Fast localization of massive black hole binaries using the coronagraphic method for the LISA mission

Hugo Alexandre

LPC Caen Junior Seminar







Introduction to GW Astronomy

From General Relativity to Gravitational Waves

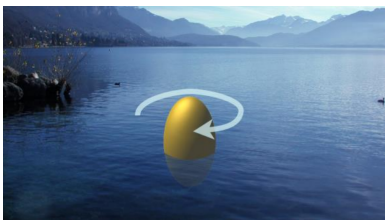
$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

→ Weak field limit: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, $|h_{\mu\nu}| \ll 1$

→ Vacuum, TT gauge: $\Box h_{\mu\nu} = 0$

From General Relativity to Gravitational Waves

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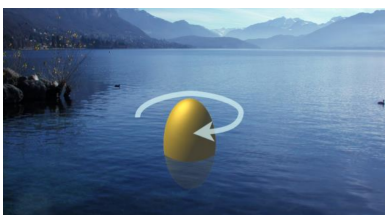
Zero quadrupole moment $\Rightarrow$ no GW

From General Relativity to Gravitational Waves

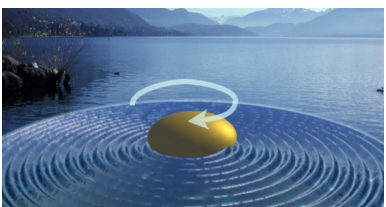
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Zero quadrupole moment ⇒ no GW



Non-zero quadrupole moment ⇒ GW

From General Relativity to Gravitational Waves

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \rightarrow$$

Weak field limit

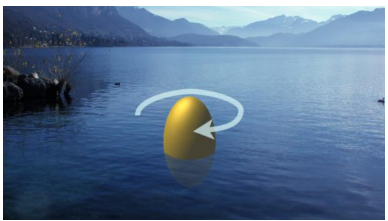
$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$|h_{\mu\nu}| \ll 1$

$\rightarrow$

Vacuum, TT gauge

$\Box h_{\mu\nu} = 0$



D. Buakulic LAPP/IN2P3/USMB

Zero quadrupole moment $\Rightarrow$ no GW



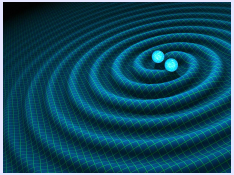
Non-zero quadrupole moment $\Rightarrow$ GW

Spacetime's stiffness is enormous ($\sim 10^{43}$ SI) $\rightarrow$ *What kind of astrophysical systems actually produce a ripple violent enough to detect?*

The Gravitational-Wave Source Zoo

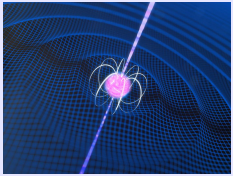
rotating mass distribution

BH* and/or NS Binary System**



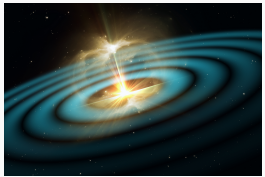
R. Hurt, Caltech/JPL

Rotating NS**



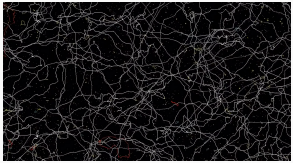
OzGrav/C. Knox

Supernova



livescience.com

Cosmic String Phase Transition



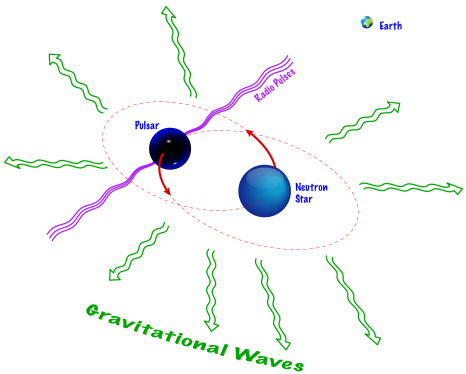
Christ Ringeval/CURL

*BH: Black Hole

**NS: Neutron Star

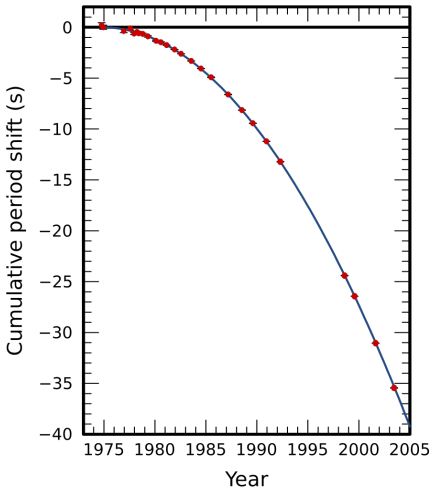
Measuring Gravitational Waves, History of Detection

Indirect Measurement: The Hulse-Taylor Pulsar

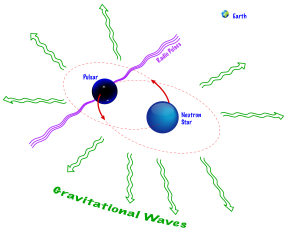


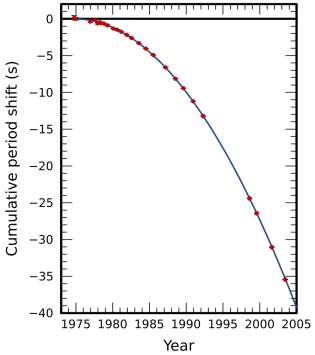
$\dot{E} < 0 \Rightarrow \dot{a} < 0 \Rightarrow \dot{P} < 0$

(orbital period shrinks)

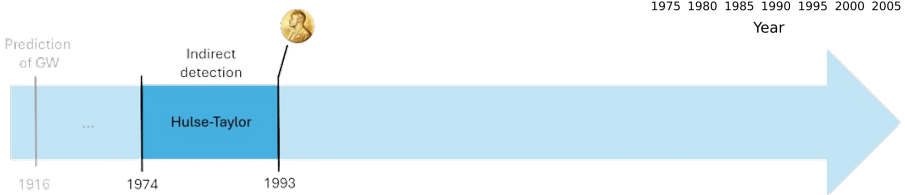


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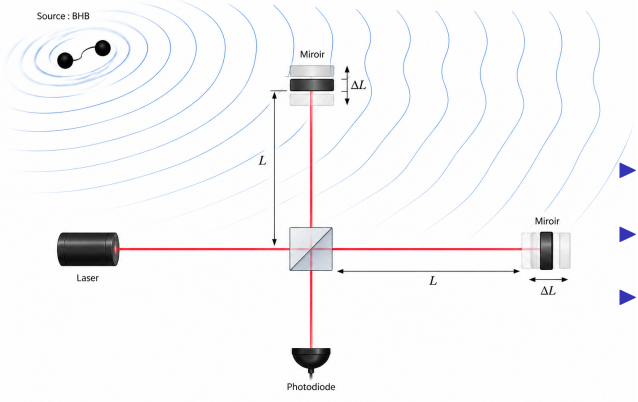




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Direct Measurement : LVK Interferometers

