Empirical Isospin Symmetry Breaking Hamiltonians for *sd* Shell

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Outline

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Theoretical Approaches

for the isospin mixing in nuclear states

- **Collective (hydrodynamic) model**
  Bohr & Mottelson (1977)
- **Hartree-Fock theory + RPA(TDA)**
  Hamamoto & Sagawa (1993); Sagawa (1995);
  Dobaczewski & Hamamoto (1995);
  Colo & Nagarajan & Van Isacker & Vitturi (1995);
  Sagawa & Van Giai & Suzuki (1996);
  Alvarez-Rodriguez & Moya de Guerra & Sarriguren & Moreno (2006);
  Petrovichi et al. (2008); Liang & Van Giai & Meng (2009);
  Rafalski et al. (2009); Satula et al. (2009)
- **Shell Model**
  Towner & Hardy (1973-2009); Ormand & Brown (1985, 1989);
  Ormand (1997); Cole (1997);
  Zuker et al. (2002); Caurier & Navratil & Ormand & Vary (2002)

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[b] Martínez-Pinedo G. *et al.*, PRC 55 (1997) p.187 – 205.

[c] Caurier E. *et al.*, Rev. Mod. Phys. 77 (2005) pp.427 – 488. ; Caurier E. & Nowacki F., Acta Phys. Pol. B 30 (1999) p.705.

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- code of higher computation power, **ANTOINE**[c], allows us to calculate without truncation

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Outline

Isospin Non-Conserving (ISB) Nucl. Hamiltonian

Isobaric Mass Multiplet Equation (IMME)

- Wigner (1957)[d], Weinberg & Treiman (1959)[e]

$$E(\psi, T, T_z) = a(\psi, T) + b(\psi, T)T_z + c(\psi, T)T_z^2,$$
$$\psi \equiv \textit{all relevant quantum numbers} + \textit{mass number}$$

[d] Wigner E. P., Proc. of the Robert A. Welsch Conf. on Chemical Research (R. A. Welsch Foundation, Houston Texas, 1957) vol. 1, p.67
[e] Weinberg S. & Treiman S. B., Phys. Rev. 116 (1959), p.465.

Isospin Symmetry Breaking (ISB) Nucl. Hamiltonian

Isobaric Mass Multiplet Equation (IMME)

- We aim to reproduce IMME, and begin with isospin symmetry invariant nuclear Hamiltonian

$$[H, T] = 0$$

$$H \left|\psi^T\right\rangle \equiv (H_0 + V) \left|\psi^T\right\rangle = E_T \left|\psi^T\right\rangle, \left|\psi^T\right\rangle = \sum_k a_k^T \left|\phi_k^T\right\rangle$$

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- Nolen-Schiffer anomaly[f]:
Coulomb force alone cannot reproduce
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- Nolen-Schiffer anomaly[f]:
  Coulomb force alone cannot reproduce
  mass difference between $T = \frac{1}{2}$ analog states;
- Extra forces are added to break the isospin symmetry,

$$V_{ISB} = s_C \cdot V_C + s_\pi \cdot V_\pi + s_\rho \cdot V_\rho + s_{IVSPE} \cdot E_{isovectorSPE}$$

s_C = Coulomb strength parameter, s_μ = Yukawa potential ($\mu = \pi, \rho$) strength parameter, ...

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- We evaluate $\left\langle\psi^T\left|V_{ISB}\right|\psi^T\right\rangle$ ... and we fit strength parameters...

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Combinations of Isospin Sym. Breaking Force

Isovector, *b* coefficients

- Coulomb + Isovector Single Particle Energies (ISV SPE)
- Coulomb + Yukawa Pion + ISV SPE
- Coulomb + Yukawa Rho + ISV SPE
- Coulomb + Yukawa Pion + Yukawa Rho + ISV SPE
- Coulomb + Nucl. Hamiltonian + ISV SPE
- ...
- (simply constructed combinations) $=>$ higher root mean square values

Combinations of Isospin Sym. Breaking Force

Isovector, *c* coefficients

Correspond to combinations in *b* coefficients

- Coulomb + Yukawa Pion
- Coulomb + Yukawa Rho
- Coulomb + Yukawa Pion + Yukawa Rho
- Coulomb + Nucl. Hamiltonian
- ...
- (simply constructed combinations) $=>$ higher root mean square values

