature

We can define two types of temperatures

- Occupational $T_N$ from $N \propto T_N^3 \propto \int E^2 f(E) \mathrm{d}E$
- Energy density $T_\rho$ from $\rho \propto T_\rho^4 \propto \int E^3 f(E) \mathrm{d}E$

It is obvious that our temperature is $T_N$.

CMB literature always refers to $T_\rho = T_N + y$, more or less (ex?)implicitely.

## STF Mutlipoles of $y$

$y(\mathbf{n}) = \sum_{\ell=0}^{\infty} y_{\underline{i_\ell}} n^{\underline{i_\ell}}$

Idem for the temperature fluctuations $\Theta$

## Evolution

The second order Boltzmann equation contains evolution for $y$ and $\Theta^{(2)}$

$$\begin{aligned}\frac{\mathrm{d}y}{\mathrm{d}\eta} = \tau' \Big[ & -\tilde{y} + \tilde{y}_\emptyset + \frac{1}{10}\tilde{y}_{ij}n^i n^j + (\Theta - v_i n^i)(\Theta - \Theta_\emptyset) \\ & -\frac{1}{10}\Theta_{ij}n^i n^j \Theta - \frac{3}{10}\Theta_i v^i - \frac{1}{10}\Theta_i v_j n^i n^j \\ & +\frac{1}{3}v_i v^i + \frac{11}{20}v_i v_j n^{\langle i} n^{j\rangle} + \dots \Big]\end{aligned} \tag{1}$$

## Hierarchy

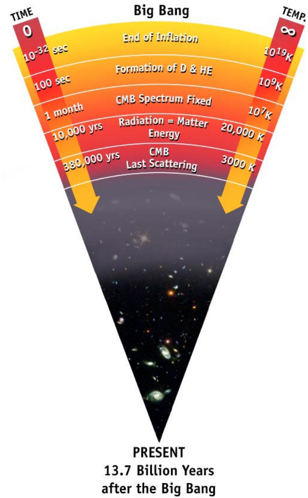
$$\frac{\partial y_{\underline{i_\ell}}}{\partial \eta} + \frac{\ell+1}{(2\ell+3)}\partial^J y_{j\underline{i_\ell}} + \partial_{\langle I_\ell} y_{\underline{i_{\ell-1}}\rangle} = \tau'(-y_{\underline{i_\ell}} + C_{\underline{i_\ell}}) \quad (2)$$

## $y$ collisions

$$C_\emptyset = y_\emptyset + \tfrac{1}{3} v_i v^i$$

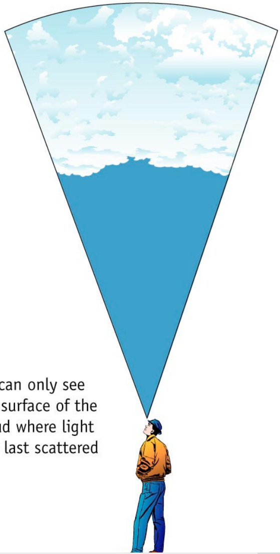
$$C_{ij} = \tfrac{1}{10} y_{ij} + \tfrac{11}{20} v_{\langle i} v_{j\rangle}$$

# Geometry of the problem

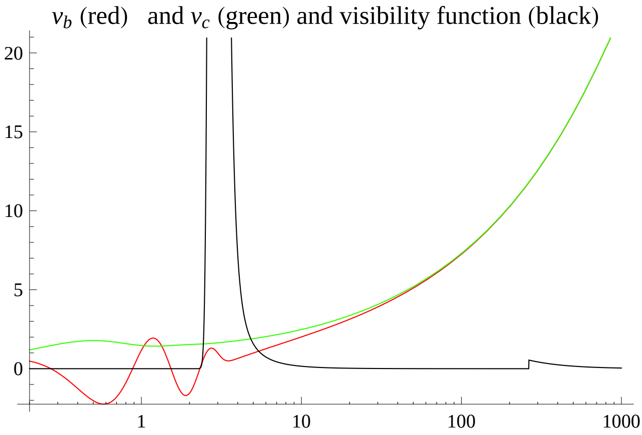



The cosmic microwave background Radiation's "surface of last scatter" is analogous to the light coming through the clouds to our eye on a cloudy day.

