

IRN Terascale @ Heidelberg

Hunting for Heavy Neutral Leptons at Future Lepton Colliders

Based on [2412.04331] & [2512.16734] with J. Kriewald, and A. M. Teixeira

Emanuelle Pinsard

DESY

HELMHOLTZ



Neutrinos beyond the SM

Standard Model lepton sector: neutrinos are strictly **massless**
⇒ **no source of charged Lepton Flavour Violation**

Neutrino oscillations: 1st laboratory **evidence of New Physics**

↪ neutrinos are **massive** & **leptons mix** $\mathcal{U}_{\alpha i}^{\text{PMNS}}$

But many new open questions:



Absolute ν **mass scale**

Ordering of the light ν spectrum

Nature (**Dirac** vs. **Majorana**)

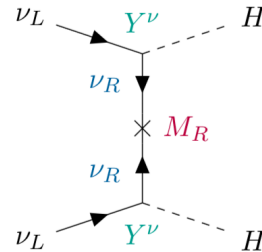
Lepton number violation

New sources of **CP violation**

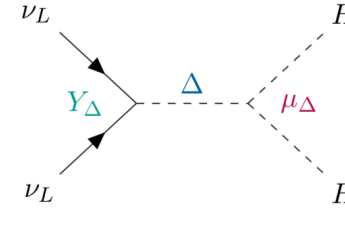
New **states to generate neutrino masses**

Generating neutrino masses

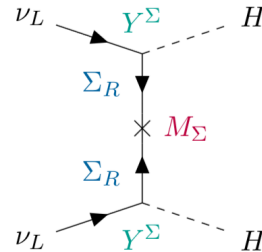
➡ **Seesaw** (and its variants):
one of the most appealing
mechanisms for m_ν **generation**



Type I



Type II

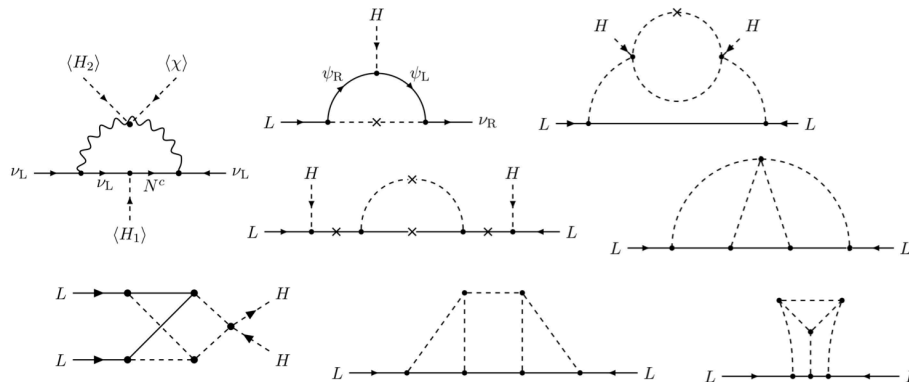


Type III

➡ **Low-scale** realisations:

Inverse seesaw, **linear seesaw**, ν **MSM**, ...

➡ Several **other well-motivated** possibilities:



Cai et al. [1706.08524]

Tree or **loop(s)** generation,

Dirac or **Majorana**,

in SUSY, GUTs, with LQs, sterile ν , ...

Addressing (or not) other **tensions**

or **observational** caveats of the **SM**

Comprehensive review, Cepedello [2105.01896]

Introducing sterile fermions

Sterile fermions: ($\sim \nu_R$ RH neutrinos)

present in SM extensions accounting for **neutrino masses & mixings**

- ↪ Interactions with SM fields only through **mixings** with active neutrinos
- ↪ No bound on the **number** of the sterile states
- ↪ Common to numerous NP models, wide range of **scales**

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⇒ **Phenomenological** implications strongly depend on their **masses**

Low-scale seesaw ↪ non-decoupled states, **modified lepton currents!**

Rich phenomenology at low-energies, high-intensity and colliders

EW precision tests, cLFV transitions and decays, $0\nu 2\beta$ decays, rare meson decays, ...

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Rich phenomenology intensity and colliders

EW precision tests, cLFV, mesons and decays, $0\nu 2\beta$ decays, rare meson decays, ...

Testability!

Impact of sterile neutrinos

How does the presence of **sterile neutrinos** affect the **SM** predictions?

Active-sterile mixing $\mathcal{U}_{\alpha i}$

Neutrino spectrum:

3 light (mostly active) neutrinos

1 **heavy non-decoupled** state,
sizeable **mixing** with active ν

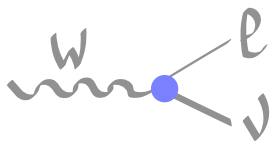
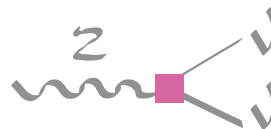
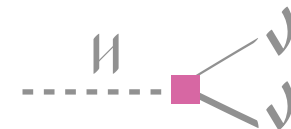
Left-handed lepton mixing $\tilde{\mathcal{U}}_{\text{PMNS}}$

3×3 sub-block **non-unitary**!

$$\mathcal{U}_{4 \times 4} = \begin{pmatrix} \begin{matrix} \mathcal{U}_{e1} & \mathcal{U}_{e2} & \mathcal{U}_{e3} & \mathcal{U}_{e4} \\ \mathcal{U}_{\mu1} & \mathcal{U}_{\mu2} & \mathcal{U}_{\mu3} & \mathcal{U}_{\mu4} \\ \mathcal{U}_{\tau1} & \mathcal{U}_{\tau2} & \mathcal{U}_{\tau3} & \mathcal{U}_{\tau4} \end{matrix} & \begin{matrix} \mathcal{U}_{s1} \\ \mathcal{U}_{s2} \\ \mathcal{U}_{s3} \\ \mathcal{U}_{s4} \end{matrix} \\ \begin{matrix} \mathcal{U}_{s1} \\ \mathcal{U}_{s2} \\ \mathcal{U}_{s3} \\ \mathcal{U}_{s4} \end{matrix} & \begin{matrix} \mathcal{U}_{s1} \\ \mathcal{U}_{s2} \\ \mathcal{U}_{s3} \\ \mathcal{U}_{s4} \end{matrix} \end{pmatrix} \begin{matrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \end{matrix}$$

Arrows from the text blocks point to the corresponding parts of the matrix: $\mathcal{U}_{\alpha i}$ points to the top-left 3×3 sub-block, and $\tilde{\mathcal{U}}_{\text{PMNS}}$ points to the bottom-left 1×3 sub-block.

$\Rightarrow$ **Modified charged & neutral** lepton currents!



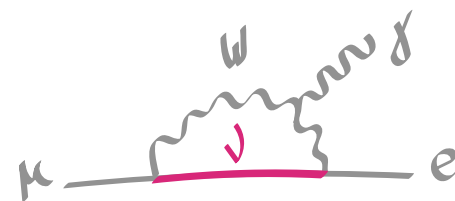
Impact of modified lepton currents

Sterile neutrinos & non-unitary **leptonic mixing**

Sizeable impact on **flavour observables** both at **tree** & **loop** level

↪ **Charged lepton flavour violation**: *Strictly forbidden in the SM*

Expect **significant contribution**
from **sterile neutrino** extensions



Negative cLFV searches

⇒ **Strong bounds** on **HNL** extensions: often **most stringent** ones

↪ **Electroweak precision observables** & **Lepton flavour universality**

Measurements in “good” agreement with **SM predictions**

⇒ **Strong bounds** on **HNL** constructions

Where to search for **sterile neutrinos**?

↪ Via searches @ **high-intensities**

e.g. MEG, COMET, Mu2e, Mu3e, Belle II, neutrino dedicated, TRIUMF, NA62, ...

Meson decay & beam dump exp.: HNL **decay products** or **missing energy**

Neutrino **oscillation** experiments, Low-energy **cLFV** signatures,

Beta decay and **Neutrinoless double beta decay**

(Mainly) probe **low masses** (together with cosmological searches)

↪ And searches @ **colliders**

e.g. LHC, future CLIC, future FCC-ee, future muon colliders

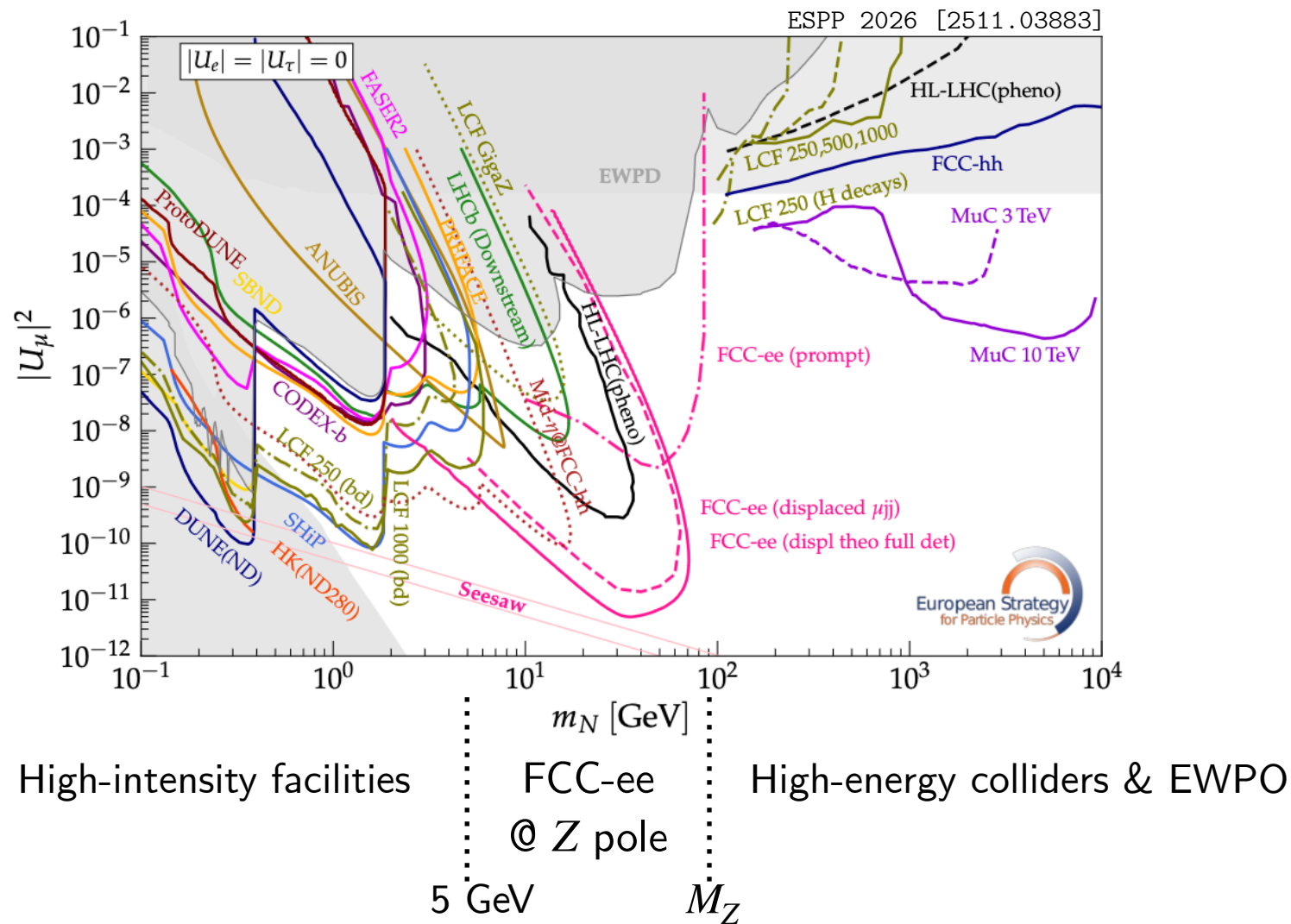
Direct production and **decay products**: $N \rightarrow W\ell$, $N \rightarrow Z\nu$, $N \rightarrow h\nu$

