QCD corrections to the electroweak sphaleron rate

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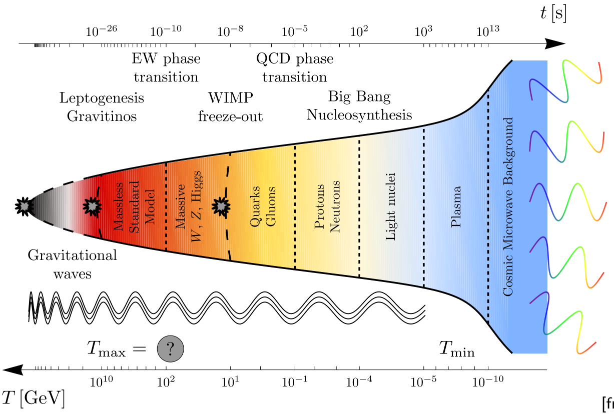
Outline

- baryon number violation in the early Universe
- electroweak sphalerons
- non-abelian plasma physics
- weak-isospin conductivity
- QCD interactions
- effect on sphaleron rate
- summary and outlook

The early Universe

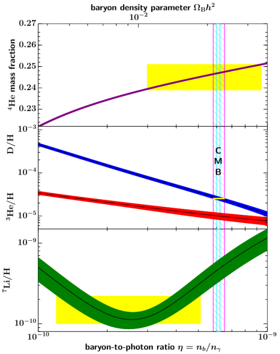


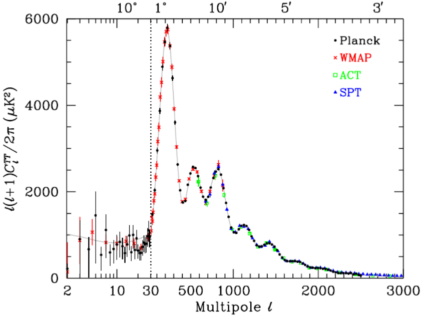
[from K. Schmitz '12]

- beautiful connection particle physics $\leftrightarrow$ early universe
- different stages of evolution: inflation, matter-antimatter asymmetry, dark matter, dark energy
- how are these phases related to extensions of the Standard Model?

Matter-antimatter asymmetry

astrophysics ⇒ no antimatter in the universe





[PDG 2019]

abundance of matter:

$$\frac{n_B}{n_\gamma} = \left[0.03 \text{ (visible) } 5.8 - 6.6 \text{ (BBN) } 6.09 \pm 0.06 \text{ (CMB)}\right] \times 10^{-10}$$

Independent measurements at very different times!

Baryogenesis

initial condition after inflation: $B = 0$

what is the origin of matter?

$$\eta_B = \frac{n_B}{n_\gamma} = (6.09 \pm 0.06) \times 10^{-10}$$

present value has to be explained!

necessary conditions for generating a matter-antimatter asymmetry [Sakharov]

1. baryon number violation
2. C and CP violation
3. deviation from thermal equilibrium

Electroweak baryon number violation

baryon and lepton number not conserved in Standard Model

divergence given by triangle anomaly

$$\begin{aligned}\partial_\mu J^\mu_B &= \partial_\mu J^\mu_L \\ &= \frac{n_f}{32\pi^2}\left(g^2 F^a_{\mu\nu}\widetilde{F}^{a\mu\nu} - g'^2 B_{\mu\nu}\widetilde{B}^{\mu\nu}\right)\end{aligned}$$

n_f: number of families,

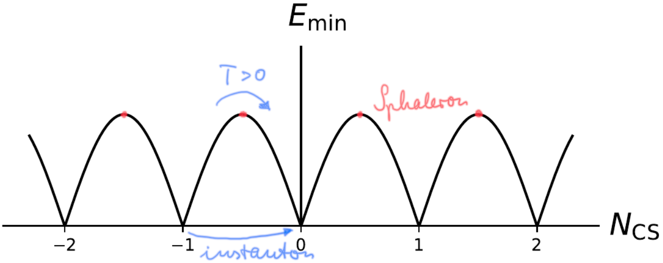
$F^a_{\mu\nu}$, $B_{\mu\nu}$ SU(2), U(1) field strengths, gauge couplings g and g'.

