Collective modes of trapped Fermi gases in the BCS-BEC crossover

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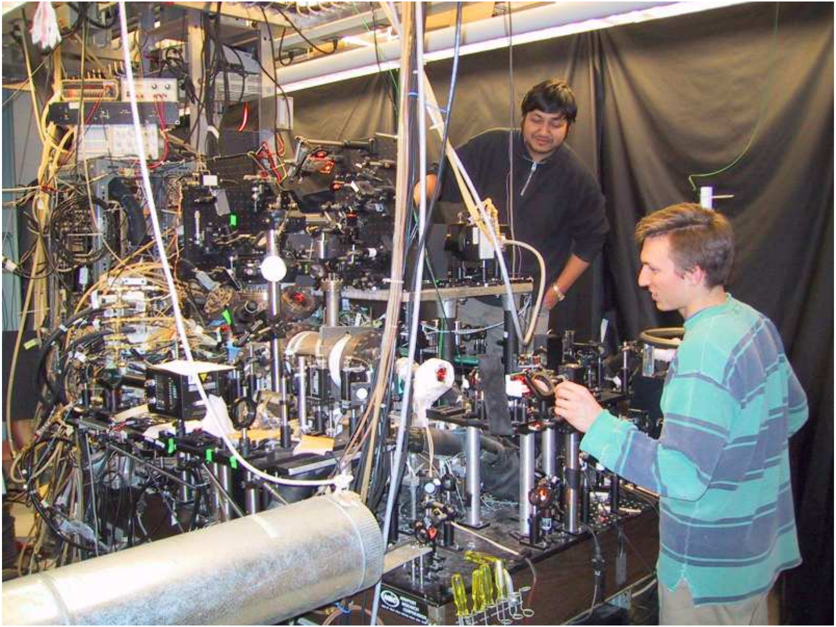
Outline

1. Brief Introduction to ultracold trapped atoms
2. BCS-BEC crossover in nuclear systems and in cold atoms
3. Collective modes of trapped Fermi gases in the BCS-BEC crossover
4. Summary and conclusions

Sodium BEC experiment (group of W. Ketterle, MIT)



Selection of experimental groups working with fermions

- ENS Paris (C. Salomon)
- Innsbruck (R. Grimm)
- Duke University (J. Thomas)
- Rice University (R. Hulet)
- MIT (W. Ketterle)
- JILA (D. Jin)

Schematic view of experiments with trapped Fermi gases

- Create trap potential
  (combining lasers and/or magnetic fields)

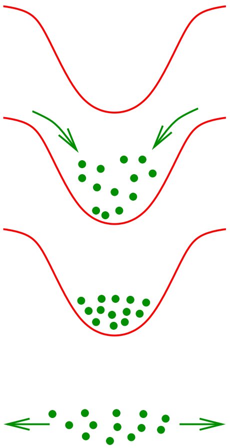
  Near its minimum: $V(\vec{r}) = \frac{1}{2} m \sum_{i=x,y,z} \omega_i^2 r_i^2$

  Typically: $\omega_z \ll \omega_x, \omega_y$ (cigar shape)

- Load the atoms into the trap

- Cool them down
  (laser cooling, evaporative cooling)

- Measure density profile by taking a picture
  (if the cloud is too small, let it first expand
  by switching off the trap)



Typical scales

system size	$\sim 100\ \mu\text{m}$	
atom number	$\sim 10^6$	
density n	$\sim 10^{18}\ \text{m}^{-3}$	(compare with air: $3 \times 10^{25}\ \text{m}^{-3}$)
distance between atoms d	$\sim 1\ \mu\text{m}$	

- thermal de Broglie wave length $\lambda = \frac{2\pi\hbar}{p} = \frac{2\pi\hbar}{\sqrt{2mk_B T}}$
- quantum statistics important if wave packets start to overlap, i.e., if $d \sim \lambda$

$$T \sim \frac{1}{2mk_B}\left(\frac{2\pi\hbar}{d}\right)^2 \sim 1\ \mu\text{K}$$

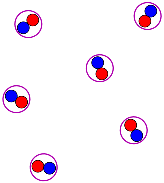
- much lower temperatures ($\sim$ 10 nK) have been reached!

