JET MOMENTUM BROADENING IN VISCOUS QCD MATTER: A MOMENT EXPANSION APPROACH

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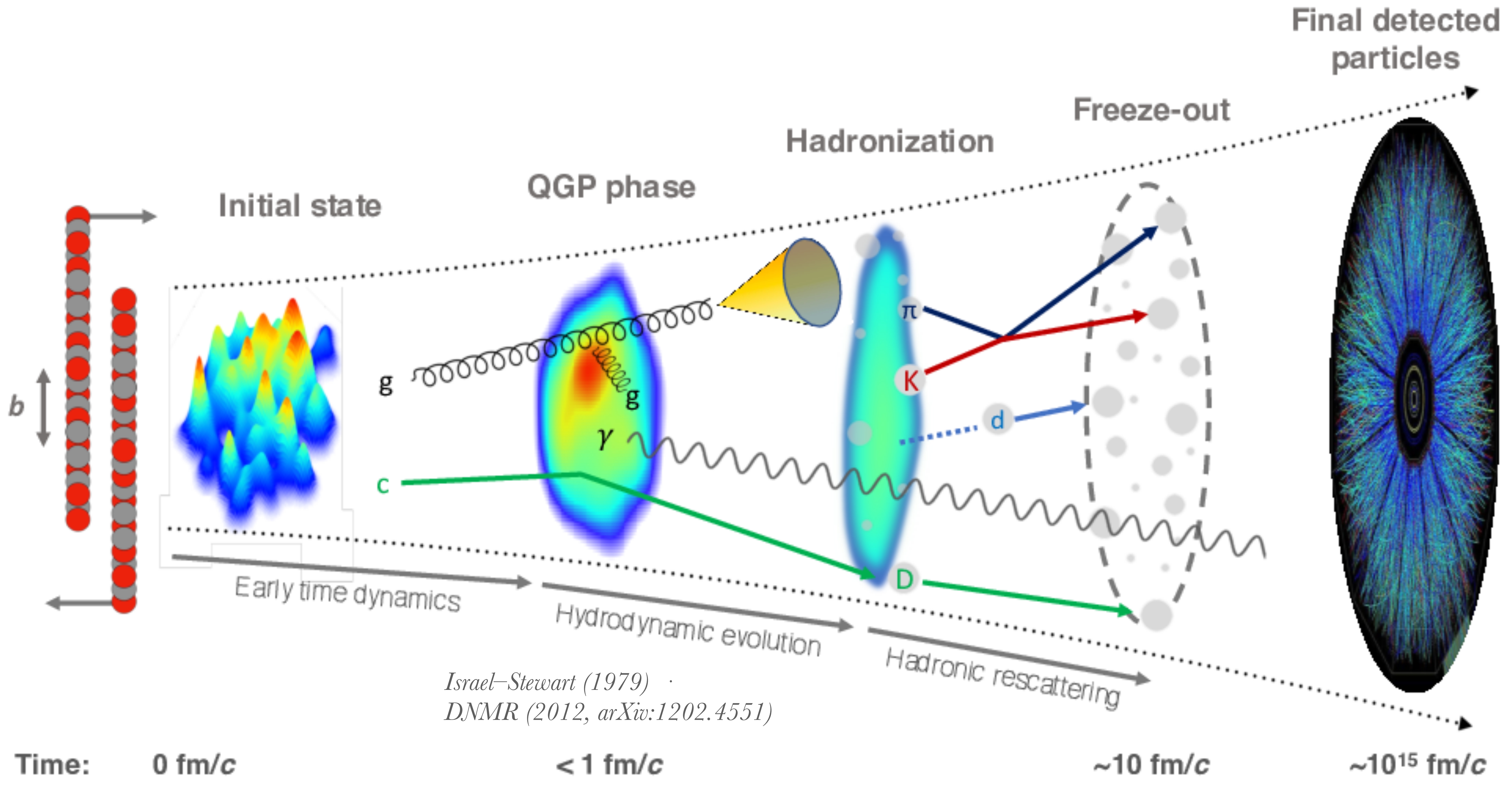
arXiv:2605.12791 [hep-ph]

What is a jet?

Relativistic Heavy-Ion Collisions

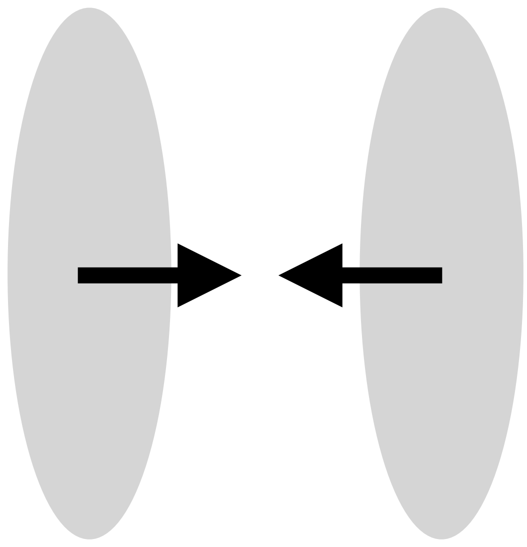


Hot QCD White Paper, M. Arslandok, S.A. Bass, A.A. Baty, I. Bautista ,C. Beattie et al.

- First, two nuclei collide at ultra-relativistic energies.
- This creates a very hot and dense system.
- Quark-Gluon Plasma is formed, where quarks and gluons are deconfined
- The system expands and cools down, finally it hadronizes
- Finally, it reaches the detector!

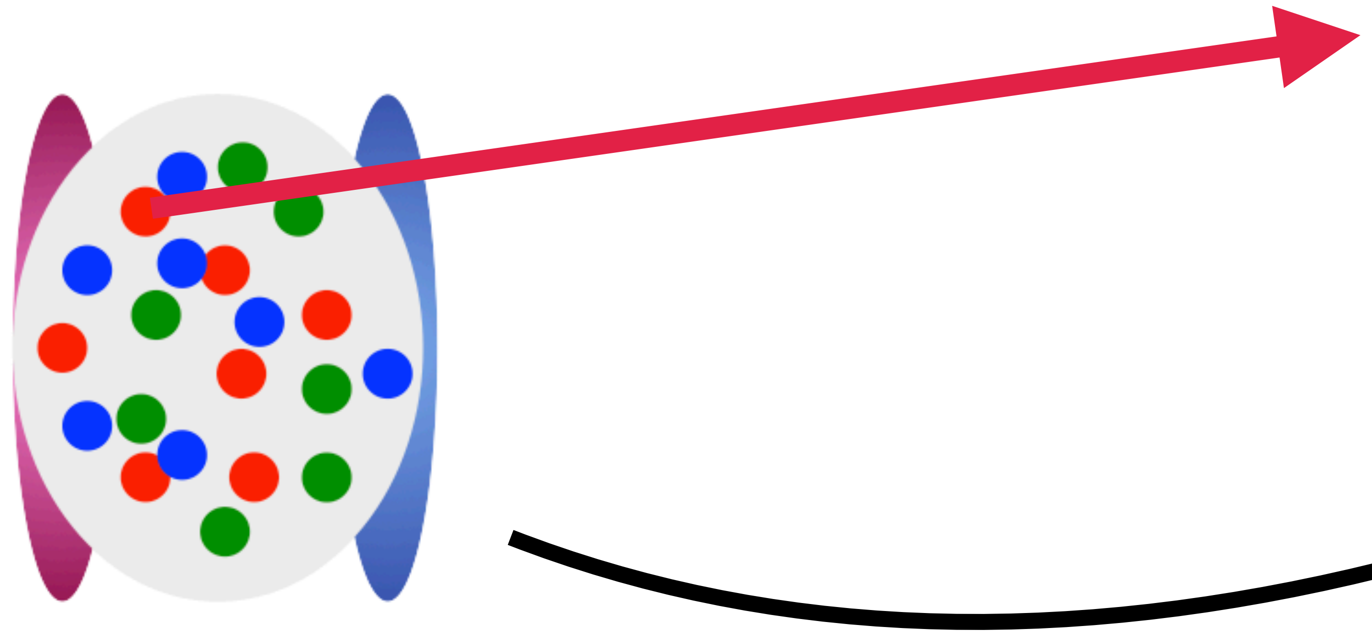
What is a jet?

Relativistic Heavy-Ion Collisions

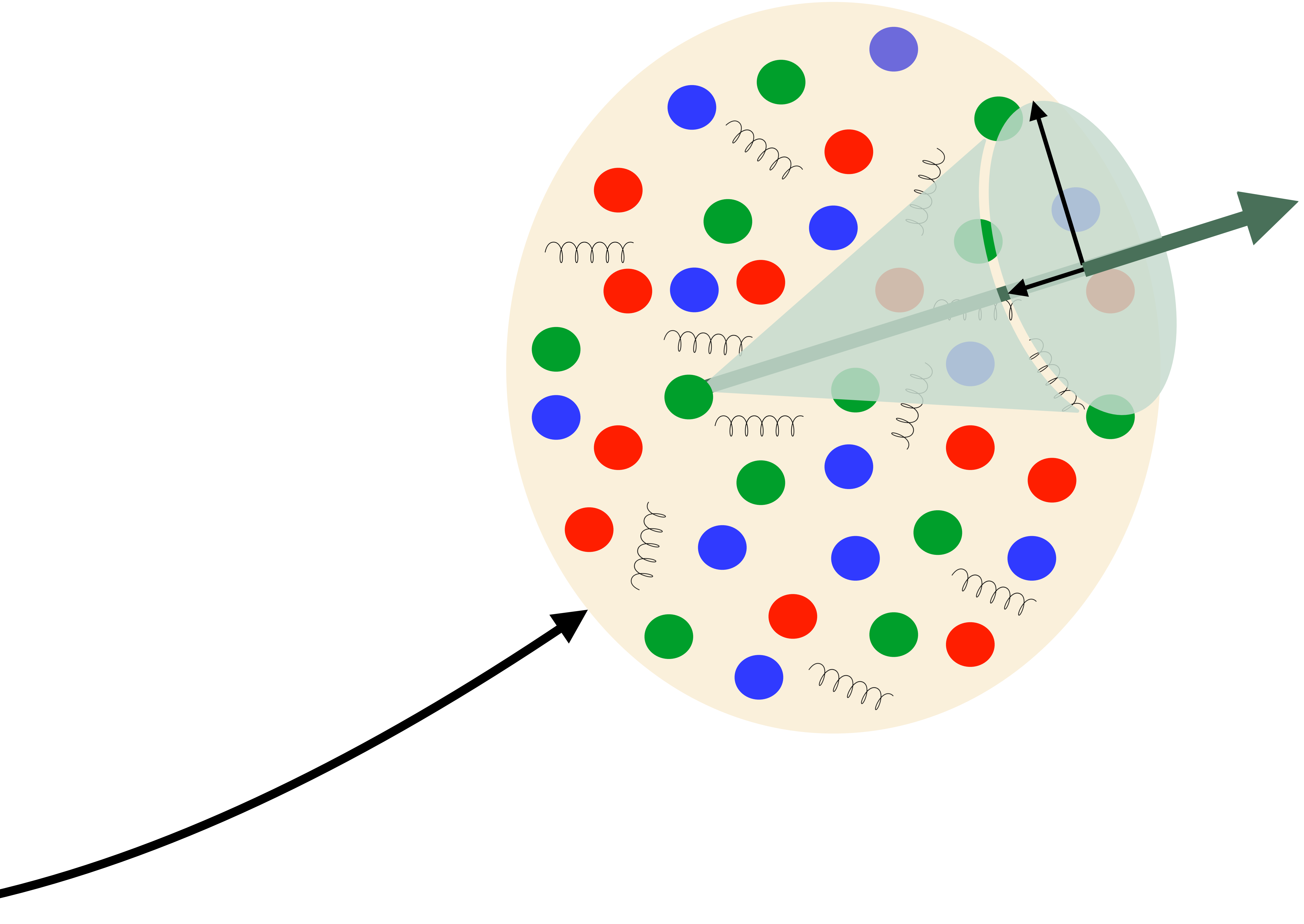


Two colliding nuclei

Hard parton traveling through the medium



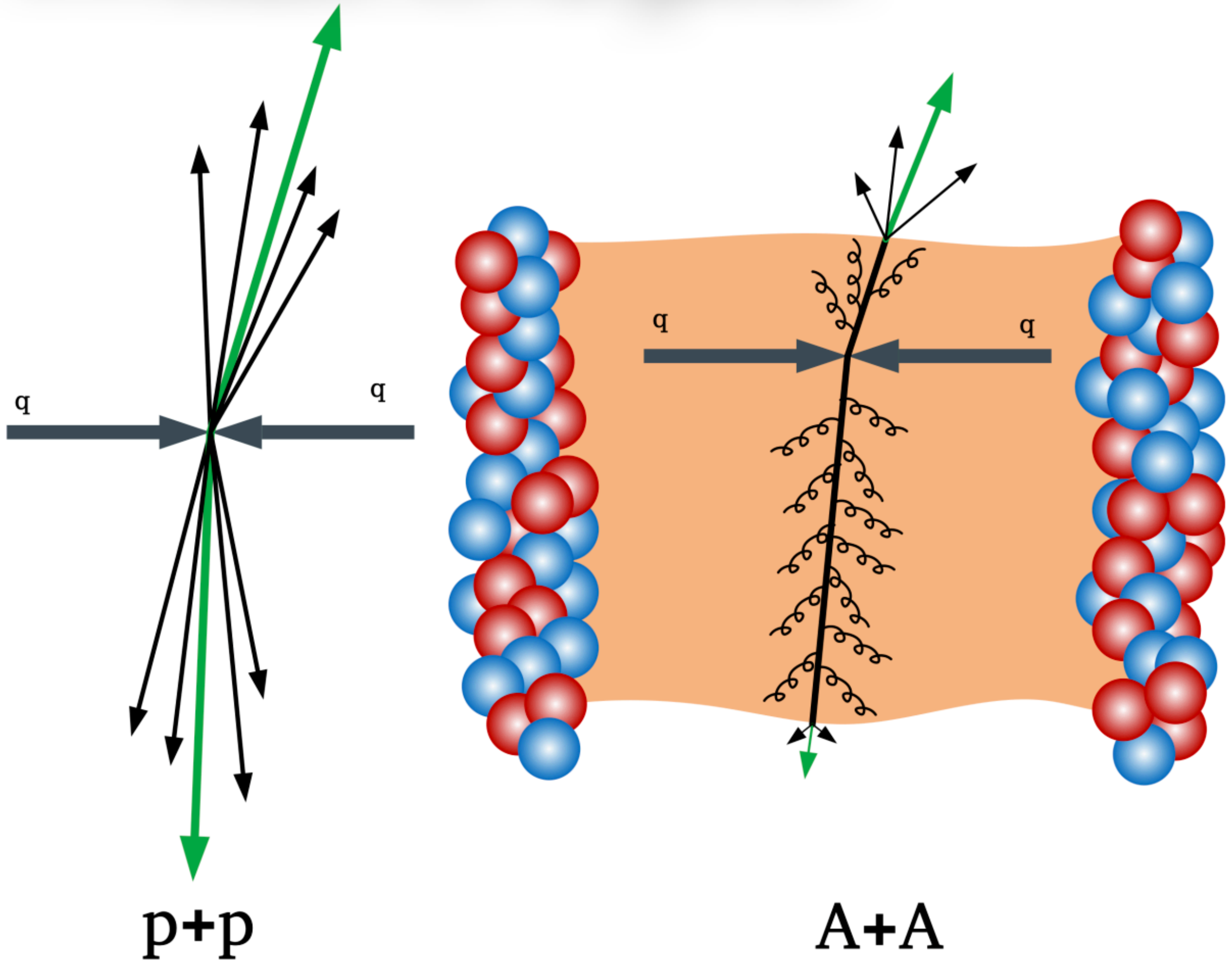
Initial State



This parton will travel as the system evolves and cools down! But why is this interesting for us?

What is a jet?

Signatures of the QGP: energy loss

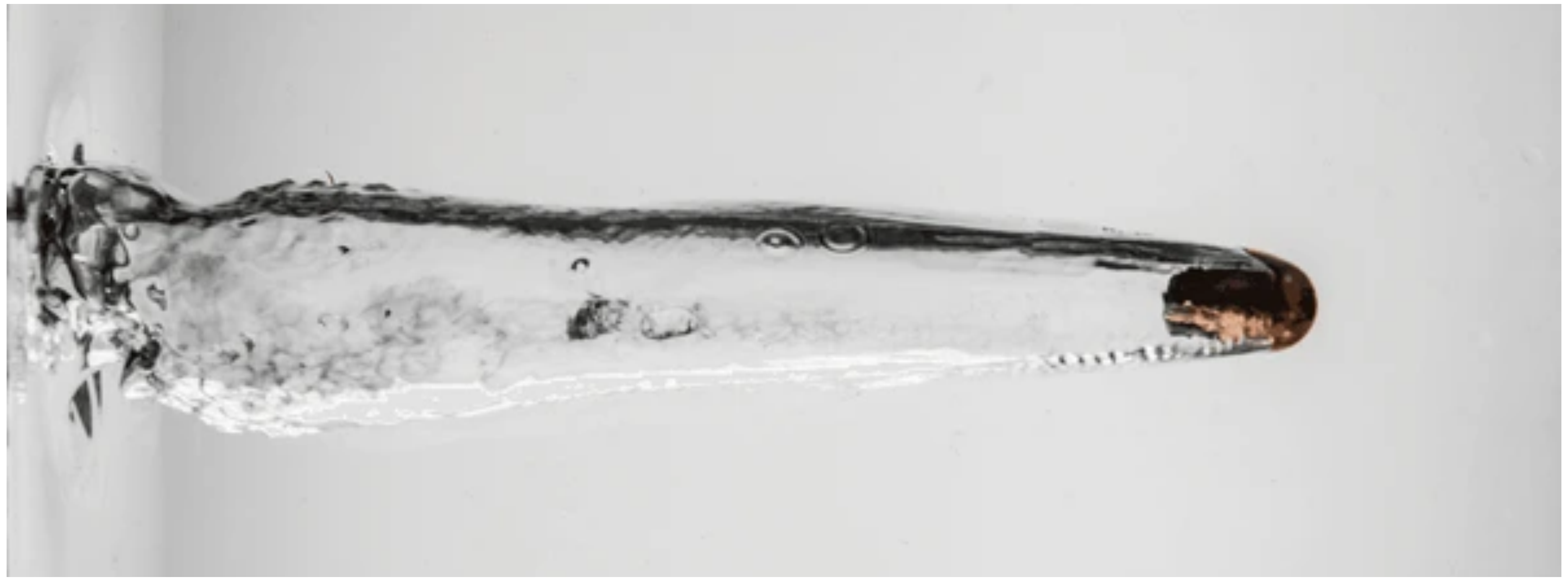


ATLAS collaboration, PRL 105, 252303 (2010)
Cao–Wang review, Rep. Prog. Phys. 84, 024301 (2021)

The yield of high-momentum jets is suppressed relative to the binary-collision-scaled pp expectation — R_{AA} falls well below one

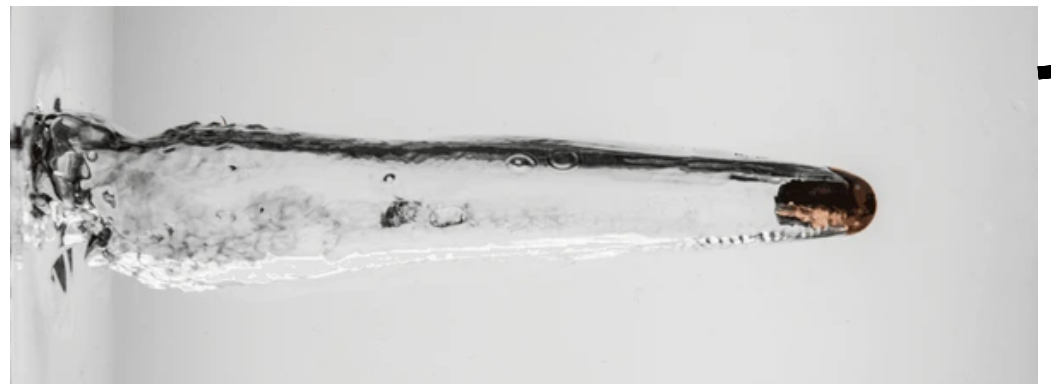
Path length of the jet $L_{Pb} > L_p$, not enough time to lose energy!

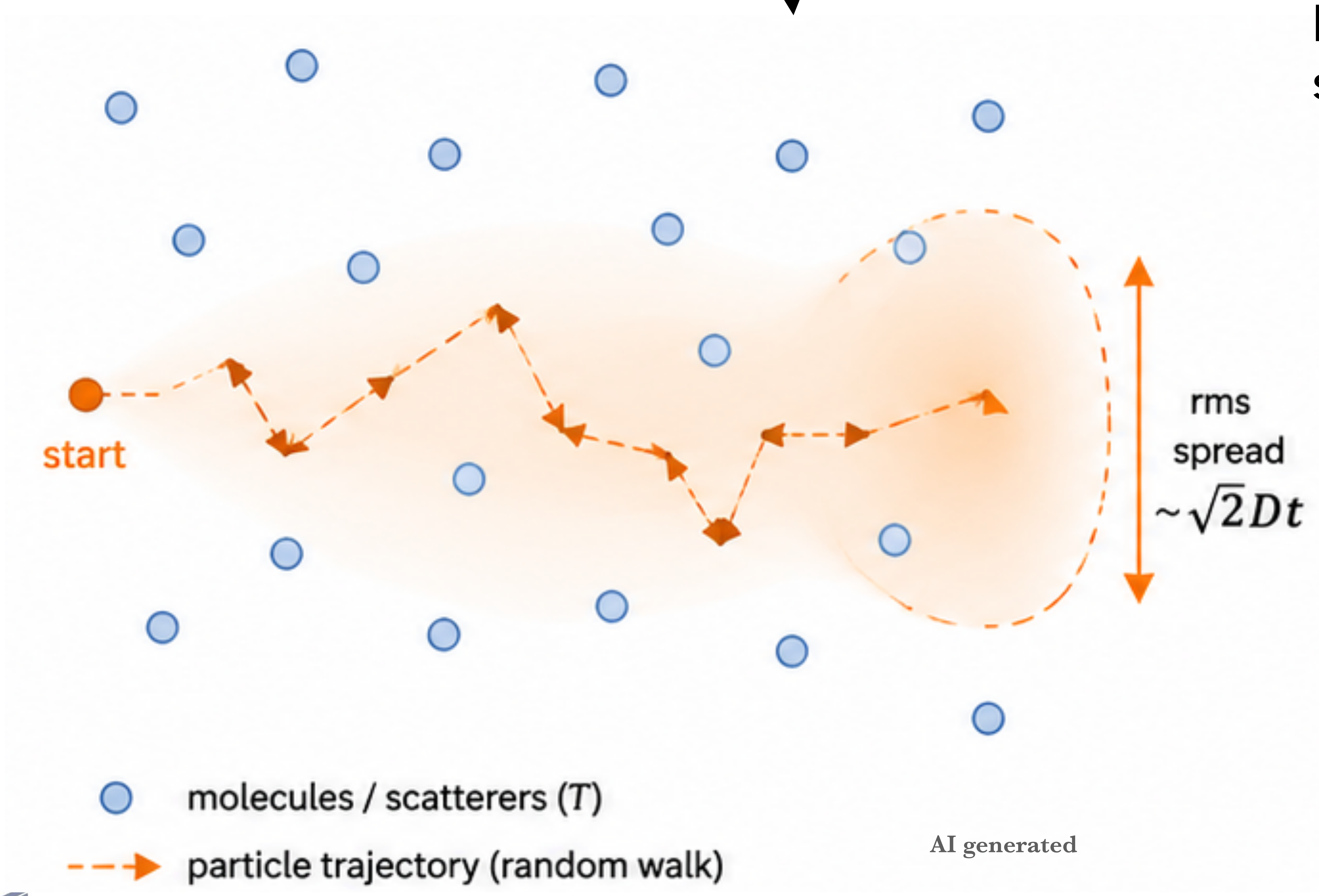
Early hard scatterings produce hard partons that travel with really high momentum through the medium!



High p_T jets lose energy getting bumped around by the strongly-interacting QGP

What is a jet?





AI generated

Brownian Motion

Diffusion in position space

In Brownian motion, a particle receives many small, random kicks from surrounding molecules.

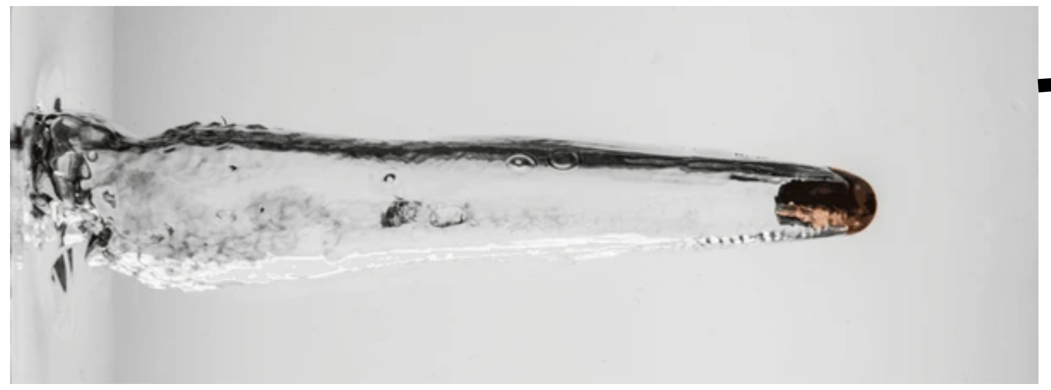
The particle performs a random walk

$$\langle\,|\mathbf{x}(t) - \mathbf{x}(0)\,|^2\,\rangle = 2Dt$$

D: encodes the microscopic medium

A. Einstein, Ann. Phys. (Leipzig) 17, 549 (1905).

What is a jet?



