# Quarkonia Propagation & Collectivity in the QGP : Elastic Diffusions, Energy losses and elliptic flow $v_2(\phi)$

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Subatech-Nantes

AGThNuc 2010 ~ Wednesday 20th Octobre 2010

# *QGP Probe : bound states of Quarkonia ...*

t = $2.10^{-22}$ s

*Hadronisation*

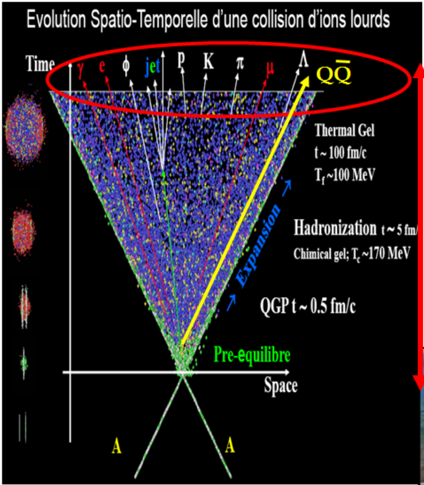
*Expansion Phase*

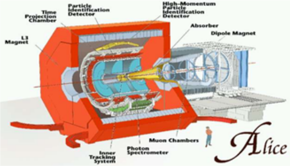
*QGP Formation*

*Pre-equilibrium*

*2 ions in collision*

t = 0 s

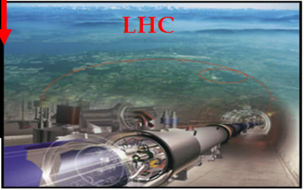



**Detectors**

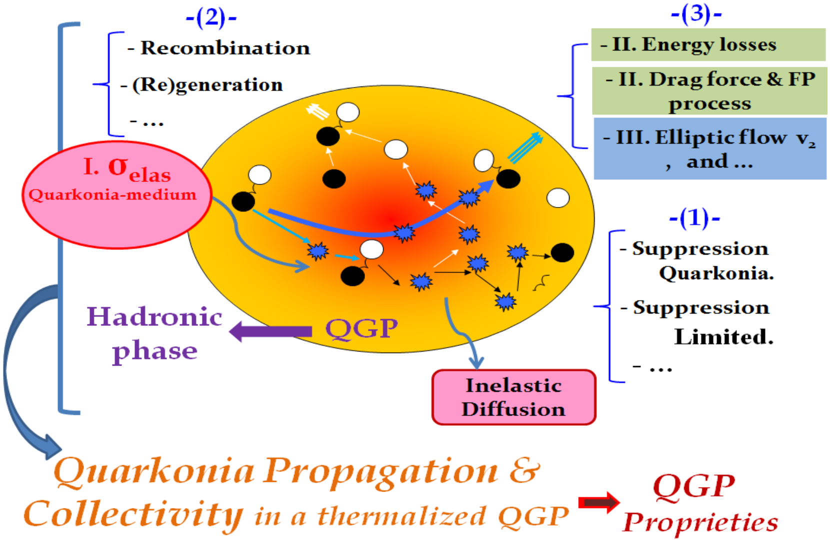
t = $2.10^{-22}$ s

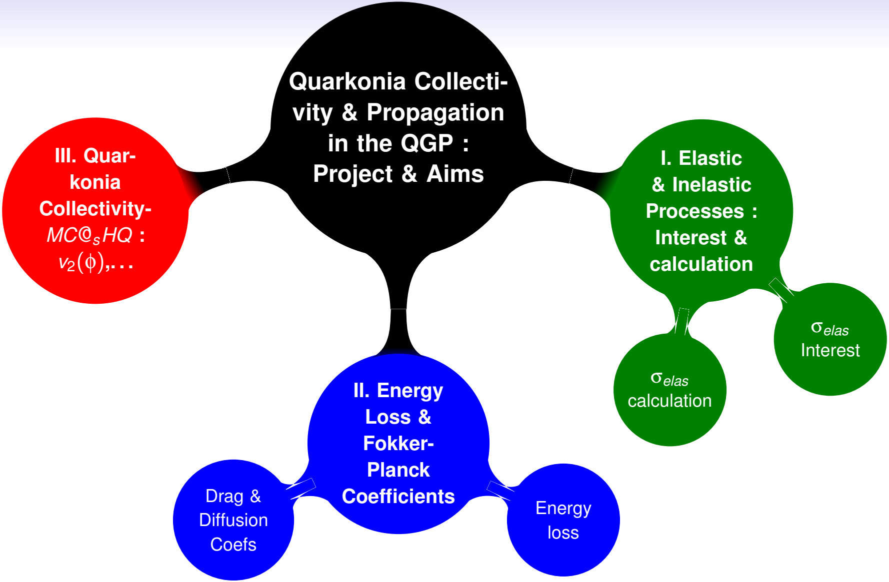
**Accelerator, Colliders**

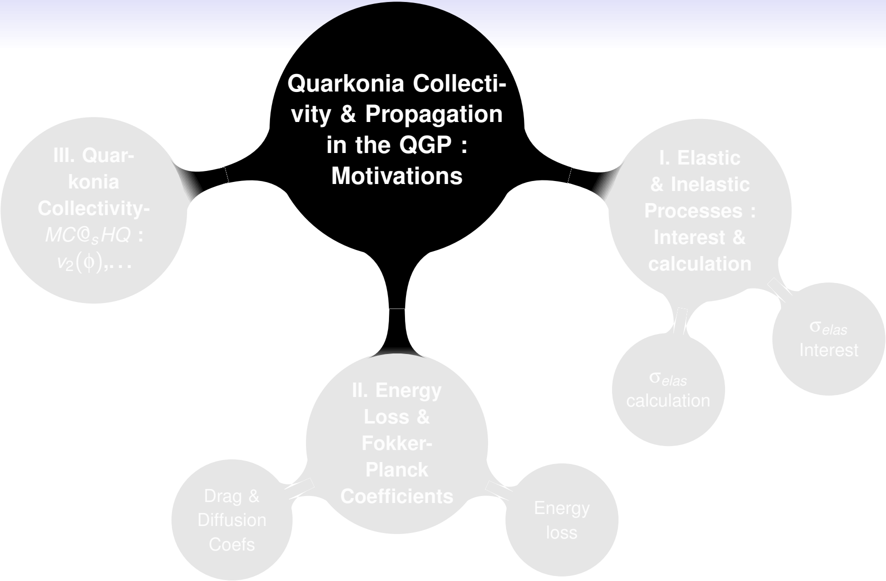



**Quarkonia : bound states $Q\bar{Q}$ ⟵ analogy positronium e+e⁻**

# Project : quarkonia propagation & collectivity . . .




Quarkonia Collectivity & Propagation in the QGP : Project & Aims
III. Quarkonia Collectivity- $MC@_sHQ$ : $v_2(\phi)$,. . .
I. Elastic & Inelastic Processes : Interest & calculation
$\sigma_{elas}$ Interest
$\sigma_{elas}$ calculation
II. Energy Loss & Fokker-Planck Coefficients
Drag & Diffusion Coefs
Energy loss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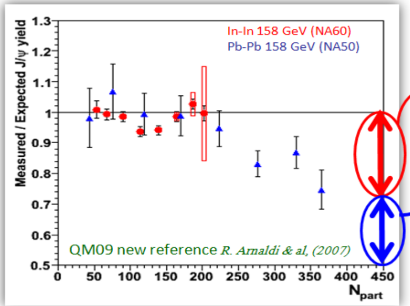
# *Motivations ...*