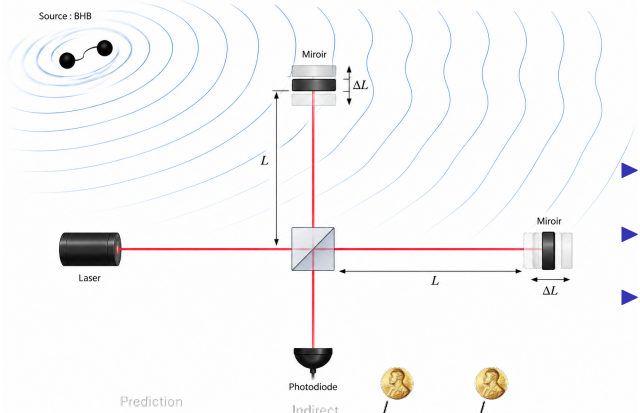


$$h = \Delta L / L \sim 10^{-21}$$

L: arm length – ΔL: relative distance variation

- Sensitivity for $f_{GW} \in [10^1, 10^4]$ Hz
- 3 plane interferometers + wave's polarization $\Rightarrow$ loc.
- We can probe the gravitational Universe

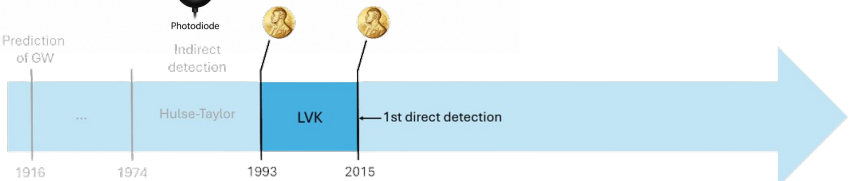
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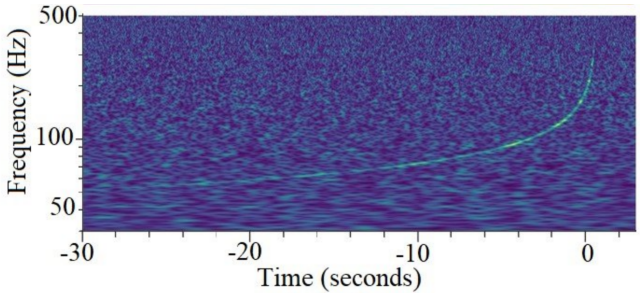
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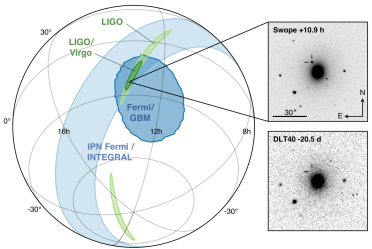
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GW170817: Multi-Messenger Astronomy

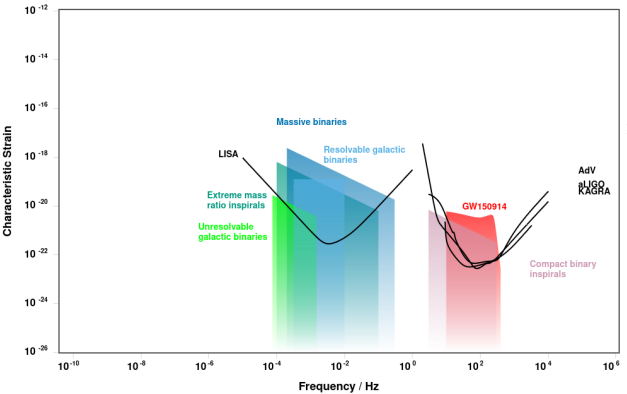


LIGO Collaboration



- Constrains H_0 via a **standard siren** measurement
- Confirms $c_{GW} = c$ to $\sim 10^{-15}$
- First observed **kilonova** $\Rightarrow$ site of heavy-element nucleosynthesis
- First **multi-messenger** event, also seen by Fermi (first signal in two independent catalogs)

Sensitivity : The Limits of LVK



Earth's size & seismic noise constrain ground-based detectors

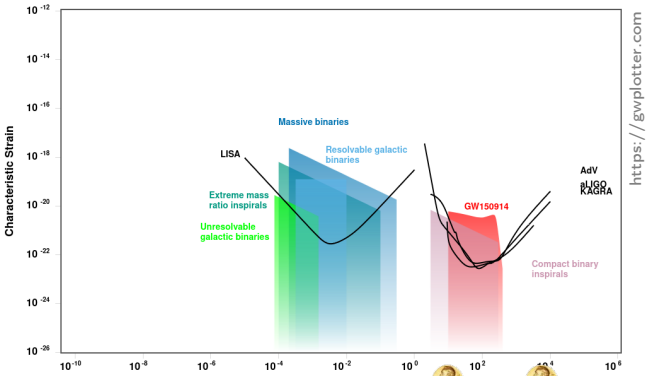
⇓

Lower frequencies need
$L \gtrsim \lambda_{GW} \Rightarrow$ a bigger detector
($\sim 10^6 \times$ km) – impossible on Earth

⇓

Send it to space ⇒ **LISA**
10^6 bigger than LVK,
no seismic noise

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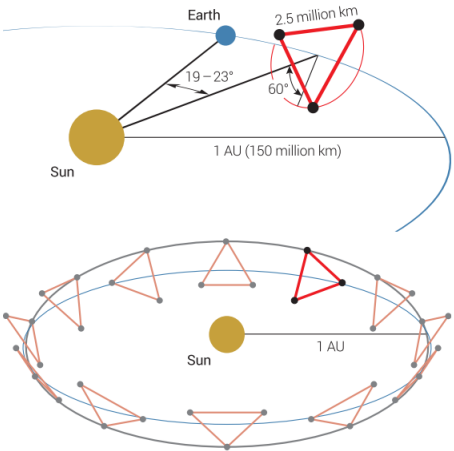
⇓

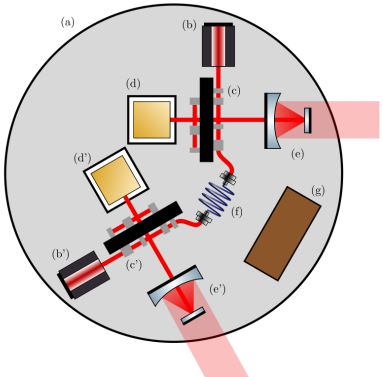
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LISA: Opening the mHz Window

The LISA Instrument





- Laser's round trip 5×10^6 km $\Rightarrow$ impossible
- $\left.\begin{array}{l} GW \sim 10^{-21} \\ laser \sim 10^{-13} \end{array}\right\} \Rightarrow$ **laser noise dominates**

(8 orders of magnitude)

Time-Delay Interferometry (1/2): The Principle

Post-processed interferometry ⇒
Time-Delay Interferometry (TDI)

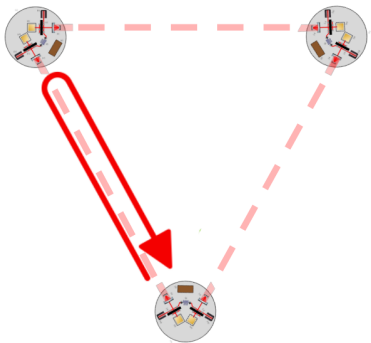
⇓

No round trip ⇒ reconstruct a **virtual photon** path

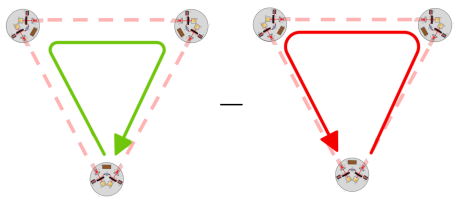
⇓

TDI cancels the laser noise*

*residual instrumental noise remains...



