

Towards precise predictions of trilinear scalar couplings and Higgs pair production in extended scalar sectors

Johannes Braathen (DESY)

IRN Terascale

Heidelberg University, Germany | 21 October 2026

HELMHOLTZ RESEARCH FOR
GRAND CHALLENGES

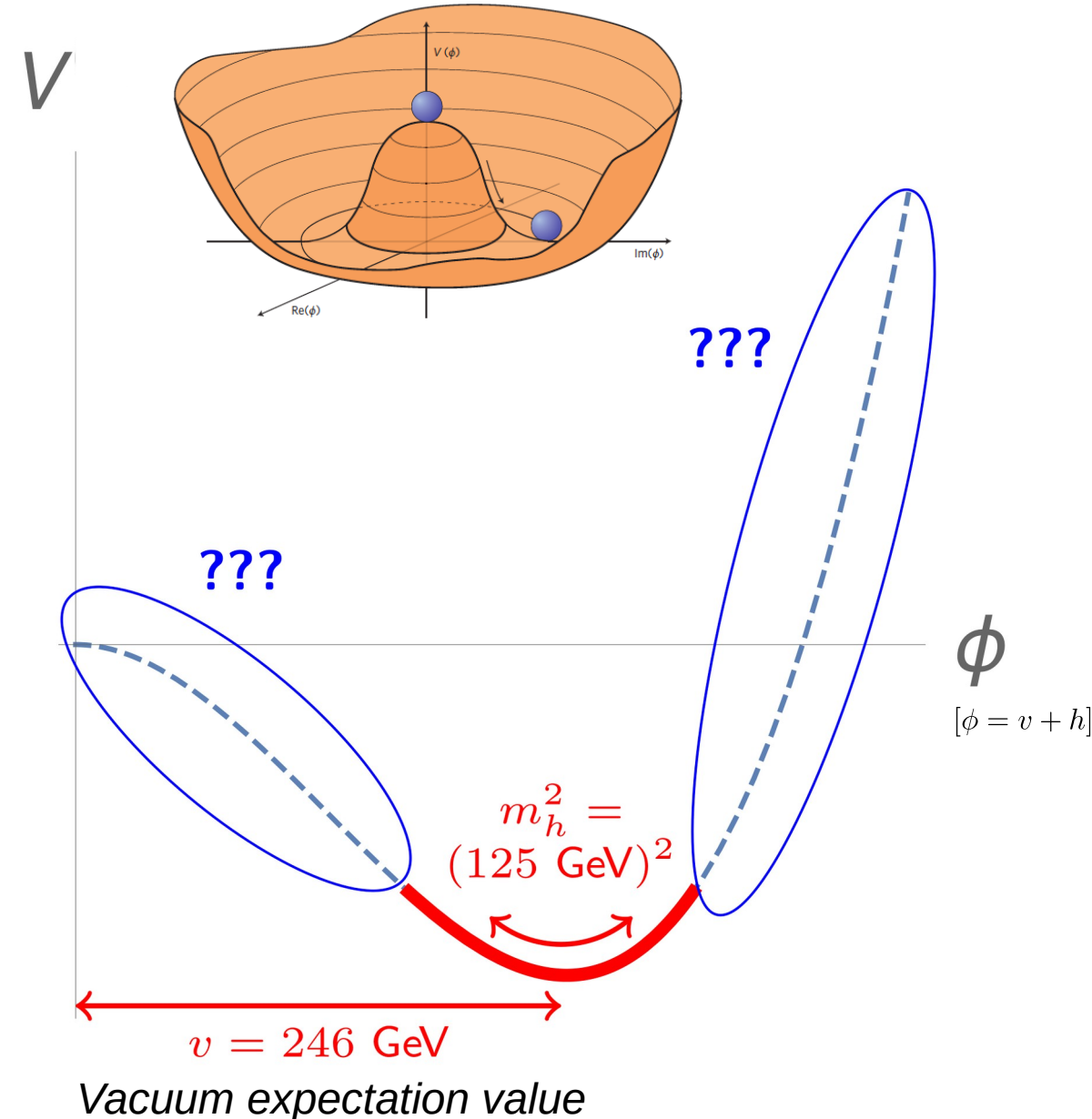
DESY.

CLUSTER OF EXCELLENCE
QUANTUM UNIVERSE



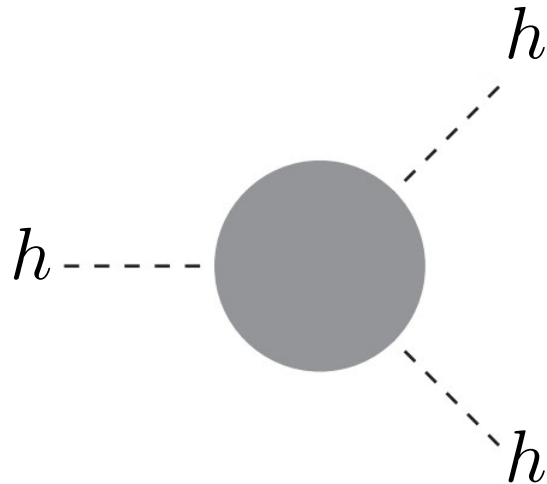
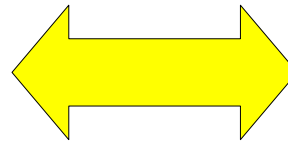
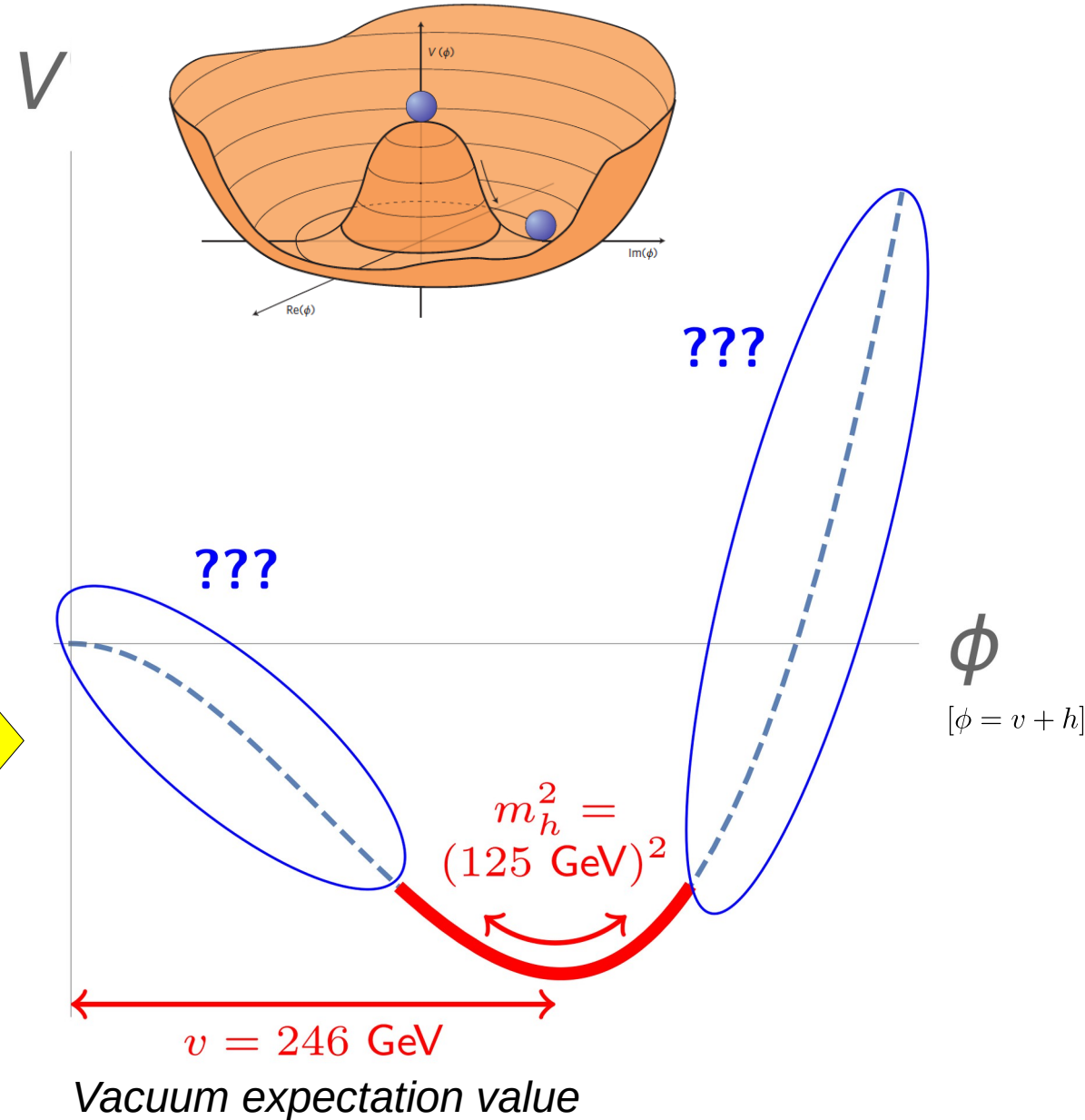
Form of the Higgs potential and trilinear Higgs coupling

- Brout-Englert-Higgs mechanism = **origin of masses of elementary particles** ...
... but very little known about the **Higgs potential** causing the **electroweak phase transition (EWPT)**



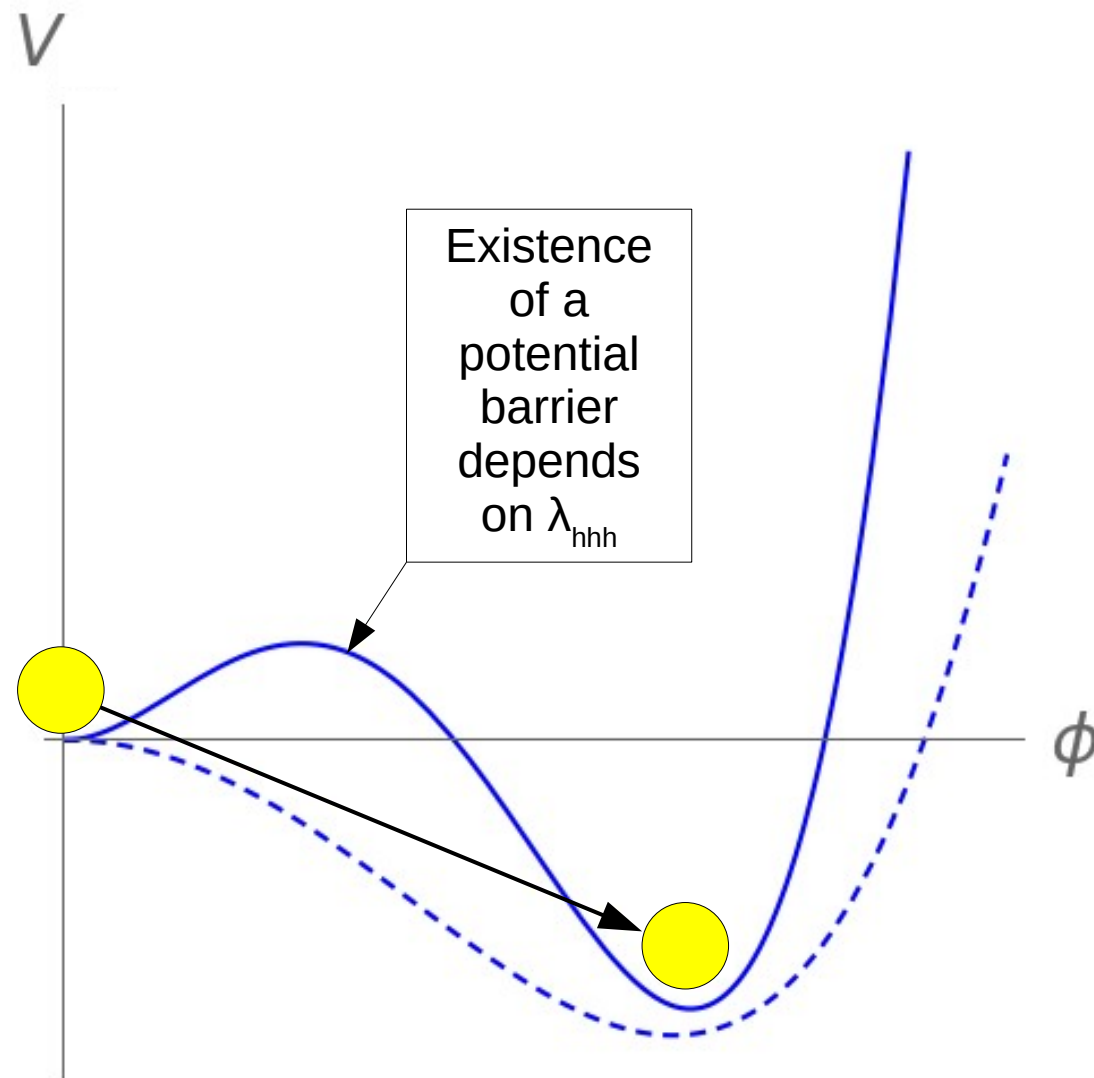
Form of the Higgs potential and trilinear Higgs coupling

- Brout-Englert-Higgs mechanism = **origin of masses of elementary particles** ...
... but very little known about the **Higgs potential** causing the **electroweak phase transition** (EWPT)
- **Trilinear Higgs coupling** λ_{hhh} crucial to understand the shape of the potential

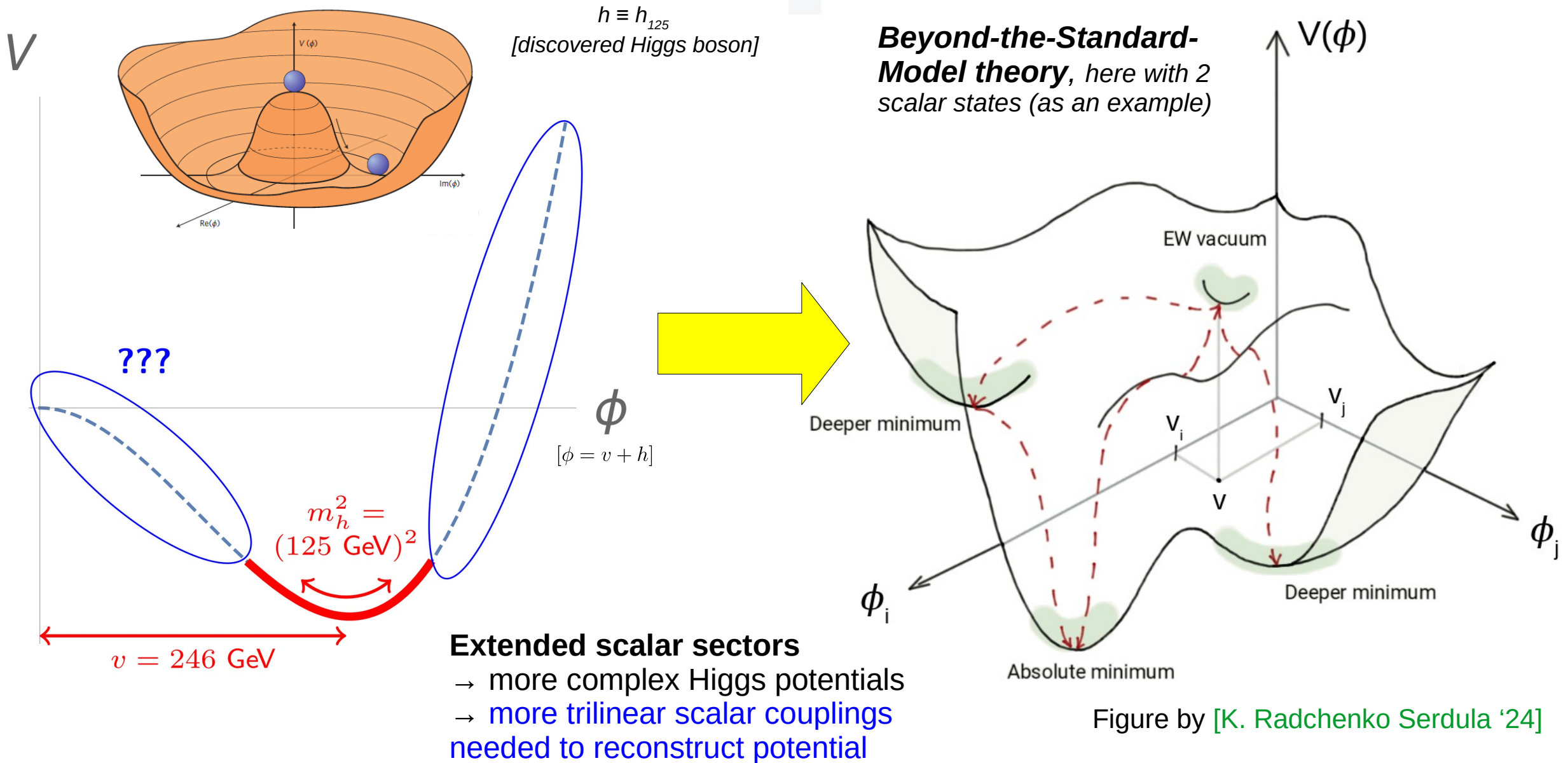


Form of the Higgs potential and baryon asymmetry

- Brout-Englert-Higgs mechanism = **origin of masses of elementary particles** ...
... but very little known about the **Higgs potential** causing the **electroweak phase transition (EWPT)**
- **Trilinear Higgs coupling λ_{hhh}** crucial to understand the shape of the potential
- Among **Sakharov conditions** necessary to explain **baryon asymmetry of the Universe via electroweak phase transition** (= *electroweak baryogenesis*):
 - **Strong first-order EWPT (SFOEWPT)**
 - barrier in Higgs potential
 - O(30-100%) deviation in λ_{hhh} from SM prediction if SFOEWPT along Higgs field direction



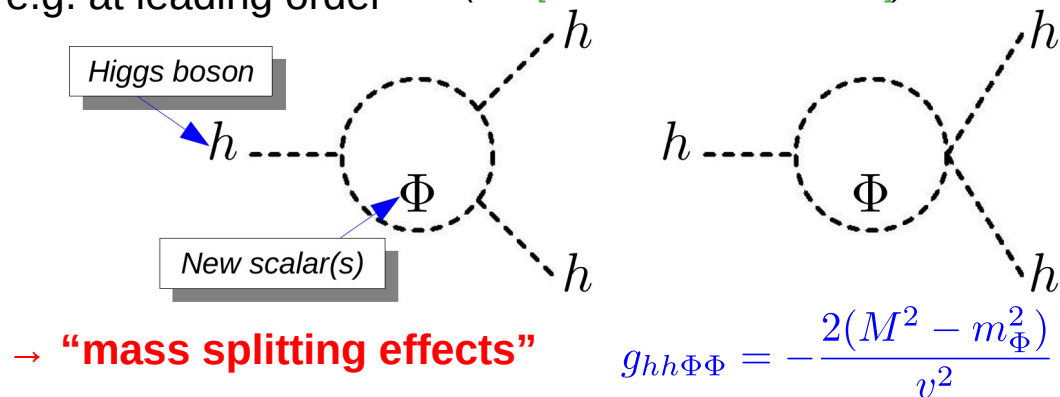
Form of the Higgs potential and trilinear scalar couplings



Probing New Physics with the trilinear Higgs coupling

[Bahl, JB, Weiglein *Phys. Rev. Lett.* '22]

- Large effects from New Physics possible in λ_{hhh} due to radiative corrections from extra scalars, e.g. at leading order (see [Kanemura et al. '02, '04])



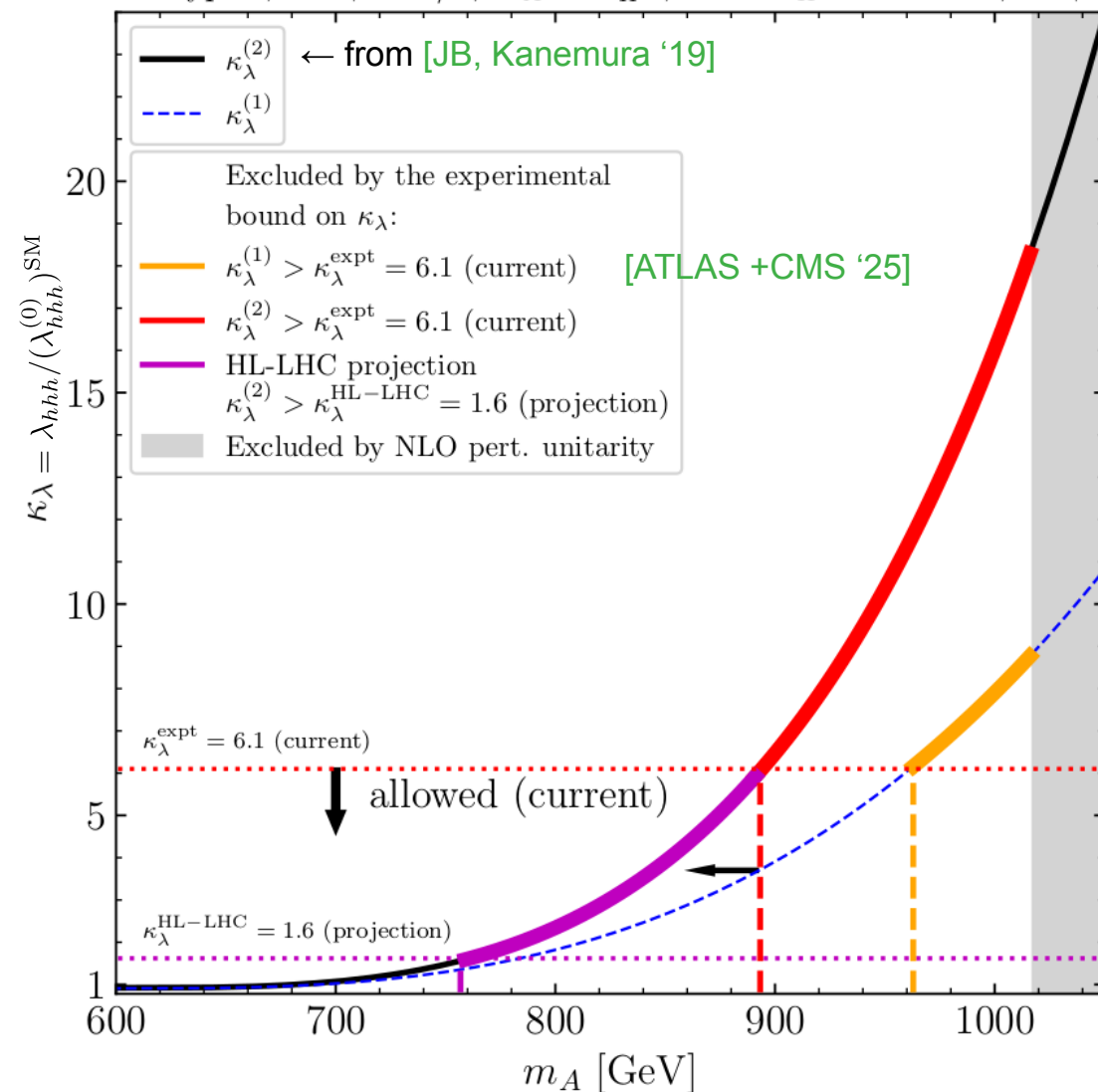
- Comparing latest exp. bounds

$$-0.7 < \kappa_\lambda = \frac{\lambda_{hhh}}{(\lambda_{hhh}^{(0)})_{\text{SM}}} < 6.1 \quad [\text{ATLAS \& CMS combination '25}]$$

with precise theory predictions for λ_{hhh} provides a **powerful new tool to constrain BSM models**

- BSM deviations in κ_λ also motivated from the point of view of early-Universe evolution (e.g. strong first-order EW phase transition)

2HDM type I, $\alpha = \beta - \pi/2$, $m_A = m_{H^\pm}$, $M = m_H = 600$ GeV, $\tan \beta = 2$



Automating predictions of the trilinear Higgs self-coupling

Calculations of λ_{hhh} are receiving increasing attention

More and more model specific results **at one loop**
SM + singlet [Kanemura et al. '16]; 2HDMs [Kanemura et al. '04], [Basler et al. '17]; N2HDM (2HDM + singlet) [Basler et al. '19]; triplet extensions [Aoki et al. '12], [Chiang et al. '18]; MSSM [Hollik, Penaranda '04]; NMSSM [Dao et al. '13]; models with classical scale invariance [Hashino, Kanemura, Orikasa '16], etc.

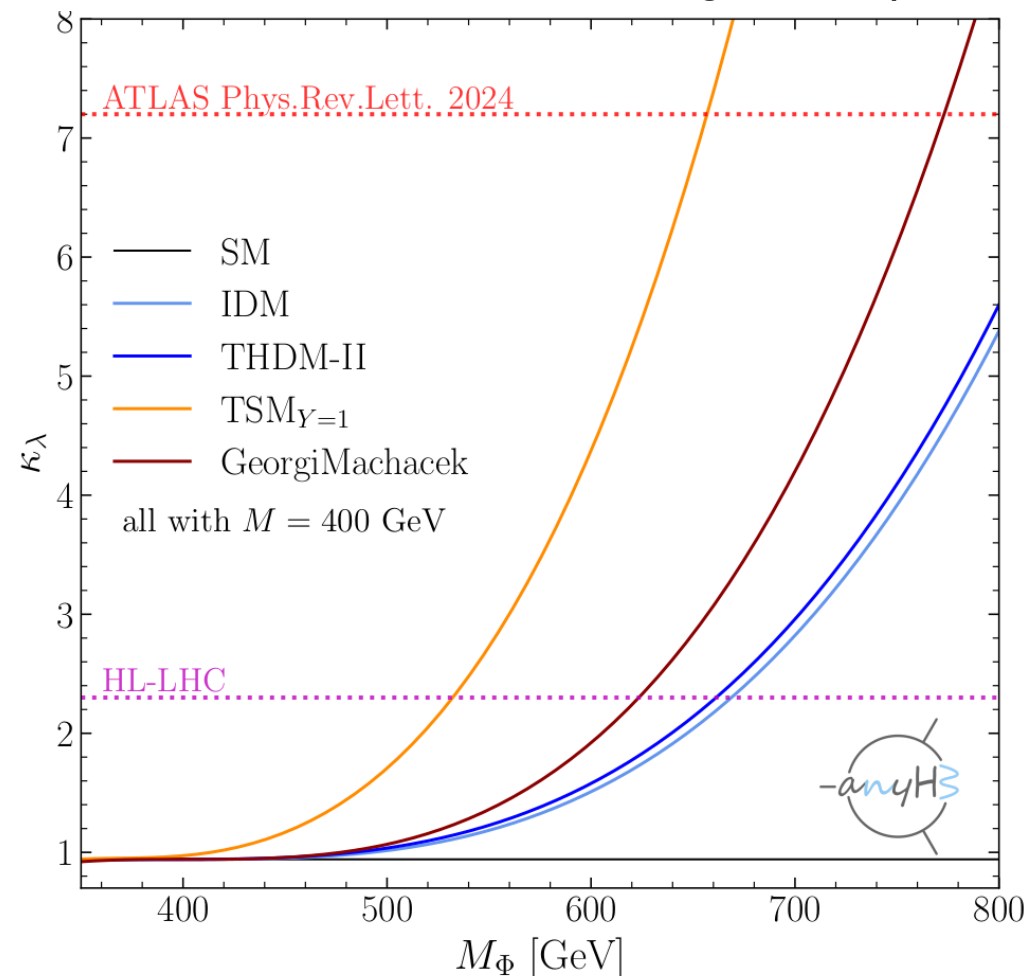
... and **at two loops**

SM + singlet [JB, Kanemura '19]; 2HDMs [Senaha '18], [JB, Kanemura '19], [Aiko, JB, Kanemura '23], [JB, Egle, Verduras Schaeidt *WIP*]; MSSM [Brucherseifer et al. '13]; NMSSM [Dao et al. '15], [Borschensky et al '22], [Dao, Gabelmann, Mühlleitner '25]; models with classical scale invariance [JB, Kanemura, Shimoda '20], etc.

but *many more models to investigate!*

→ Automation

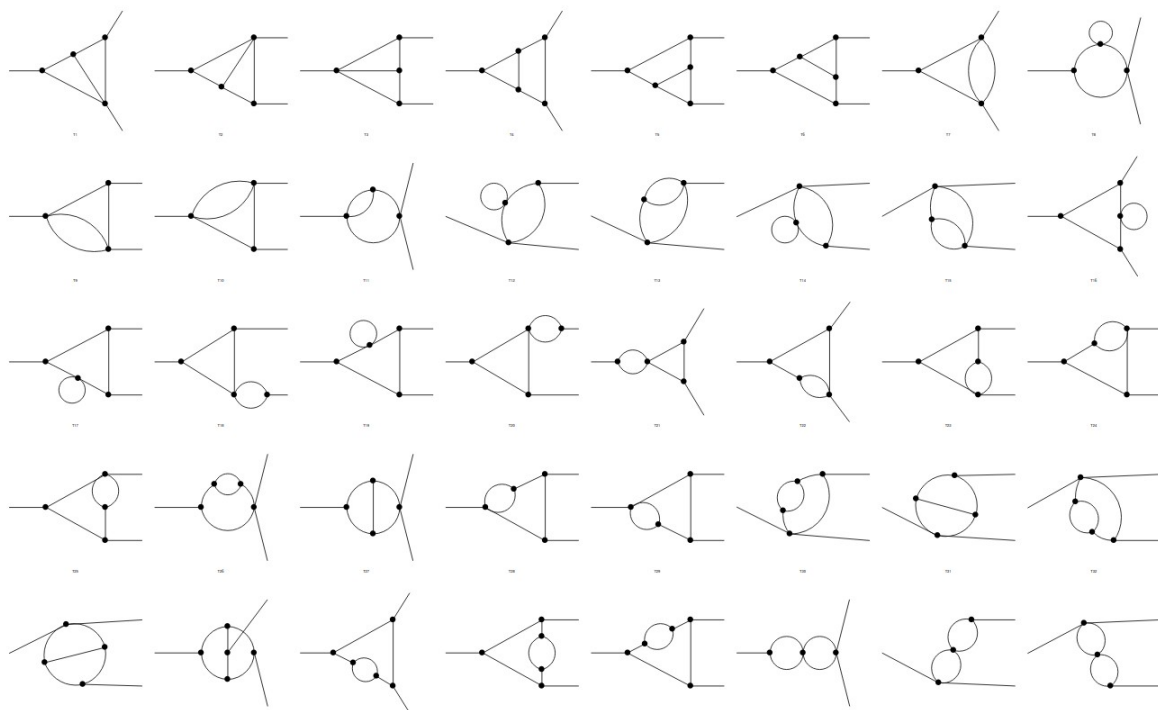
anyH3 v1 [Bahl, JB, Gabelmann, Weiglein '23]
public tool for **full one-loop calculation of λ_{hhh} in arbitrary renormalisable models and with semi-automatic renormalisation**, using UFO inputs



Automating predictions of trilinear scalar couplings

Beyond one loop: generic two-loop results for the **trilinear** (and quartic) **Higgs self-coupling**

[Bahl, JB, Gabelmann, Paßehr '25], extending on pioneering results of [Goodsell, Paßehr '19] for generic two-loop self-energies