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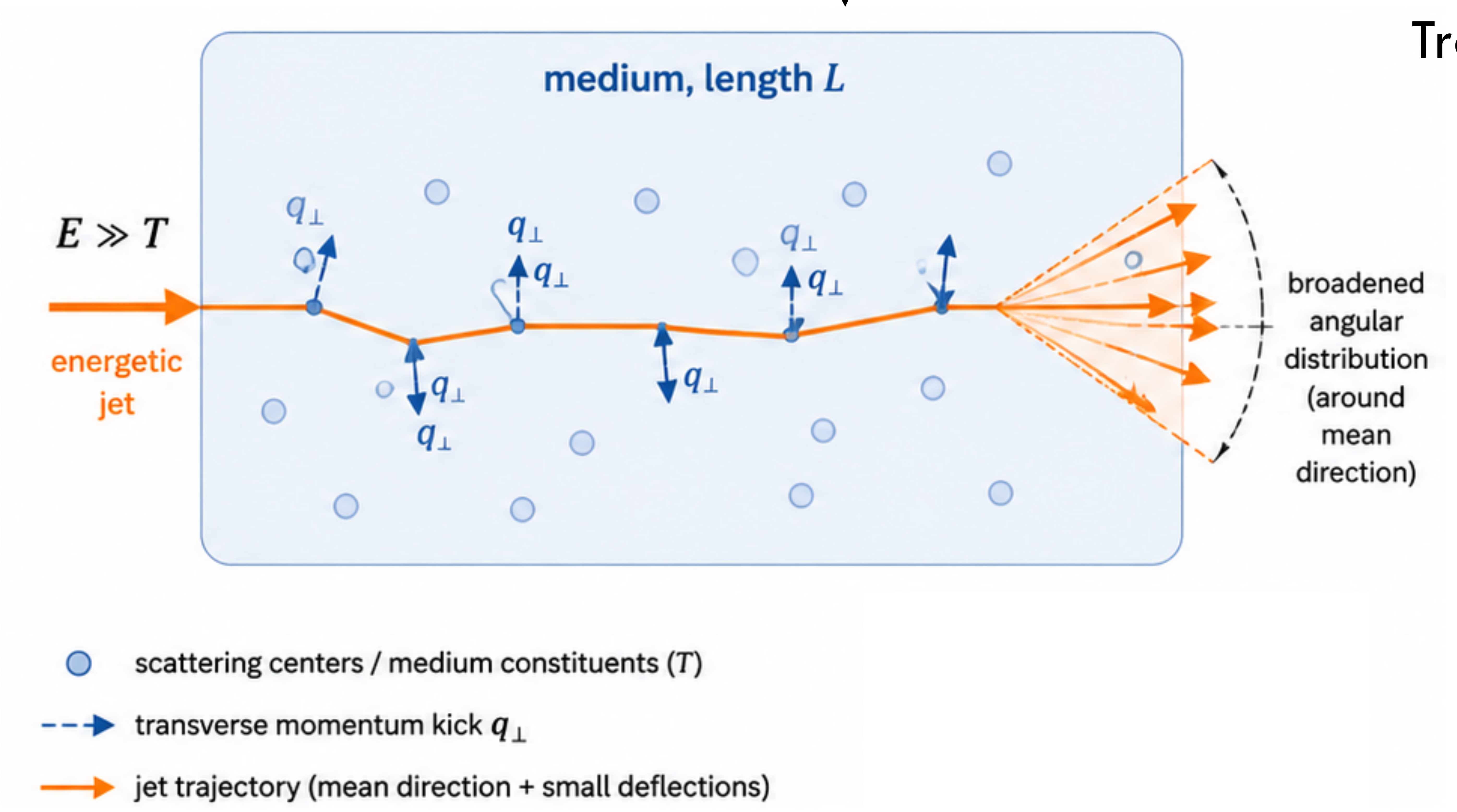


Jet Momentum Broadening

Diffusion in momentum space

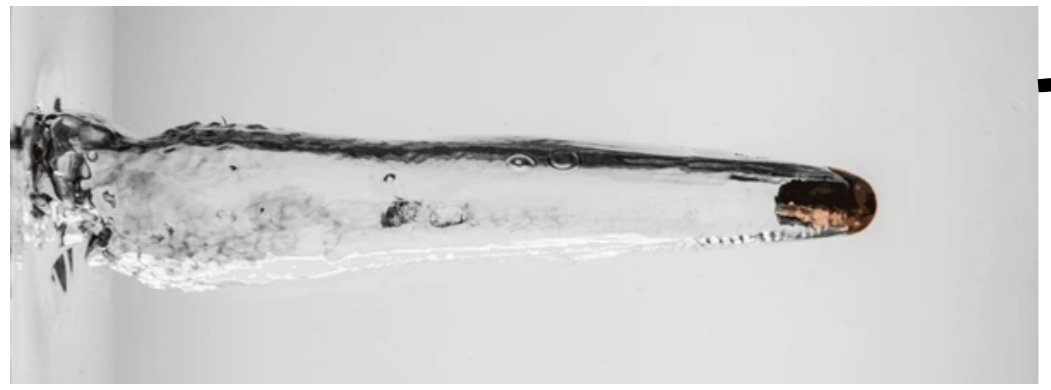
Transverse momentum $\mathbf{k}_\perp$ performs a random walk

$$\langle \mathbf{k}_\perp^2 \rangle \sim \hat{q}L$$

$\hat{q}$: transport coefficient of the jet
(describes the momentum broadening)

BDMPS: Nucl. Phys. B484, 265 (1997)

What is a jet?



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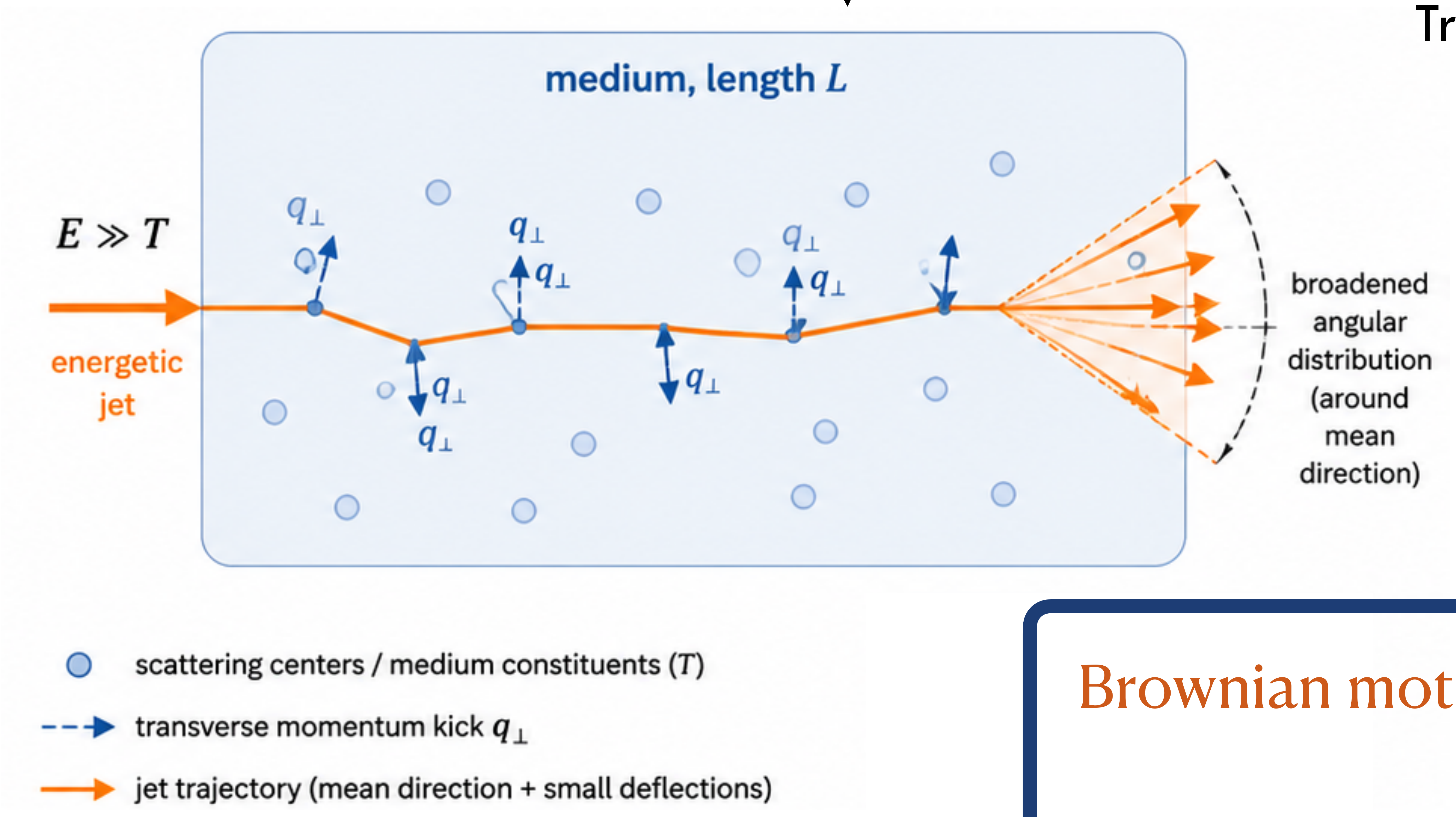

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Brownian motion ⟷ Jet momentum broadening

Same random walk idea:
Many soft, uncorrelated kicks lead to diffusion

What is a jet?

Brownian motion $\longleftrightarrow$ Jet momentum broadening

Same random walk idea:
Many soft, uncorrelated kicks lead to diffusion

$$\langle k_\perp^2 \rangle = \hat{q}\, L \quad \Leftrightarrow \quad \langle\, |\, \mathbf{x}(t) - \mathbf{x}(0)\, |^2 \,\rangle = 2Dt$$

But Einstein's diffusion coefficient D assumes an isotropic bath in equilibrium

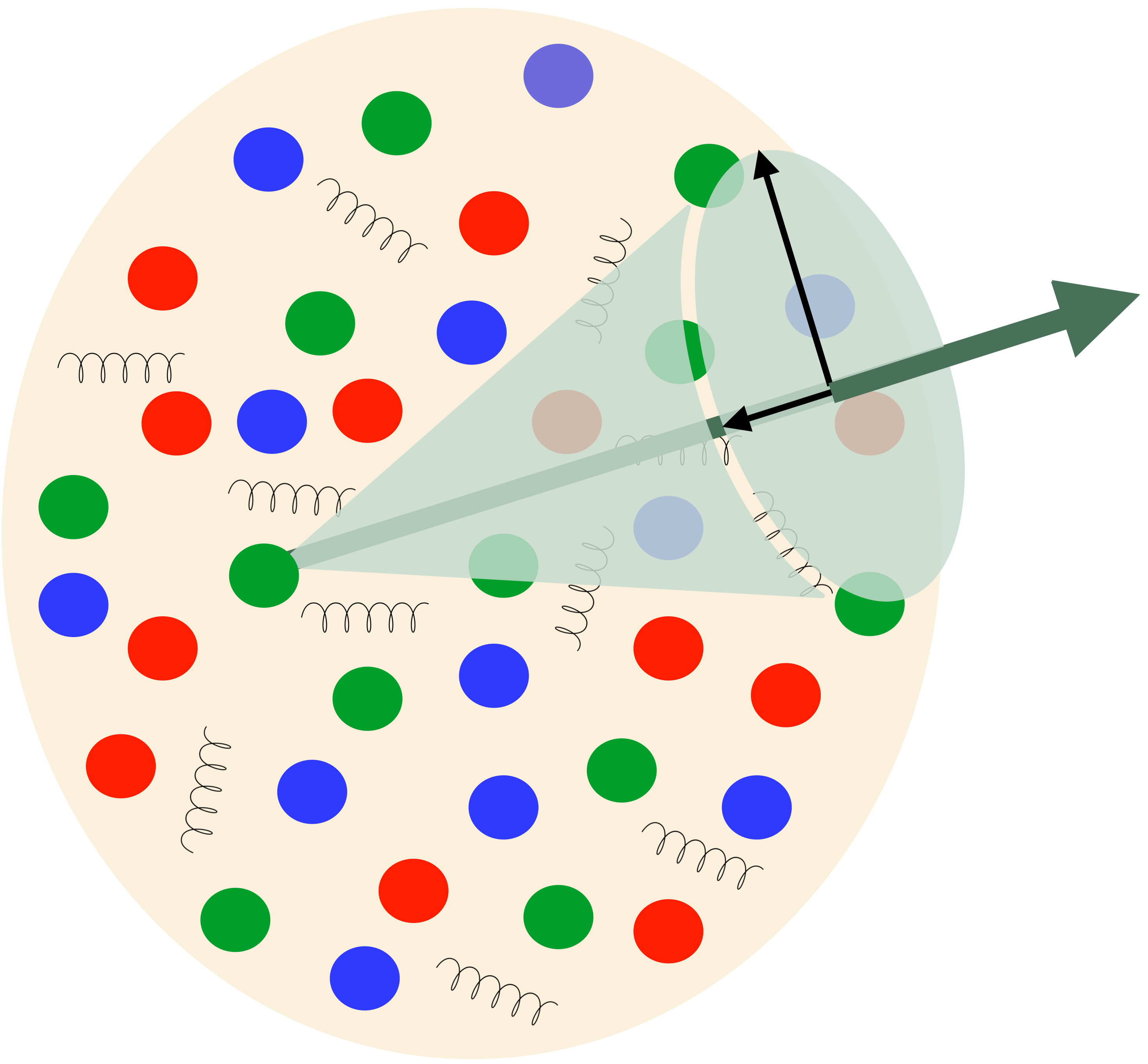
The QGP is neither: it flows, it is viscous, and the jet defines a direction....

See for example: PRD 110, 034019 (2024), arXiv:2312.00447

Effective Kinetic Theory in QCD

Introduction

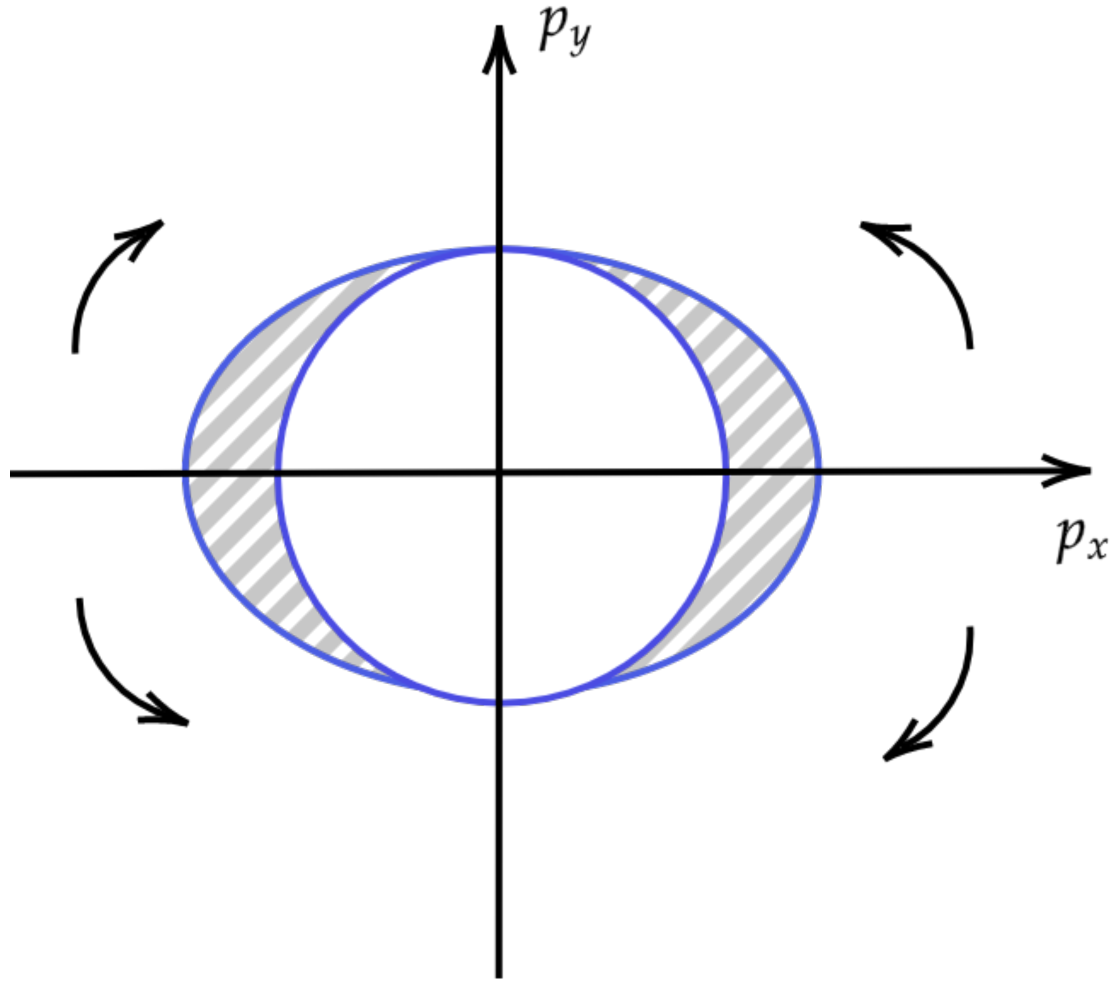
Kinetic Theory Picture



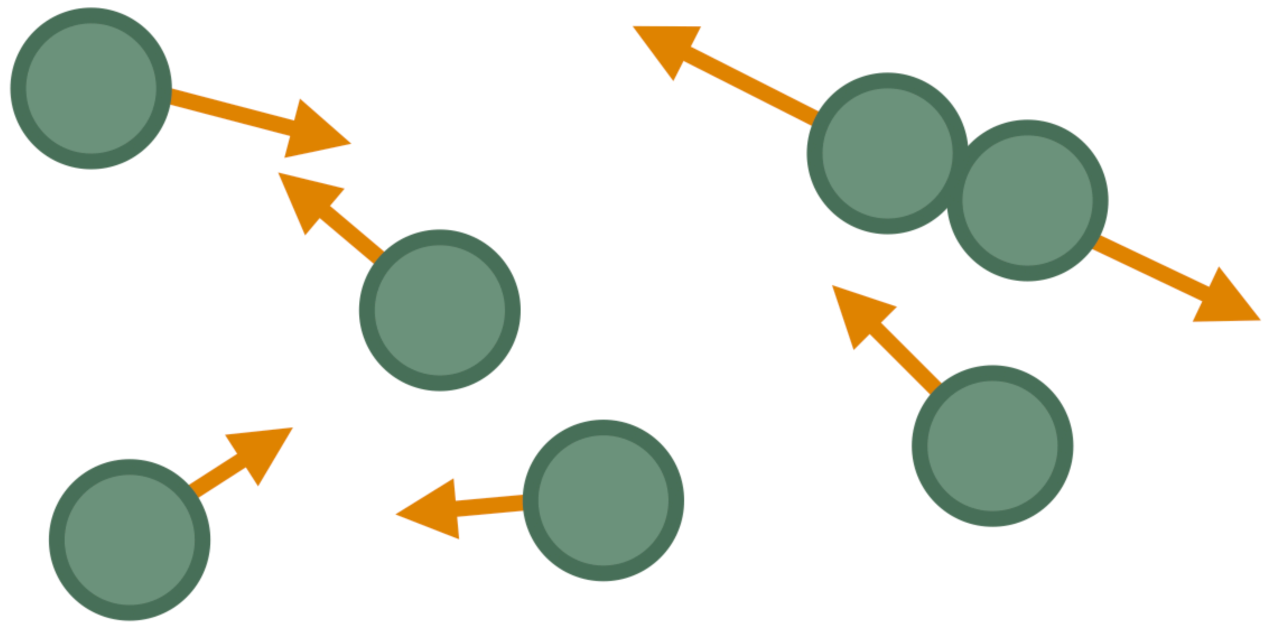
The dynamics of the system are determined from a Boltzmann equation:

$$\left[\frac{\partial}{\partial t} + \vec{v}_p\cdot\frac{\partial}{\partial \vec{x}} + \vec{F}_{\text{ext}}\cdot\frac{\partial}{\partial \vec{p}}\right] f^a(\vec{p},\vec{x},t) = -\mathscr{C}^a[f]$$

Describes propagation



Scatterings

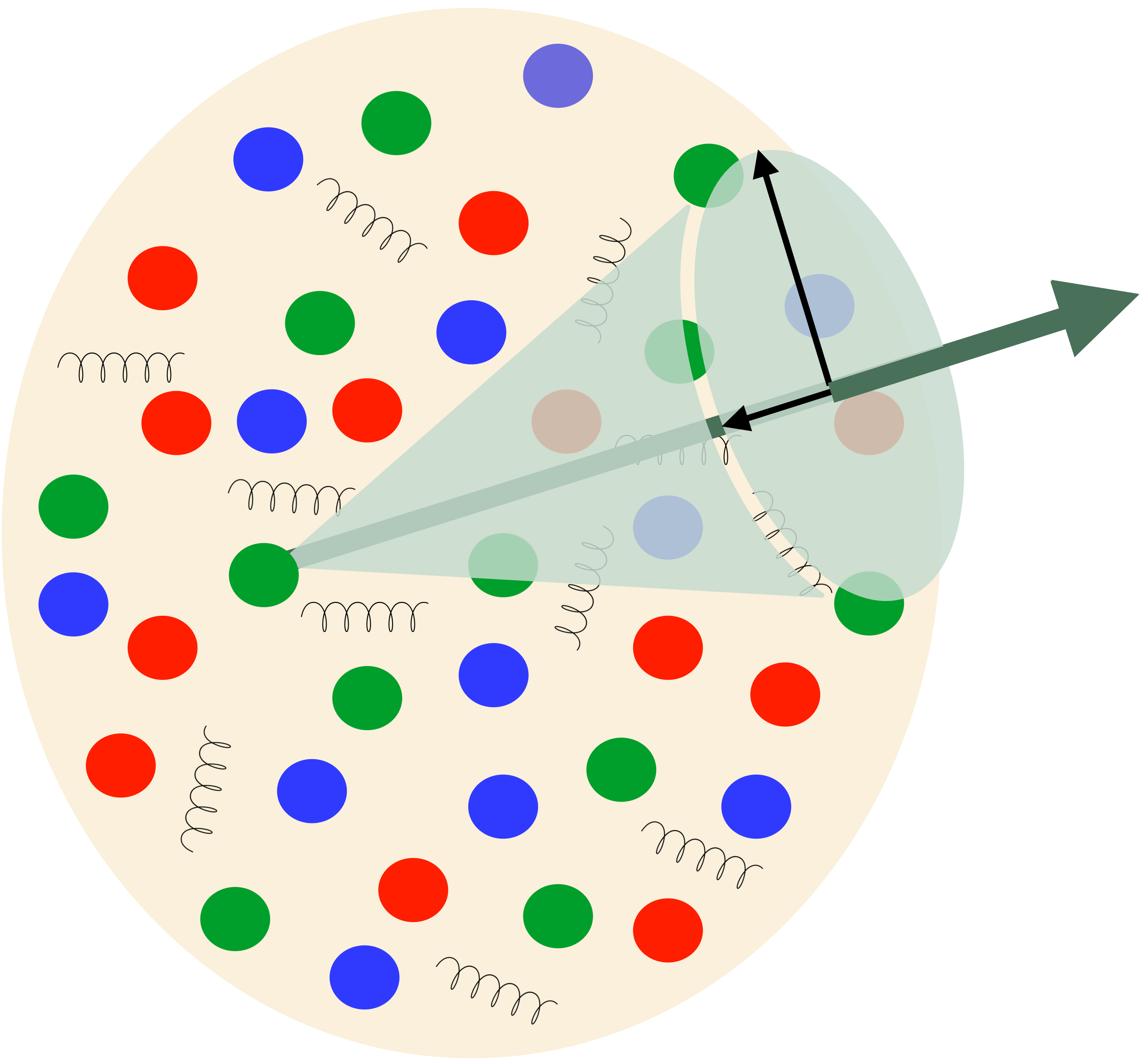


This system has a non-equilibrium distribution: $f^a = f^a_0 + \delta f^a$

Chapman, Sydney et al. "The mathematical theory of non-uniform gases." (1939).
S. Jeon, Phys. Rev. D52 (1995) 3591–3642.
Arnold, Yaffe Phys.Rev.D 57 (1998) 1178-1192
Arnold, Moore & Yaffe, JHEP 0305 (2003) 051.

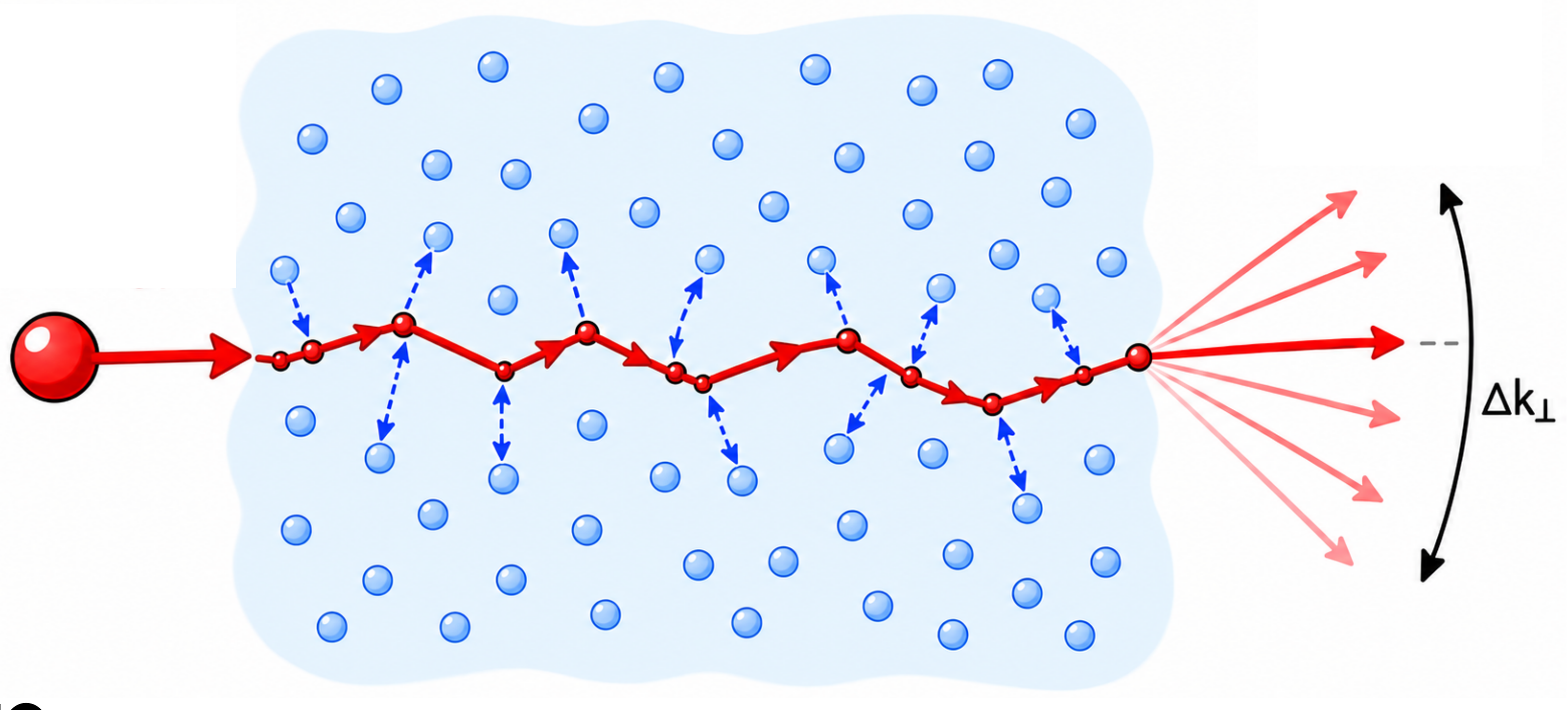
Introduction

Kinetic Theory Picture



Jet momentum broadening is generated by the cumulative effect of microscopic scatterings with medium constituents

$$\hat{q} = \int d^2\vec{q}_\perp \, \vec{q}_\perp^{\,2} \frac{d\Gamma_{a,\mathrm{el}}}{d^2\vec{q}_\perp},$$



AI generated

with the elastic scattering rate related directly to:

$$\Gamma_{a,\mathrm{el}}[\mathbf{k}] = \frac{1}{2}\int dP\, dK'\, dP'\, \mathscr{W}_a\, f_{\mathbf{k}'} \tilde{f}_{\mathbf{p}} \tilde{f}_{\mathbf{p}'}$$

where $\mathscr{W}_a = \mathscr{W}_{pp' \leftrightarrow kk'}$ denotes the relevant scattering kernel (rate) for the process.