Time-Delay Interferometry (2/2): Building the Variables

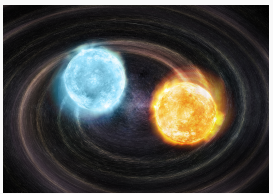
$\alpha_1 =$  $-$ $\longrightarrow$ Loops around the constellation: **Sagnac** TDI variable

Starting the construction at SC1 $\Rightarrow \tilde{\alpha}_1$ at SC2 $\Rightarrow \tilde{\beta}_1$ at SC3 $\Rightarrow \tilde{\gamma}_1$

Different photon paths through the constellation $\Rightarrow$ different TDI variables

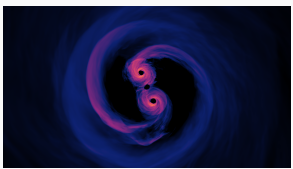
LISA: Expected Detections

Galactic Binaries (GBs)



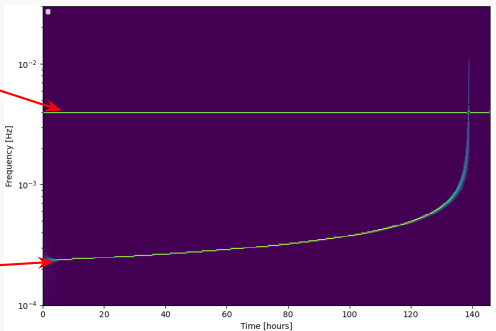
M. Weiss

Massive Black Hole Binaries (MBHBs)



NASA Goddard/Ascoli 2018

Frequency signature of sources



Other, yet unforeseen, sources may also exist

The Coronagraphic TDI Variable

We aim to build a linear combination of the TDI variables that **cancels** the GW signal, as a function of the position and frequency parameters characterizing the source:

$$\tilde{\kappa}(\alpha, \delta, f) = \vec{A}(\alpha, \delta, f) \cdot \left(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}\right)^{T}(f)$$

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Objective: find $(\alpha_\star, \delta_\star, f_\star)$ such that

$$\vec{A}(\alpha_\star, \delta_\star, f_\star) \cdot \begin{pmatrix} \tilde{\alpha} \\ \tilde{\beta} \\ \tilde{\gamma} \end{pmatrix} = 0$$

Barroso, R. C. (2025 - PhD Thesis)

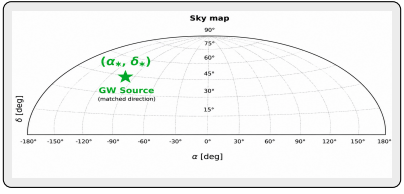
Coronagraphic Method : The Nullstream Pipeline

Principle

How to localize a source with LISA ?

Principle

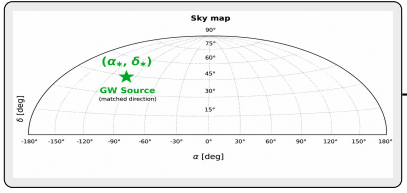
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Costa Barroso, R., Lemière, Y., Mauger, F., & Baghi, Q. (2025).

Principle

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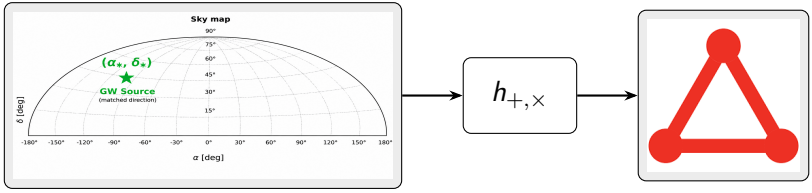


$h_{+,\times}$

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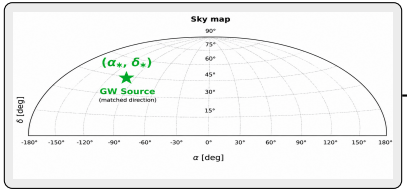
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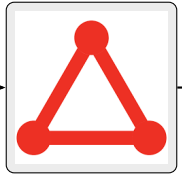


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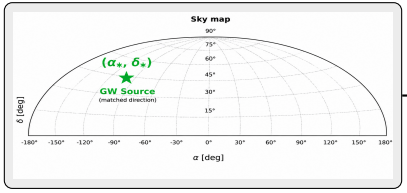
$h_{+,\times}$



$$\begin{pmatrix} \tilde{\alpha}(f) \\ \tilde{\beta}(f) \\ \tilde{\gamma}(f) \end{pmatrix}$$

Principle

How to localize a source with LISA ?



$h_{+,\times}$

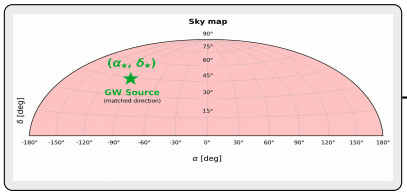
$$\begin{pmatrix} \tilde{\alpha}(f) \\ \tilde{\beta}(f) \\ \tilde{\gamma}(f) \end{pmatrix}$$

$\tilde{\kappa}$

Costa Barroso, R., Lemière, Y., Mauger, F., & Baghi, Q. (2025).

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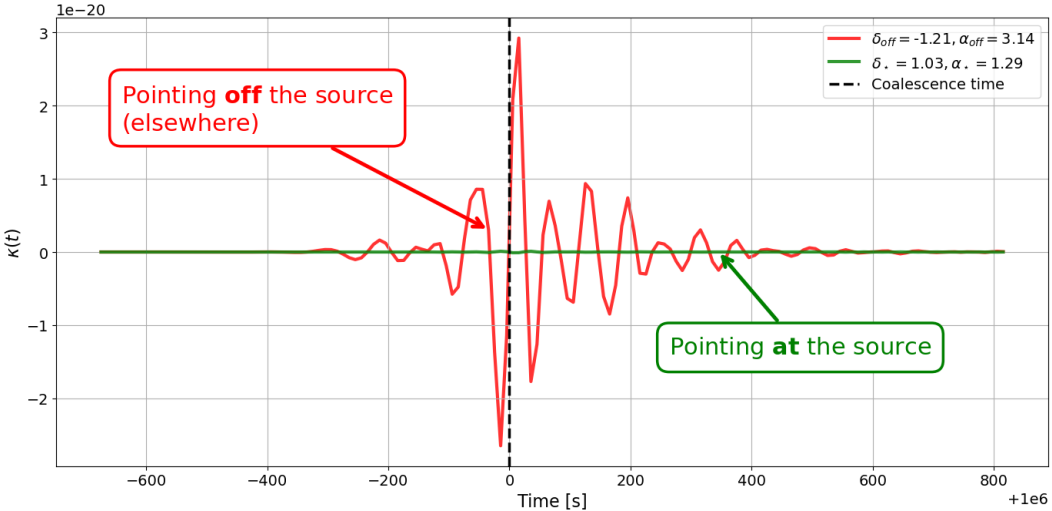
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$\tilde{\kappa}$