Displaced vertices, **LVN** signatures, **EWPO**

Allow probing **heavy masses**

Searches for HNL

Simplified scenario: **1 HNL** with **only 1 mixing** to 1 type of **active ν**



HNL mediated cLFV scattering at μ TRISTAN

μ TRISTAN future **asymmetric** μ^+e^- collider (and possibly $\mu^+\mu^+$)

➤ 3 km ring configuration

➤ Nominal **beam energies**

➤ **Polarisation** of beams

➤ $L^{\text{int}} \sim 100 \text{ fb}^{-1}$ per year
 $L^{\text{int}} \sim 1 \text{ ab}^{-1}$ in 10 years

E_{e^-} (GeV)	E_{μ^+} (GeV)	$\sqrt{s}$ (GeV)
20	200	126.5
30	1000	346.4
50	1000	447.2
100	3000	1095.4

[See e.g. Yamada et al, 2201.06664]

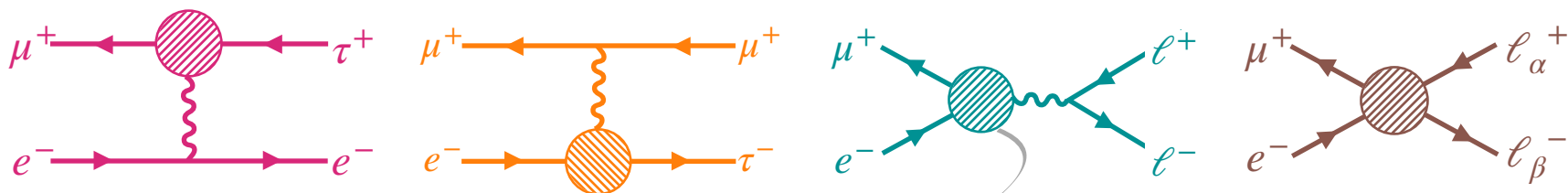
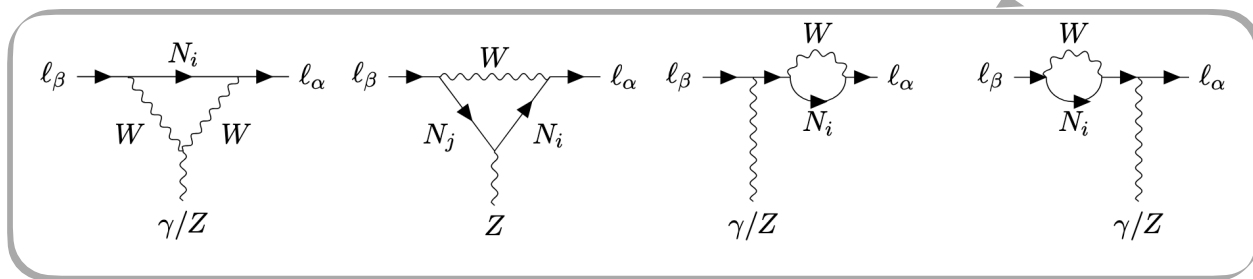
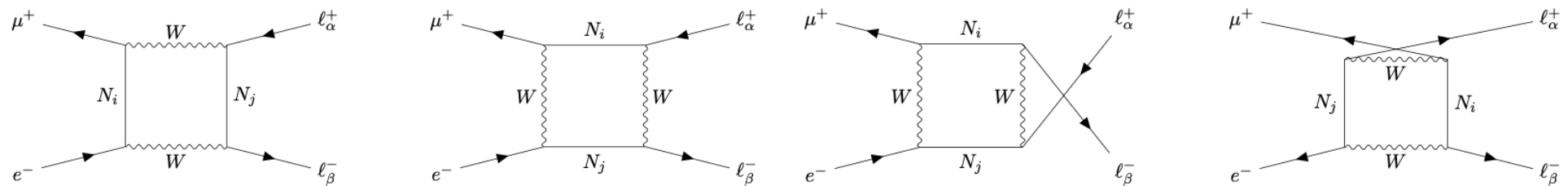
With μ^+e^- beams, **SM** processes are mainly from **higher-order**

⇒ **Reduces physical background** (vs. $\ell^+\ell^-$ colliders)

cLFV scattering at μ TRISTAN

μ TRISTAN future **asymmetric** μ^+e^- collider ($\sqrt{s} \approx 126 \text{ GeV} - 1 \text{ TeV}$)

Consider **HNL contributions** to $\mu^+e^- \rightarrow \ell_\alpha \ell_\beta$ ($\mu^+\tau^-$, τ^+e^- , $\tau^+\tau^-$, $e^+\mu^-$)



HNL mediated scattering

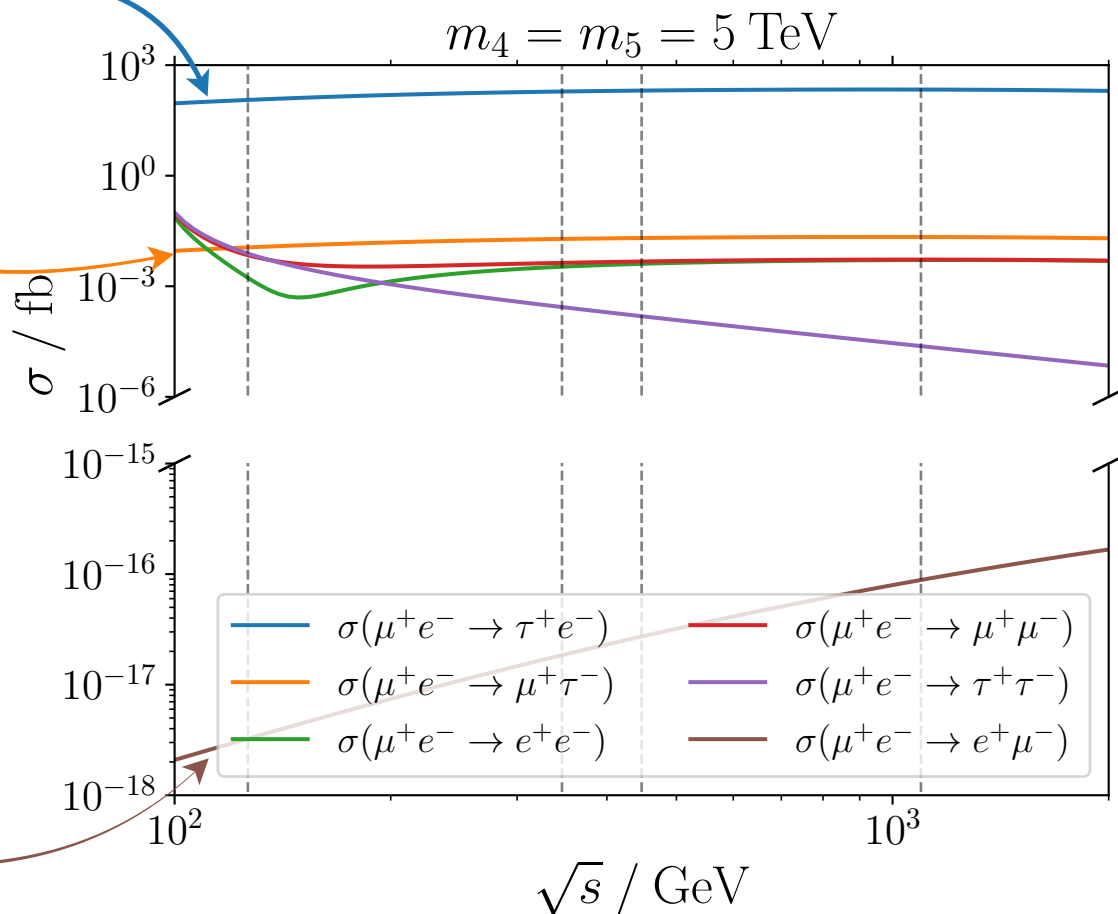
Illustration in **minimal 3+2** for a benchmark point **consistent with data**

t -channel mediated:

near **independence of s**

e^+e^- and $\mu^+\mu^-$ final states
competing s and t channels
(s at low c.o.m, interferences,
then t dominates)

Boxes subdominant

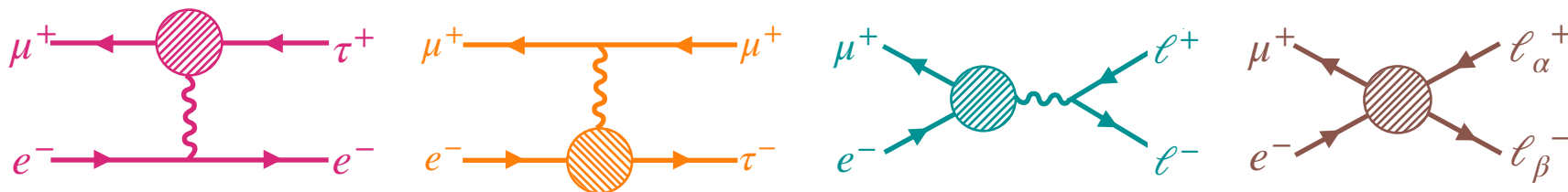


[Kriewald, EP, Teixeira, 2412.04331]

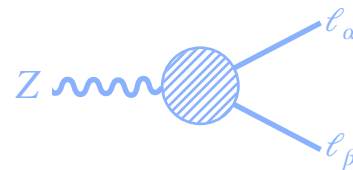
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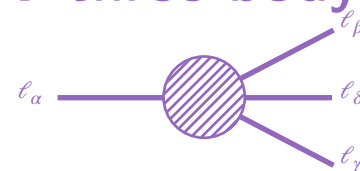
➤ Consider **HNL contributions** to $\mu^+e^- \rightarrow \ell_\alpha \ell_\beta$ ($\mu^+\tau^-$, τ^+e^- , $\tau^+\tau^-$, $e^+\mu^-$)



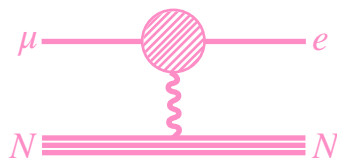
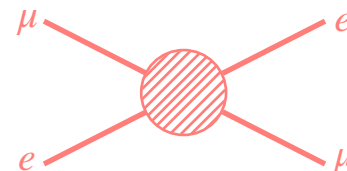
➤ Clear connection with **cLFV Z decays**