Isospin Sym. Breaking Nucl. Hamiltonian

Isobaric Mass Multiplet Equation (IMME)

$$E(\psi, T, T_z) = a(\psi, T) + b(\psi, T)T_z + c(\psi, T)T_z^2,$$

$$H_{total} = \sum_{k=0}^{2} H^{(k)}$$

$$E(\psi, T, T_z) = \langle \psi, T, T_z | H_{total} | \psi, T, T_z \rangle$$

$$= \sum_{k=0}^{2} (-1)^{T-T_z} \begin{pmatrix} T & k & T \\ -T_z & 0 & T_z \end{pmatrix} \langle \psi, T || H_{total} || \psi, T \rangle$$

$$H_{total} = \left\{ H_{nucl.Hamil.} + H_{isoscalar} \right\} + H_{ISB}$$

Isospin Sym. Breaking Nucl. Hamiltonian

Isobaric Mass Multiplet Equation (IMME)

$$E(\psi, T, T_z) = \frac{1}{\sqrt{2T+1}} \langle \psi, T \| H_0^{(0)} \| \psi, T \rangle + \frac{T_z}{\sqrt{T(2T+1)(T+1)}} \langle \psi, T \| H_0^{(1)} \| \psi, T \rangle + \frac{3T_z^2 - T(T+1)}{\sqrt{T(2T+1)(T+1)(2T+3)(2T-1)}} \langle \psi, T) \| H_0^{(2)} \| \psi, T \rangle$$

$$E(\psi, T, T_z) = \underbrace{E^{(0)}(\psi, T)} + \underbrace{E^{(1)}(\psi, T) \cdot T_z} + \underbrace{E^{(2)}(\psi, T) \times [3T_z^2 - T(T+1)]}$$

Isospin Sym. Breaking Nucl. Hamiltonian

Isobaric Mass Multiplet Equation (IMME)

rearrange...

$$a(\psi, T) = E^{(0)}(\psi, T) - T(T+1)E^{(2)}(\psi, T)$$
$$b(\psi, T) = E^{(1)}(\psi, T)$$
$$c(\psi, T) = 3E^{(2)}(\psi, T)$$

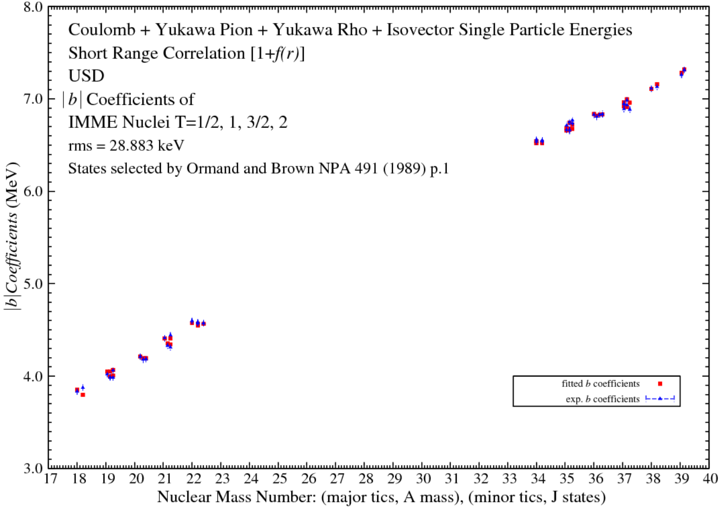
Isospin Symmetry Breaking Hamiltonian

b and *c* coefficients

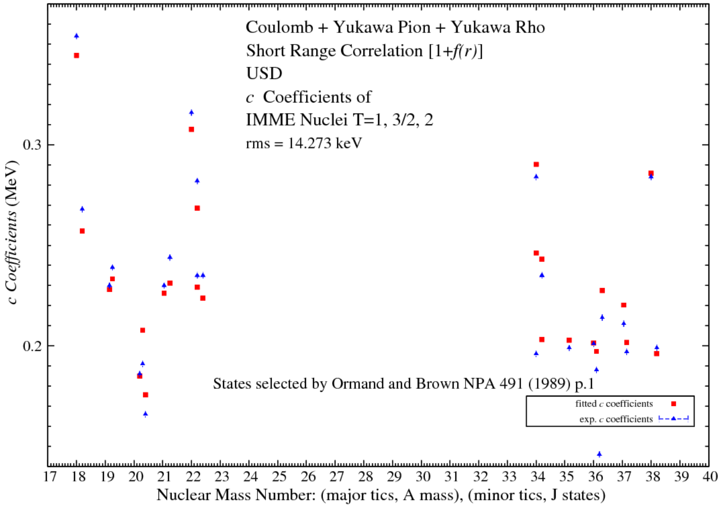
$$
\begin{aligned}
b(\psi, T) &= E^{(1)}(\psi, T) \\
&= \frac{1}{\sqrt{T(2T+1)(T+1)}} \langle \psi, T \| H_0^{(1)} \| \psi, T \rangle \\
&= \frac{3}{T(2T+1)(T+1)} \times \sum_{T_z=-T}^{T} \{ T_z \cdot E_{ISB}(\psi, T, T_z) \} \\
c(\psi, T) &= 3E^{(2)}(\psi, T) \\
&= \frac{3}{\sqrt{T(2T+1)(T+1)(2T+3)(2T-1)}} \langle \psi, T \| H_0^{(2)} \| \psi, T \rangle \\
&= \frac{15}{T(2T+1)(T+1)(2T+3)(2T-1)} \\
&\times \sum_{T_z=-T}^{T} \{ [3T_z^2 - T(T+1)] \times E_{ISB}(\psi, T, T_z) \}
\end{aligned}
$$