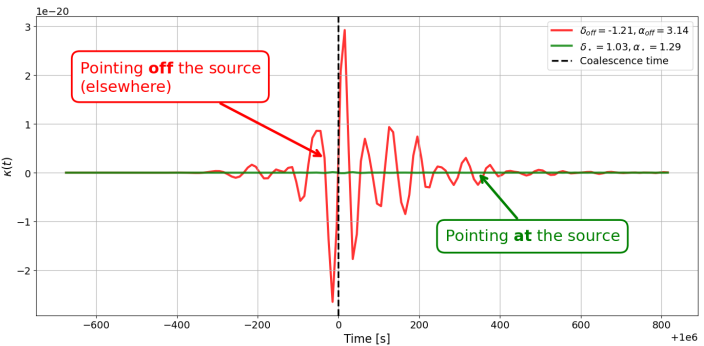
$$\tilde{\kappa}(\alpha, \delta) \begin{cases} = 0 \text{ if } (\alpha, \delta) = (\alpha_\star, \delta_\star) \\ \neq 0 \quad \text{elsewhere} \end{cases}$$

Costa Barroso, R., Lemière, Y., Mauger, F., & Baghi, Q. (2025).

Cancellation with $\kappa(t)$



Cancellation with $\kappa(t)$



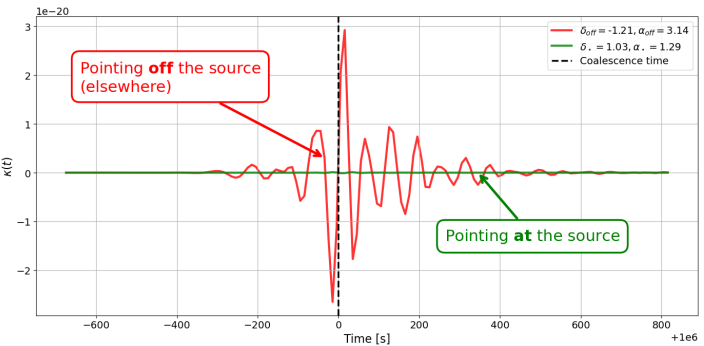
Advantages

- Model-agnostic
- Low computational cost (low-latency method)

Limitations

- Recovers only the source position
- No individual localization

Cancellation with $\kappa(t)$



Advantages

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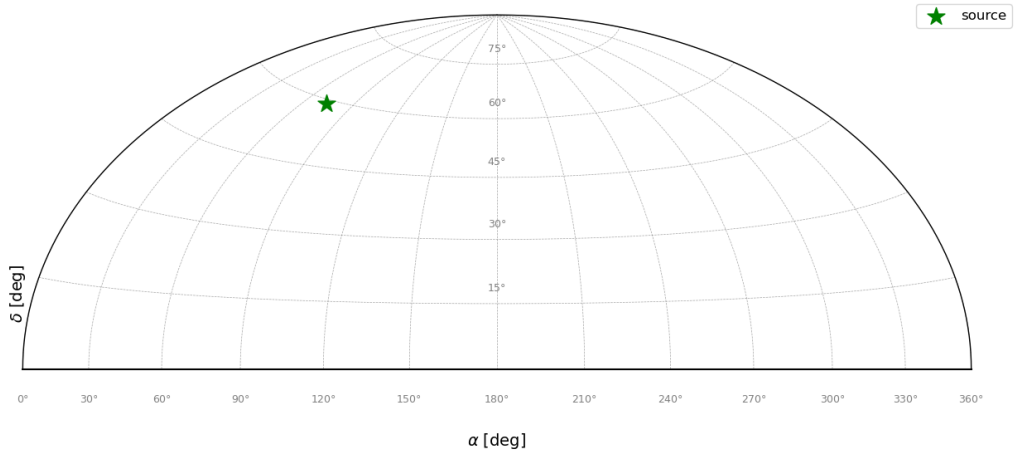
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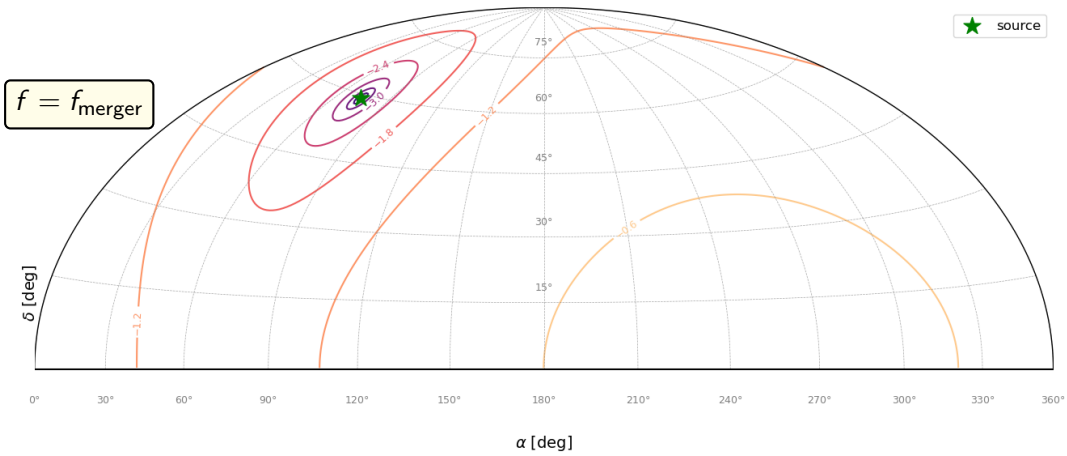
Interest of the coronagraphic method