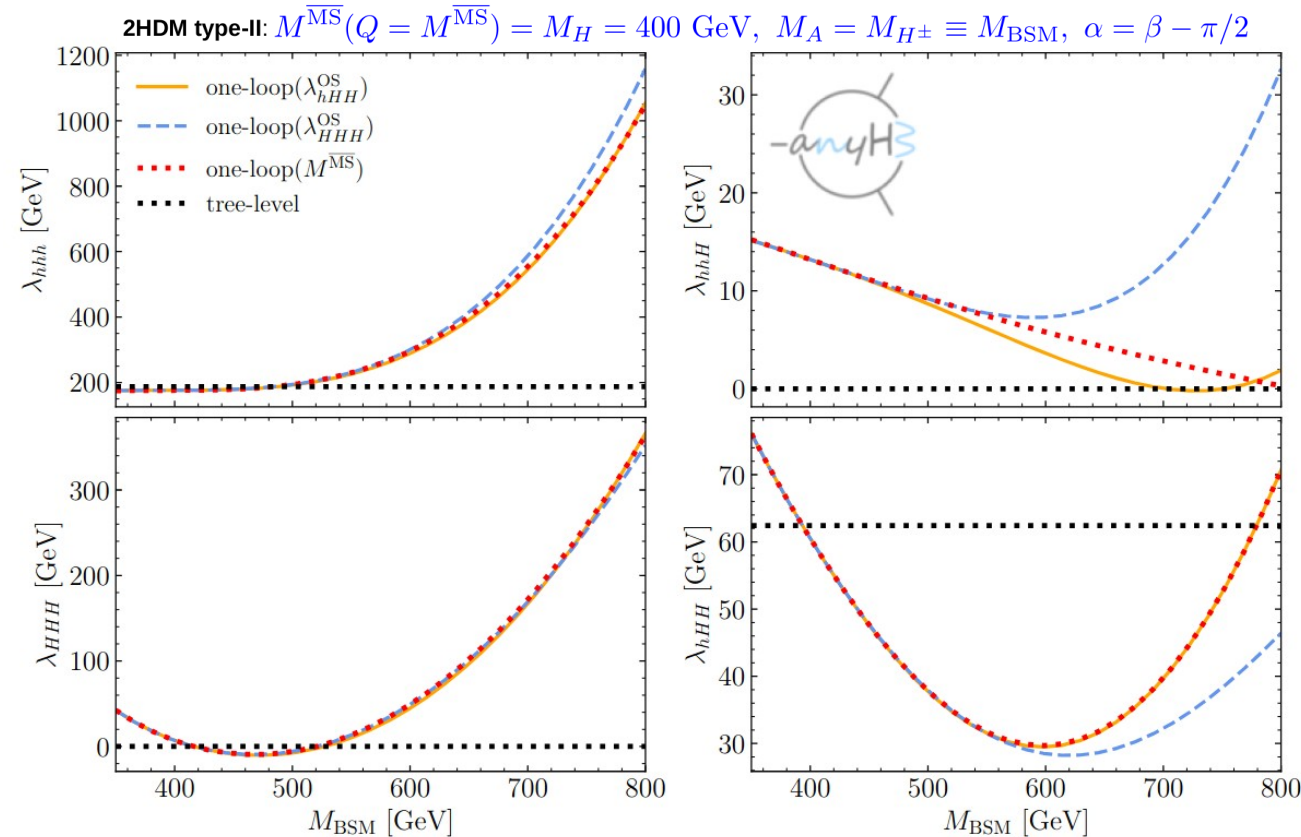


(see details and example application in backup)

Beyond λ_{hhh} : Predictions for generic trilinear scalar couplings

anyH3 v2 [Bahl, JB, Gabelmann, Radchenko Serdula, Weiglein WIP]

Example below: mass-splitting effects in generic λ_{ijk} for a benchmark example in an aligned 2HDM and with different renormalisation schemes



Model-independent higher-order corrections to λ_{hhh}

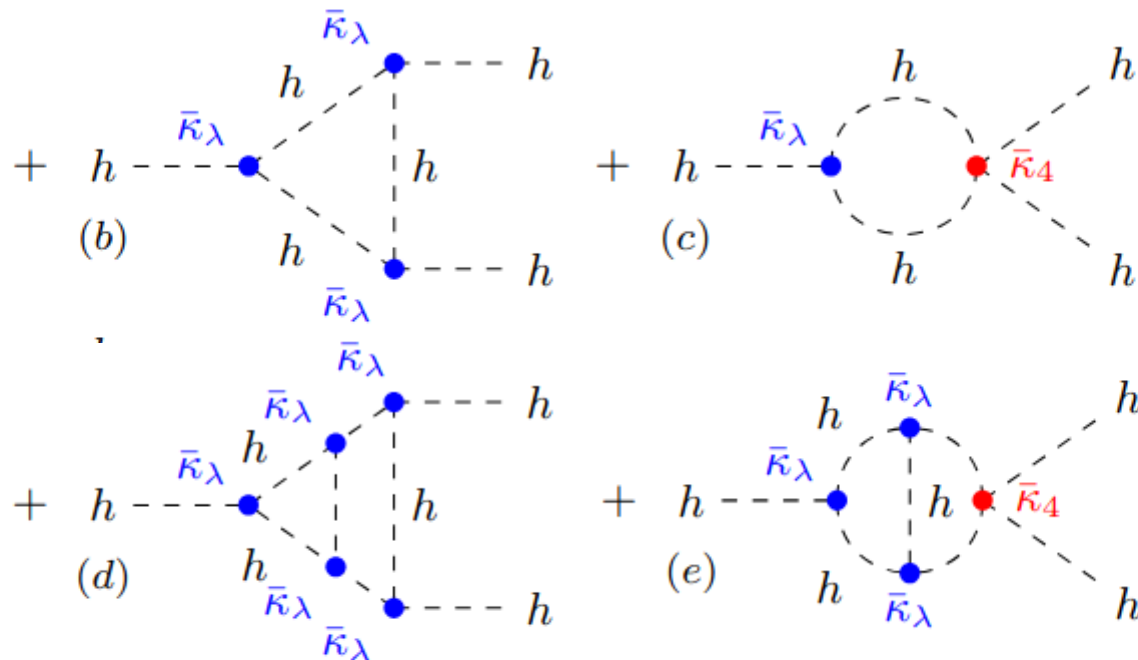
[JB, Naskar, Weiglein '26]

Higher-order effects can still be significant for $\kappa_\lambda \gg 1$

→ what happens with diagrams containing the trilinear (or quartic) couplings themselves?

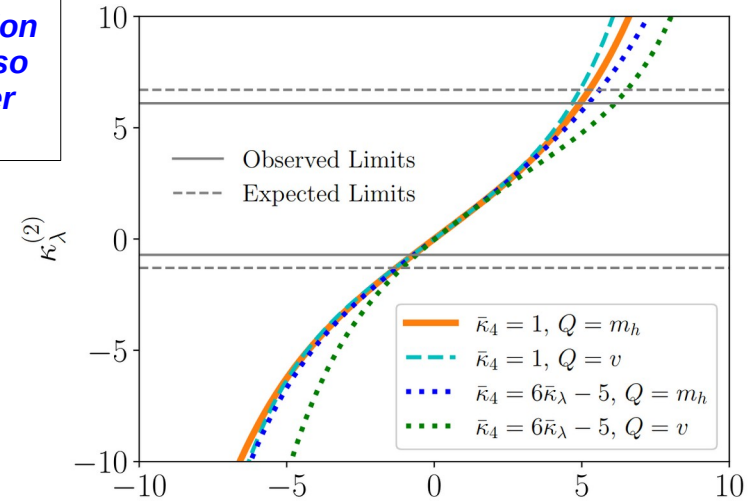
e.g.: $\kappa_\lambda^{(2)}$

(a) Following **effective approach**, inspired by scalar sector of **Higgs Effective Field Theory**, we computed diagrams with insertions of $\bar{\kappa}_\lambda$ and $\bar{\kappa}_4$ (denoted with bars to indicate inputs), up to two loops

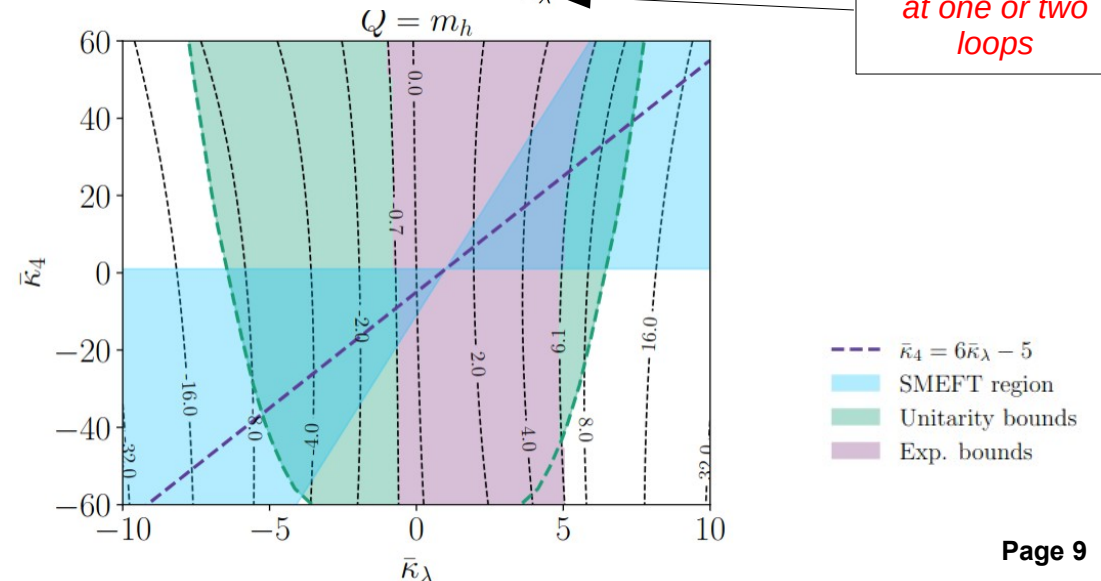


- Incorporates well-defined set of higher-order corrections
- Modifies theoretical interpretation of experimental bounds

BSM prediction including also higher order effects



BSM prediction at one or two loops



Automating calculations of Higgs pair production

Based on

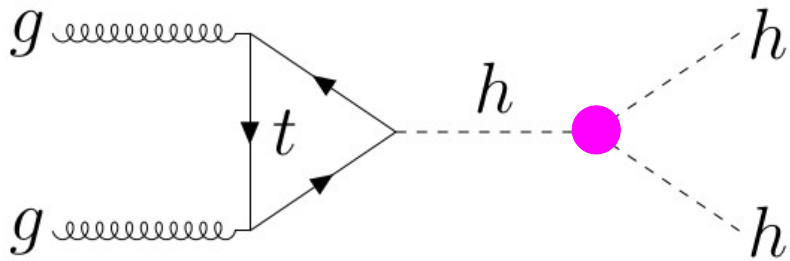
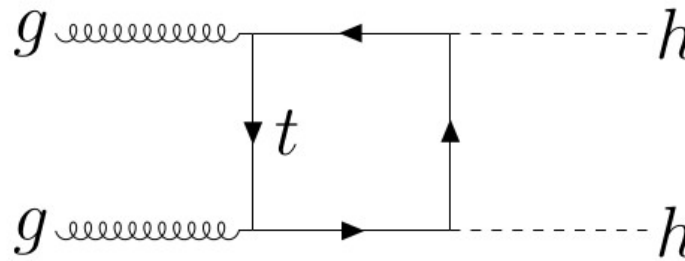
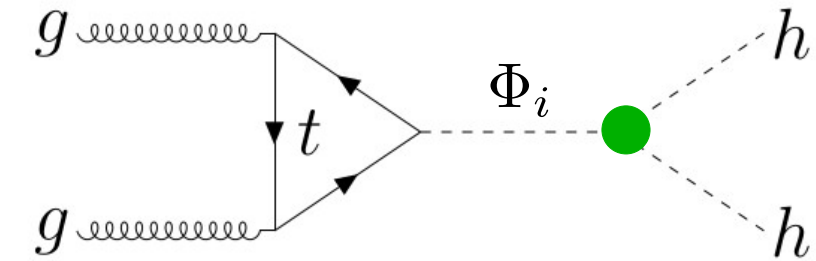
[Bahl, JB, Gabelmann, Radchenko Serdula, Weiglein WIP]

Di-Higgs production

$h \equiv h_{125}$
[discovered Higgs boson]

➤ Dominant channel for di-Higgs production at (HL-)LHC: **gluon fusion**

Here, leading order (LO) diagrams, involving top quark (dominant contribution among quark loops)



“Non-resonant contributions”

- Standard Model (SM)-like diagrams
- Involves the trilinear self-coupling of h_{125} — λ_{hhh}
 - **probe of the Higgs potential**
- Large destructive interference between triangle and box diagram
 - **suppression of cross-section in SM**
 - **large changes in di-Higgs cross-section possible from BSM effects in λ_{hhh}**

“Resonant contributions”

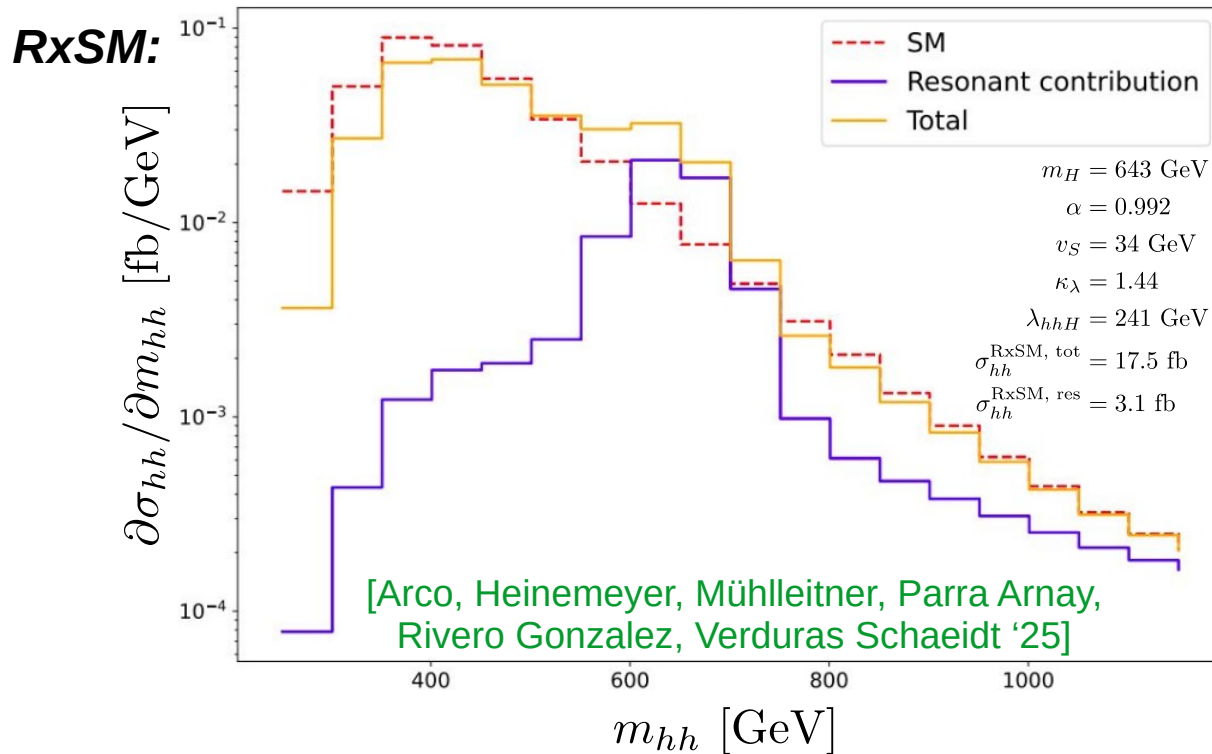
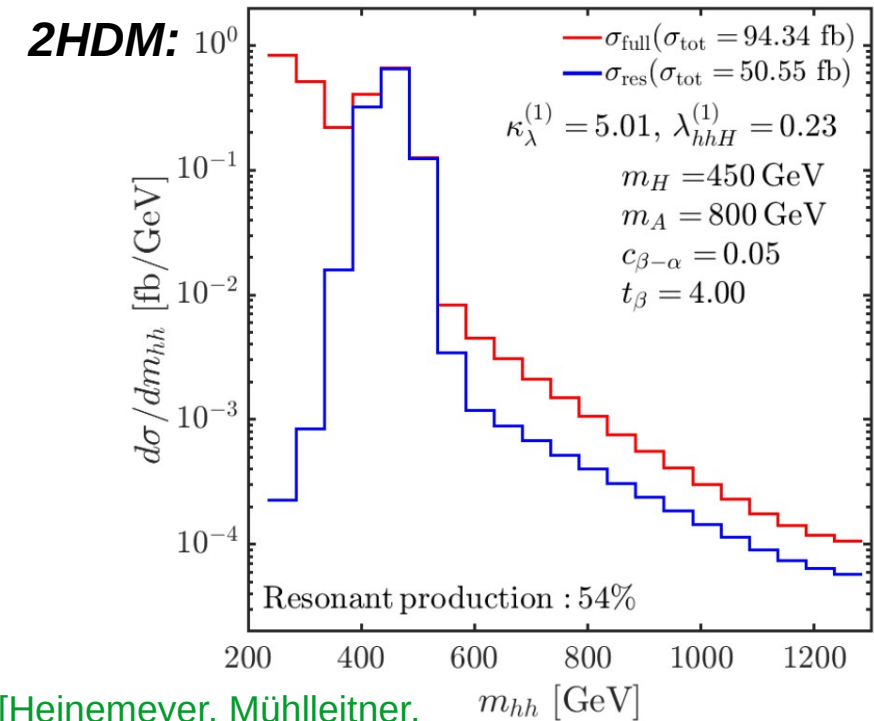
- Diagrams involving BSM scalars in s-channel (generically denoted Φ_i)
 - **collider searches for BSM scalars**
- Involve BSM trilinear scalar couplings λ_{ijk}
 - **probe of Higgs potential in extended scalar sectors**

*NB: in the following, I focus on **BSM effects (incl. higher-order corrections)** to $gg \rightarrow h_i h_j$*

→ for progress on the SM calculation, see talk by Sauro Carlotti

Interference effects: resonant contributions vs full distributions

- Including interference between non-resonant and resonant contributions can **drastically change di-Higgs invariant mass distributions**
- Relatively good agreement between peak of resonant and total m_{hh} distributions, but **away from peak they can differ by up to 2 orders of magnitude**
 - large impact of interference effects **warrants analysis of these scenarios with full predictions**
 - possible with **codes for di-Higgs predictions** (e.g. HPAIR, anyHH), or **reweighting tools**, e.g. HHReweighter [Feuerstake et al '24]



NB: this 2HDM scenario is excluded by resonant searches but allowed by non-resonant searches

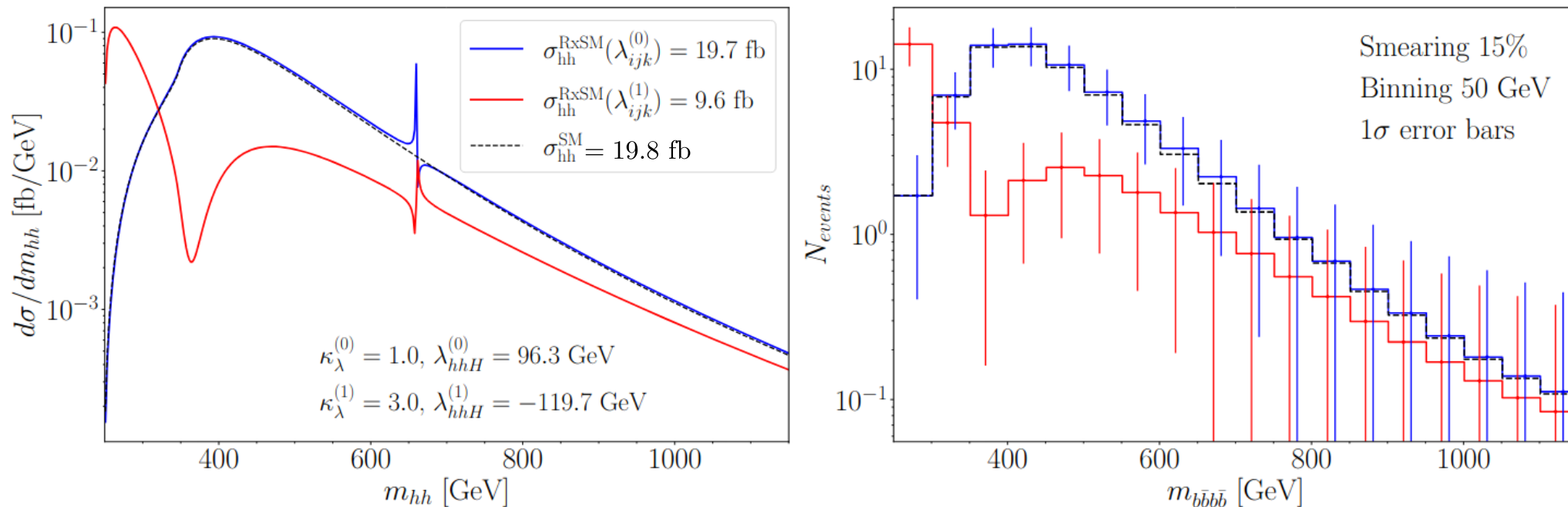
[Heinemeyer, Mühlleitner, Radchenko Serdula, Weiglein '24]

- Distributions computed with HPAIR ([Plehn, Spira, Zerwas '96], [Dawson, Dittmaier, Spira '98], [Abouabid et al. '22], [Arco et al., '25]), shown here after 15% smearing and 50 GeV binning

Impact of loop-corrected trilinears on di-Higgs production

Here: RxSM (general singlet extension of the SM) [JB, Heinemeyer, Parra Aray, Verduras Schaeidt '25]

BP1: $m_H = 659.4$ GeV, $\cos \alpha = 0.99$, $v_S = 56.1$ GeV, $\kappa_S = -880$ GeV, $\kappa_{SH} = -880$ GeV, $Z^{(0)} = 0.3$, $Z^{(1)} = 9.3$

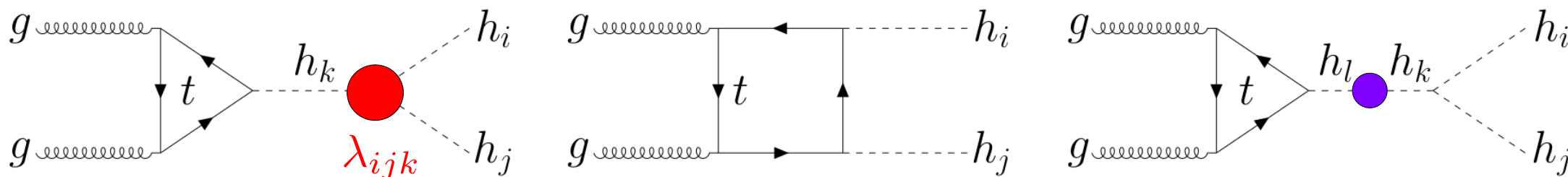


- Trilinear scalar couplings computed with anyH3 and di-Higgs production with RxSM-specific version of HPAIR [Arco et al. '25]; results shown at LO in QCD
- Large radiative corrections to both trilinear couplings, λ_{hhh} and λ_{hhH} , modify the interference pattern and shift the dip from $m_{hh} \sim 250$ GeV to 350 GeV + change the peak-dip structure at m_H into dip-peak
- After smearing and binning, analysis with tree-level trilinears ($Z^{(0)}=0.3$) would not find a deviation from the SM, but an analysis with one-loop ones can ($Z^{(1)}=9.3$)
- Similar results in 2HDM [Heinemeyer, Mühlleitner, Radchenko Serdula, Weiglein '24]

Di-Higgs production in arbitrary models: anyHH

[Bahl, JB, Gabelmann,
Radchenko Serdula,
Weiglein *WIP*]

- **anyHH: Total and differential cross-sections** (so far, at LO in QCD) for $gg \rightarrow h_i h_j$ including:
 - **1L corrections to λ_{ijk}** (computed by anyH3)
 - **BSM contributions in s-channel diagrams**
 - **momentum-dependence** both in vertex corrections and in propagators



- Takes UFO model files as inputs, as anyH3. So far limited to models without additional coloured particles.
- Good agreement found with existing results in the literature – e.g. HPAIR (*details in backup*)

Impact of NLO corrections in the 2HDM

[Bahl, JB, Gabelmann,
Radchenko Serdula,
Weiglein '26 + WIP]

Example in **aligned 2HDM**, with di-Higgs cross-section and m_{hh} distributions computed with anyHH

→ $\kappa_\lambda^{(0)} = 1$; $\lambda_{hhH}^{(0)} = 0$ (due to alignment limit)

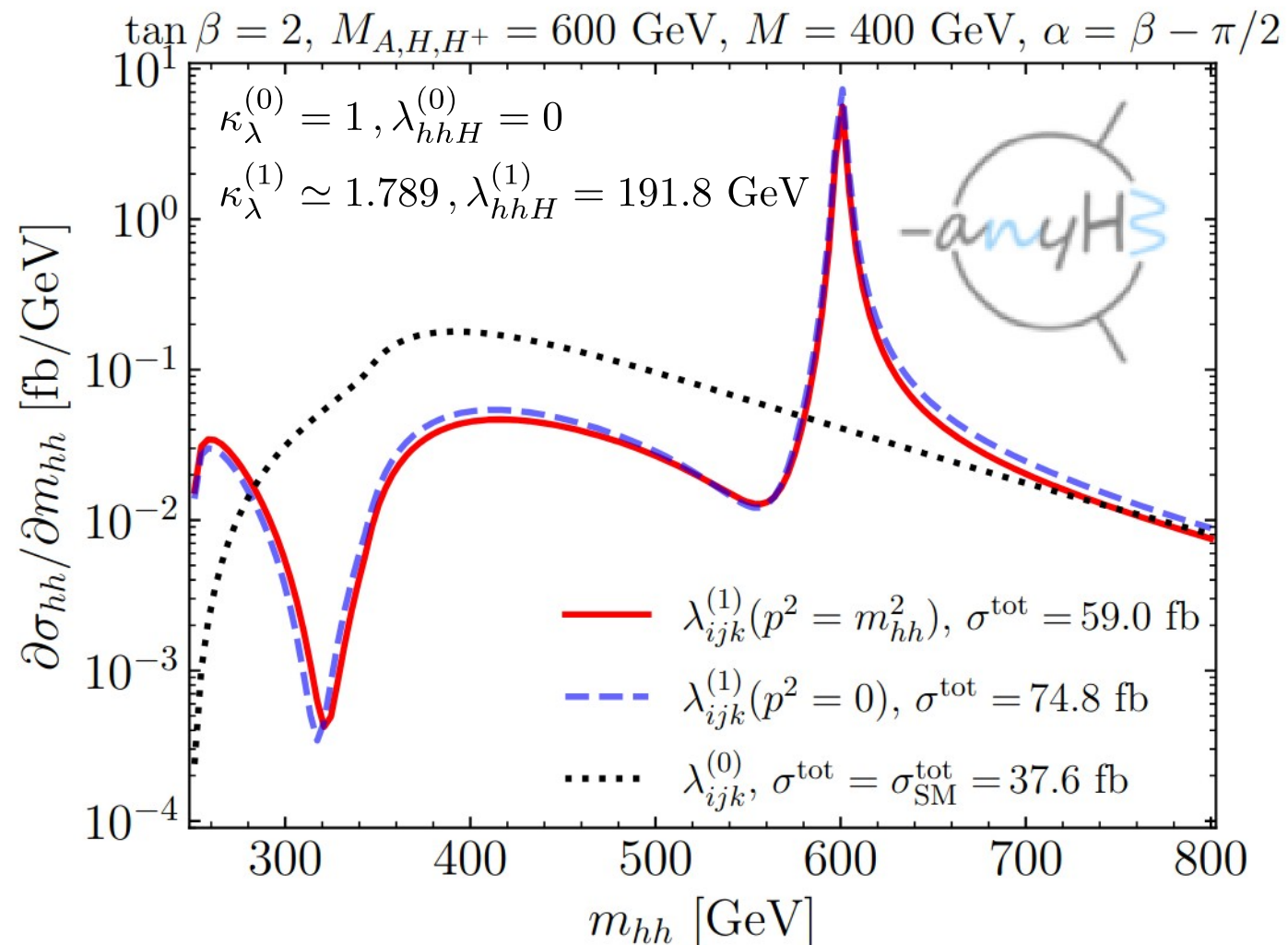
Impact of loop corrections to trilinear couplings

Black vs red curves → huge impact of loop corrections to λ_{ijk} on both cross-section and distributions

Impact of momentum in λ_{ijk} and s-channel propagators

Blue vs red curves

- shape of distribution unchanged
- O(20%) impact on total cross-section

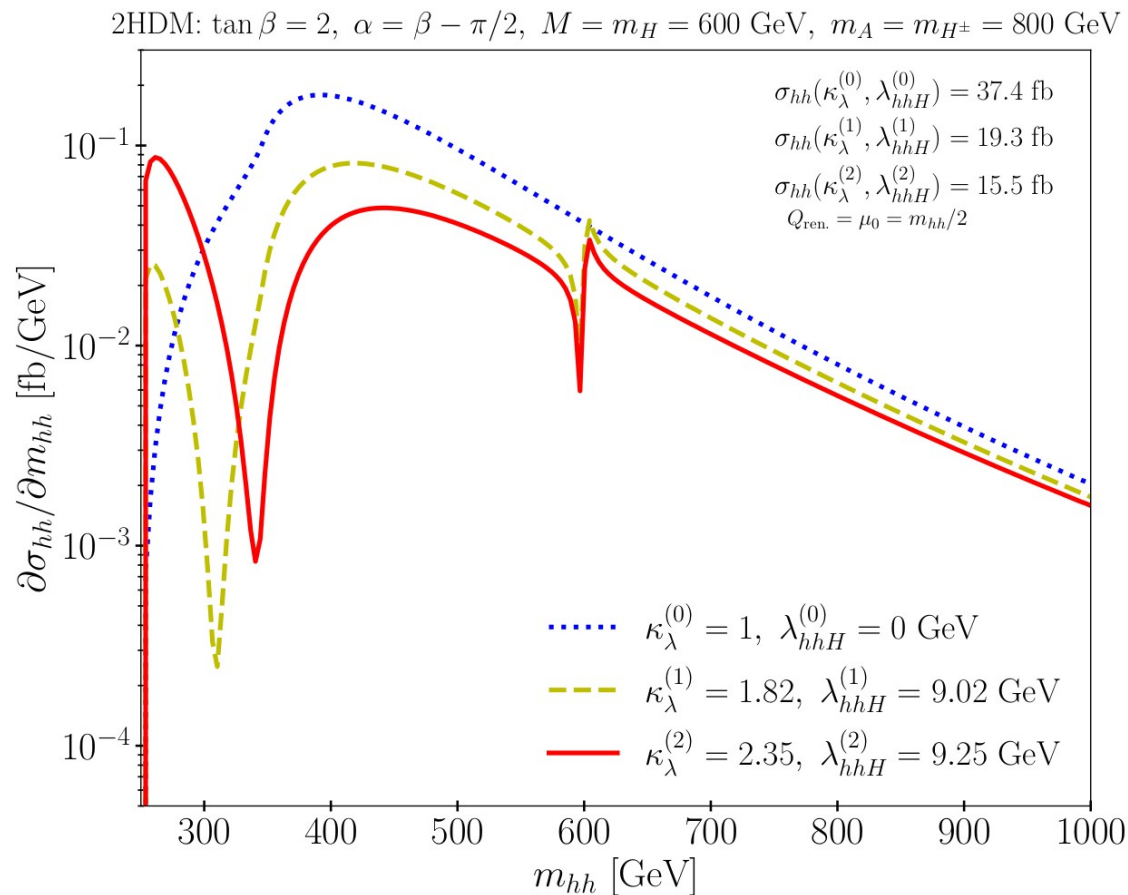
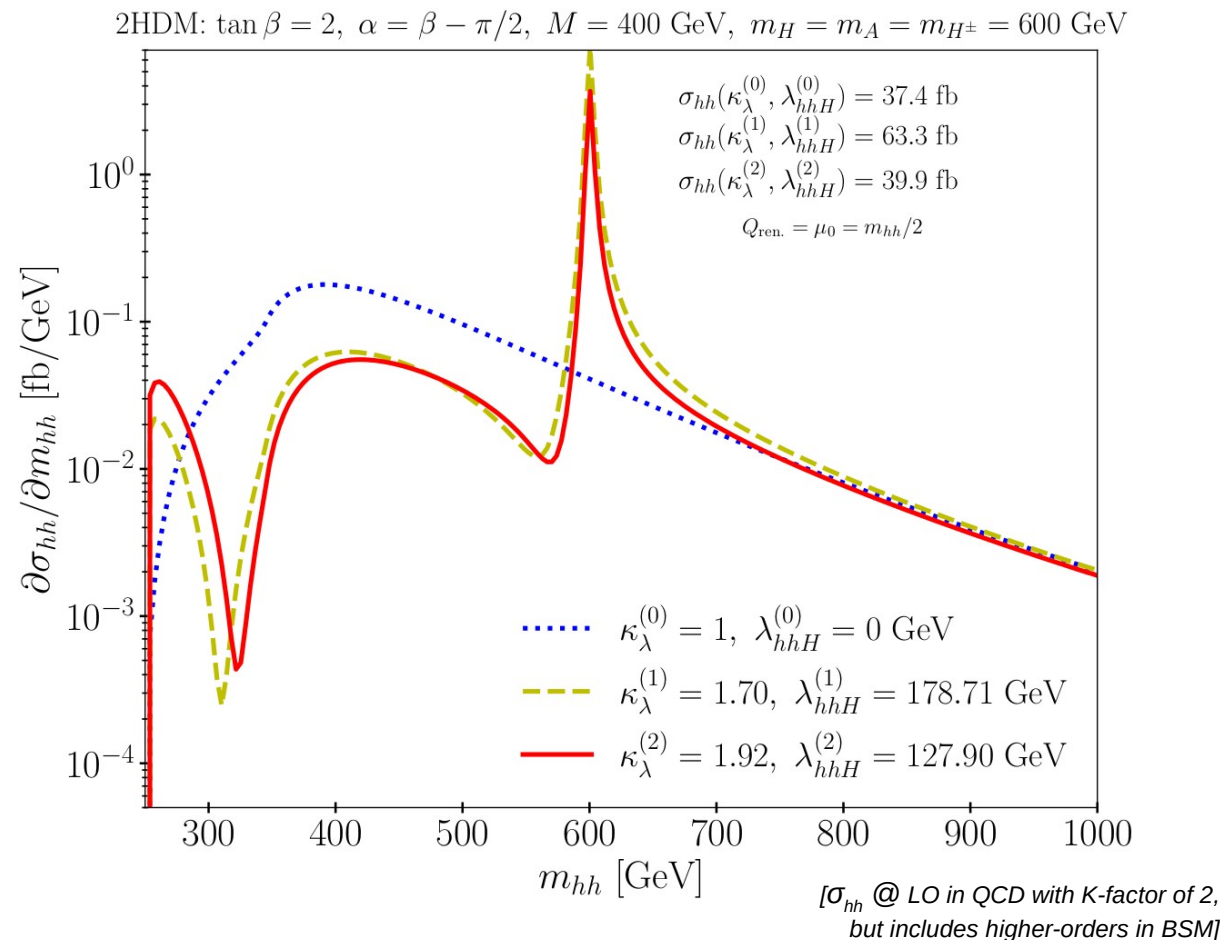


Note: similar results for impact of loop corrections also found [Heinemeyer, Mühlleitner, Radchenko Serdula, Weiglein '24]

Impact of NNLO BSM corrections in the 2HDM

What about higher orders?

