The Streaming Instability: a review

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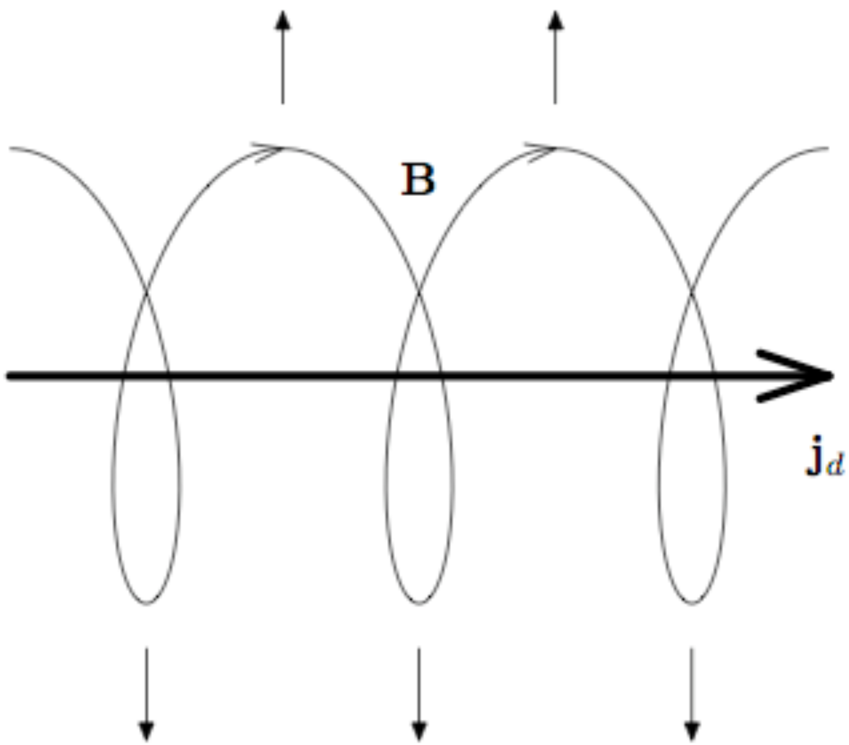
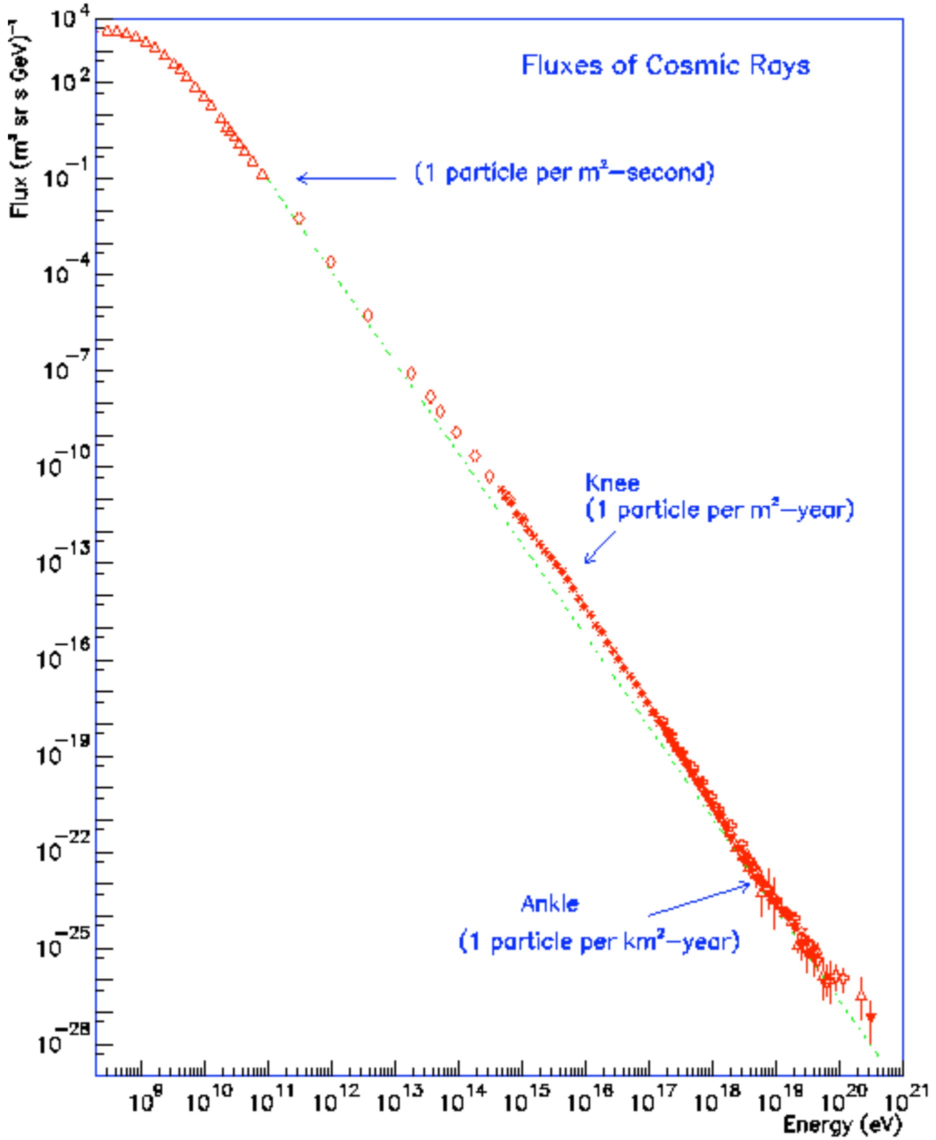
Outline

- **Why the streaming instability is interesting for the CR community**
- **What is it?**
  - Interactions between streaming particles and Alfvén waves
  - pitch angle scattering and wave growth
- **Resonant streaming instability**
  - The problem of CR confinement in the Galaxy
- **Non-resonant streaming instability**
  - The problem of CR acceleration up to the "knee"
- **Conclusions**

Whence the interest for streaming instabilities



CR acceleration



Spectrum $N(E) \propto E^{-p}$

$p=2.7 \quad 10<E_{GeV}<3\times10^6$

$p=3.1 \quad E_{GeV}>3\times10^6$

Galactic origin of CRs up to the knee

CR density: $n_{CR}\sim10^{-9}cm^{-3}$

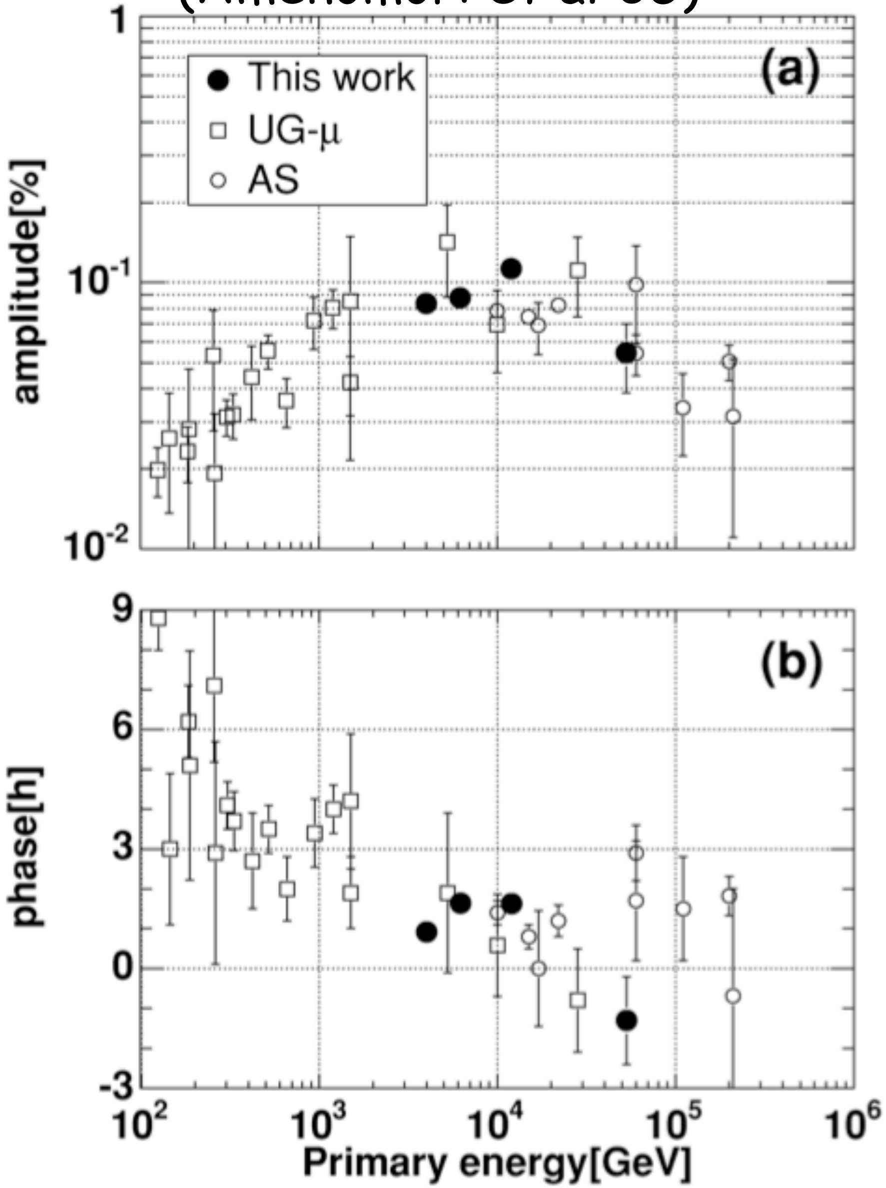
CR luminosity: $L_{CR}\sim3x10^{40}$ erg s^{-1}

SNRs best candidate
sources on energetic grounds with
10% efficiency

Best candidate mechanism
Diffusive Shock Acceleration
(Krymsky 77, Bell 78)
but efficient scattering is needed

CR confinement

(Amenomori et al 05)