& low-energy cLFV: **cLFV radiative decays** & **cLFV three-body decays**



$\mu - e$ conversion in nuclei & **muonium oscillations**



cLFV scattering at μ TRISTAN

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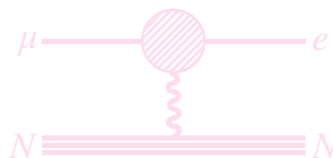
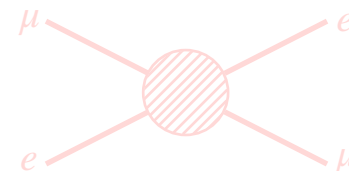
→ Consider **HNL contributions** to $\mu^+e^- \rightarrow \ell_\alpha \ell_\beta$ ($\mu^+\tau^-$, τ^+e^- , $\tau^+\tau^-$, $e^+\mu^-$)

Can $\mu^+e^- \rightarrow \ell_\alpha \ell_\beta$ scattering at μ TRISTAN be **competitive** with other **cLFV searches?** (low-energy and Z pole)

Illustrate results with **well-motivated m_ν mechanism: ISS(3,3)**



$\mu - e$ conversion in nuclei & muonium oscillations

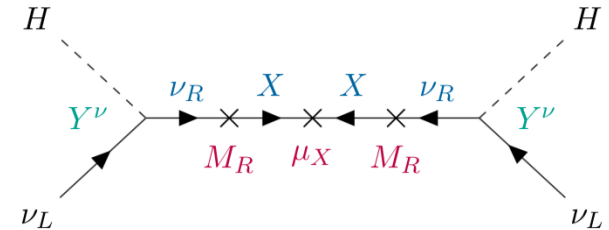


(Inverse seesaw mechanism)

Low-scale mechanism for **neutrino mass generation**

↪ **ISS(3,3)**: SM + 3 **RH neutrinos** ν_R and 3 **sterile states** X

$$\mathcal{L}_{\text{ISS}} = -\frac{1}{2} (\overline{\nu}_L^c, \overline{\nu}_R, \overline{X}^c) \begin{pmatrix} 0 & m_D^T & 0 \\ m_D & \mu_R & M_R \\ 0 & M_R^T & \mu_X \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \\ X \end{pmatrix}$$



3 **light** (mostly active) neutrinos: $m_\nu \simeq m_D^2 \frac{\mu_X}{M_R^2}$

3 **heavy pseudo-Dirac** pairs: $M_{\text{heavy}} \approx M_R \pm \mu_X$

Smallness of m_ν from interplay of 2 scales:

M_R and μ_X

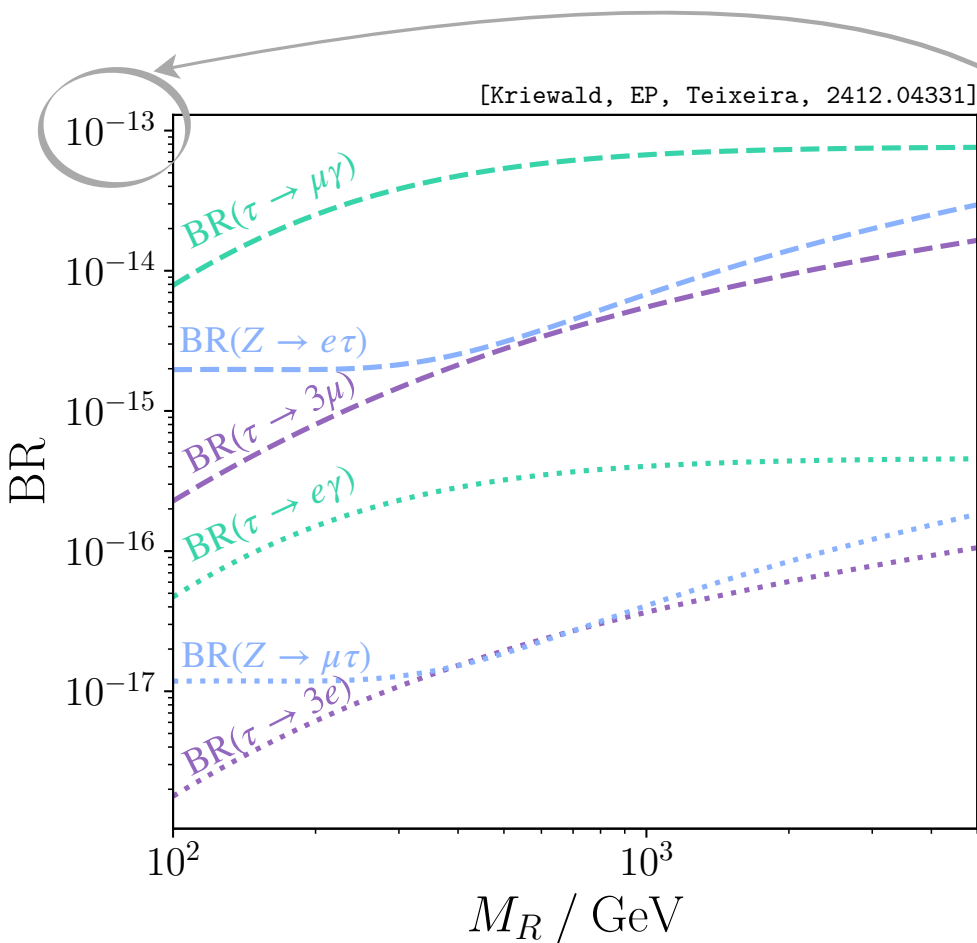
$\mu_X \rightarrow 0$ **lepton number** is restored $\Rightarrow$ Small values of μ_X technically natural

↪ **Low NP scale** $\sim M_R$ and **sizeable mixings** ($\theta \propto m_D M_R^{-1}$)

Very rich phenomenology

μ TRISTAN and τ -cLFV

Low-energy τ -cLFV and $Z \rightarrow \tau\ell$ vs. $\sigma(\mu^+e^- \rightarrow \ell_\alpha^+ \ell_\beta^-)$ at μ TRISTAN



In the **Inverse Seesaw (3,3)**
 $\Rightarrow$ **Low-energy τ -cLFV**
and **cLFV $Z \rightarrow \tau\ell$**
far below future sensitivities

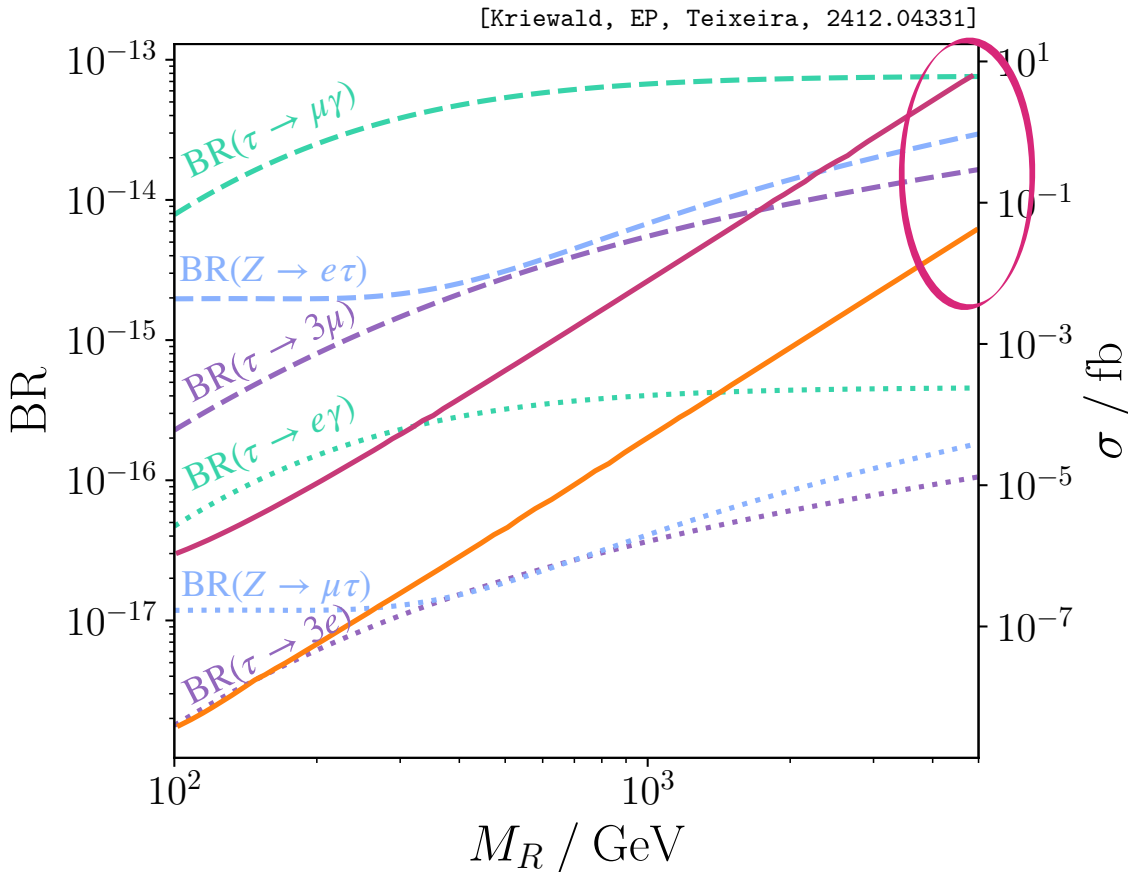
$$BR(\tau \rightarrow \ell\gamma) \lesssim 10^{-9}$$

$$BR(Z \rightarrow \tau\ell) \lesssim 10^{-10}$$

$$BR(\tau \rightarrow 3\ell) \lesssim 10^{-10}$$

μ TRISTAN and τ -cLFV

Low-energy τ -cLFV and $Z \rightarrow \tau \ell$ vs. $\sigma(\mu^+ e^- \rightarrow \ell_\alpha^+ \ell_\beta^-)$ at μ TRISTAN