- Fast Parameter Estimation
- Alert Triggering

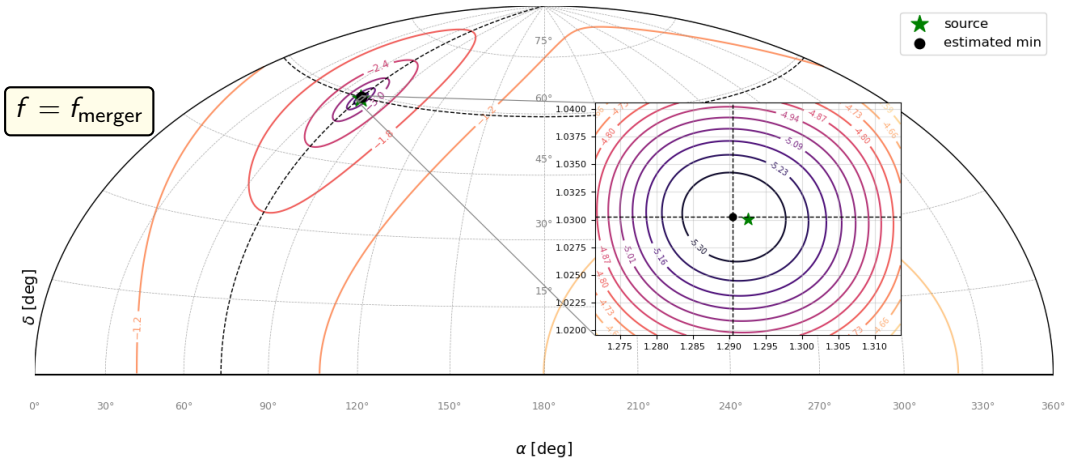
Sky Localization with κ



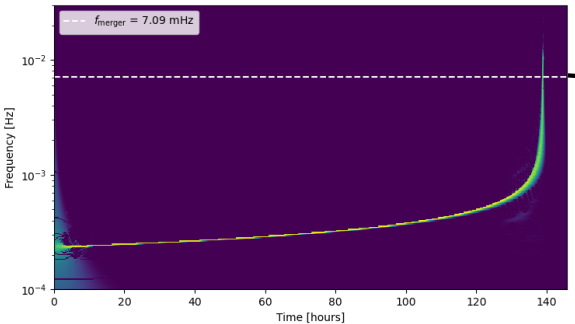
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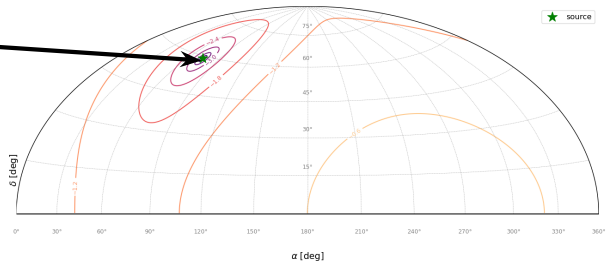


Sky Localization with κ



Beyond a Single Frequency...

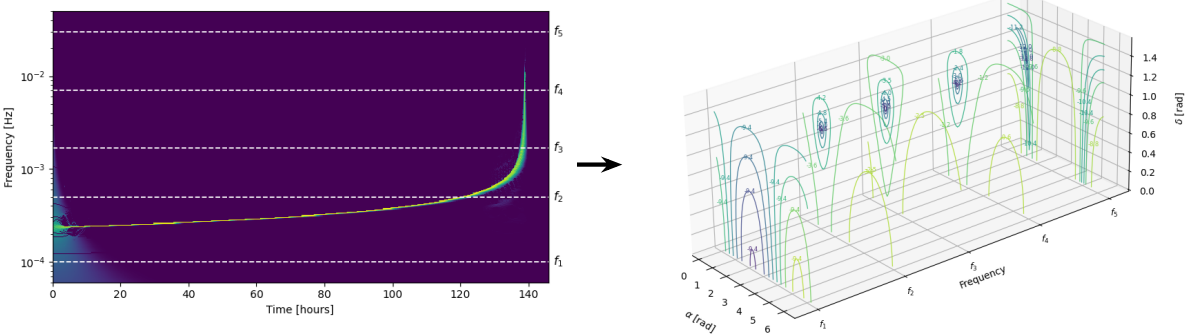




Localization done at a **single frequency** – but the MBHB chirps across many frequencies

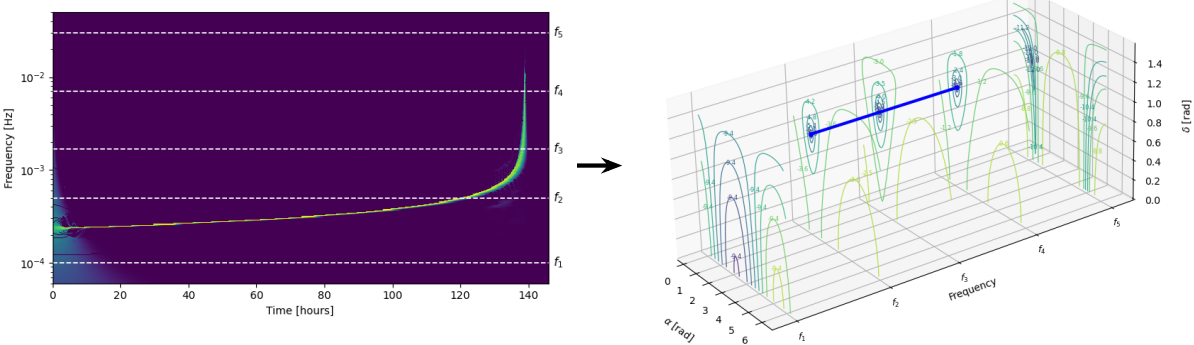
$\Rightarrow$ localizations can be accumulated over many frequencies

Scanning the MBHB Band (Noiseless)



$f_2, f_3, f_4 \in f_{\mathrm{MBHB}} \rightarrow$ **localization** but $f_1, f_5 \notin f_{\mathrm{MBHB}} \rightarrow$ **no localization**

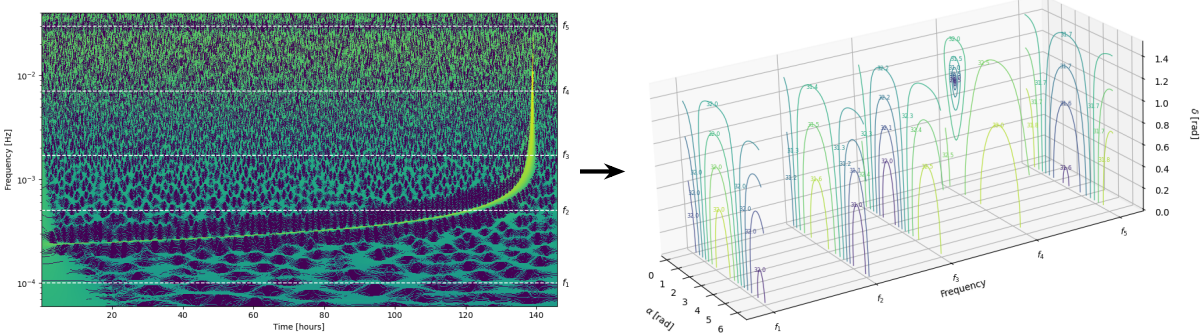
Scanning the MBHB Band (Noiseless)



$f_2, f_3, f_4 \in f_{\text{MBHB}} \rightarrow$ **localization** but $f_1, f_5 \notin f_{\text{MBHB}} \rightarrow$ **no localization**

The 3 localizations align into a segment $\Rightarrow$ pattern to identify and characterize

Scanning the MBHB Band (Noisy)

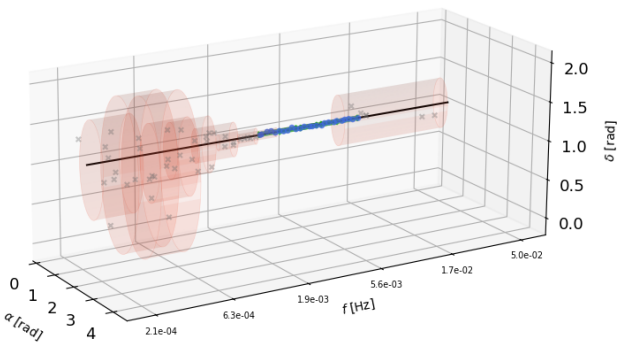


With noise, f_2, f_3 **lose their localization** – only f_4 **still localizes** (highest spectral power)

must search $f_i \in f_{\mathrm{MBHB}}$ for the frequencies that localize **and** align into a segment