Anisotropy $\delta \sim 3\times10^{-4}$

$T_{conf} \sim 2 \times 10^{7}$ yr at ~ 1GeV

$T_{conf} \propto E^{-\alpha}$

$\alpha \sim 0.3\text{-}0.6$ E>1-10 GeV

CRs are generated in violent events and then they should stream with

$v_d \sim c$

With N sources $v_d \sim c/N^{1/2}$

$$\delta \approx 3\times10^{-4} \Rightarrow v_d \approx 50\ km/s$$

Exceedingly large N

Again
Effective scattering required

Going back to the '70s

Efficient acceleration and confinement require efficient scattering

•Coulomb collisions with the gas not an option

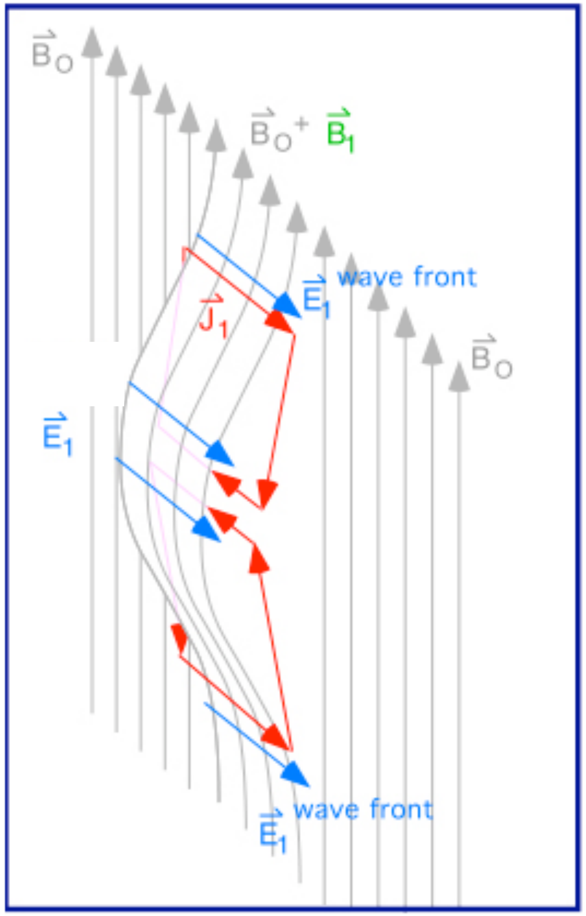
•Scattering by large scale moving B field inhomogeneities (Fermi 49): energetic problems

Resonant scattering in a static B field

energy stays constant
only direction of velocity changes

Alfvén waves

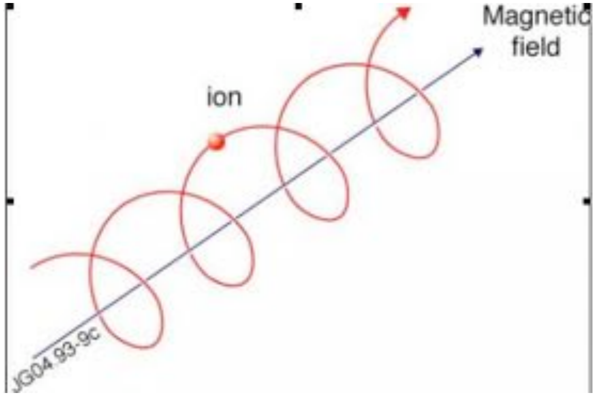
(Axford, Bell, Cesarsky, Jokipii, Kulsrud, Parker, Skilling, Wentzel)



Interaction of a cosmic ray with an Alfvén wave

$$\begin{cases} \vec{B}_0 = B_0 \vec{z} \\ \delta\vec{B}_\perp = \delta B \sin(kz - \omega t)\vec{x} \end{cases}$$

$$\begin{cases} z(t) = z_0 + v_z t \\ v_y = v_\perp \sin(\Omega t + \varphi) \end{cases}$$



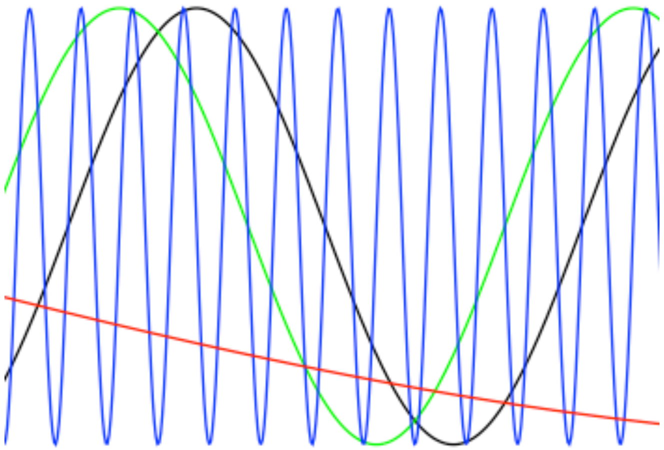
$$\Delta p_z = e \int_0^\tau dt \left(\frac{\vec{v} \wedge \vec{B}}{c} \right)_z$$

$$\tau = \frac{2\pi}{k v_z - \omega}$$

$$\left(\vec{v} \wedge \vec{B}\right)_z = \frac{v_\perp \delta B}{2} \left\{ \cos\left[\left(k v_z - \omega + \Omega\right)t + (k z_0 + \varphi)\right] - \cos\left[\left(k v_z - \omega - \Omega\right)t + (k z_0 - \varphi)\right] \right\}$$

High freq

Low freq



Interaction most effective at resonance

$\lambda \sim r_L$

$$k v_z - \omega = k(v_z - v_A) \approx k v_z \approx \Omega = \frac{e B_0}{m \gamma c}$$

$$\Delta p_z = \pi \, p_\perp \left(\frac{\delta B}{B_0} \right) \cos\Phi$$

Pitch angle scattering and isotropy of CRs

$$\Delta p_z = \pi\, p_\perp \left(\frac{\delta B}{B_0}\right)\cos\Phi \qquad \begin{cases} p_z = p\cos\vartheta \\ p_\perp = p\sin\vartheta \end{cases} \qquad \Delta\vartheta = -\pi\left(\frac{\delta B}{B_0}\right)\cos\Phi$$

$$\left\langle \Delta\vartheta^2 \right\rangle = \frac{1}{2}\pi^2 \frac{t}{\tau}\left\langle \left(\frac{\delta B}{B_0}\right)^2 \right\rangle = t\,\frac{\pi}{4}\Omega \left\langle \left(\frac{\delta B}{B_0}\right)^2 \right\rangle$$

$$D_\alpha = \frac{\pi}{8}\Omega\left\langle \left(\frac{\delta B}{B_0}\right)^2 \right\rangle$$

Isotropy in the wave frame is reached in time $T_{iso} \sim 1/D_\alpha$

$T_{iso} << T_{conf} \Rightarrow (\delta B/B_0) >> 10^{-10} E_{GeV}^{(1+\alpha)/2}$ at $\lambda \sim 10^{12} E_{GeV}$ cm

If CRs are isotropic in the wave frame then $v_d \sim v_A \sim 50$km/s and $\delta \sim v_d/c \sim 1.5\times 10^{-4}$

Associated **spatial diffusion**

$$\lambda_{mfp} = \frac{v}{2D_\alpha}$$

$$D_z = \frac{1}{3} v \lambda_{mfp} \approx \frac{v^2}{6D_\alpha} \qquad D_\perp = D_z\left(\frac{r_L}{\lambda_{mfp}}\right)^2$$

What makes the waves?

If CR are isotropized by the waves,
there must be momentum transfer between CR and waves

Before scattering: $P_{CR}=n_{CR}m\gamma_{CR}v_d$

After scattering: $P_{CR}=n_{CR}m\gamma_{CR}v_A$

$$\Rightarrow \frac{dP_{CR}}{dt} = \frac{n^*_{CR} m\gamma_{CR}(v_d - v_A)}{\tau}$$

$$\tau^{-1} = D_\alpha = \frac{\pi}{8}\Omega\left(\frac{\delta B}{B_0}\right)^2$$

Momentum has gone to the waves

$$\frac{dP_{CR}}{dt} = \frac{dP_w}{dt} = \frac{1}{v_A}\frac{dE_w}{dt} = \frac{1}{v_A}2\gamma_w\frac{(\delta B)^2}{8\pi}$$

$$\gamma_w = \frac{\pi}{4}\frac{n^*_{CR}}{n_i}\Omega_0\frac{(v_d - v_A)}{v_A}$$

with

$$n^*_{CR} = n_{CR}(p > eB/ck)$$

If there are seed Alfvén waves super-alfvenic streaming
CRs make them unstable

Formal Theory

- Take unperturbed distributions of CRs and background plasma
- Assume a small amplitude Alfvén wave perturbs them
- Find the perturbed distributions of CRs and background plasma
- Compute the currents
- Use them in linearized Maxwell's equations to compute how waves evolve in time

$$\begin{cases} \vec{\nabla} \wedge \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\vec{J} \\ \vec{\nabla} \wedge \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \end{cases} \Rightarrow \frac{\partial^2 \vec{E}}{\partial t^2} = -c^2 \vec{\nabla} \wedge \left(\vec{\nabla} \wedge \vec{E}\right) - 4\pi \frac{\partial \vec{J}}{\partial t}$$

For circularly polarized Alfvén waves (**B**=B_0**z** and **k**=k**z**)

$$J_i = \sigma_{ij} E_j = \sum_s q_s \int d^3p \, v_i^s f_1^s(\vec{p})$$

$$\left[\frac{c^2k^2}{\omega^2} - 1 - \frac{4\pi i}{\omega}\left(\sigma_{xx} \pm i\sigma_{xy}\right)\right] = 0$$

Unperturbed distribution functions

In a reference frame where CRs are isotropic

$$\begin{cases} n_i + n_{CR} = n_e + n_{ec} \\ n_i v_D = n_e v_D \end{cases}$$

Charge neutrality and no current

$$\begin{cases} n_i + n_{CR} = n_e \\ n_i v_D = n_e v_e \end{cases}$$

$$\begin{cases} f_0^i = \dfrac{n_i}{2\pi p^2}\delta(p - m_i v_D)\delta(\mu - 1) \\ f_0^e = \dfrac{n_i}{2\pi p^2}\delta(p - m_e v_D)\delta(\mu - 1) \\ f_0^{ec} = \dfrac{1}{4\pi}\dfrac{n_{CR}}{p^2}\delta(p) \\ f_0^{CR} = \dfrac{1}{4\pi} n_{CR}\,\Phi(p) \end{cases}$$