These two define the scalar:

$$\hat{q}[\mathbf{k}] = \frac{1}{2|\mathbf{k}|}\int dP\, dK'\, dP' \vec{q}_\perp \vec{q}_\perp \mathscr{W}_a f_{\mathbf{k}'} \tilde{f}_{\mathbf{p}} \tilde{f}_{\mathbf{p}'}$$

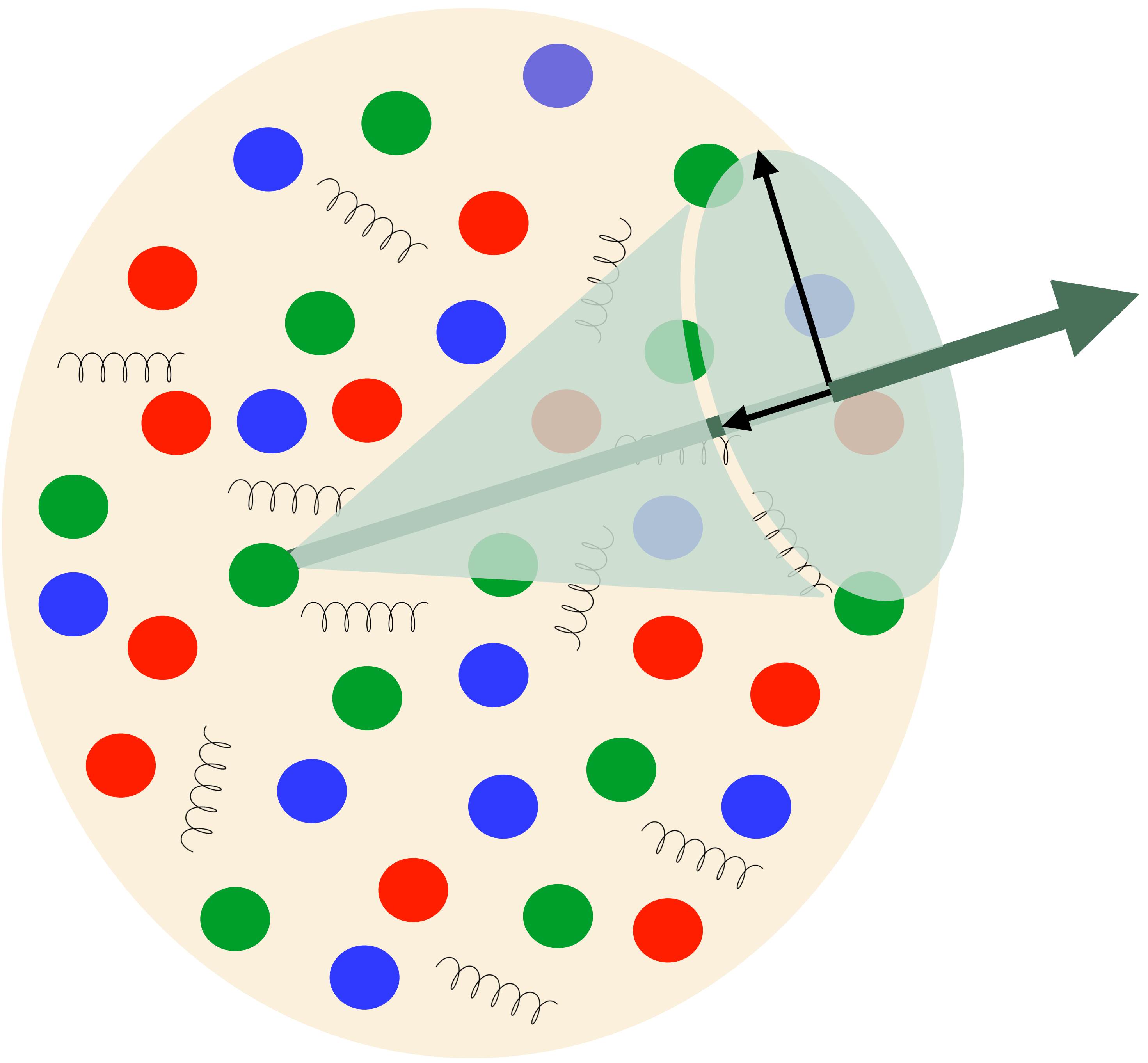
Introduction

Kinetic Theory Picture

The jet quenching parameter $\hat{q}$ is determined by the elastic scattering rate:

$$\hat{q}^{ij} = \frac{(2\pi)^4}{4p\,\nu_a} \sum_{bcd} \int d\Gamma_{kp'k'} \delta^{(4)}(P + K - P' - K') \left| \mathcal{M}^{ab}_{cd} \right|^2 q^i q^j f_b(X,K) \tilde{f}_c(X,P') \tilde{f}_d(X,K')$$

arXiv:2303.12520 [hep-ph]
arXiv:2603.00844 [hep-ph]



$\hat{q}$ is intrinsically tied to a specific frame,

and to a specific choice of transverse plane:

Isotropic medium →

Azimuthal symmetry around the beam axis:

$$\hat{q}^{12} = \hat{q}^{23} = 0$$

$$\hat{q} = \hat{q}_{11} + \hat{q}_{22}$$

$$\hat{q}^{ij} = \begin{pmatrix} q^{11} & 0 & 0 \\ 0 & q^{22} & 0 \\ 0 & 0 & q^{33} \end{pmatrix}$$

Spherical symmetry of the phase-space:

$$\hat{q}^{11} = \hat{q}^{22}$$

$$\hat{q}^{12} = \hat{q}^{13} = \hat{q}^{23} = 0$$

How does jet momentum broadening couple to the dissipative currents of viscous relativistic hydrodynamics as the QGP evolves?

Viscous relativistic hydrodynamics

Conservation laws:

$$T^{\mu\nu} \rightarrow \nabla_\mu T^{\mu\nu} = 0$$
$$J^{\mu}_{(B,S,Q,...)} \rightarrow \nabla_\mu J^{\mu} = 0$$

Large separation of scales:

- Fluid cells are much larger than mean free path

$$\frac{\ell_{\rm mfp}}{L} \ll 1$$

Many degrees of freedom (dof):

- Conserved quantities described by: T , $\vec{u}$, $\mu_{B,S,Q}$
- Large particle numbers

Equilibrium

+

small deviations

Diffusion matrix and gradients of μ/T

$$\kappa = \begin{bmatrix} \kappa_{BB} & \kappa_{BS} & \kappa_{BQ} \\ \kappa_{SB} & \kappa_{SS} & \kappa_{SQ} \\ \kappa_{QB} & \kappa_{QS} & \kappa_{QQ} \end{bmatrix}$$

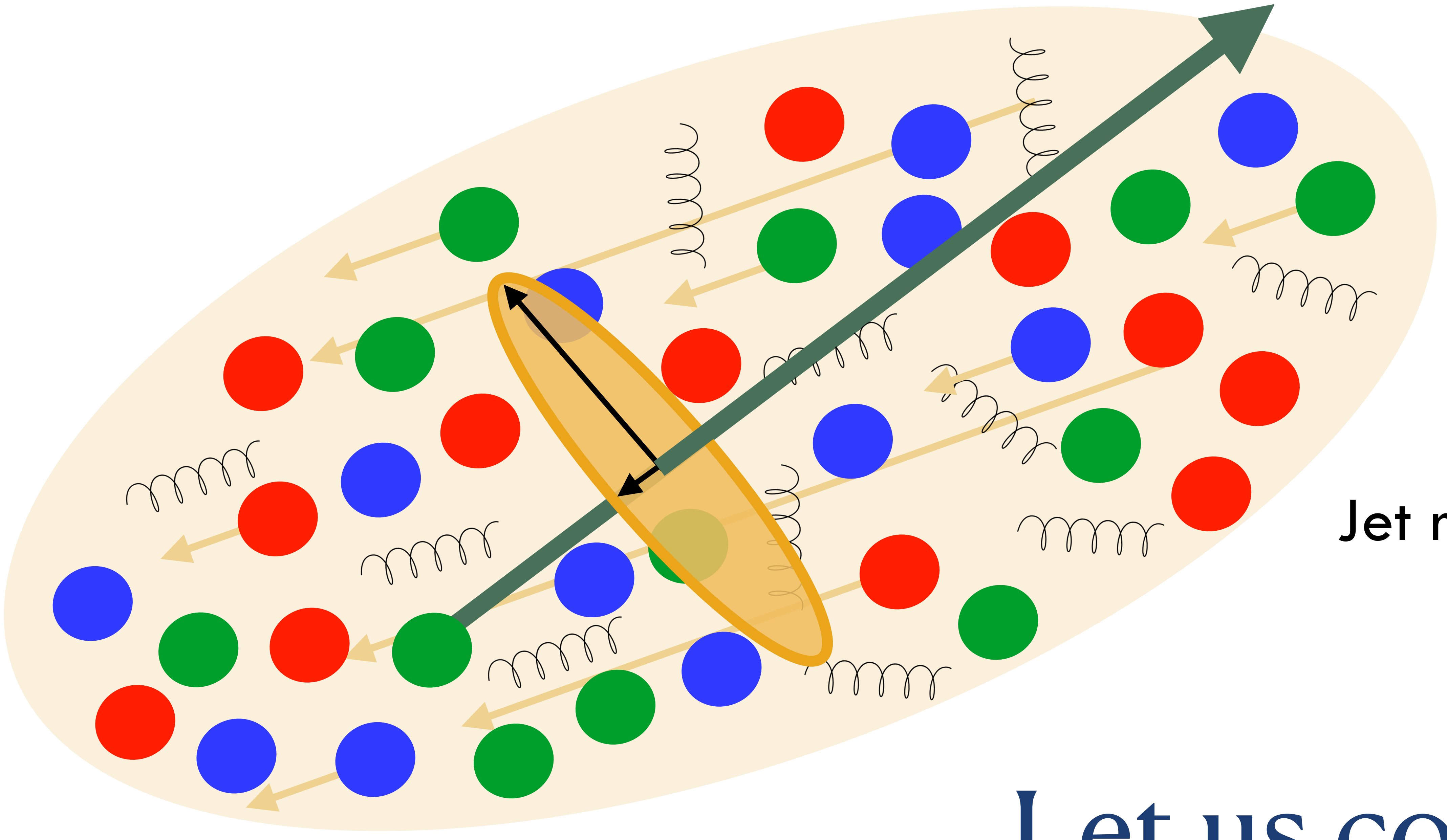
Bulk Pressure $\Pi = Tr\left[T^{\mu\nu} - T_0^{\mu\nu}\right]$

Israel–Stewart (1979) ·
DNMR (2012, arXiv:1202.4551)

Shear stress tensor $\pi^{\mu\nu} = T^{\mu\nu} - T_0^{\mu\nu} - \Pi$

The jet quenching parameter $\hat{q}$ is the transport coefficient of a jet in a viscous fluid.

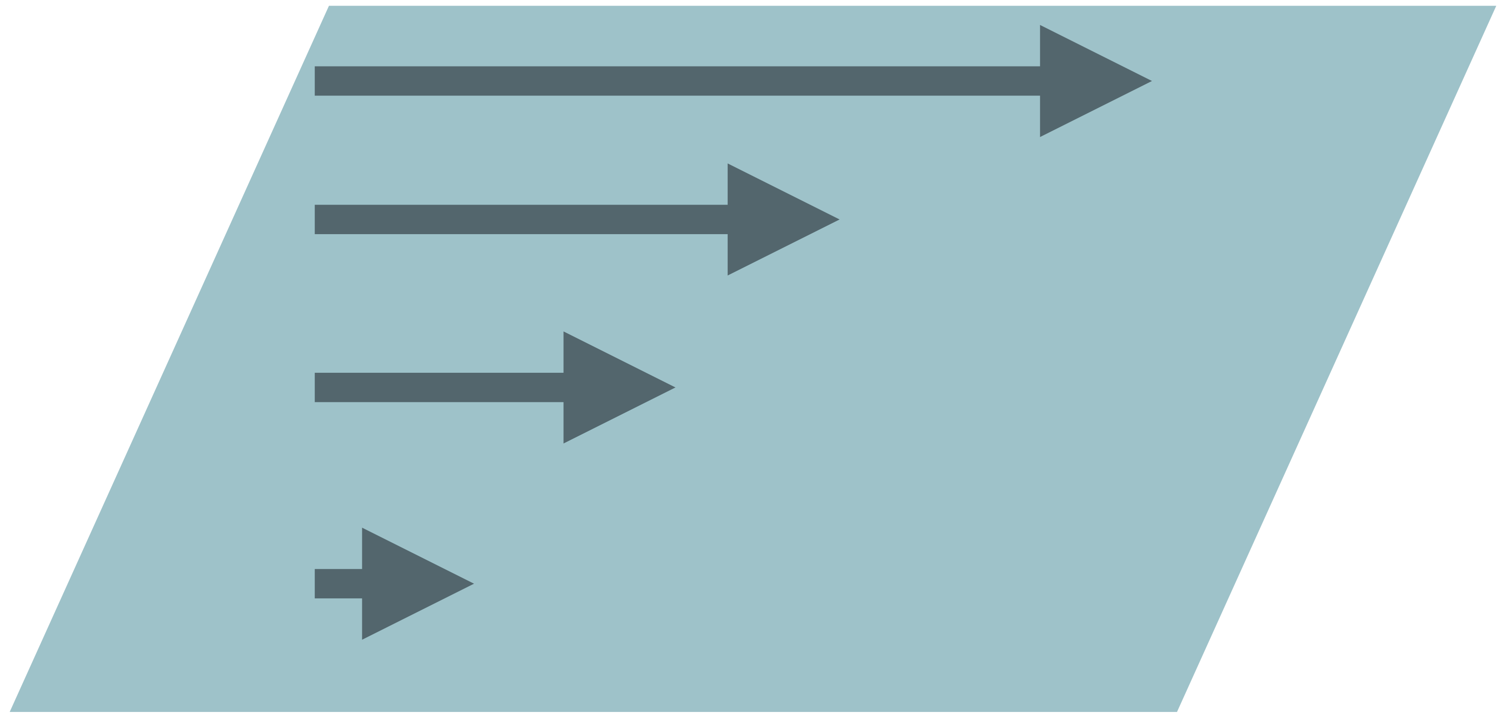
Jet momentum broadening out of equilibrium



The broadening process is intrinsically direction dependent.
No symmetry reason for momentum broadening to be a single scalar.

Jet momentum broadening becomes: $\hat{q}^{ij} = \begin{pmatrix} q^{11} & q^{12} & q^{13} \\ q^{12} & q^{22} & q^{23} \\ q^{13} & q^{23} & q^{33} \end{pmatrix}$ $\hat{q}^{12} \neq \hat{q}^{13} \neq \hat{q}^{23} \neq 0,$ $\hat{q}^{11} \neq \hat{q}^{22}.$

Let us consider a fluid with a shear flow!



General tensorial structure of $\delta\hat{q}^{ij}$

What are the symmetry constraints that this tensor has?

- $\delta\hat{q}$ is constrained by rotational symmetry in the LRF
- Symmetry under $i \leftrightarrow j$
- Linearity in the shear-stress tensor

arXiv:1401.3751 [hep-ph]
arXiv:2312.00447 [hep-ph]

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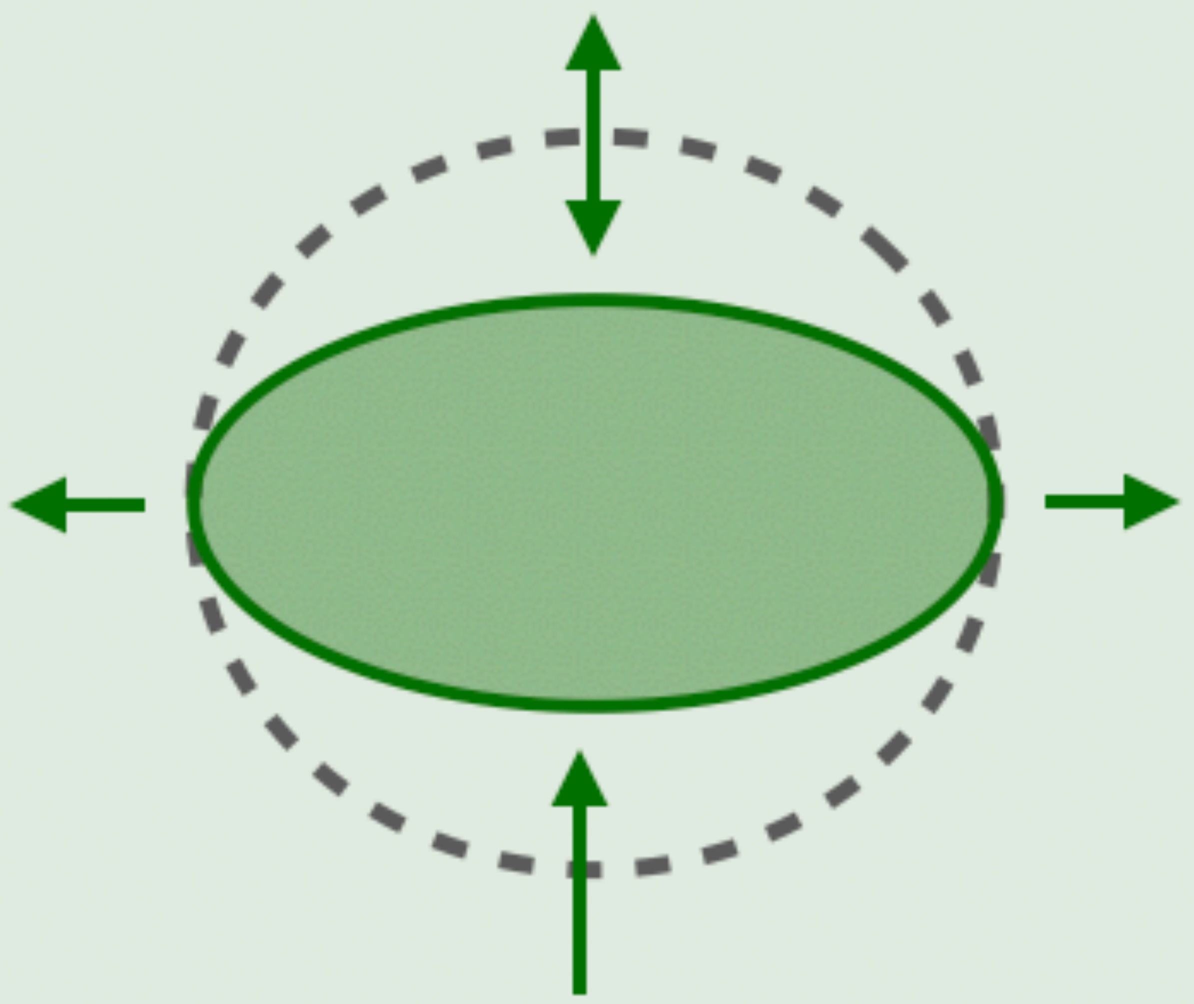
Let us begin with the simplest possible tensor structure, and write something linear in π^{ij}:

$$\delta\hat{q}^{ij} = \boxed{\alpha\, \pi^{ij}}$$

This is not enough, our system has a preferred direction $\hat{n}$, the jet direction!

Direct shear contribution

$\alpha\, \pi^{ij}$



Pure shear (traceless): Stretches in one direction and squeezes in the orthogonal with no overall size change

arXiv:1401.3751 [hep-ph]
arXiv:2312.00447 [hep-ph]

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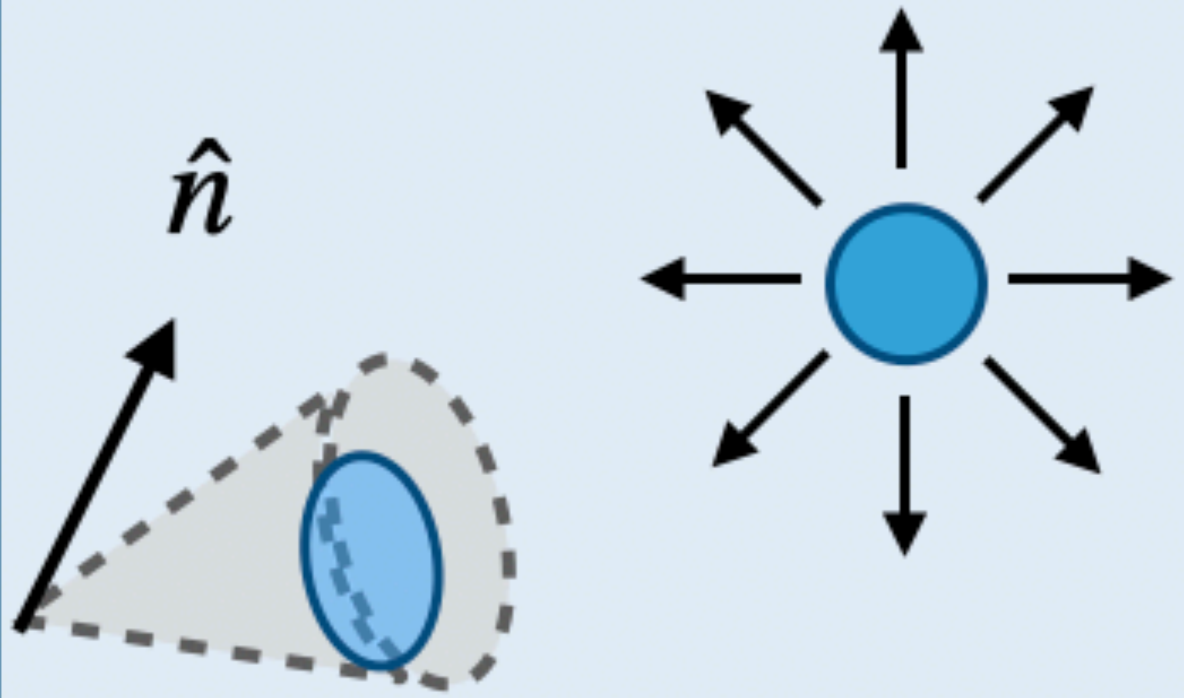
Let us now use the available tensors $\delta^{ij}$and the jet direction $\hat{n}^i$:

$$\delta\hat{q}^{ij} = \boxed{\alpha\,\pi^{ij}} + \boxed{\beta\,\delta^{ij}\,\hat{n}^a\hat{n}^b\pi_{ab}}$$

We have the direct shear + the isotropic contribution from the projection! Still one contribution missing!

Isotropic contribution from projection along $\hat{n}$

$$\beta\,\delta^{ij}\,\hat{n}^a\hat{n}^b\pi_{ab}$$



Project π^{ij} along the direction $\hat{n}$ to obtain the scalars $= \hat{n}^a\hat{n}^b\pi_{ab}$, then distribute it isotropically via δ^{ij}

arXiv:1401.3751 [hep-ph]
arXiv:2312.00447 [hep-ph]

General tensorial structure of $\delta\hat{q}^{ij}$

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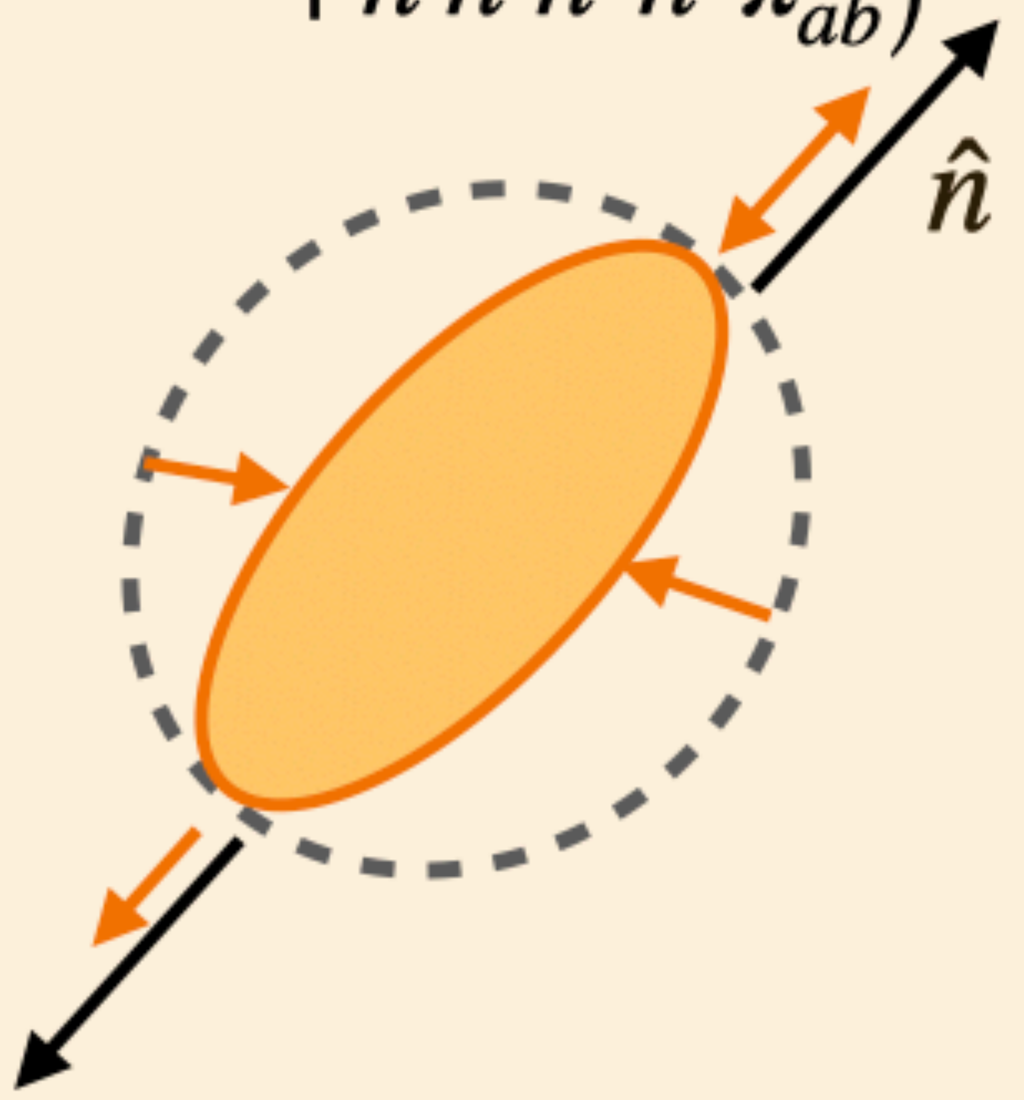
$$\boxed{+\gamma\,\left(\hat{n}^i\pi^{ja}\hat{n}_a + \hat{n}^j\pi^{ia}\hat{n}_a + \hat{n}^i\hat{n}^j\hat{n}^a\hat{n}^b\pi_{ab}\right)}$$

arXiv:2605.12791 [hep-ph]

Together, these structures exhaust the independent tensor contributions that can appear in the LRF!