We can only see the surface of the cloud where light was last scattered



# Two contributions: LSS and Reionization



## Line of sight solution

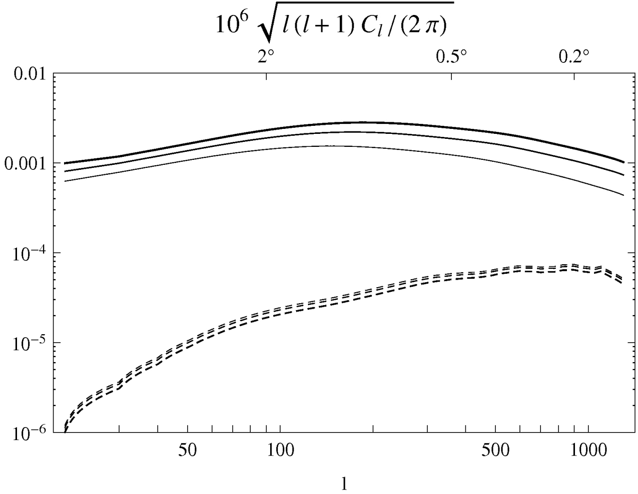
- Flat-Sky approximation. Good enough since there is no spectral distortions on large scales.
- Refined to a Limber approximation to deal with reionization.

## Orders of magnitude

The contribution from reionization is expected to dominate:
$v_b \propto (\eta - \eta_{\rm LSS})^2$
This perturbative approach corresponds to the non-linear Kinetic SZ effect in the intergalactic medium at high redshift ($z \leq 11$).

# Spectrum of the distortions $C_\ell^{yy}$

We use the thin shell for the LSS
We use Limber for the Reionization era



# The line of sight solution

## General form

$y(k,\mathbf{n}) = \int \mathrm{d}r e^{ikr\cos\theta} S(k,r,\mathbf{n})$

$S(k,r,\mathbf{n}) \equiv (\tau' e^{-\tau}) \sum_{\ell m} y_\ell^m(k) Y_\ell^m(\mathbf{k},\mathbf{n})$

where we use the $y_\ell^m$ rather than the $y_{\underline{i_\ell}}$.

## Full Sky method

- Align **k** with the azimuthal direction.
- Expand the $e^{ikr\cos\theta}$ in $Y_\ell^m(\mathbf{n})$. Coefficients are $j_\ell(kr)$
- Compose these $Y_\ell^m$ with the $Y_\ell^m$ of the sources: Clebsch Gordan coefficients.
- Perform the integral on **k**, by rotating the result to a general **k**

# The flat sky method

## Method

- Use cylindrinc coordinates in the $\int \mathrm{d}\mathbf{k}$ around an average direction $\mathbf{n}_{\rm FS}$ to compute $\xi(\theta) = \langle \Theta(\mathbf{n})\Theta(\mathbf{n}')\rangle_{\mathbf{n}.\mathbf{n}'=\cos\theta}$
- We obtain the $C_\ell$ from $C_\ell = 2\pi \int \sin\theta \mathrm{d}\theta\, P_\ell(\cos\theta)\, \xi(\theta)$

## General expression

$$C_\ell = \frac{1}{2\pi} \int \mathrm{d}r \mathrm{d}r' \mathrm{d}k_r \exp[\mathrm{i}k_r(r-r')] \frac{1}{[(r+r')/2]^2} P(k) S(k,r,\theta) S^\star(k,r',\theta)$$

with $\cos(\theta) = k_r/k$

## FS constraint

- $k_\perp(r+r')/2 = \ell$
- $k_\perp(r+r')/2 = \ell + 1/2$
- $k_\perp(r+r')/2 = \sqrt{\ell(\ell+1)}$

# Two useful limits

## The thin shell

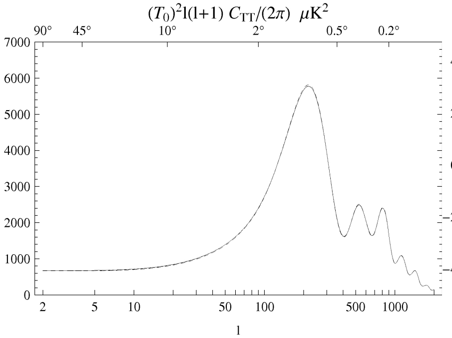
Sources confined in a narrow region around $r_{\rm LSS}$