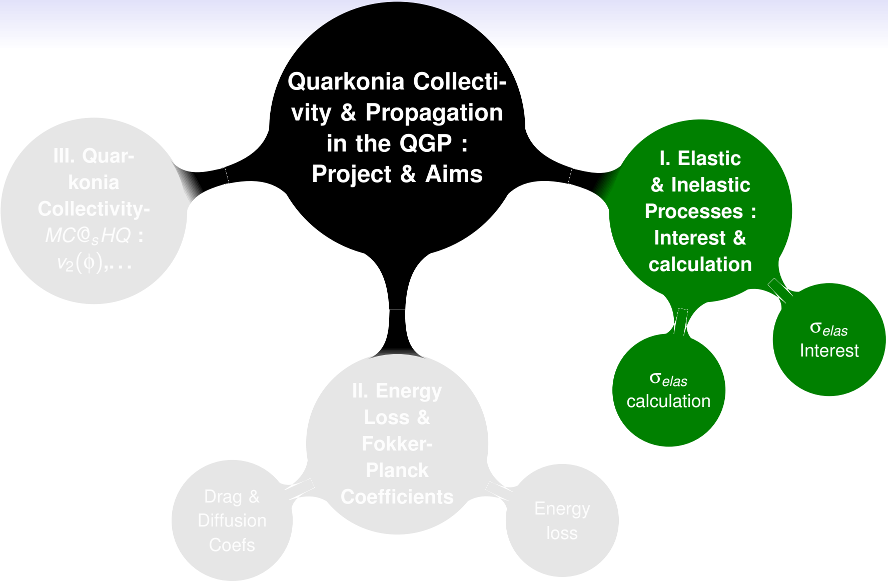



- Suppressed (studied with $\sigma_{inel}$)
- Focus on J/ψ remaining

properties modified in the plasma during the scattering J/ψ-hadron, J/ψ-gluons...

$\sigma_{elas}$ : *Elliptic flow, Energy losses ...*

+ *J/ψ Suppression studied for 20 years,*
  *... But No consistent results ....*
+ *Recent Results of RHIC* ... No significant additional suppression expected has been observed for J/ψ by increasing energy...
+ *Large fluctuations over the value of* $\sigma_{elas}$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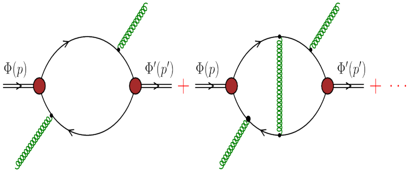
# Elastic & Inelastic Process

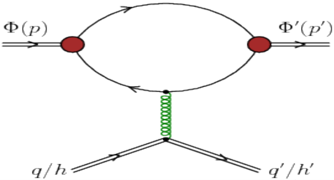
## 1 Elastic Process

- Compton Diffusion



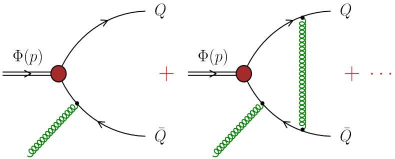

Most important process

- Hadron Diffusion



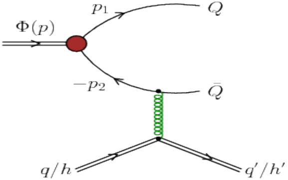

## 2 Inelastic Process

- Gluon Dissociation




Dominant process

- Hadron Dissociation



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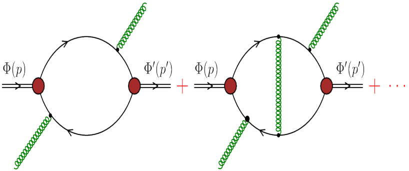


Most important process

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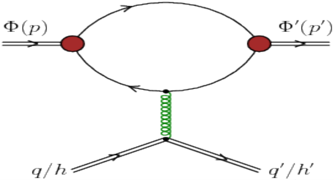


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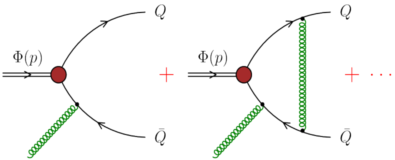


Dominant process

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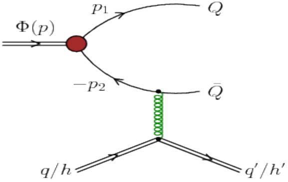
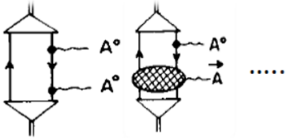


# $\sigma_{elas}$ calculation : How ?

- *Formalisms for* $\sigma_{elas}$ *calculation*

1. *Historical Calculation on Q-h/g* $\sigma_{elas}$

1. *Bhanot and Peskin formalism (79)*
   - *From OPE (operator product expansion )*
   - *Binding energy* $= e_0 \gg LQCD$




*field-dipole Interaction*

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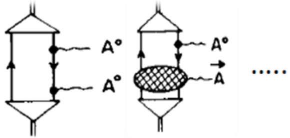
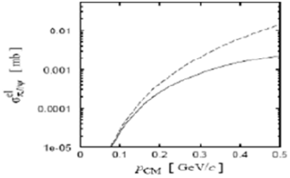



field-dipole Interaction

2. Kharzeev and Fujii (99), Povh & Hüfner
   - Short-distance QCD calculation
   - Optic theorem ...




Kharzeev and Fujii (99)

# $\sigma_{elas}$ calculation : How ?

- Formalisms for $\sigma_{elas}$ calculation

2. Rederivation of Peskin formula using Bethe-Salpeter

3. Other equivalent method in 1st order in pQCD based on :
   - Factorization Theorem + Bethe-Salpeter Amplitudes

## Factorization Formula

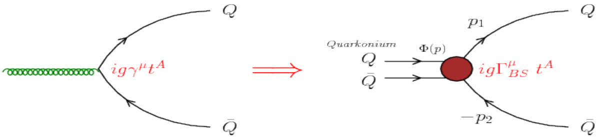
$$\sigma_{\phi h}(\nu) = \int_0^1 dx\, \sigma_{\phi g}(x\nu) \times g(x)$$

- $\sigma$ total of scattering quarkonia $\phi - h$
- Perturbative Cross Section of $\phi - g$ scattering $\Rightarrow$ BS Formalism
- Distribution function of gluons in the hadron

Y. Oh, S. Kim, S. Houng Lee,(02)

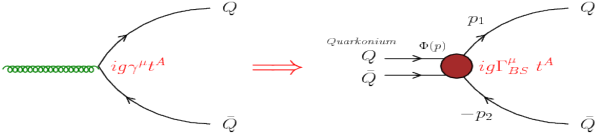
# Bethe Salpeter Formalism

1. ## Goal : Bethe Salpeter Vertex



# Bethe Salpeter Formalism

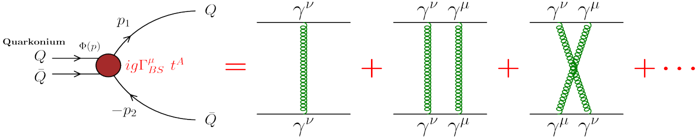
1. Goal : Bethe Salpeter Vertex




2. Bethe Salpeter Equation

$$\mathcal{M} = \mathcal{M} + \int \mathcal{V}\mathcal{G}\mathcal{V} + \int\int \mathcal{V}\mathcal{G}\mathcal{V}\mathcal{G}\mathcal{V} + \cdots + \left(\int \mathcal{V}\mathcal{G}\right)^n + \cdots = \frac{\mathcal{V}}{1 - \int \mathcal{V}\mathcal{G}}$$

$\mathcal{V}$ : kernel, $\mathcal{M}$ : amplitude, $\mathcal{G}$ : propagateur

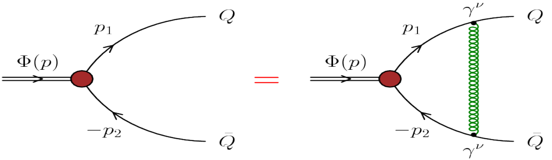



Bound states : produce a pole in $\mathcal{M}$.