Building Cylinders (1/2): Principle

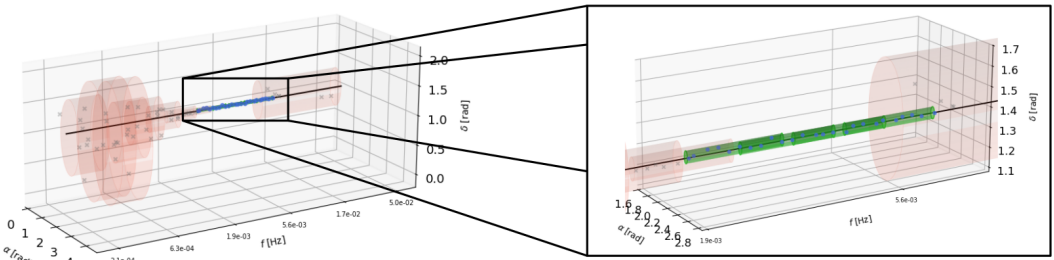
Principle: build cylinders of N points, each point is a coordinate $(\alpha_i, \delta_i, f_i)$, i.e. one localization made at one frequency



- Center: $(\hat{\mu}_\alpha, \hat{\mu}_\delta)$ mean of the sample
- Radius: $\hat{\sigma}$ std of the sample

The cylinder(s) with the smallest $\hat{\sigma}$ give the localization, with its uncertainty

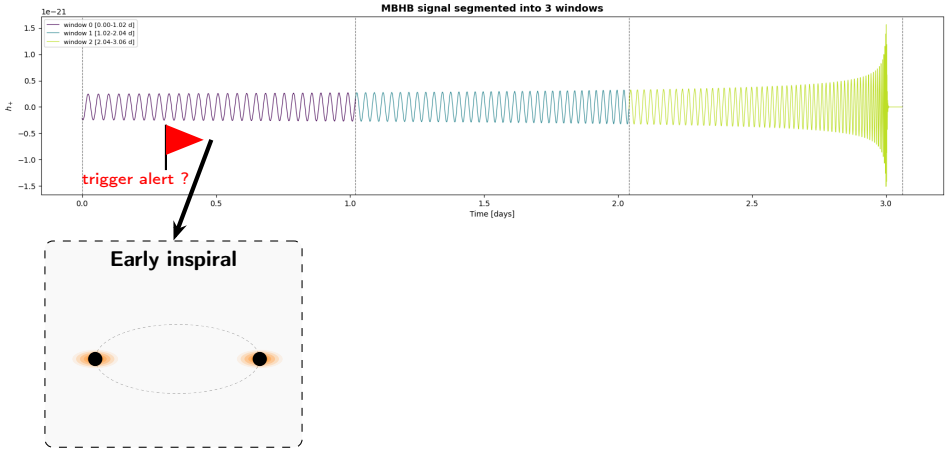
Building Cylinders (2/2): Results



We observe **aligned points** forming a pattern characterized by several cylinders of the same size, following one another

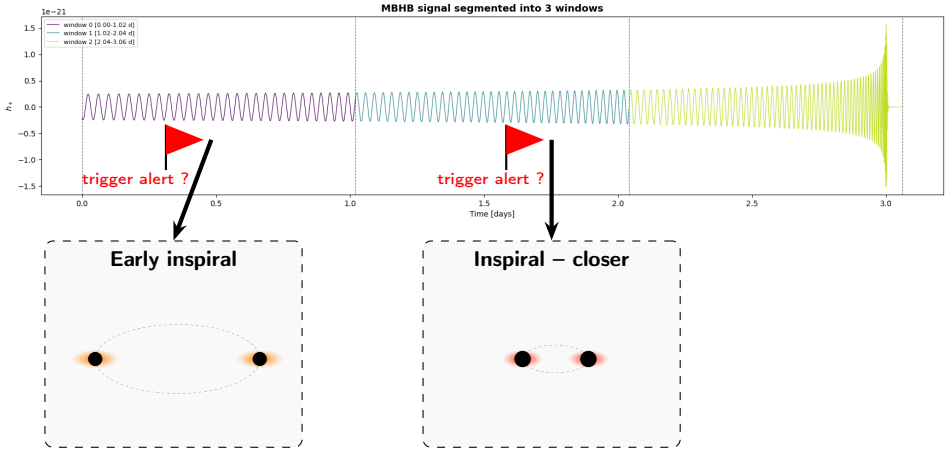
$$\Delta\alpha = 0.6013^\circ \qquad \Delta\delta = 0.0442^\circ \qquad \Delta\sigma = 0.2600^\circ$$

L2A - Catching EM Counterparts



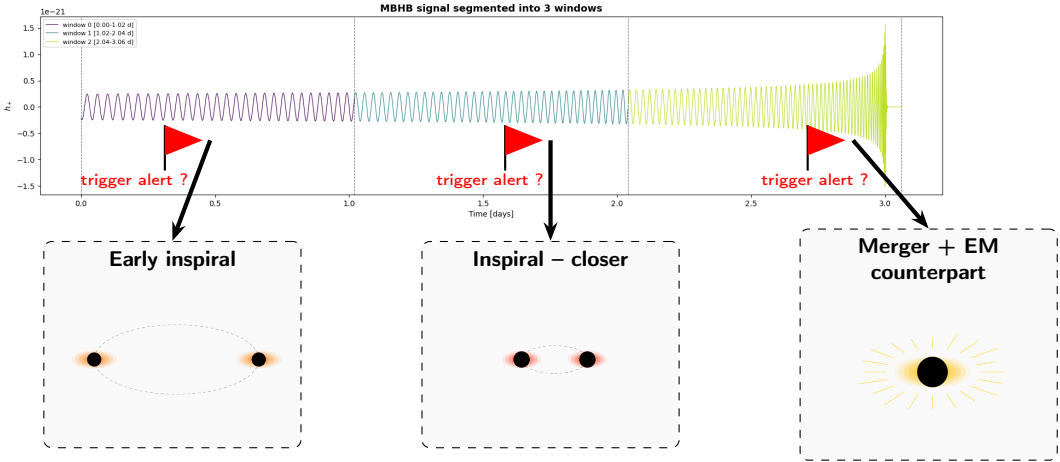
Early inspiral: signal just emerging $\Rightarrow$ a first, coarse alert

L2A - Catching EM Counterparts



Binary tightens: signal grows ⇒ localization sharpens ⇒ updated alert

L2A - Catching EM Counterparts

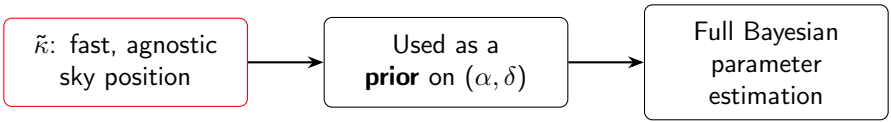


Merger: disk interactions may radiate ⇒ **final, sharpest alert** for EM follow-up

Perspectives & Conclusion

Beyond Sky Localization: Parameter Estimation

The coronagraphic method can be used as a **prior** to accelerate parameter estimation (PE)



Summary

- The coronagraphic method is an agnostic method to localize GW sources for the LISA mission.
- Validated on **noiseless** and **low-noise** MBHB simulations $\Rightarrow$ breaks down at **high noise** levels
- Tailored for **low-latency** searches: directly usable within L2A for multi-messenger alerts