Jet direction-selective

$$\gamma\,\left(\hat{n}^i\pi^{ja}\hat{n}_a + \hat{n}^j\pi^{ia}\hat{n}_a + \hat{n}^i\hat{n}^j\hat{n}^a\hat{n}^b\pi_{ab}\right)$$

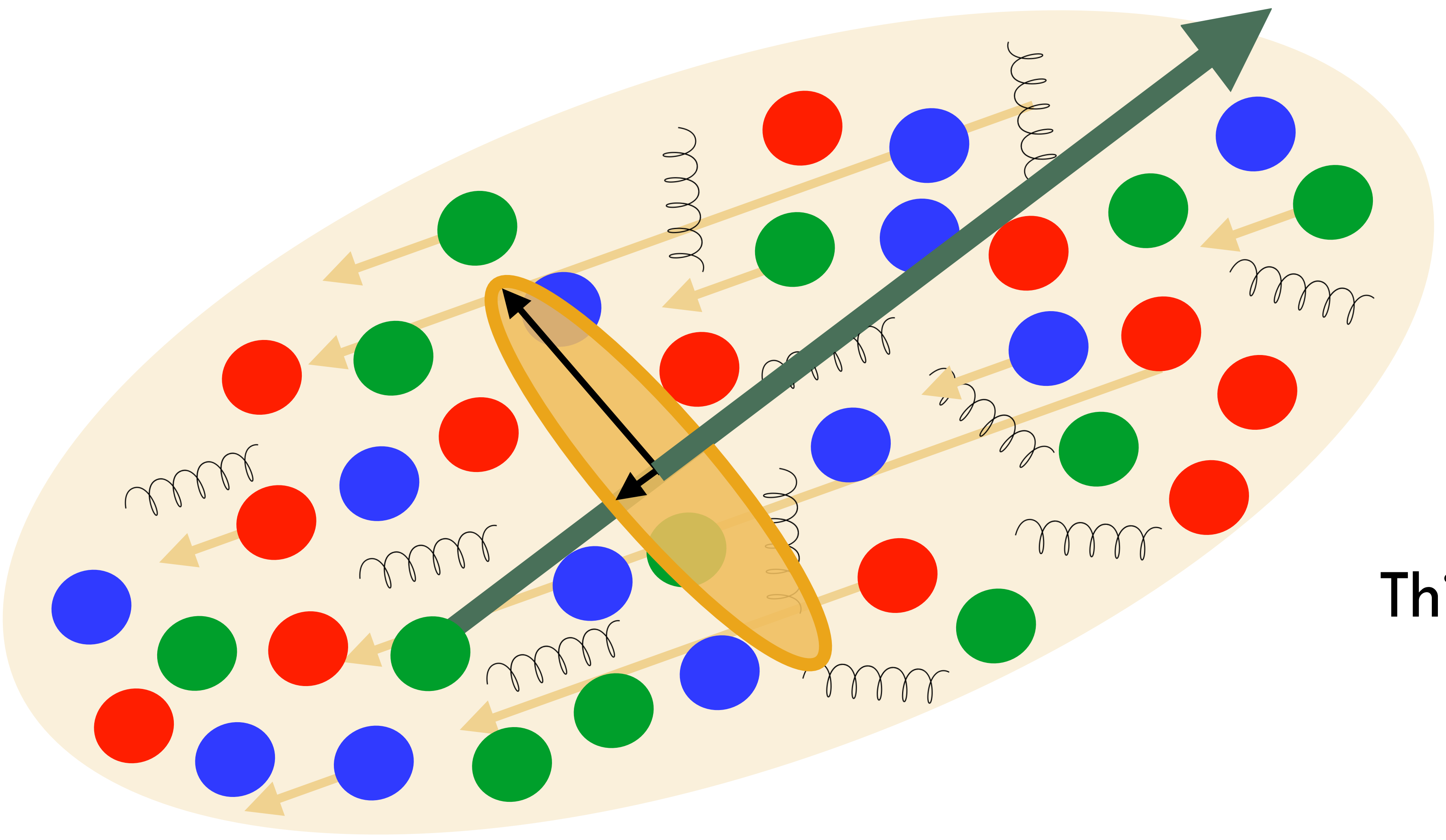


Couples shear to direction $\hat{n}$: enhances broadening along the jet direction

arXiv:1401.3751 [hep-ph]
arXiv:2312.00447 [hep-ph]

How can we obtain this tensor structure solving the Boltzmann equation?

Jet momentum broadening out of equilibrium



The broadening process is intrinsically direction dependent.
No symmetry reason for momentum broadening to be a single scalar.

This scenario has been extensively studied before, see

arXiv:2511.07519 [hep-ph].
arXiv:2312.00447 [hep-ph].
arXiv:2308.01294 [hep-ph].
arXiv:2008.04928 [hep-ph].
arXiv:2309.00683 [hep-ph].
arXiv:2502.13205 [nucl-th].
arXiv:2512.11951 [hep-ph].
arXiv:2110.03590 [hep-ph].
arXiv:2202.08847 [hep-ph].
arXiv:1203.0561 [hep-th].

arXiv:2605.12791 [hep-ph]

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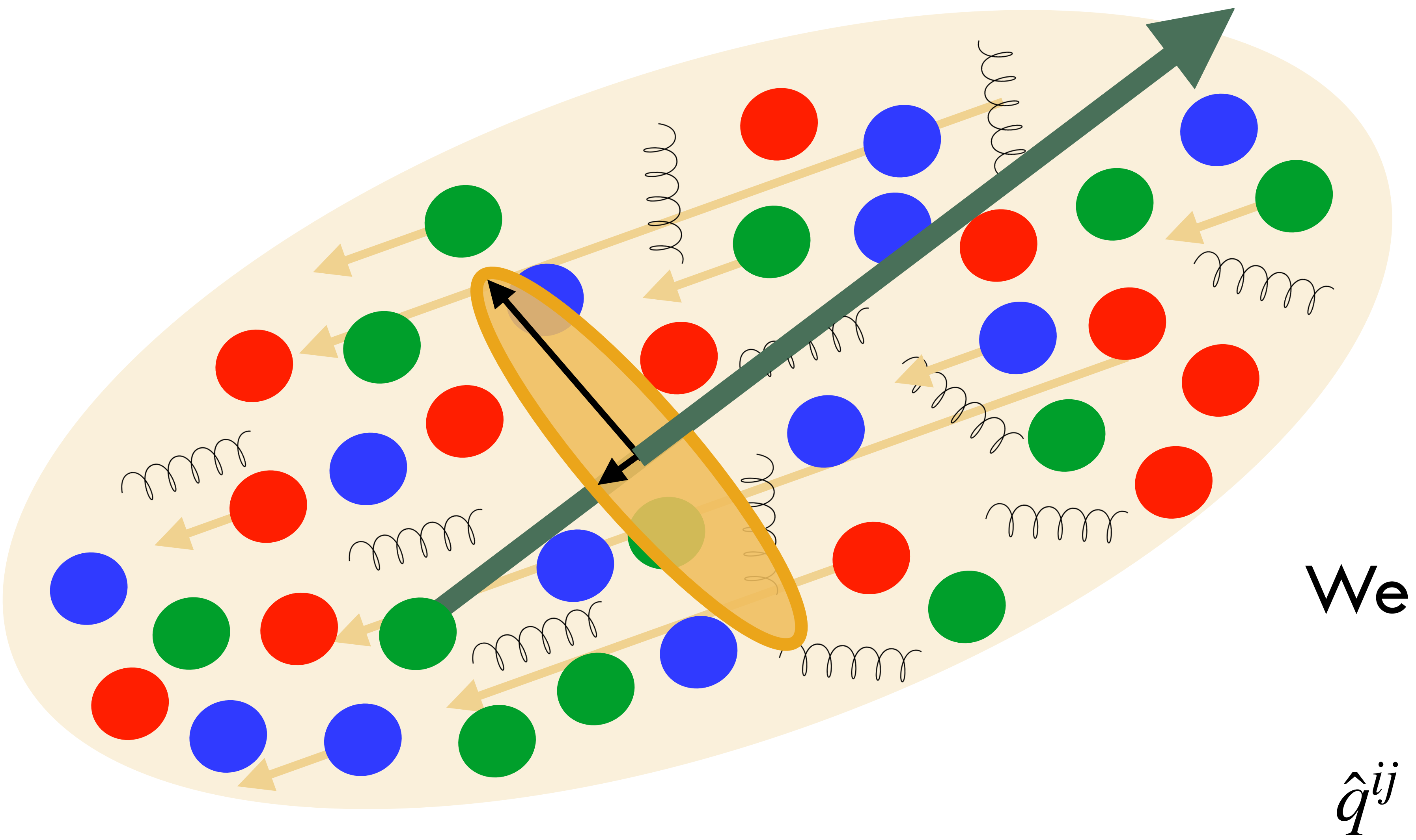

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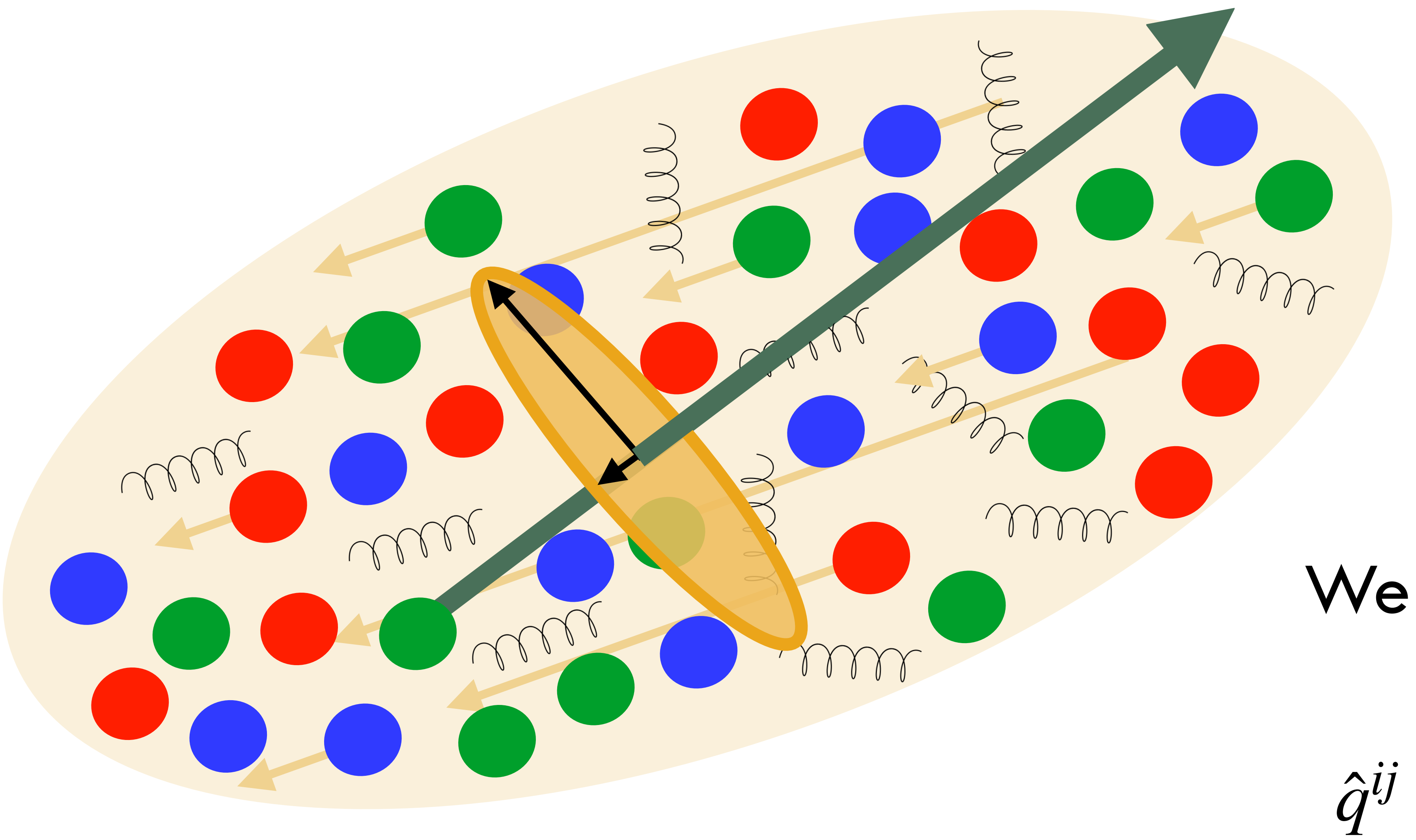
We begin with the general definition of jet momentum broadening tensor:

$$\hat{q}^{ij} = \frac{(2\pi)^4}{4p\,\nu_a} \sum_{bcd} \int d\Gamma_{kp'k'} \delta^{(4)}(P + K - P' - K') \left| \mathcal{M}^{ab}_{cd} \right|^2 q^i q^j f_b(X,K) \tilde{f}_c(X,P') \tilde{f}_d(X,K')$$

Let us go back to the general expression and introduce non-equilibrium corrections

arXiv:2605.12791 [hep-ph]

Jet momentum broadening out of equilibrium



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We incorporate the corrections from equilibrium to jet momentum broadening in a *systematic fashion*

For a generic δf: $f_b = f_{0,b} + \delta f_b = f_{0,b}\left[1 + \tilde{f}_{0,b}\phi_k\right],$ $f_{0,b} \rightarrow$ local equilibrium distribution

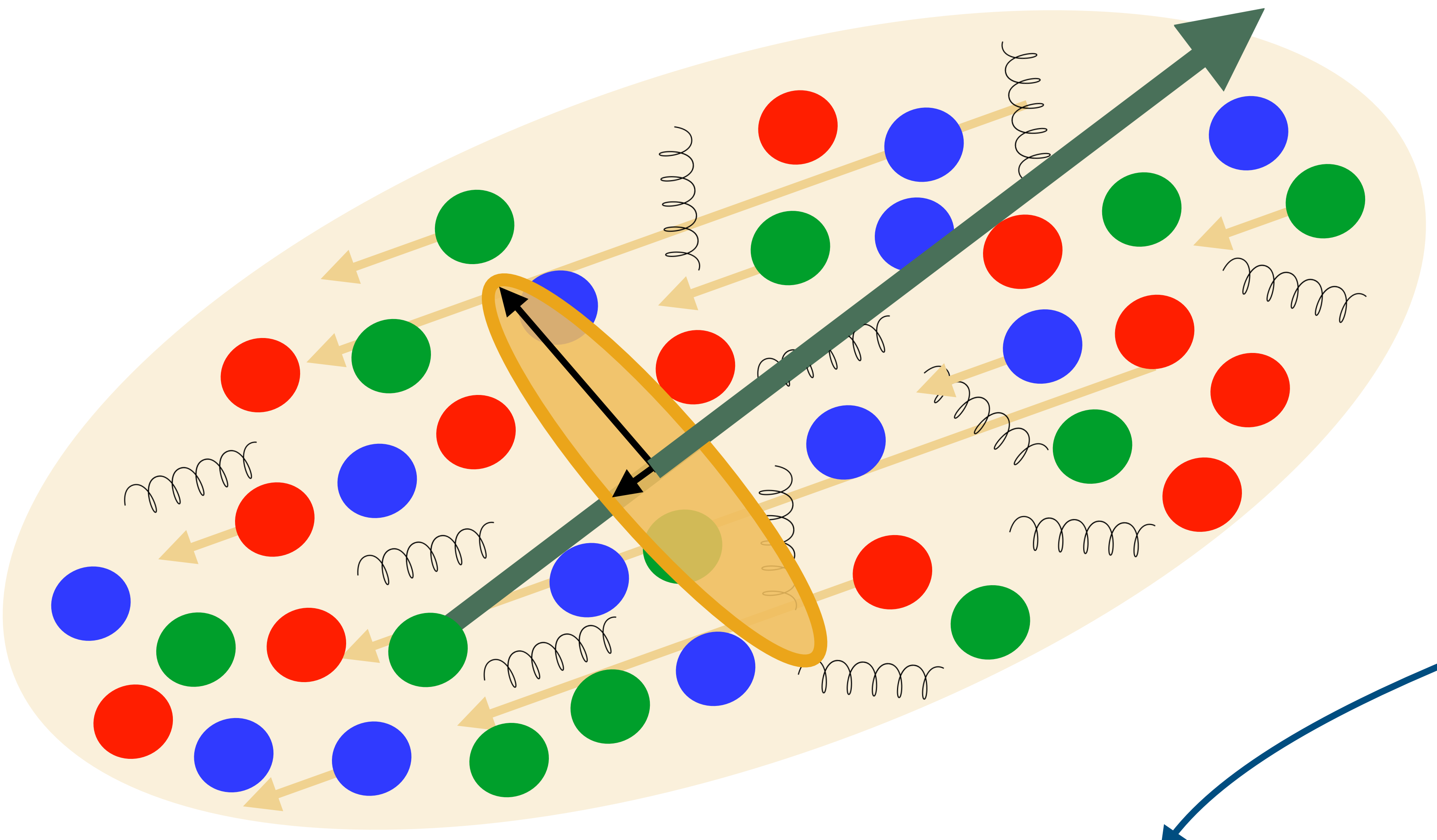
which induces the decomposition, (for a linearized collision operator)

$$\hat{q}^{ij}(P, U) = \hat{q}^{ij}_{\rm eq}(P, U) + \delta\hat{q}^{ij}(P, U)$$

U^μ is the medium's 4-velocity

arXiv:2605.12791 [hep-ph]

Jet momentum broadening out of equilibrium



$$\hat{q}^{ij}(P,U) = \boxed{\hat{q}^{ij}_{\rm eq}(P,U)} + \boxed{\delta\hat{q}^{ij}(P,U)}$$

$$\hat{q}^{ij}_{\rm eq} = \frac{1}{4p\,\nu_a}\sum_{bd}\int d\Gamma_{kp'k'}\left|\mathcal{M}^{ab}_{cd}\right|^2 q^i q^j (2\pi)^4\delta^{(4)}(P+K-P'-K') f^b_{0k}\tilde{f}^d_{0k'}$$

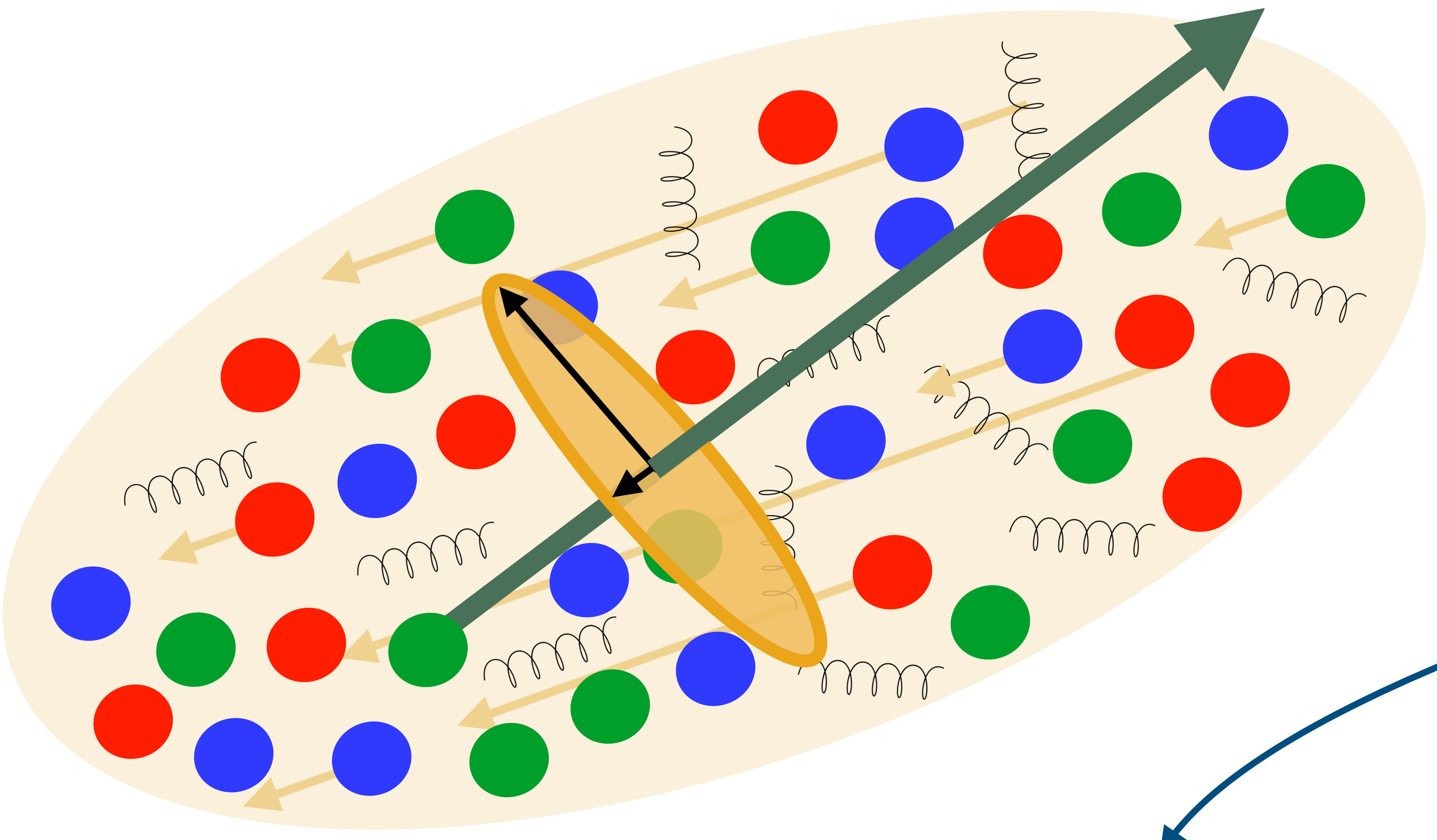
arXiv:2605.12791 [hep-ph]

$$\delta\hat{q}^{ij} = \frac{(2\pi)^4}{4p\,\nu_a}\sum_{bd}\int d\Gamma_{kp'k'}\delta^{(4)}(P+K-P'-K')\, q^i q^j \left|\mathcal{M}^{ab}_{cd}\right|^2 \left[f^b_{0k} f^d_{0k'}\tilde{f}^d_{0k'}\phi_{k'} + f^b_{0k}\tilde{f}^b_{0k}\tilde{f}^d_{0k'}\phi_k\right]$$

The scalar function $\phi_{\mathbf{k}}$ encodes all the non-equilibrium contributions coming from the medium.

The equilibrium contribution has been extensively studied before, see arXiv:0811.1603 [hep-ph], arXiv:1502.03730 [hep-ph], arXiv:2312.00447 [hep-ph], arXiv:2303.12595 [hep-ph], arXiv:1509.07773 [hep-ph].

Jet momentum broadening out of equilibrium



$$\hat{q}^{ij}(P,U) = \hat{q}^{ij}_{\rm eq}(P,U) + \delta\hat{q}^{ij}(P,U)$$

$$\hat{q}^{ij}_{\rm eq} = \frac{1}{4p\,\nu_a}\sum_{bd}\int d\Gamma_{kp'k'}\left|\mathcal{M}^{ab}_{cd}\right|^2 q^i q^j (2\pi)^4\delta^{(4)}(P+K-P'-K')f^b_{0k}\tilde{f}^d_{0k'}$$

arXiv:2605.12791 [hep-ph]

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The scalar function $\phi_{\mathbf{k}}$ encodes all the non-equilibrium contributions coming from the medium.

How do we parametrize it?

The equilibrium contribution has been extensively studied before, see arXiv:0811.1603 [hep-ph], arXiv:1502.03730 [hep-ph], arXiv:2312.00447 [hep-ph], arXiv:2303.12595 [hep-ph], arXiv:1509.07773 [hep-ph].

The Method of Moments: A General Strategy for the Boltzmann Equation

1949 — Rarefied gas dynamics

Grad expands f in tensorial Hermite polynomials → the classic 13-moment equations.

Grad, *Comm. Pure Appl. Math.* **2**, 331 (1949) · *Struchtrup & Torrilhon, Phys. Fluids* ***15***, *2668–2680 (2003)*

1965 — Plasma physics

Moment and Chapman–Enskog closures give Braginskii's transport equations and modern gyrofluid models.

Braginskii, Reviews of Plasma Physics Vol. 1 (1965) · arXiv:2201.11561

1984 — Radiative & neutrino transport

Radiation hydrodynamics used in supernova and merger simulations.

Levermore, JQSRT ***31****, 149 (1984)*

1987 — Maximum entropy / extended thermodynamics

Fix the closure by maximizing entropy at fixed moments.

Dreyer, J. Phys. A ***20****, 6505–6517 (1987) · Levermore, J. Stat. Phys.* ***83****, 1021–1065 (1996)*

1995 — Semiconductor & phonon transport

Max-entropy closure → energy-transport / hydrodynamic device models.

Anile & Muscato, Phys. Rev. B ***51****, 16728 (1995)*

2012 — Relativistic fluids / heavy-ion

Second-order moment closure → causal, stable dissipative hydrodynamics; the 14-moment truncation is the workhorse for the QGP.

Israel–Stewart (1979) · DNMR (2012, arXiv:1202.4551)

★ *our setting*

→ **2026**: this talk

The Method of Moments

W. Israel and J. M. Stewart, Phys. Lett. 58A, 213 (1976)

This provides a systematic and improvable way to solve kinetic theory

We expand ϕ_k in a ***complete and orthogonal basis*** formed by the irreducible tensors

$$1,\, K^{\langle\mu\rangle},\, K^{\langle\mu}K^{\nu\rangle},\, K^{\langle\mu}K^{\nu}K^{\lambda\rangle},\, \dots .$$

Angle brackets denote symmetric, traceless projections orthogonal to the fluid four-velocity,

$$A^{\langle\mu\rangle} = \Delta^{\mu}{}_{\nu}A^{\nu}, \qquad \Delta_{\mu\nu} = g_{\mu\nu} + U_\mu U_\nu,$$

The expansion then reads

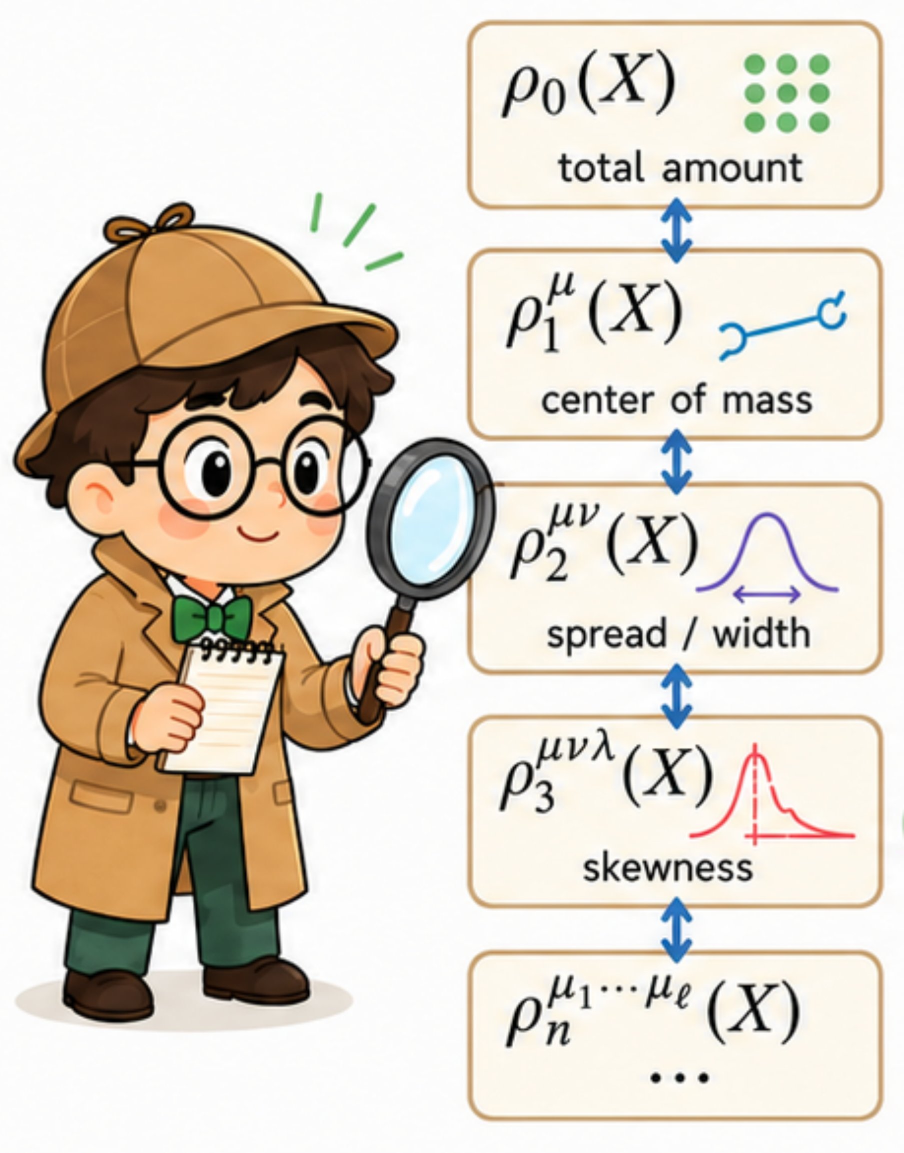
$$\phi_k = \sum_{\ell=0}^{\infty}\sum_{n=0}^{N_\ell} \mathcal{H}^{(\ell)}_{\mathbf{k}n}\rho_n^{\mu_1\cdots\mu_\ell}K_{\langle\mu_1}\cdots K_{\mu_\ell\rangle}$$

$\mathcal{H}^{(\ell)}_{\mathbf{k}n} \rightarrow$ polynomials of $E_\mathbf{k}$

$\ell \rightarrow$ rank

where the moments are defined as

$$\rho_n^{\mu_1\cdots\mu_\ell}(X) = \int \frac{d^3\mathbf{k}}{(2\pi)^3k^0}E_\mathbf{k}^n\, K^{\langle\mu_1}\cdots K^{\mu_\ell\rangle}\,\delta f(X,K),$$



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Solving this infinite hierarchy of coupled PDEs

↓

solving the Boltzmann equation

↓

used to reconstruct the full distribution function.