$\Rightarrow$ $\mathcal{M}$ eigenvector $\Gamma$ satisfies : $\Gamma = \int_k \mathcal{V}(p,k,P)\mathcal{G}(k,P)\Gamma(k,P)$

# Bethe Salpeter Formalism

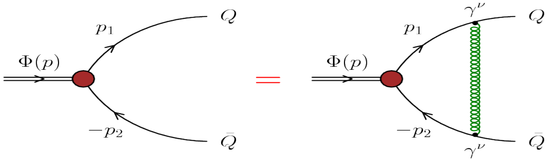
2. Bethe Salpeter Equation




$$\Gamma(p,P) = iC_{color}\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2+i\eta}\gamma_\nu \frac{1}{\not{p}_1+\not{k}-m+i\eta}\Gamma(p+k,P)\frac{1}{-\not{p}_2+\not{k}-m+i\eta}\gamma_\nu$$

# Bethe Salpeter Formalism

## 2 Bethe Salpeter Equation




$$\Gamma(p,P) = iC_{color}\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2+i\eta}\gamma_\nu \frac{1}{\not{p}_1+\not{k}-m+i\eta}\Gamma(p+k,P)\frac{1}{-\not{p}_2+\not{k}-m+i\eta}\gamma_\nu$$

## 3 Bethe Salpeter Vertices *(Case of quarkonium in the rest frame)*

- *Instantanous Interaction*

$$\Gamma_I(E,\vec{p}) \approx \frac{-ie_0(2e_0-E)}{\pi E}\gamma^0\,\frac{I+\gamma_0-\vec{\not{p}}/m}{2}.\,\phi_I^{+-}(\vec{p})\,.\frac{I-\gamma_0-\vec{\not{p}}/m}{2}.\gamma^0$$

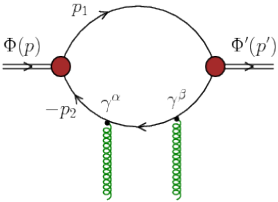
- $\phi_I^{+-}(\vec{p}) = \phi_{space}(\vec{p})\times\phi_{spin}^{ij}$ : *instantanous wave function for the bound state*

# $\sigma_{elas}$ Results & Discussion

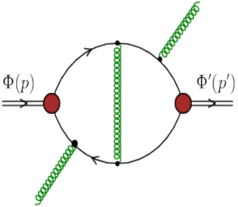
1. *Compton Diffusion Process* $J/\psi - g$

   1. *2 gluons attached, "LO"*

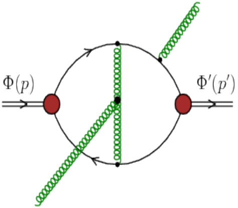
$\Rightarrow$ 6 diagrams (bb||, bbX, tt||, ttX, tb, bt)



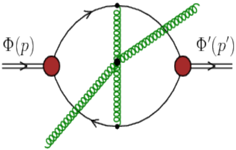

   2. *3 gluons attached, "SNLO"*




$\Rightarrow$ 4 diagrams
(bb||, btX, tt||, tbX)




$\Rightarrow$ 7 diagrams
(gluon emitted in each line)

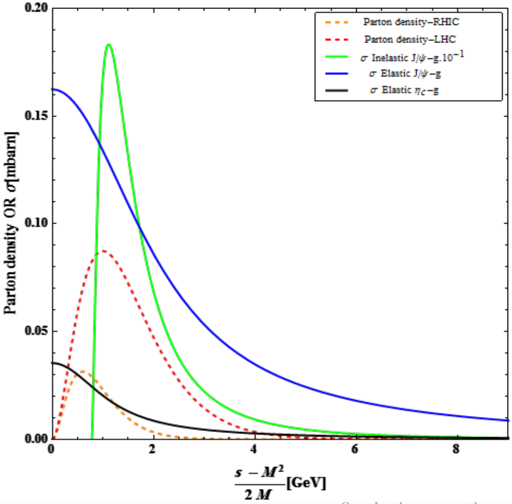



$\Rightarrow$ 1 diagram

# $\sigma_{elas}$ Interest & Discussion

1. $J/\psi - g$ : Gluon Dissociation vs Compton Diffusion

"LO" Diagrams

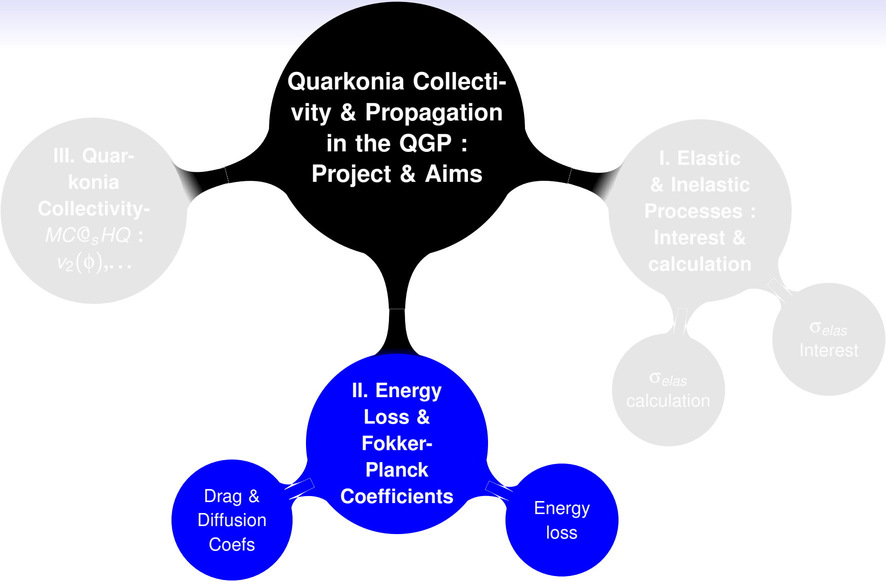



$$R = \int d^3k \; n_{mb}(k) \; \sigma_{elas/inel}(s(k))$$

$$\hookrightarrow R = \int_{M^2}^{+\infty} ds \; \tilde{n}_{mb}(k) \; \sigma_{elas/inel}(s)$$

- Inelastic cross section has a threshold
- Quantities measured are convluted by MB Distribution
- Overlap $\sigma_{elas}$ and MB ditribution larger than $\sigma_{inel}$ and MB.