Sphaleron processes

baryon and lepton number are related to the topology of the weak non-abelian gauge fields:

$$B(t) - B(0) = n_f \int_0^t dt' \int d^3x \frac{g^2}{32\pi^2} \underbrace{F^a_{\mu\nu}\widetilde{F}^{a\mu\nu}}_{=\mathbf{E}^a\cdot\mathbf{B}^a}$$
$$= n_f \left[N_{\rm CS}(t) - N_{\rm CS}(0)\right]$$

$N_{\rm CS}$: Chern-Simons number



N_{CS} diffusion

in pure gauge theory

$$\left\langle [N_{\text{CS}}(t) - N_{\text{CS}}(0)]^2 \right\rangle = \Gamma_{\text{sph}} V t$$

Γ_{sph}: CS-diffusion rate, sphaleron rate

baryon number dissipation

$$\frac{d}{dt} B = -\gamma_{B+L} B$$

fluctuation $\leftrightarrow$ dissipation

$$\gamma_{B+L} = \frac{n_f^2 V}{2\langle B^2 \rangle} \Gamma_{\text{sph}}$$

analogous process in QCD: 'strong sphalerons'
$\rightarrow$ anomalous chirality violation

Electroweak sphalerons

only relevant in early Universe:

- low T: $\Gamma_{\rm sph} \sim e^{-4\pi/\alpha} = O(10^{-154})$
- near EW scale: $\Gamma_{\rm sph} \simeq \kappa \dfrac{m_W^7}{(\alpha_W T)^3} e^{-E_{\rm sph}/T}$ $\qquad E_{\rm sph} \simeq 8\pi v(T)/g$
- $T \gtrsim T_{\rm EW}$: $v(T) = 0$, EW symmetry unbroken

  $\Gamma_{\rm sph} = \kappa T^4$ unsuppressed

$\Gamma_{\rm sph}$ enters Boltzmann equations for baryogenesis

most popular scenarios:

- electroweak baryogenesis in first-order phase transition

  [Kuzmin, Rubakov, Shaposhnikov]
- leptogenesis: create lepton asymmetry through sterile-neutrino decay, partialy converted to baryon asymmetry through sphalerons

  [Fukugita, Yanagida]

Symmetric phase of SM

$\Delta N_{\rm CS} \sim 1$ requires

$$g^2 \int d^4x \underbrace{\mathbf{E}\cdot\mathbf{B}}_{\sim\dot{\mathbf{A}}\cdot\nabla\times\mathbf{A}} \sim g^2 k^{-2} \mathbf{A}^2 \overset{!}{\sim} 1$$

avoid Boltzmann suppression:

$$\text{energy} \sim \int d^3x\, \mathbf{B}^2 \sim k^{-1}\, \mathbf{A}^2 \overset{!}{\lesssim} T$$

$$\Rightarrow \qquad k \gtrsim g^2 T, \qquad k \sim g\mathbf{A}$$

magnetic screening length $\sim (g^2T)^{-1}$

$\Rightarrow k \sim g^2 T$ non-perturbative [Linde]

$\rightarrow$ lattice

Classical fields

occupancy $\dfrac{1}{e^{k/T}-1} \simeq \dfrac{T}{k} \gg 1$ when $k \ll T$

classical field limit

solve classical field equation for magnetic scale ($\equiv$ ultrasoft) gauge fields

most field modes are hard ($k \sim T$), not classical

include current carried by hard particles

$$D_0\mathbf{E} + \mathbf{D} \times \mathbf{B} = \mathbf{J}$$

Weak-isospin conductivity

mean free path for small-angle scattering $\sim \dfrac{1}{g^2 T \ln(1/g)}$

small ange scattering

- momenta almost unchanged
- weak gauge boson exchange changes weak isospin

logarithmic approximation: $\dfrac{1}{g^2 T \ln(1/g)} \ll \dfrac{1}{g^2 T} \qquad \Rightarrow \quad \mathbf{J} = \sigma \mathbf{E}$

σ: weak-isospin conductivity

NB: not a transport coefficient!

analogous consideration in QCD $\rightarrow$ color conductivity

Effective classical theory

$\sigma\mathbf{E} = -\sigma\dot{\mathbf{A}} + \cdots$ damping term

$\sigma \sim g^2 \times \text{(mean free path)} \times T^2 \sim \dfrac{T}{\log(1/g)}$