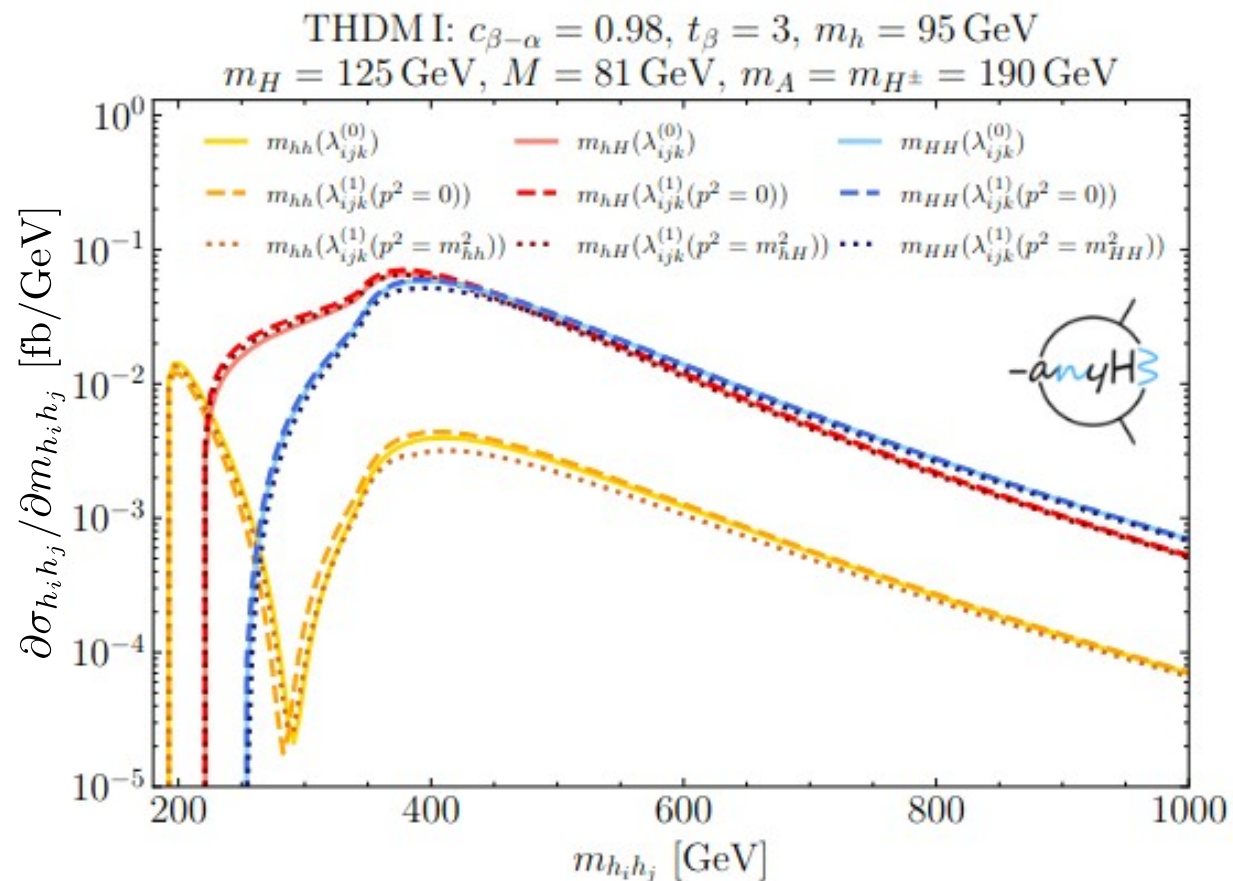
Here: example in **aligned 2HDM**, with di-Higgs cross-section and m_{hh} distributions computed with anyHH and with 2L corrections to λ_{hhh} and λ_{hhH} from [JB, Egle, Verduras Schaeidt '26 + WIP]



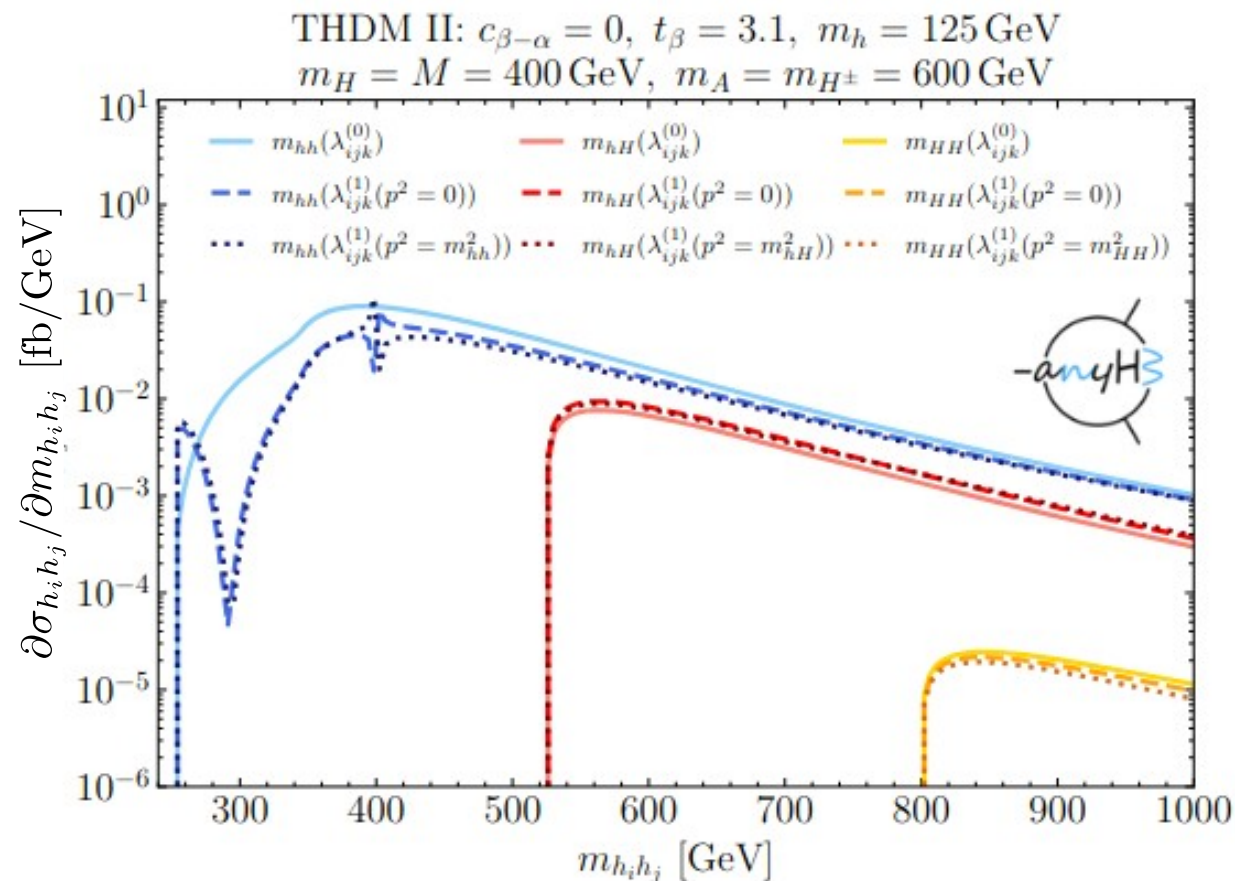
Note: see also [Degrassi, Gröber, Slavich '25] for NNLO BSM corrections to λ_{hhh} , λ_{hhH} and total di-Higgs cross-section

Results with different final states

[Bahl, JB, Gabelmann,
Radchenko Serdula,
Weiglein WIP]



Scenario 1: 2HDM type I, with a light h_{95}



Scenario 2: 2HDM type II, with heavy BSM states
 (same as in prev. slides)

Results in new models: e.g. N2HDM

[Bahl, JB, Gabelmann,
Radchenko Serdula,
Weiglein WIP]

- **N2HDM = 2HDM + real singlet**
→ 3 CP-even mass eigenstates
 h_1, h_2, h_3



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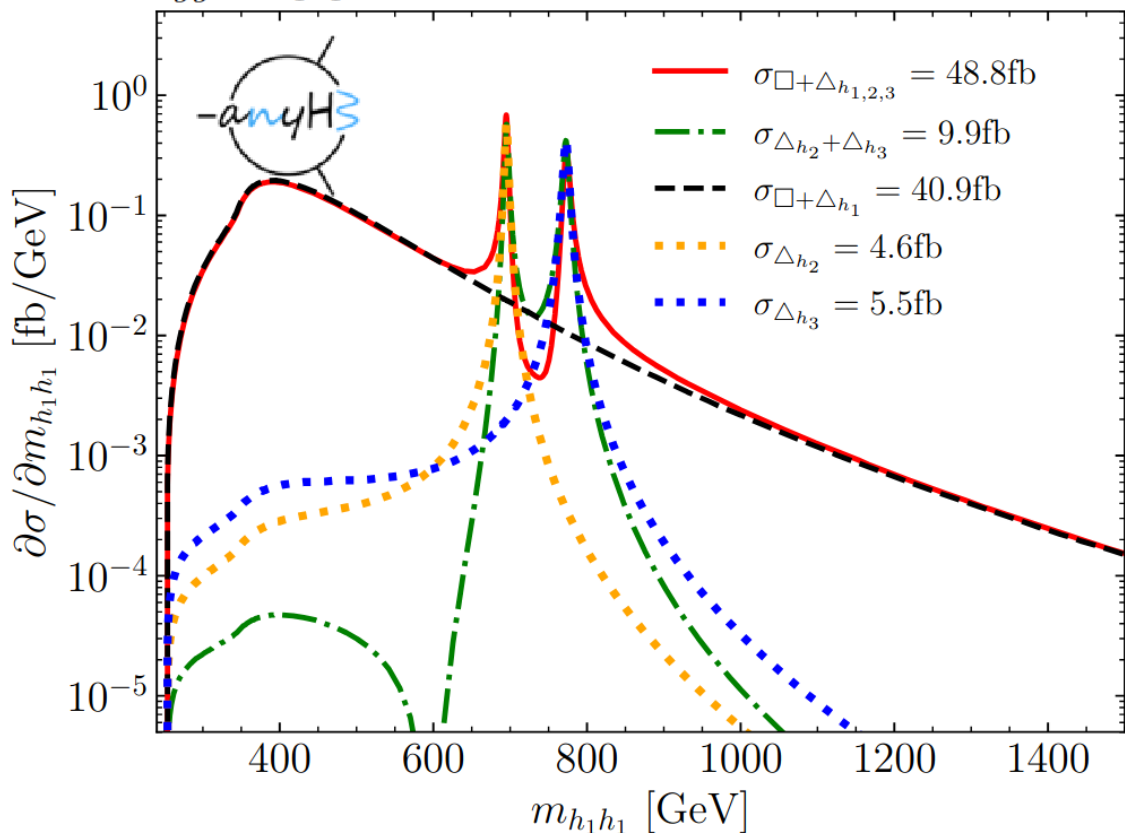
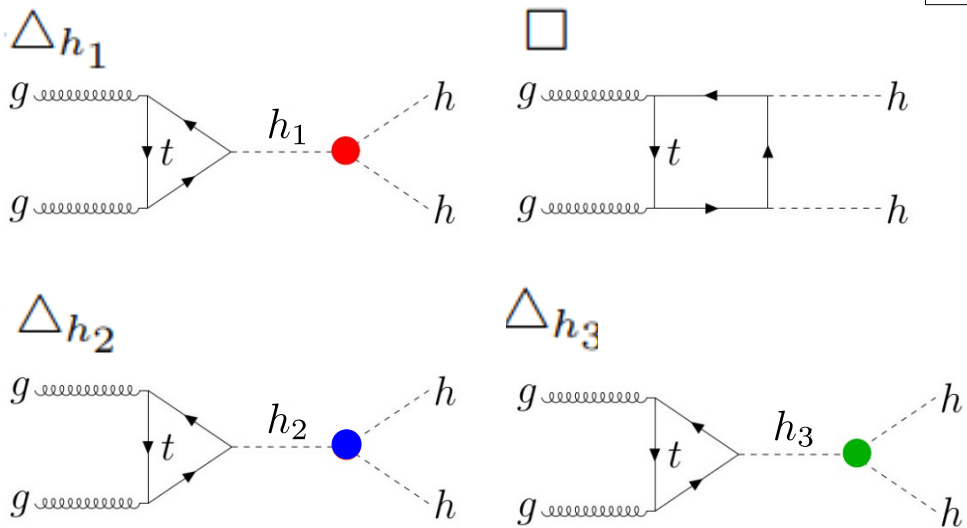


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- anyHH allows computing contributions to di-Higgs production together or separately
→ *find impact of interference*
- Here first with **tree-level trilinear scalar couplings**

$\kappa_\lambda^{(0)}$	0.97
$\kappa_\lambda^{(1)}$	2.18
$\lambda_{h_1 h_1 h_2}^{(0)}$ [GeV]	93.5
$\lambda_{h_1 h_1 h_2}^{(1)}$ [GeV]	40.3
$\lambda_{h_1 h_1 h_3}^{(0)}$ [GeV]	160.5
$\lambda_{h_1 h_1 h_3}^{(1)}$ [GeV]	342.8

$gg \rightarrow h_1 h_1$ in the NTHDM at 14 TeV: tree-level trilinears



N2HDM benchmark definition:

$$\begin{aligned}\alpha_1 &= 1.019, \\ M_{h_1} &= 125 \text{ GeV}, \\ M_A &= 673 \text{ GeV},\end{aligned}$$

$$\begin{aligned}\alpha_2 &= -0.076, \\ M_{h_2} &= 695 \text{ GeV}, \\ M_{H^\pm} &= 762 \text{ GeV}, \\ v_S &= 415.4 \text{ GeV}.\end{aligned}$$

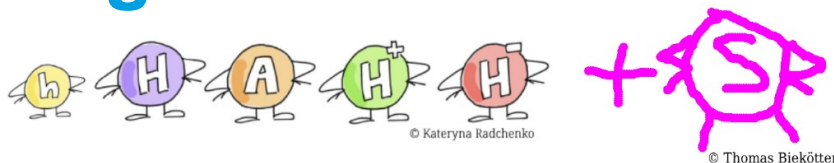
$$\begin{aligned}\alpha_3 &= 0.960, \\ M_{h_3} &= 773 \text{ GeV}, \\ M_{12} &= 367 \text{ GeV},\end{aligned}$$

*Allowed by flavour,
Higgs phys. EWPO,
vacuum stability,
perturbative unitarity*

Results in new models: e.g. N2HDM

[Bahl, JB, Gabelmann, Radchenko Serdula, Weiglein WIP]

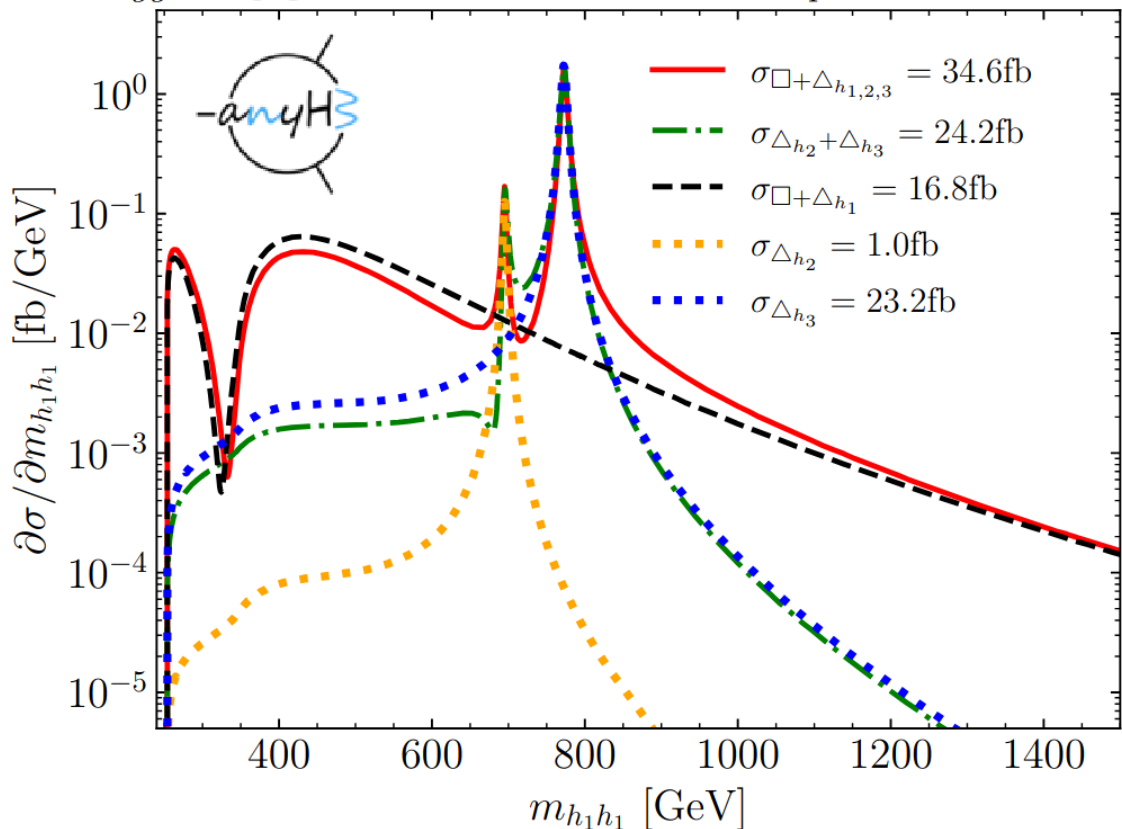
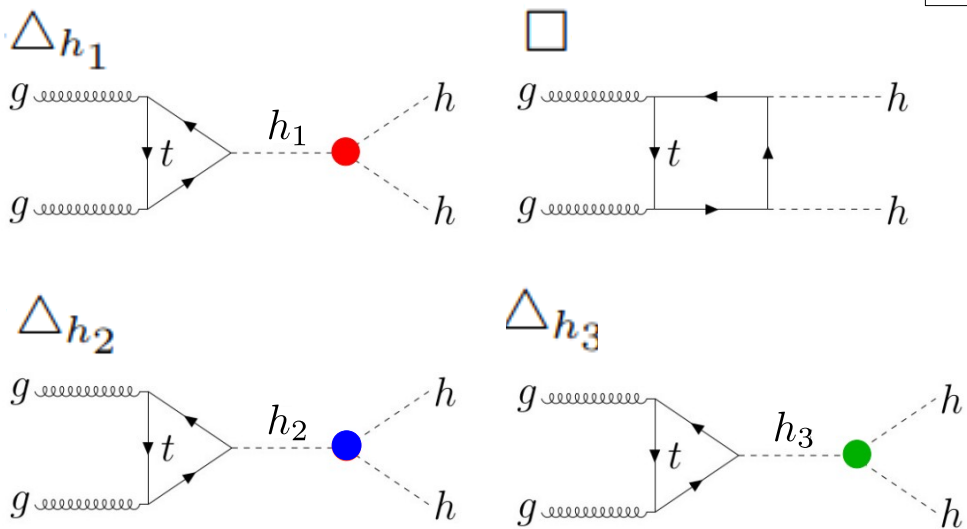
- N2HDM = 2HDM + real singlet
→ 3 CP-even mass eigenstates h_1, h_2, h_3



- anyHH allows computing contributions to di-Higgs production together or separately
→ *find impact of interference*
- Now instead with **loop-corrected trilinear scalar couplings**

$\kappa_\lambda^{(0)}$	0.97
$\kappa_\lambda^{(1)}$	2.18
$\lambda_{h_1 h_1 h_2}^{(0)}$ [GeV]	93.5
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$gg \rightarrow h_1 h_1$ in the NTHDM at 14 TeV: loop-corrected trilinears



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 $M_{12} = 367 \text{ GeV},$

Allowed by flavour,
Higgs phys. EWPO,
vacuum stability,
perturbative unitarity

Summary

- Accessing trilinear scalar couplings is crucial to reconstruct the **shape of the Higgs potential** realised in Nature, and probe **indirect signs of New Physics**
- Large BSM deviations in λ_{hhh} , as well as large loop corrections to other BSM trilinear scalar couplings, can occur for points allowed by theoretical (esp. perturbative unitarity) and experimental constraints
 - such scenarios are **strongly motivated by phenomenology and by early-Universe evolution** (e.g. possibility of a strong first-order EW phase transition)
- Higher-order corrections can be significant → **full two-loop corrections to λ_{hhh} (and λ_{hhhh}) and full one-loop corrections to any λ_{ijk} publicly available for general renormalisable theories**
- Radiative corrections to λ_{ijk} can have a **strong impact on total and differential di-Higgs cross-sections**
 - the **inclusion of these loop effects in theoretical and experimental analyses is paramount**
 - **public tools** anyH3/anyHH being developed to make this possible (link to Monte Carlo generators under development)

Thank you very much for your attention!

Contact

DESY. Deutsches
Elektronen-Synchrotron

www.desy.de

Johannes Braathen
DESY Theory group
Building 2a, Room 208a
johannes.braathen@desy.de

Backup slides

Two-loop corrections to λ_{hhh} in general renormalisable theories

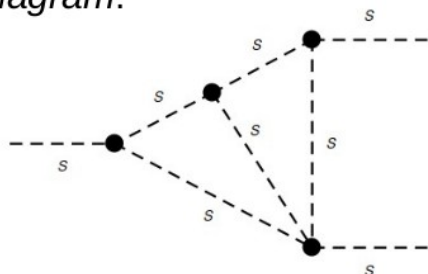
Based on

[Bahl, JB, Gabelmann, Paßehr '25]

Generic two-loop calculation of λ_{hhh}

- All 2L contributions to Higgs/scalar self-energies computed in [Goodsell, Paßehr '19]
- In [Bahl, JB, Gabelmann, Paßehr '25] we **generalised this with**
 - **new results for full 2L corrections to λ_{iii} and λ_{iiii} (with $p^2 = 0$)**
 - **results for 0-, 1-, 2-point functions in same conventions**
- Generic diagrams generated with FeynArts, computed with TwoCalc and OneCalc [Weiglein '93+]

E.g. for the following 2L diagram:



Generic couplings
(here SSS & SSSS = trilinear and quartic scalar couplings)

T-integral
[Weiglein, Scharf, Böhm '93]

Our corresponding generic expression:

```
{
  {
    {1, 1, 1},
    {edge[iv[1], v[4], S[i3]], edge[iv[2], v[5], S[i1]],
     edge[iv[3], v[6], S[i2]], edge[v[4], v[5], -S[i7]],
     edge[v[4], v[6], -S[i8]], edge[v[4], v[7], -S[i6]],
     edge[v[5], v[7], -S[i4]], edge[v[6], v[7], -S[i5]]},
    -((1fac^2*SSS[i1, i4, -i7, 1]*SSS[i2, i5, -i8, 1]*
      SSS[-i4, -i5, -i6, 1]*SSSS[i3, i6, i7, i8, 1]*
      T[Df[k1, MS[i5]]*Df[k1, MS[i8]]*Df[k3, MS[i4]]*Df[k3, MS[i7]]*
      Df[k4, MS[i6]]])/SF[1, 1/3, 1])
  }
}
```

“Edge list” → defines diagram in terms of propagators (edges)

Symmetry factor

$$\text{with } T_{i_1 \dots i_n} = \int \frac{d^d q_1 d^d q_2}{[i \pi^2 (2 \pi \mu)^{d-4}]^2} \frac{1}{(k_{i_1}^2 - m_{i_1}^2) \cdots (k_{i_n}^2 - m_{i_n}^2)}$$

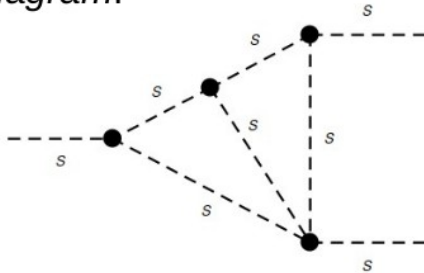
$$k_1 = q_1, \quad k_2 = q_1 + p, \quad k_3 = q_2 - q_1, \quad k_4 = q_2, \quad k_5 = q_2 + p,$$

