CSI Presentation 2026

Amplitude Analysis of $B \rightarrow K\pi\pi\gamma$ *decays*

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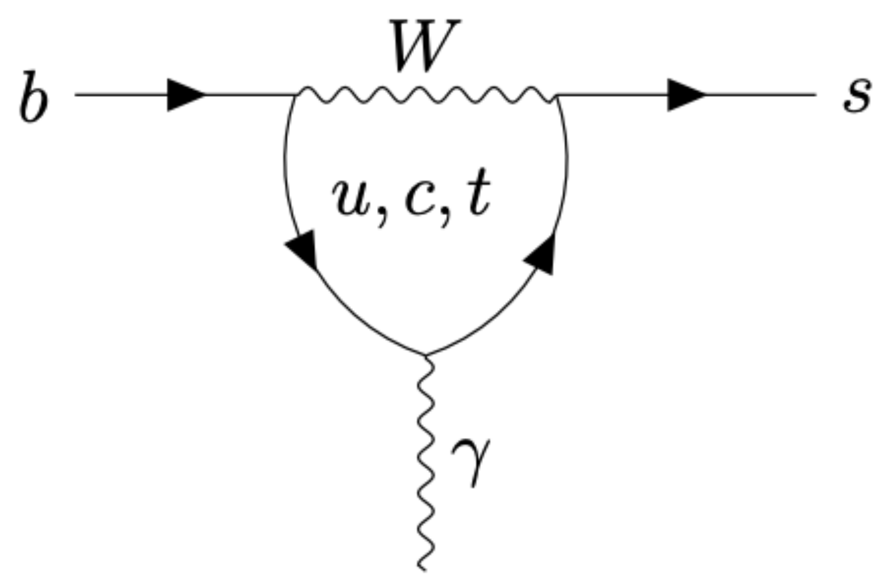
$b \to s\gamma$ transitions

- $b \to s\gamma$ transitions are sensitive to BSM physics due to absence of flavour-changing neutral currents at the tree level in the SM



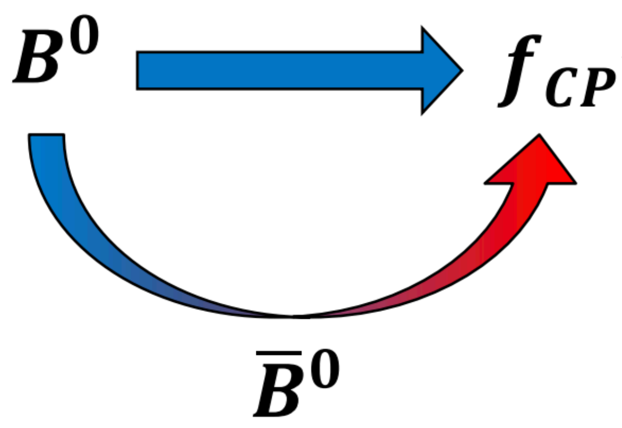
Penguin diagram for $b \to s\gamma$

- The **time dependent CP asymmetry** is given by :

$$\mathscr{A}_{CP}(\Delta t) = \frac{\Gamma(\bar{B}^0(\Delta t) \to f_{CP}\gamma) - \Gamma(B^0(\Delta t) \to f_{CP}\gamma)}{\Gamma(\bar{B}^0(\Delta t) \to f_{CP}\gamma) + \Gamma(B^0(\Delta t) \to f_{CP}\gamma)} = \mathscr{S}_{CP}\sin(\Delta m \Delta t) - \mathscr{C}_{CP}\cos(\Delta m \Delta t)$$

$$f_{CP} = K_S^0\rho^0(\pi^\pm\pi^\mp)$$

- $K_S^0\rho^0(\pi^\pm\pi^\mp)$ is a **CP eigenstate** as both B^0 and $\bar{B}^0$ decay to the final state

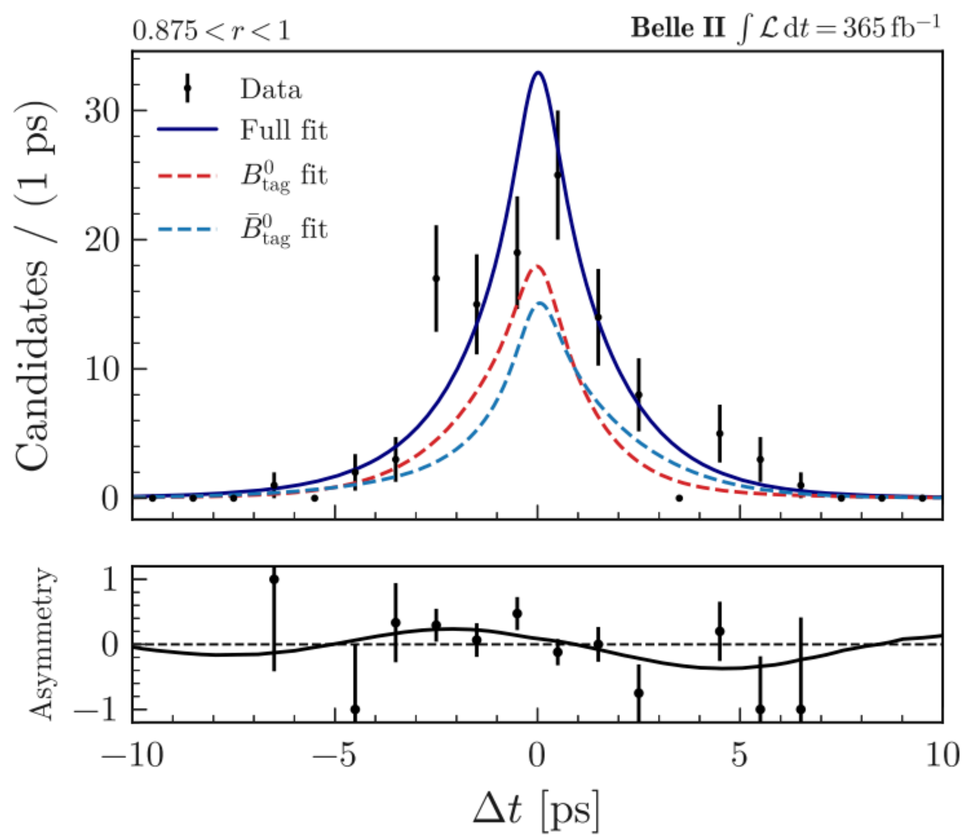


CP eigenstate

- $\mathscr{S}_{CP} \neq 0$ implies **BSM contributions**

- **Effective** $\mathscr{S}_{B\to K_S\pi\pi\gamma}$ was measured with Belle and Belle-II data: JHEP01(2026)134

- It is also a null test of the SM, and the measured value was –0.29 ± 0.11 ± 0.05



Time-dependent CP asymmetry

Dilution Factor

- Due to presence of **non-CP eigenstates**, the $\mathscr{S}_{CP}$ can get diluted -

$$\mathscr{S}_{B^0 \to K_S^0 \pi\pi\gamma} = \mathscr{D} \cdot \mathscr{S}_{CP}$$

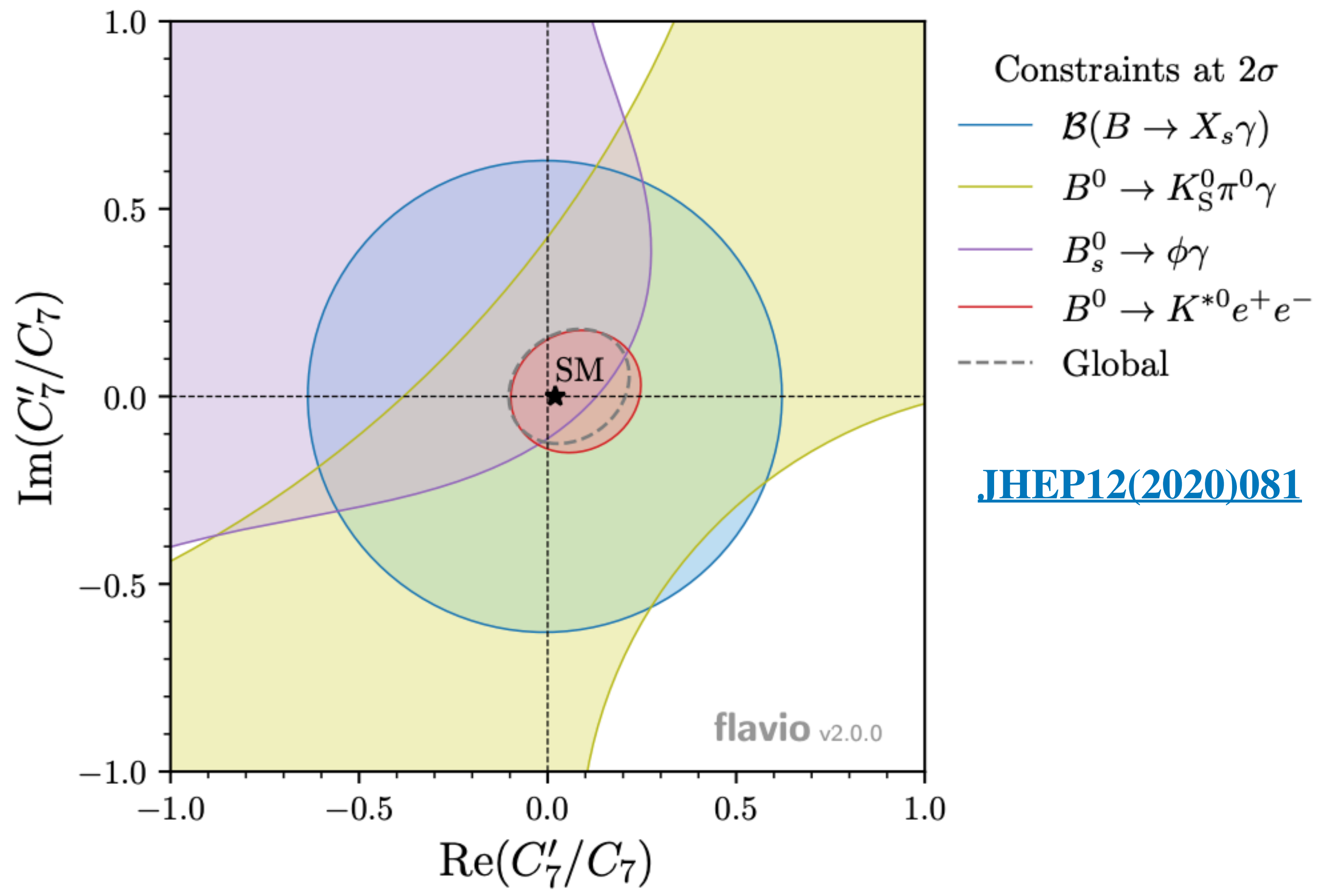
Measured *Dilution Factor (Goal)*

- Measurement of $\mathscr{D}$ by **amplitude analysis** -
  - Obtain $\mathscr{S}_{CP}$ Constrain Wilson co-efficients C_7'/C_7
- **Amplitude analysis** of isospin partner $B^+ \to K^+\pi^+\pi^-\gamma$