(e.g. Zweibel 03)

or

$$\begin{cases} f_0^i = \dfrac{n_i}{p^2}\delta(p - m_i v_D)\delta(\mu - 1) \\ f_0^e = \dfrac{n_i + n_{CR}}{p^2}\delta(p - m_e v_D)\delta(\mu - 1) \\ f_0^{CR} = \dfrac{1}{2} n_{CR}\,\Phi(p) \end{cases}$$

(e.g. Achterberg 83)

Result is the same to order $O[(n_{CR}/n_i)^2]$ (Amato & Blasi 09)

Perturbed distribution functions

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial \vec{x}}(\vec{v}f) + \frac{\partial}{\partial \vec{p}}\left[e\left(\vec{E} + \frac{\vec{v} \wedge \vec{B}}{c}\right)f\right] = 0$$

Vlasov Equation
Continuity equation in momentum
Phase space

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{x}} + e\left(\vec{E} + \frac{\vec{v} \wedge \vec{B}}{c}\right)\frac{\partial f}{\partial \vec{p}} = 0$$

f is constant along the particle trajectory

$$-i\omega f_1 + ikv_z f_1 + \frac{e}{c}\left(\vec{v} \wedge \vec{B}_0\right) \cdot \frac{\partial f_1}{\partial \vec{p}} = -e\left[\vec{E} + \frac{\vec{v} \wedge \left(\vec{k} \wedge \vec{E}\right)}{\omega}\right] \cdot \frac{\partial f_0}{\partial \vec{p}}$$

For circularly polarized waves

$$\sigma = \frac{e^2}{2}\frac{4\pi}{\omega}\int dp_\perp p_\perp dp_z \frac{v_\perp}{\omega - kv_z \pm \Omega}\left[\left(1 - \frac{kv_z}{\omega}\right)\frac{\partial f_0}{\partial p_\perp} + \frac{kv_\perp}{\omega}\frac{\partial f_0}{\partial p_z}\right]$$

$$d\varphi\, dp_z\, dp_\perp p_\perp \rightarrow d\varphi\, d\mu\, dp\, p^2$$

$$\sigma = \frac{2\pi e^2}{\omega}\int_0^\infty dp \int_{-1}^{1} d\mu\, \frac{p^2 v(1-\mu^2)}{\omega - kv\mu \pm \Omega}\left[\frac{\partial f_0}{\partial p} + \left(\frac{kv}{\omega} - \mu\right)\frac{1}{p}\frac{\partial f_0}{\partial \mu}\right]$$

The CR response

$$\varepsilon^{CR} = \frac{2\pi e^2}{\omega}\int_0^\infty dp \int_{-1}^{1} d\mu \, \frac{p^2 v(1-\mu^2)}{\omega - kv\mu \pm \Omega_i}\left[\frac{\partial f_0}{\partial p} + \left(\frac{kv}{\omega} - \mu\right)\frac{1}{p}\frac{\partial f_0}{\partial \mu}\right]$$

$$\omega \approx kv_A << \Omega$$

$$\int_{-1}^{1} d\mu \, \frac{(1-\mu^2)}{-kv\mu \pm \Omega_i} = -i\pi \int_{-1}^{1} d\mu \,(1-\mu^2)\,\delta\left(-kv\mu \pm \Omega_i\right) + \mathrm{P}\int_{-1}^{1} d\mu \frac{(1-\mu^2)}{-kv\mu \pm \Omega_i}$$

Classical resonant part

Non-resonant term

Resonant term is non-zero only if

$$\frac{\Omega}{kv} \le 1 \Leftrightarrow p \ge p_1 = \frac{eB_0}{ck}$$

$$\varepsilon^{CR} = -\frac{4\pi e^2 n_{CR}}{\omega\, m_i \Omega_i^0}\left[\frac{i\pi}{4} p_1 \int_{p_1}^{\infty}\left(p^2 - p_1^2\right)\frac{d\Phi}{dp}dp + \frac{P}{4}\int_0^\infty dp \int_{-1}^{1} d\mu \, \frac{p^2 p_1 (1-\mu^2)}{\mu \mp p_1/p}\frac{d\Phi}{dp}\right]$$

Resonant and non-resonant CRs

$$I_{res} = \frac{\pi}{2}\int_{p_1}^{\infty} dp\, p\, p_1\, \Phi(p)$$

$$\varepsilon^{CR} = \frac{c^2}{v_A^2}\frac{\Omega_0}{\omega}\frac{n_{CR}}{n_i}\left(iI_{res} \mp I_{nr}\right)$$

$$I_{nr}^{\pm} = \pm I_{nr}$$

$$I_{nr} = \frac{p_1}{4}\int_0^{\infty} dp \left[\left(p^2 - p_1^2\right)\ln\left|\frac{1+p/p_1}{1-p/p_1}\right| + 2pp_1\right]\frac{d\Phi}{dp}$$

$\Phi(p) \propto p^{-\alpha}$ — α=4.7 for CRs in the Galaxy; α=4 for CRs at shocks

resonant term

$$I_{res}(k) = \frac{\pi}{2(\alpha-2)} p_1^{3-\alpha} = \frac{\pi}{2}\frac{\alpha-3}{\alpha-2}\frac{n_{CR}^*}{n_{CR}} \qquad n_{CR}^* = n_{CR}\left(p > eB_0/ck\right)$$

Traditionally considered since the '70s

non-resonant term

$$I_{nr}(k) = -\frac{p_1^3}{2}\int_{s_{min}}^{s_{max}} ds\, s\, \Phi(s)\ln\left|\frac{1+s}{1-s}\right| \qquad s = p/p_1$$

Its possible importance at large k first appreciated by Bell in 2004

The plasma response

Cold plasma dispersion relation
(e.g. Krall & Trivelpiece)

$$\varepsilon^p = -\frac{4\pi e^2}{\omega^2}\frac{n_i}{m_i}\left[\frac{\omega + kv_D}{\omega + kv_D \pm \Omega_i^0} + \frac{m_i}{m_e}\frac{\omega + kv_D}{\omega + kv_D \pm \Omega_e^0} + \frac{n_{CR}}{n_i}\frac{m_i}{m_e}\frac{\omega}{\omega \pm \Omega_e^0}\right]$$

Cold drifting ions | Cold drifting electrons | Cold isotropic electrons

$$\omega << \Omega_i^0 = -\frac{m_e}{m_i}\Omega_e^0 << \left|\Omega_e^0\right|$$

$$\varepsilon^p = \mp\frac{4\pi e^2}{\omega^2}\frac{n_i}{m_i}\left[(\omega + kv_D)\frac{1}{\Omega_i^0}\left(\cancel{1} \mp \frac{\omega + kv_D}{\Omega_i^0} + \frac{m_i}{m_e}\frac{\Omega_i^0}{\Omega_e^0}\left(\cancel{1} \mp \frac{\omega + kv_D}{\Omega_e^0}\right)\right) + \frac{n_{CR}}{n_i}\frac{m_i}{m_e}\frac{\omega}{\Omega_e^0}\right]$$

In low freq. waves the plasma sets in **ExB** drift: no net current

$$\varepsilon^p = \frac{4\pi e^2}{\omega^2\left(\Omega_i^0\right)^2}\frac{n_i}{m_i}\left[(\omega + kv_D)^2 \pm \frac{n_{CR}}{n_i}\omega\Omega_i^0\right] = \frac{c^2}{\omega^2 v_A^2}\left[(\omega + kv_D)^2 \pm \frac{n_{CR}}{n_i}\omega\Omega_i^0\right]$$

The full dispersion relation

$$\varepsilon^{p} = \frac{c^2}{\omega^2 v_A^2}\left[(\omega + k v_D)^2 \pm \frac{n_{CR}}{n_i}\omega\Omega_i^0\right]$$

$$\varepsilon^{CR} = \frac{c^2}{v_A^2}\frac{\Omega_0}{\omega}\frac{n_{CR}}{n_i}\left(iI_{res} \pm I_{nr}\right)$$

res CRs

$$\frac{c^2k^2}{\omega^2} = \cancel{1} + \frac{c^2(\omega + k v_D)^2}{\omega^2 v_A^2} + \frac{c^2}{v_A^2}\frac{\Omega_0}{\omega}\frac{n_{CR}}{n_i}\left(iI_{res} \pm (I_{nr} + 1)\right)$$

Compensating electrons

non-res CRs

Move to the lab frame $\omega' = \omega + k v_D$

$$v_A^2 k^2 = \omega^2 + \Omega_0(\omega - k v_D)\frac{n_{CR}}{n_i}\left(iI_{res} \pm (1 + I_{nr})\right)$$

(Zweibel03, Bell 04, Amato & Blasi 09)

Lower sign corresponds to right-hand circular polarization
Electric field rotation is clockwise ($\mathbf{E}=E(\mathbf{e}_x+i\mathbf{e}_y)$)
Here is where the non-resonant term becomes important

But when will it become important?