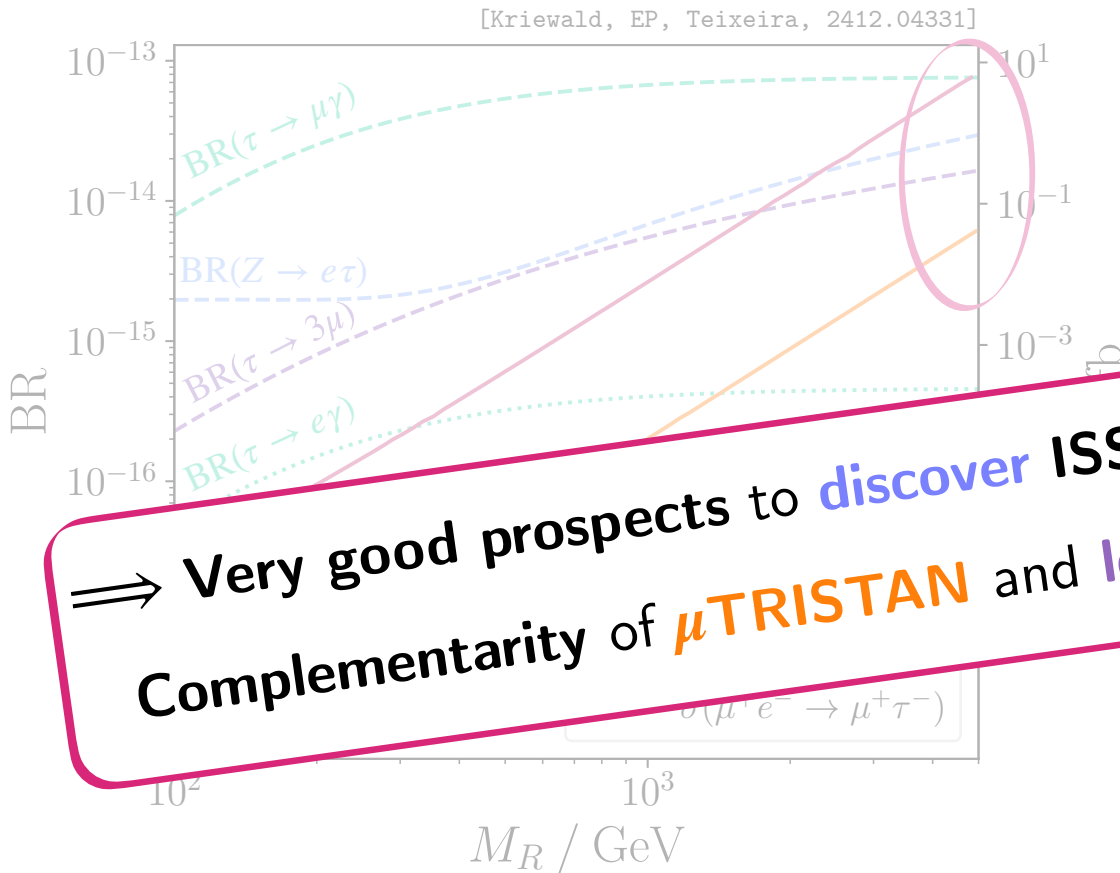
In the **Inverse Seesaw (3,3)**
 $\Rightarrow$ **Low-energy τ -cLFV**
and **cLFV $Z \rightarrow \tau \ell$**
far below future sensitivities

BUT sizeable $\mu^+ e^- \rightarrow \tau^+ e^-$
& $\mu^+ e^- \rightarrow \mu^+ \tau^-$
Leading up to **100(0)** events

Competitive prospects for
 τ -cLFV searches at μ TRISTAN

μ TRISTAN and τ -cLFV

Low-energy τ -cLFV and $Z \rightarrow \tau\ell$ vs. $\sigma(\mu^+e^- \rightarrow \ell_\alpha^+\ell_\beta^-)$ at μ TRISTAN



In the **Inverse Seesaw (3,3)**
 $\Rightarrow$ **Low-energy τ -cLFV**
and **cLFV $Z \rightarrow \tau\ell$**
far below future sensitivities

$\Rightarrow$ **Very good prospects to discover ISS(3,3) via τ -cLFV scattering**
Complementarity of μ TRISTAN and low-energy τ -cLFV searches

Leading up to 100(0) events

Competitive prospects for τ -cLFV searches at μ TRISTAN

ISS(3,3) at μ TRISTAN

Constraining the ISS(3,3) parameter space (μ_X , M_R) with μ TRISTAN?

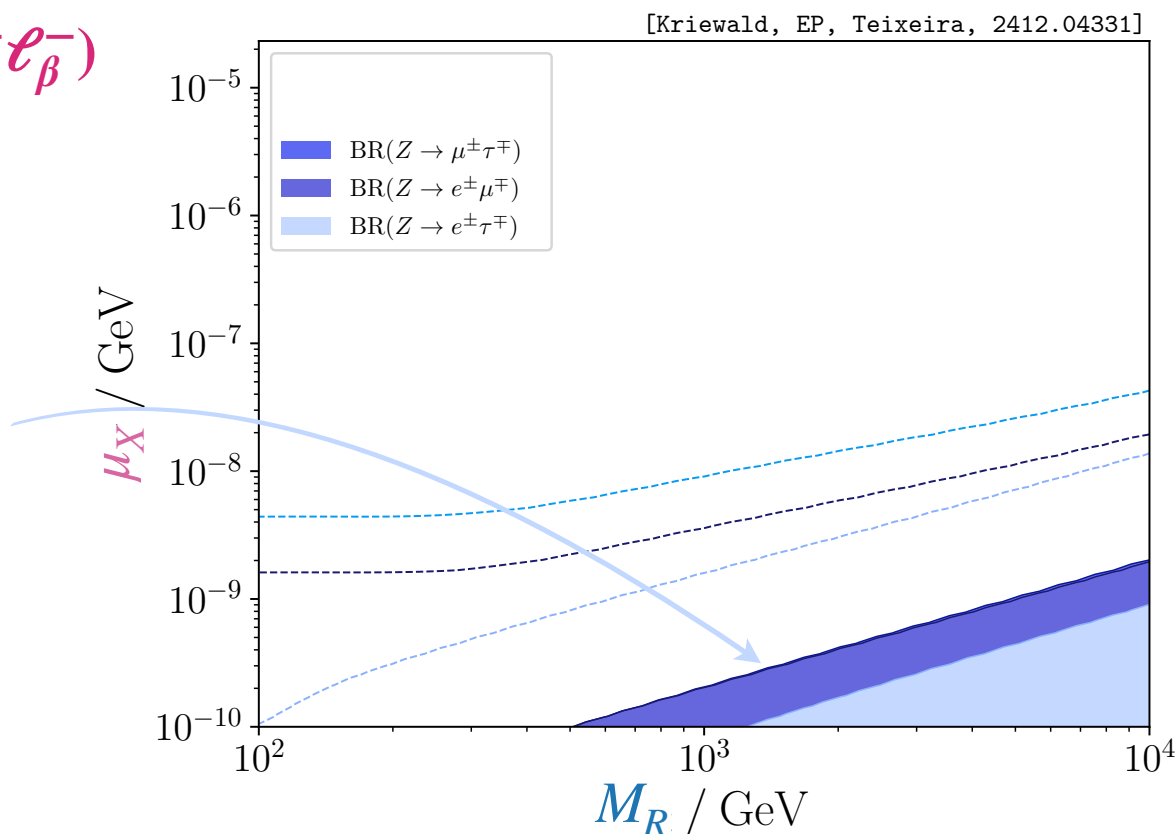
$$Z \rightarrow \ell_\alpha \ell_\beta \text{ vs. } \sigma(\mu^+ e^- \rightarrow \ell_\alpha^+ \ell_\beta^-)$$

$Z \rightarrow \ell_\alpha \ell_\beta$ constrains **small part** of the ISS(3,3) parameter space:

very **small** μ_X & **large** M_R

Future sensitivity:

expect some improvement



ISS(3,3) at μ TRISTAN

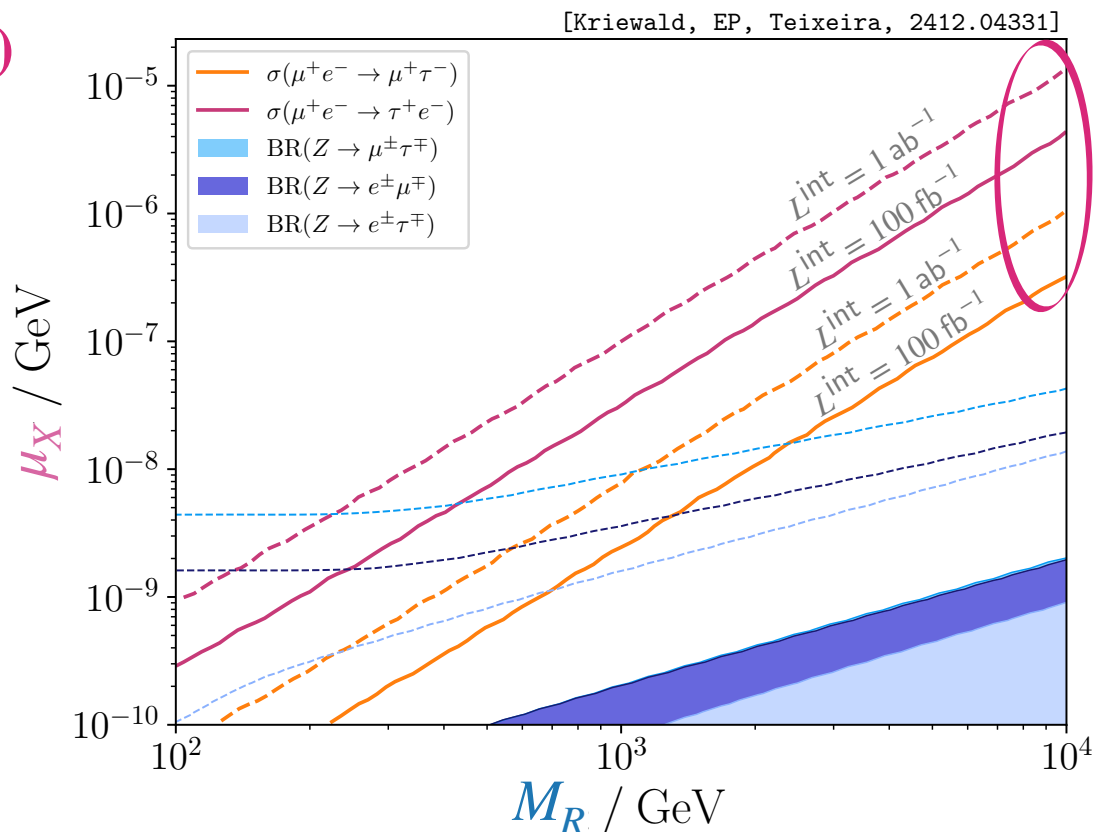
Constraining the ISS(3,3) parameter space (μ_X , M_R) with μ TRISTAN?

$$Z \rightarrow \ell_\alpha \ell_\beta \text{ vs. } \sigma(\mu^+ e^- \rightarrow \ell_\alpha^+ \ell_\beta^-)$$

Assume **1% signal** efficiency

Require **10 events** detection

$\Rightarrow$ **Probe** important region
the ISS(3,3) parameter space
with large μ_X



μ TRISTAN can vastly outperform future FCC-ee sensitivities

Complementarity with **low-energy cLFV** and **Z pole** searches

Stronger probes for cLFV involving τ **leptons**

Rich prospects for the **observation** of $\tau - \ell$ flavour violation (even in the absence of other cLFV signals)