Y. Oh, S. Kim, S. Houng Lee,(02)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# Energy losses & Transport Coefficient

1. *Energy losses, given by Bjorken* ($\phi(M, E, p) \rightsquigarrow$ "$i$" $(m, e, q)$)

$$\frac{dE}{d\tau} = \frac{dE}{dt} \times \frac{E}{M} = \sum_i \int d^3q\, n_i(\vec{q})\, \frac{\sqrt{(p.q)^2 - M^2 m^2}}{Me} \int dt\, \frac{d\sigma_{elas}}{dt} (E' - E),$$

# *Energy losses & Transport Coefficient*

1. ***Energy losses*, *given by Bjorken*** $(\phi(M,E,p) \rightsquigarrow "i"(m,e,q))$

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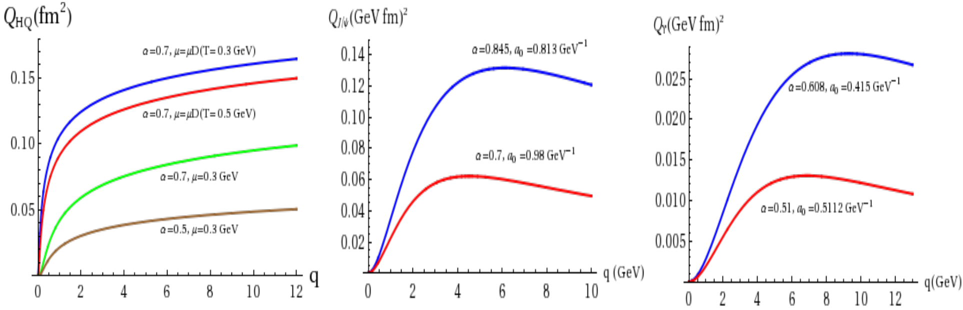
2. ***Transport coefficient*** $\hat{Q}(s) = \int \frac{d\sigma_{elas}}{dt}\, t\, dt$ *: cf. Part I*



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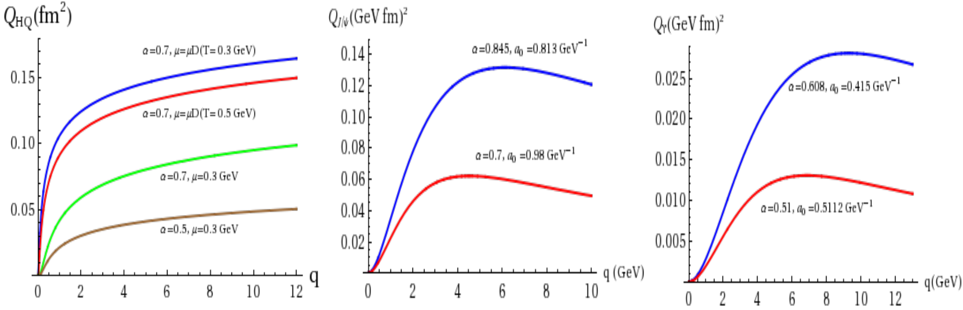
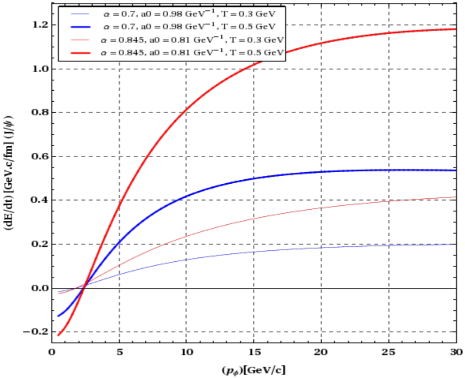


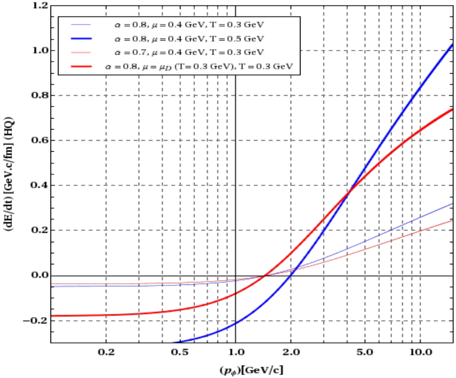
# Energy losses (HQ, $J/\psi$, $\Upsilon$)

## 1 Quarkonia ($J/\psi$)




- Behaviour : decrease at p ↗
- Two regimes, $P \approx \sqrt{MT} \Rightarrow \frac{dE_{J/\psi}}{dt} = 0$
- $\frac{dE}{dt}$ variation vs α and $T$, $\frac{dE}{dt}$ ↗ with T ↗

## 3 Heavy Quark (C)




- Behaviour : log increase vs p
- $P \approx \sqrt{MT} \Rightarrow \frac{dE_{HQ}}{dt} = 0$
- $\frac{dE}{dt}$ ↗ with T ↗

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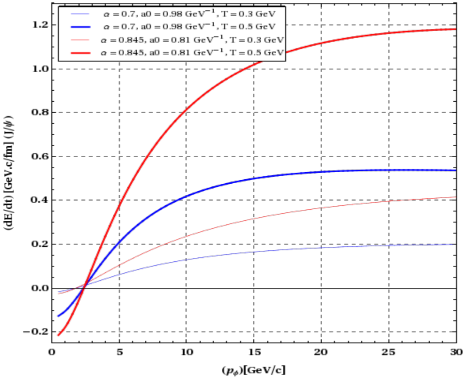


- Behaviour : decrease at p $\nearrow$
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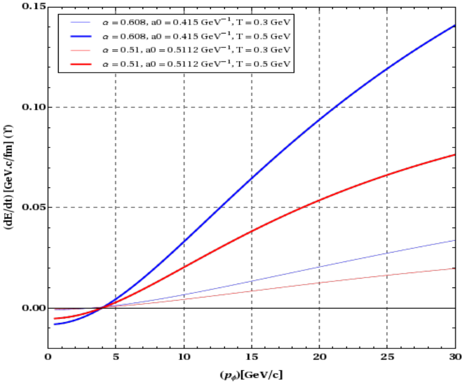
## 3 Quarkonia ($\Upsilon$)




- Behaviour : decrease at p $\nearrow$
- Two regimes, $P \approx \sqrt{MT} \Rightarrow \frac{dE_{\Upsilon}}{dt} = 0$
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# Drag Force & coefficient (HQ, $J/\psi$, $\Upsilon$)

1. **Drag Coefficient**, *for* : ($\phi(M,E,p) \rightsquigarrow$ "$i$" $(m,e,q)$)

$$A_i = \frac{d<p>}{dt} = \sum_i \int d^3q\, n_i(\vec{q})\, \frac{\sqrt{(p.q)^2 - M^2 m^2}}{Ee} \int dt\, \frac{d\sigma_{elas}}{dt}\, \frac{\left\langle (\vec{P}-\vec{P}').\,\vec{P} \right\rangle}{\|\vec{P}\|},$$