Propagation in the ISM

$$v_A^2 k^2 = \omega^2 + \Omega_0(\omega - k v_D)\frac{n_{CR}}{n_i}\left(iI_{res} \pm (1 + I_{nr})\right)$$

In the ISM CR density: $n_{CR} \sim 10^{-9} cm^{-3}$ and $v_D \sim v_A$

Perturbation will be small $\omega = k v_A + \omega_1$

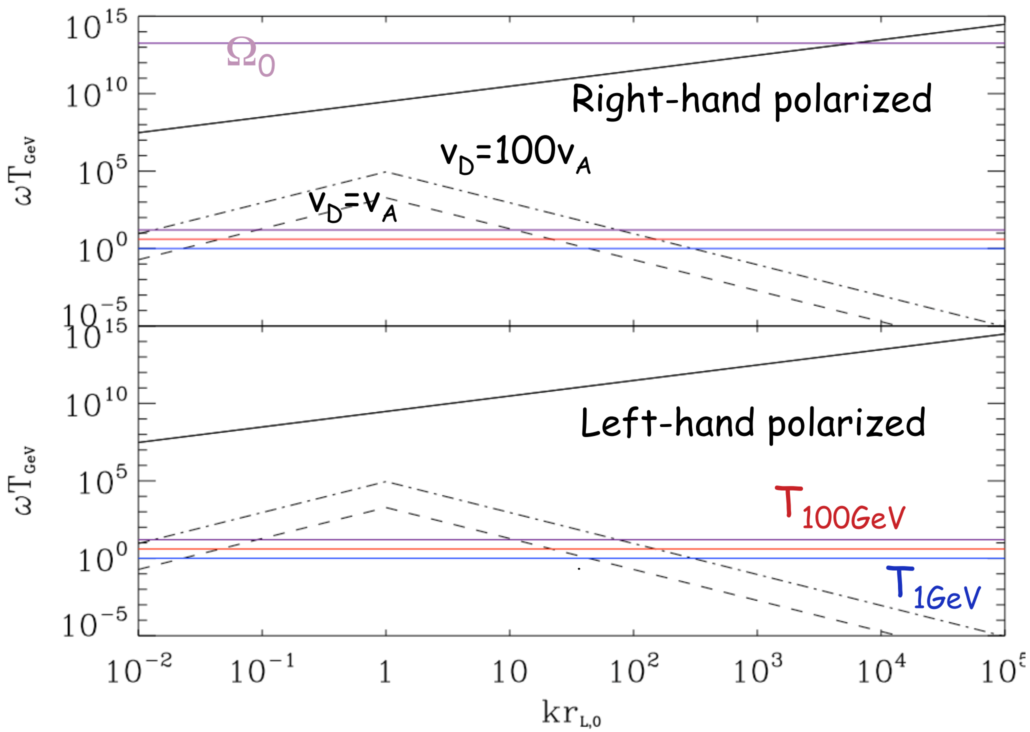
$$\omega_1 = -\frac{\Omega_0}{2}\left(1 - \frac{v_D}{v_A}\right)\frac{n_{CR}}{n_i}\left(iI_{res} \pm (1 + I_{nr})\right)$$

$$\begin{cases} \mathrm{Re}(\omega_1) << k v_A \\ \Gamma_g = \mathrm{Im}(\omega) = -\frac{\Omega_0}{4\pi}\left(1 - \frac{v_D}{v_A}\right)\frac{n_{CR}(p > eB_0/ck)}{n_i} \end{cases}$$

✓Left and right hand polarized modes are identical

✓Non-resonant instability is irrelevant

✓Growth rates are fast enough for isotropization if no damping



Wave damping

(Wentzel 74)

In partially neutral ISM damping due to ion-neutral collisions

$$\Gamma_d = -R n_H,$$

$$\frac{1}{R} \approx 5\left(\frac{T}{10^3 K}\right)^{-0.4} yr\, cm^{-3}$$

(Kulsrud & Cesarsky 71)

$$\Gamma_d = \Gamma_g$$

$$\Downarrow$$

$$v_D = v_A + R n_H \frac{n_i}{n_{CR}^*} \frac{v_A}{\Omega_0} \approx v_A + 100\left(\frac{p}{mc}\right)^{1.7}\left(\frac{T}{10^3 K}\right)^{0.4}\left(\frac{n_H}{0.1 cm^{-3}}\right) km/s$$

100GeV CRs too fast, $T_{conf} \propto E^{-1.7}$

In fully ionized ISM damping due to wave-wave interactions

(Non-linear Landau damping)

$$\Gamma_d = \frac{\sqrt{\pi}}{8} \frac{v_i}{c} \left(\frac{\delta B}{B_0}\right)^2 \Omega \propto \left(\frac{n_{CR}^*}{LB_0}\right)^{1/2}$$

$$v_D = v_A + \alpha_{NL} \frac{n_i}{n_{CR}^*} \left(\frac{n_{CR}^*}{LB_0}\right)^{1/2} = v_A + 55\left(\frac{100\, pc}{L}\right)^{1/2}\left(\frac{p}{mc}\right)^{0.85} km/s$$

(Cesarsky & Kulsrud 81)

100GeV CRs still too fast, $T_{conf} \propto E^{-0.85}$

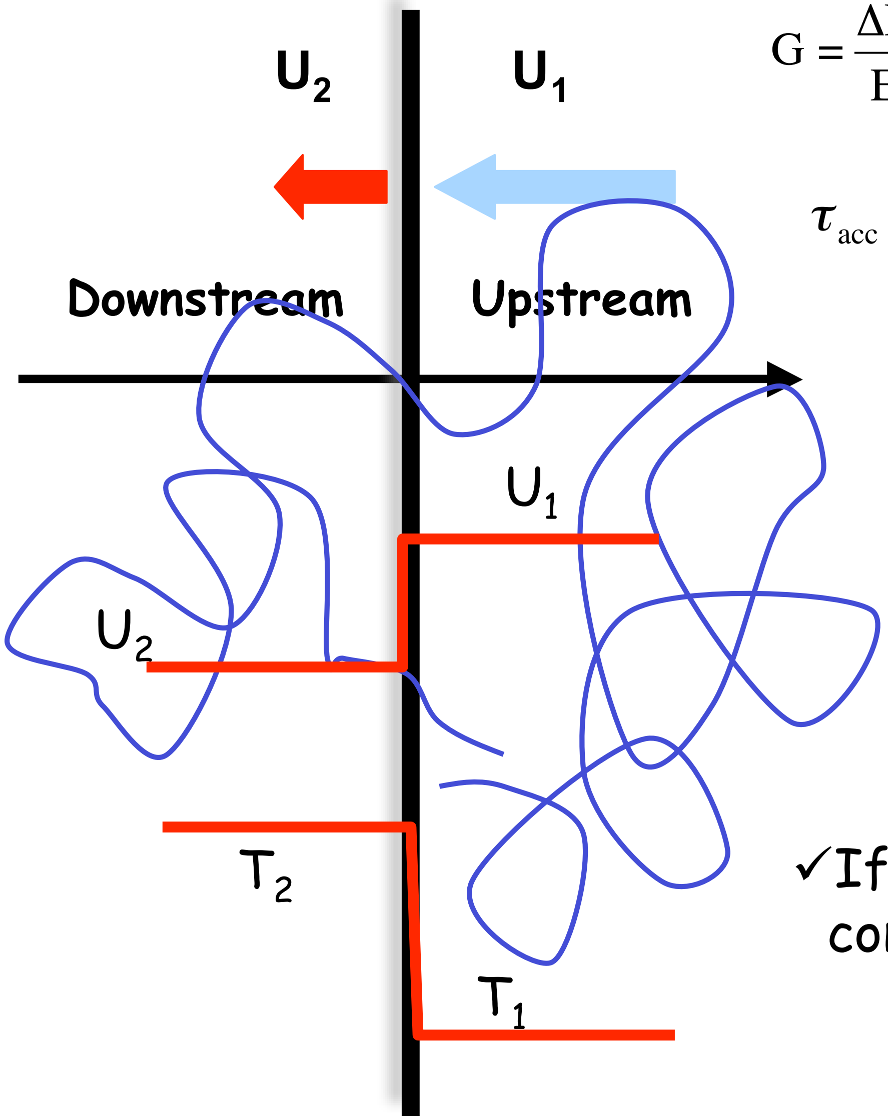
Observations + Theory hint at $T_{conf} \propto E^{-1/3}$ (Kolmogorov type turbulence)

(Blasi & Amato, submitt., Ptuskin et al, 06)

Much more in Ptuskin's talk

Shock Acceleration

How do we get CRs accelerated up to the knee?



$$G = \frac{\Delta E}{E} = \frac{4}{3}(u_1 - u_2)$$

$T_{acc}(E_{max})=\min(T_{age},T_{loss})$

$$\tau_{acc} = \frac{3}{U_1 - U_2}\left\{\frac{D_1}{U_1} + \frac{D_2}{U_2}\right\}$$

High E_{max} only if efficient scattering

Pitch angle scattering implies spatial diffusion

$$D_\alpha = \frac{\pi}{8}\Omega\left\langle\left(\frac{\delta B}{B_0}\right)^2\right\rangle$$