Generic two-loop calculation of λ_{hhh}

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**Done once
and for all!**

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     edge[v[4], v[6], -S[i8]], edge[v[4], v[7], -S[i6]],
     edge[v[5], v[7], -S[i4]], edge[v[6], v[7], -S[i5]]},
    -((1fac^2*SSS[i1, i4, -i7, 1]*SSS[i2, i5, -i8, 1]*
      SSS[-i4, -i5, -i6, 1]*SSSS[i3, i6, i7, i8, 1]*
      T[Df[k1, MS[i5]]*Df[k1, MS[i8]]*Df[k3, MS[i4]]*Df[k3, MS[i7]]*
      Df[k4, MS[i6]]])/SF[1, 1/3, 1])
  }
}
```

"Edge list" → defines diagram
in terms of propagators (edges)

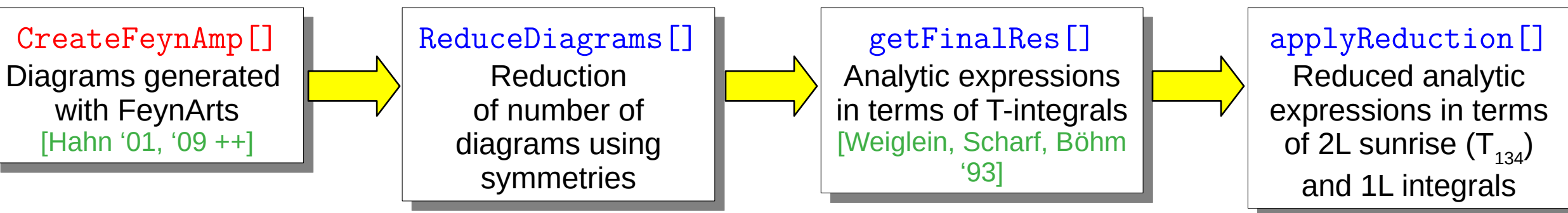
Symmetry factor

$$\text{with } T_{i_1 \dots i_n} = \int \frac{d^d q_1 d^d q_2}{[i \pi^2 (2 \pi \mu)^{d-4}]^2} \frac{1}{(k_{i_1}^2 - m_{i_1}^2) \cdots (k_{i_n}^2 - m_{i_n}^2)}$$

$$k_1 = q_1, \quad k_2 = q_1 + p, \quad k_3 = q_2 - q_1, \quad k_4 = q_2, \quad k_5 = q_2 + p,$$

Generic two-loop calculation of λ_{hhh} – application to models

- All 2L contributions to Higgs/scalar self-energies computed in [Goodsell, Paßehr '19]
- In [Bahl, JB, Gabelmann, Paßehr '25] we **generalised this with**
 - **new results for full 2L corrections to λ_{iii} and λ_{iiii} (with $p^2 = 0$)**
 - **results for 0-, 1-, 2-point functions in same conventions**
- Generic diagrams generated with FeynArts, computed with TwoCalc and OneCalc [Weiglein '93+]
- Application of generic results – for both **genuine 2L diagrams** and **2L subloop renormalisation diagrams** – to concrete models using FeynArts output and self-made routines (*see details in backup*)

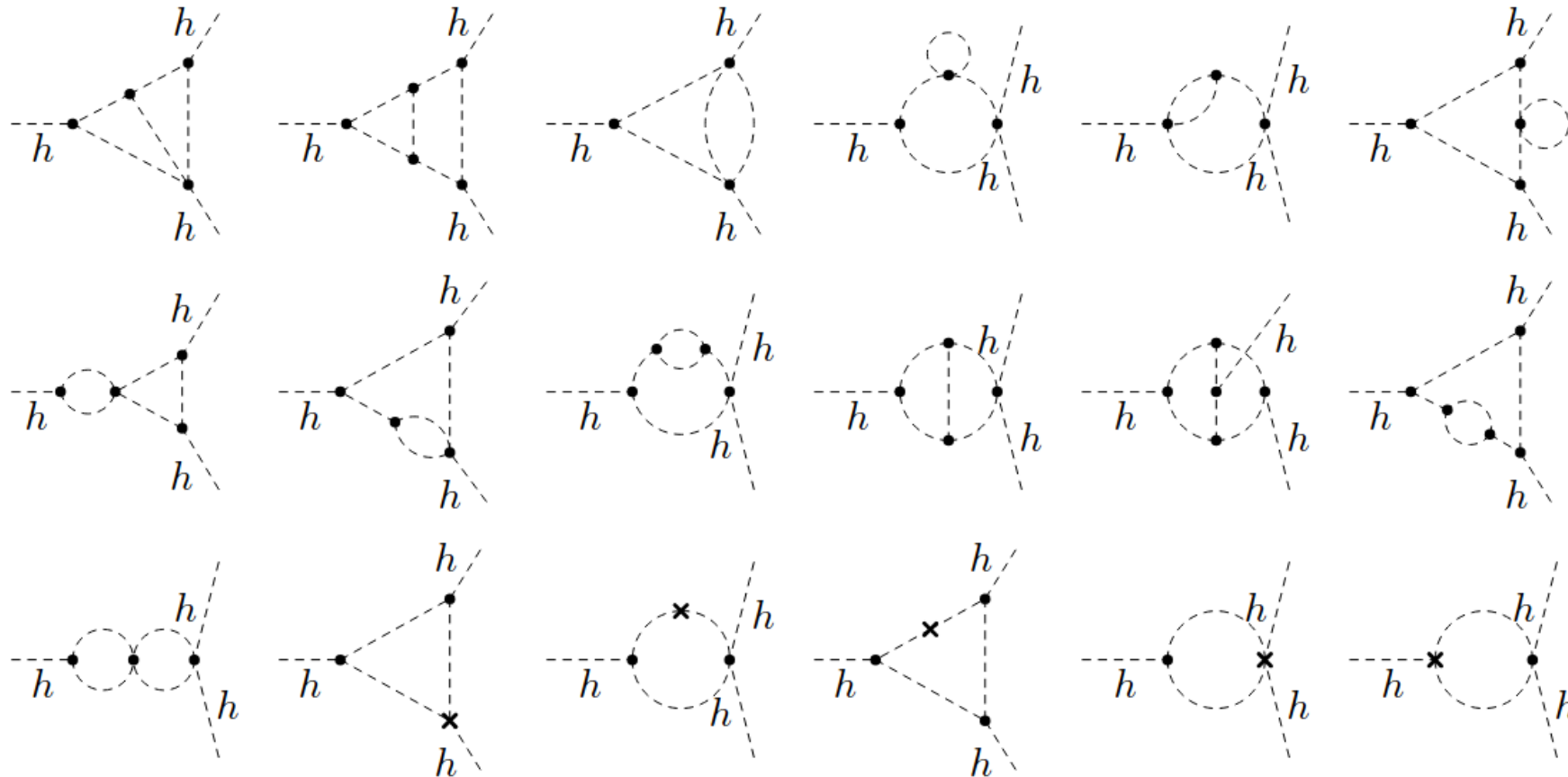


New results: leading two-loop corrections in the RxSM

- **RxSM, or SSM** = Real-singlet extension of SM, without Z_2 symmetry

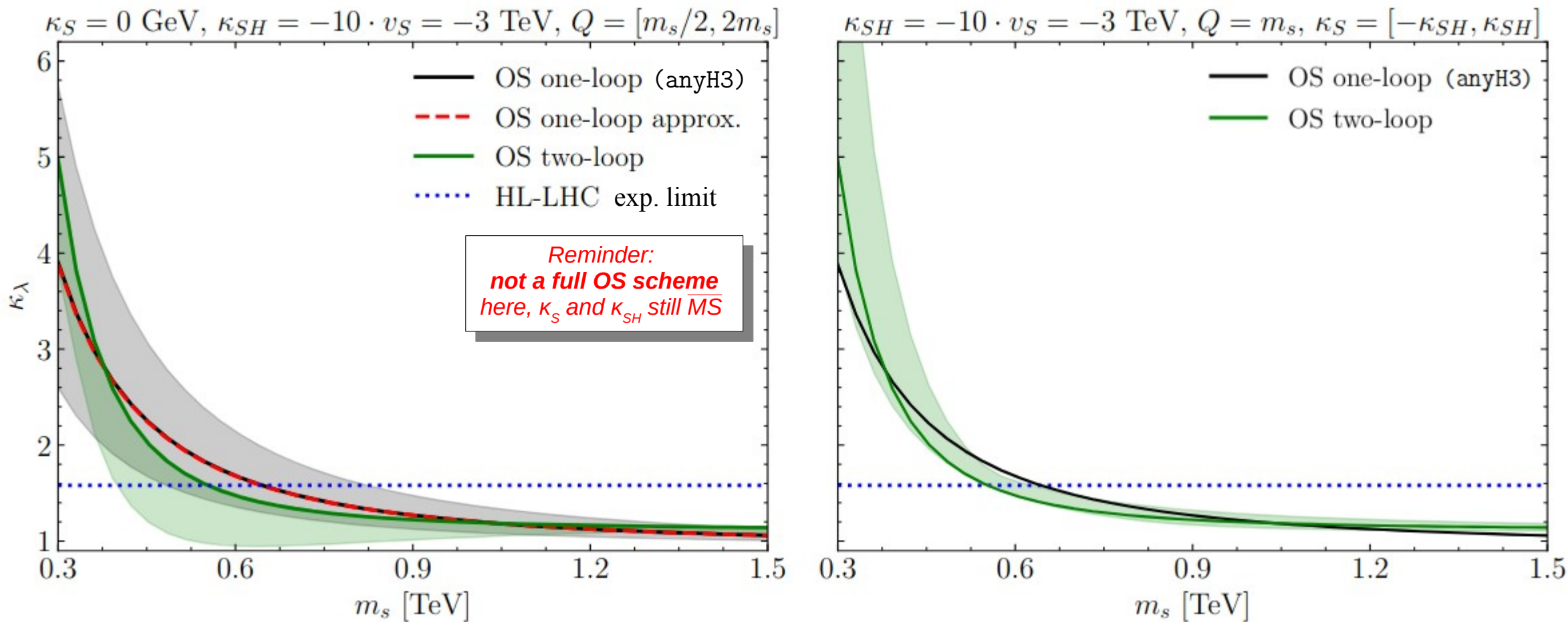
$$V_{\text{SSM}} = \mu^2 |\Phi|^2 + \frac{\lambda_H}{2} |\Phi|^4 + \frac{m_S^2}{2} S^2 + \frac{\kappa_S}{3} S^3 + \frac{\lambda_S}{2} S^4 + \kappa_{SH} S |\Phi|^2 + \frac{\lambda_{SH}}{2} S^2 |\Phi|^2$$

- Diagrams (reduced) for 2L contributions to λ_{hhh} from BSM scalar S:



New results: leading two-loop corrections in the RxSM

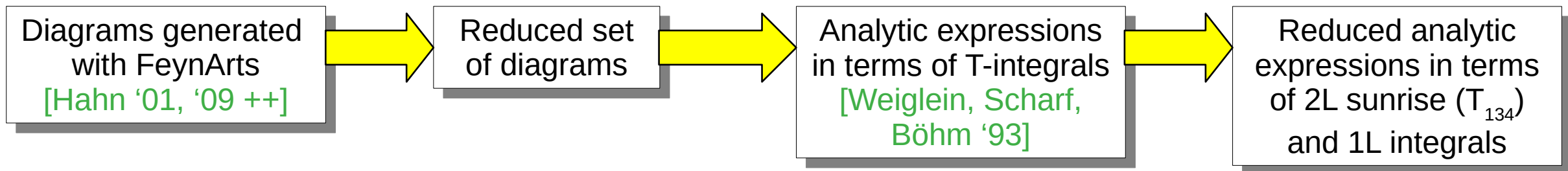
Numerical results, in decoupling limit $m_s \rightarrow \infty$



→ inclusion of 2L corrections significantly reduces the renormalisation-scale dependence

Computing λ_{hhh} in general renormalisable theories: method

- All 2L contributions to Higgs/scalar self-energies computed in [Goodsell, Paßehr '19]
- In [Bahl, JB, Gabelmann, Paßehr '25] we **generalise this with**
 - **new results for full 2L corrections to λ_{iii} and λ_{iiii} (with $p^2 = 0$)**
 - **results for 0-, 1-, 2-point functions in same conventions**
- Generic diagrams generated with FeynArts, computed with TwoCalc and OneCalc [Weiglein '93+]
- Generic results then mapped to specific models via private routines (using FeynArts diagrams and amplitudes; could in principle also be done starting from UFO model)
- Topologies and diagrams reduced, taking benefit of symmetries → details in next slides
- Application of generic results to concrete models:
(for genuine 2L diagrams, and 2L subloop renormalisation diagrams)



Reduction of T-integrals

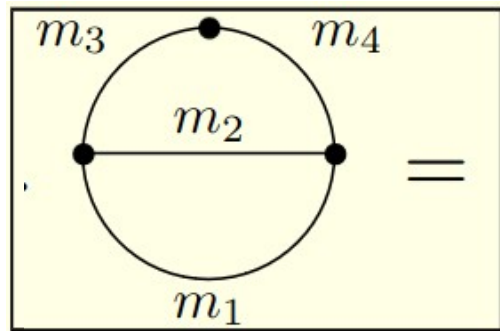
Diagrams generated
with FeynArts
[Hahn '01, '09 ++]

Reduced set
of diagrams

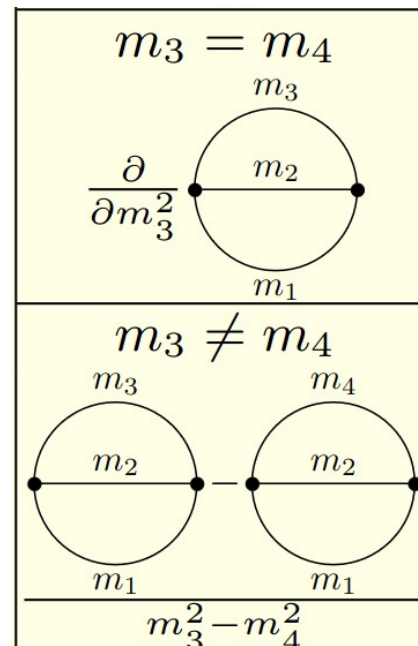
Analytic expressions
in terms of T-integrals
[Weiglein, Scharf,
Böhm '93]

Reduced analytic
expressions in terms
of 2L sunrise (T_{134})
and 1L integrals

E.g. reduction of
 T_{1134} integral



Analytically
or
on-the-fly
during
numerical
evaluation



- Reduction of T-integrals to 2L sunrise (T_{134}) and products of 1L integrals
- Can be done in with our Mathematica routines to handle and use the generic results
- Or can be done on-the-fly during numerical evaluation in our Python routines (similar to what is implemented in FeynHiggs now [Paßehr '22])

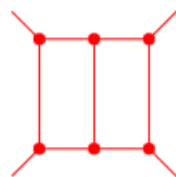
Reducing topologies and diagrams using symmetries

In our setting:

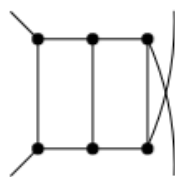
- External states are identical ($\rightarrow \lambda_{\text{iii}}, \lambda_{\text{iii}}$)
- All external momenta set to zero

} $\rightarrow$ **many diagrams are identical**

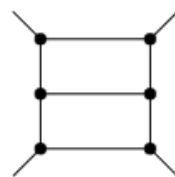
For example, double box diagram,
with fermions and scalars:
 $\rightarrow$ 6 topologies, 4 diagrams/topology



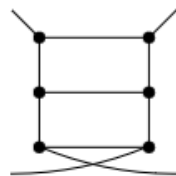
T1



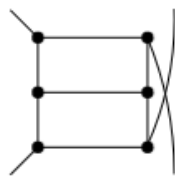
T2



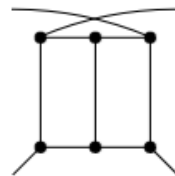
T3



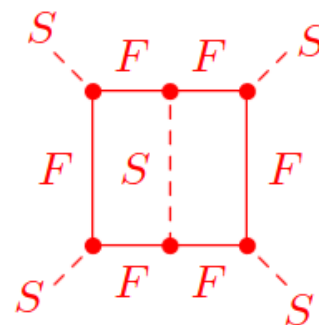
T4



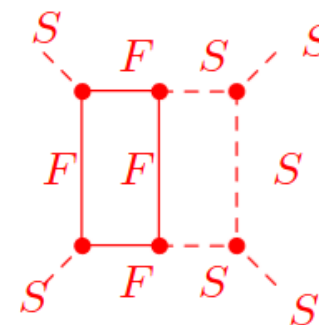
T5



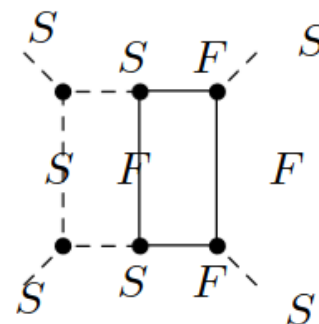
T6



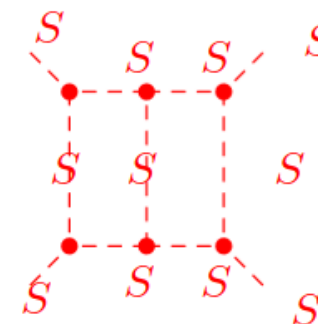
T1 G1 N1



T1 G2 N2



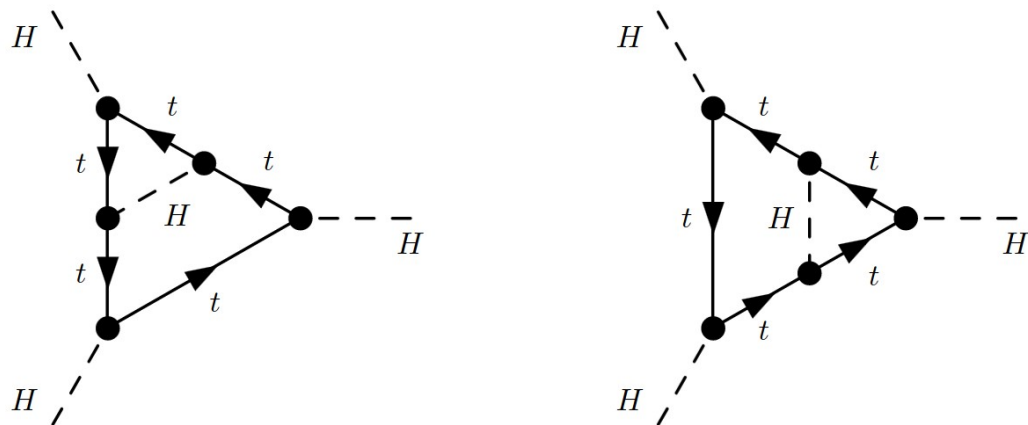
T1 G3 N3



T1 G4 N4

$\rightarrow$ **Only need to compute 3 diagrams, rather than 24!**

Automating the reduction



Unique representation of diagram

→ "**canonical edges**"

- list of "edges" (= lines in diagram)
- identical diagrams $\leftrightarrow$ permutations of edges
- canonical form = one particular choice of ordering

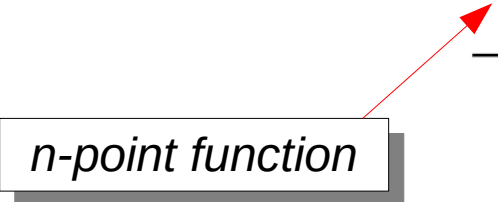
Reduction algorithm with "canonical edges" in pseudo code:

- identify internal and external indices
- generate permutations of external indices
- generate permutations of internal indices
- combine permutations of internal and external indices
- permute edge list following the combined list of permutations
- sort list of permuted edge lists
- return first element of sorted list of "edge lists"

```
edgelist1 = {edge[v[1], v[4], S[1]], edge[v[2], v[5], S[1]],
             edge[v[3], v[6], S[1]], edge[v[4], v[7], -F[3]],
             edge[v[4], v[8], F[3]], edge[v[5], v[6], F[3]],
             edge[v[5], v[8], -F[3]], edge[v[6], v[7], F[3]],
             edge[v[7], v[8], S[1]]}