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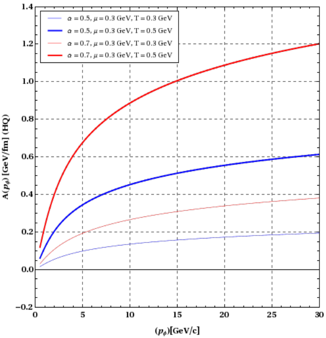
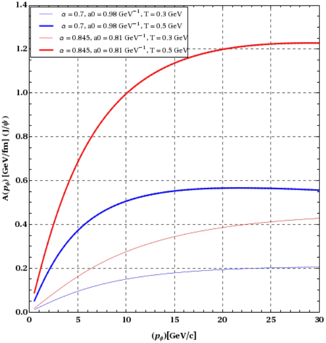
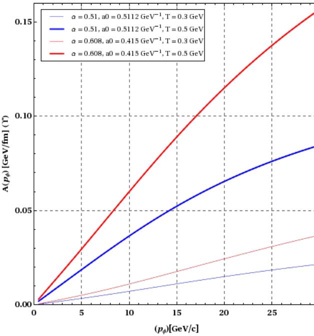
1. Heavy Quark




2. Quarkonia ($J/\psi$)




3. Quarkonia ($\Upsilon$)



# *Diffusion coefficient (HQ, $J/\psi$, $\Upsilon$ )*

1. *Diffusion Coefficient*, *for : ($\phi(M,E,p) \rightsquigarrow$ " i" $(m,e,q)$)*

$$B(E) = \int_E^{+\infty} dE' \, A_i(E') \times \frac{E'}{P'} \, e^{-(E'-E)/T}, \; with: \; B_\perp = B_\parallel = B, \; B \leftrightarrow A \text{ relation},$$

- Fokker-Planck equation : $\frac{\partial f}{\partial t} = \frac{\partial}{\partial p_i}\left(A_i f + \frac{\partial}{\partial p_i} B_{ij} f\right) = -\vec{\nabla}_p . \vec{\rho}$, (homogenous background)
- Einstein relation : $\left[\vec{A}f + \vec{\nabla}_p(Bf)\right]_i = 0, \; f = e^{-E/T}$, (stationary, D.B Walton, J.Rafelski(99))

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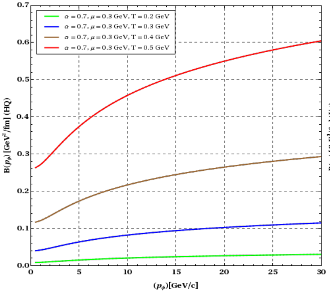
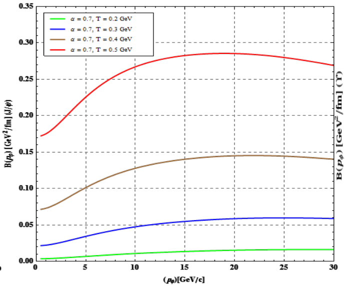
- Einstein relation : $\left[\vec{A}f + \vec{\nabla}_p(Bf)\right]_i = 0, \ f = e^{-E/T}$, (stationary, D.B Walton, J.Rafelski(99))

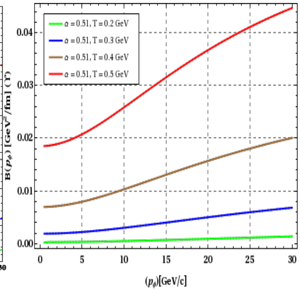
### 1 Heavy Quark

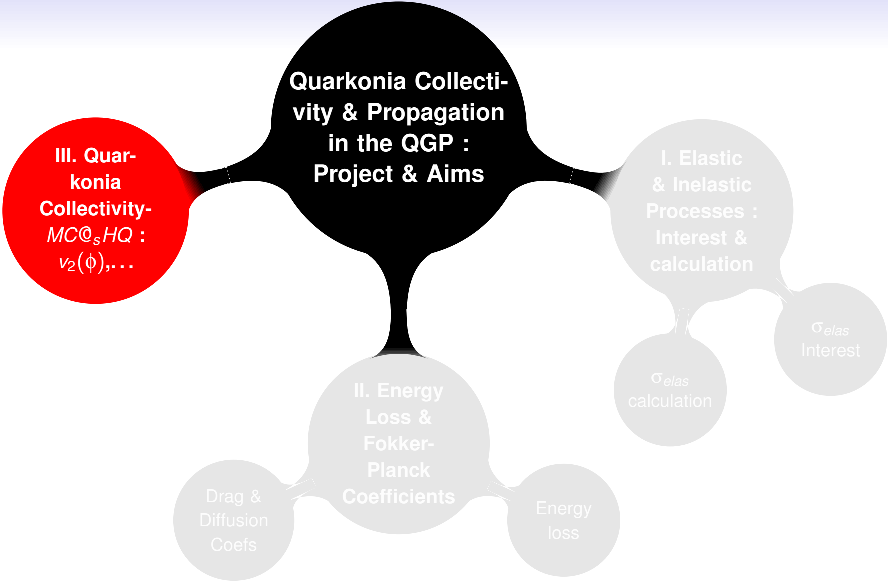



### 2 Quarkonia ($J/\psi$)



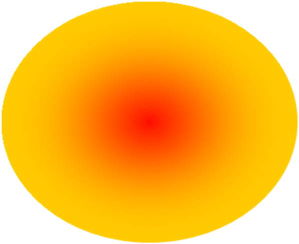

### 3 Quarkonia ($\Upsilon$)





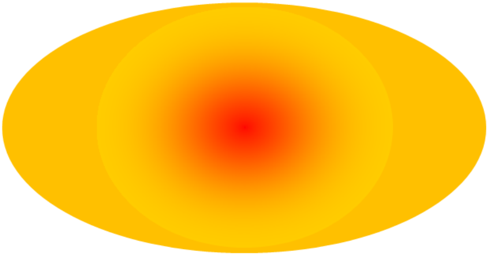
Quarkonia Collectivity & Propagation in the QGP : Project & Aims
III. Quarkonia Collectivity-$MC@_sHQ$ : $v_2(\phi)$,...
I. Elastic & Inelastic Processes : Interest & calculation
$\sigma_{elas}$ Interest
$\sigma_{elas}$ calculation
II. Energy Loss & Fokker-Planck Coefficients
Drag & Diffusion Coefs
Energy loss