$D_t\mathbf{E} \ll \sigma\mathbf{E}$ over-damping

effective classical equation of motion [DB]

$$\mathbf{D} \times \mathbf{B} = \sigma\mathbf{E} + \boldsymbol{\zeta}$$

ζ: Gaussian white noise

Hot sphaleron rate

$$\Gamma_{\rm sph} = q\frac{2\pi T}{3\sigma}\alpha_W^5 T^4, \qquad \alpha_W \equiv \frac{g^2}{4\pi}$$

lattice: $q = 10.0 \pm 0.3$ [Moore; Annala, Rummukainen]

so far: only EW interactions included in σ

QCD scattering

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almost half of the weak-isospin current carried by quarks

quarks scatter through strong interaction

mean free path $\sim \dfrac{1}{g_S^4 \log(g_S^{-1}) T}$

for $g_S^4 \ll g^2$ this would be much larger than the mean free path for EW small-angle scattering

however: at $m_Z \qquad g_S^4 = 2.2, \qquad g^2 = 0.44$

Vlasov-Boltzmann equation

linearized Vlasov equation

electroweak interactions only:

$$(D_t + \mathbf{v} \cdot \mathbf{D})\, \delta f_\pm \pm g\mathbf{v} \cdot \mathbf{E}\, f'_{\mathrm{eq}} = \hat{C}_W \delta f_\pm$$

non-abelian $\mathbf{E} = \mathbf{E}^a \tau^a / 2$ (adjoint rep. of SU(2))

contribution to weak-isospin current:

$$\mathbf{J}^a = \frac{g}{2} N_{\mathrm{fam}} N_{\mathrm{c}} \int \frac{d^3p}{(2\pi)^3} \left[\delta f_+^a - \delta f_-^a\right] \mathbf{v}$$

Solving the Boltzmann equation

$$\underbrace{(D_t + \mathbf{v}\cdot\mathbf{D})}_{g^2 T}\,\delta f_\pm \pm g\mathbf{v}\cdot\mathbf{E}\,f'_{\mathrm{F}} = \underbrace{\hat{C}_W}_{g^2\log(1/g)T}\,\delta f_\pm$$

leading log approximation: neglect convective term

$\to$ solution

$$\delta f_\pm = \frac{\mp g\mathbf{v}\cdot\mathbf{E}}{c_W} f'_{\mathrm{F}}$$

insert into current $\to$

$$\sigma = \frac{m_W^2}{3\,c_W}$$

we may choose $\mathbf{E}$ diagonal in flavor space

$$\mathbf{E} = \frac{1}{2}\mathbf{E}^3 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$\Rightarrow \delta f_\pm$ diagonal

for left-handed (LH) quarks:

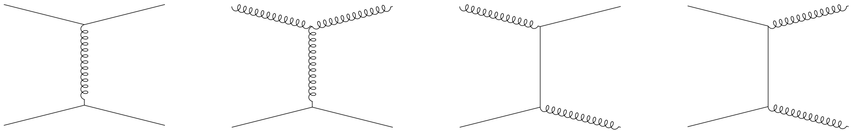
$$\delta f_+ = f_\mathrm{F}(1 - f_\mathrm{F}) \begin{pmatrix} \eta_{uL} & 0 \\ 0 & \eta_{dL} \end{pmatrix}, \qquad \eta_{dL} = -\eta_{uL}$$

RH quarks, LH antiquarks, and gluons remain in equilibrium

$$\pm g \mathbf{v} \cdot \mathbf{E}\, f_\mathrm{F}' = \left[\hat{C}_W + \hat{C}_S\right] \delta f_\pm$$

QCD collision term

leading log contributions to $\hat{C}_S$:



diffusion approximation [Arnold,Moore,Yaffe; Ghiglieri,Moore,Teaney]

$$\left[\hat{C}_S \delta f_+\right]_{\alpha,\mathsf{gl}} = \frac{g_S^2 \log(g_S^{-1})}{6\pi} m_G^2 T \partial_{p^i} \left\{ f_{\mathrm{F}}(p_0)[1 - f_{\mathrm{F}}(p_0)] \partial_{p^i} \eta_\alpha \right\} + \text{gain terms}$$

gain terms cancel in sum over α

Solving the Boltzmann equation

collition term commutes with rotations $\Rightarrow \delta f_\pm$ and η_α have same angular dependence as force term