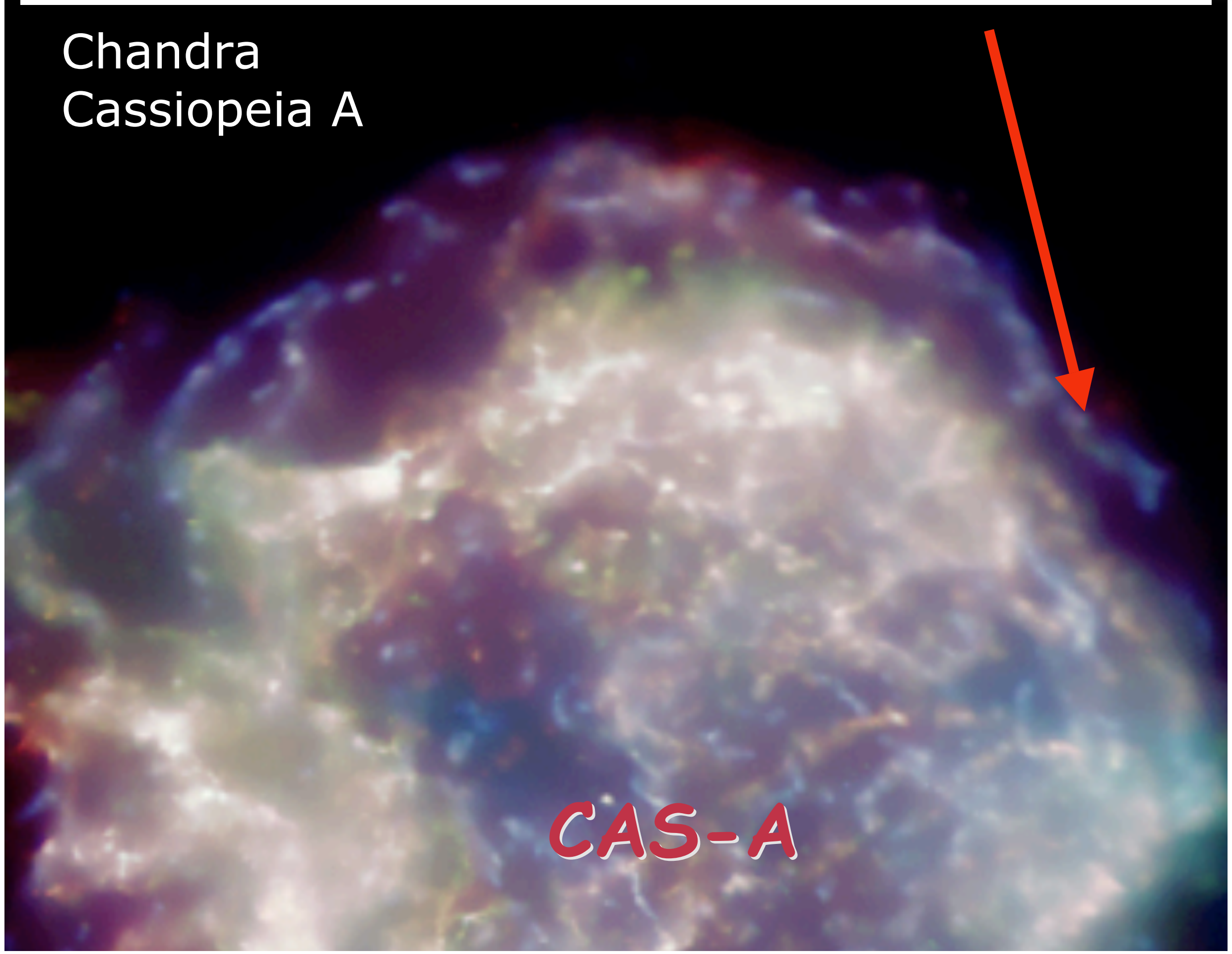
$$D(p) = \frac{c^2}{6D_\alpha} = \frac{4}{3\pi}\left(\frac{B_0}{\delta B}\right)^2 c r_L$$

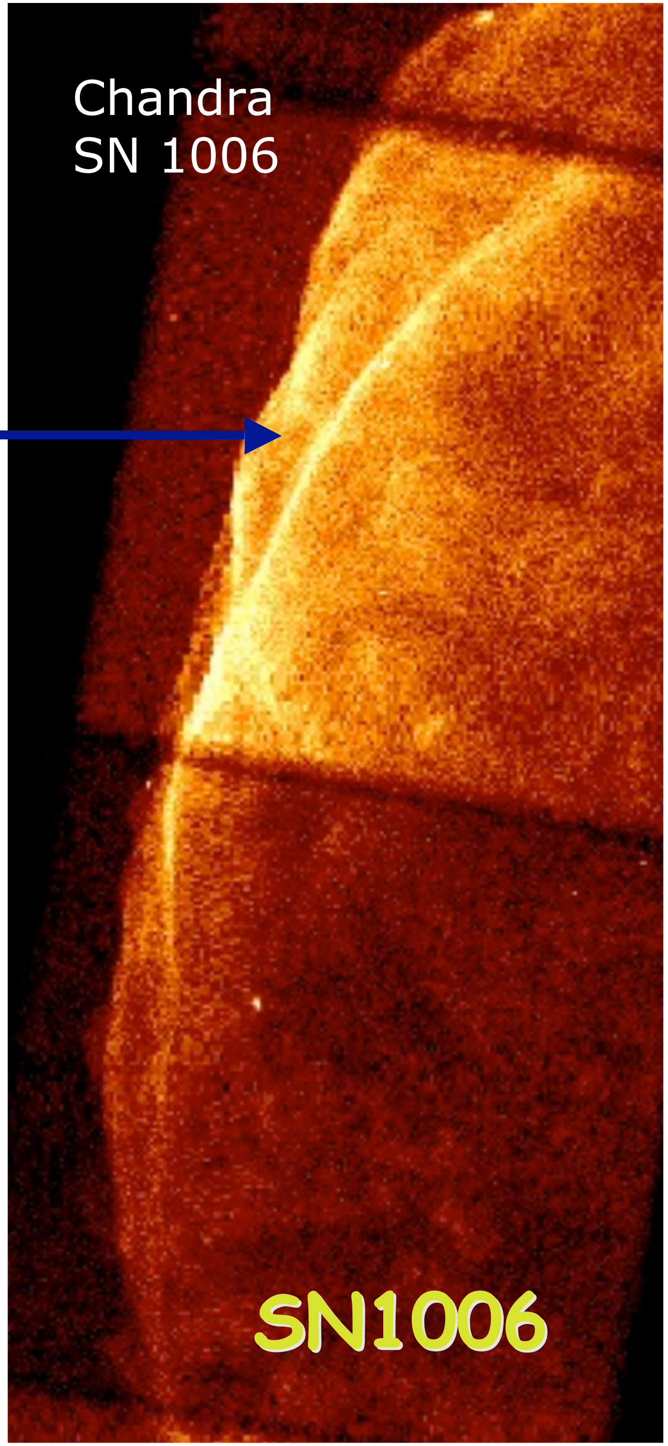
✓If δB is the same responsible for CR confinement in the Galaxy E_{max}~GeV

✓If δB ~B_0, E_{max}~10^4-10^5 GeV (Lagage & Cesarsky 83)

Hints from Observations

Detection of amplified B fields in SNRs: B~100μG





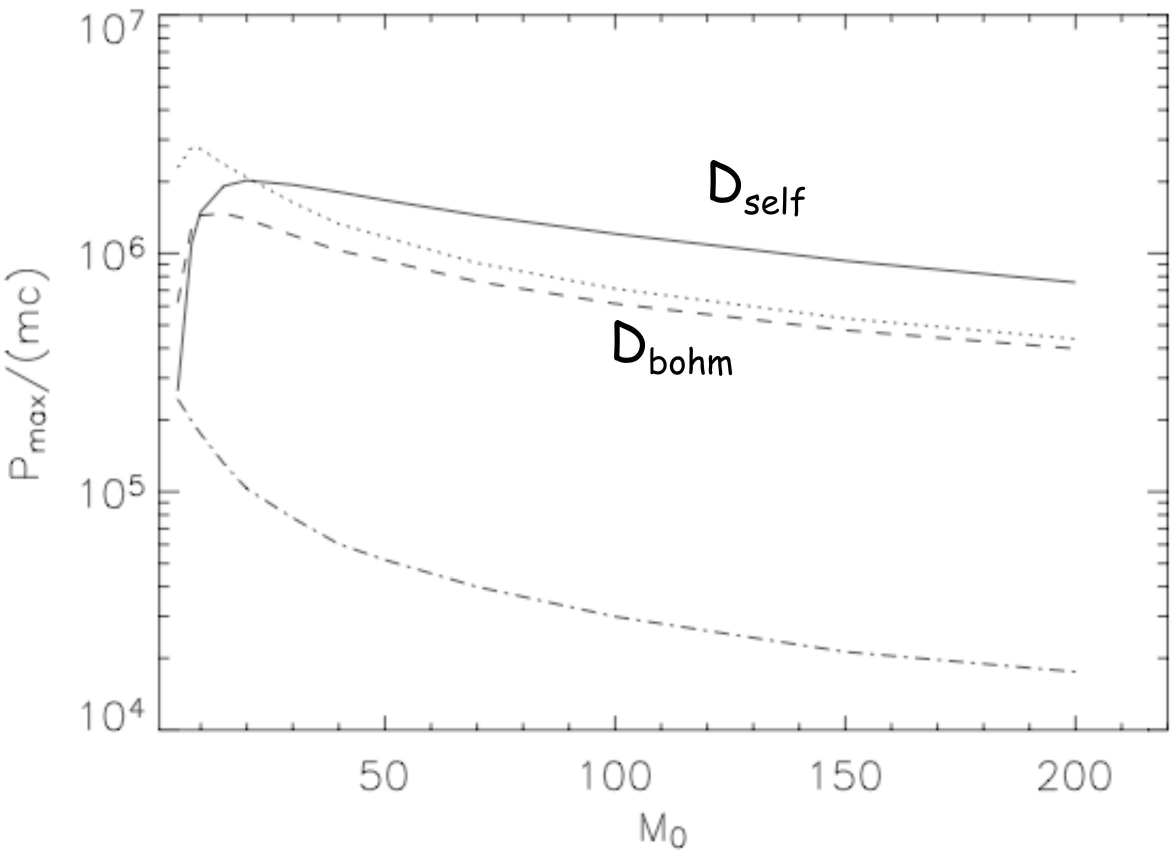
E_{max} with resonant modes alone

If you are not afraid of applying quasi-linear theory even for $\delta B \gg B_0$

$$\left(\frac{\delta B_{res}}{B_0}\right)^2 = 2\frac{v_D}{v_A}\frac{P_{CR}}{n_i m_i v_D^2}$$

Modified shocks would be close to the knee at transition between Free expansion and Sedov