- $\mathscr{D}$ can be measured from the **amplitudes** of all intermediate contributions (JHEP09(2019)034)
- $\mathscr{D}_{K_S^0\rho\gamma} = -0.78^{+0.19}_{-0.17}$ measured with few intermediate resonances [BaBar: 1512.03579]

$$\mathscr{D} = \frac{2\int \mathbb{A}^*(p_0,p_1,p_2)\mathbb{A}(p_1,p_0,p_2)d\Phi}{\int \left(|\mathbb{A}(p_0,p_1,p_2)|^2 + |\mathbb{A}(p_1,p_0,p_2)|^2 \right) d\Phi}$$

- Where $\mathbb{A}(p_0,p_1,p_2)$ is the amplitude of $B^0 \to K_S^0(p_2)\pi^+(p_0)\pi^-(p_1)\gamma$ decay



JHEP12(2020)081

Analysis Pipeline

Amplitude model

- Isobar model
- Intermediate decay pathways
- Resonance lineshapes, spin formalism

Event generation

- Include interference and angular structures
- Belle II simulation

Reconstruction and selection

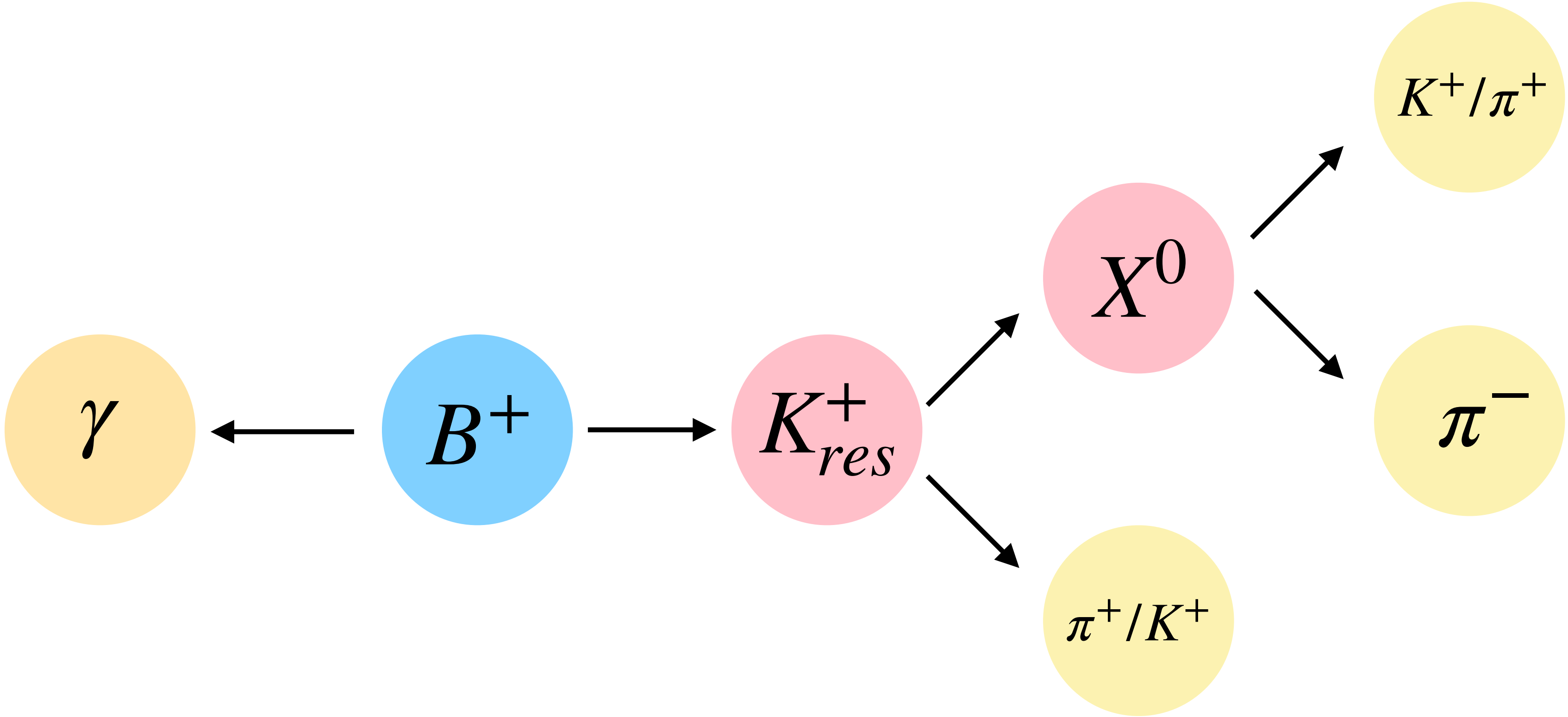
- Signal selection
- Background subtraction

(M. Maushart)

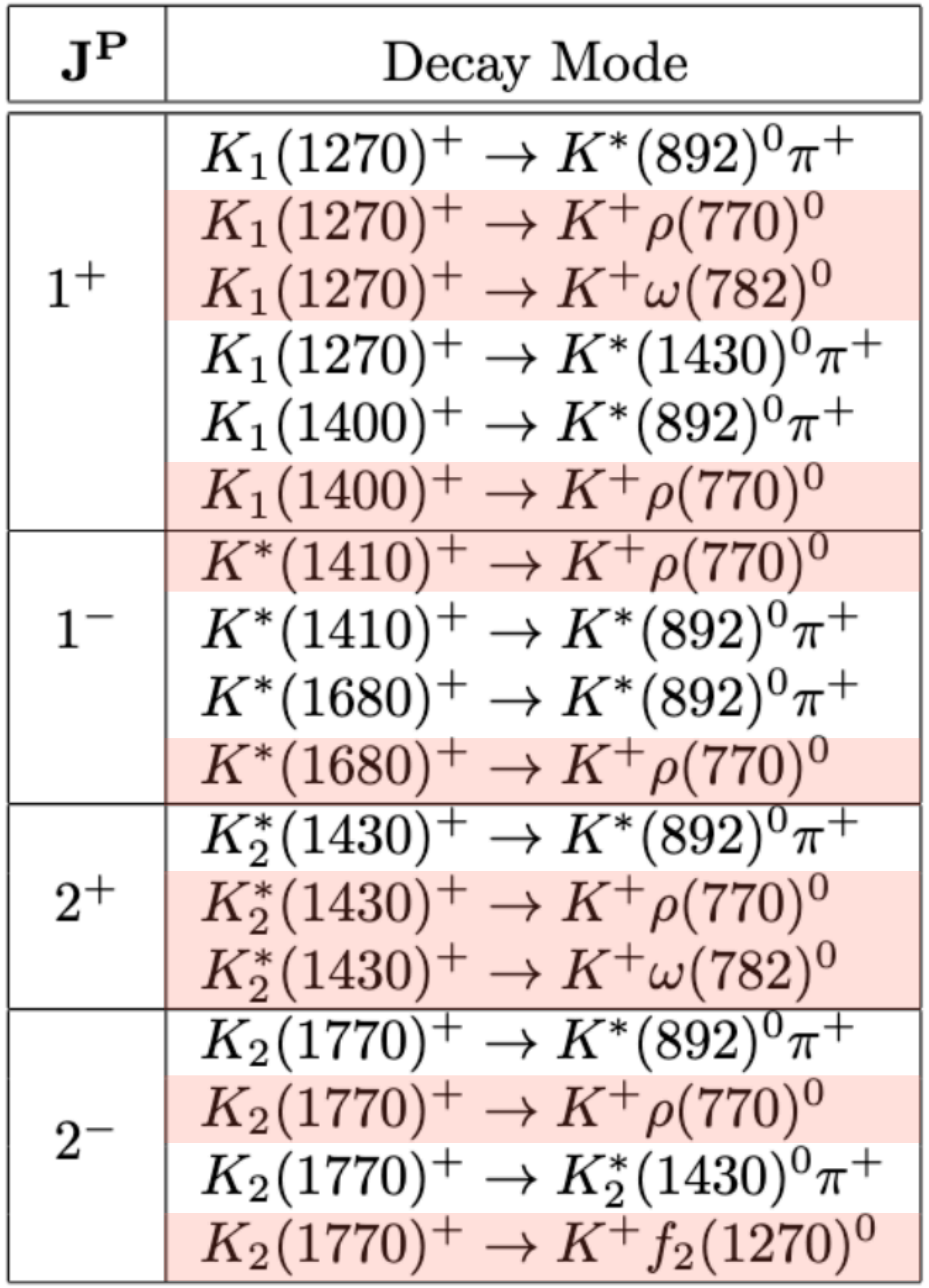
Fit to Belle II data

- Fit PDF
- Acceptance Effects
- Toy studies on MC
- Measurement of observables

Contributions to the $B^+ \to K^+\pi^+\pi^-\gamma$ decay



- The decay proceeds through 2 intermediate **resonances**
- Broadly, we have to consider $K_{res} \to \left(K^*/\rho/(K\pi)_S\right) P_1 \to K\pi\pi$ modes
- K^+_{res} can have $J^P = 1^\pm/2^\pm$ and they **interfere** with each other
- The CP-eigenstate and non-CP eigenstates must be disentangled
- The decays marked in red in the table are the CP-eigenstates
- We construct the full amplitude using the **Isobar-model.**



J^P	Decay Mode
1^+	$K_1(1270)^+ \to K^*(892)^0\pi^+$
	$K_1(1270)^+ \to K^+\rho(770)^0$
	$K_1(1270)^+ \to K^+\omega(782)^0$
	$K_1(1270)^+ \to K^*(1430)^0\pi^+$
	$K_1(1400)^+ \to K^*(892)^0\pi^+$
	$K_1(1400)^+ \to K^+\rho(770)^0$
1^-	$K^*(1410)^+ \to K^+\rho(770)^0$
	$K^*(1410)^+ \to K^*(892)^0\pi^+$
	$K^*(1680)^+ \to K^*(892)^0\pi^+$
	$K^*(1680)^+ \to K^+\rho(770)^0$
2^+	$K_2^*(1430)^+ \to K^*(892)^0\pi^+$
	$K_2^*(1430)^+ \to K^+\rho(770)^0$
	$K_2^*(1430)^+ \to K^+\omega(782)^0$
2^-	$K_2(1770)^+ \to K^*(892)^0\pi^+$
	$K_2(1770)^+ \to K^+\rho(770)^0$
	$K_2(1770)^+ \to K_2^*(1430)^0\pi^+$
	$K_2(1770)^+ \to K^+f_2(1270)^0$