14-Moment Approximation

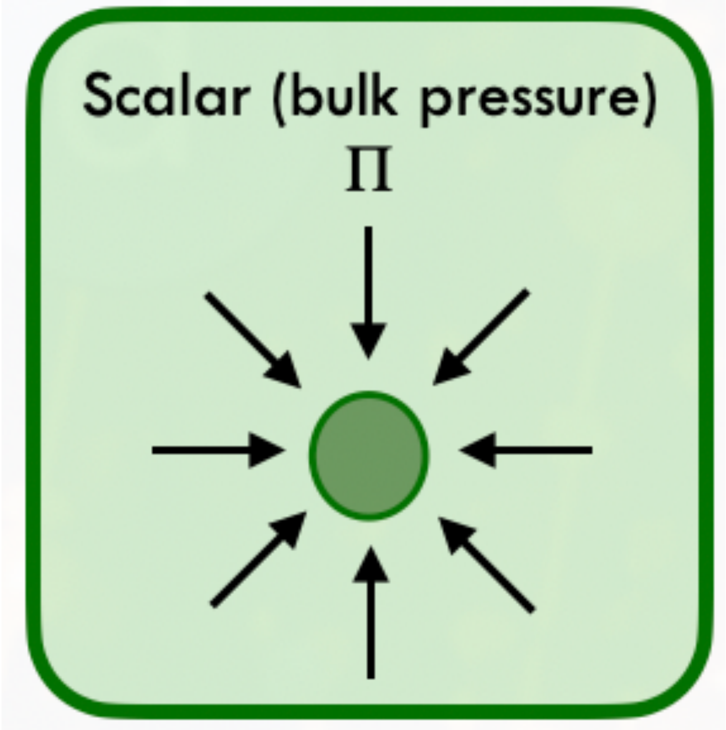
We adopt the lowest-order truncation compatible with ***causal and stable*** hydrodynamic evolution, namely the 14-moment approximation, in which the series is truncated at rank two so that moments with $\ell \geq 3$ are neglected,

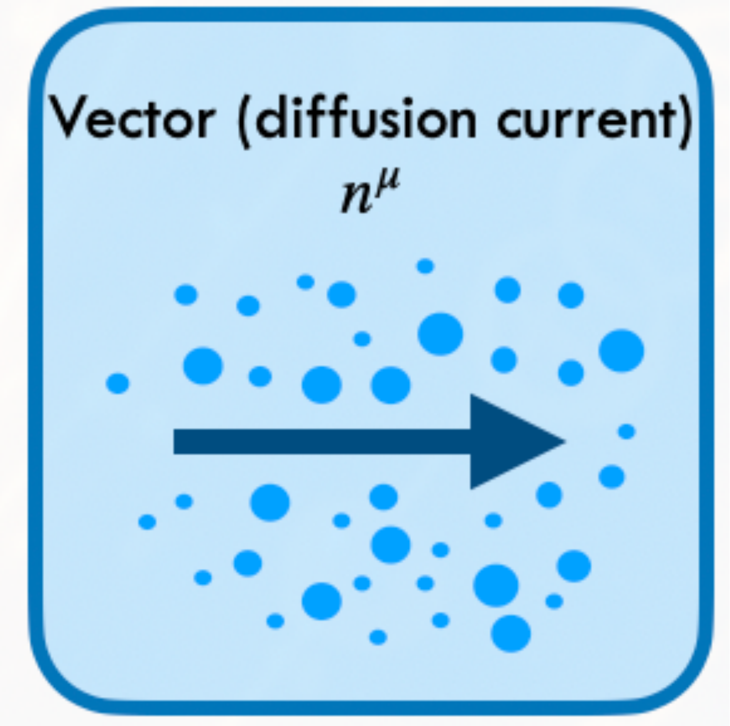
arXiv:2311.15063 [nucl-th]

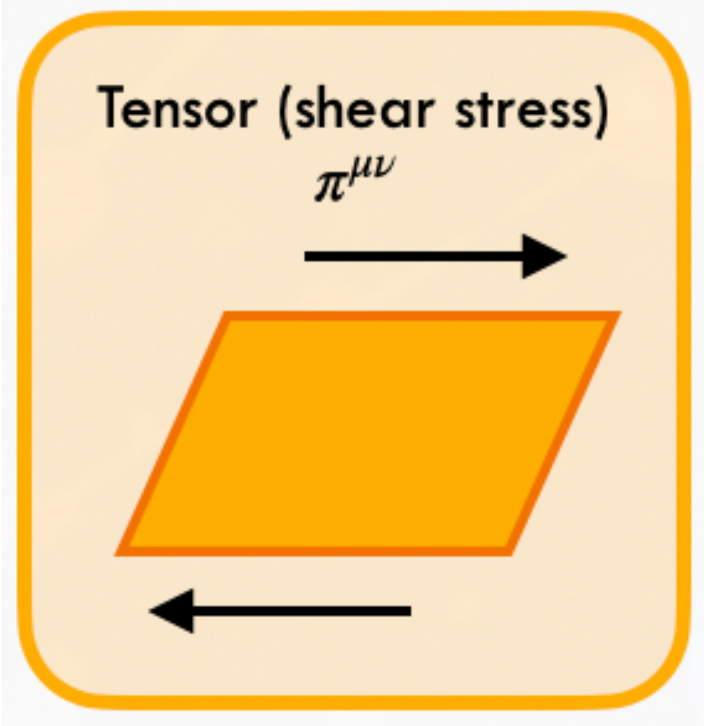
$$\rho_r^{\mu_1 \cdots \mu_\ell} = 0, \qquad \ell \geq 3.$$

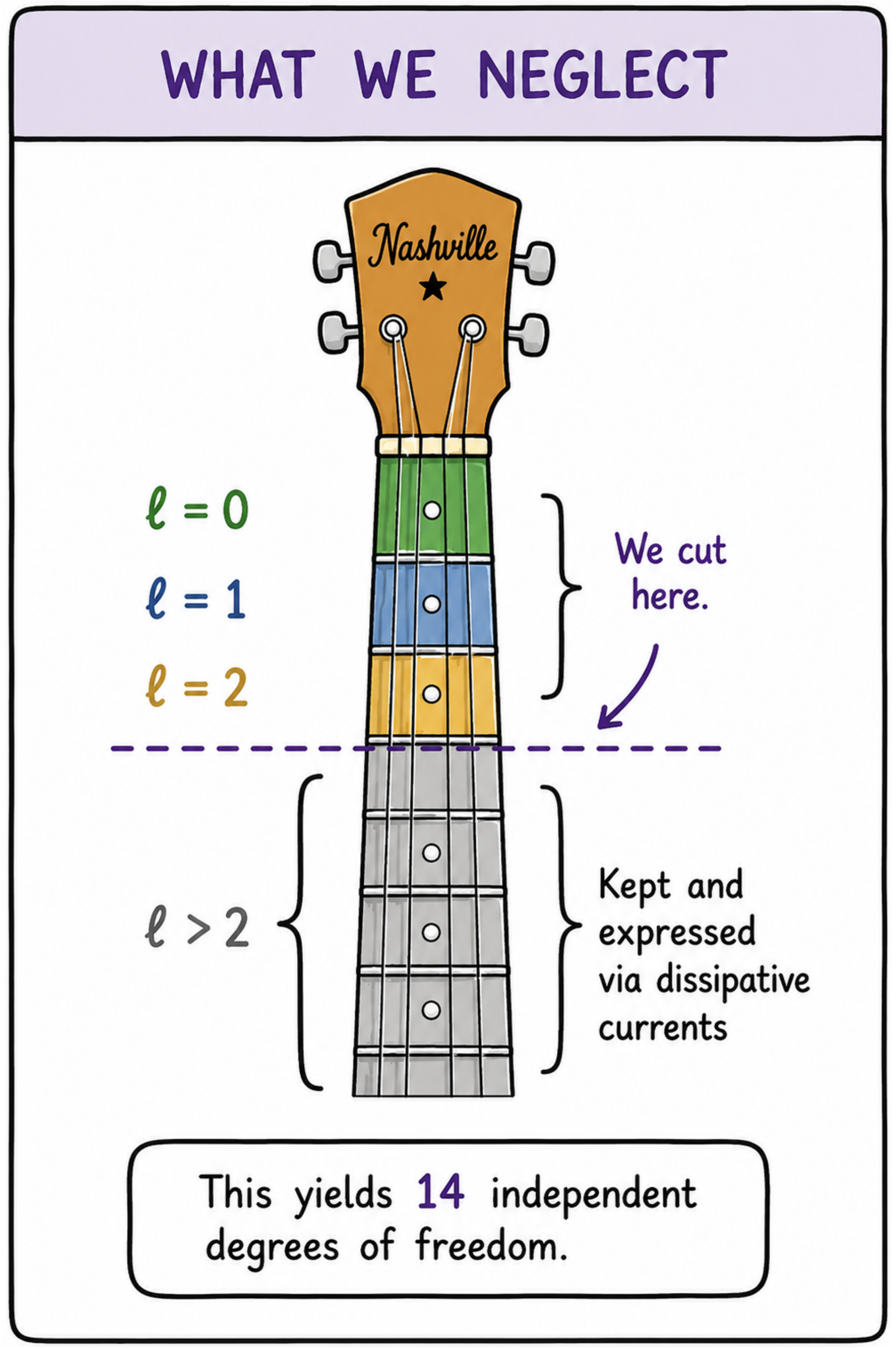
This retains only the scalar ($\ell = 0$), vector ($\ell = 1$), and tensor ($\ell = 2$) moments,

$$\phi_k = \quad [E_0 + B_0 m^2 - D_0 E_k - 4B_0 E_k^2]\Pi \qquad + \lambda_n n_\alpha K^\alpha \quad + B_2 \pi_{\alpha\beta} K^\alpha K^\beta$$









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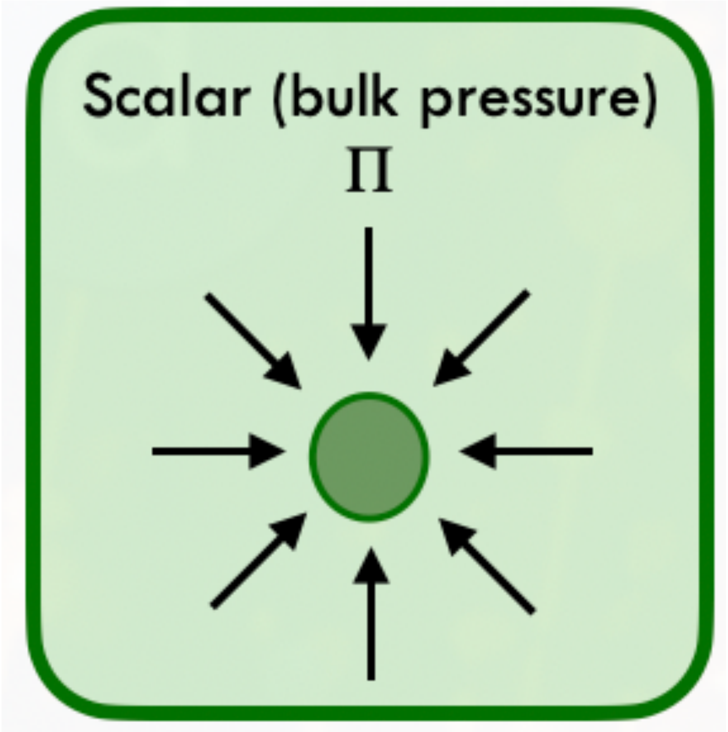
arXiv:1202.4551 [nucl-th]

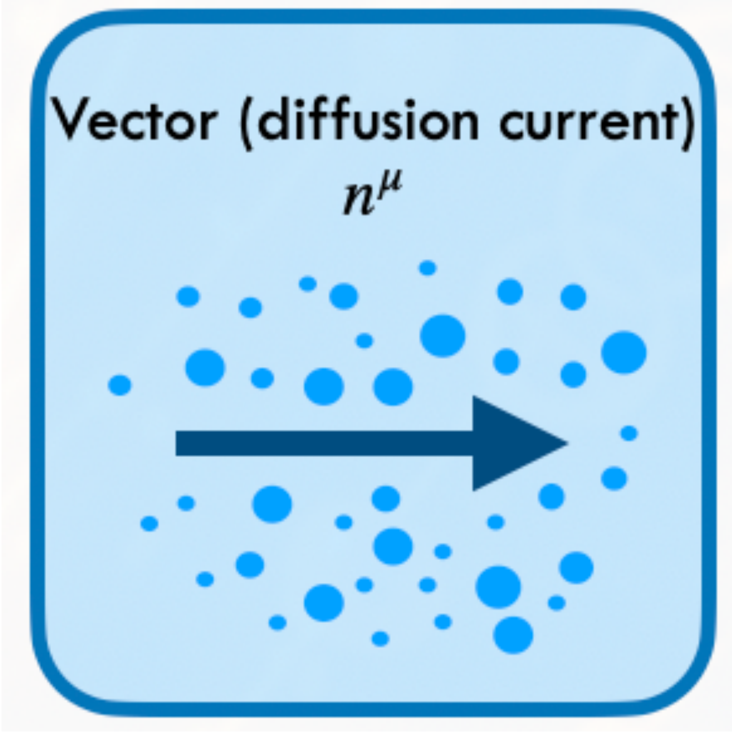
The expression can be systematically improved by including higher-rank moments!

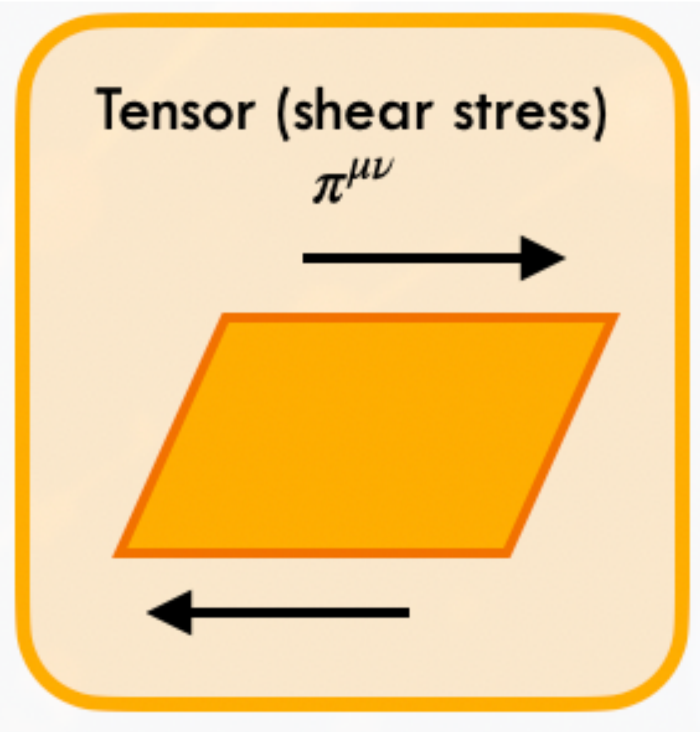
Connection with Hydrodynamics

$$\phi_k = \quad [E_0 + B_0 m^2 - D_0 E_k - 4B_0 E_k^2]\Pi \qquad +\lambda_n n_\alpha K^\alpha \qquad +B_2 \pi_{\alpha\beta} K^\alpha K^\beta$$

arXiv:2311.15063 [nucl-th]







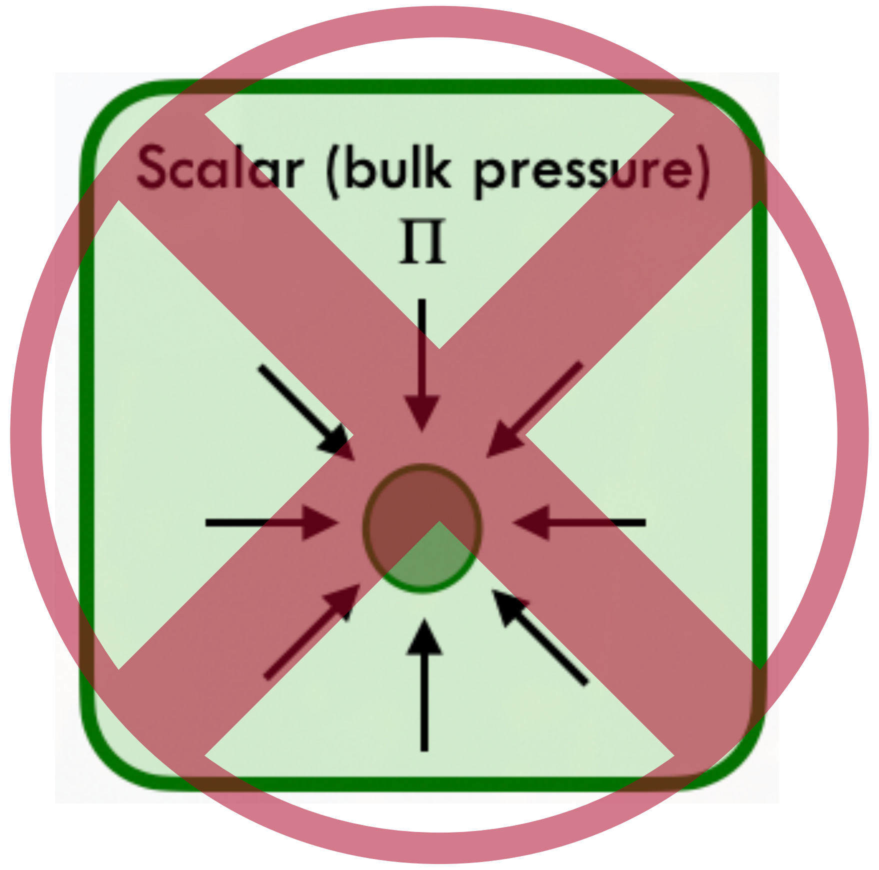
We consider massless particles at zero baryon chemical potential.

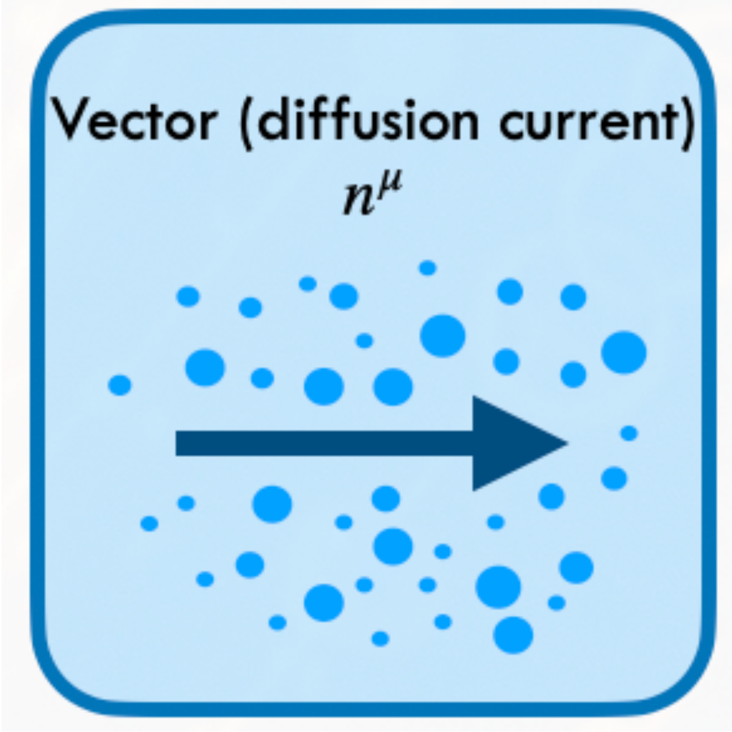
arXiv:2605.12791 [hep-ph]

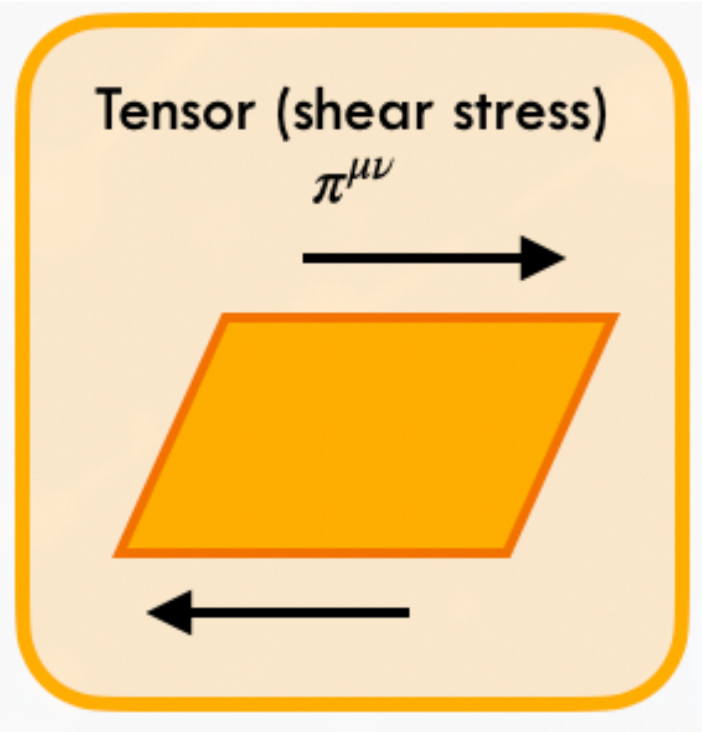
Connection with Hydrodynamics

$$\phi_k = \quad [E_0 + B_0 m^2 - D_0 E_k - 4B_0 E_k^2]\Pi \qquad + \lambda_n n_\alpha K^\alpha \quad + B_2 \pi_{\alpha\beta} K^\alpha K^\beta$$

arXiv:2311.15063 [nucl-th]







We consider massless particles at zero baryon chemical potential.

For our massless system the bulk viscous scalar $\Pi = 0$.

(we neglect running-coupling corrections that generate a nonzero bulk viscosity coefficient)

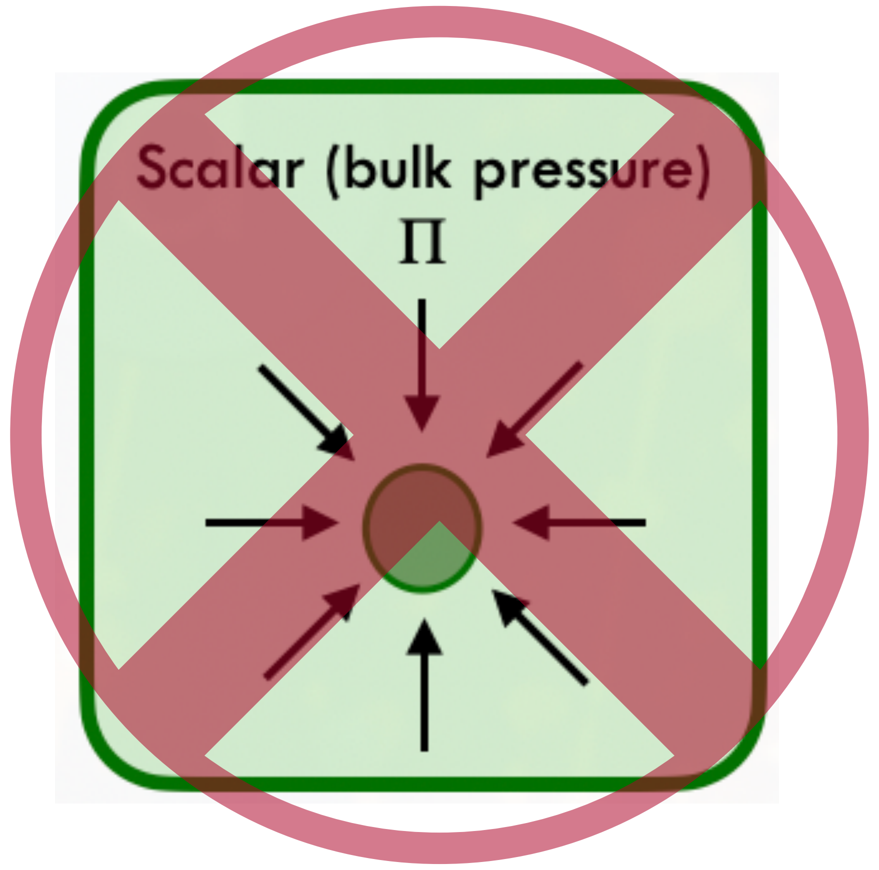
arXiv:hep-ph/0608012.