Backup

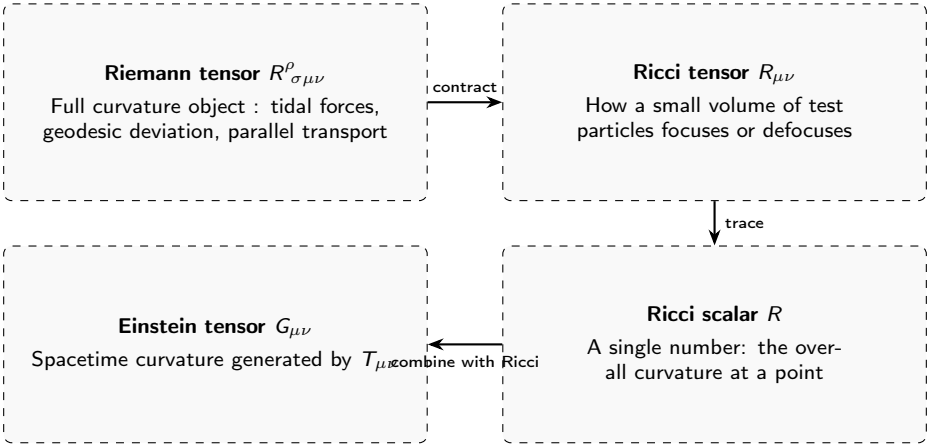
Angular Localization Error

The sky localization error is defined as the angular separation between the true and estimated source positions:

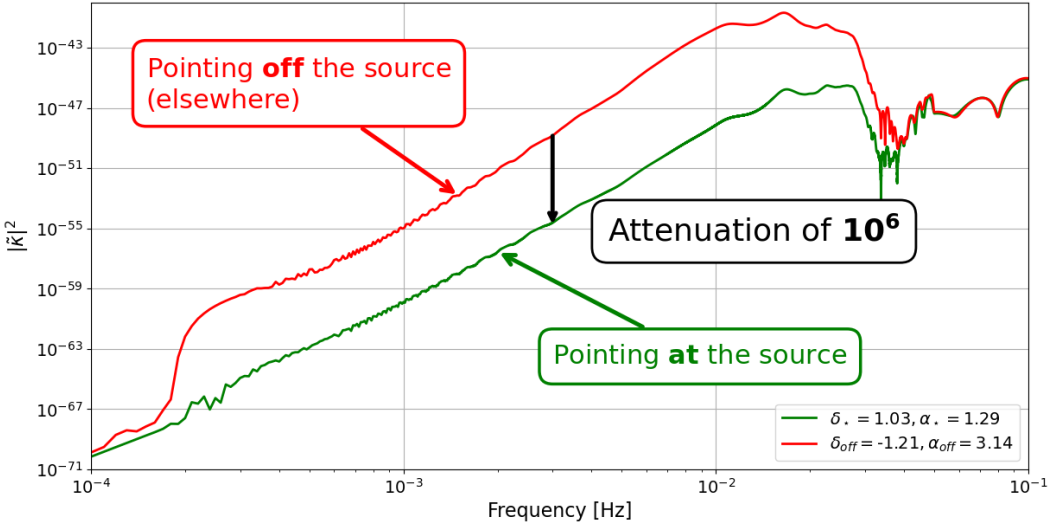
$$\Delta\sigma = \arccos\left[\sin(\delta_{\text{test}})\sin(\delta_\star) + \cos(\delta_{\text{test}})\cos(\delta_\star)\cos(\alpha_{\text{test}} - \alpha_\star)\right]$$

$\Rightarrow$ measures the angular distance on the celestial sphere between the estimated position $(\alpha_{\text{test}}, \delta_{\text{test}})$ and the true position $(\alpha_\star, \delta_\star)$

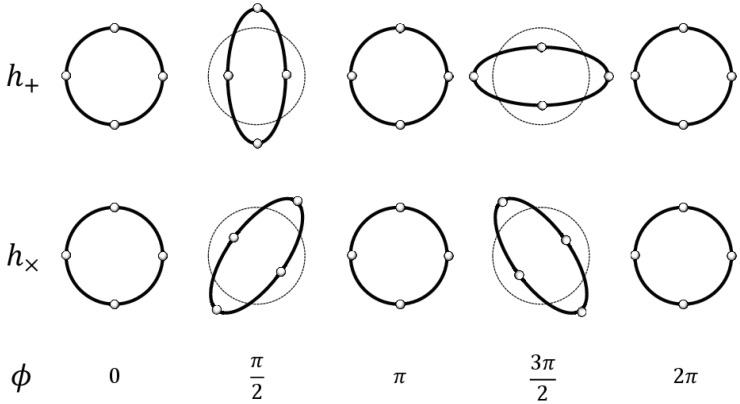
Curvature Tensors: Telling Them Apart



Cancellation with $\kappa(\tilde{f})$ TDI



Quadrupole Radiation Pattern



A ring of test particles stretches and squeezes along the $+$ and $\times$ polarizations as the wave passes

Standard Sirens: Measuring H_0 with GW170817

A **standard siren** is the GW analogue of a standard candle: the waveform directly encodes the luminosity distance D_L

GW signal

GR gives the luminosity distance directly:

$D_L \approx 40$ Mpc

EM counterpart

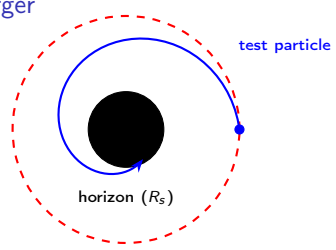
Host galaxy identified (NGC 4993) $\Rightarrow$ redshift from its spectrum:

$z \approx 0.097$

From Hubble's law

$$D_L \approx \frac{cz}{H_0} \quad \Longrightarrow \quad H_0 = \frac{cz}{D_L} \approx 70 \text{ km/s/Mpc}$$

The Merger Frequency: f_{merger}



Beyond the ISCO, circular orbits are no longer stable – the particle plunges in. The GW frequency at that point sets f_{merger}:

$$f_{\text{merger}} \approx f_{\text{ISCO}} = \frac{c^3}{6^{3/2}\,\pi\,GM}$$

Building the Vector $\vec{A}$

For a generator, e.g.

$$\tilde{\alpha}_{gw} = \alpha_+ \tilde{h}_+ + \alpha_\times \tilde{h}_\times$$
where $\alpha_{+,\times} = \sum_{ij} \alpha_{ij} \xi_{ij}^{+,\times}$

$\xi_{ij}^{+,\times}$: **antenna response** – how each link ij responds to $h_+, h_\times$

$$\vec{P}_{+,\times} = (\alpha_{+,\times}, \beta_{+,\times}, \gamma_{+,\times})^T$$
Cancelling $\tilde{\kappa}_{gw}$ for any $\tilde{h}_{+,\times}$:
$\vec{A} \cdot \vec{P}_+ = 0 \quad \vec{A} \cdot \vec{P}_\times = 0$

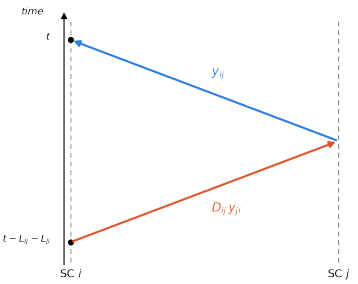
(orthogonality condition)

$$\vec{A} = \vec{P}_+ \times \vec{P}_\times = \begin{pmatrix} \beta_+\gamma_\times - \gamma_+\beta_\times \\ \gamma_+\alpha_\times - \alpha_+\gamma_\times \\ \alpha_+\beta_\times - \beta_+\alpha_\times \end{pmatrix} \quad (3.1.27)$$