edgelist2 = {edge[v[1], v[4], S[1]], edge[v[2], v[5], S[1]],
             edge[v[3], v[6], S[1]], edge[v[4], v[5], F[3]],
             edge[v[4], v[7], -F[3]], edge[v[5], v[8], F[3]],
             edge[v[6], v[7], F[3]], edge[v[6], v[8], -F[3]],
             edge[v[7], v[8], S[1]]}
```

Reducing topologies and diagrams in practice



n	topology-level	field-level
0	$2 \rightarrow 2$	$11 \rightarrow 11$
1	$3 \rightarrow 3$	$25 \rightarrow 25$
2	$9 \rightarrow 8$	$121 \rightarrow 92 (102)$
3	$40 \rightarrow 13$	$936 \rightarrow 229 (291)$
4	$265 \rightarrow 29$	$10496 \rightarrow 698 (928)$

- Count **number of two-loop diagrams before and after reduction** of diagrams using canonical edges algorithm
 - ➔ At the level of topologies
 - ➔ At the level of generic field insertions*[Numbers in brackets → for models with CP violation]*

- **Reduction of up to one order of magnitude!**

NB: numbers shown here at generic level, not considering model-specific particle insertions nor summation over generation indices

RxSM

The RxSM

General singlet extension of the SM (no Z_2 symmetry)

➤ Potential

$$V(\Phi, S) = \mu^2(\Phi^\dagger\Phi) + \frac{\lambda}{2}(\Phi^\dagger\Phi)^2 + \kappa_{SH}(\Phi^\dagger\Phi)S + \frac{\lambda_{SH}}{2}(\Phi^\dagger\Phi)S^2 + \frac{M_S^2}{2}S^2 + \frac{\kappa_S}{3}S^3 + \frac{\lambda_S}{2}S^4.$$

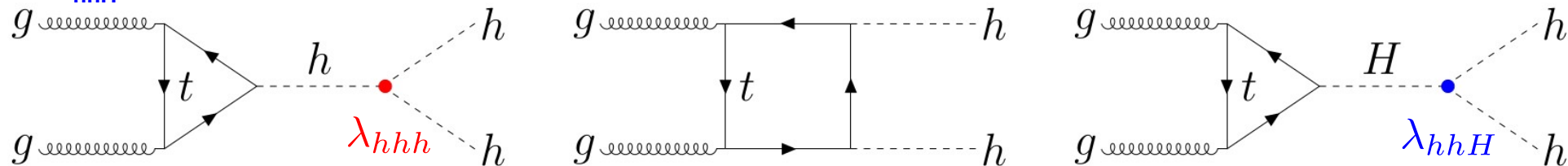
➤ Field content (after EWSB) $\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}G^+ \\ v + \phi + iG^0 \end{pmatrix}, \quad S = s + v_S$

➤ Mixing among 2 CP-even Higgs bosons: Gauge basis: $\phi, s \xrightarrow{\text{mixing angle } \alpha} \text{Mass basis: } h, H$

➤ RxSM scalar sector parametrised in terms of $\underbrace{m_h^2, v}_{\text{known}}, \underbrace{m_H^2, \alpha, v_S, \kappa_S, \kappa_{SH}}_{\text{free}}$

➤ Three leading diagrams in di-Higgs production, with significant dependence on 2 relevant trilinears:

λ_{hhh} and λ_{hhH}



Full on-shell renormalisation of the RxSM

[JB, Heinemeyer, Parra Arnay,
Verduras Schaeidt *WIP*]

- Masses: m_h^2, m_H^2

Renormalization of two-point functions

- EW VEV: v

SM-like electroweak sector

- Singlet VEV: v_S

No divergences

- Mixing angle: α

Rotation matrix: [Kanemura, Kikuchi, Yagyu, '15]

- Tadpoles: t_ϕ, t_s

OS/Standard scheme

- Kappas: κ_S, κ_{SH}

$$\hat{\lambda}_{hHH}^{(1)} \stackrel{!}{=} \lambda_{hHH}^{(0)} \quad \hat{\lambda}_{HHH}^{(1)} \stackrel{!}{=} \lambda_{HHH}^{(0)}$$

Slide by A. Verduras Schaeidt

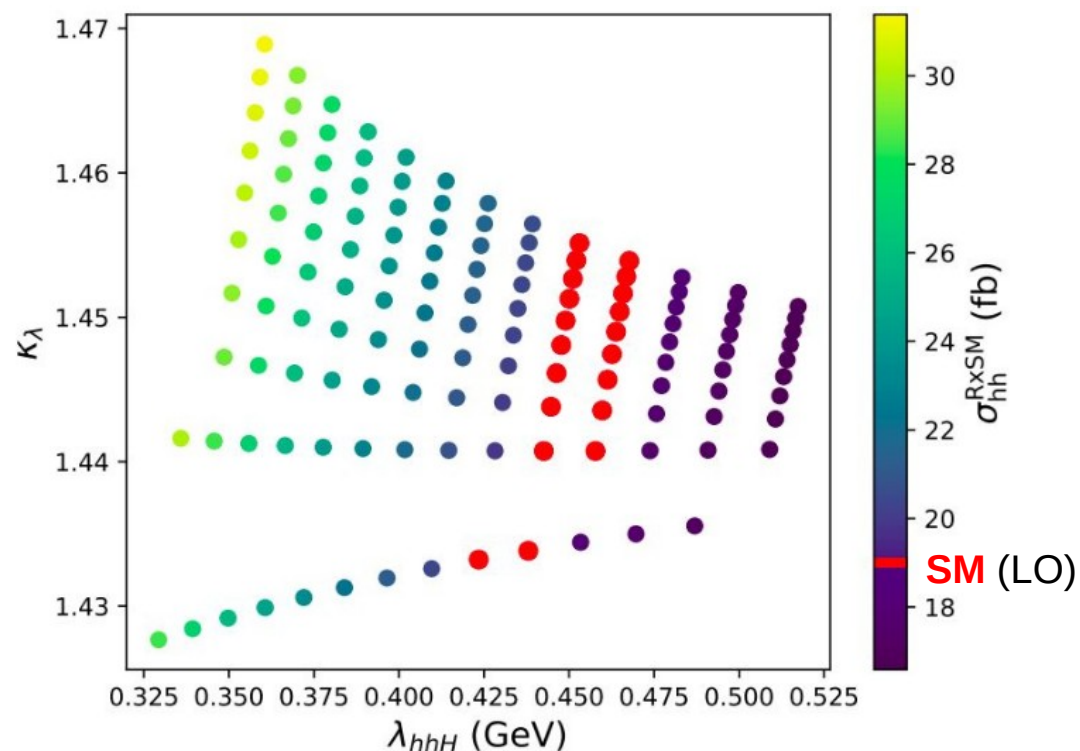
Interference effects in the RxSM

Di-Higgs production cross-sections: total result vs resonant only

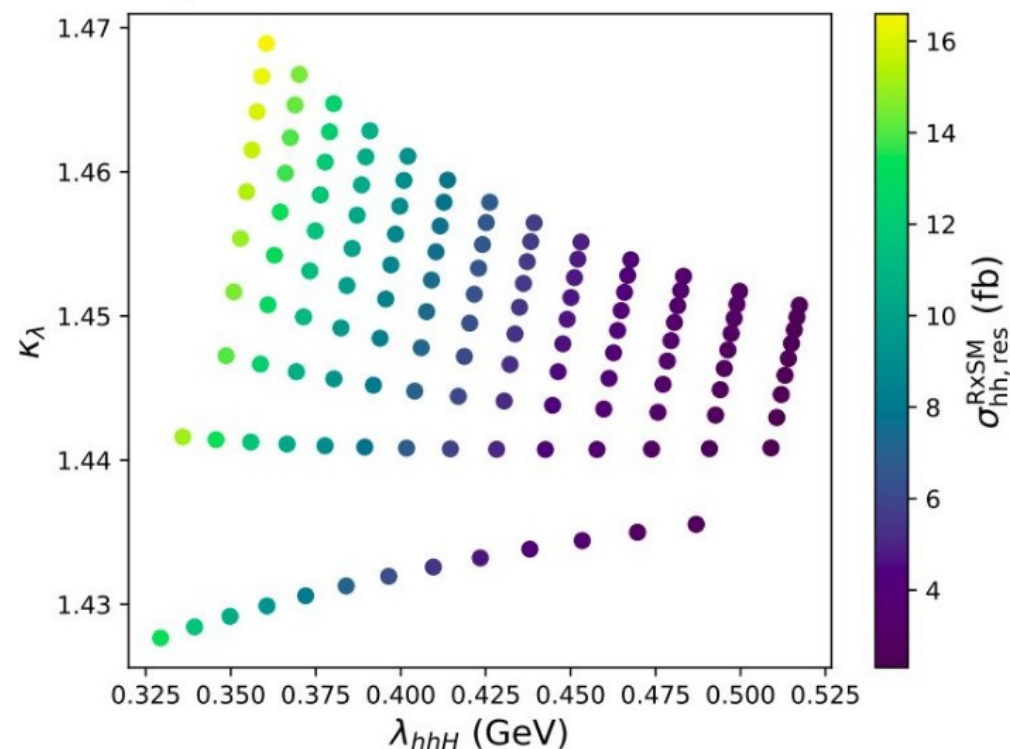
[Arco, Heinemeyer, Mühlleitner,
Parra Aray, Rivero Gonzalez,
Verduras Schaeidt '25]

- Di-Higgs predictions computed with RxSM-specific version of HPAIR (see [Plehn, Spira, Zerwas '96], [Dawson, Dittmaier, Spira '98], [Abouabid et al. '22]) → results up to NLO QCD for cross-sections and LO for distributions
- Results shown here for RxSM scan points at LO QCD, for consistency with distributions shown in next slides
- With HPAIR: $(\sigma_{hh}^{SM})^{LO} = 19.76 \text{ fb}$ while $(\sigma_{hh}^{SM})^{NLO} = 38.24 \text{ fb}$

Full calculation



Only H resonant contribution

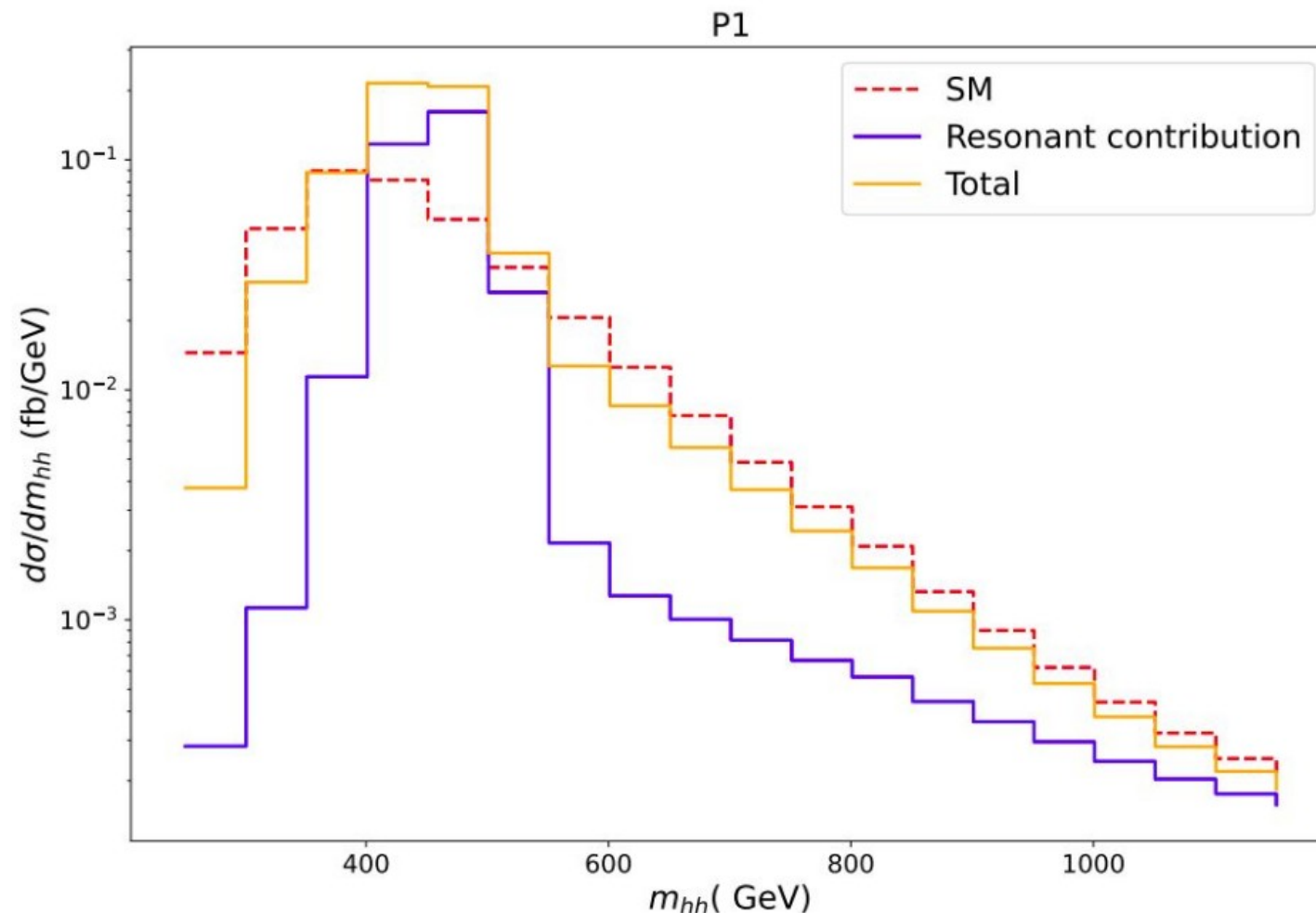


- **Resonant contribution only amounts to 20-50% of the total cross-section!**

Interference effects in the RxSM

Di-Higgs production differential distributions: total result vs resonant only

[Arco, Heinemeyer, Mühlleitner,
Parra Aray, Rivero Gonzalez,
Verduras Schaeidt '25]



Benchmark P1

$$m_H = 459 \text{ GeV}$$

$$\alpha = 0.984$$

$$v_S = 46 \text{ GeV}$$

$$\kappa_\lambda = 1.47$$

$$\lambda_{hhH} = 177 \text{ GeV}$$

$$\sigma_{hh}^{\text{RxSM, tot}} = 31.1 \text{ fb}$$

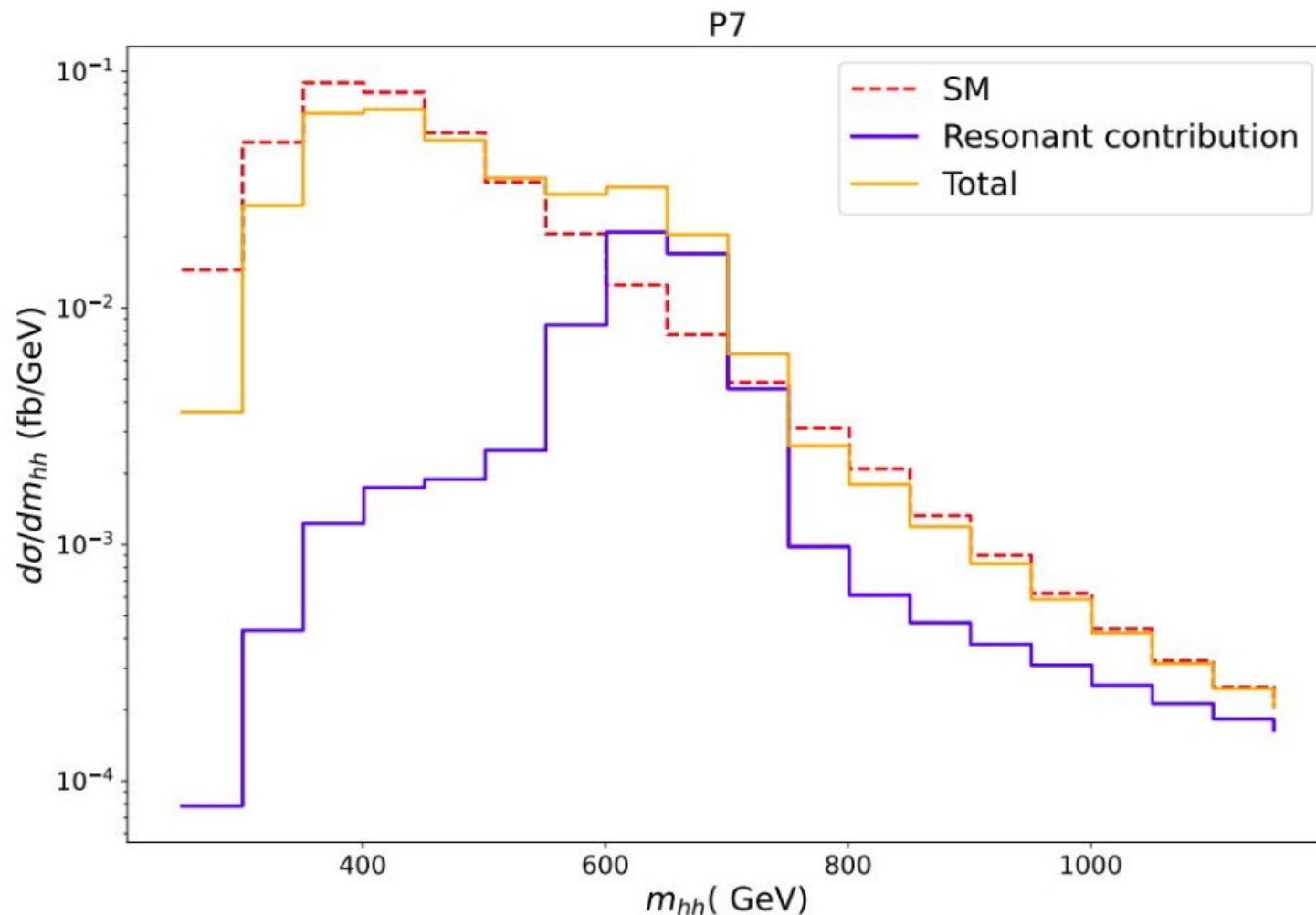
$$\sigma_{hh}^{\text{RxSM, res}} = 16.3 \text{ fb}$$

- Distributions shown here after **experimental effects** are taken into account (**15% smearing and 50 GeV binning**)
- **Relatively good agreement between peak of resonant and total distributions, but tails are far apart**

Interference effects in the RxSM

Di-Higgs production differential distributions: total result vs resonant only

[Arco, Heinemeyer, Mühlleitner,
Parra Aray, Rivero Gonzalez,
Verduras Schaeidt '25]



Benchmark P7

$$m_H = 643 \text{ GeV}$$

$$\alpha = 0.992$$

$$v_S = 34 \text{ GeV}$$

$$\kappa_\lambda = 1.44$$

$$\lambda_{hhH} = 241 \text{ GeV}$$

$$\sigma_{hh}^{\text{RxSM, tot}} = 17.5 \text{ fb}$$

$$\sigma_{hh}^{\text{RxSM, res}} = 3.1 \text{ fb}$$

- The distributions differ by up to 2 orders of magnitude (esp. for low m_{hh})
- The peak in the total distribution is significantly broadened

Impact of NLO corrections in the RxSM

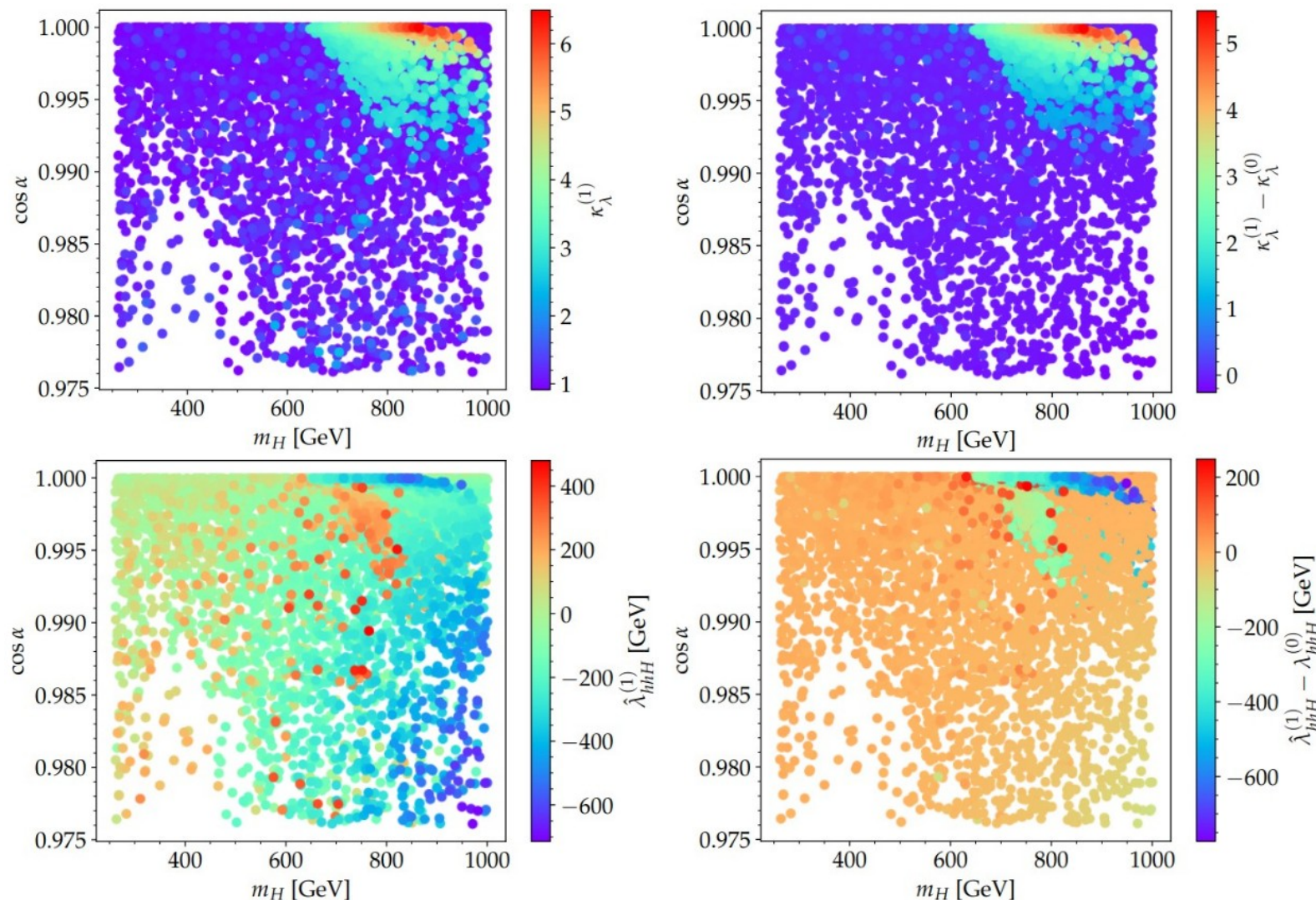
[JB, Heinemeyer, Parra Aray,
Verduras Schaeidt *WIP*]

Loop corrections

to κ_λ and λ_{hhH} :

- Complete one-loop (1L) calculations of the trilinear scalar couplings λ_{hhh} and λ_{hhH} , using anyH3 (v2) [Bahl, JB, Gabelmann, Radchenko, Serdula, Weiglein *WIP*]
- Full on-shell renormalisation of all RxSM parameters (*c.f. backup*)
- Region for large $m_H > \sim 750$ GeV, small $v_S < 50$ GeV, and small mixing features **large radiative corrections to the trilinears**

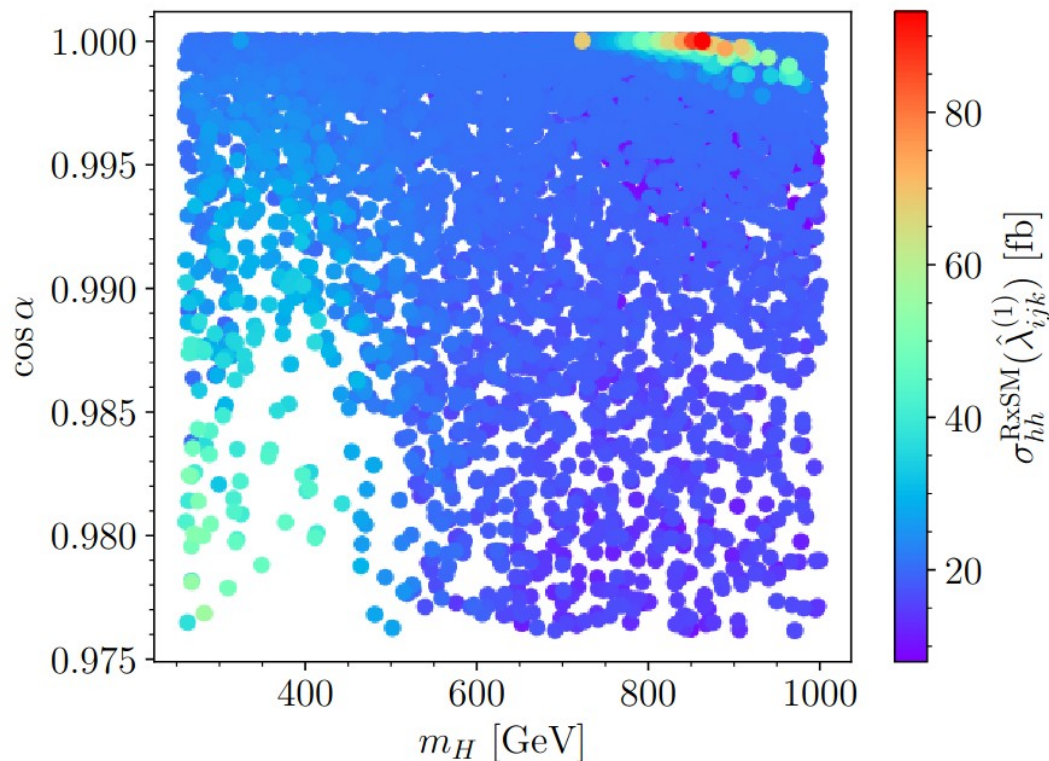
5D scan over BSM param. ($m_H, v_S, \kappa_S, \kappa_{SH}, \alpha$) of RxSM parameter space, including state-of-the-art theory and exp. constraints – shown here in $\{m_H, \cos\alpha\}$ plane



Impact of NLO corrections in the RxSM

[JB, Heinemeyer, Parra Aray,
Verduras Schaeidt *WIP*]

Impact on di-Higgs production



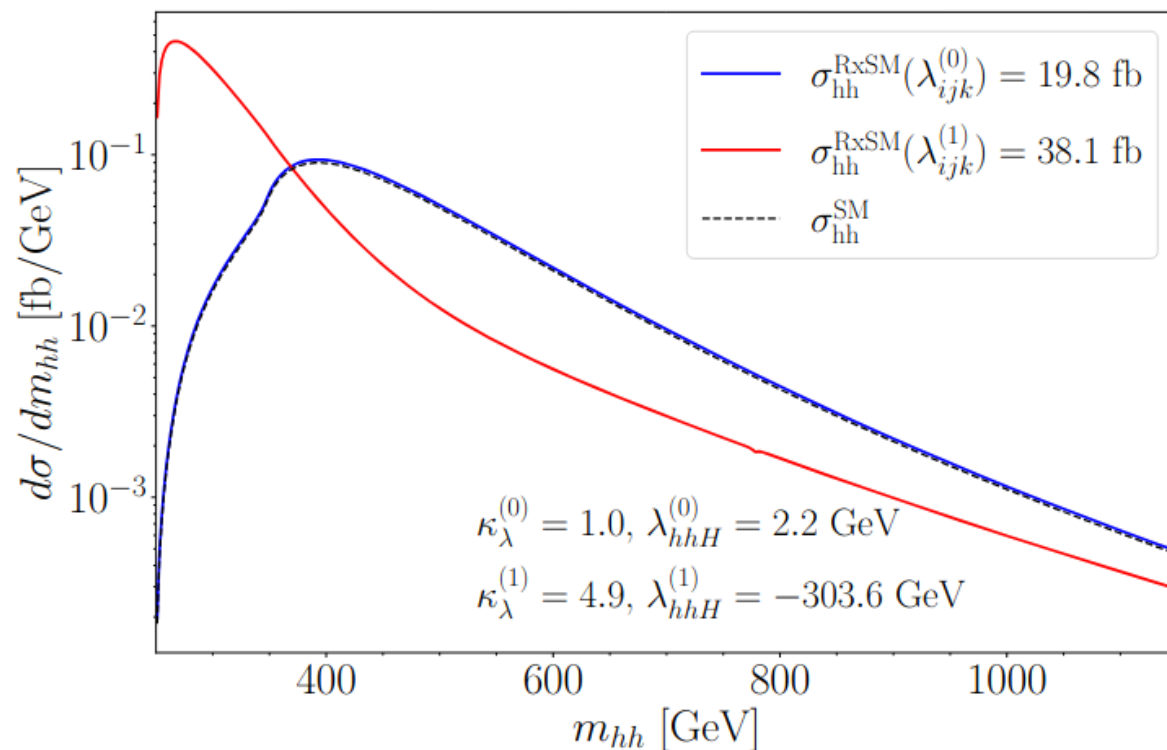
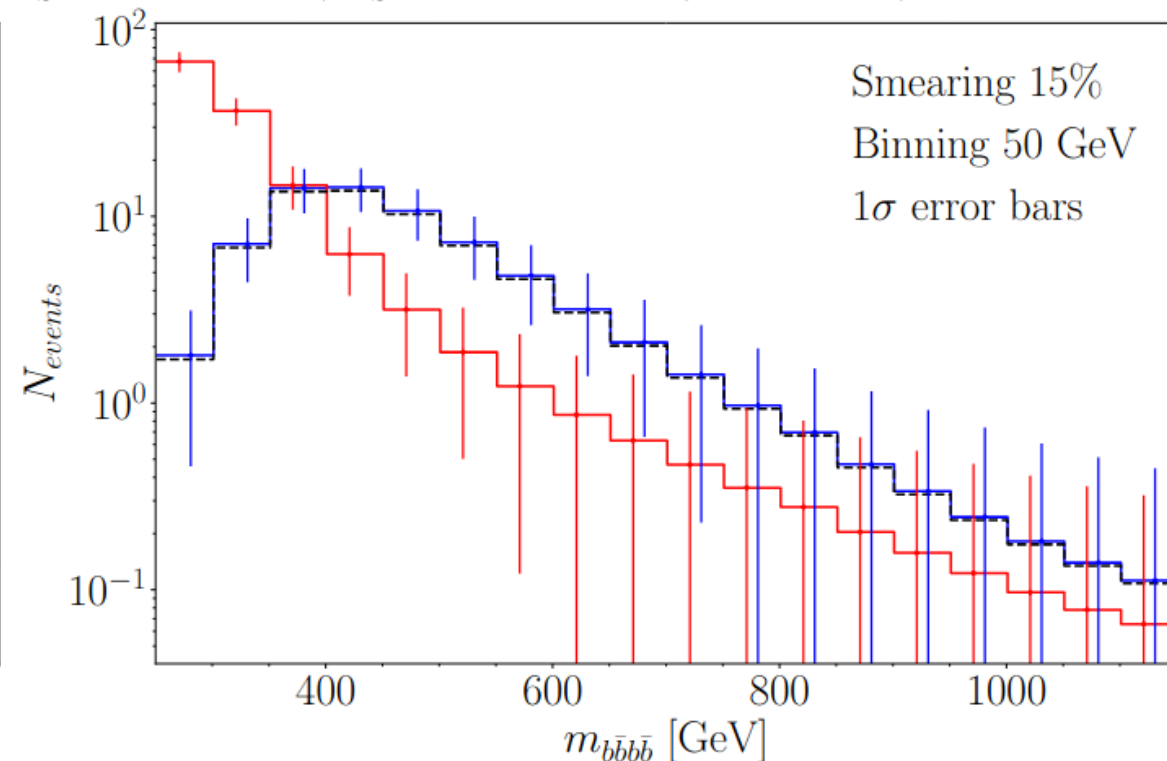
- Cross-sections computed with RxSM-specific version of HPAIR, see [Plehn, Spira, Zerwas '96], [Dawson, Dittmaier, Spira '98], [Abouabid et al. '22], [Arco et al. '25], results again shown at LO (in QCD)
- Largest increases in $\sigma_{hh}^{\text{RxSM}}$ w.r.t. SM for $m_H > 750$ GeV, $v_s < 50$ GeV and small mixing → **loop induced effects**
- Points with smaller increases in $\sigma_{hh}^{\text{RxSM}}$ (up to $\sim 2.5 \sigma_{hh}^{\text{SM}}$) for low m_H → **effect from resonant H contribution (already at tree level)**

Impact of NLO corrections in the RxSM