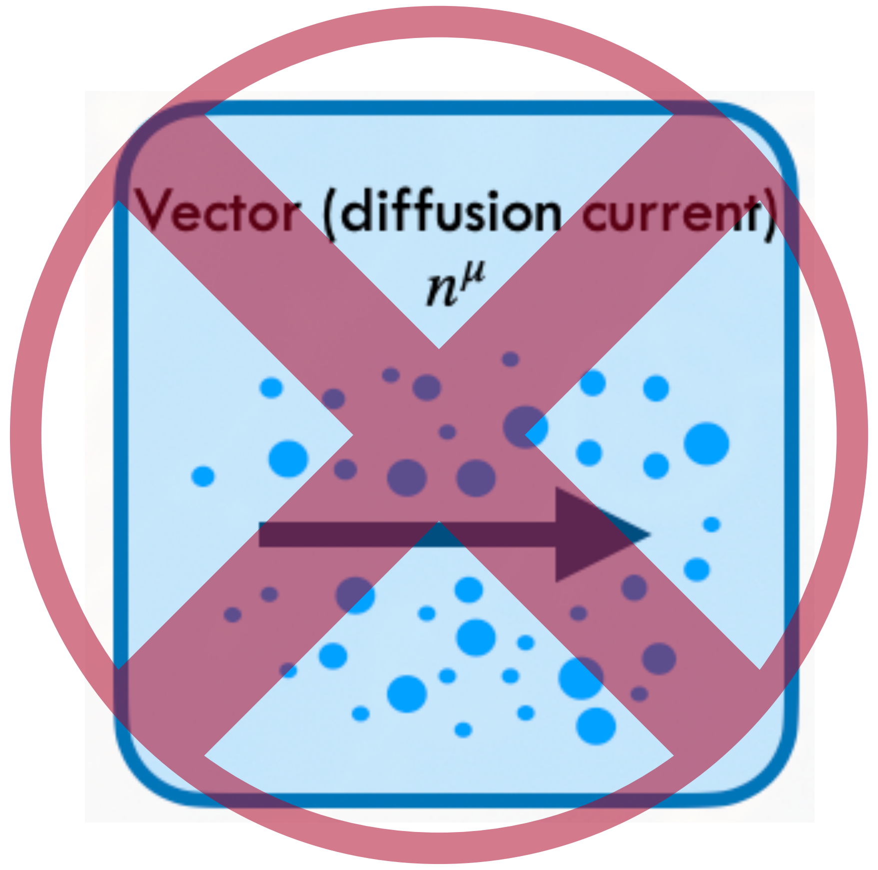
arXiv:2605.12791 [hep-ph]

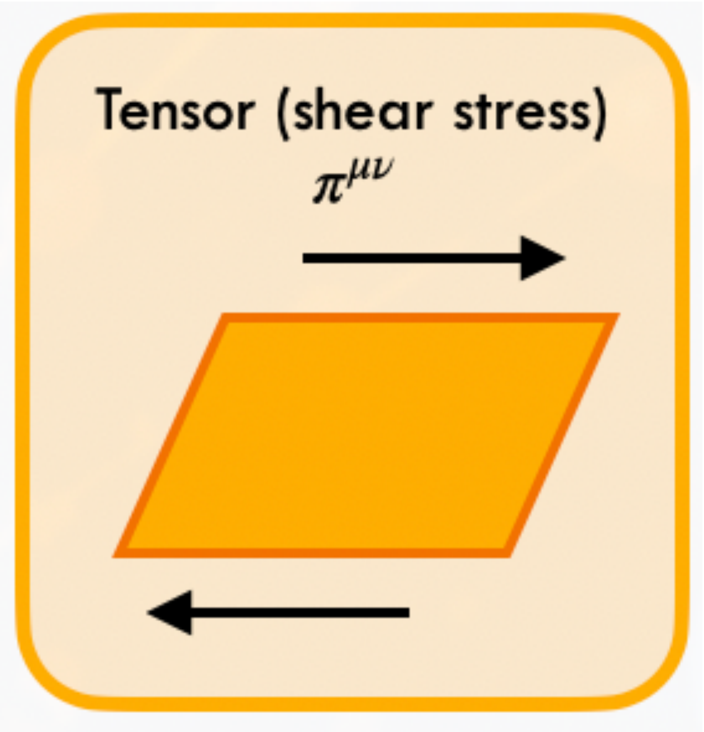
Connection with Hydrodynamics

$$\phi_k = \quad [E_0 + B_0 m^2 - D_0 E_k - 4B_0 E_k^2]\Pi \qquad + \lambda_n n_\alpha K^\alpha \quad + B_2 \pi_{\alpha\beta} K^\alpha K^\beta$$

arXiv:2311.15063 [nucl-th]







We consider massless particles at zero baryon chemical potential.

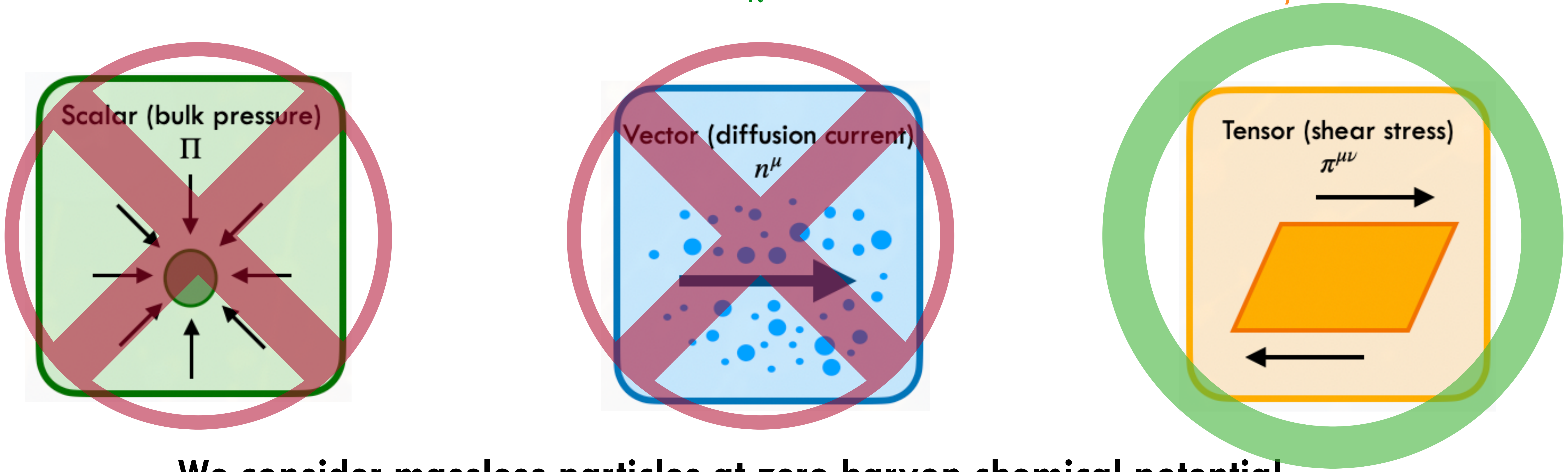
Landau frame → energy-diffusion current vanishes.
At zero μ_B the charge-diffusion current $\mathcal{J}^\mu$ vanishes

arXiv:2605.12791 [hep-ph]

Connection with Hydrodynamics

$$\phi_k = \quad [E_0 + B_0 m^2 - D_0 E_k - 4B_0 E_k^2]\Pi \qquad + \lambda_n n_\alpha K^\alpha \qquad + B_2 \pi_{\alpha\beta} K^\alpha K^\beta$$

arXiv:2311.15063 [nucl-th]



We consider massless particles at zero baryon chemical potential.

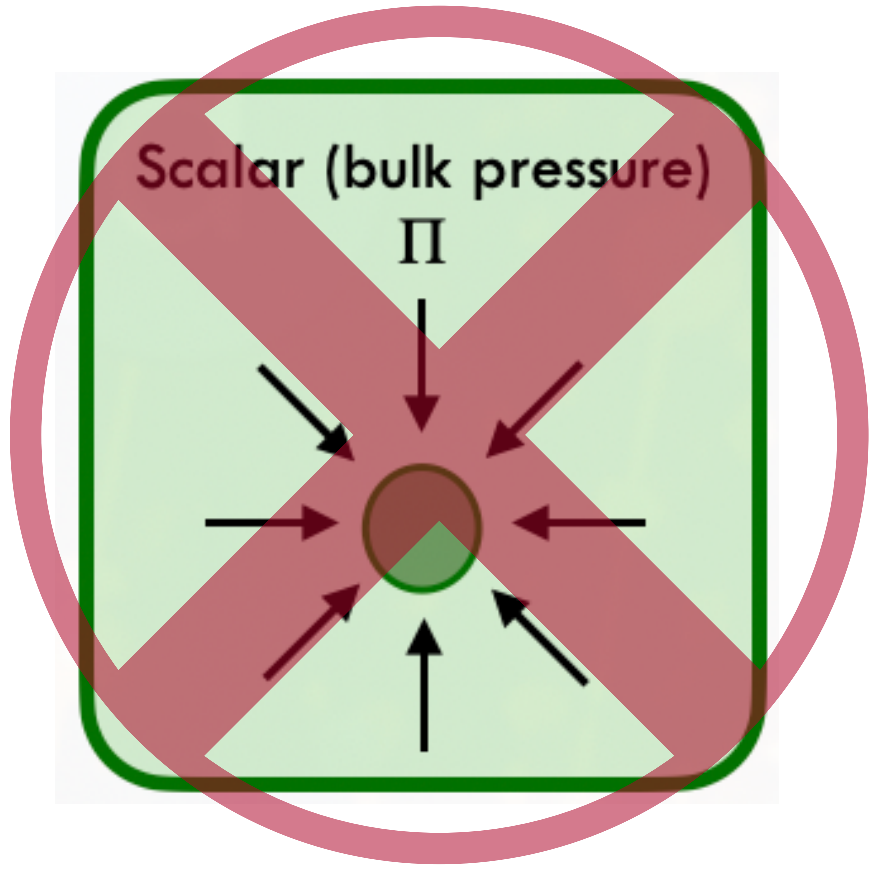
$$\boxed{\phi_k = \frac{1}{2J_{42}} \pi_{\mu\nu} K^\mu K^\nu}$$

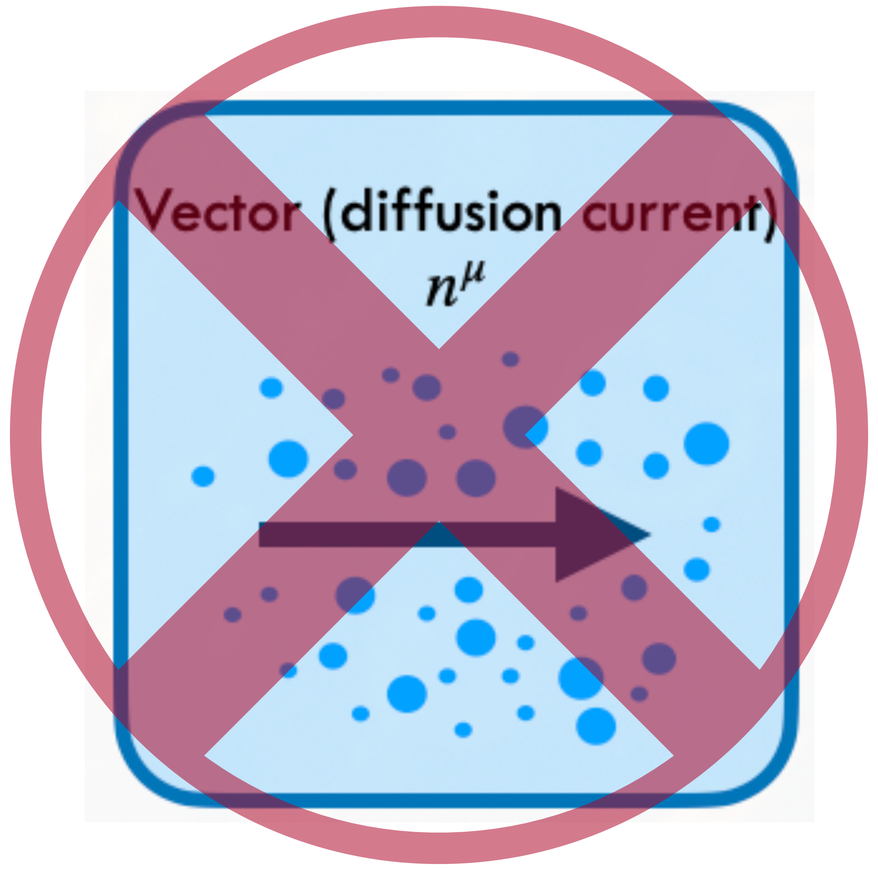
arXiv:2605.12791 [hep-ph]

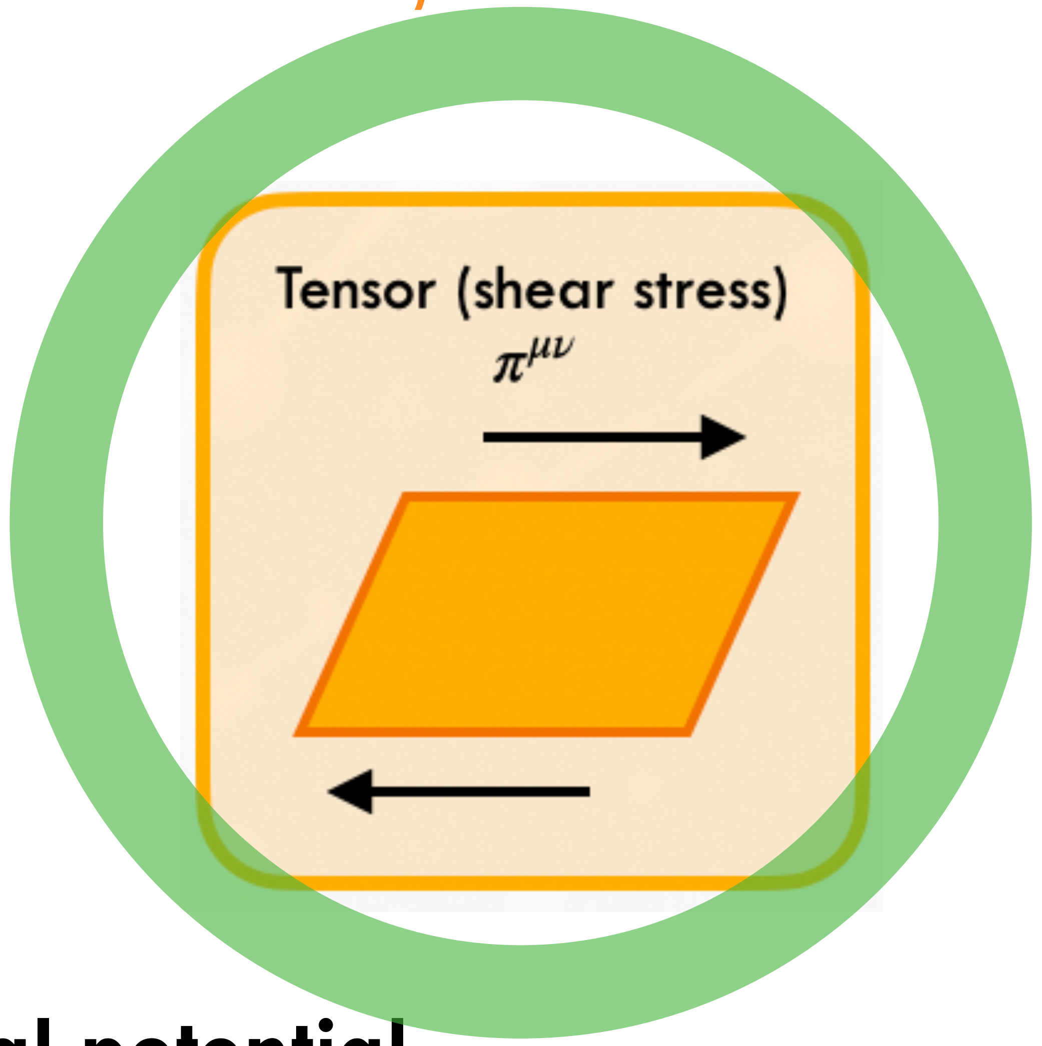
Connection with Hydrodynamics

$$\phi_k = \quad [E_0 + B_0 m^2 - D_0 E_k - 4B_0 E_k^2]\Pi \qquad + \lambda_n n_\alpha K^\alpha \qquad + B_2 \pi_{\alpha\beta} K^\alpha K^\beta$$

arXiv:2311.15063 [nucl-th]







We consider massless particles at zero baryon chemical potential.

$$\boxed{\phi_k = \frac{1}{2J_{42}} \pi_{\mu\nu} K^\mu K^\nu}$$

Working in linear response around local equilibrium

$$\delta \hat{q}^{ij} = \boxed{\mathscr{K}^{ijab}} \pi_{ab}$$

Fixed by the collision kernel!

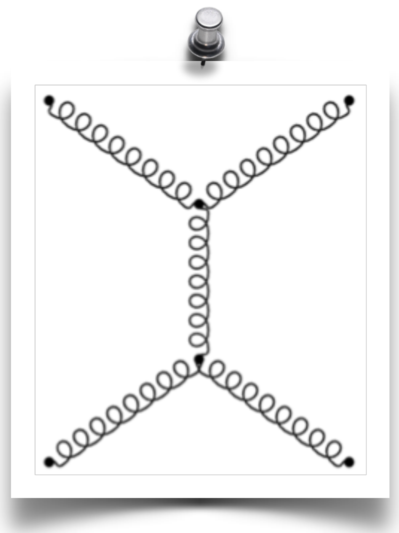
Microscopics of the system!

arXiv:2605.12791 [hep-ph]

General tensorial structure of $\delta\hat{q}^{ij}$

Computed with screened t-channel gluon exchange!

arXiv:1401.3751 [hep-ph]
arXiv:2312.00447 [hep-ph]



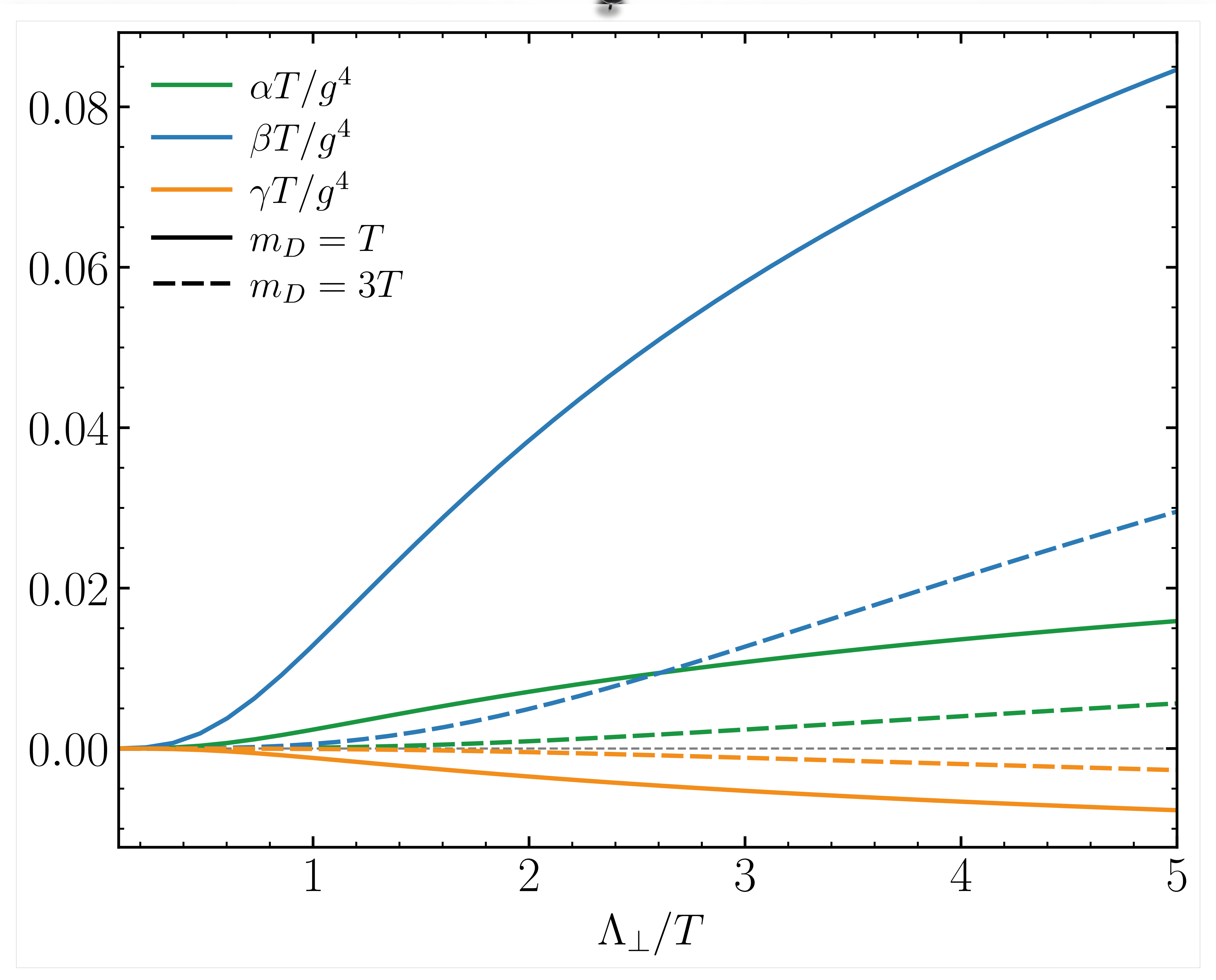
$\delta\hat{q}$ is then given by,

$$\delta\hat{q}^{ij} = \frac{1}{4\nu_a(2\pi)^5}\sum_{bd}\int d\Gamma_{kp'k'}\, q^i q^j (2\pi)\delta(p+k-p'-k')$$
$$\times\,|\mathscr{M}^{ab}_{cd}(P,K;P',K')|^2 f_{0k}\tilde{f}_{0k'}\left[\frac{f_{0k'}}{2J_{42}}\pi_{\mu\nu}K'^{\mu}K'^{\nu} + \frac{\tilde{f}_{0k}}{2J_{42}}\pi_{\mu\nu}K^{\mu}K^{\nu}\right]$$

Performing all angular integrations one arrives to the exact same tensorial structure as before,

$$\delta\hat{q}^{ij} = \begin{pmatrix} \alpha\pi^{xx}+\beta\pi^{zz} & \alpha\pi^{xy} & (\alpha+\gamma)\pi^{xz} \\ \alpha\pi^{xy} & \alpha\pi^{yy}+\beta\pi^{zz} & (\alpha+\gamma)\pi^{yz} \\ (\alpha+\gamma)\pi^{xz} & (\alpha+\gamma)\pi^{yz} & (\alpha+\beta+3\gamma)\pi^{zz} \end{pmatrix}$$

Coefficients $\alpha T, \beta T,$ and γT defined as functions of the transverse momentum cutoff $\Lambda_\perp/T$ for a purely gluonic medium



arXiv:2605.12791 [hep-ph]

Summary Part 1

✓ Out-of-equilibrium jet momentum broadening in QCD effective kinetic theory through a moment expansion of the medium distribution function.

✓ Within the 14-moment approximation, we demonstrate that viscous corrections to jet momentum broadening are governed by the hydrodynamic shear-stress tensor.

✓ Our work provides a **systematic** route to incorporate realistic medium evolution into jet transport calculations, offering an alternative to simplified homogeneous ("brick") backgrounds commonly employed in studies of jet-medium interactions.

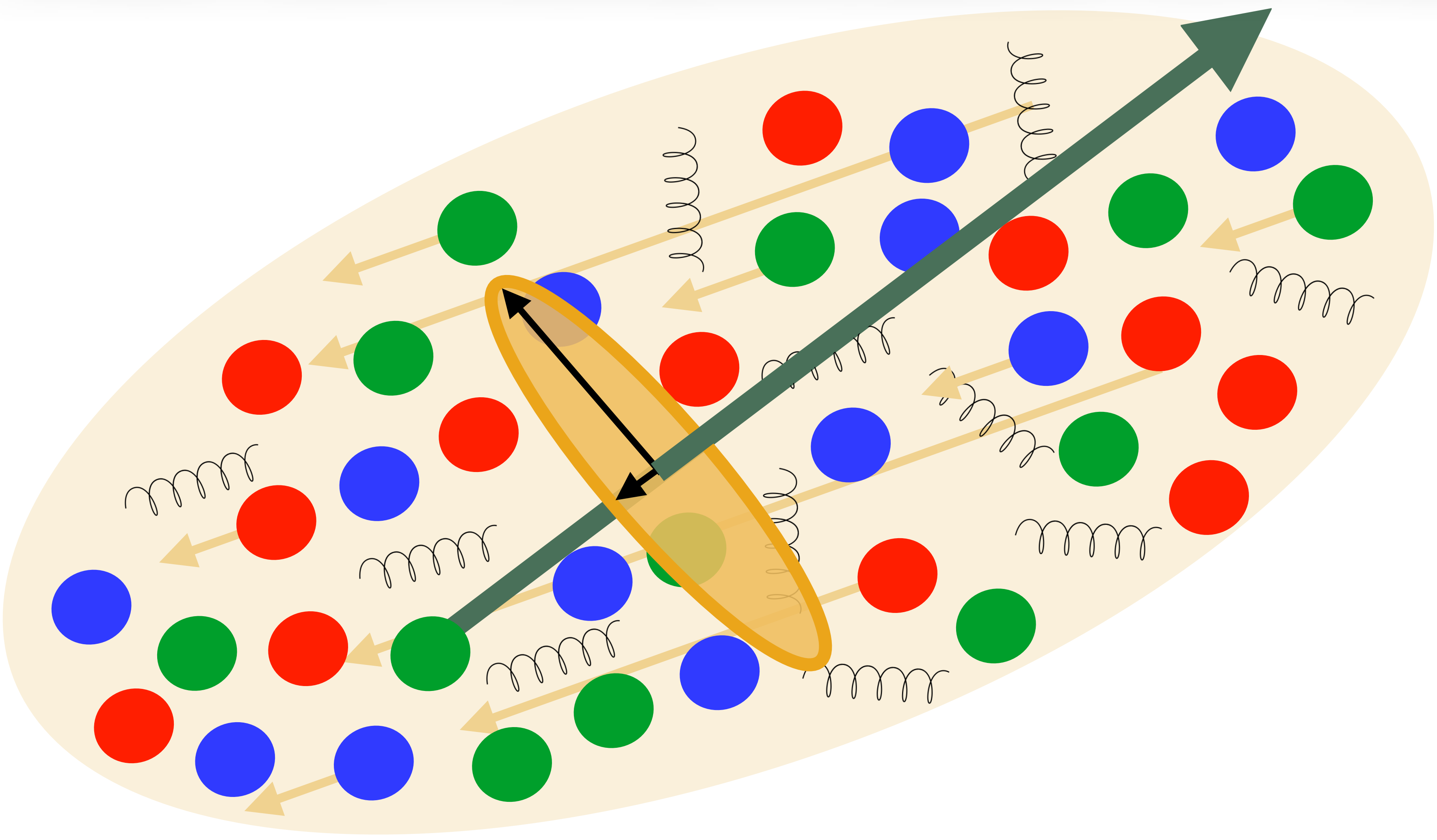
This is a plug and play formula that can be used to understand out of equilibrium viscous hydro corrections to $\hat{q}$!

arXiv:2605.12791 [hep-ph]

What about a fully covariant version of $\hat{q}$?