Amplitude Model of $B^+ \to K^+\pi^+\pi^-\gamma$ decay

- We use a **coherent sum** of amplitudes of each contributing decay path (indexed with i) -

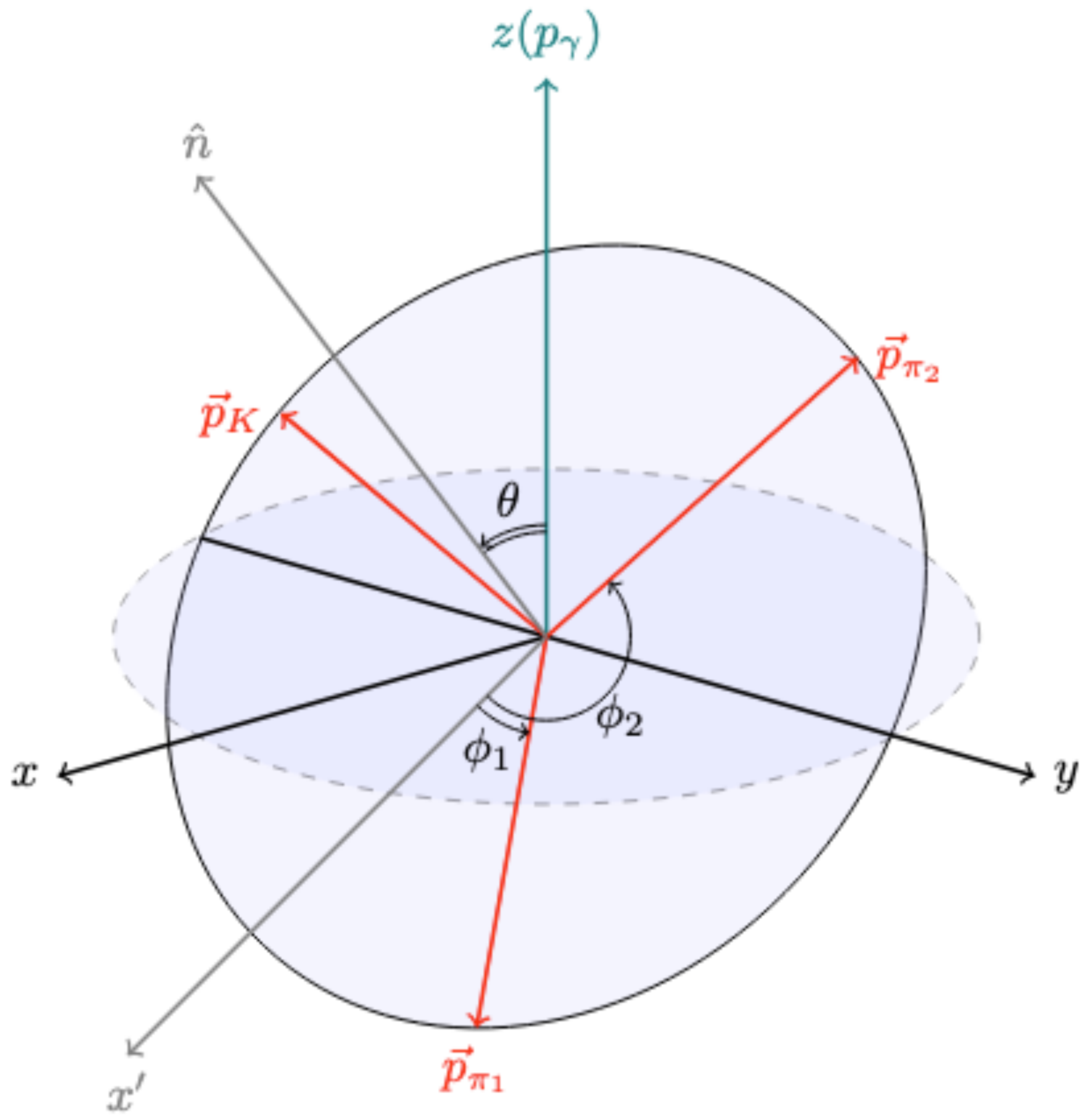
$$\mathbb{A} = \sum_i c_i \mathbb{A}_i\,, \quad d\Gamma = \frac{|\,\mathbb{A}\,|^2}{2m_B} d\Phi$$

$$\mathbb{A}_i(P) = (S_B \cdot B_{L_B}) \cdot (T_{K_{res}} \cdot S_{K_{res}} \cdot B_{L_{K_{res}}}) \cdot (T_X \cdot S_X \cdot B_{L_X})$$

- $c_i = a_i + ib_i \Rightarrow$ complex weight of each amplitude $\mathbb{A}_i$
- Angular momentum factors $\Rightarrow S_i$, Barrier factors $\Rightarrow B_i$, Lineshapes $\Rightarrow T_i$
- S_B includes radiative corrections. $S_{K_{res}/X} \Rightarrow$ **covariant tensor** formalism.

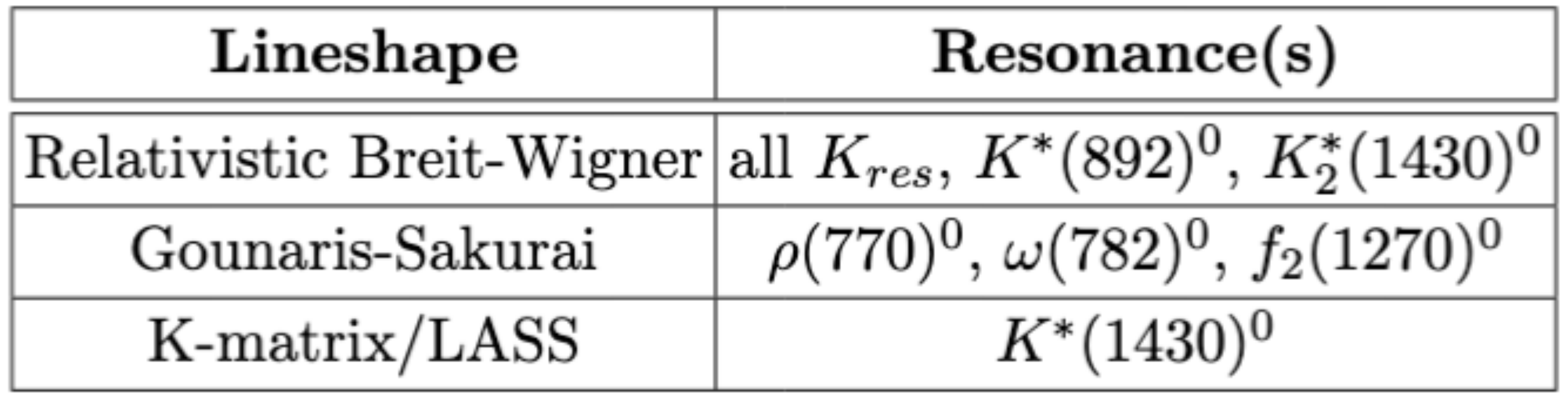
- Amplitude described in K_{res} rest frame with **5 variables** $\{m_{K^+\pi^+\pi^-},\, m_{K^+\pi^-},\, m_{\pi^+\pi^-},\, \cos\theta, \phi_1\}$
- The **Lorentz invariant phase space** is then -

$$d\Phi = \frac{1}{(2\pi)^8} \frac{m_B^2 - m_{K^+\pi^+\pi^-}^2}{16 m_B^2\, m_{K^+\pi^+\pi^-}} m_{K^+\pi^-}\, m_{\pi^+\pi^-} \cdot dm_{K^+\pi^+\pi^-}\, dm_{K^+\pi^-}\, dm_{\pi^+\pi^-}\, d\cos\theta\, d\phi_1$$



Kinematics of $B^+ \to K^+\pi^+\pi^-\gamma$ decay in K_{res} rest-frame

Resonance lineshapes



Lineshape	**Resonance(s)**
Relativistic Breit-Wigner	all K_{res}, $K^*(892)^0$, $K_2^*(1430)^0$
Gounaris-Sakurai	$\rho(770)^0$, $\omega(782)^0$, $f_2(1270)^0$
K-matrix/LASS	$K^*(1430)^0$

- All K_{res} are **Relativistic Breit-Wigner** lineshapes
- The resonances decaying to $\pi\pi$ state are considered **Gounaris-Sakurai** (53152)

- The third type of contribution to the final state comes from the $(K\pi)_S^0$ which is modelled by **K-matrix** (0705.2248)
- This mode has a pole at 1.430 GeV
- There are other lineshapes like **LASS/GLASS** which can be used and compared

Vertices for B decay

- The $B \to (V/A/T)\gamma$ vertices are taken from 0909.4627 (Covariant light-front approach)
- Comparison has been performed with 0804.3198 (Light-cone QCD sum rules)
- **Instead of using form-factors for each decay step, all form factors are absorbed into the complex weight** c_i

Detector acceptance and Selection

- Including detector acceptance effects, the fit PDF is -

$$P(x) = \frac{|\mathbb{A}(x)|^2 \epsilon(x)\Phi(x)}{\int |\mathbb{A}(x)|^2 \epsilon(x)\Phi(x)dx} = \frac{1}{N} \cdot |\mathbb{A}(x)|^2 \epsilon(x)\Phi(x)$$