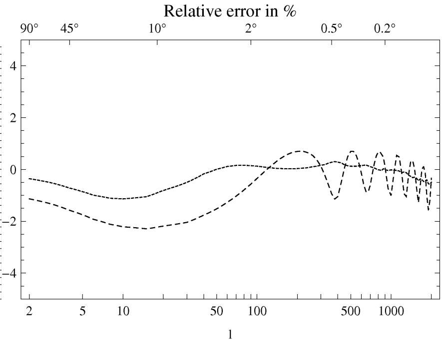
$$C_\ell = \frac{1}{r_{\rm LSS}^2}\int \frac{\mathrm{d}k_r}{2\pi}\left|\int \mathrm{d}r\,\exp(\mathrm{i}k_r r)S(k,r,\theta)\right|^2 P(k)$$

## The Limber approximation

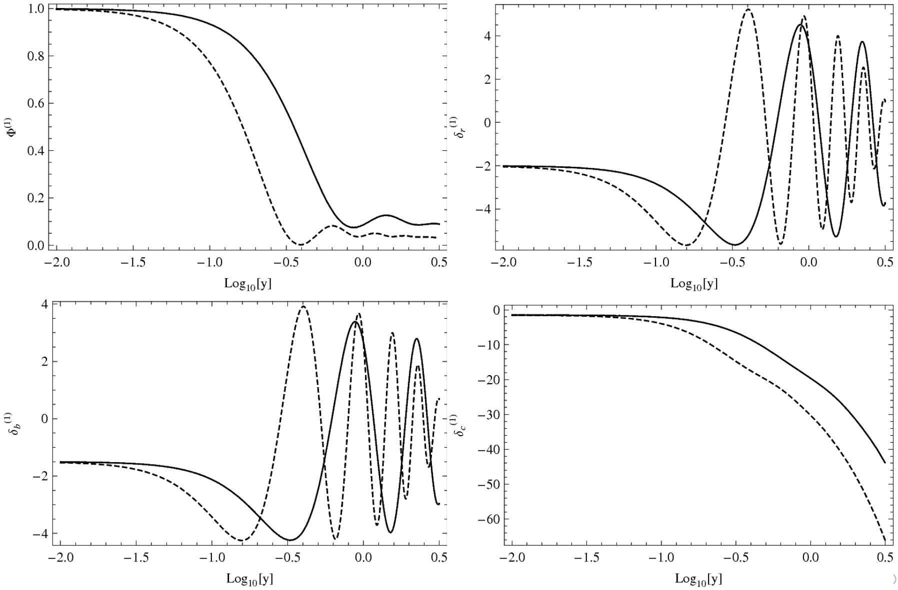
Sources in a large range of values, $\int \mathrm{d}k_r \exp[\mathrm{i}k_r(r-r')] \to 2\pi\delta(r-r')$

$$C_\ell = \int \mathrm{d}r\left|\frac{S(k,r,0)}{r}\right|^2 P(k)$$

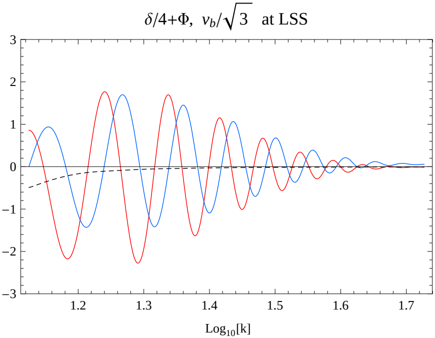

(T0)2l(l+1) CTT/(2π) μK2
90°
45°
10°
2°
0.5°
0.2°
7000
6000
5000
4000
3000
2000
1000
0
2
5
10
50
100
500
1000
l

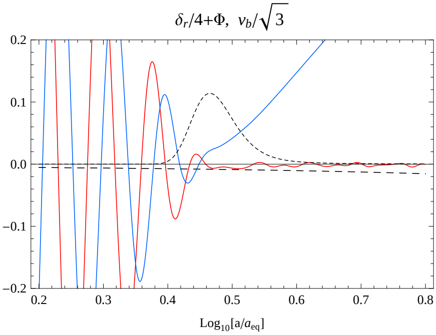

Relative error in %
90°
45°
10°
2°
0.5°
0.2°
4
2
0
−2
−4
2
5
10
50
100
500
1000
l

# Linear evolution of perturbations (order 1)



# Perturbations on the LSS




- Red: Intrinsic $\Theta$ and Einstein effect
- Blue: Doppler effect

## Linear response of radiation

$$\left[(1+R)\frac{\delta_r^{(1)\prime}}{4}\right]' + \mathrm{visc} + \frac{k^2}{3}\frac{\delta_r^{(1)}}{4} \simeq -\frac{k^2}{3}(1+R)\Phi^{(1)} + \left[(1+R)\Phi^{(1)\prime}\right]'$$

with $R = 3\bar{\rho}_b/(4\bar{\rho}_r)$.