Jet momentum broadening out of equilibrium

Danhoni, Mullins and Noronha, [arXiv:2603.00844 [hep-ph]]



We begin with our $\hat{q}^{ij}$ and upgrade it to a covariant tensor:

$$\hat{q}^{ij}(\mathbf{p}) = \frac{1}{2|\mathbf{p}|}\int dK\, dK'\, dP'\, q^i q^j \mathscr{W}_a f_{\mathbf{k}'} \tilde{f}_{\mathbf{k}} \tilde{f}_{\mathbf{p}'}$$

$$\Downarrow$$

$$\hat{q}^{\mu\nu}(P^\mu) = \frac{1}{2u_\mu P^\mu}\int dK\, dK'\, dP'\, q^\mu q^\nu \mathscr{W}_a f_{\mathbf{k}'} \tilde{f}_{\mathbf{k}} \tilde{f}_{\mathbf{p}'}$$

Jet Momentum Broadening as a Diffusion Tensor

$$f_{\mathbf{k+q}} = f_{\mathbf{k}} + q^\alpha \frac{\partial f_{\mathbf{k}}}{\partial K^\alpha} + \frac{1}{2} q^\alpha q^\beta \frac{\partial^2 f_{\mathbf{k}}}{\partial K^\alpha \partial K^\beta} + \cdots$$

$$C_a[f_{\mathbf{k}}] \simeq \int d^4 q\, W_a(K,q) \left[q^\alpha \frac{\partial f_{\mathbf{k}}}{\partial K^\alpha} + \frac{1}{2} q^\alpha q^\beta \frac{\partial^2 f_{\mathbf{k}}}{\partial K^\alpha \partial K^\beta} \right]$$

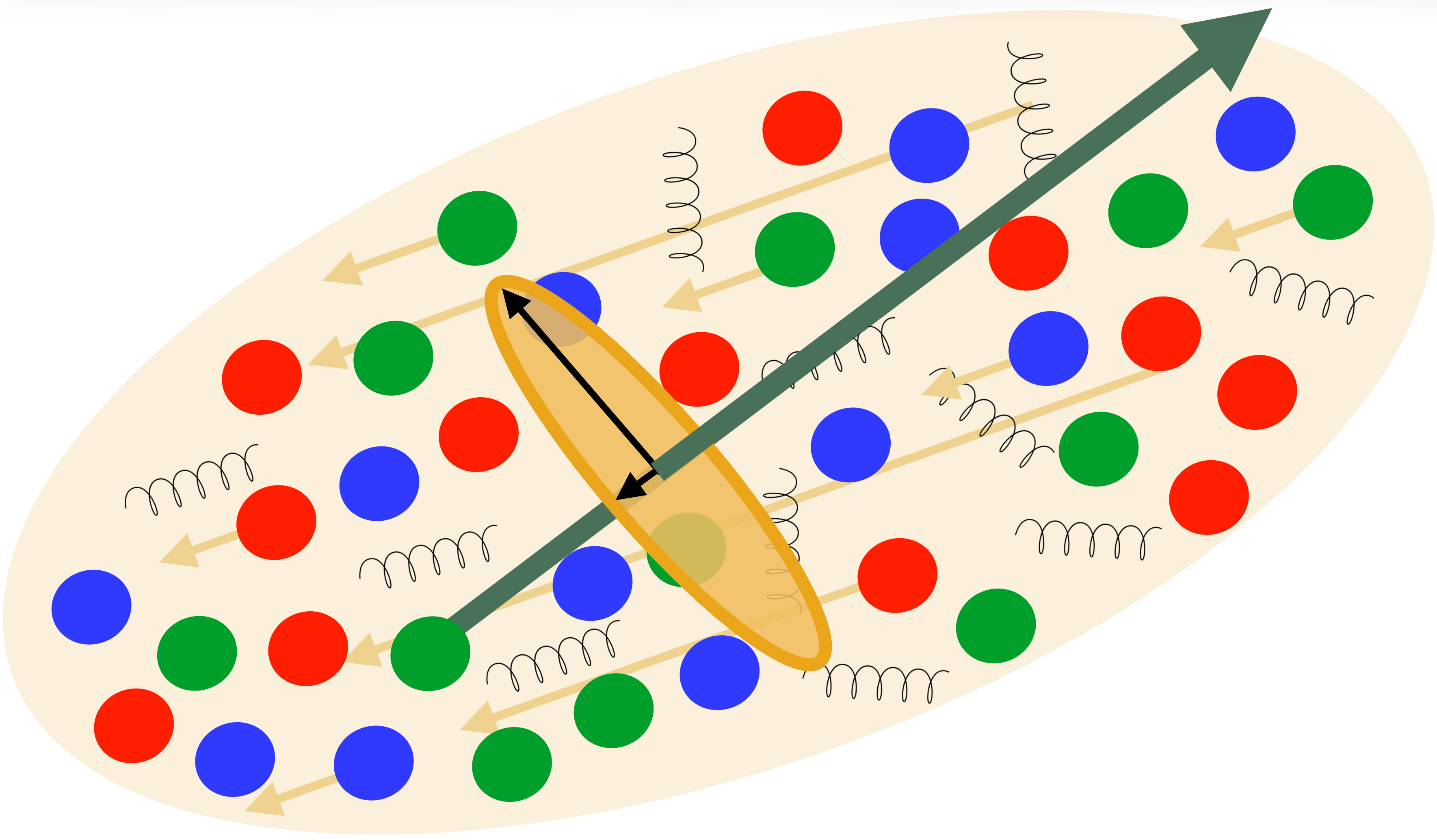
$$= \frac{\partial}{\partial K^\alpha}\left[A_a^\alpha(K) f_{\mathbf{k}}\right] + \frac{1}{2}\frac{\partial^2}{\partial K^\alpha \partial K^\beta}\left[B_a^{\alpha\beta}(K) f_{\mathbf{k}}\right]$$

To understand the physical meaning of $B^{\alpha\beta}$, we parametrize the jet trajectory $x^\mu(\lambda)$ by an affine parameter, λ,

$$\frac{dx^\mu}{d\lambda} = K^\mu, \qquad \frac{d}{d\lambda} f(\lambda; K) = \frac{dx^\mu}{d\lambda}\partial_\mu f_{\mathbf{k}} = K^\mu \partial_\mu f_{\mathbf{k}}$$

$$\frac{d}{d\ell}\langle \Delta K^\mu \Delta K^\nu \rangle \simeq \frac{1}{E_k} B^{\mu\nu}(K) = \hat{q}^{\mu\nu}(K,u)\,.$$

Jet momentum broadening out of equilibrium



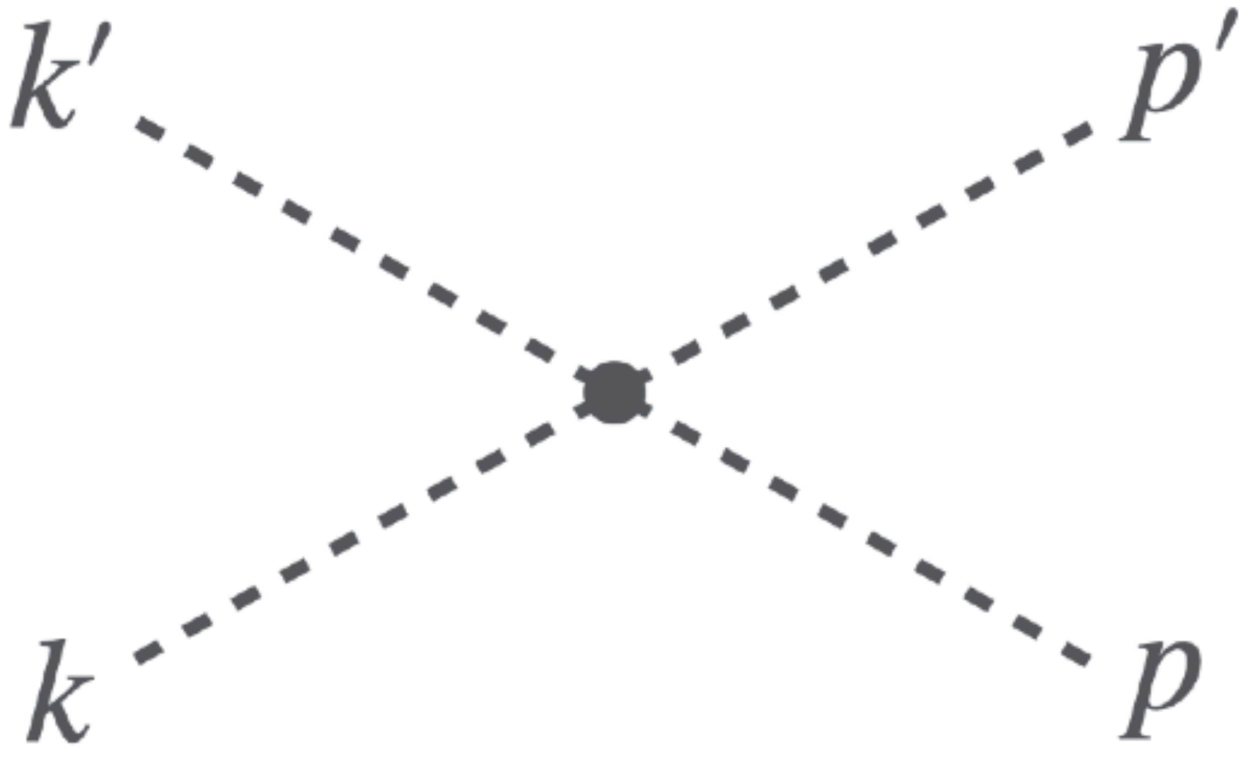
Let us begin with an isotropic non-equilibrium in a toy model:

We consider a system of massless scalar particles governed by the Lagrangian density:

$$\mathscr{L} = \frac{1}{2}\partial_\mu \varphi\ \partial^\mu \varphi - \frac{\lambda \varphi^4}{4!}$$

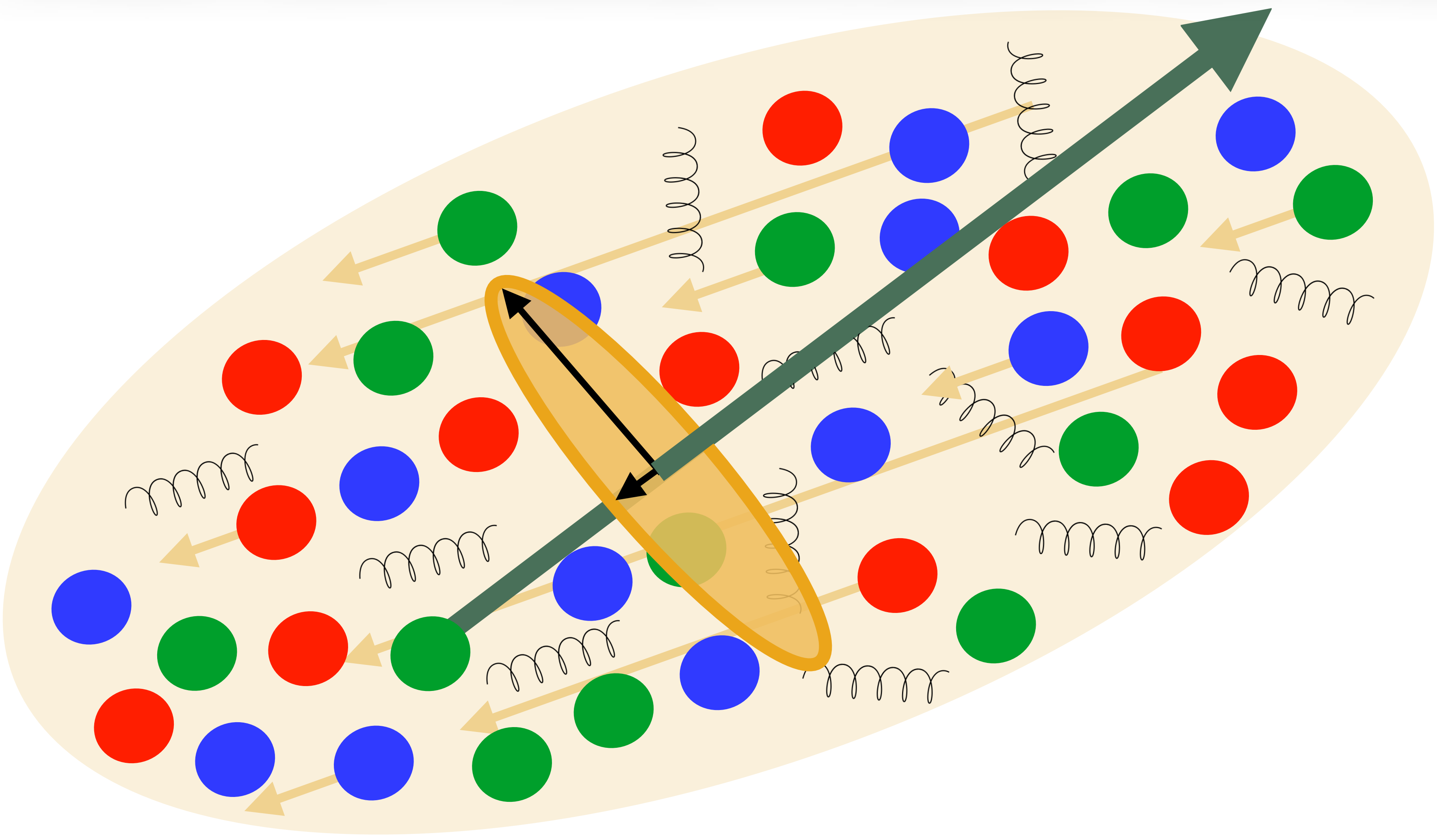
We assume a classical Boltzmann distribution. At leading order in the coupling constant, this interaction is simply given by

M. E. Peskin and D. V. Schroeder, An Introduction to quantum field theory



$$\sigma_T(s) = \frac{1}{2}\int d\Phi\, d\Theta\ \sin\Theta\, \sigma(s,\Theta) = \frac{\lambda^2}{32\pi s} \equiv \frac{g}{s}$$

Jet momentum broadening out of equilibrium



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The covariant definition of jet momentum broadening is written as,

$$\hat{q}^{\mu\nu}(K) = \frac{1}{2\,u_\alpha K^\alpha}\int dP\,dK'\,dP'\,q^\mu q^\nu\,\mathscr{W}_a\,f_{\mathbf{k}'}\tilde{f}_{\mathbf{p}}\tilde{f}_{\mathbf{p}'}$$

We then arrive at a simple tensorial structure that can be written as,

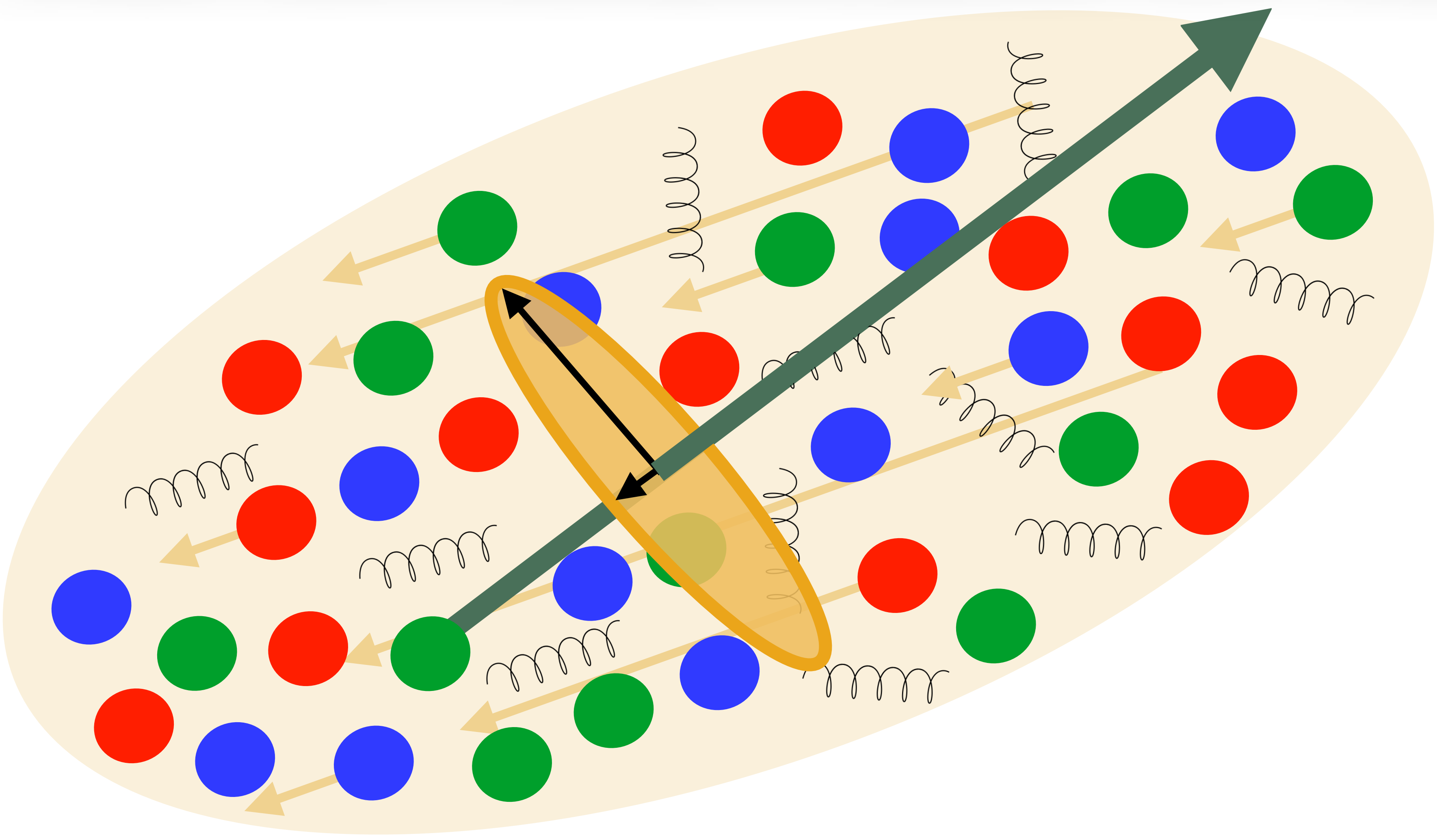
$$\hat{q}^{\mu\nu}(K^\mu) = \frac{g}{K^\beta u_\beta}\left[-\frac{2}{3}\left(K^\mu J^\nu + K^\nu J^\mu\right) + \frac{2}{3}T^{\mu\nu} - \frac{K^\alpha J_\alpha}{3}g^{\mu\nu} + \frac{1}{6}K^\mu K^\nu\ m_\varphi^2\right]$$

Effective screening–like scale modified by non-equilibrium effects

$$m_\varphi^2 = \int dK' f_{\mathbf{k}'}$$

Danhoni, Mullins and Noronha, [arXiv:2603.00844 [hep-ph]]

Jet momentum broadening out of equilibrium



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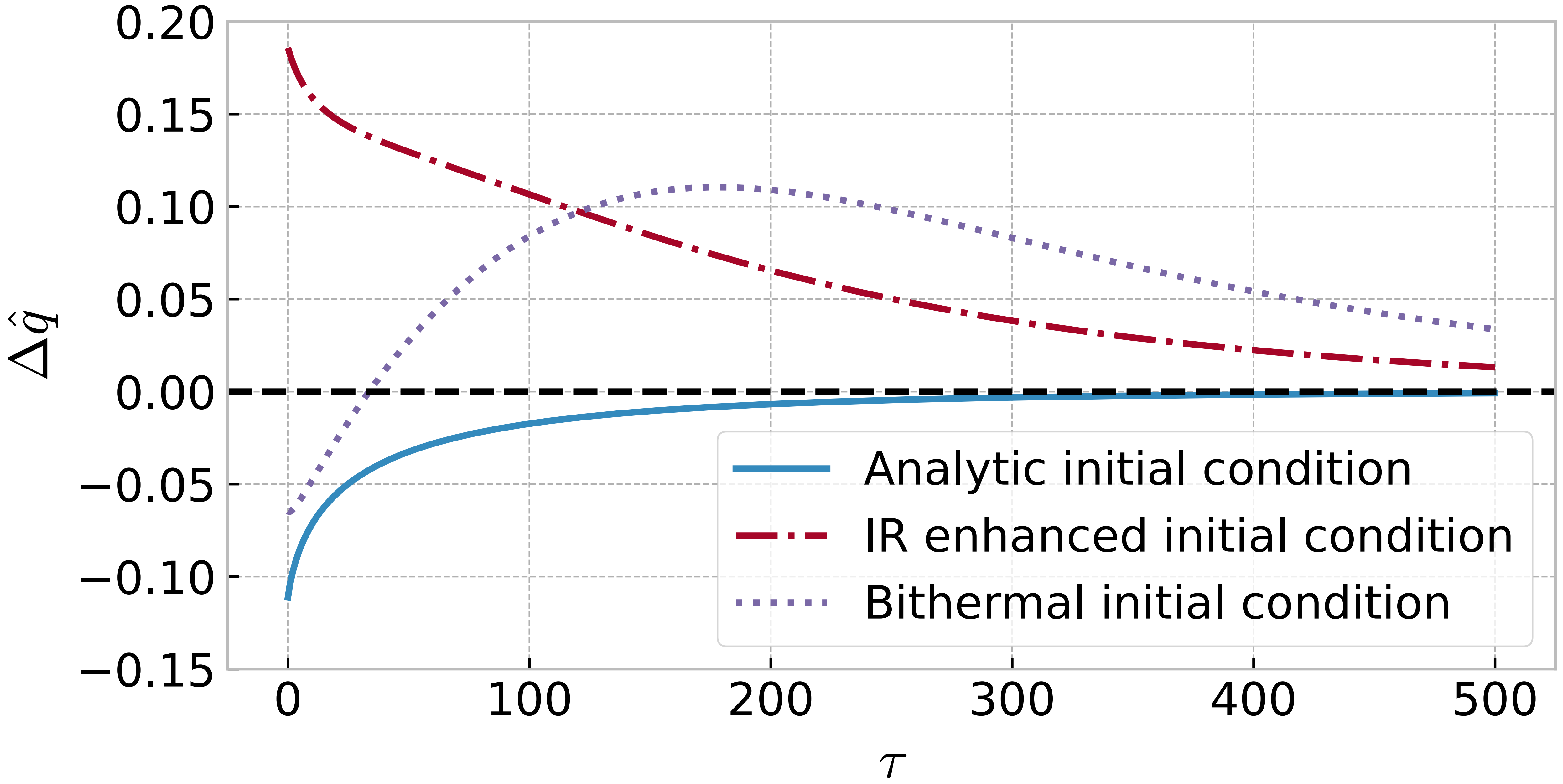
We consider a system of massless scalar particles governed by the Lagrangian density:

$$\mathscr{L} = \frac{1}{2}\partial_\mu\varphi\ \partial^\mu\varphi - \frac{\lambda\varphi^4}{4!}$$

$$\Delta\hat{q} = \frac{|\hat{q}| - |\hat{q}|_{\rm eq}}{|\hat{q}|_{\rm eq}}$$

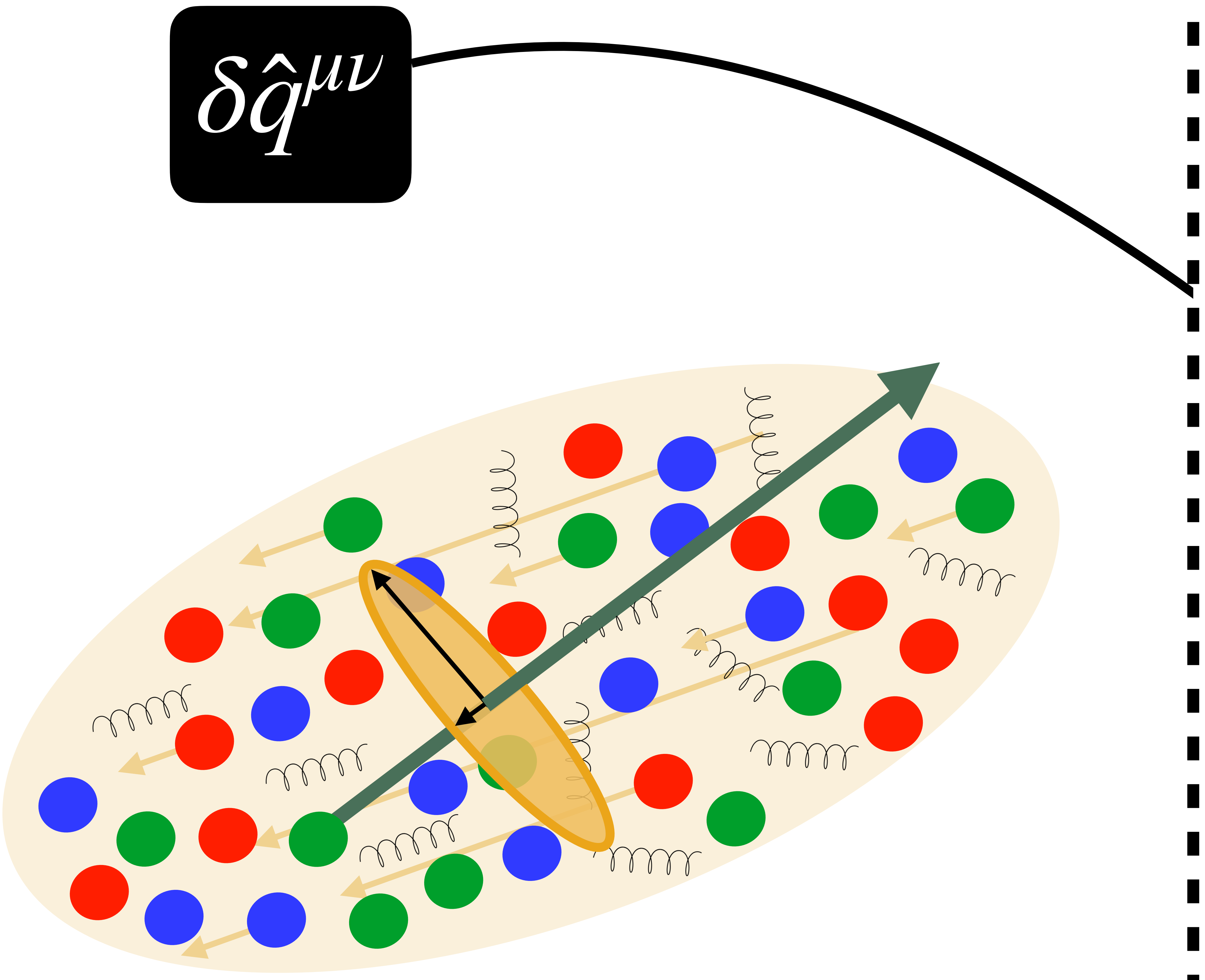


Danhoni, Mullins and Noronha, [arXiv:2603.00844 [hep-ph]]

Can we also do that for QCD?

Covariant QCD jet momentum broadening

The momentum-broadening coefficient is promoted to a rank-two tensor $\delta\hat{q}^{\mu\nu}$ depending on the jet momentum, the local fluid 4-velocity, and the dissipative currents. The tensor has additional information and can be described as,



Momentum broadening: $$\hat{q}^{ij} = \begin{pmatrix} q^{11} & q^{12} & q^{13} \\ q^{12} & q^{22} & q^{23} \\ q^{13} & q^{23} & q^{33} \end{pmatrix}$$

$+$

Energy broadening: $\hat{q}^{00}$

$+$

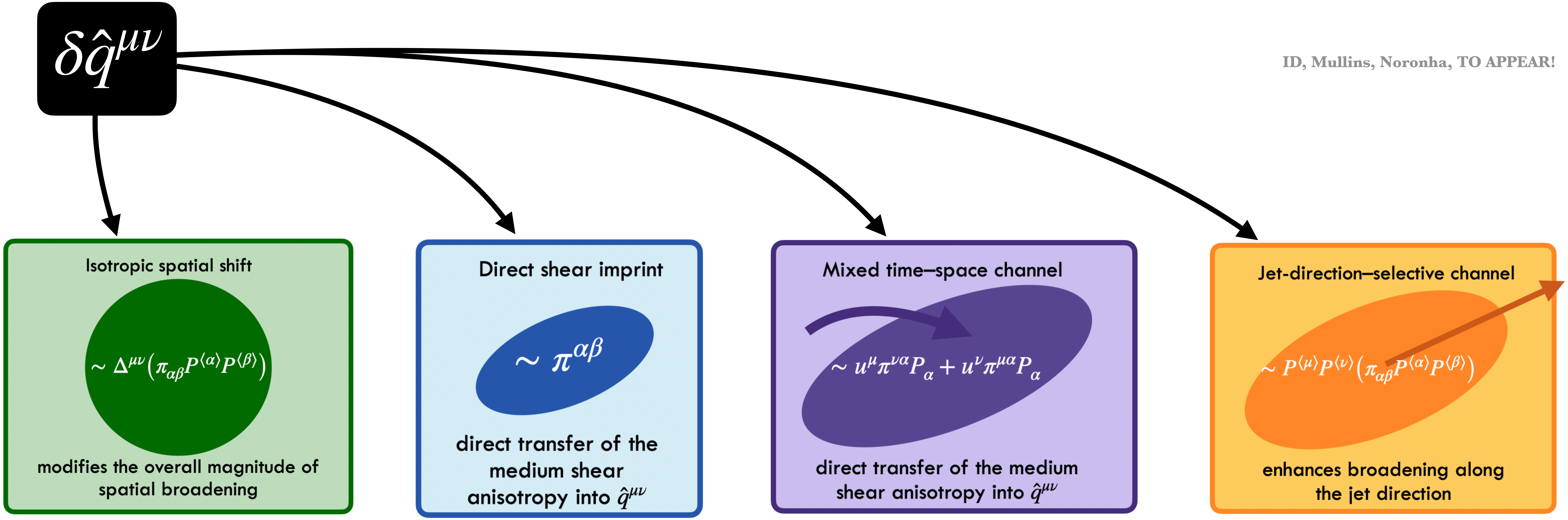
Energy-momentum correlations: $\hat{q}^{0i}$

ID, Mullins, Noronha, TO APPEAR!