$$\log(P(x)) = \log(|\mathbb{A}(x)|^2) - \log(N) + \log(\epsilon(x)\Phi(x))$$

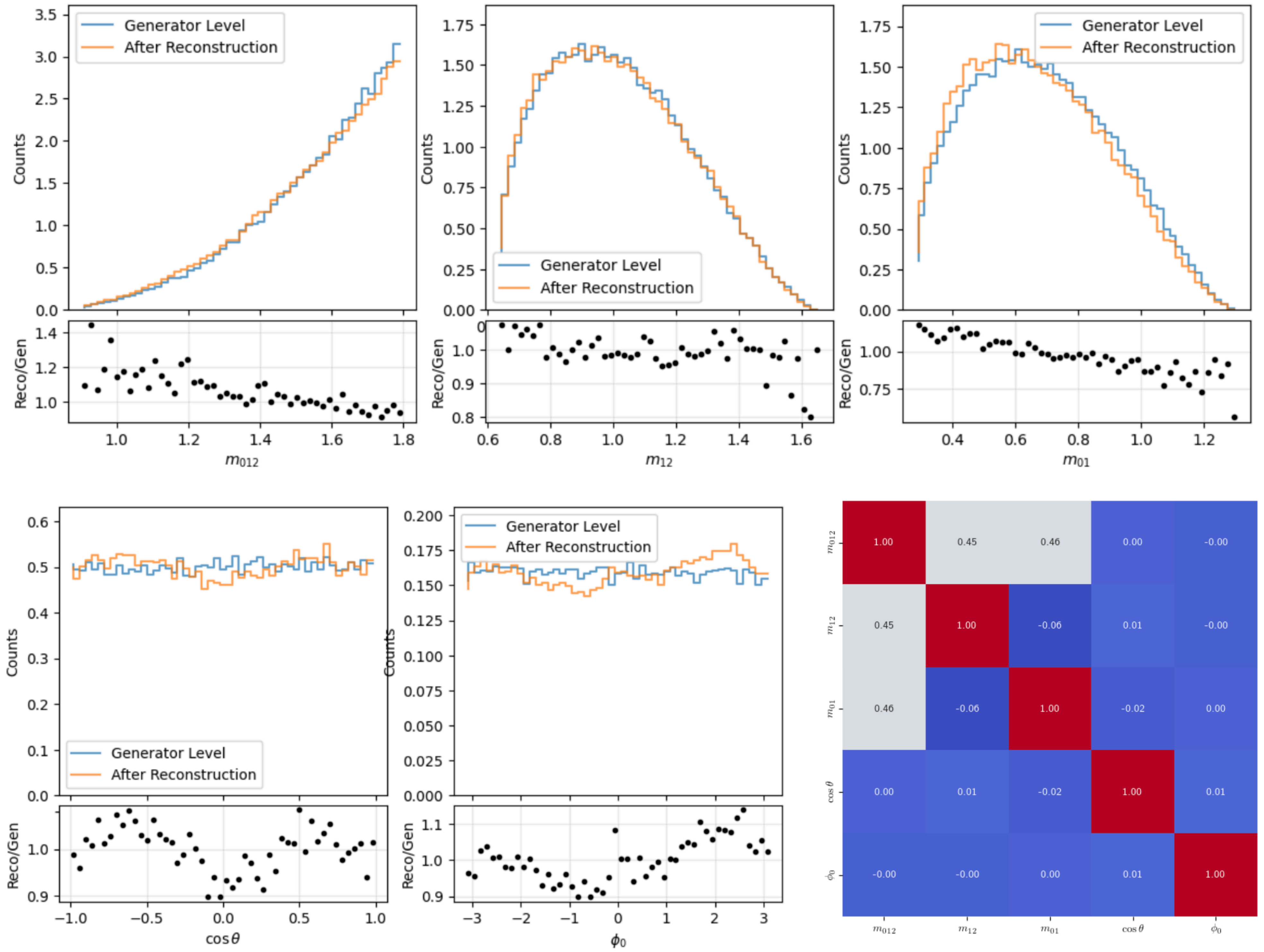
$\epsilon(x)$ is the efficiency of a point x in the phase space.

- The last term in the equation is parameter free, so we can include all acceptance and pre-selection effects into the normalisation integral.
- N can be calculated using MC integration -

$$N = \sum_{i,j} c_i c_j^* \mathbb{A}_i \mathbb{A}_j^* = \sum_{i,j} c_i c_j^* N_{ij}$$

$$N_{ij} = \frac{1}{N_{gen}} \cdot \left(\int \Phi(x) d(x) \right) \cdot \left(\sum_k^{N_{acc}} \mathbb{A}_i(x_k) \mathbb{A}_j^*(x_k) \right)$$

- The fit visualisation can be done by importance sampling on reconstructed phase space events with $|\mathbb{A}(x)|^2$



- Vertex quality cut (transformed $\chi^2 \to$ effect on $m_{\pi^+\pi^-}$ and ϕ_0
- Reconstruction → dip around 0 in $\cos\theta$, changes in ϕ_0
- $(m_{K^+\pi^+\pi^-}, m_{K^+\pi^-})$,$(m_{K^+\pi^+\pi^-}, m_{\pi^+\pi^-})$ - correlated

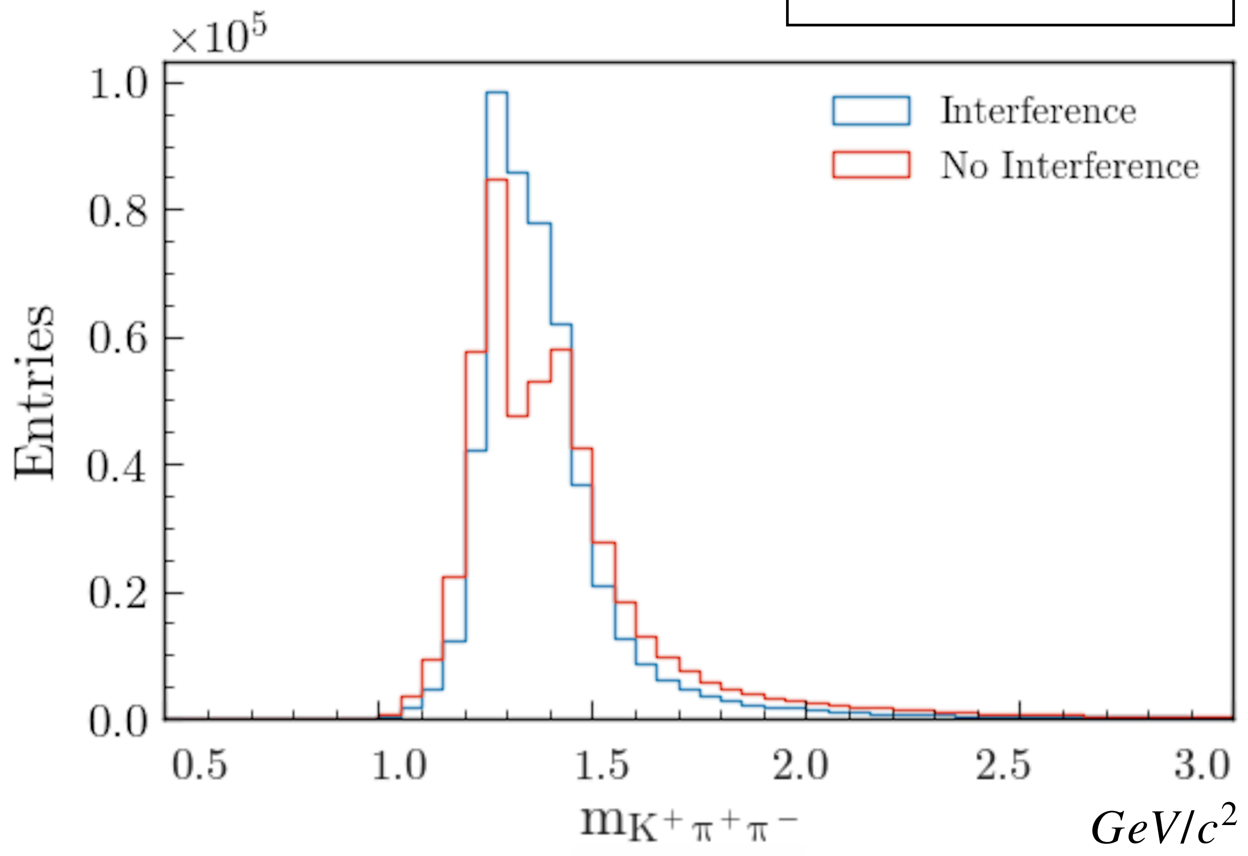
Analysis strategy

- The generic EvtGen Monte-Carlo does not include **interference** and proper **angular structure** of the decay
  - Use AmpGen to generate our signal, Belle-II **reconstruction** and **simulation** on AmpGen signal
- Signal reconstruction and selection *(M. Maushart)*. Validated with a control channel.
- After selection, to get background-free sample ⇒ sPlot *(M. Maushart)*
- Unbinned likelihood fit of Belle II data . Simultaneous amplitude fit of 5 fit variables -
  - Pole masses and widths of resonances ⇒ fixed
  - Complex weights $c_i = a_i + ib_i$ are free parameters
  - Express contribution of interfering and non-interfering parts as **fit fractions**

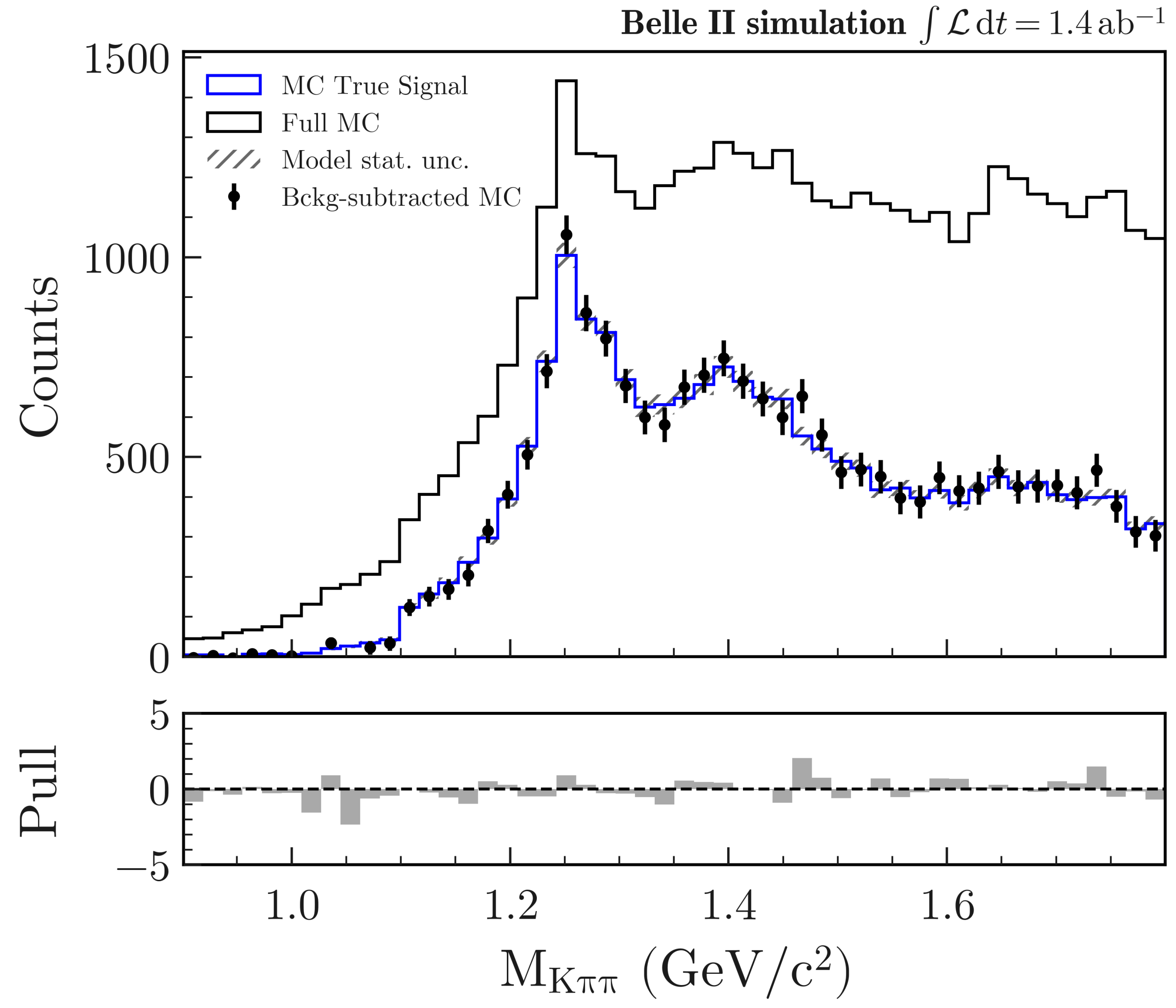
$$\text{Non-interfering: } FF_i = \frac{\int |c_i \mathbb{A}_i|^2 d\Phi}{\int |\sum_i c_i \mathbb{A}_i|^2 d\Phi} \text{Interfering:} FF_{ij,(i<j)} = \frac{\int 2\mathrm{Re}(c_i c_j^* \mathbb{A}_i \mathbb{A}_j^*) d\Phi}{\int |\sum_i c_i \mathbb{A}_i|^2 d\Phi}$$

  - Find **branching fractions** of all decay paths.
  - Find **dilution factor** $\mathscr{D}$.