Blasi, Amato & Caprioli 2008

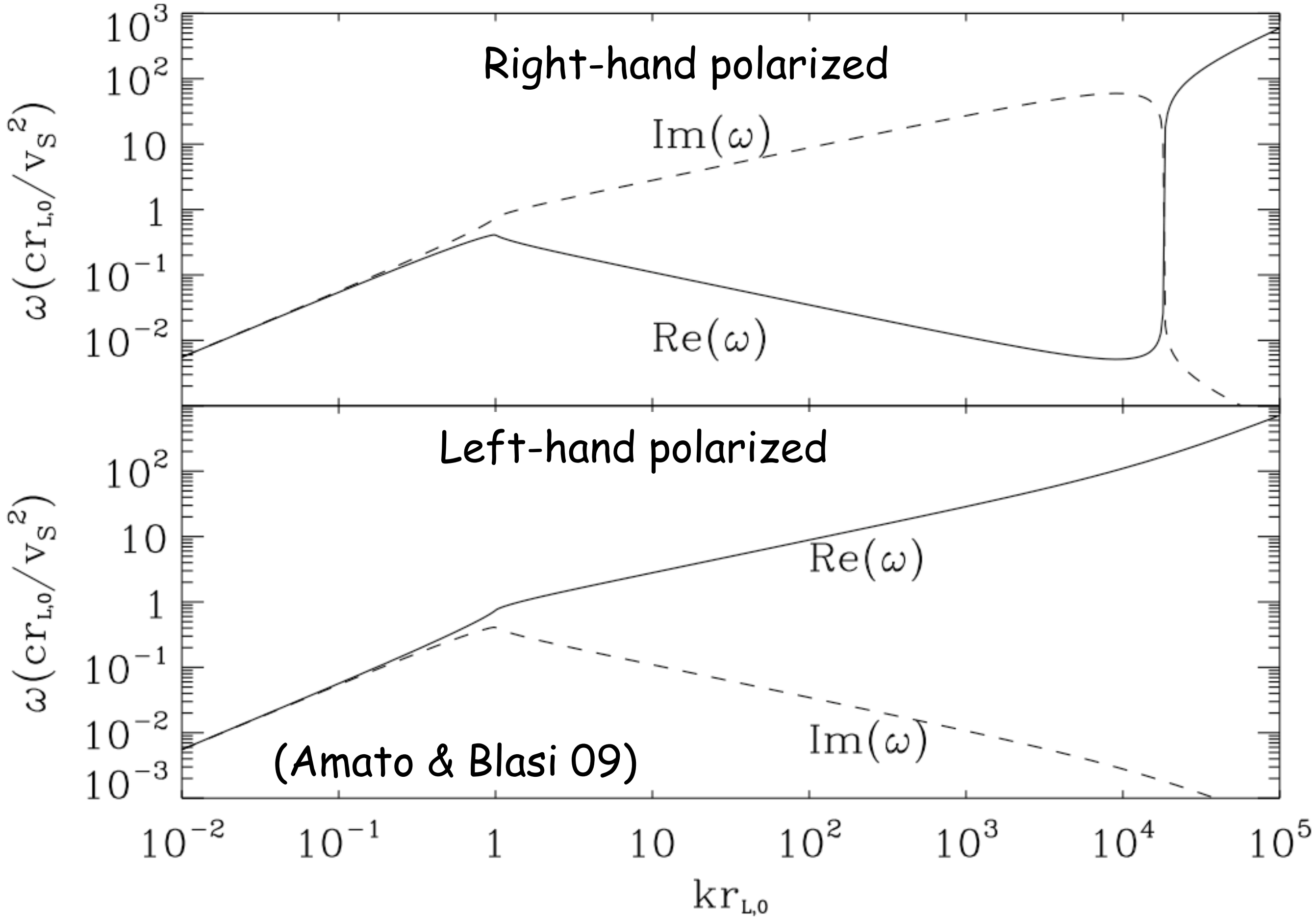


Bell instability

$$v_A^2 k^2 = \omega^2 + \Omega_0(\cancel{\omega} - k v_D)\frac{n_{CR}}{n_i}\left(iI_{res} \pm (1 + I_{nr})\right)$$

Strongly current driven regime

At SNR shocks B$_0$~1μG, η~0.1, v$_S$~10^{4}km/s



$$\frac{n_{CR}}{n_i}\frac{k v_S \Omega_0}{k^2 v_A^2} > 1$$

$$\Updownarrow$$

$$\frac{4\pi J_{CR}}{c}\frac{1}{kB_0} = \frac{k_c}{k} > 1$$

$\Gamma_{nr} \gg \Gamma_{res}$

$$k_c = \frac{4\pi J_{CR}}{cB_0}$$

Existence of non-resonant modes

$$v_A^2 k^2 = \omega^2 - v_S \Omega_0 \frac{n_{CR}}{n_i}\left(ikI_{res} \pm k(1+I_{nr})\right)$$

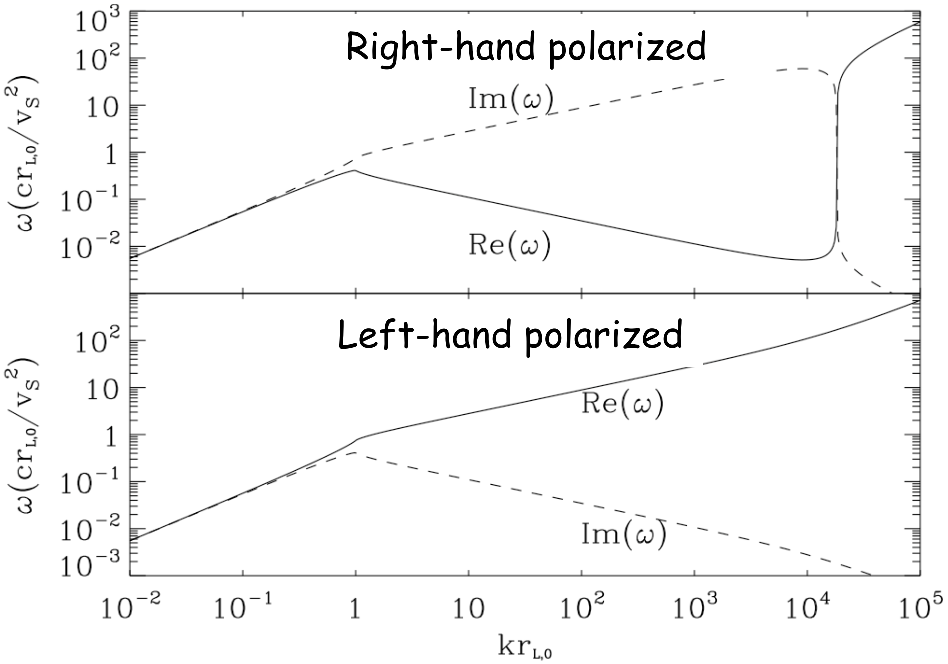
$\propto k^2$ $\qquad$ $\propto k^0$ $\qquad$ $\propto k$

$$kr_{L0} >> 1 \quad \Rightarrow \begin{cases} I_{res} \approx (\pi/4)/(kr_{L0}) \\ I_{nr} \approx -(1/3)(kr_{L0})^2 \end{cases}$$

$$k_1 r_{L0} = \sqrt{\frac{\pi}{4}\frac{n_{CR}}{n_i}\frac{v_S c}{v_A^2}} \qquad k_2 r_{L0} = \frac{n_{CR}}{n_i}\frac{v_S c}{v_A^2}$$

$$k_2 > k_1 \quad \Leftrightarrow \quad \frac{4\pi J_{CR}}{c}\frac{r_{L0}}{B_0} = k_c r_{L0} > \frac{\pi}{4}$$

Also $U_{CR}/U_B > c/v_D$



$$k_{max} = \frac{k_2}{2} = \frac{k_c}{2} = \frac{4\pi J_{CR}}{2cB_0}$$

Large by definition

~ equilibrium between current and magnetic tension

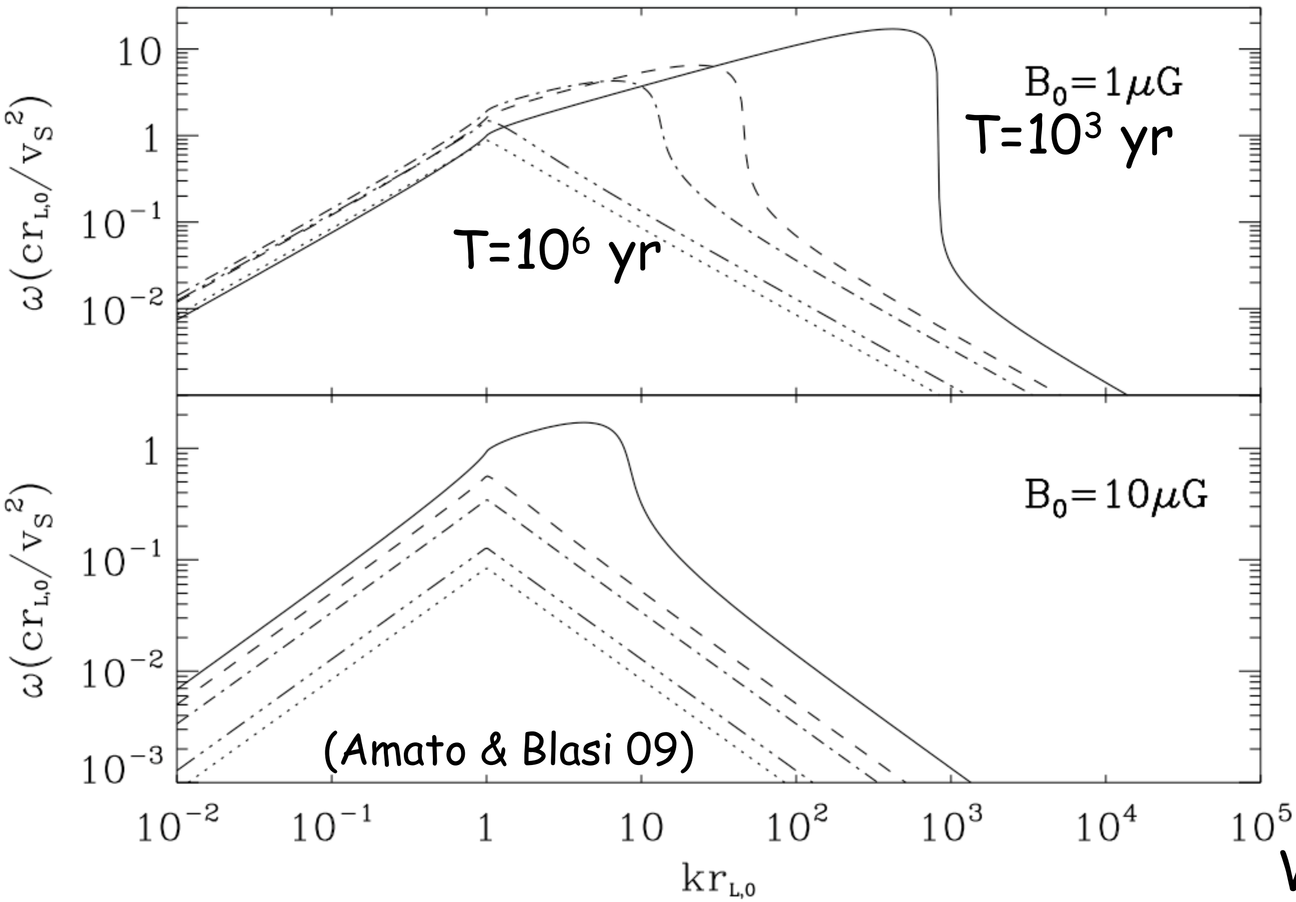
$$\Gamma_{max} = \Gamma\left(\frac{k_2}{2}\right) \approx v_A k_c$$