- range of the atom-atom interaction (van der Waals): $R \sim 1$ Å
- since $R/d \sim 10^{-4}$, the interaction can be replaced by a δ function
- interaction fully characterized by the s-wave scattering length a

BCS-BEC crossover in symmetric nuclear matter

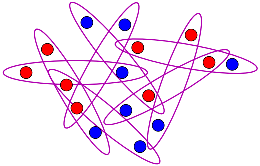
BEC limit:

- at low density and temperature: protons and neutrons are bound in deuterons
- deuterons form a Bose-Einstein condensate (BEC) if $T < T_{BEC}$
- deuterons are dissociated if $T \gtrsim E_B = 2.2$ MeV



BCS limit:

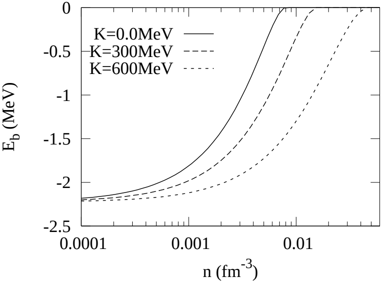
- at high density: deuterons are not bound because of Pauli blocking
- nevertheless nucleons form Cooper pairs at $T < T_{BCS}$
- temperature of Cooper pair formation and condensation is the same

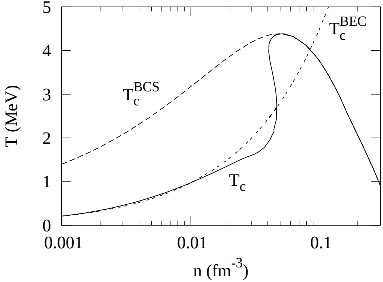


Nuclear matter within the Nozières-Schmitt-Rink scheme

M. Jin, M.U., P. Schuck: PRC 82, 024911 (2010)

- calculate in-medium T matrix (separable Yamaguchi potential)
- NSR scheme $\widehat{=}$ density calculated from Green's function with one self-energy insertion in T-matrix approximation
- here: mean-field calculated with Gogny force, use T-matrix only for correlation contribution
- deuteron binding energy E_B and superfluid critical temperature T_C as functions of the density n:



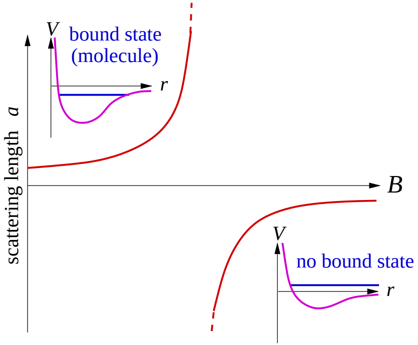


Unitary limit as "simple" model for dilute neutron matter

- neutron-neutron $^1S_0$ scattering length $a = -18$ fm
  much larger than range of interaction ($R \sim 1$ fm)
- at low density, one can simultaneously satisfy $k_F R \ll 1$ and $k_F|a| \gg 1$
- simplified model: consider the "unitary limit" $k_F R \to 0$ and $k_F|a| \to \infty$
- at $T = 0$: only length scale in the system: $1/k_F \quad (k_F = (3\pi^2 n)^{1/3})$
- energy per particle: $\left(\dfrac{E}{A}\right)_{T=0} = \xi \dfrac{3}{5} \dfrac{\hbar^2 k_F^2}{2m_n}$
- $\xi =???$ dimensionless parameter ("Bertsch parameter")
- measurements with cold atoms ($\longrightarrow$next slide), QMC calculations: $\xi \approx 0.42$

Back to cold atoms: Feshbach resonance

- consider Fermionic atoms trapped in two hyperfine states $\uparrow, \downarrow$ with equal numbers $N_\uparrow = N_\downarrow$
- low temperature ($\longrightarrow$ low energy): interaction in s wave dominant $\longrightarrow$ interaction only between atoms of opposite spin
- depending on the B field, two atoms can have a bound state or not
- scattering length a can be tuned
- fermionic atoms $\leftrightarrow$ molecules
- on resonance ($B = 834$ G in the case of ^{6}Li): unitary limit $a \to \infty$
- at zero temperature: crossover from BEC (molecules) to BCS superfluid (Cooper pairs)

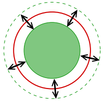


- from now on: concentrate on the attractive ($a \leq 0$) side of the resonance

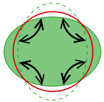
Collective modes

- Small oscillations of the cloud size or shape around equilibrium
- Experiments done at Duke, Innsbruck, ENS
- Examples (cut through the xy plane):