MC Simulation



Effect of Interference in -
$K_1(1270) \rightarrow K^*(892)^0\pi^+$
$K_1(1400) \rightarrow K^*(892)^0\pi^+$



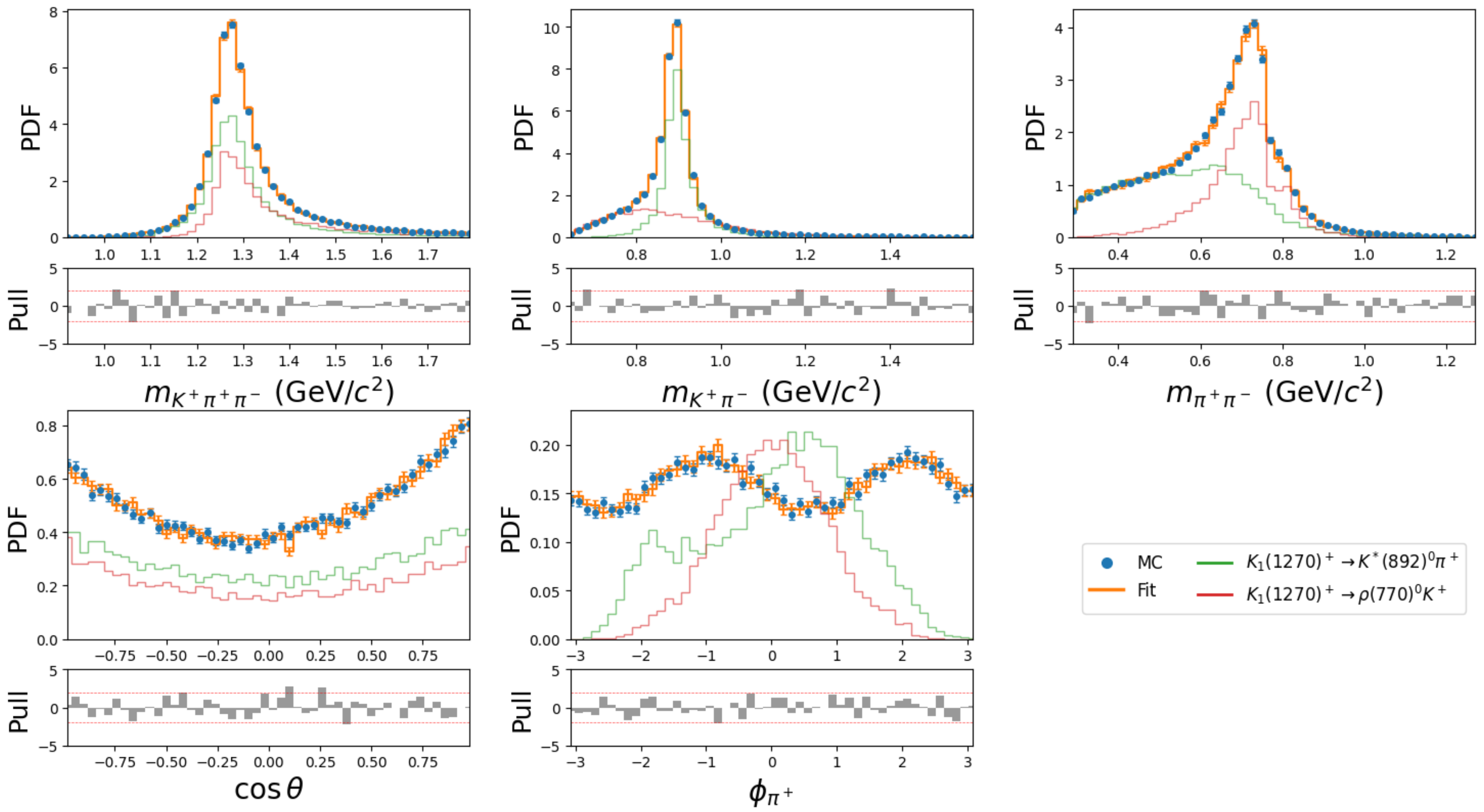
Background subtraction with sPlot
(on Belle-II generic MC)

Normalisation and Scaling

- Scaling of AmpGen amplitudes and our fit model is different
- Each decay path amplitude is normalised $\int \mathbb{A}_i \, d\Phi = 1$. Obtain $\mathbb{A}'_i$.
- The total amplitude then has a normalisation factor

$$I = \sum c_i c_j^* \int \mathbb{A}'_i \, \mathbb{A}'^*_j d\Phi$$

- This method does not allow pole masses and widths of resonances to float



An example of combination of two channels

- This causes ambiguity in parameters and fit fractions when directly comparing fit result and AmpGen truth
- Self-validations performed (sampling from fit PDF and fitting the sample)

- Combination of-
  1. $K_1(1270)^+ \to K^*(892)^0\pi^+$ 2. $K_1(1270)^+ \to \rho(770)^0K^+$
- Ampgen value: $c_1 = c_2 = 1.0 + 0.0i$
- Fit value: $c_1 = 1.0 + 0.0i$ (fixed), $c_2 = -0.74 + 0.02i$
- χ^2/dof per variable ranges from $0.85 - 1.0$

Measured Observables

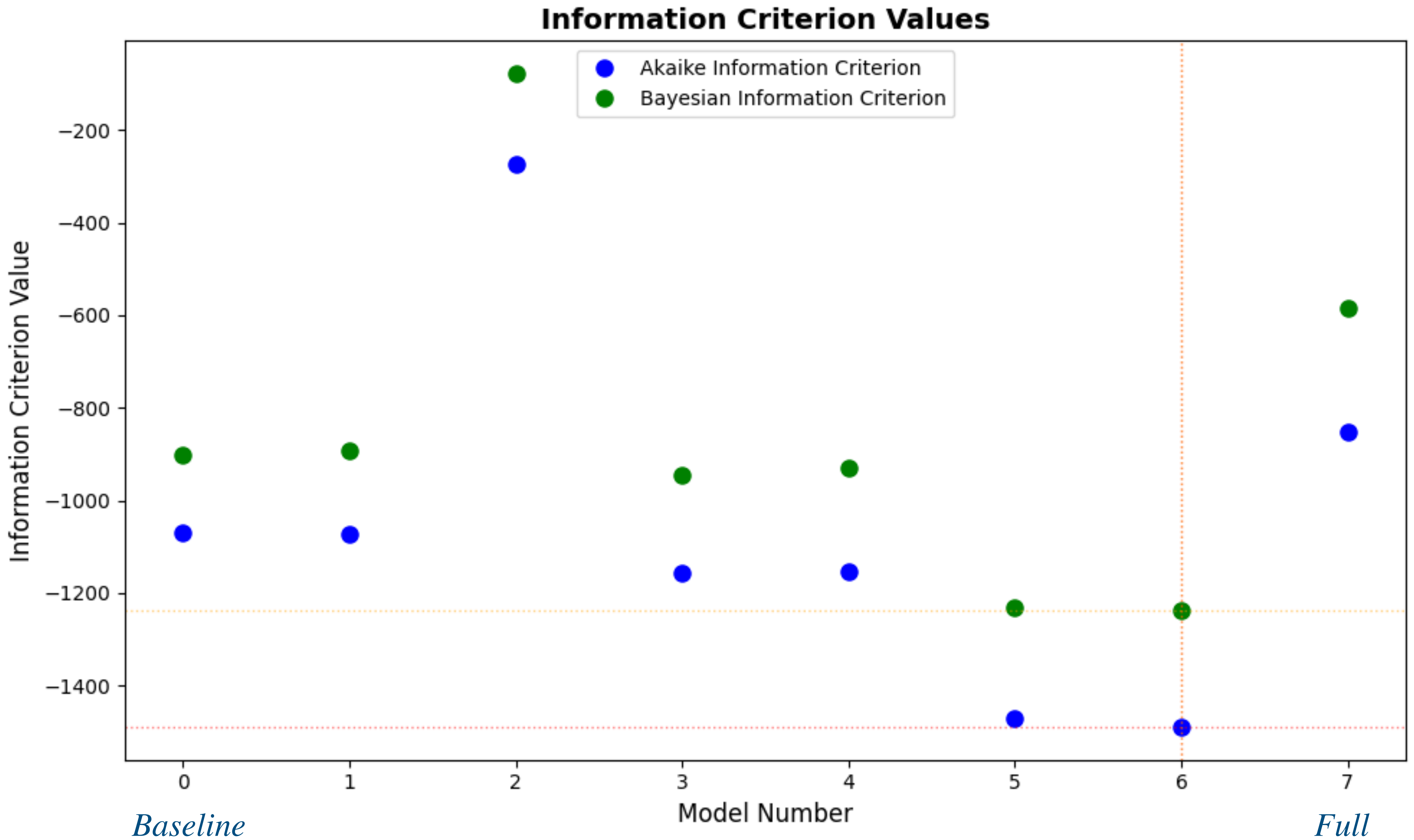
- **Dilution Factor:**
  - Calculation of $\mathscr{D}$ in JHEP09(2019)034 assumes integrating over variables $\cos\theta, \phi$ to cancel interferences over the whole phase space
  - We have a mass cut $(0.9\,\text{GeV} \leq m_{K^+\pi^+\pi^-} \leq 1.8\,\text{GeV})$. So, even after integrating they will not cancel out.
  - We find $\mathscr{D}$ in this range

- **Branching Fractions:**
  - $\mathbb{B}(B^+ \to K^+\pi^+\pi^-\gamma) = \dfrac{N_{sig}^{CR}}{\epsilon N_{B^+}}, F_i = \dfrac{\int |c_i\mathbb{A}_i|^2 d\Phi}{\int \sum_i |c_i\mathbb{A}_i|^2 d\Phi}$ (coherent fit fraction)
  - $\mathbb{B}(B^+ \to K^+_{res}\gamma \to K^+\pi^+\pi^-\gamma) = F_{K_{res}} \cdot \mathbb{B}(B^+ \to K^+\pi^+\pi^-\gamma)$
  - Weighted mean of the efficiency, $\epsilon = \sum_i \epsilon_i \dfrac{F_i}{\sum_k F_k}$
- The completion of this measurement may require further studies which may not be feasible as a part of this analysis.