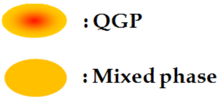
# MC@sHQ- as a cartoon



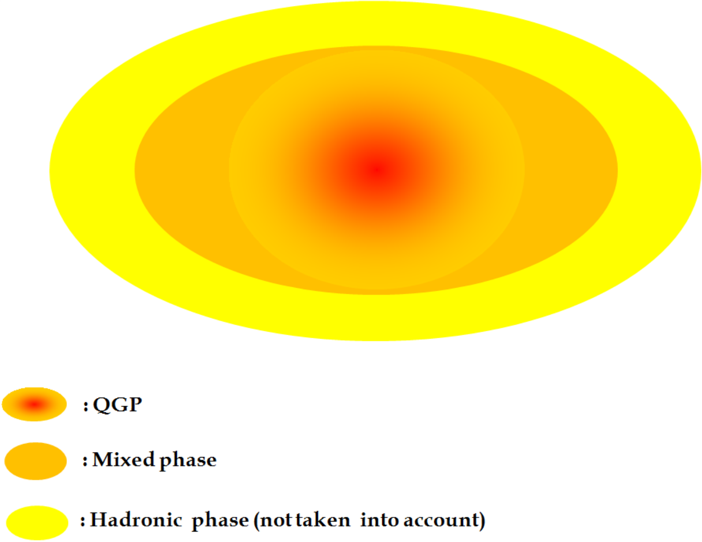

: QGP

# MC@sHQ- as a cartoon

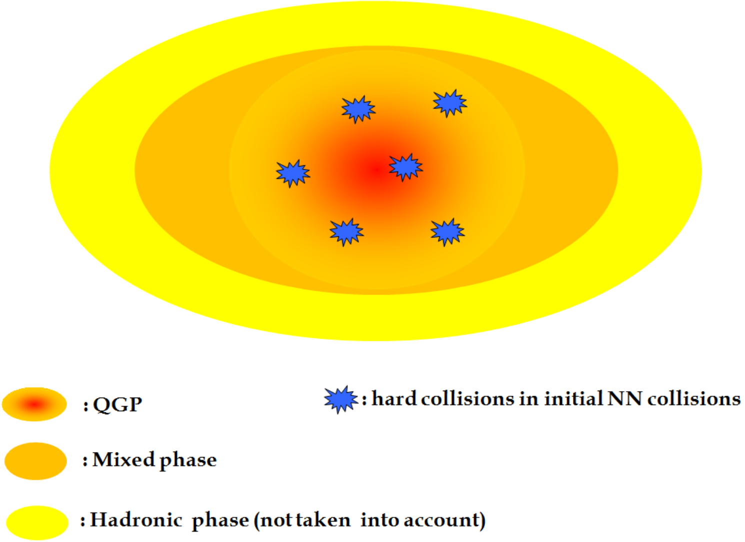




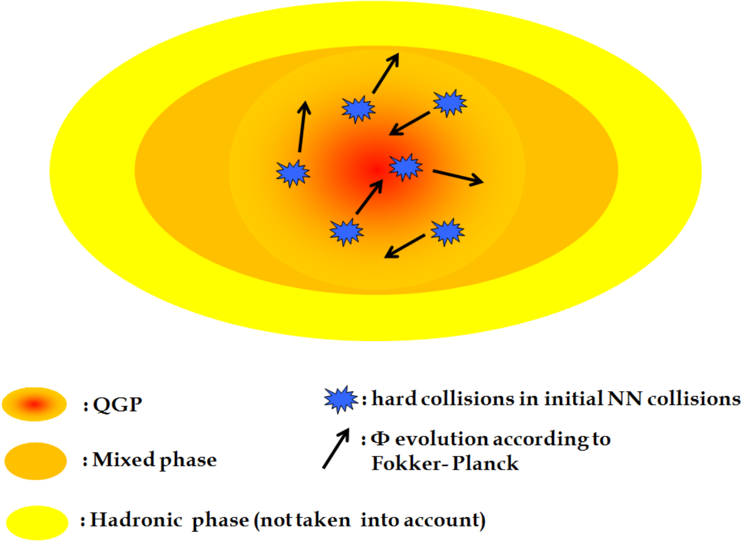
# *MC@sHQ- as a cartoon*



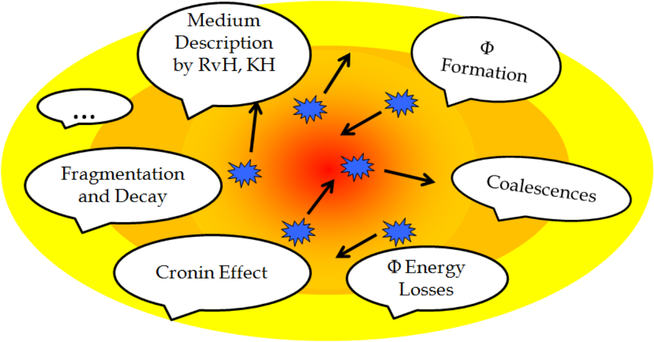
# MC@sHQ- as a cartoon



# MC@sHQ- as a cartoon



# MC@sHQ- as a cartoon




: QGP

: Mixed phase

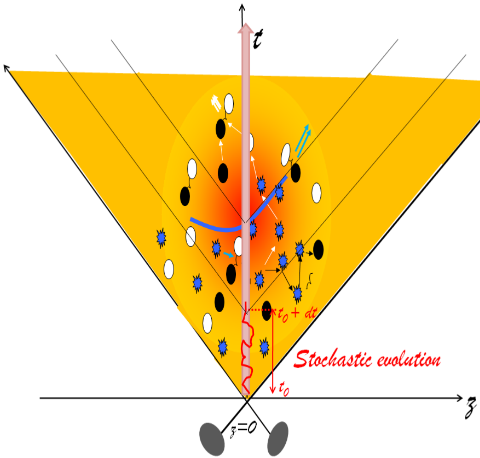
: Hadronic phase (not taken into account)

: hard collisions in initial NN collisions

: Φ evolution according to Fokker-Planck

# MC@sHQ-Keywords

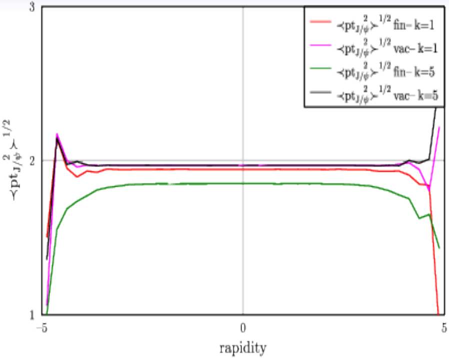
## ① Stochastic evolution of $Q\bar{Q}$

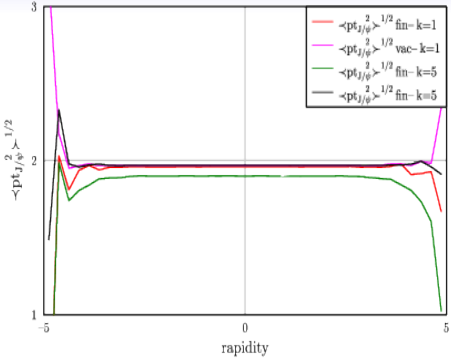



## ② MC@sHQ in few words

- **Stochastic Evolution :** Boltzman evolution or FP evolution
- $J/\psi$ **suppression :**- by T, - cold nuclear effects, -Inelastic processes.
- Au-Au/Pb-Pb collisions at $\sqrt{s} = 200$ GeV or $\sqrt{s} = 5.5$ TeV
- Coalescence and fragmentation mechanism . . .
- Plasma type and . . .
- **study many observables :** $v_2(\phi)$, $< p_t^2 >$, $\frac{dN}{d\phi_{rel}}$, $R_{AA}$, $p_t$ spectra, y spectra, $\phi$-spectra . . .

# Mean $< p_t^2 >^{1/2}$ of $J/\psi$ vs $y$



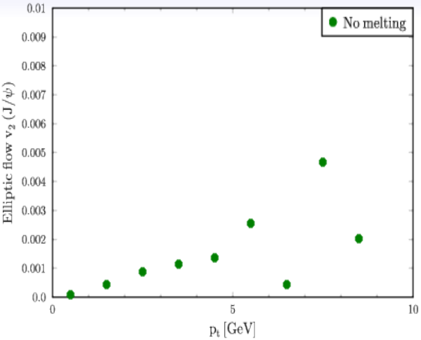



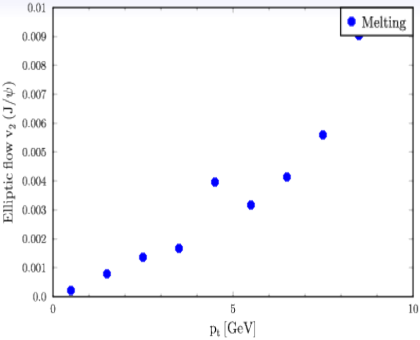

- Au-Au, $\sqrt{s} = 200GeV$, Central collisions
- No Melting. tuning factor k=1,5.