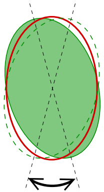
- Radial breathing mode
  - Depends on compressibility $\longrightarrow$ equation of state



- Radial quadrupole mode
  - Deformation of the Fermi sphere in a collisionless gas $\longrightarrow$ distinguish collisionless and hydrodynamic regimes
  - No compression $\longrightarrow$ independent of equation of state in the hydrodynamic regime

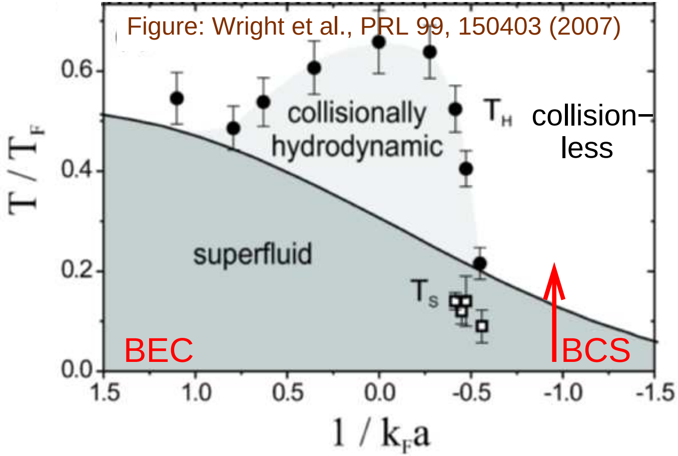


- Scissors mode
  - Rotate back and forth in a triaxial trap
  - Relation to moment of inertia $\longrightarrow$ distinguish between superfluid or normal fluid behaviour



Temperature effects in collective modes in the BCS phase

- Transition from BCS superfluidity to the collisionless normal regime



Description of collective modes in the collisionless regime

(a) Quasiparticle Random-Phase Approximation (QRPA)

- Limited to numbers of atoms $N \lesssim 10^4$
- Only available for spherically symmetric traps
- Interpretation in terms of macroscopic quantities difficult

(b) Semiclassical approaches

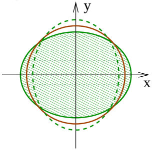
- $T = 0$: Superfluid hydrodynamics
  (coherence length $\xi \ll$ system size)
- $T \geqslant T_c$: Collisionless regime: Landau-Vlasov equation
  ($1/k_F \ll$ system size, collision rate $\ll$ trap frequency ω)
- $0 < T < T_c$: Some Cooper pairs are thermally broken,
  "mixture" of superfluid and normal components

Here:

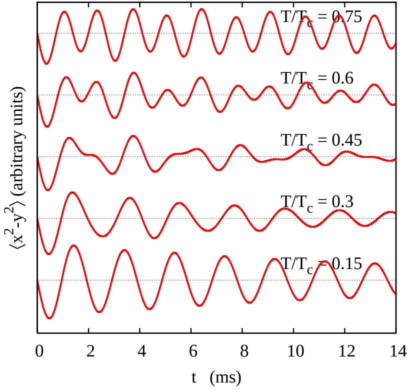
Quasiparticle transport theory by Betbeder-Matibet and Nozières (1969)
hydrodynamics and Vlasov equation as limits for $T \to 0$ and $T \to T_c$

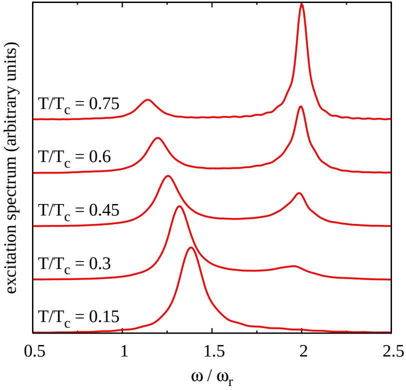
Radial quadrupole mode as function of temperature

- Cigar-shaped trap potential: $\omega_r \approx 17\omega_z$
- Limit $T \to 0$: hydrodynamic mode at $\omega = \sqrt{2}\omega_r$



- Limit $T \to T_c$: Vlasov equation predicts $\omega = 2\,\omega_r$ (Hartree field neglected)
- Results for intermediate temperatures ($N = 4 \times 10^5$, $1/k_F a = -1.5$):





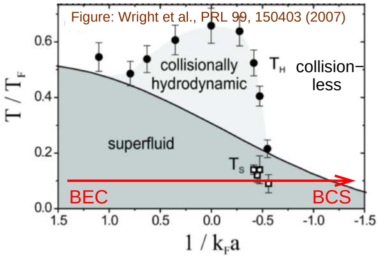
Radial quadrupole mode as function of interaction strength