Covariant QCD jet momentum broadening

The momentum-broadening coefficient is promoted to a rank-two tensor $\delta\hat{q}^{\mu\nu}$ depending on the jet momentum, the local fluid 4-velocity, and the dissipative currents. The tensor has additional information and can be described as,

ID, Mullins, Noronha, TO APPEAR!



isotropic / magnitude → jet-aligned / directional

Summary Part 2

✓ Fully covariant definition of jet momentum broadening: $\hat{q}^{\mu\nu}$ — momentum diffusion along the jet trajectory, valid for non-equilibrium viscous medium, can be used in hydrodynamics evolution.

✓ In a $\lambda\varphi^4$ toy model, non-equilibrium effects enter through an effective screening scale; the approach of $\Delta\hat{q}$ to equilibrium depends strongly on the initial condition.

✓ In QCD, the scalar $\hat{q}$ is promoted to a full tensor: momentum broadening $\hat{q}^{ij}$+ energy broadening $\hat{q}^{00}$+ energy–momentum correlations $\hat{q}^{0i}$.

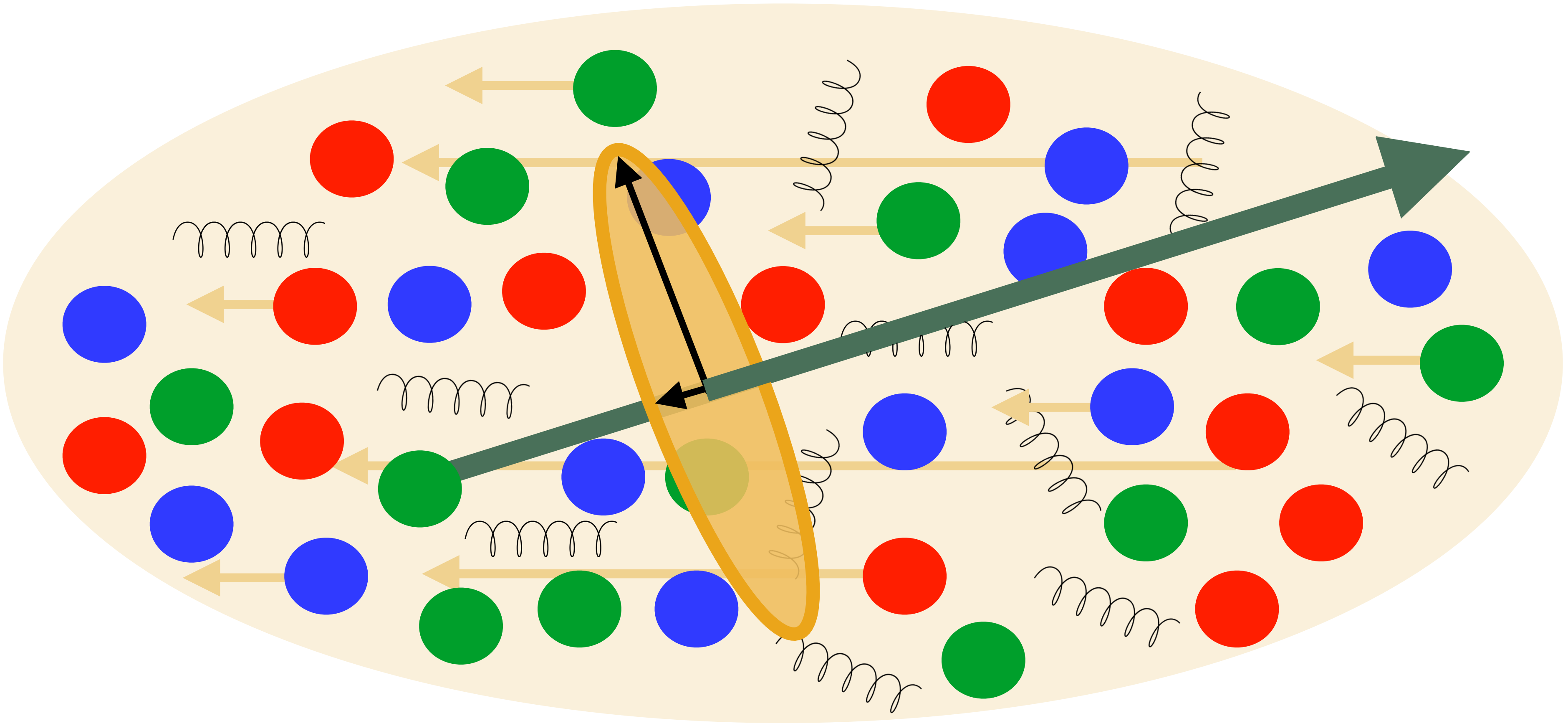
✓ Dissipative currents imprint distinct channels: **isotropic shift**, **direct shear imprint**, **mixed time–space**, and a **jet-direction-selective enhancement**

Danhoni, Mullins and Noronha, [arXiv:2603.00844 [hep-ph]]

arXiv:2605.12791 [hep-ph]

ID, Mullins, Noronha, TO APPEAR!

Non-equilibrium QGP + pQCD



Example equilibrium contribution:

$$\hat{q}^{\mu\nu}_{eq} = \frac{1}{4p}\int_{\mathbf{k}',\mathbf{k}} q^\mu q^\nu (2\pi)\delta(p+k-p'-k')\,|\mathcal{M}|^2 f_{0k}\tilde{f}_{0k'}$$

$$\hat{q}^{11}_{\rm eq} \sim \int_{q_\perp<\Lambda_\perp} d^2\vec{q}_\perp\, q_\perp^2 \int_0^\infty dk\, k^2 f_k \tilde{f}_{k'} \frac{m_D^2}{q_\perp^2\left(q_\perp^2+m_D^2\right)}$$

$$\hat{q}^{11}_{eq} = \frac{g^2}{4\pi} C_A T m_D^2 \ln\left(1+\frac{\Lambda_\perp^2}{m_D^2}\right)$$

Reproduce results from Boguslavski, Kurkela, Lappi, Lindenbauer and Peuron.

Example non-equilibrium contribution:

$$\hat{q}^{\mu\nu}_{neq} = \frac{1}{4p}\int_{\mathbf{k}',\mathbf{k}} q^\mu q^\nu (2\pi)\delta(p+k-p'-k')\,|\mathcal{M}|^2 f_{0k}\tilde{f}_{0k'}$$
$$\times\left\{\frac{f_{0k'}}{2J_{42}}\pi_{\alpha\beta}k'^\alpha k'^\beta + \frac{\tilde{f}_{0k}}{2J_{42}}\pi_{\alpha\beta}k^\alpha k^\beta\right\}$$

$$\delta\hat{q}^{11} \sim \int_{\mathbf{k},\mathbf{k}'} q^1 q^1 |\mathcal{M}|^2 \pi_{\alpha\beta}k^\alpha k^\beta \times (\cdots)$$

$$\delta\hat{q}^{11} \sim \pi_{\alpha\beta}\int dq\, d\omega\, dk\; q^1 q^1\, \boxed{\mathcal{F}(k,\omega,q)}\,\boxed{\mathcal{T}^{\alpha\beta}}$$

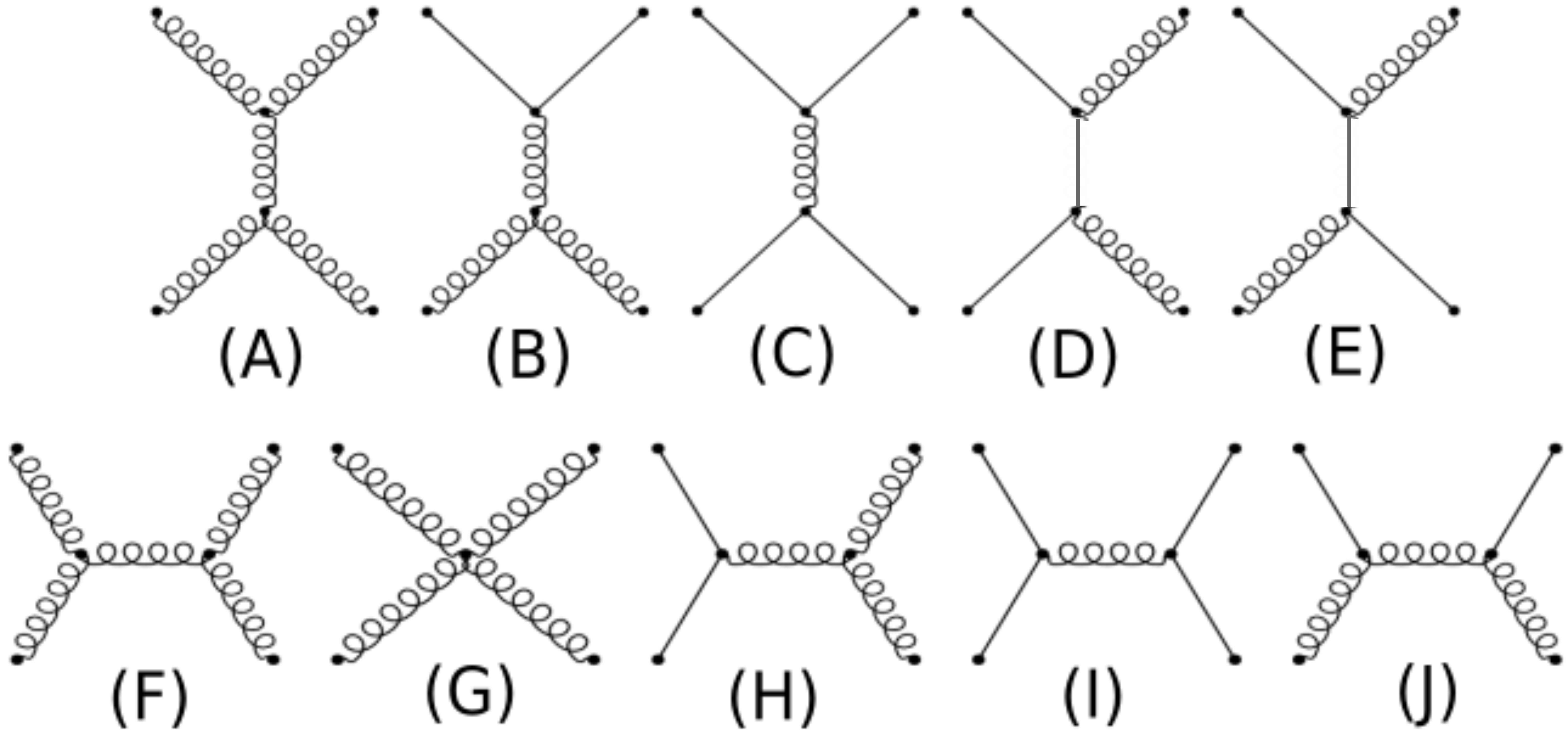
$$\int_{\phi,\theta} |\mathcal{M}|^2 f_{0k}\tilde{f}_{0k'}\tilde{f}_{0k} \qquad\qquad k^\alpha k^\beta$$

For a given π^{ij} we compute numerically the correction as a function of m_D^2 and Λ_T.

$2 \leftrightarrow 2$ Processes

Arnold, Moore and Yaffe, JHEP 05 (2003) 051.

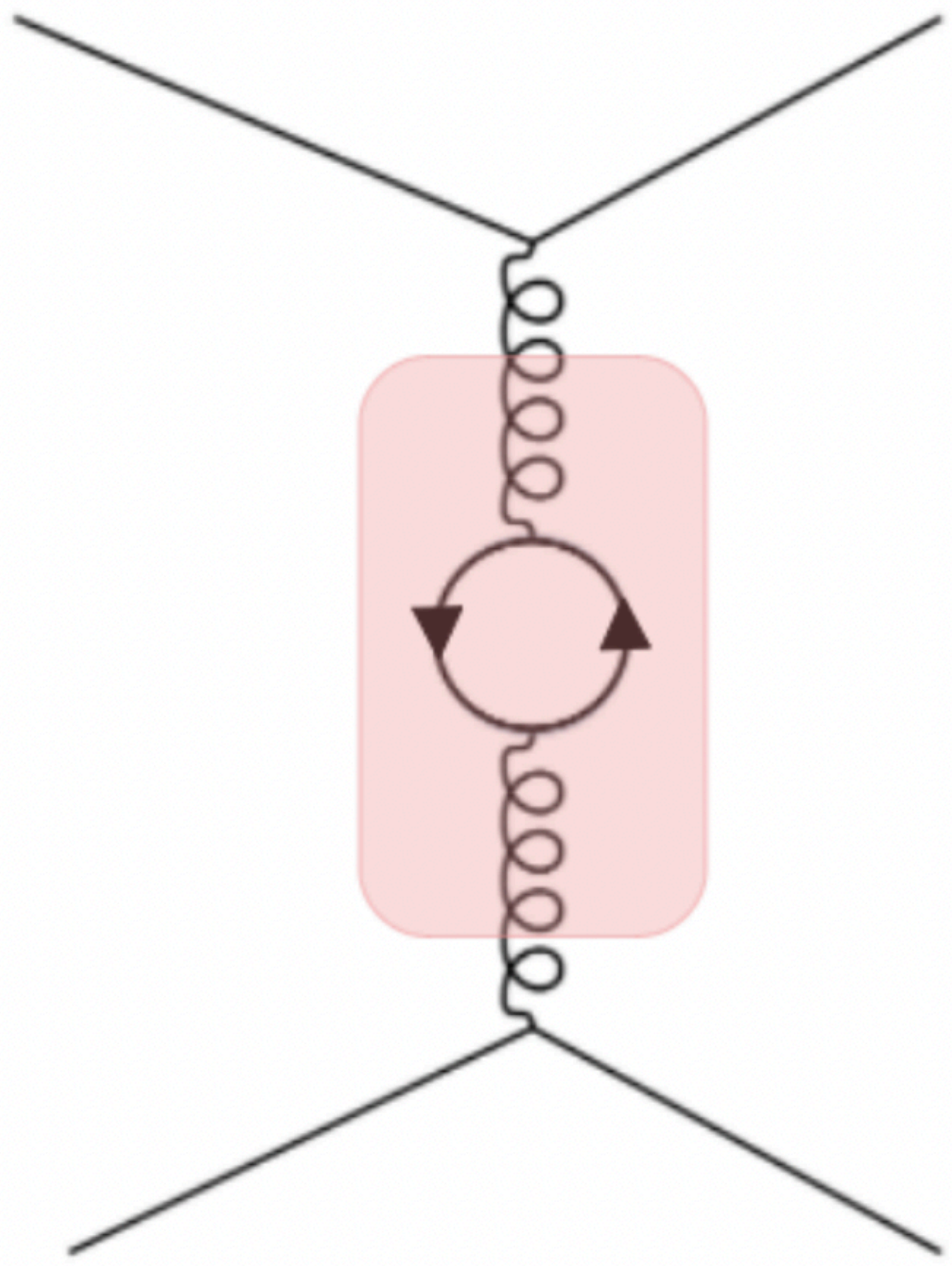
$$\mathscr{C}_a^{2\leftrightarrow 2}[f](\vec{p}) = \sum_{bcd} \int_{\vec{k},\vec{p},\vec{k}'} \frac{|\mathscr{M}_{abcd}(P,K,P',K')|^2}{2p^0\,2k^0\,2p'^0\,2k'^0} \frac{(2\pi)^4}{2} \delta^4(P+K-P'-K')$$
$$\times \left\{ f^a(\vec{p}) f^b(\vec{k})[1 \pm f^c(\vec{p'})][1 \pm f^d(\vec{k'})] - f^c(\vec{p'}) f^d(\vec{k'})[1 \pm f^a(\vec{p})][1 \pm f^b(\vec{k})] \right\}$$



Soft gluon exchange - diagrams (A)-(C)

Arnold, Moore and Yaffe, JHEP 05 (2003) 051.

- At high μ the most significant contribution comes from diagram (C):



- we have assumed $|\omega| < q$, which is the only case of relevance.

- Self-energies only matter when the exchange momentum is soft.

- The matrix element is $(s^2+u^2)/t^2$:

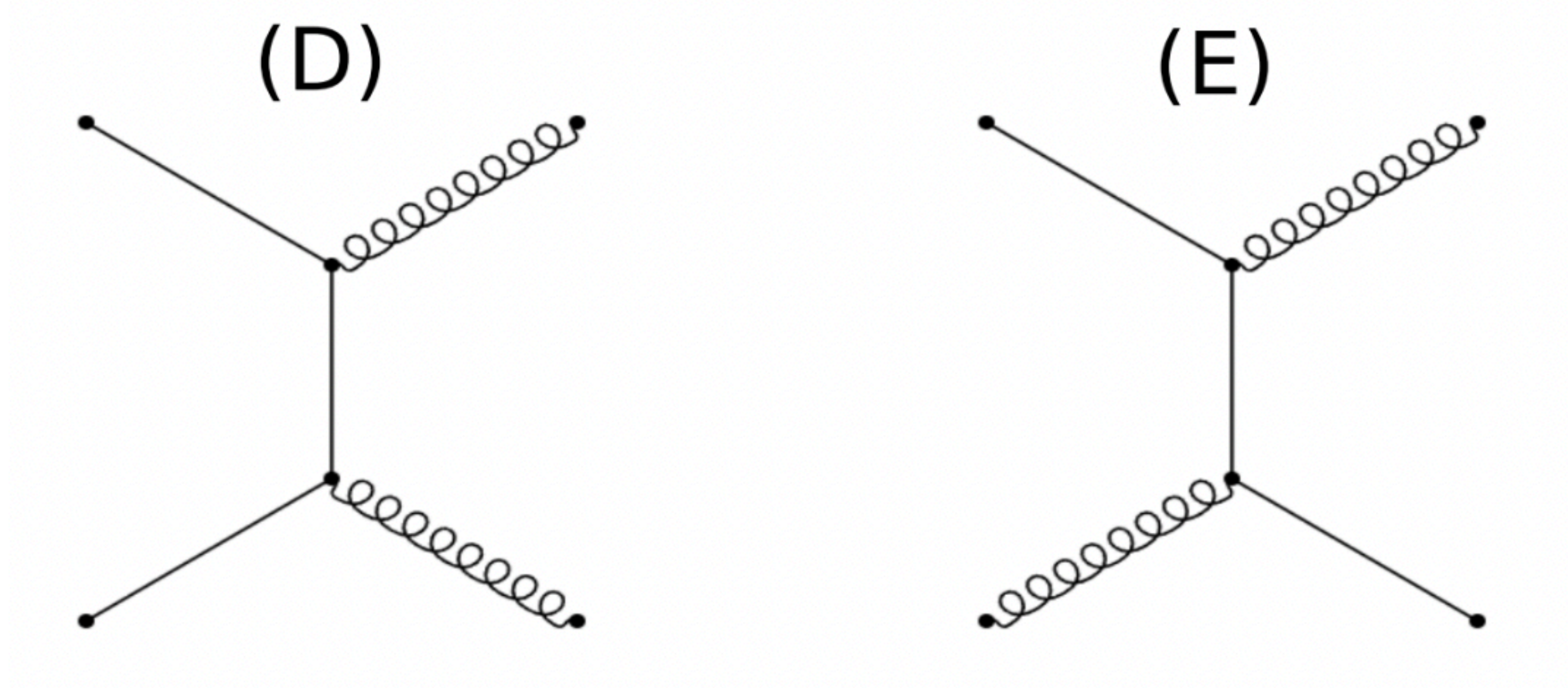
$$\frac{1}{8} D^{\text{ret}}_{\mu\alpha} D^{\text{adv}}_{\nu\beta}\, \text{Tr}(\not{p}\gamma^\mu \not{k}\gamma^\nu)\text{Tr}(\not{p}'\gamma^\alpha \not{k}'\gamma^\beta)$$

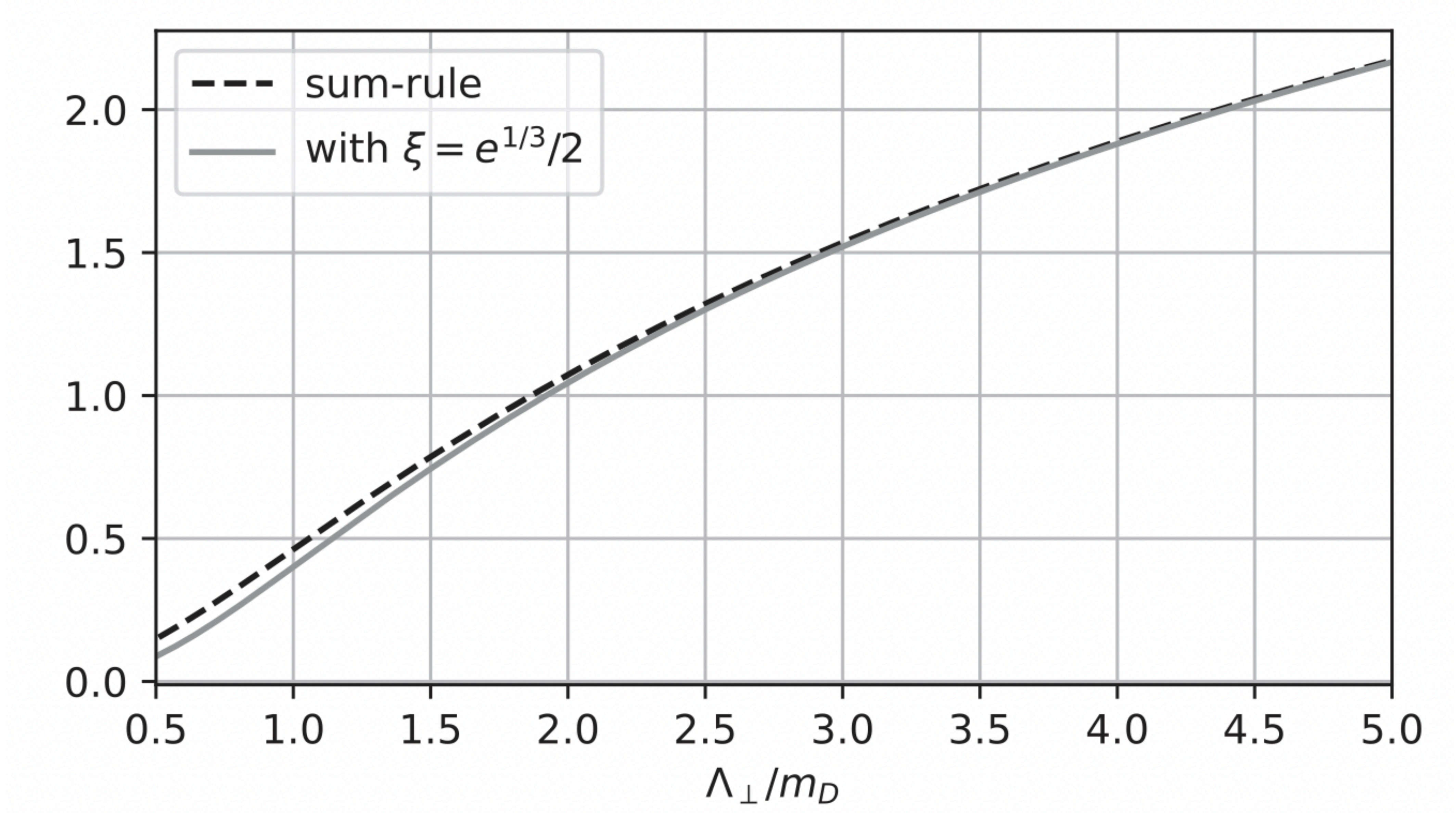
retarded gauge boson propagator

$$\begin{cases} D^{\text{ret}}_{ij}(\omega,\mathbf{q}) = \dfrac{\delta_{\mathbf{ij}} - \hat{\mathbf{q}}_{\mathbf{i}}\hat{\mathbf{q}}_{\mathbf{j}}}{\mathbf{q}^2 - \omega^2 + \boldsymbol{\Pi}^{\text{ret}}_{\mathbf{T}}(\omega,\mathbf{p})} \\ D^{\text{ret}}_{00}(\omega,\mathbf{q}) = \dfrac{-\mathbf{1}}{\mathbf{q}^2 - \boldsymbol{\Pi}^{\text{ret}}_{\mathbf{00}}(\omega,\mathbf{q})} \\ D^{\text{ret}}_{0i}(\omega,\mathbf{q}) = \mathbf{D}^{\text{ret}}_{\mathbf{i0}}(\omega,\mathbf{q}) = \mathbf{0} \end{cases}$$

- where $\Pi^{\text{ret}}_{00}(\omega,\mathbf{q})$and $\Pi^{\text{ret}}_{T}(\omega,\mathbf{p})$ are directly dependent on m_D^2.

Diagrams (D) and (E)





Boguslavski, Kurkela, Lappi, Lindenbauer and Peuron, Phys. Rev. D 110, no.3, 034019 (2024)

- For a soft fermion exchange the divergent term $(u-s)/t \sim 1/q^2$ is replaced by IR regulated term:

$$\frac{u-s}{t} \to \frac{u-s}{t}\frac{q^2}{q^2+\xi_q^2 m_q^2}$$

Kurkela and Mazeliauskas, Phys. Rev. D 99 (2019) 054018

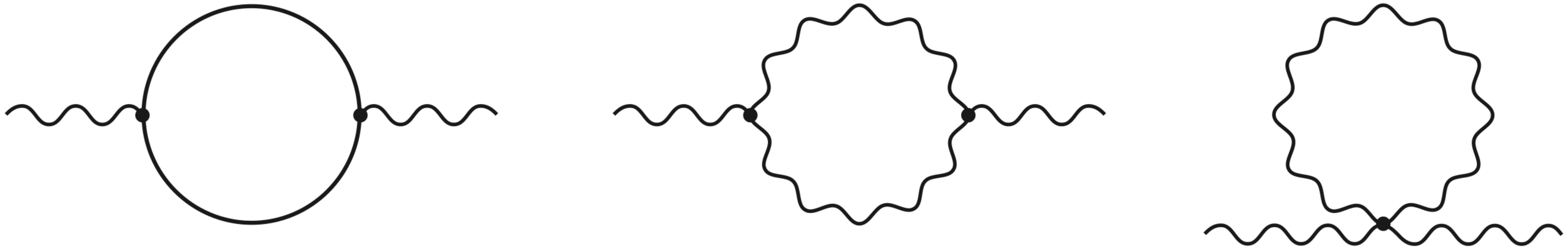
- where q is the exchanged momentum, $\xi_q = e/2 \simeq 1.359$, and m_q is:

$$m_q^2 = \frac{g^2}{3}\left(T^2 + \frac{\mu^2}{\pi^2}\right)$$

- the cumulative effect of these diagrams is suppressed $\propto e^{-\mu/T}$ in the large μ/T region.

Danhoni and Moore, JHEP 02, 124

HTL



The form of the HTLs are identical for high density as for high temperature:

$$m_D^2 = \frac{N_c}{3} g^2 T^2 + \frac{N_f}{6} g^2 \left(T^2 + \frac{3}{\pi^2} \mu^2 \right)$$

- T (or μ) sets a “hard” energy scale, while the external energy and momentum are “soft”.
- This separation of scales forms the basis of the HTL- (or HDL-) resummation scheme.

Rischke, Prog. Part. Nucl. Phys. 52, 197-296 (2004)