Very good prospects to **discover**

ISS heavy sterile states through **cLFV** scattering at **μ TRISTAN**

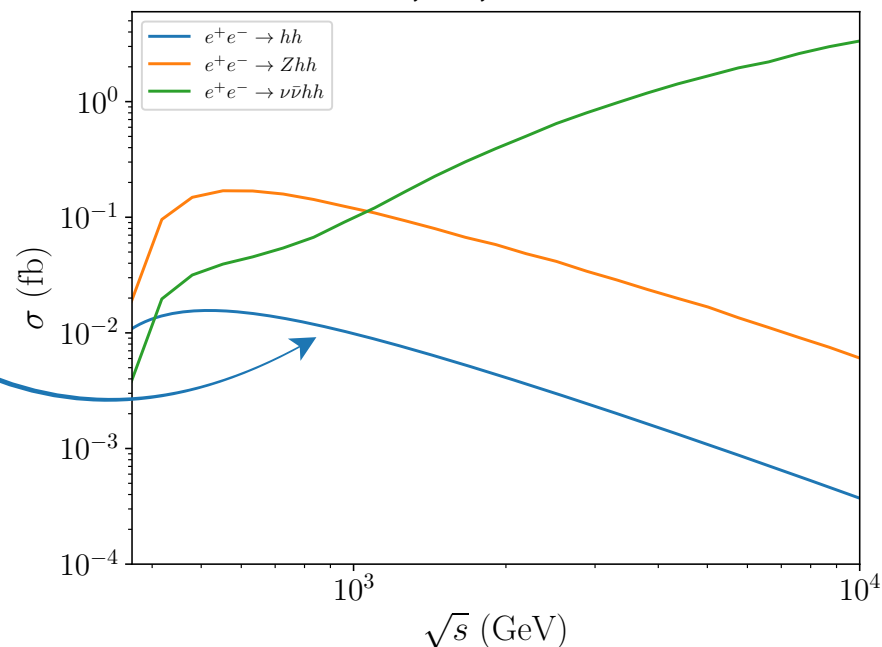
**Di-Higgs production at future
high-energy lepton colliders**

Di-Higgs production in the SM

In the **SM**, the **di-Higgs production** at e^+e^- colliders occurs via

- Double VBF $e^+e^- \rightarrow W^+W^- \rightarrow hh\nu\nu$
- Double Higgs-strahlung $e^+e^- \rightarrow Zhh$
- (Mostly) loop-induced $e^+e^- \rightarrow hh$
 $\Rightarrow$ **Subdominant process**

Kriewald, EP, Teixeira [2512.16734]



Di-Higgs production in the SM

In the **SM**, the **di-Higgs production** at e^+e^- colliders occurs via

Can be distinguished due to the **distinct pseudo-rapidity & transverse-momentum distributions**

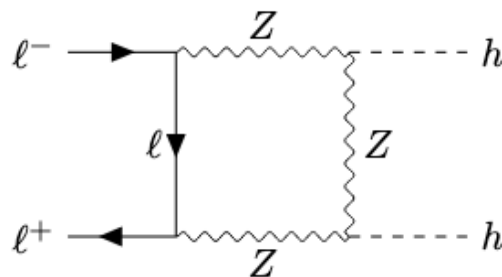
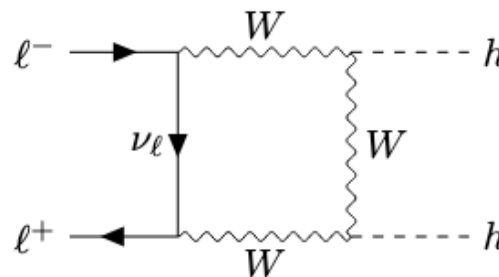
- (Mostly) loop-induced $e^+e^- \rightarrow hh$
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Loop-suppressed

Vanishing Z^, γ^* triangles (no CPV)*

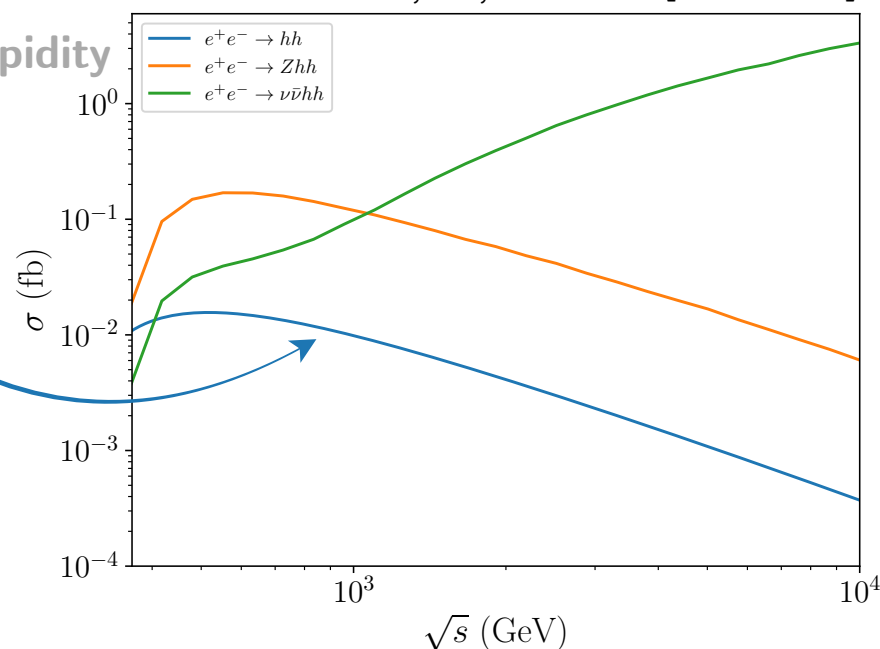
Tree & triangles $\propto m_e$

Only **SM**
 contributions:



+ crossed legs

Kriewald, EP, Teixeira [2512.16734]

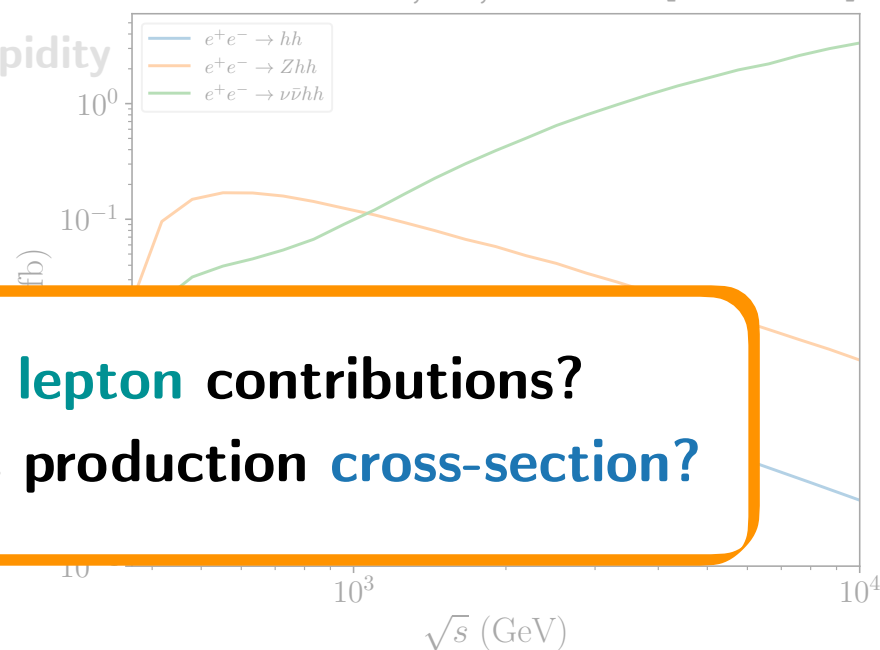


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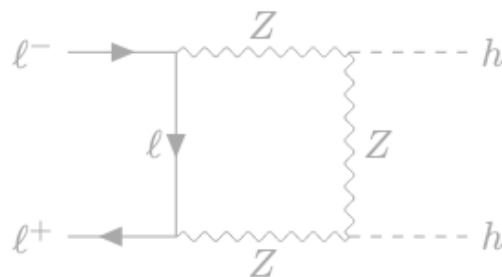
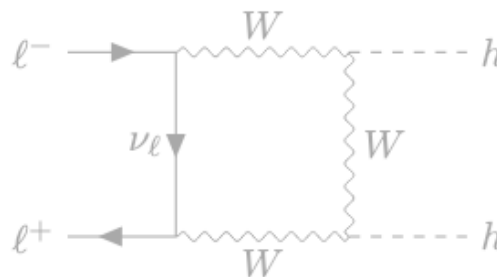


What about **heavy neutral lepton** contributions?
Can they **enhance** the **di-Higgs production cross-section**?

Vanishing Z^*, γ^* triangles (no CPV)

Tree & triangles $\propto m_e$

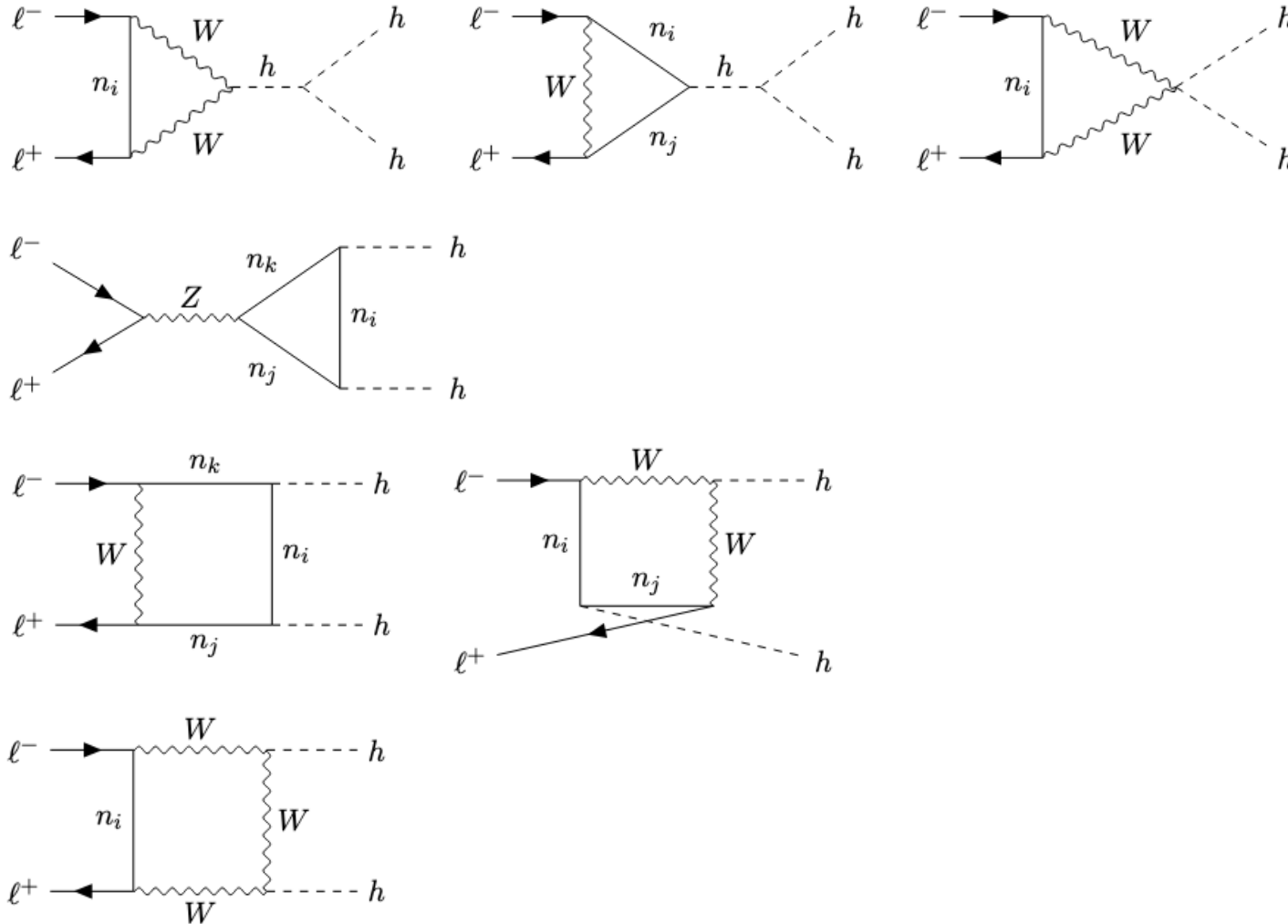
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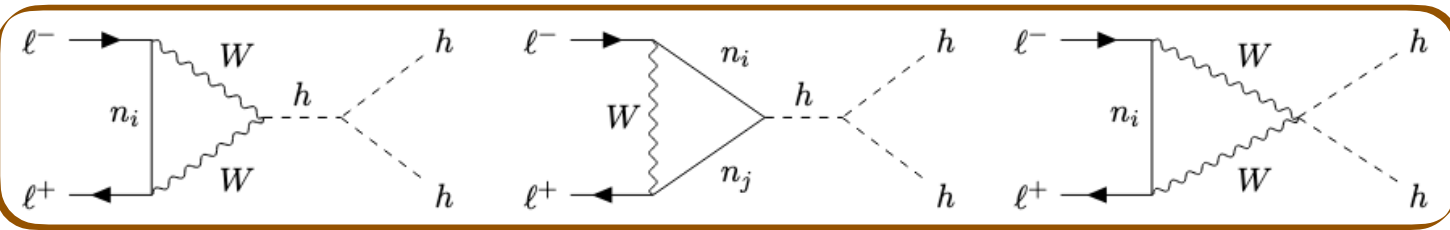
HNL contributions