$$\eta_{uL} = \frac{g}{2T}\,\mathbf{v}\cdot\mathbf{E}^3\,\chi(p_0)\ .$$

$\rightarrow$ ordinary differential equation for χ

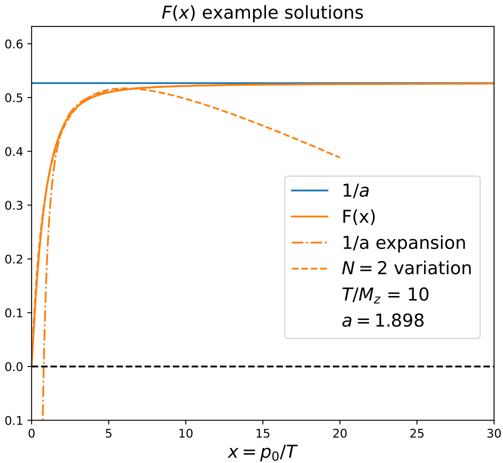
boundary value problem

follows from variational problem $\delta Q[F] = 0$ [Heiselberg; Arnold, Moore, Yaffe]

ansatz for F in terms of N basis functions

already $N = 2$ gives good approximation

Results: variational approach

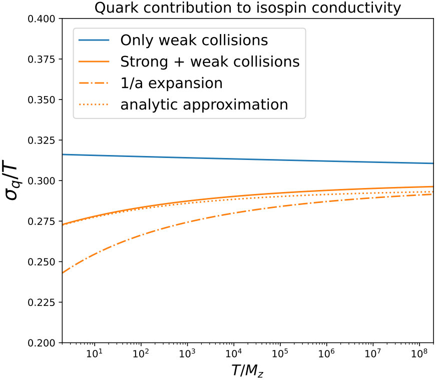


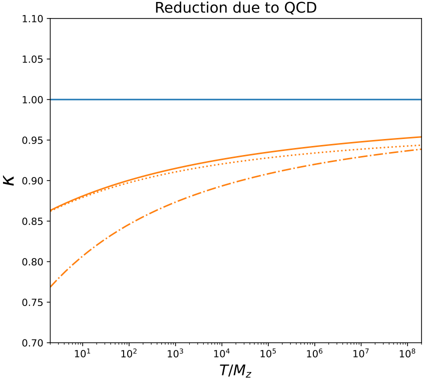
$$a = \frac{3\pi c_W T}{g_s^2 m_G^2 \log\left(\mu_* / m_G\right)}$$

Results: condutivity

quark contribution to weak-isospin conductivity

$$\sigma_q = \frac{m_{W\mathsf{q}}^2}{3\,c_W}\kappa$$





Results: electroweak sphaleron rate

QCD reduces σ_q by 10 to 15%

$\rightarrow$ electroweak sphaleron rate increases by 6%

modest increase, considering the size of g_S

Summary

- baryon number B is rapidly violated in the symmetric phase of the Standard Model
- rate for B violation enters computation of baryonesis in the early Universe
- rate determined by non-perturbative W boson dynamics
- rate computed with effective classical theory
- overdamped due to large weak-isospin conductivity σ
- we included QCD collisions in LL approximation, decrease σ
- sphaleron rate decreased by 6%

Outlook

- go beyond LL approximation
- full leading order in QCD scattering
- order g_S corrections
- include masses near EW crossover
- beyond LL in electroweak interaction: new operators in effecitve equation of motion

$$c_W = \gamma + \frac{g^2 T}{2\pi} K \ , \qquad\qquad K = 3.0410 \ ,$$

where γ is defined implicitly as a solution of the equation

$$\gamma = \frac{g^2 T}{2\pi} \log\left(\frac{m_W}{\gamma}\right)$$

$$m_W^2 = \left(\frac{2}{3} + \frac{N_s}{6} + \frac{(1+N_\mathrm{c})N_\mathrm{fam}}{12}\right) g^2 T^2$$

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maximize the integral

$$Q[F] = \int_0^\infty \frac{x^2 e^x}{(e^x+1)^2} \left\{ F - \frac{1}{4} F'^2 - \left(\frac{a}{2} + \frac{b}{4x} \frac{e^x+1}{e^x-1} + \frac{1}{2x^2} \right) F^2 \right\}$$

$$b = \frac{m_\infty^2}{m_G^2} = \frac{1}{6}\,, \qquad x = \frac{p_0}{T}\,, \qquad F(x) = \frac{c_W\, \chi(p_0)}{a}\,. \tag{1}$$

$$a = \frac{3\pi c_W T}{g_s^2 m_G^2 \log\left(\mu_*/m_G\right)}\,, \quad b = \frac{m_\infty^2}{m_G^2} = \frac{1}{6}\,, \quad x = \frac{p_0}{T}\,, \quad F(x) = \frac{c_W\, \chi(p_0)}{a}\,. \tag{2}$$