Evolution with SNR age

During Sedov-Taylor evolution of a SNR

$$k_2 > k_1 \quad \Leftrightarrow \frac{n_{CR}}{n_i}\frac{v_S c}{v_A^2} > \frac{\pi}{4}$$

η=0.1, E_{SN} =10^{51}erg



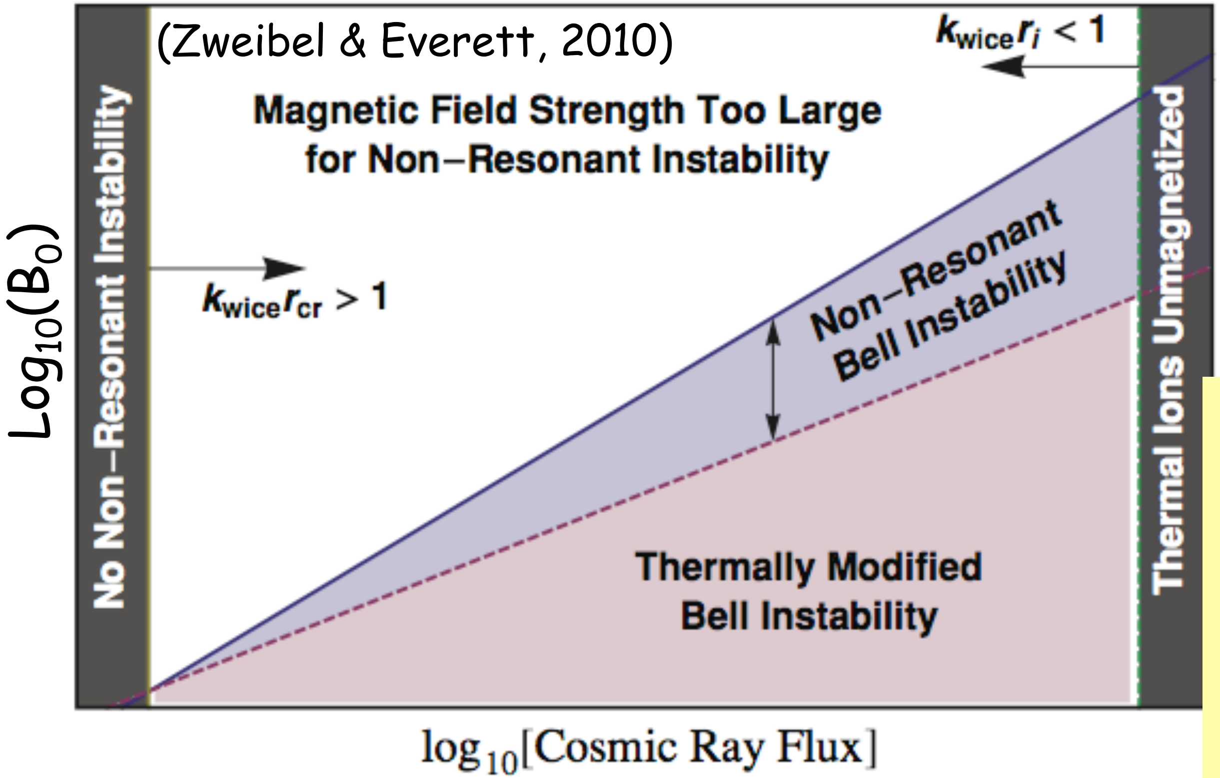
After 5×10^3-10^4 yr
The resonant mode disappears

Earlier when the field is higher
(U_{CR}/U_B>c/v_D)

Bell modes could be Important at beginning of Sedov phase
(see also Pelletier et al 06)

When is E_{max} reached?

Where is it relevant?



Thermal ions modify dispersion relation (cyc. Res. + gyrovisc.)

$$\frac{n_{CR} v_D}{n_i v_i} > \left(\frac{v_A}{v_i}\right)^3 \quad \text{(to compare with } \frac{n_{CR}}{n_i}\frac{v_S c}{v_A^2} > \frac{\pi}{4}\text{)}$$

K_{max} and Γ_{max} determined by v_D vs finite r_L

Cold ISM
$n_i=1cm^{-3}$, $T=10^4K$
$v_S \sim 10^4 km/s$, $n_{CR} \sim 10^{-5}$
$1\mu G < B_0 < 87\mu G$

Hot low density bubble
$n_i=3\times10^{-3}\ cm^{-3}$, $T=3\times10^6K$
$0.4\mu G < B_0 < 4.7\mu G$
thermally modified
$\Gamma > 1/T_{adv}$ only for $B_0 < 1.3\mu G$
Increasing n_{CR} would help

Hot but denser bubble
$n_i=3\times10^{-2}\ cm^{-3}$, $T=3\times10^6K$
$1.2\mu G < B_0 < 15\mu G$
$\Gamma > T_{adv}$ only for $B_0 < 4.1\mu G$

Non-linear evolution

Numerical studies:

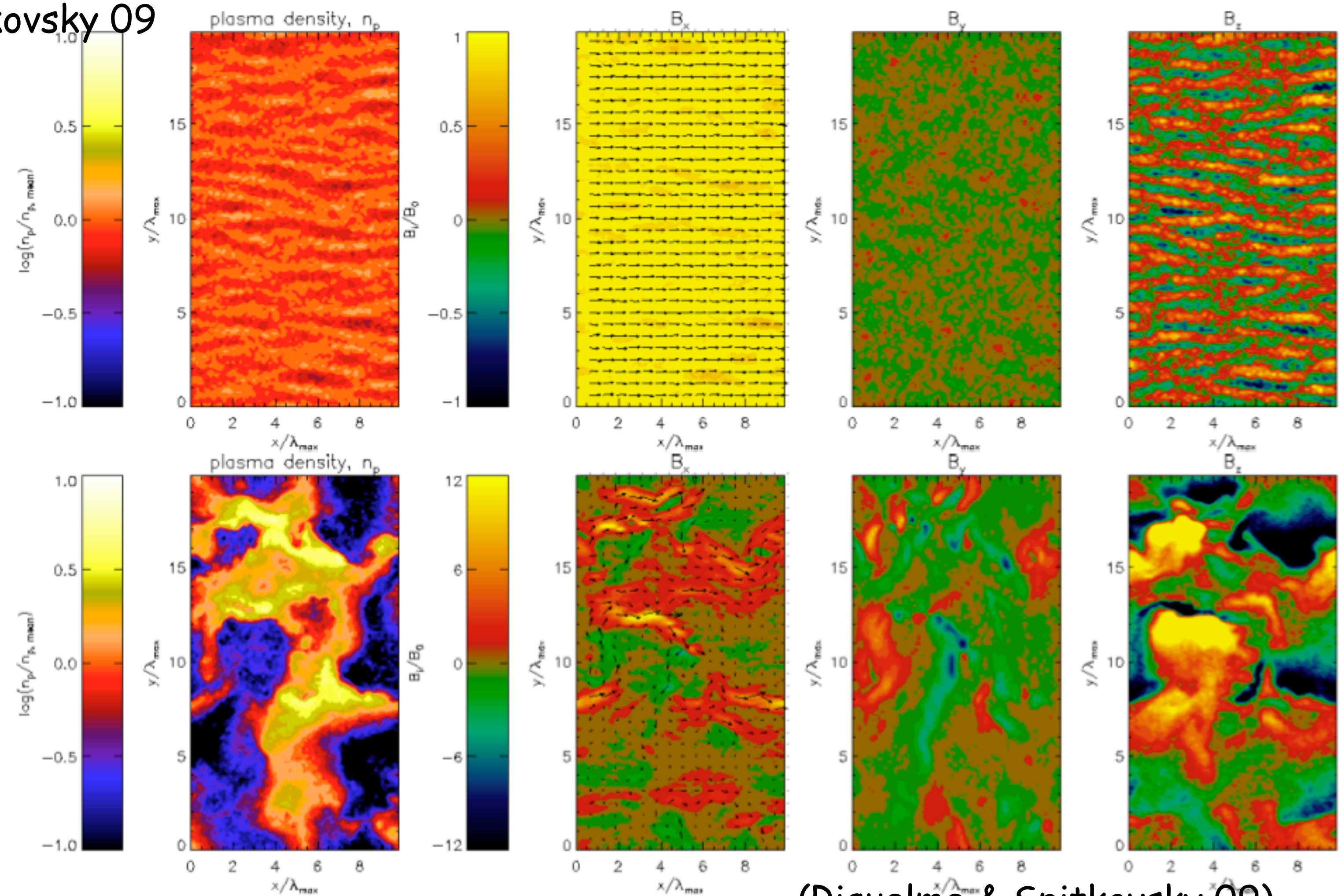
MHD: Bell 04,05; Reville et al 08, Zirakashvili et al 08
PIC: Niemec et al 08
Riqeulme & Spitkovsky 09
Ohira et al 09

$$\frac{\delta B^2}{8\pi} = \frac{v_D}{c} U_{CR}$$ (Bell 04)