Relevant **one-loop HNL** contributions to $\ell^+\ell^- \rightarrow hh$ ($\ell = e, \mu$)

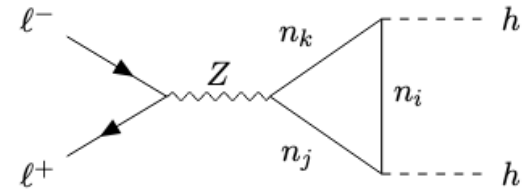
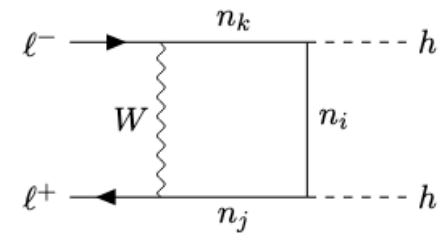
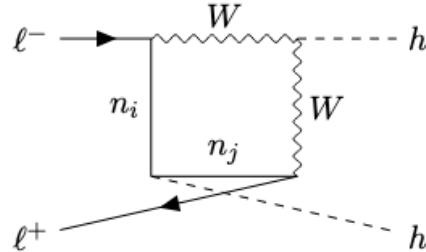
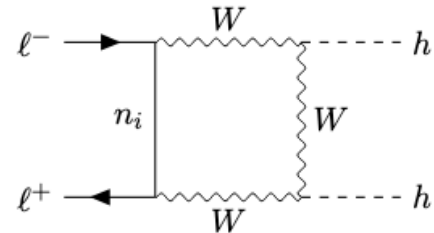


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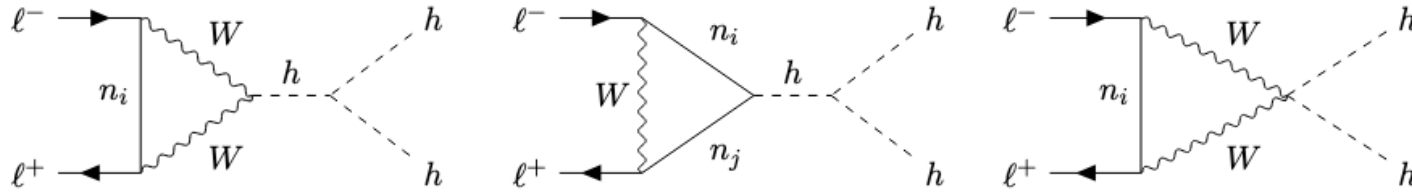
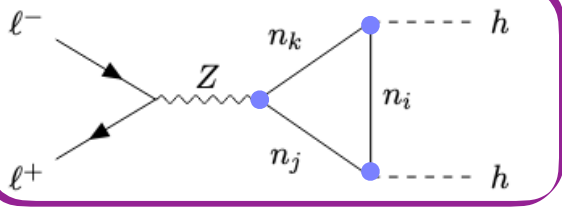


$$\propto m_\ell \rightarrow 0$$



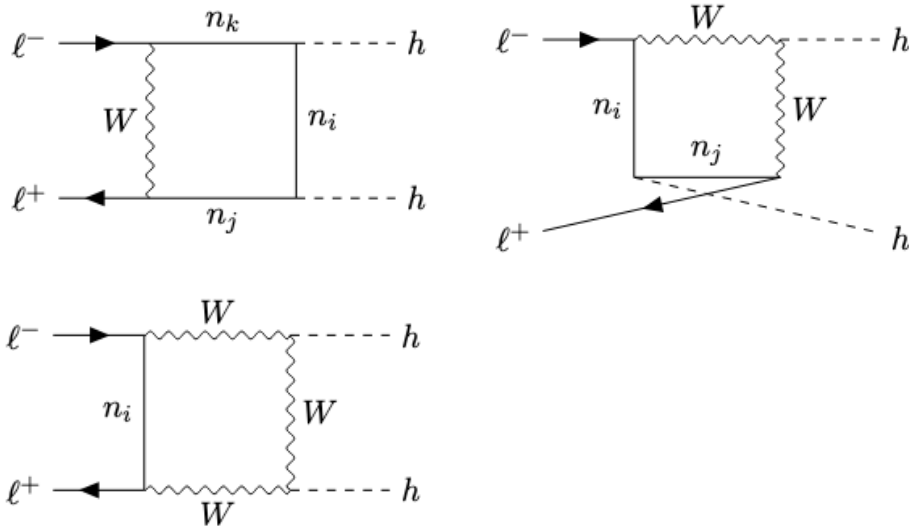
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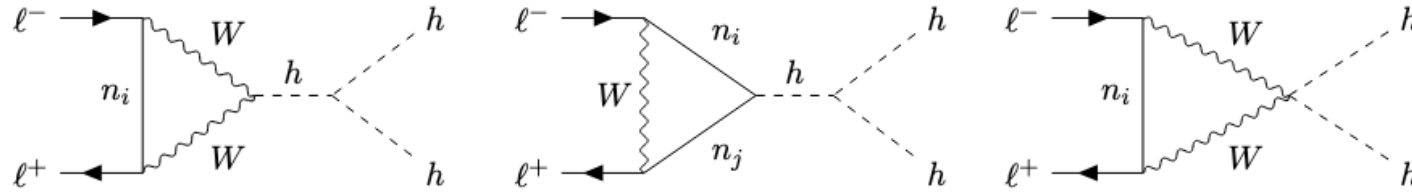
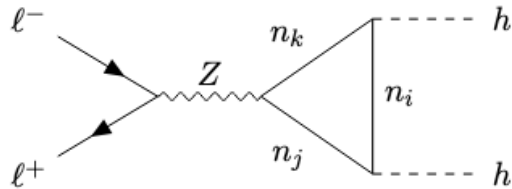
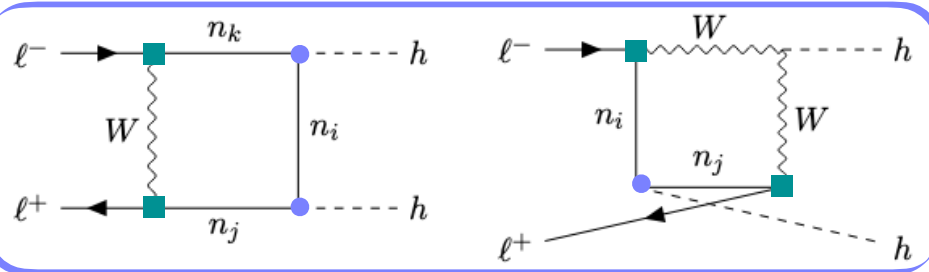
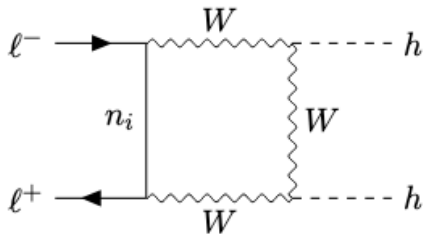
$$\propto \frac{M_N}{S} C_{ij} C_{jk} C_{ki} \rightarrow \text{Negligible}$$

$$C_{ij} = \sum_{\rho=1}^3 \mathcal{U}_{i\rho}^\dagger \mathcal{U}_{j\rho}$$



HNL contributions

Relevant **one-loop HNL** contributions to $\ell^+\ell^- \rightarrow hh$ ($\ell = e, \mu$)

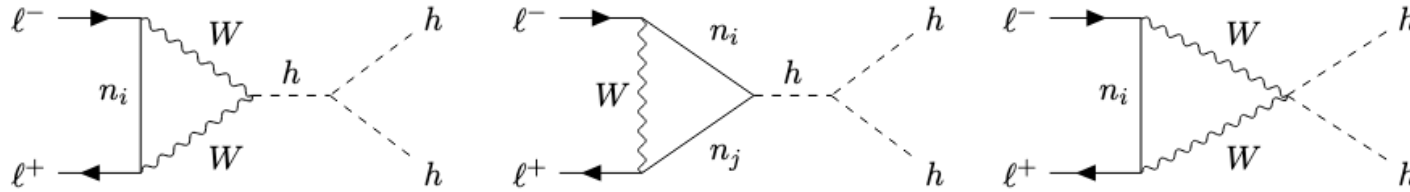
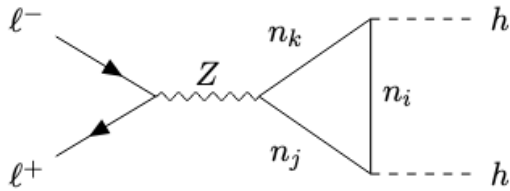
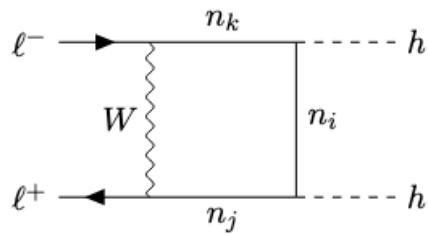
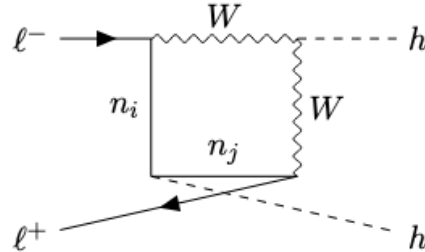
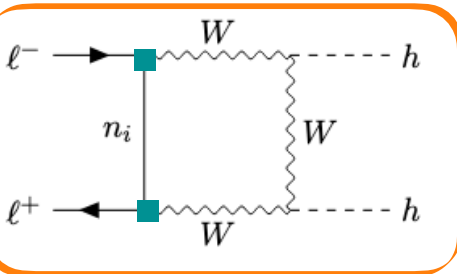


$$\propto \mathcal{U}_{\ell k}^* \mathcal{U}_{\ell j} C_{ki} C_{ij} \text{ and } \propto \mathcal{U}_{\ell i}^* \mathcal{U}_{\ell j} C_{ij}$$