## Because of viscosity, at small scale:

$\Theta^{(1)} \simeq \frac{\delta_r^{(1)}}{4} + \Phi^{(1)} \rightarrow -R\Phi^{(1)}$

# Primary and secondary effects

A mode $k$ correlates points separated at most by $\Delta r = r_{\rm LSS}/\ell$

## Primary effects

Located on the LSS. Intrinsic $\Theta$, $\Phi$, Doppler ....

$$\langle \Theta^{(2){\rm intr}}(n^i_1, r_{\rm LSS}) \Theta^{(1)}(n^i_2, r_{\rm LSS}) \Theta^{(1)}(n^i_3, r_{\rm LSS}) \rangle$$

## Secondary effects

$$\Theta^{(2){\rm lensed}}(n^i, r_{\rm LSS}) = \nabla_i \Theta^{(1)}(n^i, r_{\rm LSS}) \int_0^{r_{\rm LSS}} \nabla^i \phi(n^i, r') {\rm d}r'$$

Coupling between the lensing potential $\phi(r')$ and a late-time effect.

$$\int_0^{r_{\rm LSS}} {\rm d}r' \langle \nabla^i \phi(n^i_1, r') \nabla_i \Theta^{(1)}(n^i_1, r_{\rm LSS}) \Theta^{(1)}(n^i_2, r') \Theta^{(1)}(n^i_3, r_{\rm LSS}) \rangle$$

# Dynamics of primary effects

## Non-linear evolution (order 2)

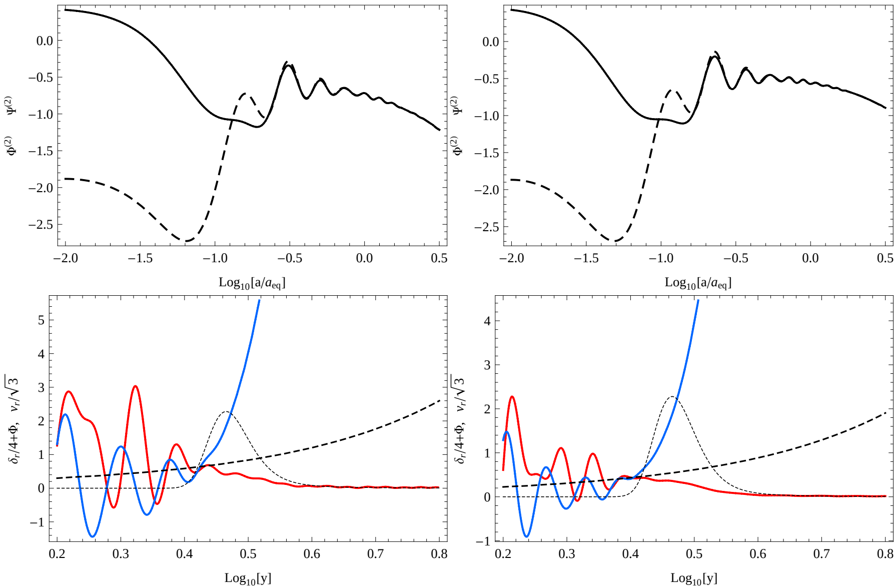
$$\left[(1+R)\frac{\delta_r^{(2)\prime}}{4}\right]' + \mathrm{visc} + \frac{k^2}{3}\frac{\delta_r^{(2)}}{4} \simeq$$
$$-\frac{k^2}{3}(1+R)\Phi^{(2)} + \left[(1+R)\Phi^{(2)\prime}\right]' + \mathrm{quadr}$$

## Behaviour on small scales (viscosity)

$$\Theta^{(2)\mathrm{intr}} \simeq \frac{\delta_r^{(2)}}{4} + \Phi^{(2)} \to -R\Phi^{(2)}$$