Outline

Isovector, *b* coefficients (42 data points)



Isotensor, *c* coefficients (25 data points)



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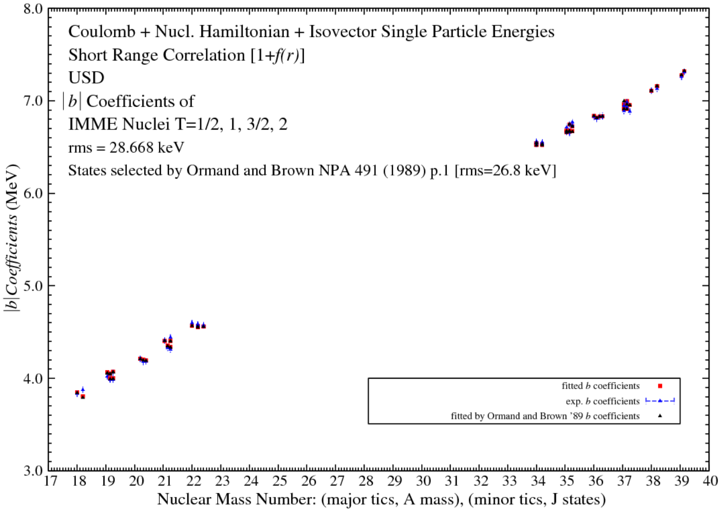

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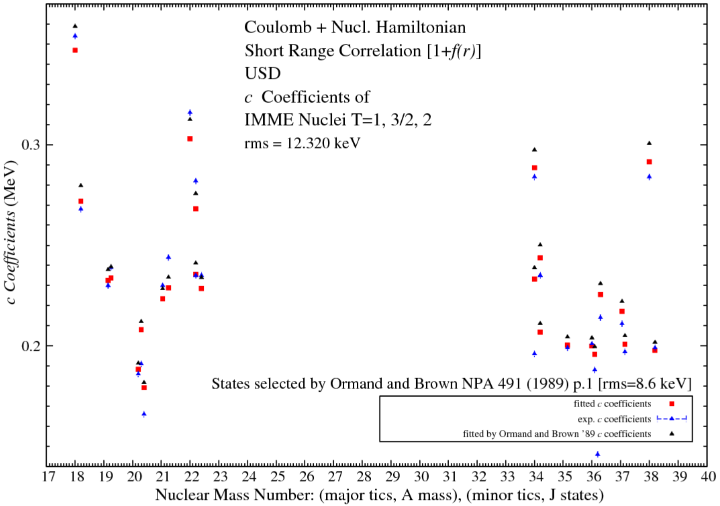

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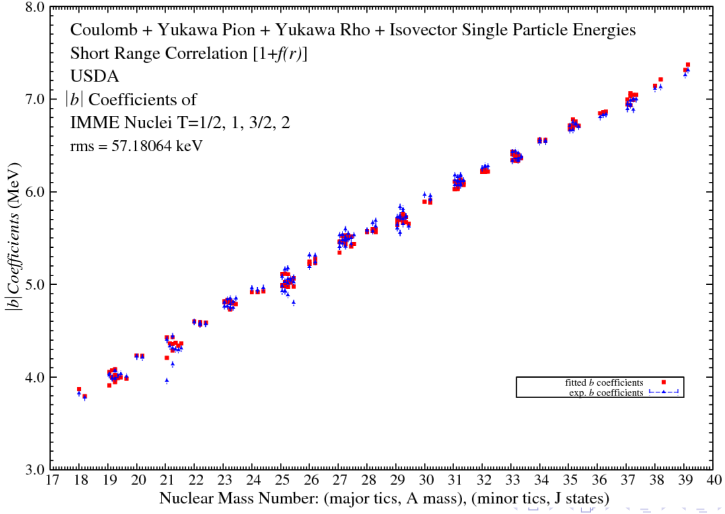