$\Rightarrow$ **Mixing suppressions**

HNL contributions

Relevant **one-loop HNL** contributions to $\ell^+\ell^- \rightarrow hh$ ($\ell = e, \mu$)



$\propto \mathcal{U}_{\ell i}^* \mathcal{U}_{\ell i}$
 $\Rightarrow$ **Dominates**

Same kinematical dependence,
 p_T and η distributions as **SM**

Possible **enhancement**
of the **cross-section?**

HNL contributions

Relevant one-loop **HNL** contributions to $\ell^+\ell^- \rightarrow hh$ ($\ell = e, \mu$)

Illustrate our results with an **Inverse Seesaw (3,3) realisation** leading to **charged lepton flavour conservation** (*up to oscillation data*)

Parametrisation encoding low-energy data & cLFV into Yukawa couplings

J. Kriewald & A. M. Teixeira [2412.13696]

Look at **di-Higgs production** $\ell^+\ell^- \rightarrow hh$ ($\ell = e, \mu$) at future **colliders**

FCC-ee ($\sqrt{s} = 365$ GeV), **LCF** ($\sqrt{s} = 1,3$ TeV), **PWA** ($\sqrt{s} = 10$ TeV)

Muon Collider ($\sqrt{s} = 3, 10$ TeV)



$\Rightarrow$ **Dominates**

Possible **enhancement** of the **cross-section**?

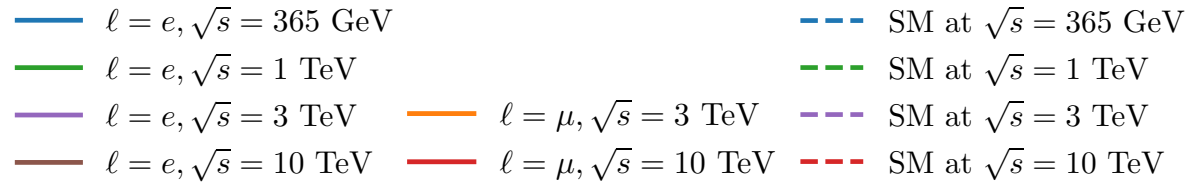
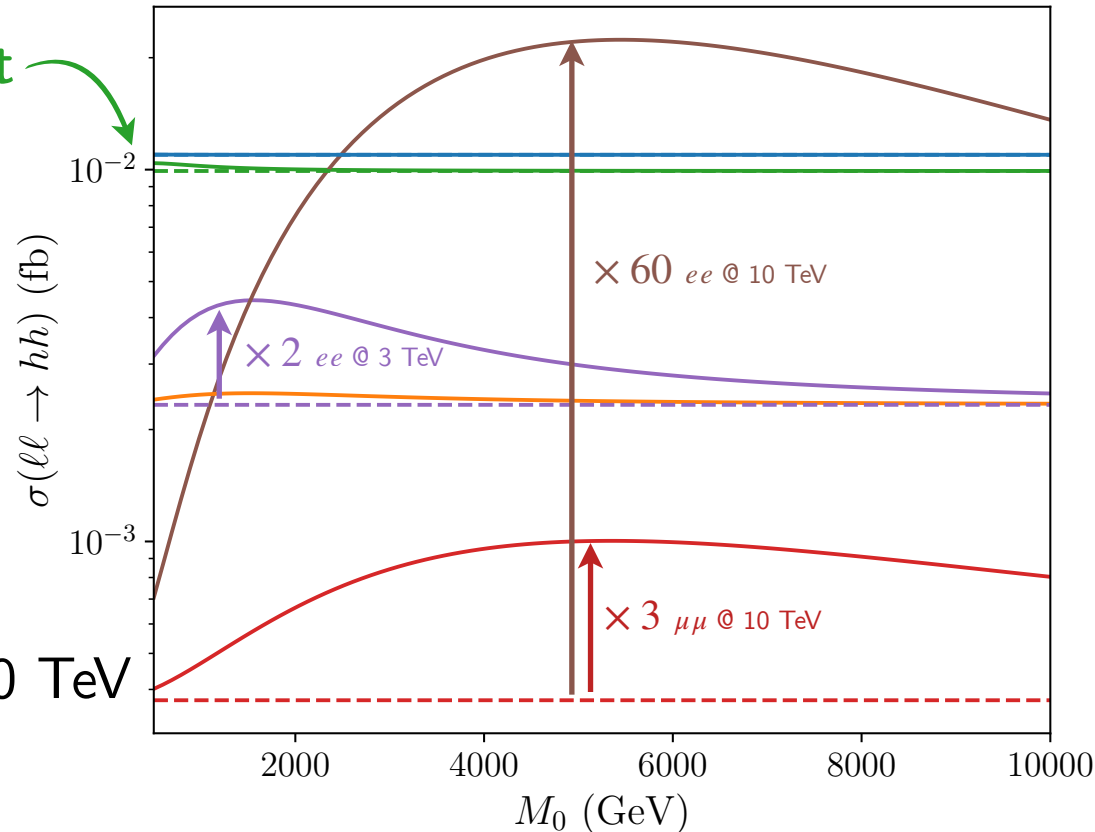
$\sigma(\ell^+\ell^- \rightarrow hh)$ with HNL

For $\sqrt{s} \leq 1\text{TeV}$

Small (%-level) **enhancement**

Difference electron vs. muon
due to the ν mixing bounds
(10 times **stronger** for muons)

$\sigma(\mu\mu \rightarrow hh)$ up to $3 \times \sigma_{\text{SM}}$ at 10 TeV

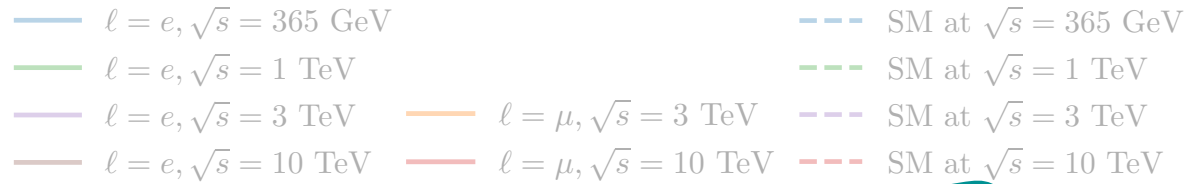


@ **PWA** can have an **enhancement** of $\sigma(e^+e^- \rightarrow hh)$ up to a **factor 60!**

$\sigma(\ell^+\ell^- \rightarrow hh)$ with HNL

For $\sqrt{s} \leq 1\text{TeV}$

Small (%-level) enhancement



HNLs can **sizeably enhance** $\sigma(\ell^+\ell^- \rightarrow hh)$!

Complementary window to probe this **class of low-scale seesaw**
at **high-energy lepton colliders**
even **in absence of cLFV**

& for **HNL masses too heavy** to be directly produced

$M_0 \text{ (GeV)}$

@ **PWA** can have an **enhancement** of $\sigma(e^+e^- \rightarrow hh)$ up to a **factor 60!**

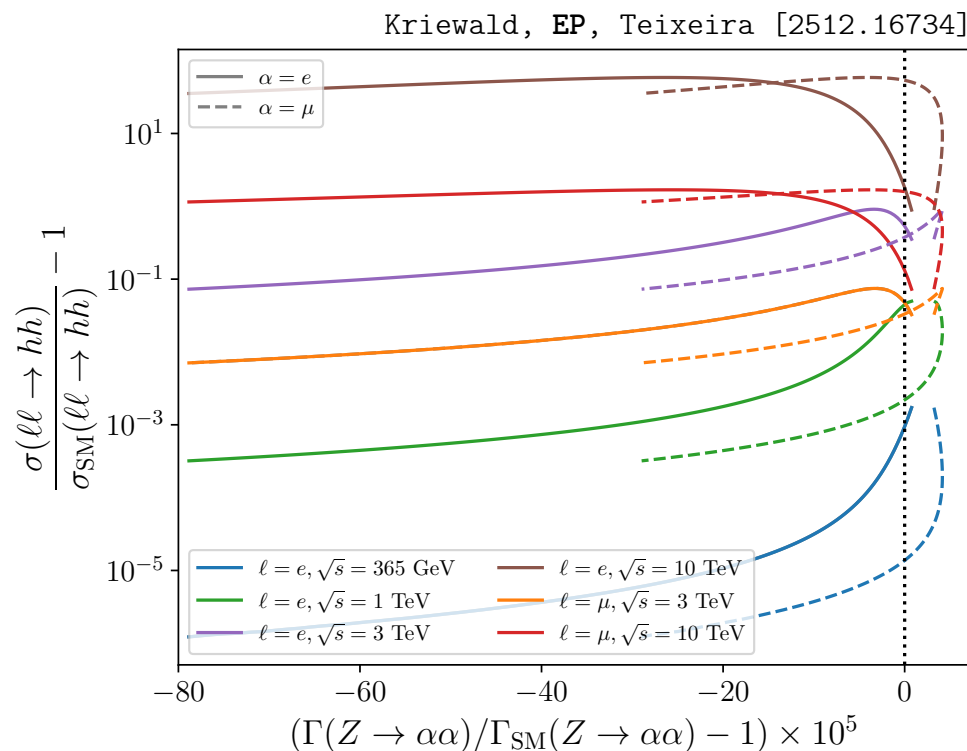
Complementarity di-Higgs and LFC

Large mixings can lead to important **contributions** to **LFC observables**

What is the **impact** on $Z \rightarrow \ell_\alpha \ell_\alpha$?