[JB, Heinemeyer, Parra Aray,
Verduras Schaeidt *WIP*]

Impact on di-Higgs invariant mass distributions

BP2: $m_H = 777.6$ GeV, $\cos \alpha = 1.0$, $v_S = 44.7$ GeV, $\kappa_S = -931$ GeV, $\kappa_{SH} = -931$ GeV, $Z^{(0)} = 0.3$, $Z^{(1)} = 21.3$



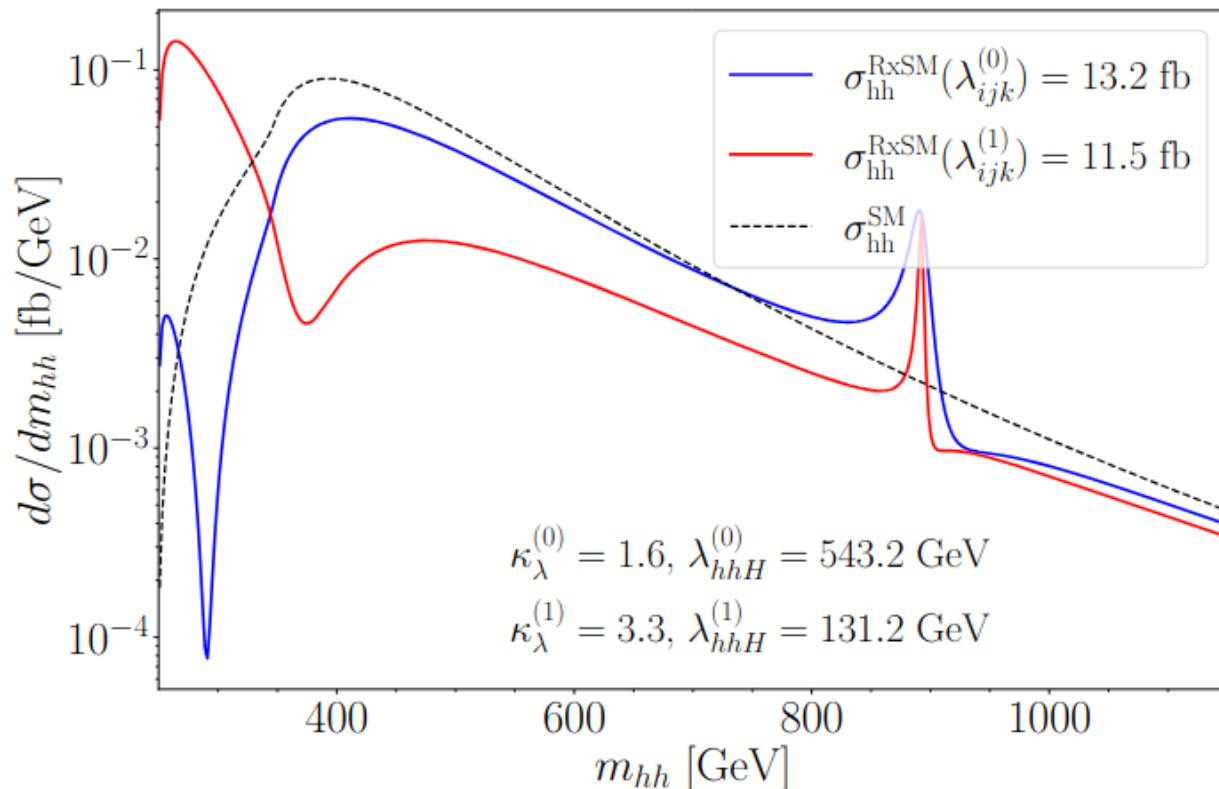
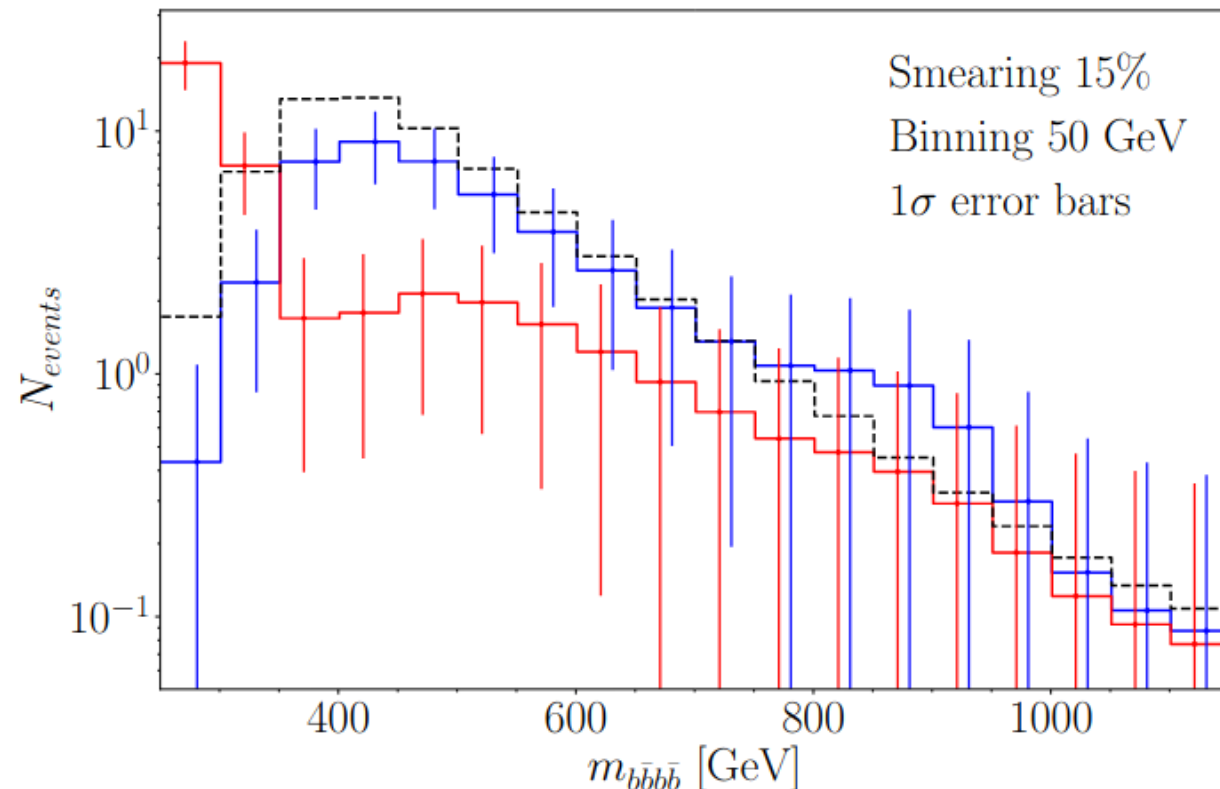
- Similar to BP1, main effect from loop correction to κ_λ (resonant peak suppressed by high m_H)
- As for BP1, for BP2, an analysis with tree-level trilinears ($Z^{(0)}=0.3$) would not be able to show a deviation from the SM, but an analysis with one-loop ones would ($Z^{(1)}=21.3$)

Impact of NLO corrections in the RxSM

[JB, Heinemeyer, Parra Aray,
Verduras Schaeidt *WIP*]

Impact on di-Higgs invariant mass distributions

BP4: $m_H = 891.1$ GeV, $\cos \alpha = 0.99$, $v_S = 37.0$ GeV, $\kappa_S = -993$ GeV, $\kappa_{SH} = -933$ GeV, $Z^{(0)} = 3.5$, $Z^{(1)} = 10.5$



- Total cross-section is not modified very significantly
- Large (positive) correction to λ_{hhh} changes the distribution drastically for low m_{hh} , while large negative correction to λ_{hhH} makes the peak at m_H narrower
- Change at low m_{hh} is substantial enough to allow distinguishing the RxSM from the SM with an analysis using one-loop trilinear scalar couplings

Statistical significance to distinguish RxSM from SM

[JB, Heinemeyer, Parra Arnay,
Verduras Schaeidt *WIP*]

$$Z = \sqrt{\sum_i 2 \left[(s_i + b_i) \log \left(1 + \frac{s_i}{b_i} \right) - s_i \right]}$$

with $s_i = N_i^{\text{RxSM}} - N_i^{\text{SM}}, \quad b_i = N_i^{\text{SM}}$

Likelihood ratio

$$\lambda(\mu) = \frac{L(\mu)}{L(\hat{\mu})}$$

maximum-likelihood
estimators (MLEs)

$$\hat{\mu}$$

Likelihood

$$L(\mu) = \prod_i \frac{(\mu s_i + b_i)^{n_i}}{n_i!} e^{-(\mu s_i + b_i)}$$

We assume that the
likelihood can be approximated
by a Gaussian distribution

$$Z = \sqrt{-2 \log(\lambda(\text{SM}))}$$

[Slide by A. Verduras Schaiedt]

2HDM

The Two-Higgs-Doublet Model

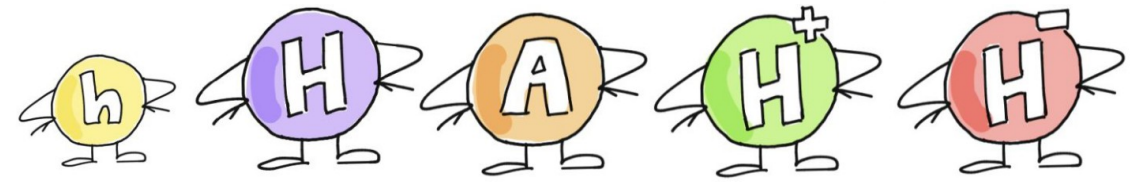


Figure by [K. Radchenko Serdula '24]

- 2 $SU(2)_L$ doublets $\Phi_{1,2}$ of hypercharge $1/2$
- CP-conserving 2HDM, with softly-broken Z_2 symmetry ($\Phi_1 \rightarrow \Phi_1$, $\Phi_2 \rightarrow -\Phi_2$) to avoid tree-level FCNCs

$$V_{2\text{HDM}}^{(0)} = m_1^2 |\Phi_1|^2 + m_2^2 |\Phi_2|^2 - m_3^2 (\Phi_2^\dagger \Phi_1 + \Phi_1^\dagger \Phi_2) \\ + \frac{\lambda_1}{2} |\Phi_1|^4 + \frac{\lambda_2}{2} |\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_4 |\Phi_2^\dagger \Phi_1|^2 + \frac{\lambda_5}{2} \left((\Phi_2^\dagger \Phi_1)^2 + \text{h.c.} \right) \\ v_1^2 + v_2^2 = v^2 = (246 \text{ GeV})^2$$

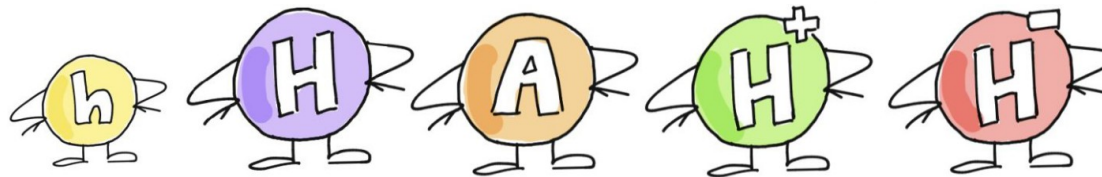
- **Mass eigenstates:**
 h, H : CP-even Higgs bosons ($h \rightarrow 125\text{-GeV SM-like state}$); A : CP-odd Higgs boson;
 $H^\pm$: charged Higgs boson
- **BSM parameters:** 3 BSM masses $m_H, m_A, m_{H^\pm}$, BSM mass scale M (defined by $M^2 \equiv 2m_3^2/s_{2\beta}$), angles α (CP-even Higgs mixing angle) and β (defined by $\tan\beta = v_2/v_1$)
- **BSM-scalar masses** take form $m_\Phi^2 = M^2 + \tilde{\lambda}_\Phi v^2$, $\Phi \in \{H, A, H^\pm\}$
- We take the **alignment limit** $\alpha = \beta - \pi/2 \rightarrow$ all Higgs couplings are SM-like at tree level
 $\rightarrow$ compatible with current experimental data

Interference effects in the 2HDM

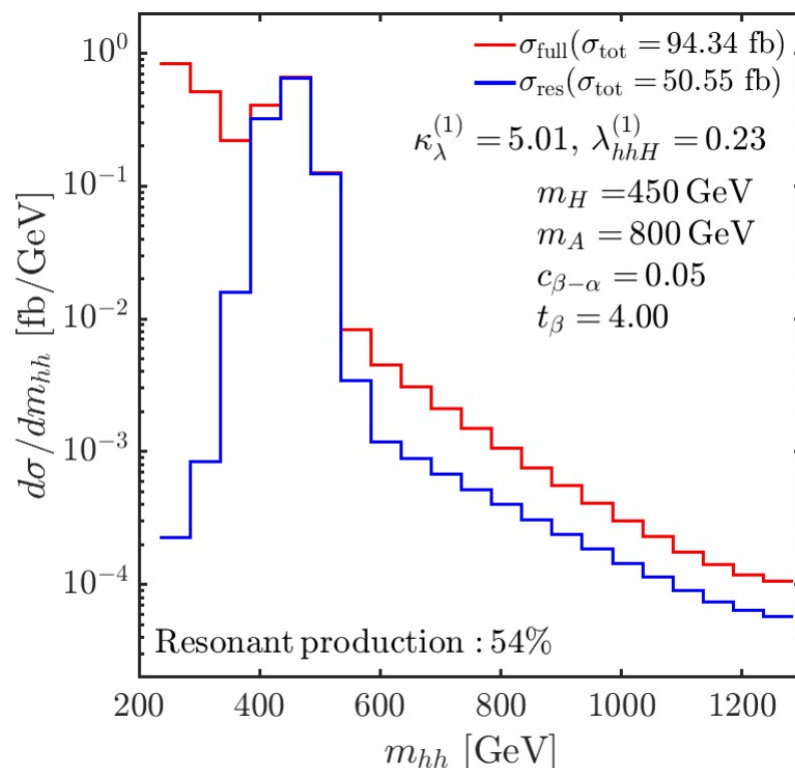
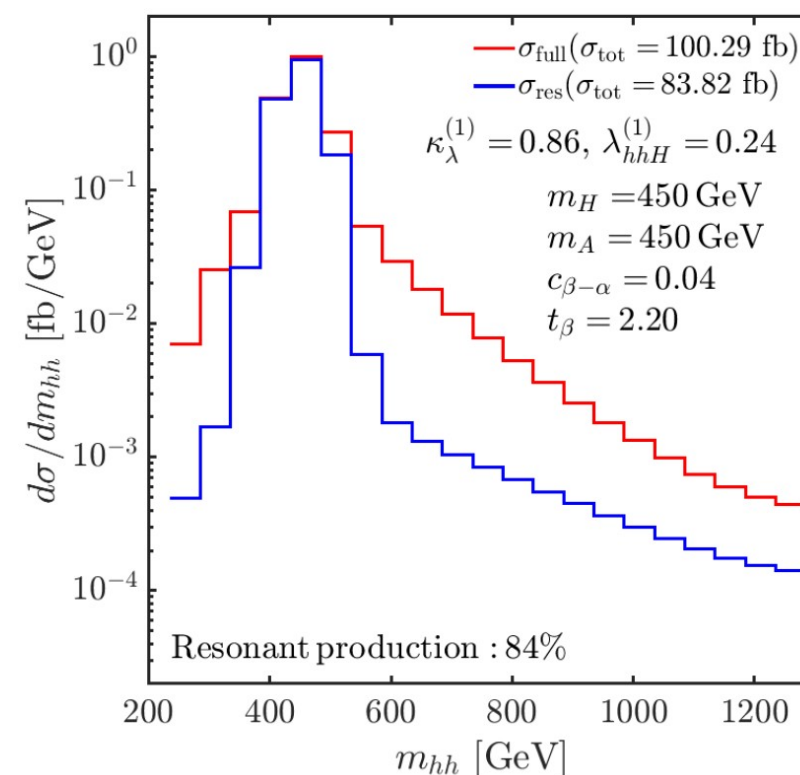
[Heinemeyer, Mühlleitner,
Radchenko Serdula, Weiglein '24]

Resonant only vs total distributions

- 2HDM adds second doublet to SM
- Interference between non-resonant and resonant contributions as well as loop corrections to trilinear scalar couplings can again drastically change di-Higgs invariant mass distributions
- 2 benchmark scenarios *excluded by resonant searches but allowed by non-resonant searches* [ATLAS '22]
→ large impact of interference effects **warrants analysis of these scenarios with full calculation**



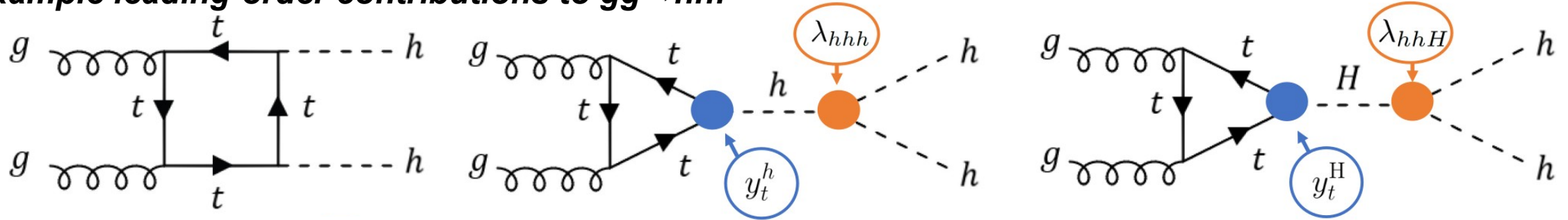
Drawing by
[K. Radchenko
Serdula '24]



anyH3 (v2) / anyHH

Ongoing developments in anyBSM: anyLambdaijk and anyHH

Example leading-order contributions to $gg \rightarrow hh$:

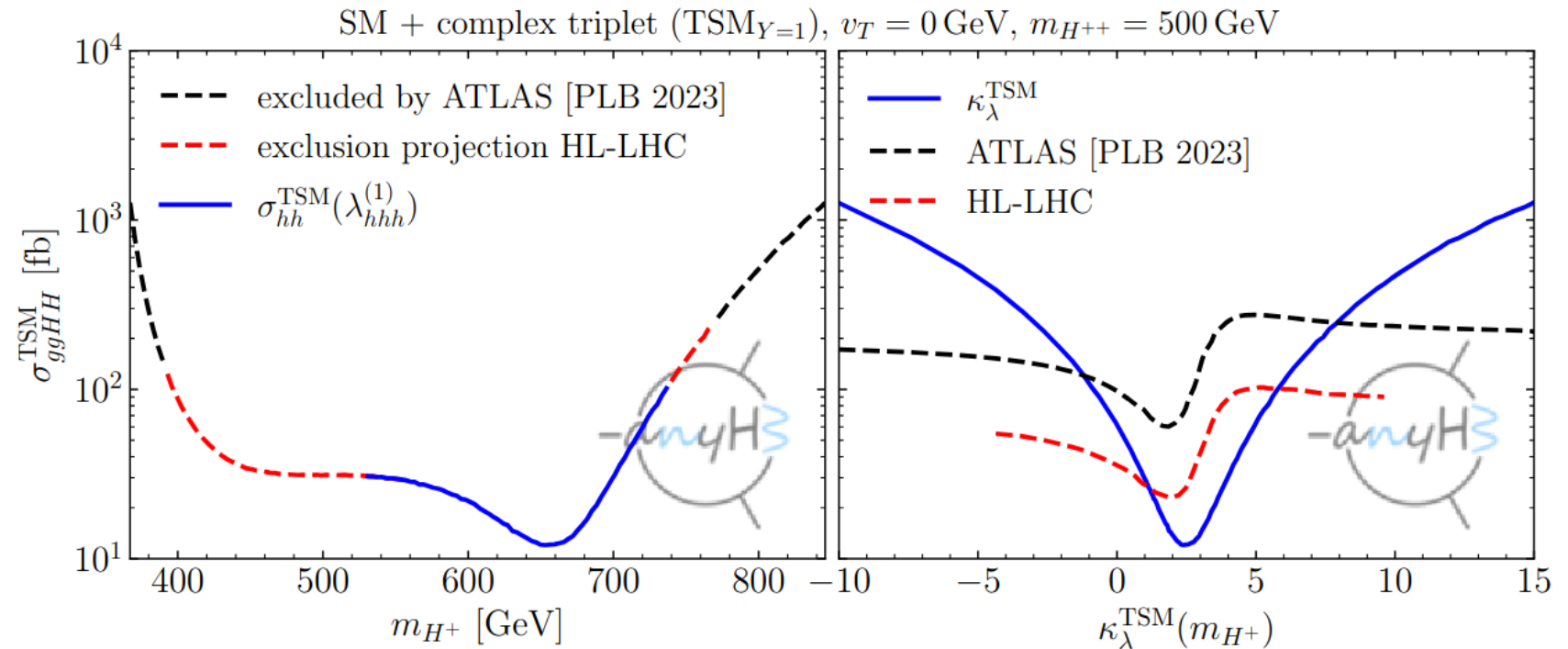


[Diagrams by A. Verduras Schaeidt]

Having predictions for di-Higgs production, including **all (i.e. resonant + non-resonant) contributions + 1L corrections to trilinear scalar couplings in arbitrary models** would be highly desirable

→ new modules anyLambdaijk and anyHH

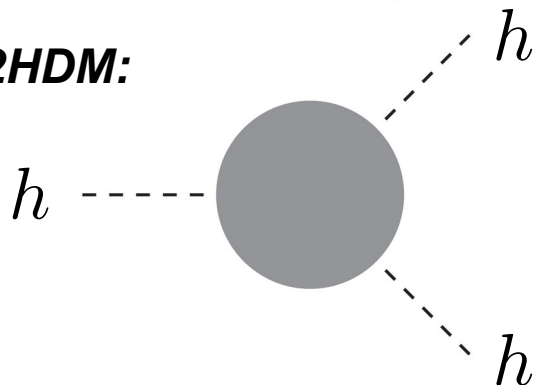
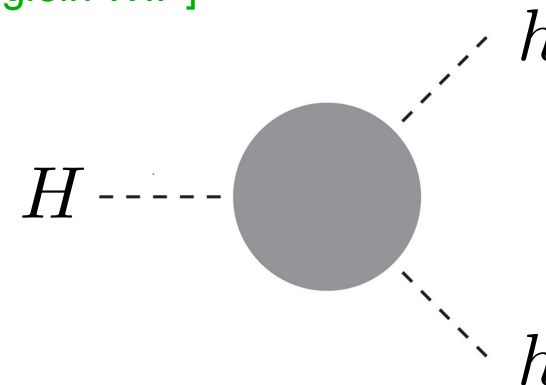
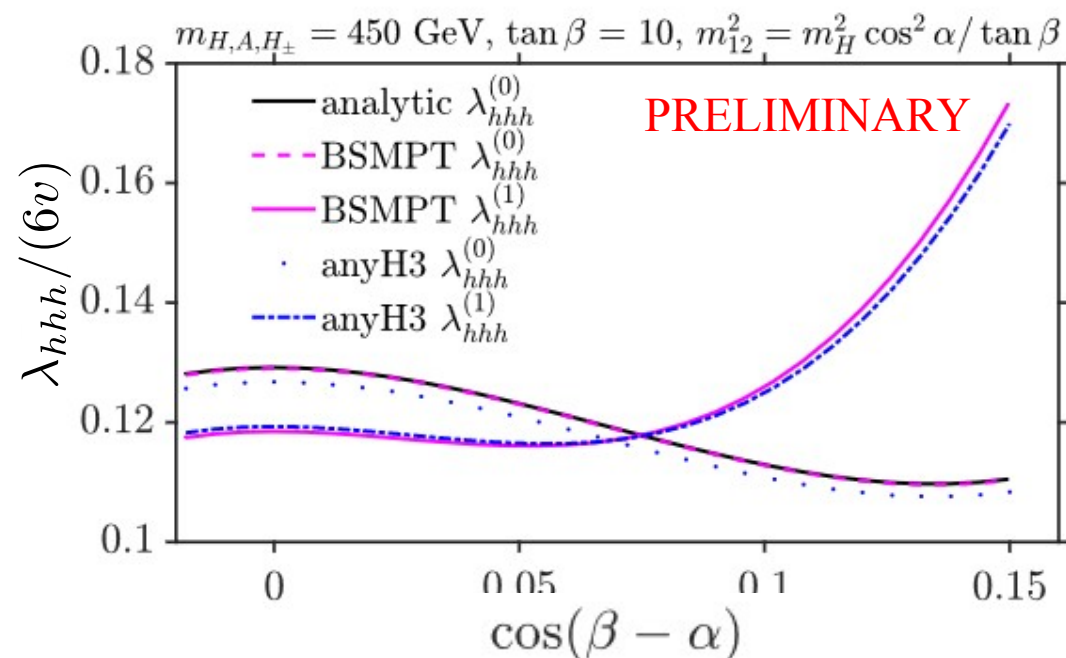
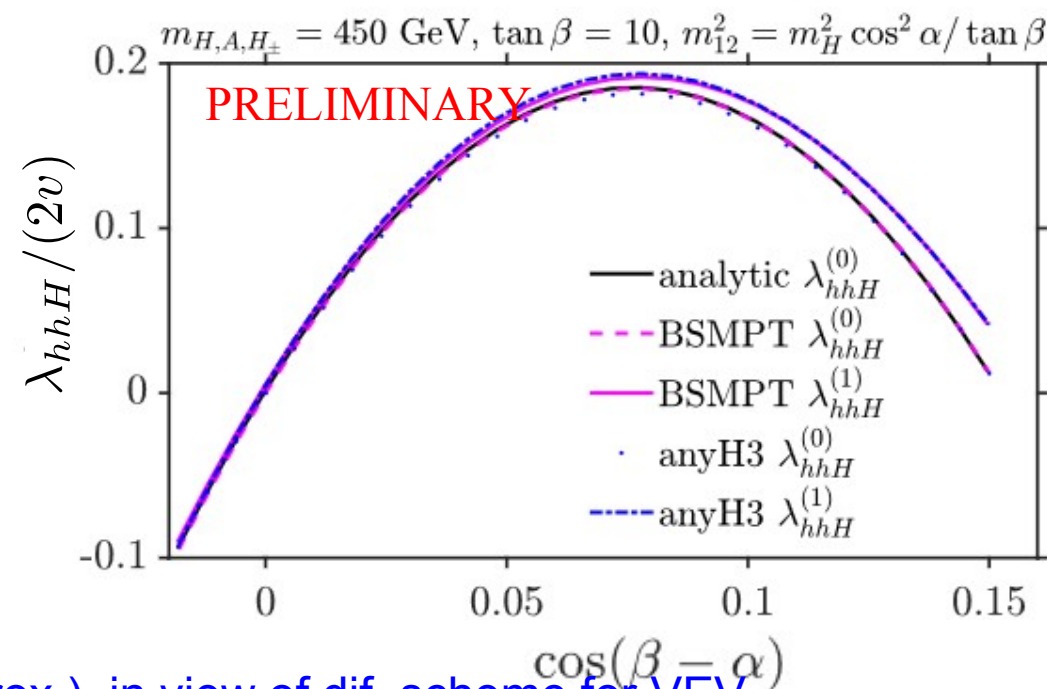
[Bahl, Braathen, Gabelmann, Radchenko Serdula, GW WIP]



Ongoing developments: anyH3 v2

[Bahl, JB, Gabelmann, Radchenko
Serdula, Weiglein *WIP*]

Example in a 2HDM:

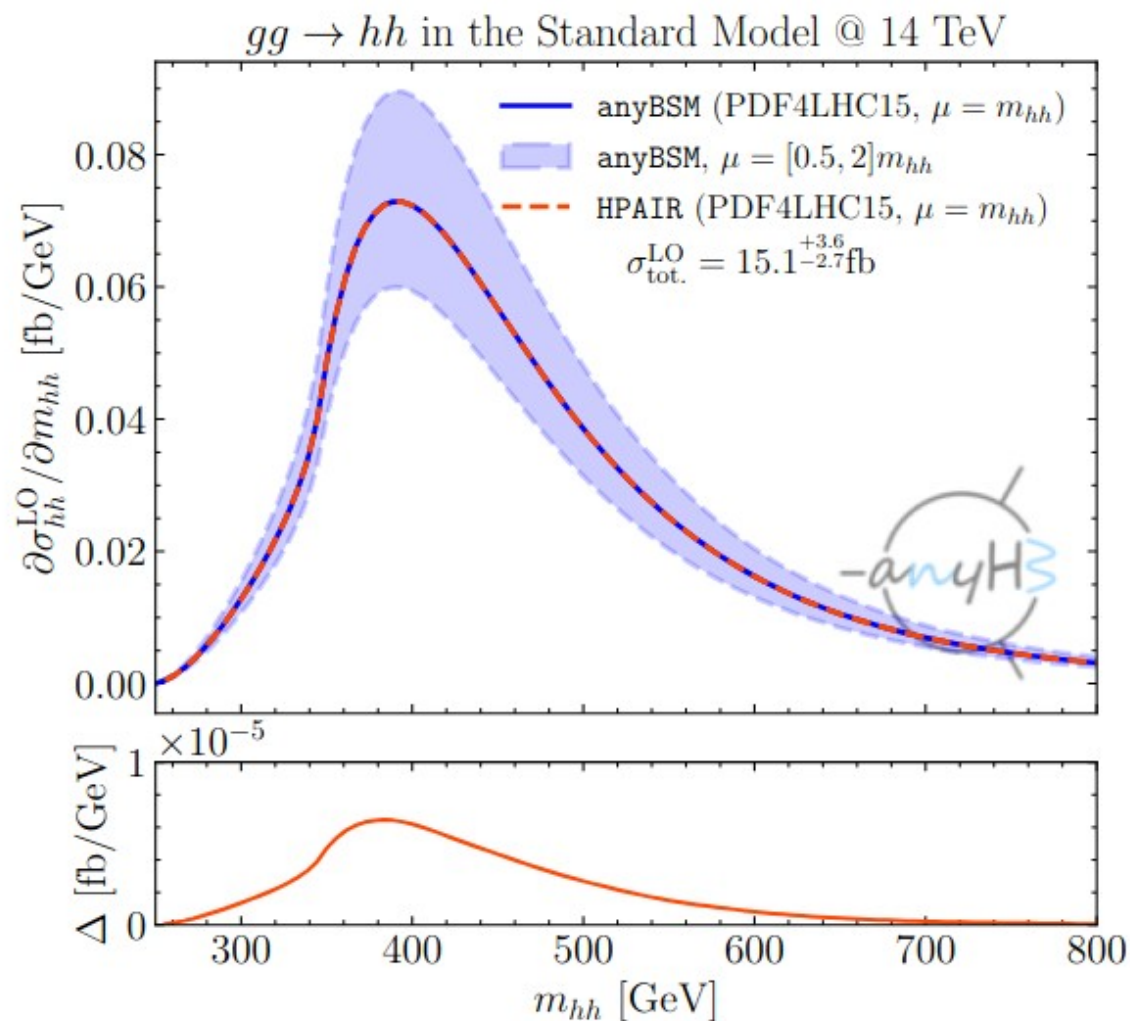
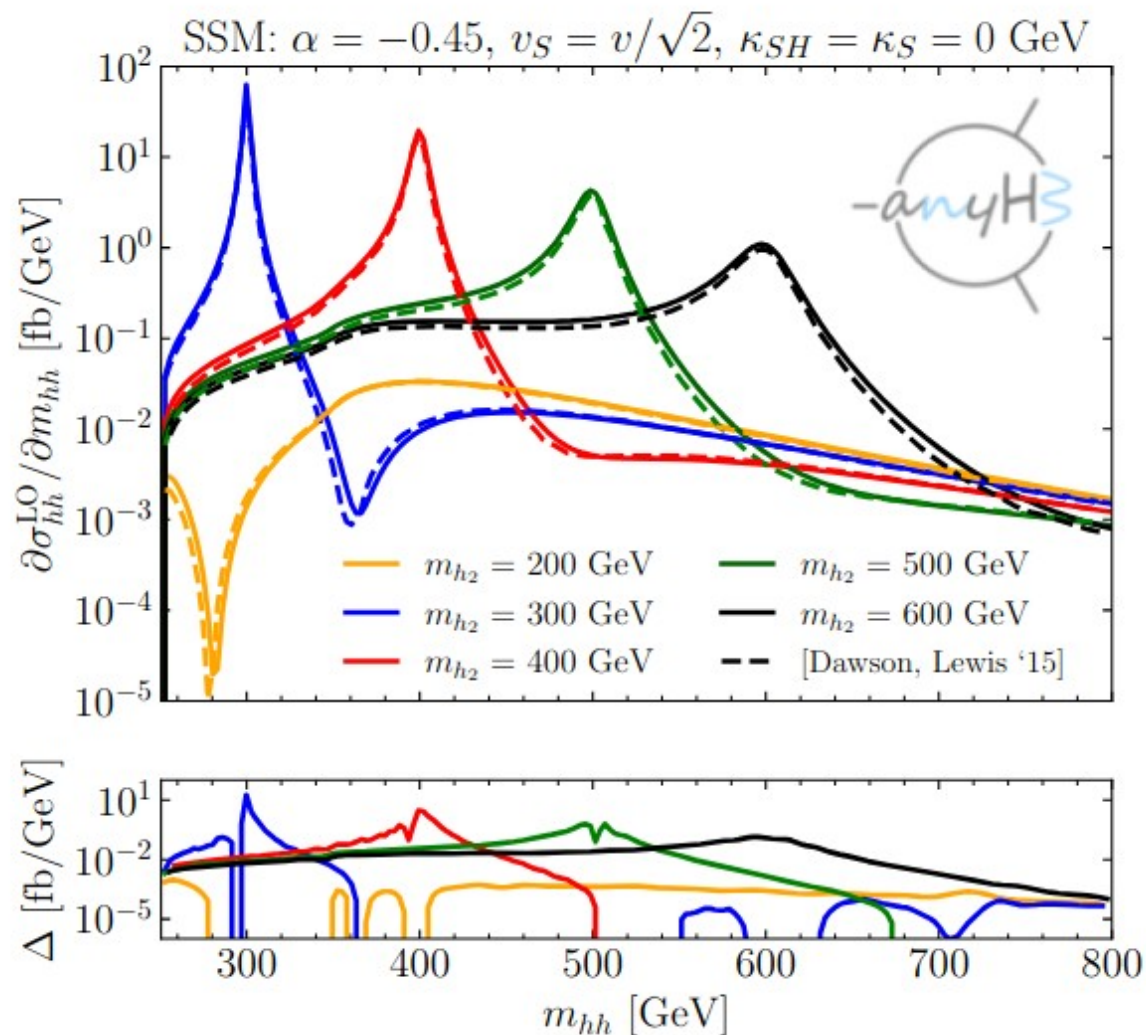


→ excellent agreement with BSMPT results (in eff. pot. approx.), in view of dif. scheme for VEV

→ full OS schemes for λ_{hhh} and λ_{hhH} couplings worked out in 2HDM [Bahl, JB, Gabelmann, Radchenko Serdula, Weiglein], RxSM [JB, Heinemeyer, Verduras Schaeidt], and more [Bosse, JB, Gabelmann, Hannig, Weiglein]!

Ongoing developments: tests of anyHH with leading order trilinear couplings

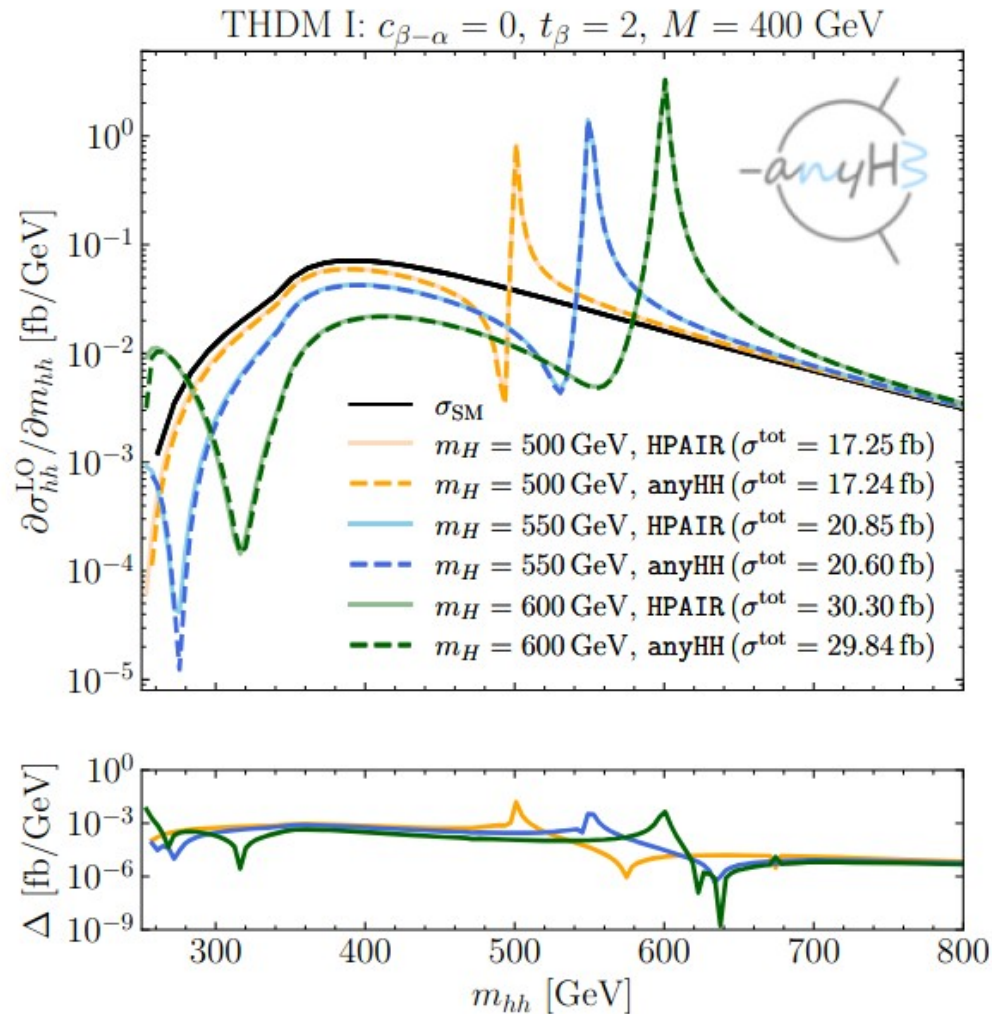
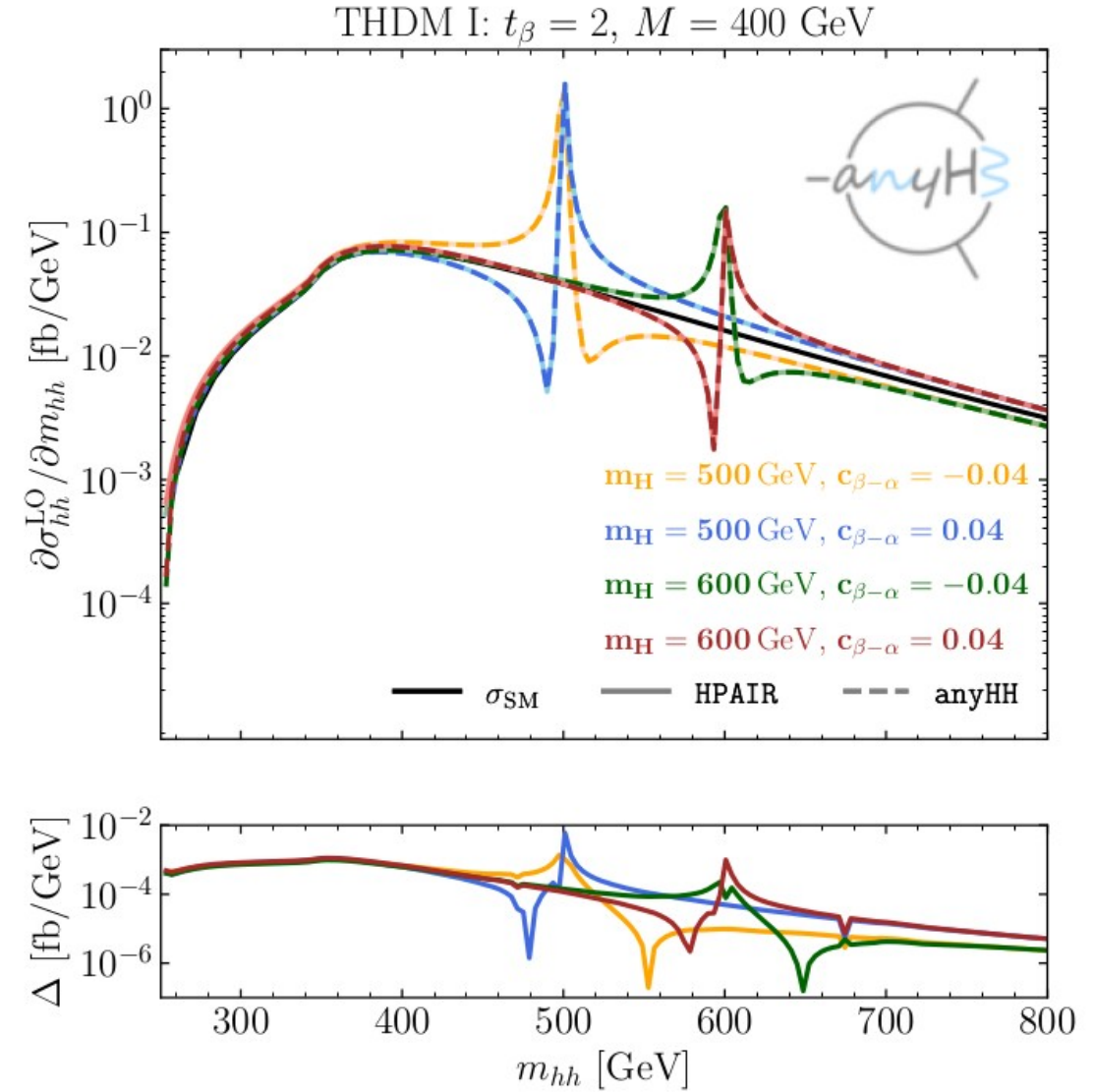
$$\Delta \equiv |\partial\sigma_{hh}^{\text{LO}}/\partial m_{hh}(\text{HPAIR}) - \partial\sigma_{hh}^{\text{LO}}/\partial m_{hh}(\text{anyHH})|$$



- Excellent agreement with LO HPair result, once one ensures that running of α_s + choice of PDFs are same

- Very good agreement results of [Dawson, Lewis '15] for singlet extension of SM (remaining difference because PDF sets can't be taken to be the same)

Ongoing developments: comparisons with HPAIR in the 2HDM



Very good agreement with HPair, using one-loop trilinear scalar couplings computed by anyH3/anyLambdaijk, for 2HDM benchmarks (here in alignment limit)

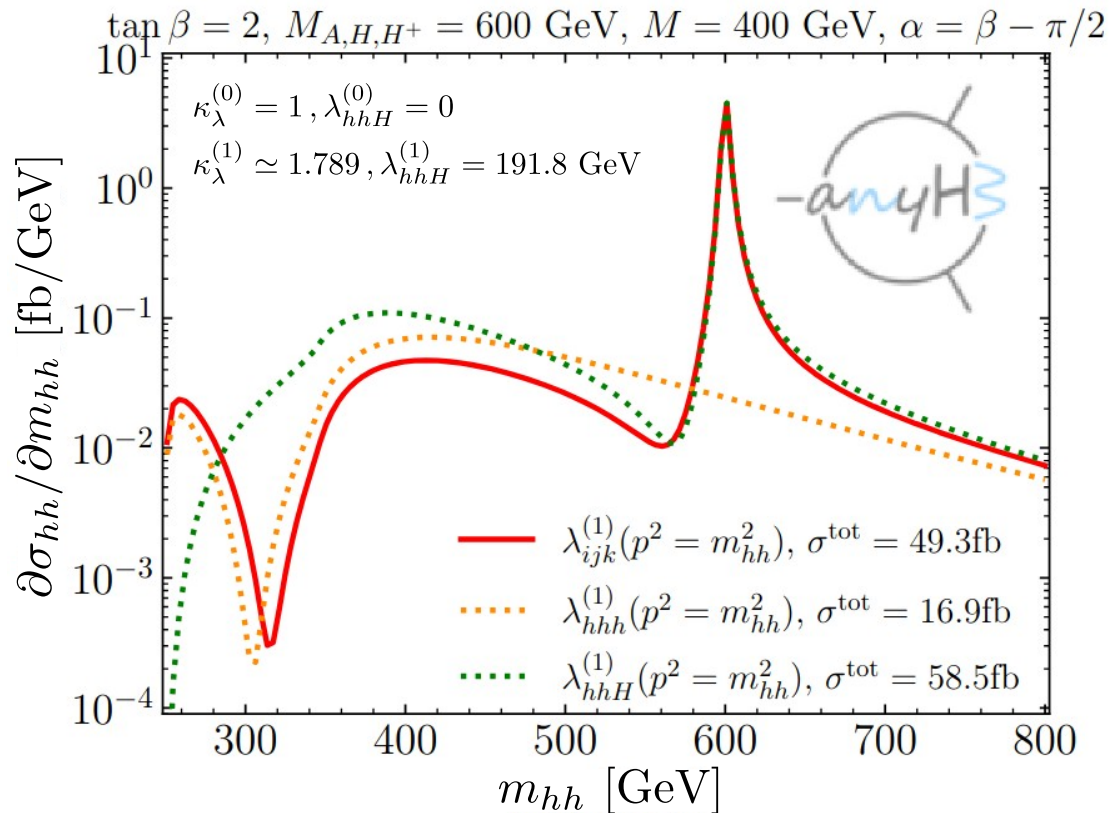
Impact of loop-corrected trilinears in the 2HDM

[Bahl, JB, Gabelmann,
Radchenko Serdula,
Weiglein WIP]

Example in **aligned 2HDM**, with di-Higgs cross-section and m_{hh} distributions computed with anyHH