Building the TDI Operators (1/2): The Delay Operator

The **delay operator** D_{ij} delays a measurement by the light travel time $\tau_{ij}(t)$ along link ij:

$$D_{ij}\, x(t) := x\big(t - L_{ij}\big), \quad \text{chained: } D_{ijk} := D_{ij} D_{jk}$$



Telescoping: a simple round trip already cancels part of the laser noise

$$y_{ij}\big|_{\text{laser}} = C_j(t - L_{ij}) - C_i(t)$$

$$D_{ij}\, y_{ji}\big|_{\text{laser}} = C_i(t - L_{ij} - L_{ji}) - C_j(t - L_{ij})$$

The same term appears in both, with opposite sign, it cancels (telescopes) in the sum

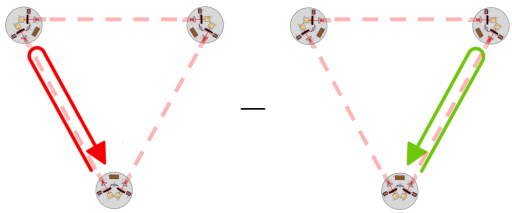
$$y_{ij} + D_{ij}\, y_{ji} = -C_i(t) + C_i(t - L_{ij} - L_{ji})$$

Only the **local** laser noise C_i remains, at two different times a full TDI variable pairs *two* such round trips of equal delay so this residual cancels too

Building the TDI Operators (2/2): Example – X

X is the difference of two round trips from SC1, one on each adjacent arm:

$$X = \underbrace{\left[y_{12} + D_{12}\, y_{21}\right]}_{\text{round trip SC1}\leftrightarrow\text{SC2}} - \underbrace{\left[y_{13} + D_{13}\, y_{31}\right]}_{\text{round trip SC1}\leftrightarrow\text{SC3}}$$

$X =$  $-$

$\Rightarrow$ vanishes only under the **equal-arm** assumption $\ell_2 = \ell_3$ (first generation, simplified): both terms become $C_1(t - 2\ell)$ and cancel, $X|_{\text{laser}} = 0$

Using the round-trip result from (1/2), with $L_{12} = L_{21} = \ell_3$ and $L_{13} = L_{31} = \ell_2$ (arm opposite the third satellite):

Red: $y_{12} + D_{12} y_{21}\big|_{\text{laser}} = -C_1(t) + C_1(t - 2\ell_3)$

Green: $y_{13} + D_{13} y_{31}\big|_{\text{laser}} = -C_1(t) + C_1(t - 2\ell_2)$

$$X\big|_{\text{laser}} = C_1(t - 2\ell_3) - C_1(t - 2\ell_2)$$

TDI Generations

First generation

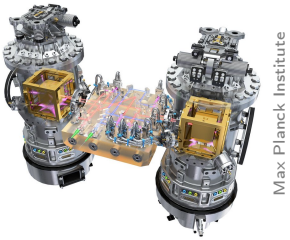
- **Static and symmetric** arms: $L_{ij}(t) = L_{ji}(t) = \text{const}$ (round-trip delay equal; the three arms can still differ from each other)
- Delay operators **commute** $\Rightarrow$ laser noise cancels exactly under this assumption
- Word length: 6 links (Sagnac), 8 links (Michelson)

Second generation

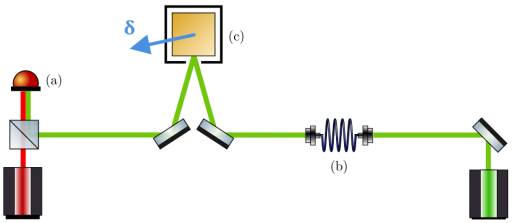
- Accounts for the real, **time-varying** arm lengths: $\dot{L}_{ij}/c \sim 10^{-8}$ (rotation, flexing)
- Delay operators no longer commute $\Rightarrow$ each virtual beam retraces its loop a second time
- Word length doubles: 12 links (Sagnac), 16 links (Michelson) – residual laser noise drops from $\mathcal{O}(\dot{L})$ to $\mathcal{O}(\dot{L}^2)$

LISA's constellation slowly rotates and flexes – second-generation TDI is required to meet the noise budget

The Test Masses



Max Planck Institute



J.B. Bayle (PhD Thesis - 2019)

- Each spacecraft hosts **two** free-falling test masses (one per MOSA), shielded from every non-gravitational force
- **Drag-free control**: the spacecraft itself is steered to follow the test mass, not the other way around
- The TMI (test-mass interferometer) measures the test mass's displacement δ **along the optical axis**, relative to its own optical bench isolating the local test-mass motion in the data

Residual Noise: Test-Mass & Optical Metrology

Once TDI cancels the laser noise, two secondary noises set LISA's final sensitivity

Test-mass acceleration noise

$$\sqrt{S_a} \sim 3 \frac{\mathrm{fm/s^2}}{\sqrt{\mathrm{Hz}}}$$

Physically: a residual **acceleration noise** disturbing free-fall (equivalent to a force smaller than the weight of a virus acting on the 2 kg test mass)

Dominates **low frequencies**

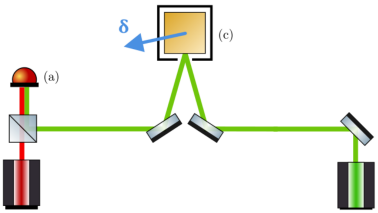
Optical metrology noise (OMS)

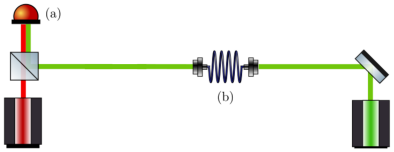
$$\sqrt{S_x} \sim 15 \frac{\mathrm{pm}}{\sqrt{\mathrm{Hz}}}$$

Physically: the shot-noise-limited **measurement precision** of the optical readout ($\sim 1/7$ of a hydrogen atom's diameter)

Dominates **high frequencies**

The Three Interferometers: ISI, TMI, RFI

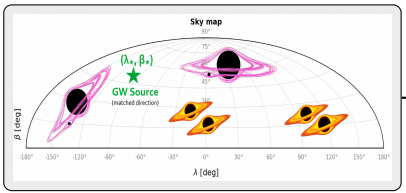




- **ISI** (Inter-Spacecraft Interferometer): local laser × beam received from the distant spacecraft (∼8.3 s later). The *only* one of the three that carries the GW signal
- **TMI** (Test-Mass Interferometer): local laser reflected off the local test mass. It measures the test mass's motion relative to the bench
- **RFI** (Reference Interferometer): local laser × the other on-board laser, via an internal backlink fiber, touches no test mass

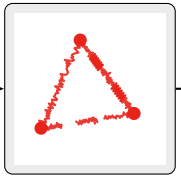
Perspectives & Future Work

How to localize MBHBs in realistic conditions in the context of the L2A ?



Overlapping sources

$h_{+,\times}$



Instrumental noise

TDI
+ noise

$\tilde{\kappa}$

$\tilde{\kappa}(\alpha, \delta)$ $\begin{cases} \approx 0 \text{ if } (\alpha, \delta) = (\alpha_\star, \delta_\star) ? \\ \not\approx 0 \quad \text{elsewhere ?} \end{cases}$

L2A pipeline