HNL induce **destructive**
interferences in $Z \rightarrow \ell_\alpha \ell_\alpha$
 $\rightsquigarrow \mathcal{O}(10^{-4})$ **suppression**

Can be **probed** at **FCC-ee**
 $\mathcal{O}(10^{-4(5)})$ precision at Z-pole



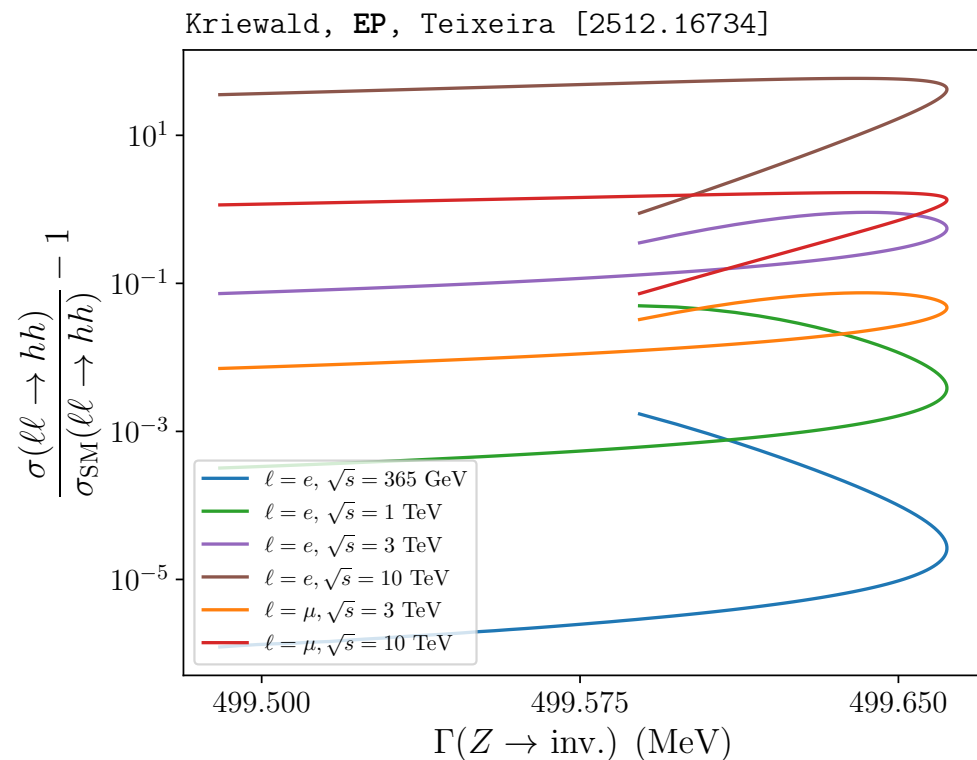
Even **without** any large **deviations of LFC**, **di-Higgs** can be **enhanced!**

Complementarity **di-Higgs** production and $Z \rightarrow \ell_\alpha \ell_\alpha$ at **FCC-ee**

Di-Higgs vs. EWPO

HNL with **large mixings** can lead to important **contributions** to **EWPO**

What is the **impact** on $Z \rightarrow \text{inv.}$?



Strong effect in $\Gamma(Z \rightarrow \text{inv.})$

→ Predictions **significantly smaller** than **SM**
for **all** $\sigma(\ell\ell \rightarrow hh)$

Testable at FCC-ee

Waiting for very high-energy PWA or muon collider,

FCC-ee could **test such extensions** with improved sensitivity on **EWPO**

HNL mediated di-Higgs production

Sizeable enhancements - up to a **factor 60!** - of the cross-section even for **HNL** extensions **without cLFV**

Offers a complementary window to probe this **class of models** even **in absence of cLFV** and **without deviation** in **LFC boson decays**

Complementarity with future **FCC-ee** ($Z, h \rightarrow \ell\ell$ and $Z \rightarrow \text{inv.}$)

Concluding remarks

Neutrino oscillations: 1st laboratory evidence of **New Physics**

HNL: very appealing hypothesis to explain **neutrino masses** & **mixings**
strong impact on flavoured observables & **very constrained** by **cLFV**

Hunting for sterile neutrinos at future lepton colliders:
Complementary windows

- **Flavour imprints:** cLFV can reveal the presence of HNL
- **Higgs sector:** HNL-enhancement of Higgs-related processes

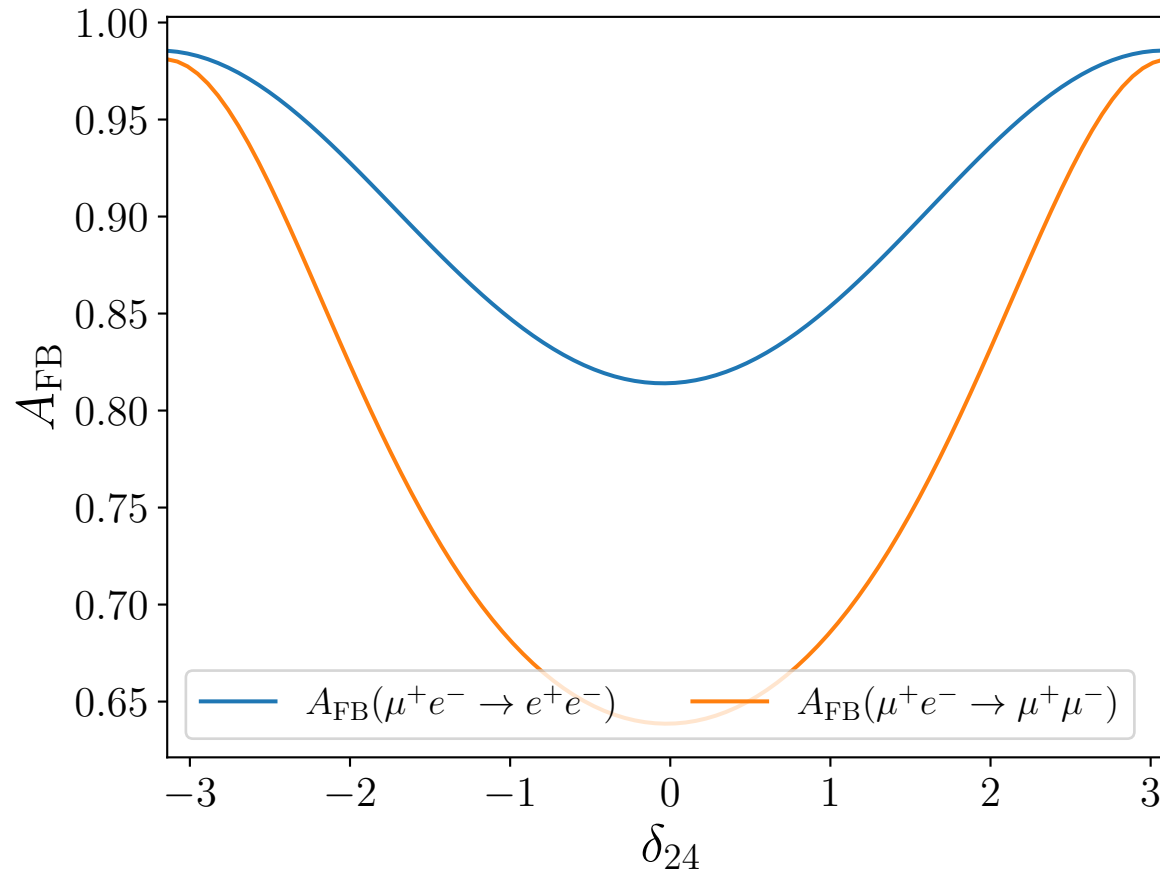
Among many other signatures: cLFV decays, LFU(V), EWPO, ...

Very good prospects to probe HNL at future colliders

Thank you for your attention

CPV phases and A_{FB}

Considering $\sqrt{s} = 126.5$ GeV and $m_4 = m_5 = 1$ TeV (fixed mixing angles)



Clear **impact** of the **Dirac CPV phase** on A_{FB} (suppression of the Z-penguins, altering the interferences between s- and t-channel penguins)

Estimation for μ TRISTAN

Total cross-section: No 4π -coverage $\Rightarrow$ **Cut on the pseudo-rapidity $\eta \leq 4$**
envisaged for ATLAS HL-LHC upgrade

Background from $\mu^+e^- \rightarrow \ell_\alpha^+ \ell_\beta^- \nu\nu$ ($\sigma \simeq 21$ fb)

$\Rightarrow$ Select events with $E_T \leq 10$ GeV + in the forward region

$\Rightarrow$ Require $2.0 \leq \eta_\ell \leq 4.0$

Leading to $\sigma_{\text{bkg,cut}} \simeq 6 \times 10^{-4}$ fb = **1 background event** for $L_{\text{int}} = 1 \text{ ab}^{-1}$

Signal efficiency 25% + τ -tagging of 40%: Estimate signal efficiency 10%
Consider signal loss (misid., contamination decaying μ) $\Rightarrow$ **Assume 1%**

Require **10 signal events** $\Rightarrow$ **total significance** of $\mathcal{S} = \frac{S}{\sqrt{S+B}} \sim 3\sigma$

SVD parametrisation

Deviation from PMNS unitarity:

$$U_{\text{PMNS}} \rightarrow \tilde{U}_{\text{PMNS}} = (1 - \eta) U_{\text{PMNS}}$$

$$\mathcal{L}_{\text{ISS}} = -\frac{1}{2} (\overline{\nu_L^c}, \overline{\nu_R}, \overline{X^c}) \begin{pmatrix} 0 & m_D^T & 0 \\ m_D & \mu_R & M_R \\ 0 & M_R^T & \mu_X \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \\ X \end{pmatrix}$$

Singular value decomposition of the matrix $Y_D = m_D/v$ $\eta \sim \frac{1}{2} m_D^* (M_R^{-1})^\dagger (M_R^{-1}) m_D^T$
 Parametrisation of the the Yukawa coupling

$$Y_D = \frac{1}{v} \mathcal{V}_1 \text{diag}(y_1, y_2, y_3) \mathcal{V}_2^\dagger M_R^T \quad \text{with } \mathcal{V} \text{ unitary matrices}$$

parametrised via 3 Euler angles θ , 3 Dirac phases δ and 3 Majorana phases φ :

$$\mathcal{V} = O_{23} O_{13} O_{12} \text{diag}(e^{i\varphi_1}, e^{i\varphi_2}, e^{i\varphi_3}) \quad \text{and} \quad O_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_{23} & \sin \theta_{23} e^{i\delta_{23}} \\ 0 & -\sin \theta_{23} e^{-i\delta_{23}} & \cos \theta_{23} \end{pmatrix}$$

$$\text{with } \eta = \frac{1}{2} \mathcal{V}_1^* \text{diag}(y_1^2, y_2^2, y_3^2) \mathcal{V}_1^T = \begin{pmatrix} \eta_{ee} & \sqrt{\eta_{ee}\eta_{\mu\mu}} a e^{i\delta_{12}} & \sqrt{\eta_{ee}\eta_{\tau\tau}} b e^{i\delta_{13}} \\ \sqrt{\eta_{ee}\eta_{\mu\mu}} a e^{-i\delta_{12}} & \eta_{\mu\mu} & \sqrt{\eta_{\mu\mu}\eta_{\tau\tau}} c e^{i\delta_{23}} \\ \sqrt{\eta_{ee}\eta_{\tau\tau}} b e^{-i\delta_{13}} & \sqrt{\eta_{\mu\mu}\eta_{\tau\tau}} c e^{-i\delta_{23}} & \eta_{\tau\tau} \end{pmatrix}$$

that **encodes low-energy data** $\Rightarrow$ **Control cLFV!**