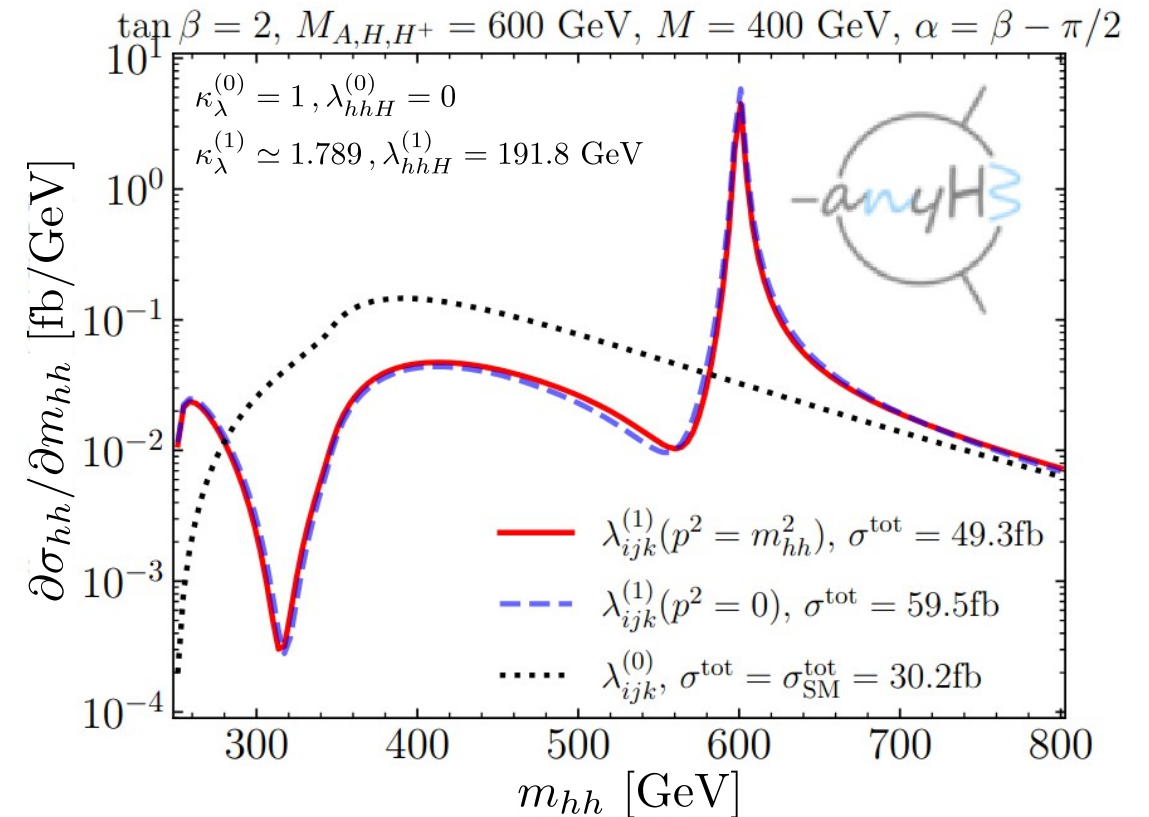
Impact of loop corrections to trilinear couplings

- $\kappa_\lambda^{(0)} = 1$; $\lambda_{hhH}^{(0)} = 0$ (due to alignment limit)
- huge impact of loop corrections to λ_{ijk} on both cross-section and distributions



Impact of momentum in s-channel diagrams

Blue vs red curves → O(20%) impact of momentum in λ_{ijk}



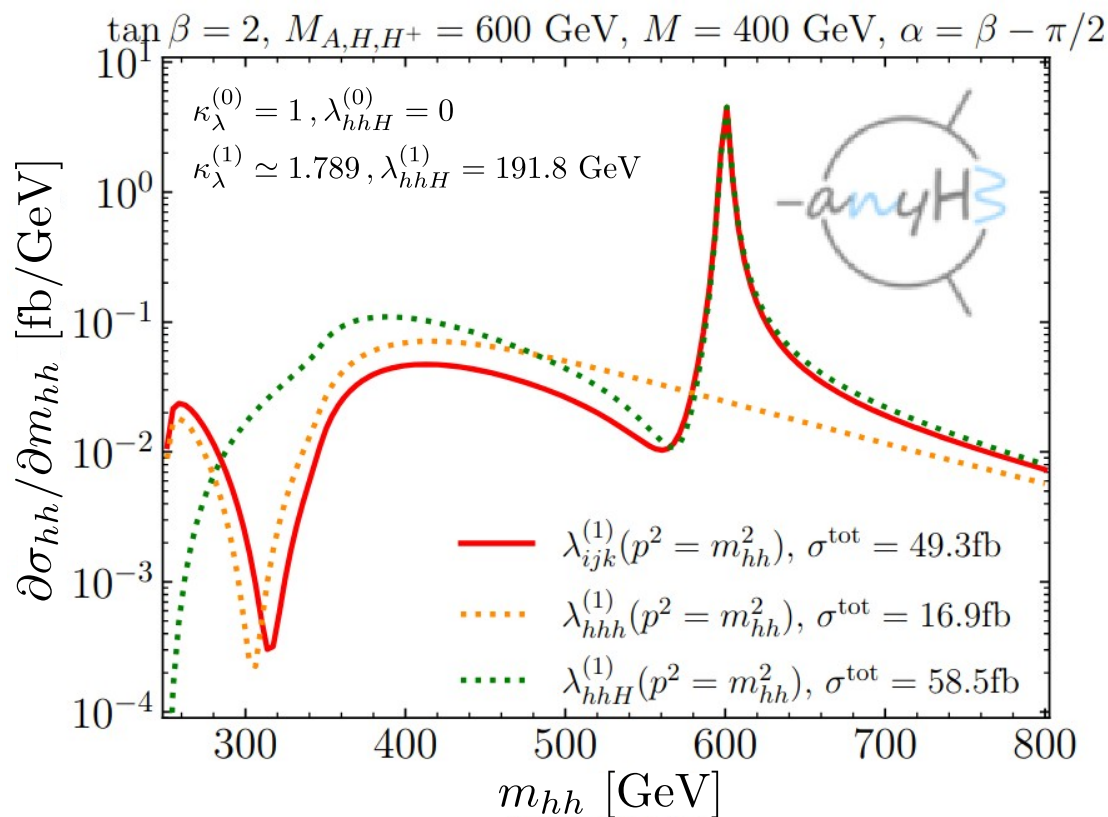
Impact of loop-corrected trilinears in the 2HDM

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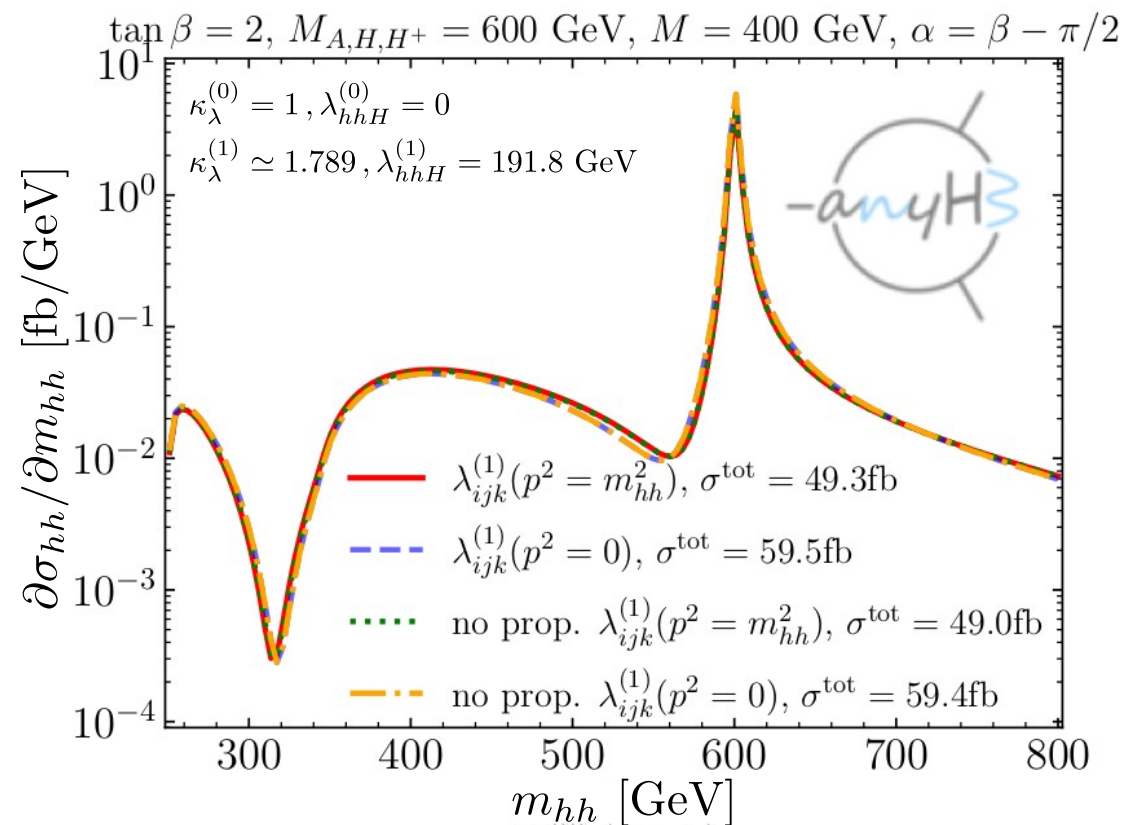
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Impact of momentum in s-channel diagrams

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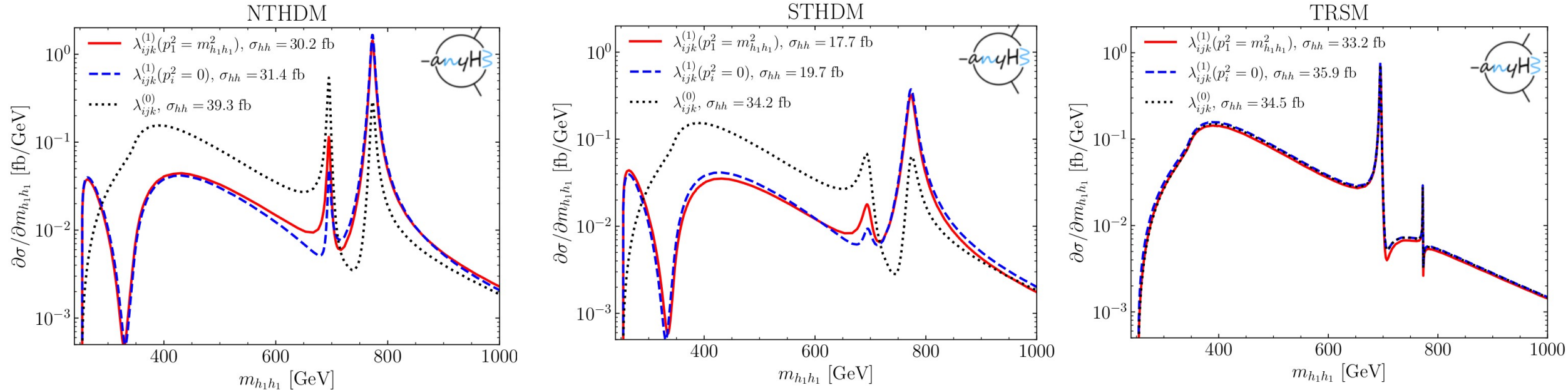
Only small effect from propagator corrections



anyHH: status and ongoing developments

[Bahl, JB, Gabelmann,
Radchenko Serdula,
Weiglein *WIP*]

- Results available in **various new models** for the first time!
(NTHDM = 2HDM + real singlet; STHDM = 2HDM + complex singlet DM; TRSM: two-real singlet model)

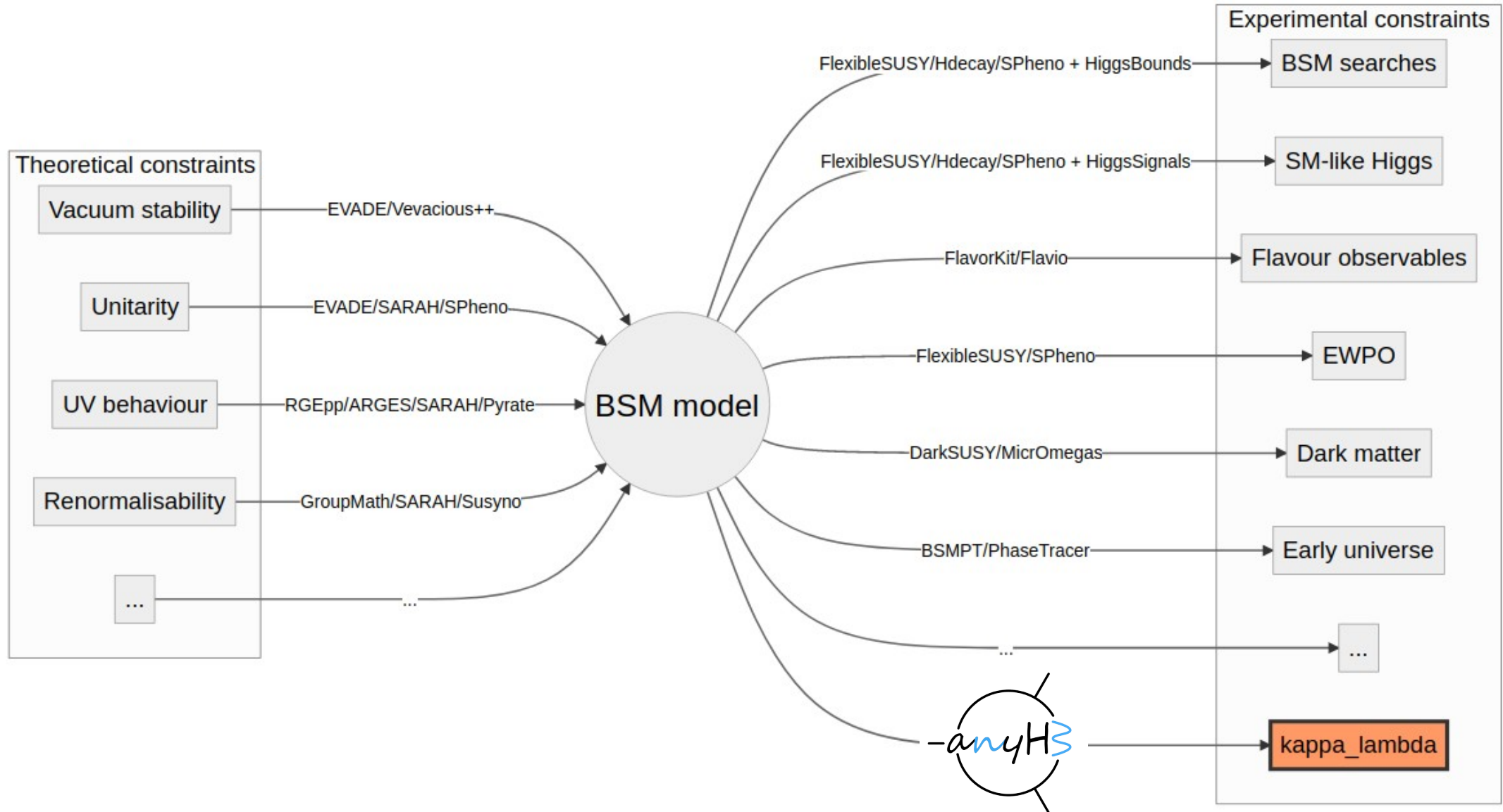


(NB: these preliminary plots are meant for illustrations purposes only; not for phenomenological studies)

- Under development:**
 - Link with Monte Carlo generators (e.g. MadGraph) for integration of results for loop-corrected trilinear scalar couplings into existing simulation setups

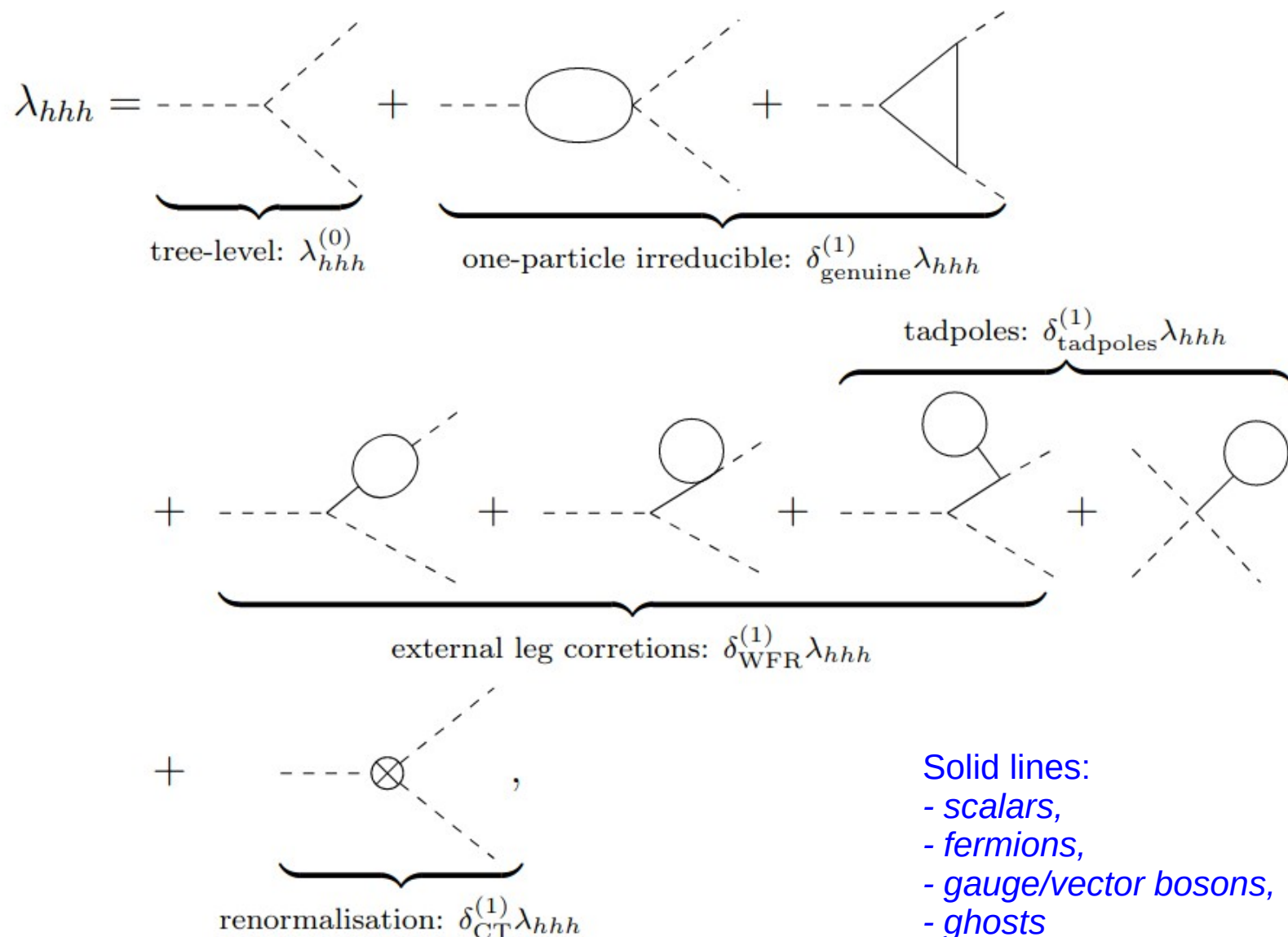
anyH3 (v1)

λ_{hhh} within the landscape of automated tools



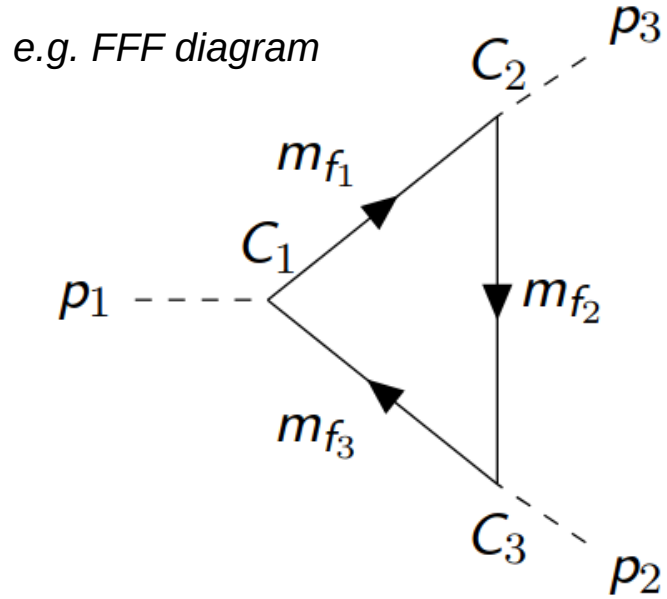
Full one-loop calculation of λ_{hhh} with anyH3: how does it work?

- Generic results applied to concrete (B)SM model, using inputs in UFO format
[Degrande et al., '11], [Darmé et al. '23]
- Loop functions evaluated via COLLIER [Denner et al '16] interface, pyCollier
- Restrictions on **particles** and/or **topologies** possible
- Renormalisation performed automatically** (*more in backup*)



Computing λ_{hhh} in general renormalisable theories: method

Our method: we derive and implement analytic results for **generic diagrams**, i.e. assuming generic



› Couplings $C_i = C_i^L P_L + C_i^R P_R$, where $P_{L,R} \equiv \frac{1}{2}(1 \mp \gamma_5)$

› Masses on the internal lines m_{fi} , $i=1,2,3$

› External momenta p_i , $i=1,2,3$

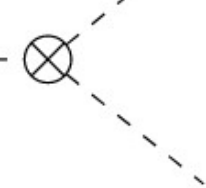
For evaluation:

- › Apply to concrete (B)SM model, using inputs in UFO format [Degrande et al., '11], [Darmé et al. '23]
- › Evaluate loop functions via COLLIER [Denner et al '16] interface, pyCollier
- › All included in public tool anyH3 [Bahl, JB, Gabelmann, Weiglein '23]

$$\begin{aligned}
 &= 2\mathbf{B0}(p_3^2, m_2^2, m_3^2)(C_1^L(C_2^L C_3^R m_{f1} + C_2^R C_3^R m_{f2} + C_2^R C_3^L m_{f3}) + C_1^R(C_2^R C_3^L m_{f1} + \\
 &C_2^L C_3^L m_{f2} + C_2^L C_3^R m_{f3})) + m_{f1} \mathbf{C0}(p_2^2, p_3^2, p_1^2, m_1^2, m_3^2, m_2^2)((C_1^L C_2^L C_3^R + \\
 &C_1^R C_2^R C_3^L)(p_1^2 + p_2^2 - p_3^2) + 2(C_1^L C_2^L C_3^L + C_1^R C_2^R C_3^R)m_{f2} m_{f3} + \\
 &2m_{f1}(C_1^L(C_2^L C_3^R m_{f1} + C_2^R C_3^R m_{f2} + C_2^R C_3^L m_{f3}) + C_1^R(C_2^R C_3^L m_{f1} + C_2^L C_3^L m_{f2} + \\
 &C_2^L C_3^R m_{f3}))) + \mathbf{C1}(p_2^2, p_3^2, p_1^2, m_1^2, m_3^2, m_2^2)(2p_2^2(C_1^L C_3^R(C_2^L m_{f1} + C_2^R m_{f2}) + \\
 &C_1^R C_3^L(C_2^R m_{f1} + C_2^L m_{f2})) + (p_1^2 + p_2^2 - p_3^2)((C_1^L C_2^L C_3^R + C_1^R C_2^R C_3^L)m_{f1} + \\
 &(C_1^L C_2^R C_3^L + C_1^R C_2^L C_3^R)m_{f3})) + \mathbf{C2}(p_2^2, p_3^2, p_1^2, m_1^2, m_3^2, m_2^2)((p_1^2 + p_2^2 - \\
 &p_3^2)(C_1^L C_3^R(C_2^L m_{f1} + C_2^R m_{f2}) + C_1^R C_3^L(C_2^R m_{f1} + C_2^L m_{f2})) + 2p_1^2((C_1^L C_2^L C_3^R + \\
 &C_1^R C_2^R C_3^L)m_{f1} + (C_1^L C_2^R C_3^L + C_1^R C_2^L C_3^R)m_{f3}))
 \end{aligned}$$

(B0, C0, C1, C2: loop functions)

Flexible choice of renormalisation schemes

$$\delta_{\text{CT}}^{(1)} \lambda_{hhh} = \text{---} \bigcirc \text{---} = ?$$


➤ **1L calculation** → renormalisation of all parameters entering λ_{hhh} at tree-level

➤ In general:

$$(\lambda_{hhh}^{(0)})^{\text{BSM}} = (\lambda_{hhh}^{(0)})^{\text{BSM}} \left(\underbrace{m_h \simeq 125 \text{ GeV}, v \simeq 246 \text{ GeV}}_{\text{SM sector}}, \underbrace{m_{\Phi_i}}_{\text{BSM masses}}, \underbrace{\alpha_i}_{\text{BSM mixing angles}}, \underbrace{v_i}_{\text{BSM VEVs}}, \underbrace{g_i}_{\text{indep. BSM coups.}} \right)$$

➤ Most automated codes: $\overline{\text{MS}}/\overline{\text{DR}}$ only

➤ **anyH3**: much more flexibility, following **user choice**:

- **SM sector** (m_h, v): fully OS or $\overline{\text{MS}}/\overline{\text{DR}}$
- **BSM masses**: OS or $\overline{\text{MS}}/\overline{\text{DR}}$
- **Additional couplings/vevs/mixings**: by default $\overline{\text{MS}}$, but **user-defined ren. conditions** also possible!

$$\delta_{\text{CT}}^{(1)} \lambda_{hhh} = \sum_x \left(\frac{\partial}{\partial x} (\lambda_{hhh}^{(0)})^{\text{BSM}} \right) \delta^{\text{CT}} x, \quad \text{with } x \in \{m_h, v, m_{\Phi_i}, v_i, \alpha_i, g_i, \text{etc.}\}$$

Renormalised in $\overline{\text{MS}}$, OS, in custom schemes, etc.

(Default) Renormalization choice of $(v^{\text{SM}})^{\text{OS}}$ and $(m_i^2)^{\text{OS}}$

- > $v^{\text{OS}} \equiv \frac{2M_W^{\text{OS}}}{e} \sqrt{1 - \frac{M_W^{2\text{OS}}}{M_Z^{2\text{OS}}}}$ with
 - $\delta^{(1)} M_V^{2\text{OS}} = \frac{\Pi_V^{(1),T}}{M_V^{2\text{OS}}} (p^2 = M_V^{2\text{OS}}), V = W, Z$
 - $\delta^{(1)} e^{\text{OS}} = \frac{1}{2} \dot{\Pi}_\gamma(p^2 = 0) + \text{sign}(\sin \theta_W) \frac{\sin \theta_W}{M_Z^2 \cos \theta_W} \Pi_{\gamma Z}(p^2 = 0)$
- > attention (i): $\rho^{\text{tree-level}} \neq 1 \rightarrow$ further CTs needed (depends on the model)
 $\rightarrow$ ability to define *custom* renormalisation conditions
- > scalar masses: $m_i^{\text{OS}} = m_i^{\text{pole}}$
 - $\delta^{\text{OS}} m_i^2 = -\widetilde{\text{Re}} \Sigma_{h_i}^{(1)}|_{p^2=m_i^2}$
 - $\delta^{\text{OS}} Z_i = \widetilde{\text{Re}} \frac{\partial}{\partial p^2} \Sigma_{h_i}^{(1)}|_{p^2=m_i^2}$
- > attention (ii): scalar mixing may also require further CTs/tree-level relations

All bosonic one- & two-point functions and their derivatives for general QFTs are required for flexible OS renormalisation.

Features of anyH3, so far

- Import/conversion of any UFO model
- Definition of renormalisation schemes