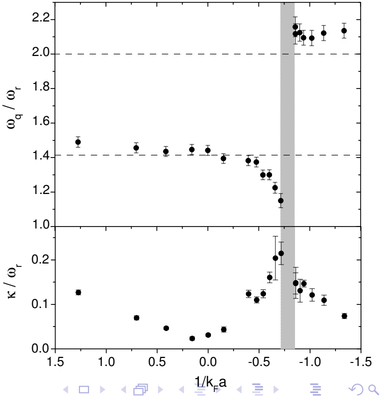
Experiment at Innsbruck
[Altmeyer et al., PRA 76, 033610 (2007)]

- 4×10^5 ^{6}Li atoms, $T = 0.1\,T_F$, vary B around Feshbach resonance (837 G)
- Excite axial quadrupole mode by switching off an initial deformation in the xy plane
- Determine frequency ω_q and damping κ by fitting $\langle(x^2 - y^2)\rangle(t)$ by

$$\boxed{A\cos(\omega t + \phi)e^{-\kappa t} + Ce^{-\xi t}}$$

  - Jump from hydrodyn. to collisionless
  - Strong damping around the jump
  - Downshift of hydrodyn. frequency
  - Upshift of collisionless frequency

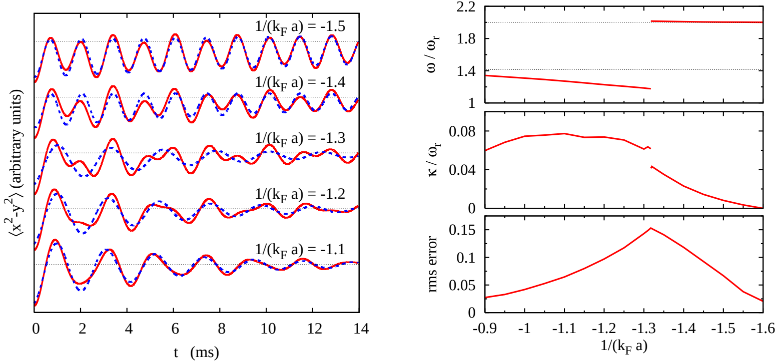




Possible interpretation of the experimental result

Try to simulate the experiment

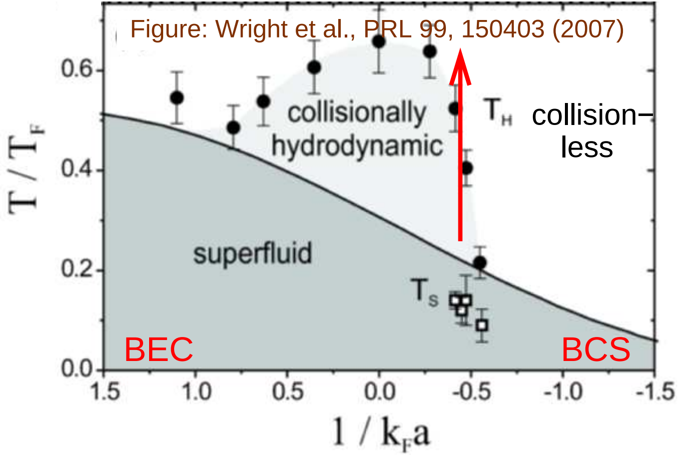
- Theory limited to the weakly-interacting BCS regime
  $\longrightarrow$ choose lower temperature $T = 0.046\,T_F$
  $\longrightarrow$ transition happens at higher value of $-1/k_F a$
- Fit numerical results as in the experiment:



- Jump of the frequency (before T reaches T_c!) due to fitting procedure
- Downshift of the hydrodynamic mode before the jump
- Strong damping around the BCS superfluid $\rightarrow$ normal transition

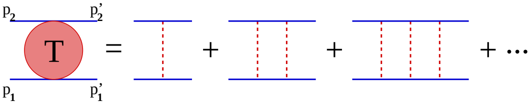
Collisional effects in collective modes in strongly interacting Fermi gases

- Consider the normal phase only ($T > T_c$): Boltzmann equation
- Transition from collisional hydrodynamics to the collisionless regime