## Potential created by the collapse of cold dark matter

$$\frac{1}{2}\Phi^{(2)}(\mathbf{k},\eta) \simeq -\frac{1}{6}K(\mathbf{k}_1,\mathbf{k}_2)\left(\frac{k_1 k_2 \eta}{k}\right)^2 \Phi(k_1)\Phi(k_2), \qquad (3)$$


$\Psi^{(2)}$
$\Phi^{(2)}$
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−2.0
−1.5
−1.0
−0.5
0.0
0.5
$\mathrm{Log}_{10}[a/a_{\mathrm{eq}}]$
$\mathrm{Log}_{10}[a/a_{\mathrm{eq}}]$
$\delta_r/4+\Phi,\ \ v_r/\sqrt{3}$
$\mathrm{Log}_{10}[y]$
$\mathrm{Log}_{10}[y]$

## Estimator of primordial non-Gaussianity

- We build an estimator for $f_{\rm NL}$ using all $\langle \Theta_{\ell_1 m_1} \Theta_{\ell_2 m_2} \Theta_{\ell_3 m_3} \rangle$ up to $\ell_{\rm max}$ without taking into account the non-linear dynamics.
- If $f_{\rm NL} = 0$, what measures this estimator? Answer : $f_{\rm NL}^{\rm eq}$

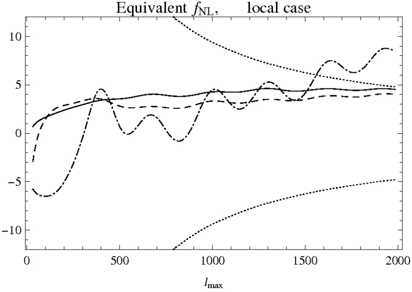
## Observational constraints on $f_{\rm NL}$ (WMAP-5)

- Local NG: $-9 < f_{\rm NL} < 111$
- Equilateral NG: $-150 < f_{\rm NL} < 253$

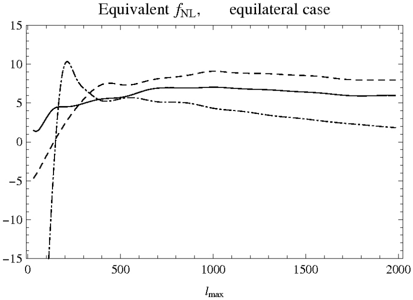
*Planck* is going to increase $\ell_{\rm max}$, thus reducing the cosmic variance limitation, thus increasing the precision.

# Bispectrum generated by primary effects

Local NG



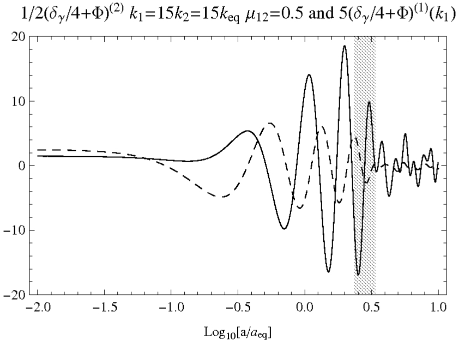

Equilateral NG

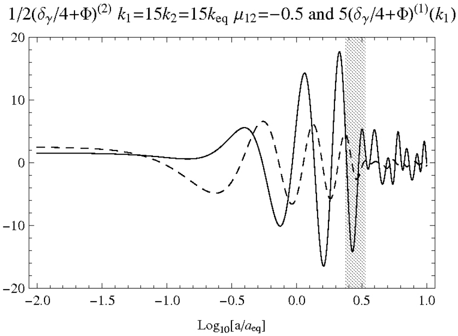



Conclusion

- Secondary effect add up (same sign)
- Non-linear evolution has to be taken into account in future constraints on non-Gaussianity.

Thanks a lot for your attention.


$1/2(\delta_\gamma/4+\Phi)^{(2)}$ $k_1=15k_2=15k_{eq}$ $\mu_{12}=0.5$ and $5(\delta_\gamma/4+\Phi)^{(1)}(k_1)$
20
10
0
−10
−20
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
$\mathrm{Log}_{10}[a/a_{eq}]$


$1/2(\delta_\gamma/4+\Phi)^{(2)}$ $k_1=15k_2=15k_{eq}$ $\mu_{12}=-0.5$ and $5(\delta_\gamma/4+\Phi)^{(1)}(k_1)$
20
10
0
−10
−20
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
$\mathrm{Log}_{10}[a/a_{eq}]$