Fit strategy and Unblinding

- On MC simulation:
  - Generate a large signal sample with all possible decay channels (~8000 events for 500 fb^{-1} of Belle II data)
  - Fit the sample with the combination of all possible decay channels.
    - If fit does not converge/bad reduced $\chi^2 \Rightarrow$ Reduce channels with lower contributions until baseline model
    - Choose the number of decay modes to keep (a baseline model will consist of 10 modes)
  - Perform bootstrap tests and toy studies to see stability of fit results
- The fit strategy will remain the same for data (current plan)
- Blind the dilution factor/branching fractions/fit fractions and check the validity of the fit only through goodness of fit metrics

Fit model selection



Baseline model

Full model

- In progress: A metric to finalise the selection that includes reduced χ^2 as well as information criterion.

- **Goal:** *Select a set of intermediate decay pathways in the fit PDF for the best fit*

- The same signal sample has been fitted with models of different complexities (8000 events)
- **Higher model index includes more of decay channels**
- We use two metrics:
    - Akaike Information Criterion: $2k - 2\ln(\hat{L})$
    - Bayesian Information Criterion: $k\ln(n) - 2\ln(\hat{L})$

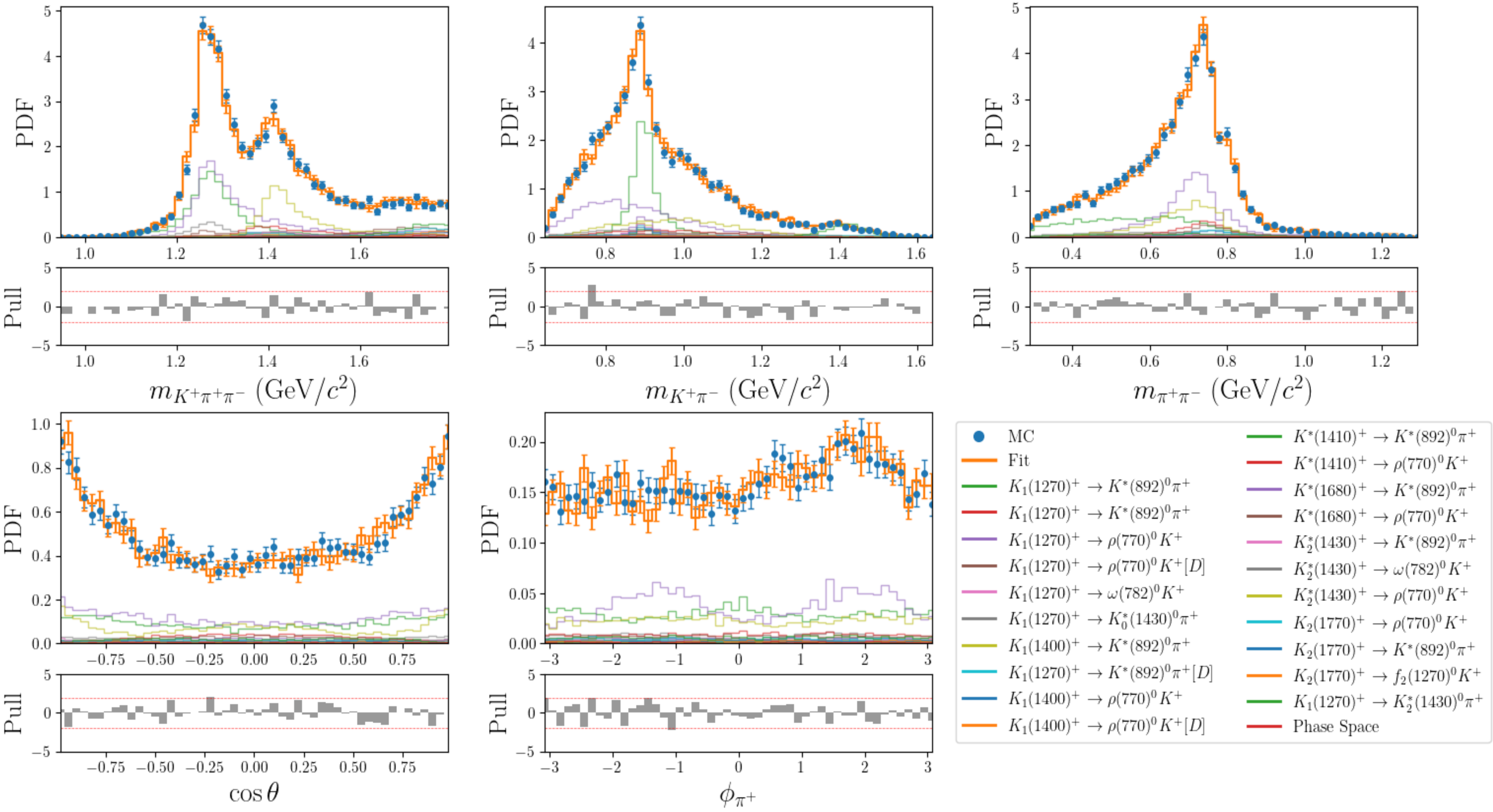
$$k = \textit{Number of free parameters},$$

$$n = \textit{Number of events}$$

$$\hat{L} = \textit{Likelihood function}$$

- Minimum values imply better selection of parameters

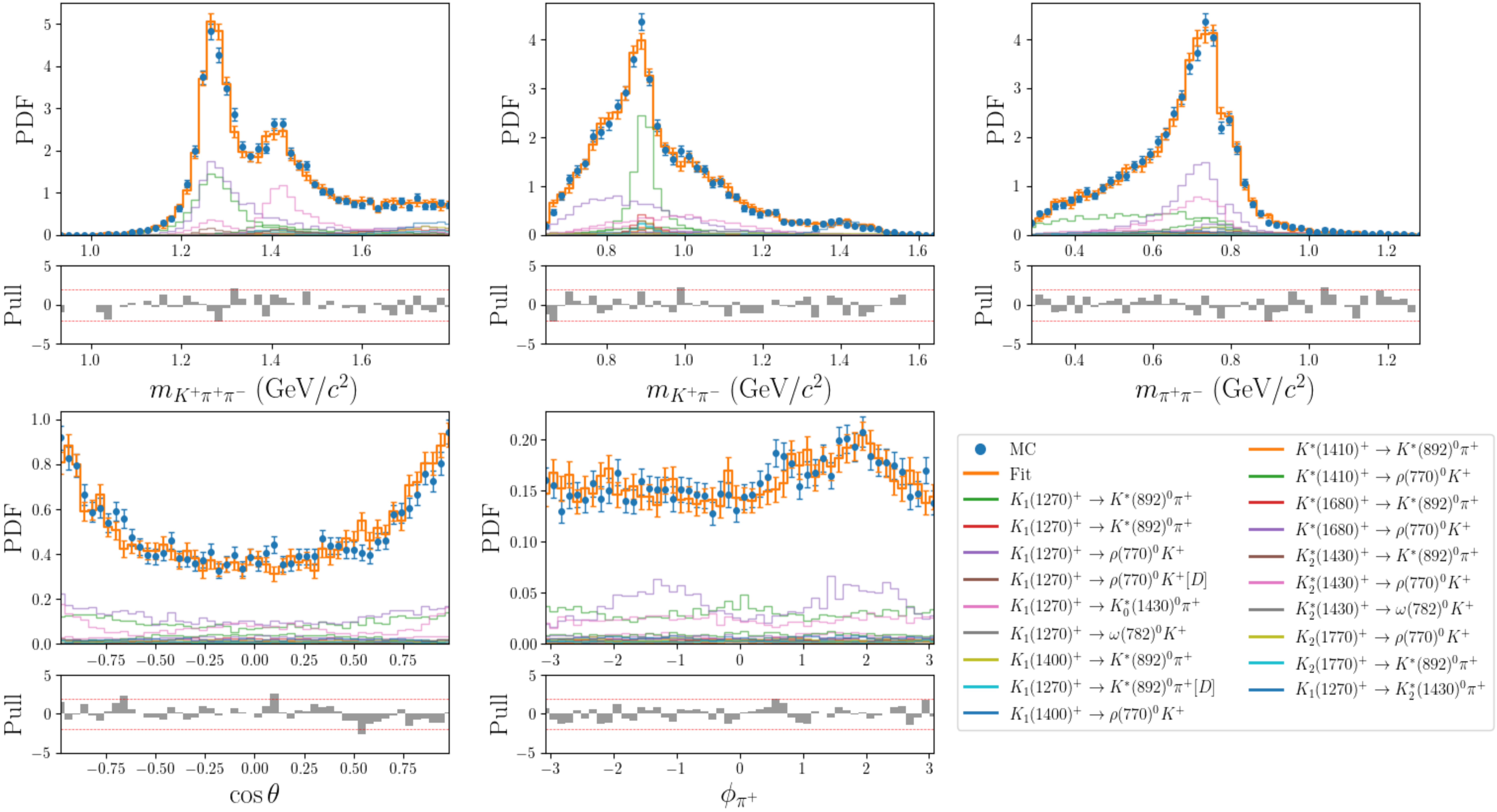
Fit result on MC : Full model



Fit on signal with fit PDF including all possible decays

- $n_{\textbf{free parameters}} = 40$
- $n_{\textbf{events}} = 8000$
- $\chi^2/dof\,(\text{mass, 3D})$ = 1.020
- $\chi^2/dof\,(\cos\theta) = 0.887$
- $\chi^2/dof\,(\phi_{\pi^+}) = 1.140$