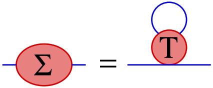


Medium effects: in-medium cross section and mean field

- Scattering cross section in free space: $\sigma_0 = \dfrac{4\pi a^2}{1+(qa)^2}$
- Calculate in-medium T matrix in ladder approximation



- At low T, in-medium σ strongly enhanced compared to free one, σ_0 (precursor of the pole in the T matrix at $T = T_c$)
  [same effect in nuclear physics: Alm et al., PRC 50, 31 (1994)]
- Mean field in Hartree approximation: $U_{\mathrm{Hartree}} = \dfrac{4\pi a}{m}\rho$
  breaks down for in the strongly interacting case ($a \to \infty$)
- Calculate mean field as on-shell self-energy at the Fermi surface, $U = \Sigma(\omega = 0, k = k_\mu)$



- U is weaker than U_{Hartree} and stays finite for $|a| \to \infty$
- U strongly affects the density profiles

Approximate solutions of the Boltzmann equation

(a) Method of moments

- Linearise Boltzmann equation for small deviations from equilibrium
- Write distribution function as $f(\vec{r}, \vec{p}, t) = f_{\mathrm{eq}}(\vec{r}, \vec{p}) + \frac{df_{eq}}{d\mu}\Phi(\vec{r}, \vec{p}, t)$
- Ansatz for Φ: polynomial in $\vec{r}$ and $\vec{p}$ with time-dependent coefficients
- Determine time-dependence by taking moments of the Boltzmann equation

(b) Numerical solution using the test-particle method

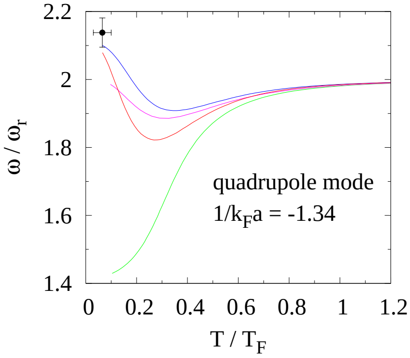
- Very common method for the simulation of heavy-ion collision
- Distribution function is replaced by an ensemble of "test particles"
- Solve classical trajectories in the trap (+ mean field) potential
- Collisions with probability in depending on the (in-medium) cross section and the occupancy of the final states (Pauli blocking)

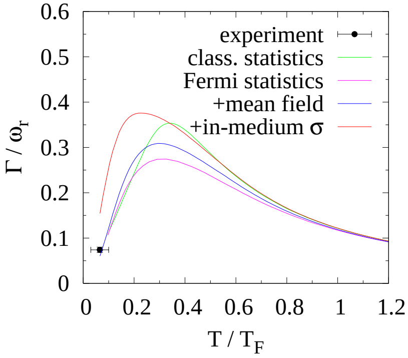
Here:

Method of moments including polynomials of second order in $\vec{r}$ and $\vec{p}$.

Radial quadrupole mode ($1/k_F a = -1.34$)

"Most collisionless" data point of Altmeyer et al., PRA 76, 033610 (2007)

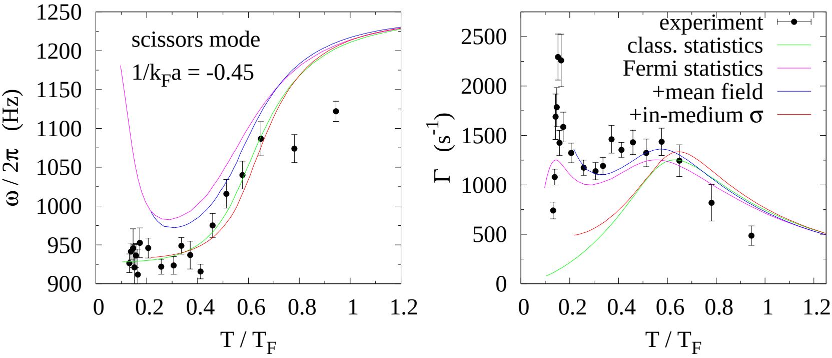




- Pauli blocking of collisions leads to collisionless behaviour at low T
- Mean field explains the upward shift of ω above the ideal gas result ($2\omega_r$)
- In-medium cross section seems to deteriorate the agreement with the data

Scissors mode ($1/k_F a = -0.45$)

Experiment Wright et al., PRL 99, 150403 (2007)



- Mean field improves agreement with the data.
- In-medium σ is too strong and compensates Pauli blocking effect [Bruun and Smith, PRA 75, 04612 (2007); Riedl et al., PRA 78, 053609 (2008)]

Problem of in-medium σ or of 2nd-order moments?