- Au-Au, $\sqrt{s} = 200GeV$, Central collisions
- Melting ($T_c$ ($J/\psi$, $\Upsilon$)= 0.3 GeV), k=1,5

- Interaction with the medium $\Rightarrow$ reduce their mean $p_t \prec pt^2_{J/\psi} \succ^{1/2}$
- Less visible effect with Melting & more visible with $\nearrow \sigma_{elas}$

# $J/\psi$ Elliptic flow vs $p_t$ at mid rapidity



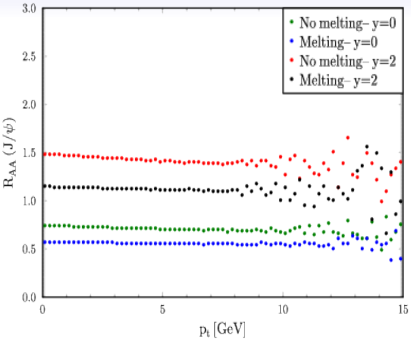



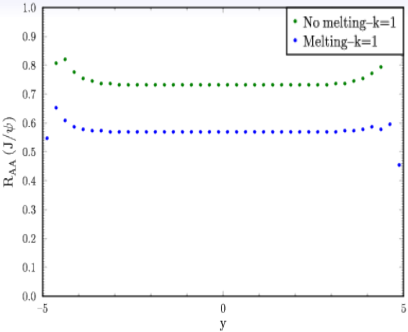

- Au-Au, $\sqrt{s} = 200 GeV$, Min bias collisions
- No Melting, 50M collisions

- Au-Au, $\sqrt{s} = 200 GeV$,Min bias collisions
- Melting ($T_c$ $(J/\psi, \Upsilon)$ = 0.3 GeV), 50M collisions

  - Non zero elliptic flow
  - Dissociation by T $\Rightarrow v_2 \nearrow$... Recombination ?
  - Influence of elastic processes, increase of $\sigma_{elas} \Rightarrow v_2 \nearrow$

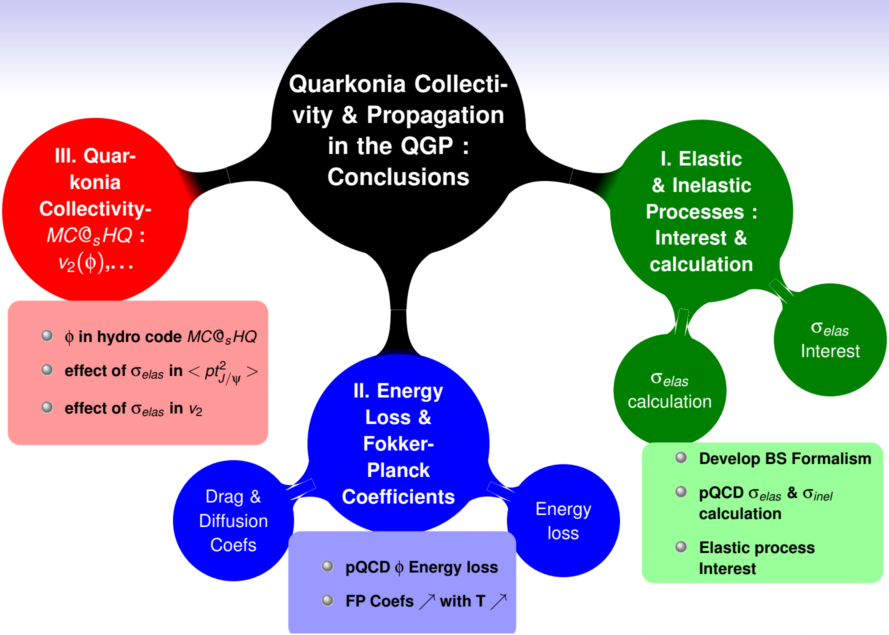
# $R_{AA}$ of $J/\psi$ vs $p_t$ at mid/average rapidity & vs y




- Au-Au, $\sqrt{s} = 200 GeV$, Min bias collisions
- No Melting, 50M collisions
- Au-Au, $\sqrt{s} = 200 GeV$,Min bias collisions
- Melting ($T_c$ ($J/\psi$, $\Upsilon$) = 0.3 GeV), 50M collisions

  - Dissociation by T $\Rightarrow R_{AA}(J/\psi) \searrow$
  - Dissociation by T $\Rightarrow R_{AA}(J/\psi)$ variation vs $p_t \approx$ small
  - Influence of elastic processes, increase of $\sigma_{elas} \Rightarrow v_2 \nearrow$ ?


Quarkonia Collectivity & Propagation in the QGP : Conclusions
III. Quarkonia Collectivity- $MC@_sHQ$ : $v_2(\phi)$,...
$\phi$ in hydro code $MC@_sHQ$
effect of $\sigma_{elas}$ in $< pt^2_{J/\psi} >$
effect of $\sigma_{elas}$ in $v_2$
I. Elastic & Inelastic Processes : Interest & calculation
$\sigma_{elas}$ Interest
$\sigma_{elas}$ calculation
Develop BS Formalism
pQCD $\sigma_{elas}$ & $\sigma_{inel}$ calculation
Elastic process Interest
II. Energy Loss & Fokker-Planck Coefficients
Drag & Diffusion Coefs
Energy loss
pQCD $\phi$ Energy loss
FP Coefs $\nearrow$ with T $\nearrow$

# Quarkonia Collectivity & Propagation in the QGP : Perspectives

## I. Elastic & Inelastic Processes : Interest & calculation

- $\sigma_{elas}$ Interest
- $\sigma_{elas}$ calculation

- **Investigate the role of $\phi$ wave function**
- **fine structure of BS vertex**

## II. Energy Loss & Fokker-Planck Coefficients

- Drag & Diffusion Coefs
- Energy loss

- **Elastic & inelastic interaction rate**
- $E_{loss}$, $A_{Drag}$ $(\phi - q)$ ...

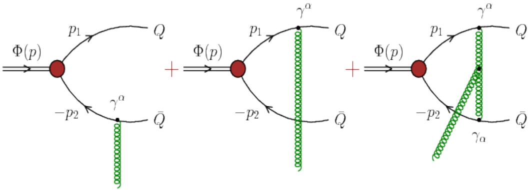
## III. Quarkonia Collectivity- *MC@$_s$HQ* : $v_2(\phi)$,...