Fit result on MC: Optimal model

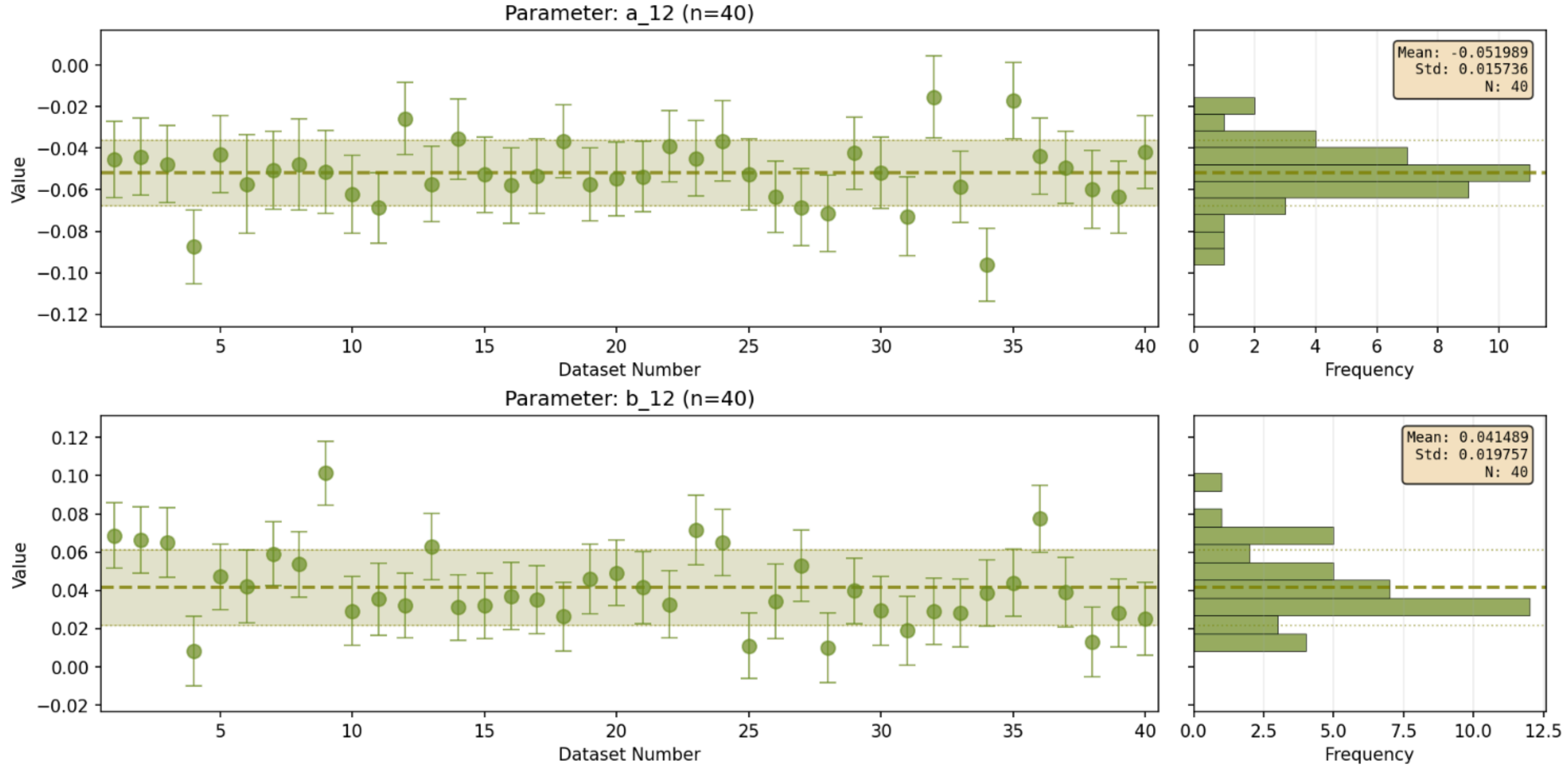


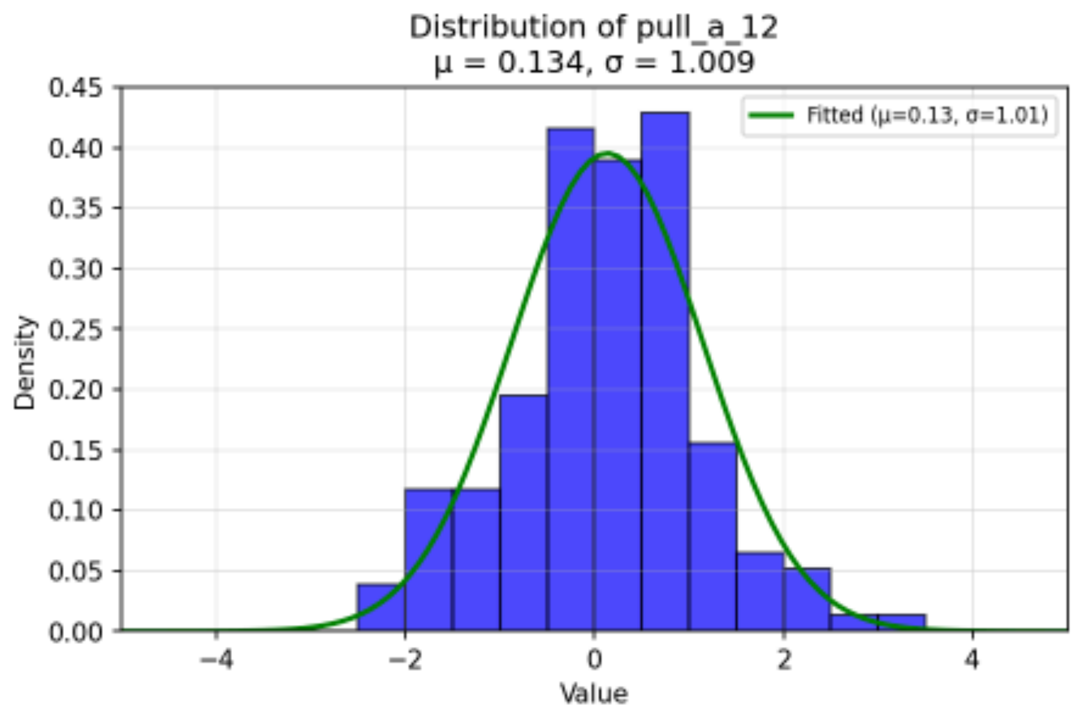
Fit on signal with best model obtained from Bayesian Information Criterion

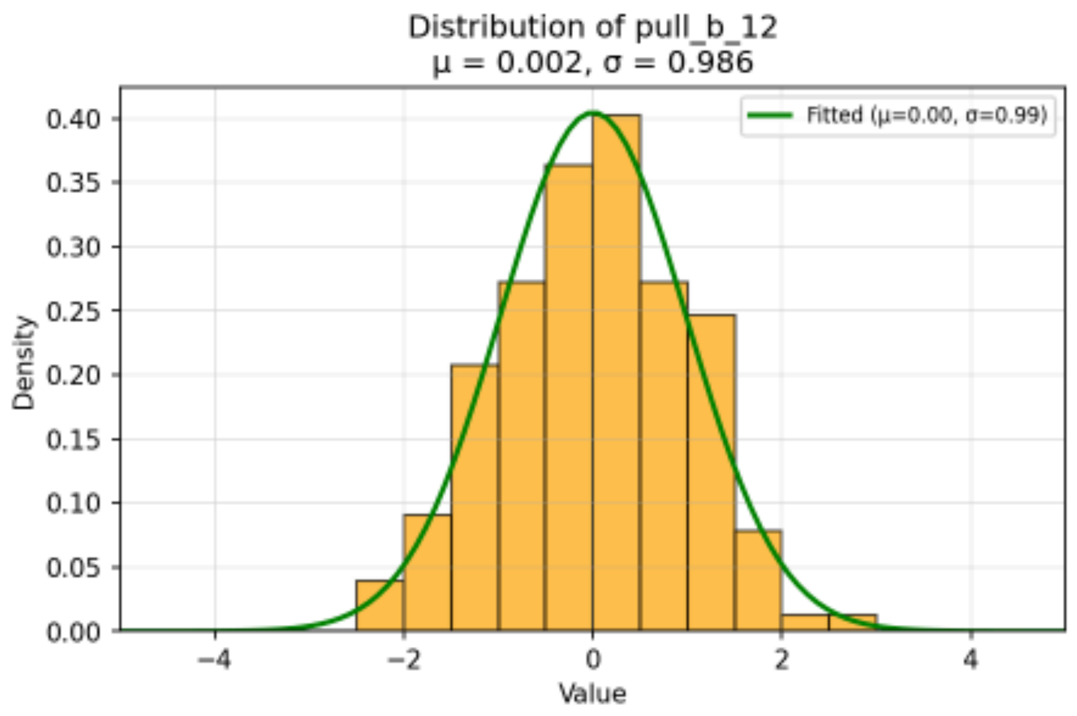
- $n_{\textbf{free parameters}} = 36$
- $n_{\textbf{events}} = 8000$
- χ^2/dof (mass, 3D) **=** 1.079
- $\chi^2/dof\,(\cos\theta) = 1.228$
- $\chi^2/dof\,(\phi_{\pi^+}) = 0.941$

Validations: Toy MC

- **Pull plots** were made for a PDF including all 20 intermediate decay paths with random blinded c_i
- $\text{Pull}_i = \dfrac{c_{i,truth} - c_{i,fit}}{\sigma_{i,fit}}$ (i is the index for each intermediate path)
- We get a gaussian distribution for all pulls $\mu \sim 0.0, \;\; \sigma \sim 1.0$
- Similar observations for fit-fraction pull plots