$T=3/\Gamma_{max}$

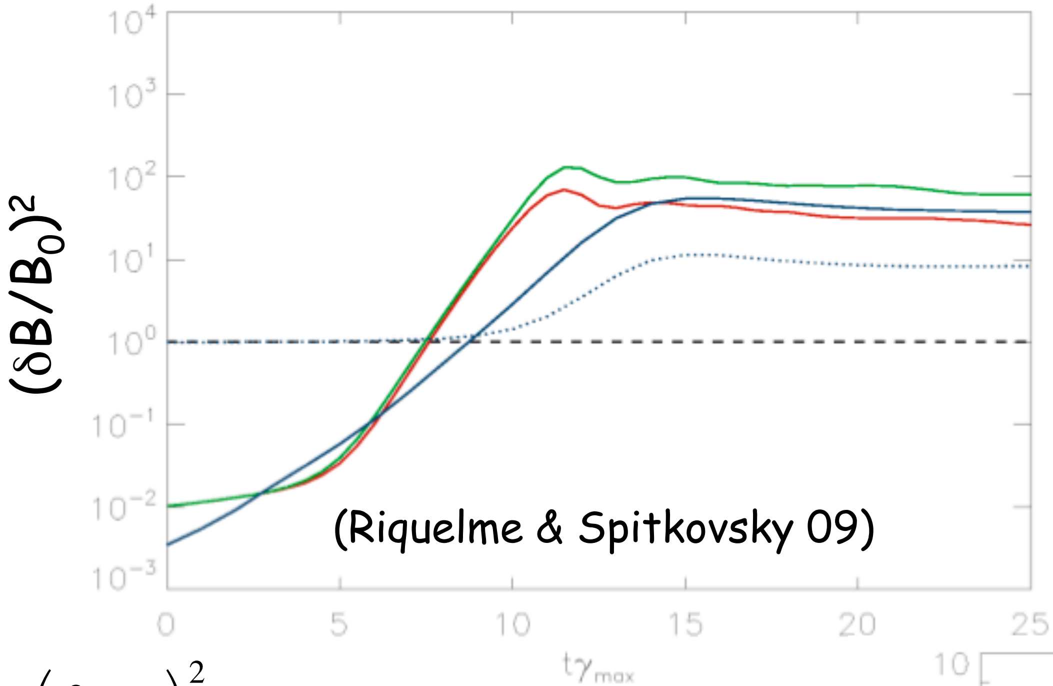
$T=11/\Gamma_{max}$

<λ> has grown



(Riquelme & Spitkovsky 09)

Saturation



For constant CR current
Saturation for $v_{A1} \sim v_D$

$$\delta B = 400\left(\frac{v_D}{10^4\,km/s}\right)\left(\frac{n_i}{cm^3}\right)^{1/2}\mu G$$

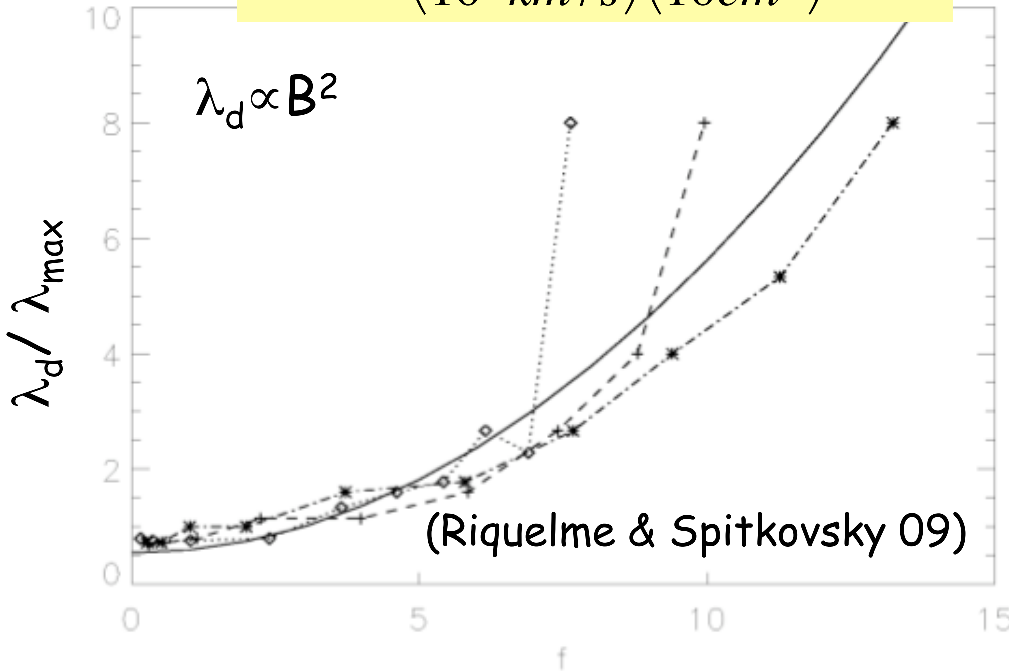
With mono-energetic CRs
Saturation can come earlier if
$r_L \sim \lambda_d$

$$\delta B = 30\left(\frac{v_D}{10^4\,km/s}\right)\left(\frac{n_{CR}}{10cm^3}\right)^{1/2}\mu G$$

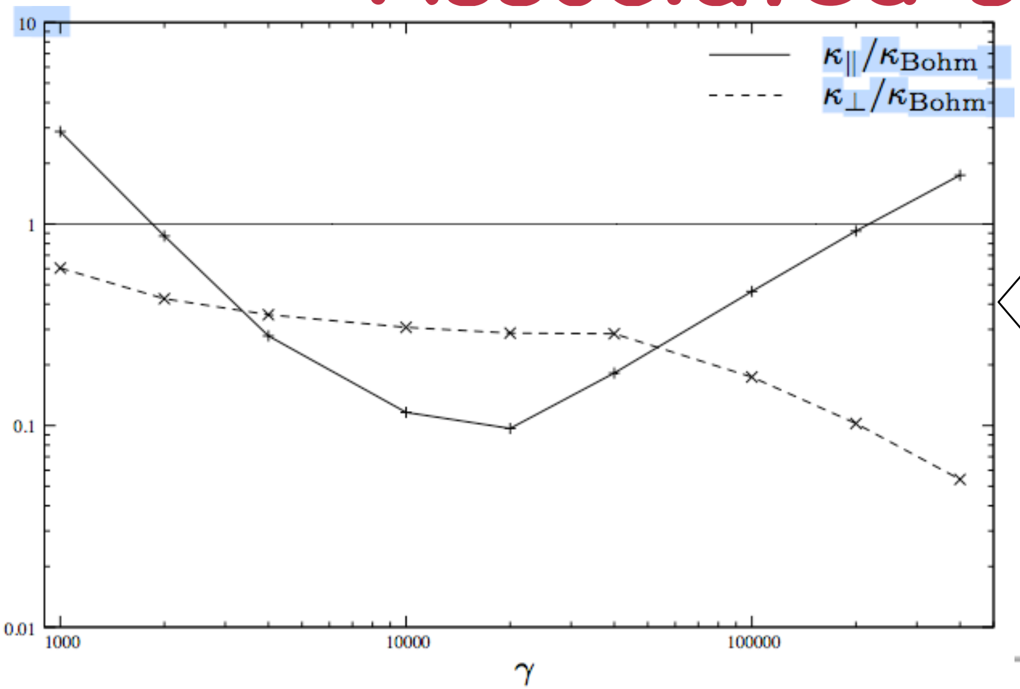
$$\left(\frac{\delta B_{res}}{B_0}\right)^2 = 2\frac{v_D}{v_A}\frac{P_{CR}}{n_i m_i v_D^2}$$

(e.g. Amato&Blasi 06)

$$\frac{\delta B_{Bell}}{\delta B_{res}} \approx \sqrt{\frac{v_D}{\eta v_A}} \quad or \quad \sqrt{\frac{v_D}{v_A}}$$



Associated scattering



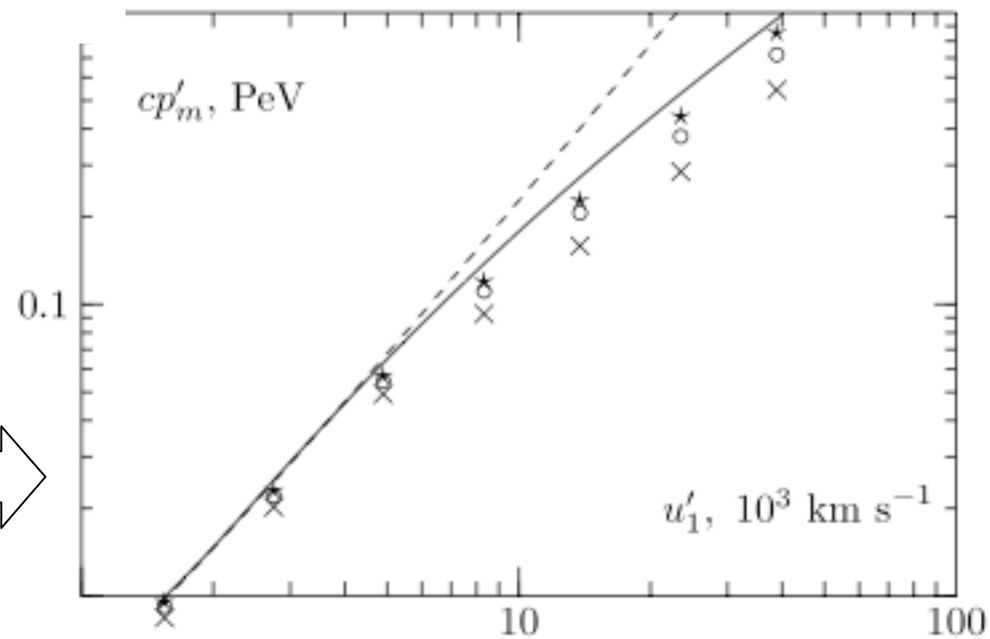
Too few waves at large λ
⇒D_Z~E^2
subdiffusive behaviour
for some range of E

K_{max} decreasing with time
but saturation not reached

For efficient scattering
Inverse cascading
vs
Advection time

When small scale turbulence
and finite time
Are taken into account….

(Zirakashvili & Ptuskin 08)



Summary

- Non-resonant modes highlighted by Bell 2004 generate potentially much larger fields than classically thought
- They can easily account for amplified fields observed in SNRs
- But can they help us make progress in CR physics?
- Approximation at the resonance very good to describe propagation in the Galaxy
- Currently it is not clear that they can provide sufficient scattering for CRs to reach the knee in SNR shocks
- An instability that has been studied for more than 40 years can still reserve surprises
- The two main problems to drive the study are still not completely understood