- **$\phi$ in hydro code *MC@$_s$HQ***
- **Inclusion of recombination ...**
- **$J/\psi$, $\Upsilon$ ...other observable**

# *Contents*

# $\sigma_{inel}$ Results & Discussion

## 1 Gluon Dissociation Process $J/\psi - g$

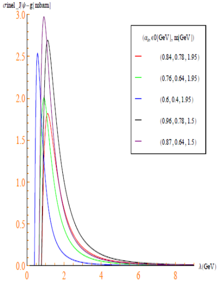



$$|\mathcal{M}|^2 = \frac{4g^2 m^2 M_\phi k_0^2}{3N_c} \, |\nabla \psi(\vec{p})|^2$$

$$\Downarrow$$

$$\sigma_{\phi g}(\lambda) = \frac{128 g^2}{3N_c} \, a_0^2 \frac{(\lambda/\varepsilon_0 - 1)^{3/2}}{(\lambda/\varepsilon_0)^5}$$

$$\text{with}: \lambda = \frac{q.k}{M_\phi} = \frac{s - M_\phi^2}{2M_\phi}$$

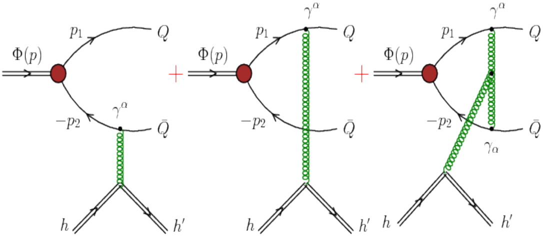



- *Dependence* $\sigma_{inel}$ *vs* $\varepsilon$ *et* $m$

*Oh, S. Kim, S. Houng Lee,(02)*

# $\sigma_{inel}$ Results & Discussion

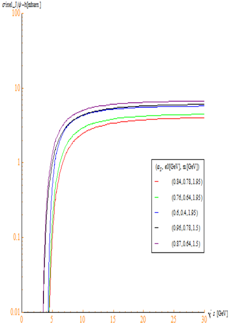
## 2 *Hadron Dissociation Process* $J/\psi - h$




*Factorization Theorem*

$$\sigma_{\phi h}(\nu) = \int_0^1 dx\, \sigma_{\phi g}(x\nu) \times g(x)$$

- *Cross section of* $J/\psi - h$
- *Cross section of* $J/\psi - g$
- *GDF* : $g(x) = 0.5(\eta+1)\frac{(1+x)^{\eta}}{x}$, $\eta = 5(BP)$




- *Dependence* $\sigma_{inel}$ *vs* $\varepsilon$ *et* $m$

*Oh, S. Kim, S. Houng Lee,(02)*

# $\sigma_{elas}$ Results & Discussion

## Compton Diffusion Process $J/\psi - g$

### 2 gluons attached, "LO"

- *Soft Gluons* $(k \approx mg^4)$ : *columbic case*

$$\mathcal{M}(k,k' \approx mg^4) \approx -2\alpha g^2 \frac{\delta^{ab}}{2N_c}\, \varepsilon_{\lambda 1}(k_1).\varepsilon_{\lambda 2}(k_2)$$

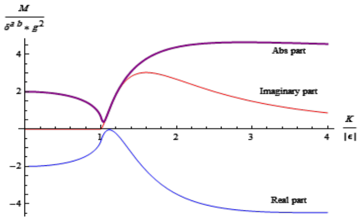
  - *Opening of the imaginary part for* $K < |\varepsilon|$
  - *Opening of the imaginary part for* $K = |\varepsilon|$

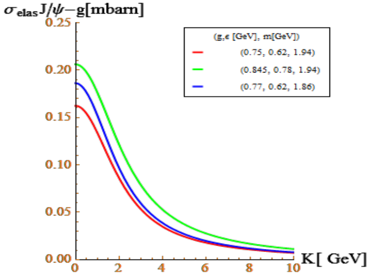
- *Hard Gluons* $(k \approx mg^2)$

$$\mathcal{M}(k,k' \equiv mg^2) \approx -\frac{4g^2\delta_{ij}\delta^{ab}}{N_c} \times \frac{1}{\left(1+\left(\frac{a_0|\mathbf{k}-\mathbf{k}'|}{4}\right)^2\right)^2}$$

  - *Form Factor*

$$\Rightarrow \sigma_{elas} \approx \frac{g^4}{2\pi m^2 N_c^2} \times \frac{1+\frac{K^2}{8m|\varepsilon|}+\frac{1}{2}\left(\frac{K^2}{8m|\varepsilon|}\right)^2}{\left(1+\frac{K^2}{8m|\varepsilon|}\right)^3}$$

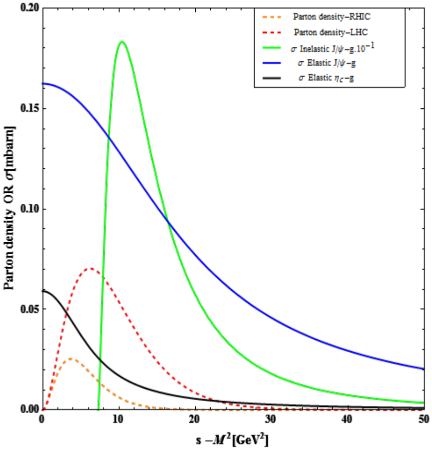


# $\sigma_{elas}$, $\sigma_{inel}$ Results & Discussion

## 1 $J/\psi - g$ : Gluon Dissociation vs Compton Diffusion

"LO" Diagrams




$$R = \int d^3k \; n_{mb}(k)\, \sigma_{elas/inel}(s(k))$$

$$\hookrightarrow R = \int_{M^2}^{+\infty} ds \; \tilde{n}_{mb}(k)\, \sigma_{elas/inel}(s)$$

- $\rightharpoonup$ Parton density at LHC energy
- $\rightharpoonup$ Parton density at RHIC energy
- $\rightharpoonup$ $\sigma$ Inelastic $J/\psi - g$
- $\rightharpoonup$ $\sigma$ Elastic $J/\psi - g$
- $\rightharpoonup$ $\sigma$ Inelastic $\eta_c - g$

# Diffusion coefficient (HQ, $J/\psi$, $\Upsilon$ )

1. **Diffusion Coefficient**, *for : ($\phi(M,E,p) \rightsquigarrow$ " i" $(m,e,q)$)*

$$B(E) = \int_E^{+\infty} dE' \, A_i(E') \times \frac{E'}{P'} \, e^{-(E'-E)/T}, \; with: \; B_\perp = B_\parallel = B \quad B \leftrightarrow A \text{ relation},$$

- Fokker-Planck equation : $\frac{\partial f}{\partial t} = \frac{\partial}{\partial p_i}\left(A_i f + \frac{\partial}{\partial p_i} B_{ij} f\right) = -\vec{\nabla}_p . \vec{\rho}$, (homogenous background)
- Einstein relation : $\left[\vec{A}f + \vec{\nabla}_p(Bf)\right]_i = 0, \; f = e^{-E/T}$, (stationary case)

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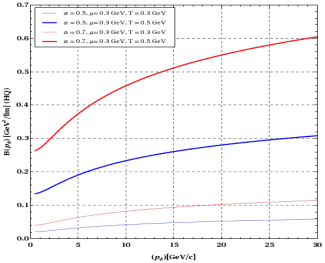


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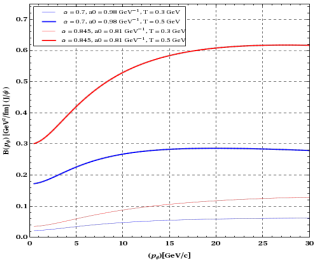


### 3 Quarkonia ($\Upsilon$)