- With the in-medium σ, the obtained relaxation time τ is too short.
- Is this a defect of the underlying theory or just of the approximate solution of the Boltzmann equation (method of moments)?
- Example: ansatz for the quadrupole mode

$$\Phi = c_1(x^2 - y^2) + c_2(p_x^2 - p_y^2) + c_3(xp_x - yp_y)$$

$\longrightarrow$ Fermi sphere deformation is the same everywhere in the trap

- But collisions are much more frequent in the centre than in the low-density regions of the cloud

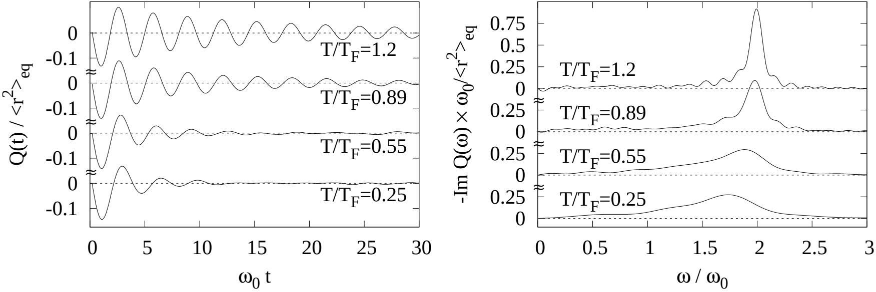
$\longrightarrow$ Fermi surface deformation should be much smaller in the centre!

- Try two solutions:
  (a) Numerical solution of the Boltzmann equation (test particles)
  (b) Include higher-order moments into the method of moments (e.g. 4th order, including a term $\propto r^2(p_x^2 - p_y^2)$)

Numerical simulation of the quadrupole mode

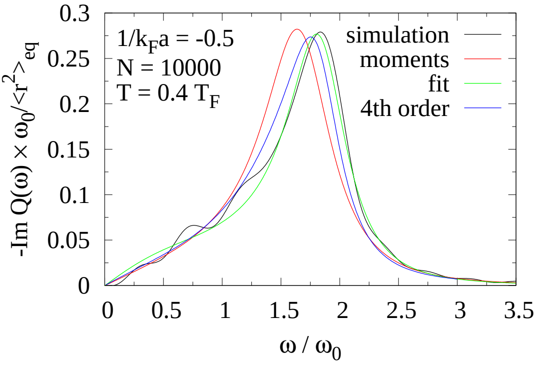
- Initially, test particles are distributed according to a Fermi distribution
- Excitation of the mode at $t = 0$
- Results for $Q(t) = \langle x^2 - y^2 \rangle(t)$ and its Fourier transform $Q(\omega)$

$(N = 10000$ atoms, $1/k_F a = -0.5)$



Comparison with the method of moments

- Compare response functions obtained from the method of moments (2nd order), from the numerical simulation (test particles), and from the extended method of moments (up to 4th-order moments):



- Detailed comparison of extended moments method and numerical simulation with experimental data is in progress.

Conclusions (1)

- Trapped atomic gases allow to study interesting many-body systems whose hamiltonians are known and can experimentally be adjusted.
- Ultracold Fermi gases with a Feshbach resonance are of great interest also for nuclear theory (BEC-BCS crossover in dilute nuclear matter, unitarity-limited Fermi gas as model for dilute neutron matter)
- Trapped gases exhibit collective modes similar to nuclei.
- Methods from nuclear theory (QRPA, Vlasov and Boltzmann equations, method of moments, test-particle method, . . .) are very useful in the study of trapped Fermi gases

Conclusions (2)

- Collective modes in ultracold trapped Fermi gases show the transition from hydrodynamic to collisionless behaviour
- On the $a < 0$ side of the BEC-BCS crossover, distinguish two regimes:

1. Weakly interacting:
   superfluid $\longrightarrow$ collisionless normal fluid
2. Strongly interacting:
   collisionally hydrodynamic normal fluid $\longrightarrow$ collisionless normal fluid

Open questions

- Up to now no theoretical description of the dynamics of the superfluid phase in the strongly interacting regime
  $\longrightarrow$include collisions into the quasiparticle transport theory
- Generalisation to asymmetric systems ($N_\uparrow \neq N_\downarrow$)