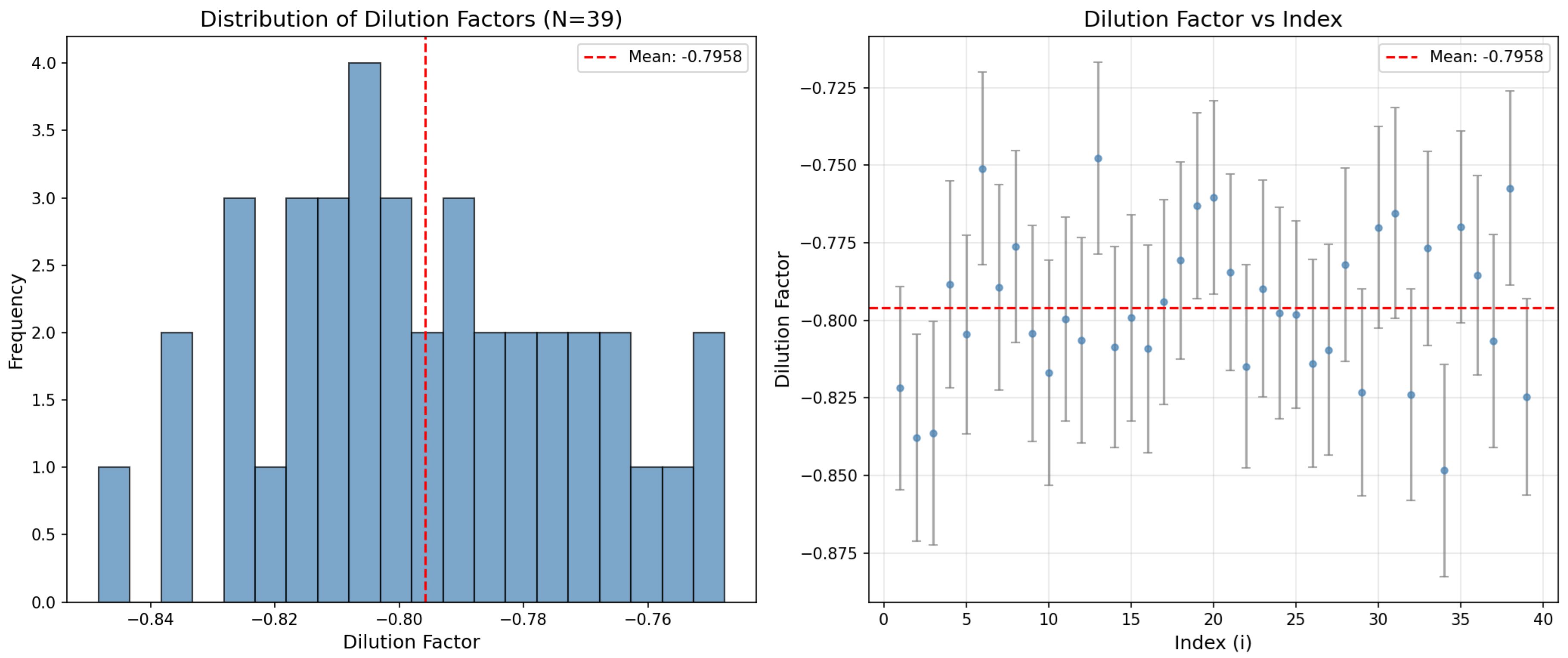






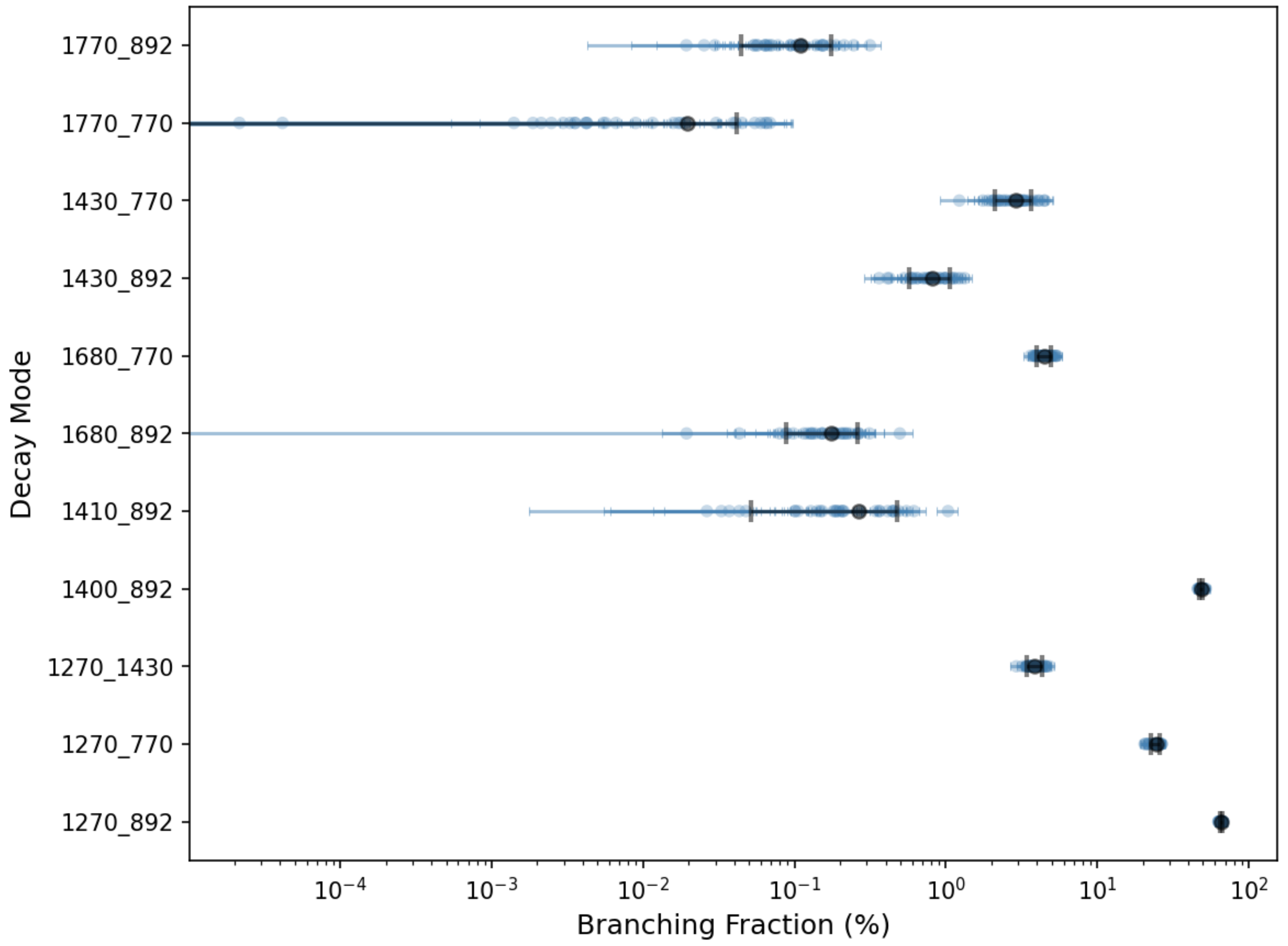
- **Stability** of fit parameters in 40 signal datasets generated with the same parameters

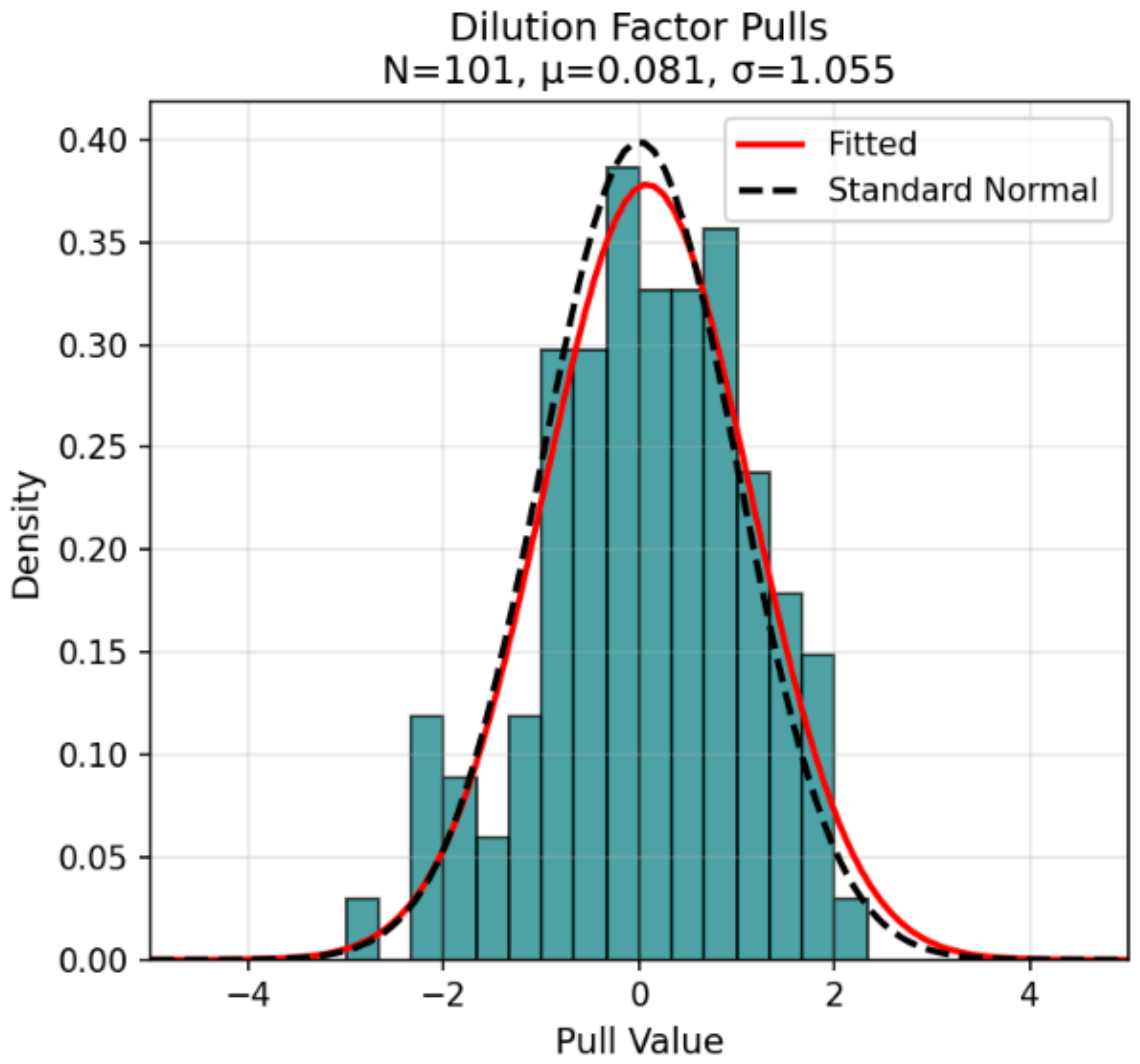
Validations: Observables



- Dilution factor $\mathscr{D}$ calculated after fit to 39 MC samples generated with same parameters
- $\mathscr{D}$ = -0.79 ± 0.0053
- The value displayed does not correspond to final expectations

- Branching fractions of different modes after fit to 39 datasets generated with same parameters





- Pull plot of dilution factor: Fit on generator-level data vs truth values

Trainings

- All hours of scientific training and transversals have been completed.
- Attended 2 physics schools - IDAPSC School (IJCLab, Orsay), ESHEP (CERN, Benasque)
- Conference: Moriond EW proceeding: 2604.26315

Belle II Data Production

- Role: Data Processing Manager
- More than 20 processing requests completed
- Involves processing/reprocessing of Run-1 and Run-2 data, testing new software releases, and production of background overlays.

Outlooks

- Moriond EW Proceeding: 2604.26315
- Belle II internal note in preparation. (~September 2026).
- From ~October 2026, focus will be on collaboration internal review.
- Thesis report preparation commences ~March 2027.
- We expect to finish obtaining results after unblinding and a possible publication by ~November 2027.
- For analysis:
    - Possible improvement in resonance lineshapes (Eg: $K_1(1270)^+$, $(K\pi)^0_S$)
    - Improvement of detector acceptance modelling with higher statistics
    - Re-performing toy-studies after finalisation of reconstruction and selection
    - Fit checks on data with sWeights (stability, fit-fraction pulls)