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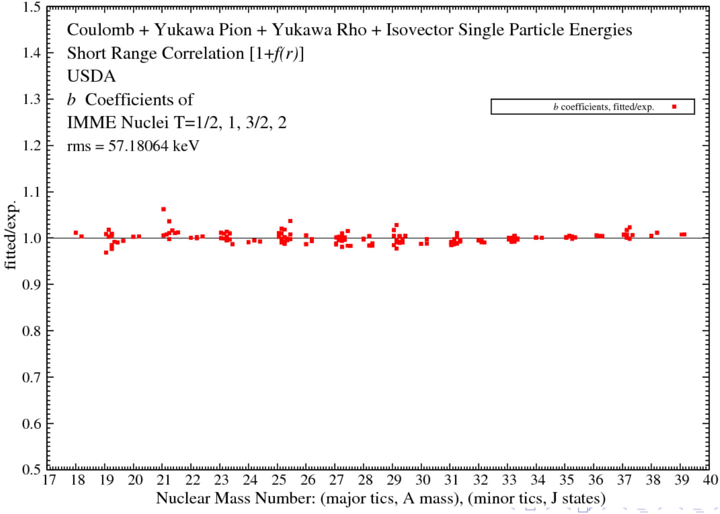

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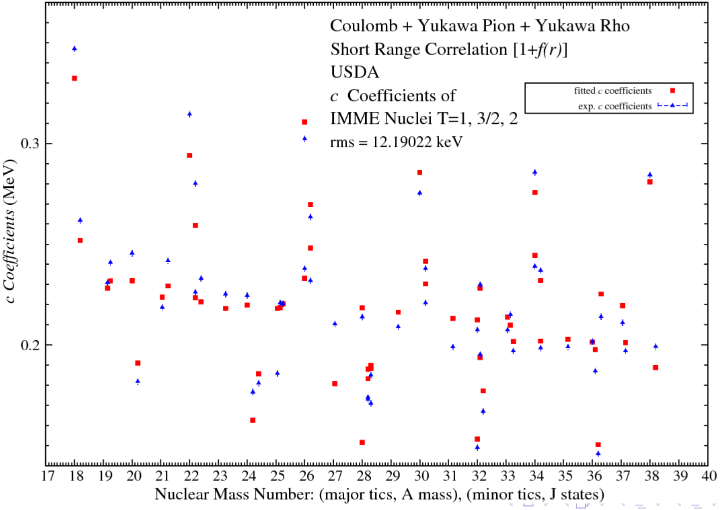

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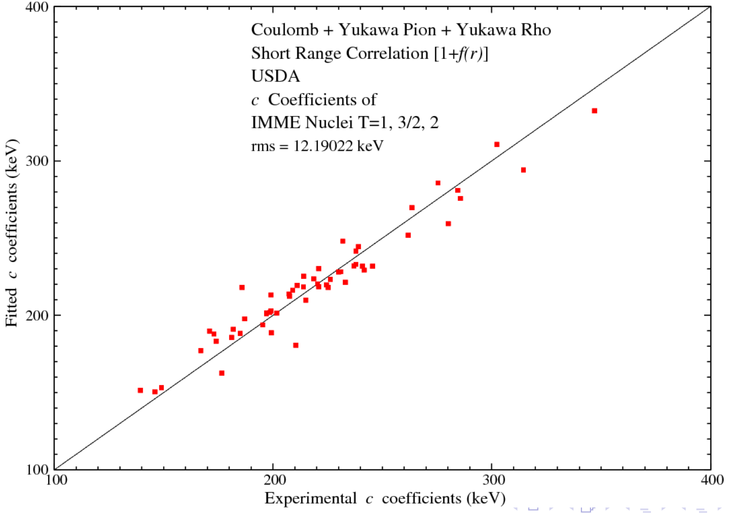

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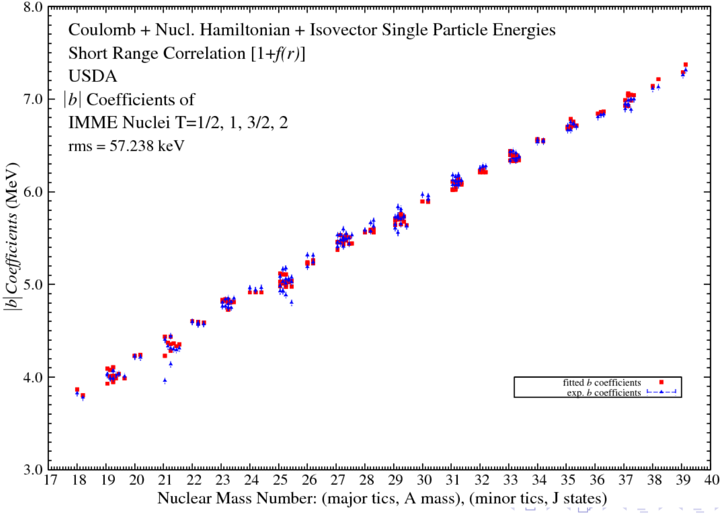

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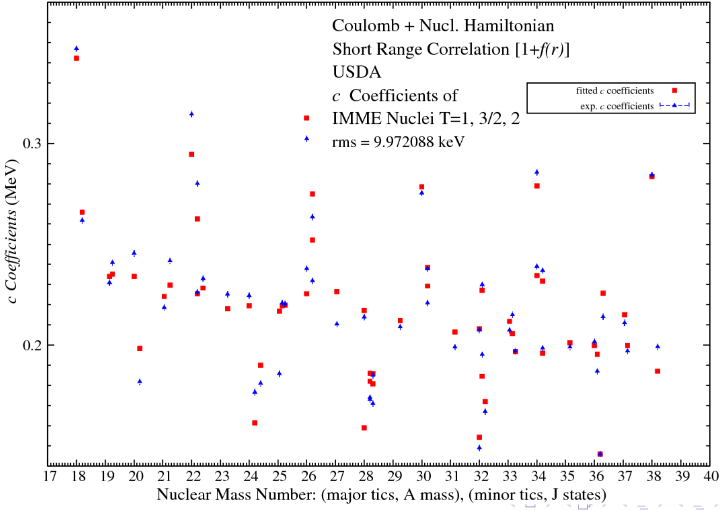

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Spectrum of IMME nuclei A=25

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  Coulomb + Yukawa Pion + Yukawa Rho + Isovector Single Particle Energies
  Isotensor
  Coulomb + Yukawa Pion + Yukawa Rho

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  Isotensor
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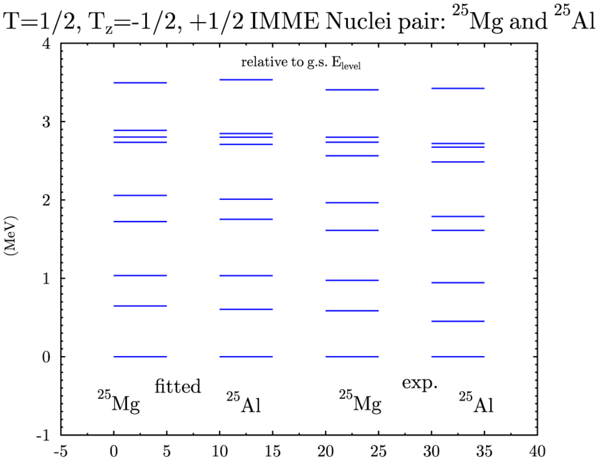

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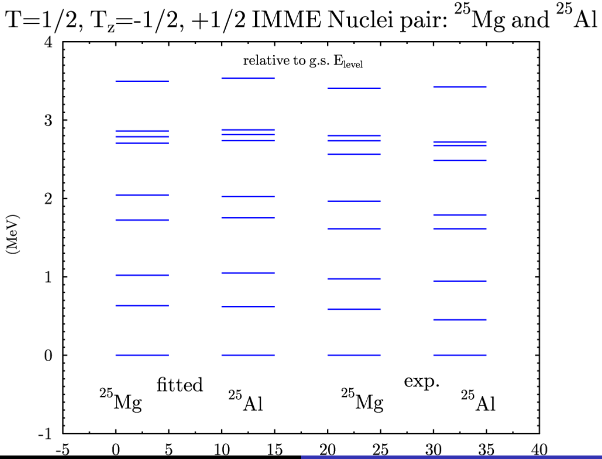

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- Improve the fitting method...

Acknowledgements

- Blank B., Caillon J-C,
  CNRS/IN2P3, Centre d'Etudes Nucléaires de Bordeaux Gradignan,
  Chemin du Solarium, BP120, 33175 Gradignan, France.
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  Institut de Recherches Subatomiques et Université Louis Pasteur
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