```
# schemes.yml

renormalization_schemes:
  MS:
    SM_names:
      Higgs-Boson: h1
    VEV_counterterm: MS
    mass_counterterms:
      h1: MS
      h2: MS
  OS:
    SM_names:
      Higgs-Boson: h1
    VEV_counterterm: OS
    custom_CT_hhh: 'dbetaH =
f"({Sigma(''Hm1'', ''Hm2'', momentum='''0''))} +
{Sigma(''Hm1'', ''Hm2'', momentum='''MHm2**2'')})/ -
(2*MHm2**2)"

    dTanBeta = f"({dbetaH})/cos(betaH)**2"

...
```

(extract from
schemes.yml
for 2HDM)

- Analytical / numerical / LaTeX outputs
- **3 user interfaces:**
 - Python library

```
from anyBSM import anyH3
myfancymodel = anyH3('path/to/UFO/model')
result = myfancymodel.lambdahhh()
```
 - Command line
 - Mathematica interface
- **Perturbative unitarity checks** available (at tree level and in high-energy limit for now)
- Can be used together with a spectrum generator and **handles SLHA format**
- Efficient **caching** available
- Lots more!

New results I: mass-splitting effects in various BSM models

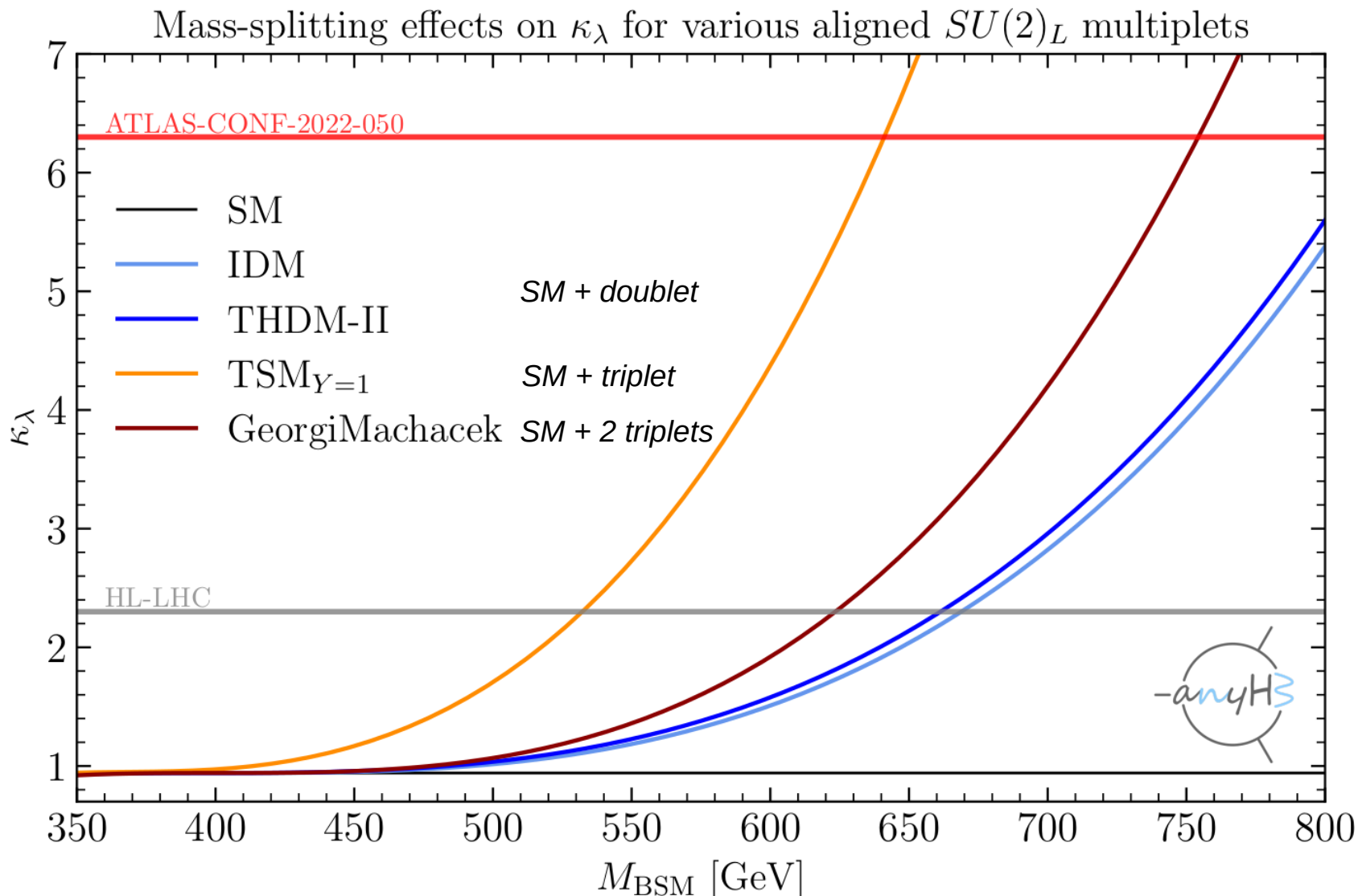
- Consider the non-decoupling limit in several BSM models

$$M_{\text{BSM}}^2 = \mathcal{M}^2 + \tilde{\lambda} v^2$$

- Increase M_{BSM} , keeping $\mathcal{M}$ fixed
 - large mass splittings
 - **large BSM effects!**

- Perturbative unitarity checked with anyPerturbativeUnitarity

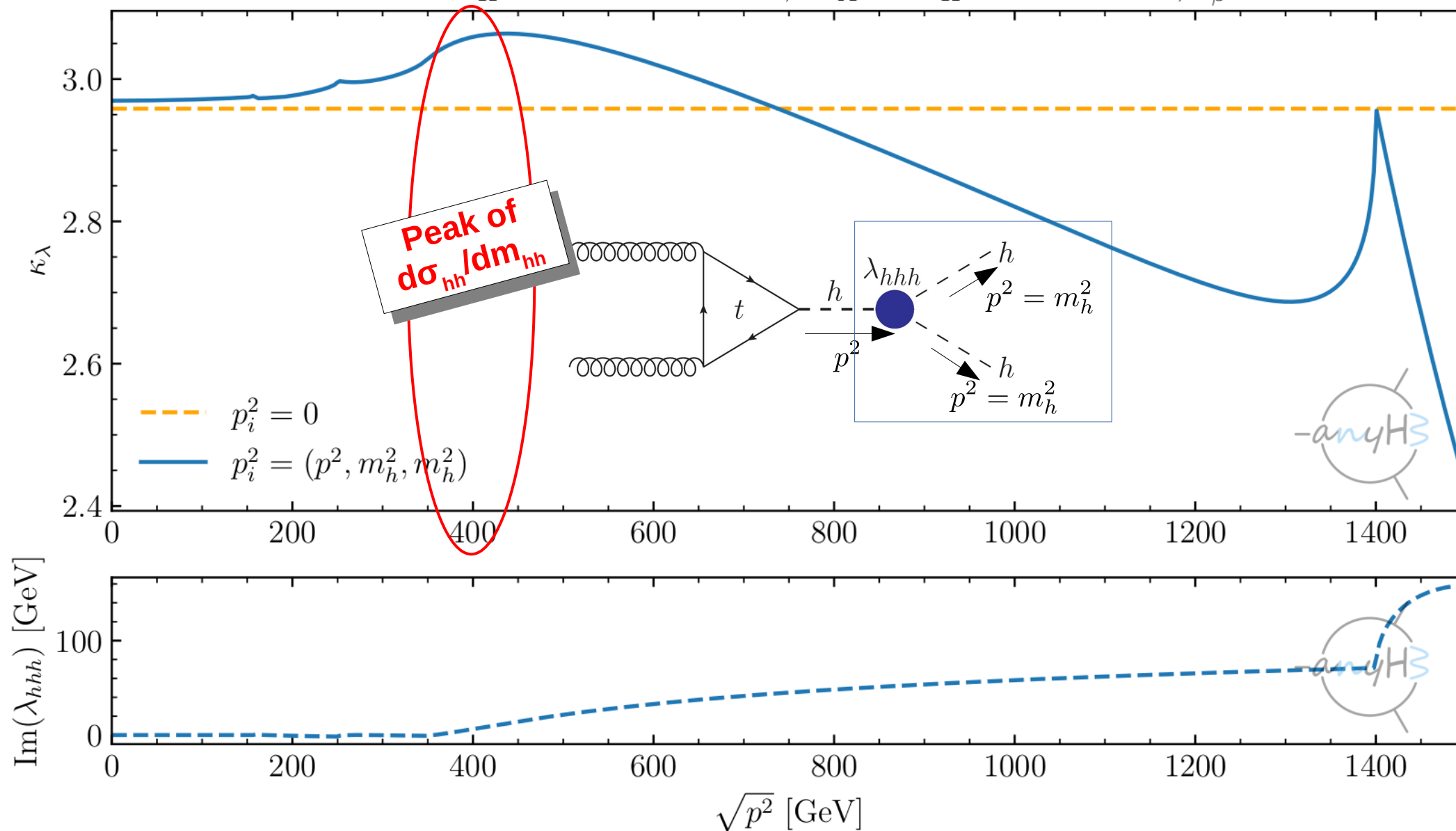
- Constraints on BSM parameter space!**



Here: scenarios with lightest BSM scalar mass & BSM mass param. at 400 GeV; other BSM scalar masses = M_{BSM}

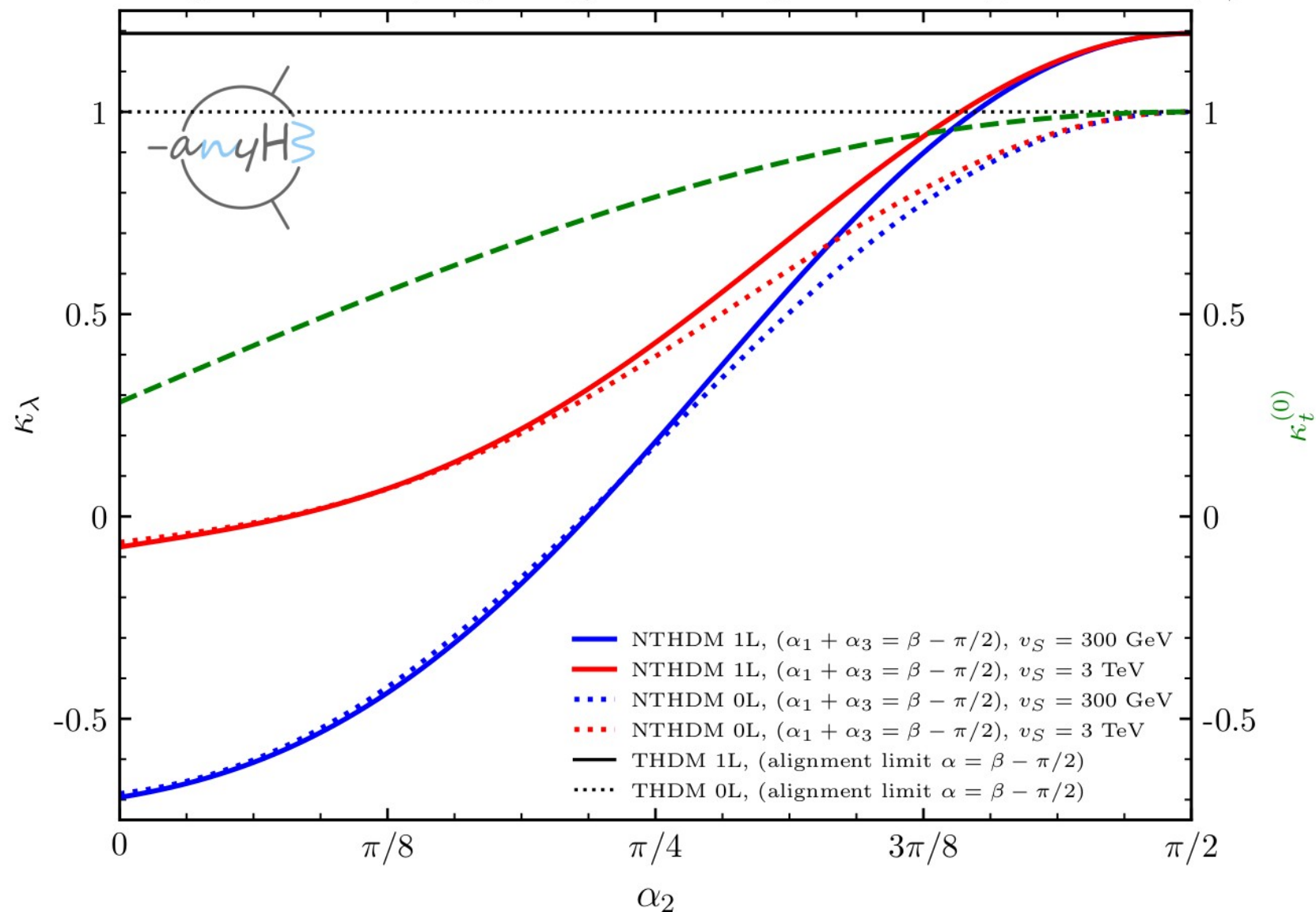
New results II: momentum dependence in the 2HDM

THDM-I: $m_H = M = 400 \text{ GeV}$, $m_A = m_{H^\pm} = 700 \text{ GeV}$, $t_\beta = 2$



More new results with anyH3: an example in the N2HDM

NTHDM: $m_{h_2} = 125.1$ GeV, $m_{h_1} = m_{h_3} = m_A = m_{H^\pm} = 300$ GeV, $\tilde{\mu} = 100$ GeV, $t_\beta = 2$



➤ N2HDM = 2HDM + real singlet

➤ CP-even sector: 3 states

h_1, h_2, h_3 ,

with 3 mixing angles $\alpha_1, \alpha_2, \alpha_3$

$\kappa_t^{(0)}$

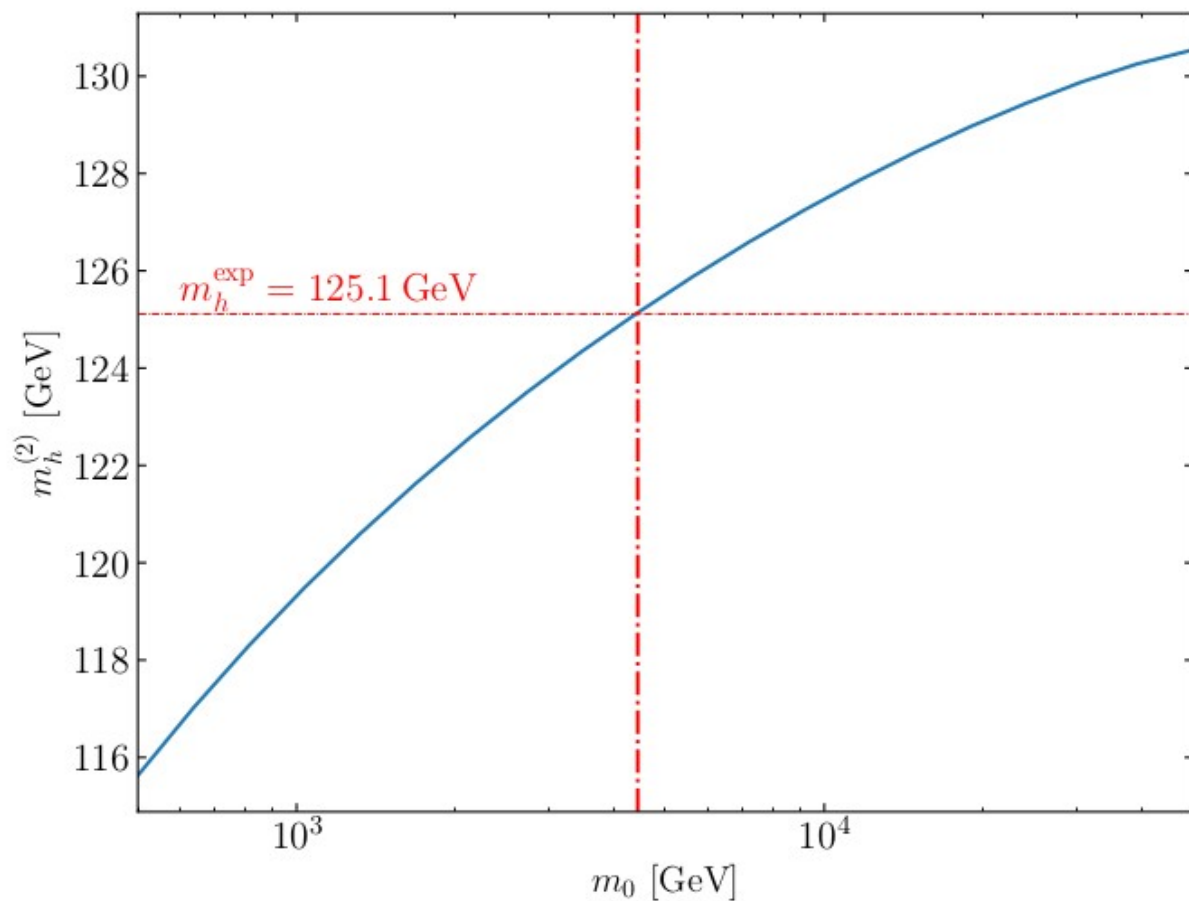
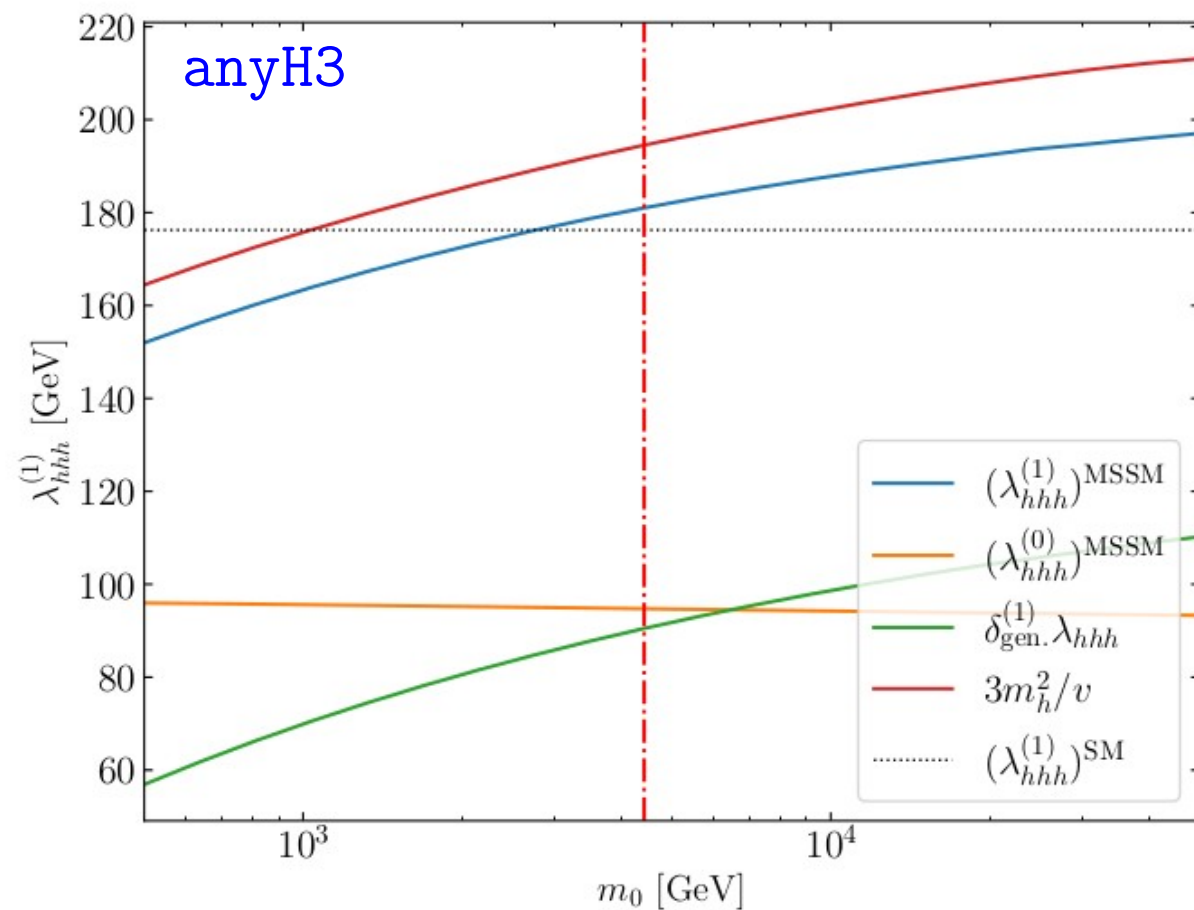
➤ Here $\alpha_2 \rightarrow \pi/2 \rightarrow$ recover 2HDM (itself in alignment limit)

➤ We can study e.g. the relative sign of κ_λ and $\kappa_t \rightarrow$ affects double-Higgs production

➤ κ_t too far away from 1 excluded

Full one-loop calculation of λ_{hhh} in the MSSM

CMSSM, $m_0 = m_{1/2} = -A_0$, $\tan\beta = 10$, $\text{sgn}(\mu) = 1$, with m_h computed at 2L in SPheno

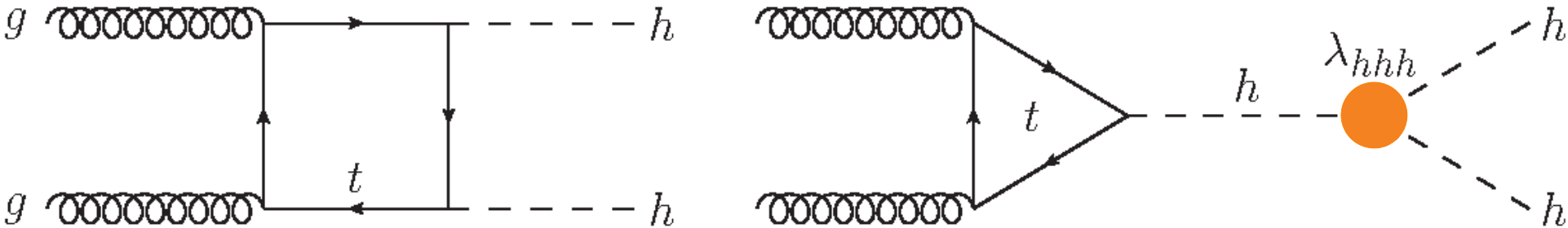


- Example for a very simple version of the constrained MSSM → BSM parameters m_0 , $m_{1/2}$, A_0 , $\text{sgn}(\mu)$, $\tan\beta$
- For each point, M_h computed at 2L with SPheno, and SLHA output of SPheno used as input of anyH3

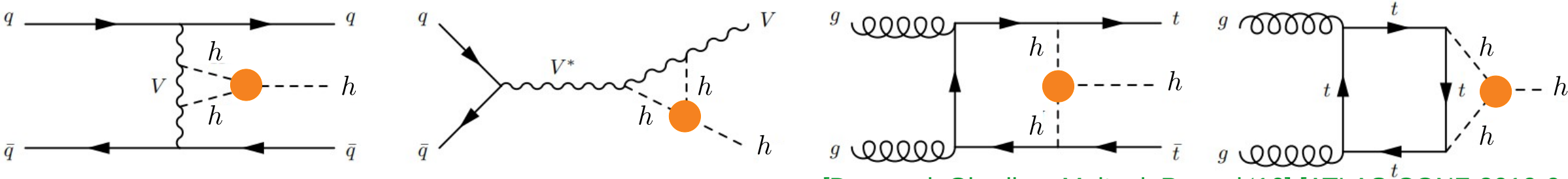
Trilinear Higgs self-coupling

Experimental probes of λ_{hhh}

- **Double-Higgs production** $\rightarrow \lambda_{hhh}$ enters at leading order (LO) $\rightarrow$ **most direct probe!**

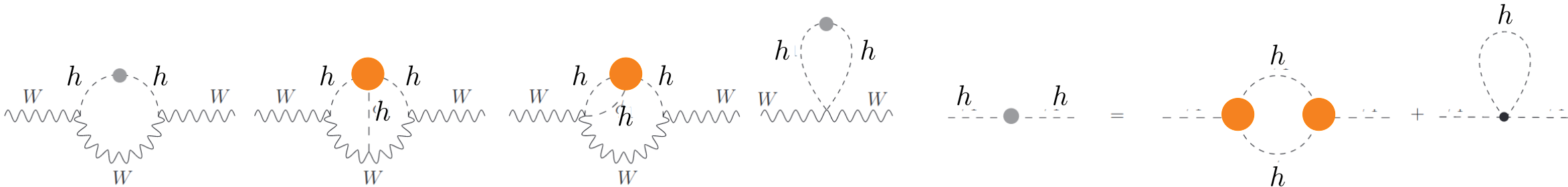


- **Single-Higgs production** $\rightarrow \lambda_{hhh}$ enters at NLO



[Degrassi, Giardino, Maltoni, Pagani '16] [ATLAS-CONF-2019-049]

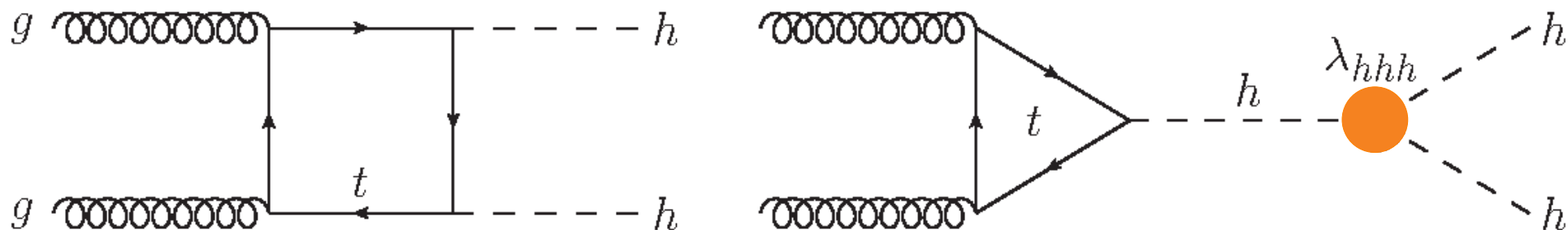
- **Electroweak Precision Observables (EWPOs)** $\rightarrow \lambda_{hhh}$ enters at NNLO



[Degrassi, Fedele, Giardino '17]

Accessing λ_{hhh} via di-Higgs production

- **Di-Higgs production** $\rightarrow \lambda_{hhh}$ enters at leading order (LO) $\rightarrow$ **most direct probe of λ_{hhh}**



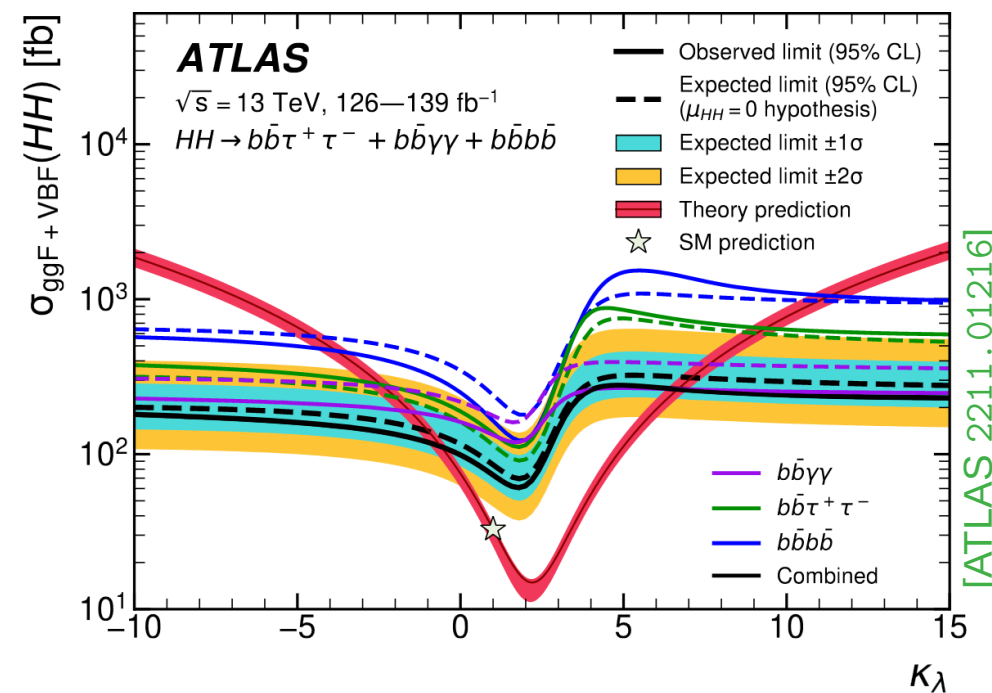
[Note: Single-Higgs production (EW precision observables) $\rightarrow \lambda_{hhh}$ enters at NLO (NNLO)]

- Box and triangle diagrams **interfere destructively**
 $\rightarrow$ small di-Higgs cross-section σ_{hh} in SM

$\rightarrow$ BSM deviation in λ_{hhh} can **significantly alter di-Higgs production!**

- Upper limit on di-Higgs cross-section
 $\rightarrow$ **limits on $\kappa_\lambda \equiv \lambda_{hhh} / (\lambda_{hhh}^{(0)})^{SM}$**

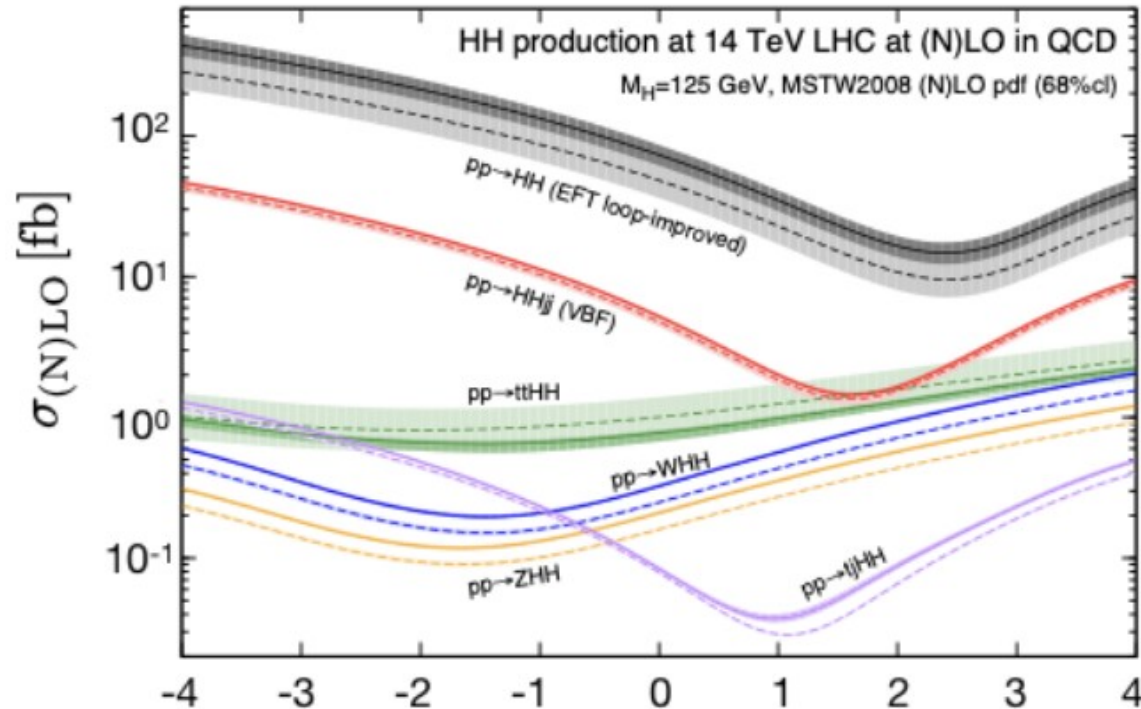
- κ_λ as an *effective coupling*: $\mathcal{L} \supset -\kappa_\lambda \times \frac{3m_h^2}{v^2} \cdot h^3 + \dots$



Di-Higgs production cross-sections as a function of λ_{hhh}

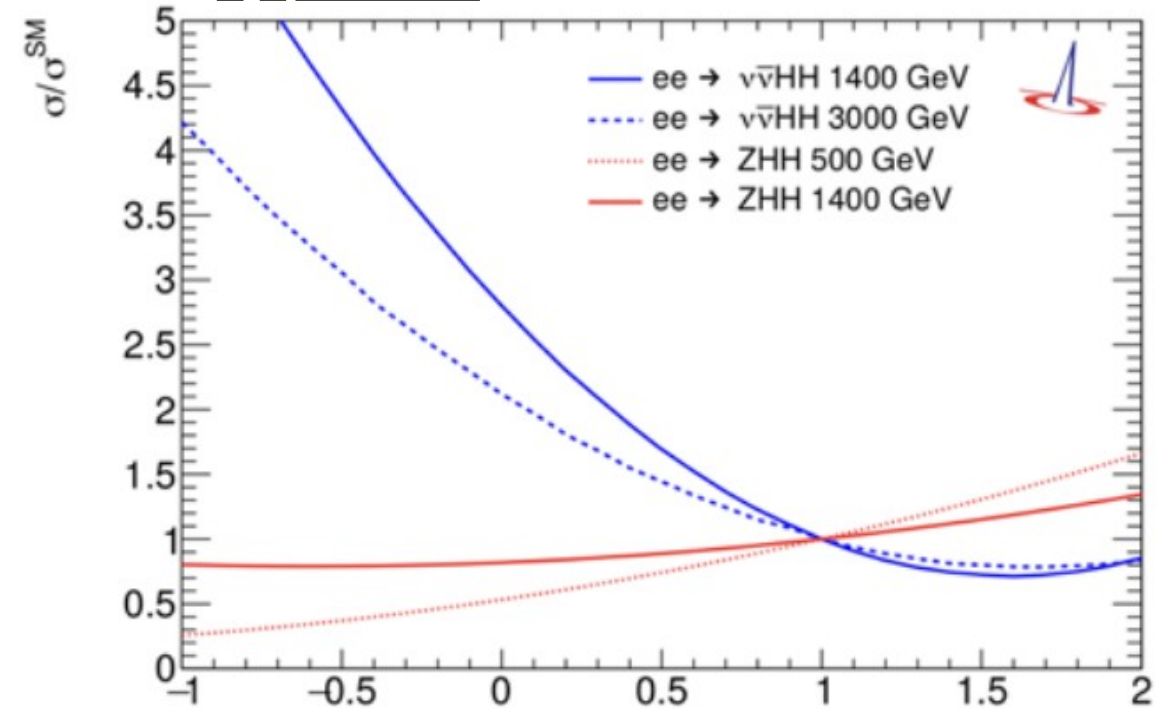
Plots taken from
[de Blas et al., 1905.03764]

Hadron collider



[Frederix et al., 1401.7340] $\lambda_3/\lambda_3^{SM} = \kappa_\lambda$

e^+e^- collider

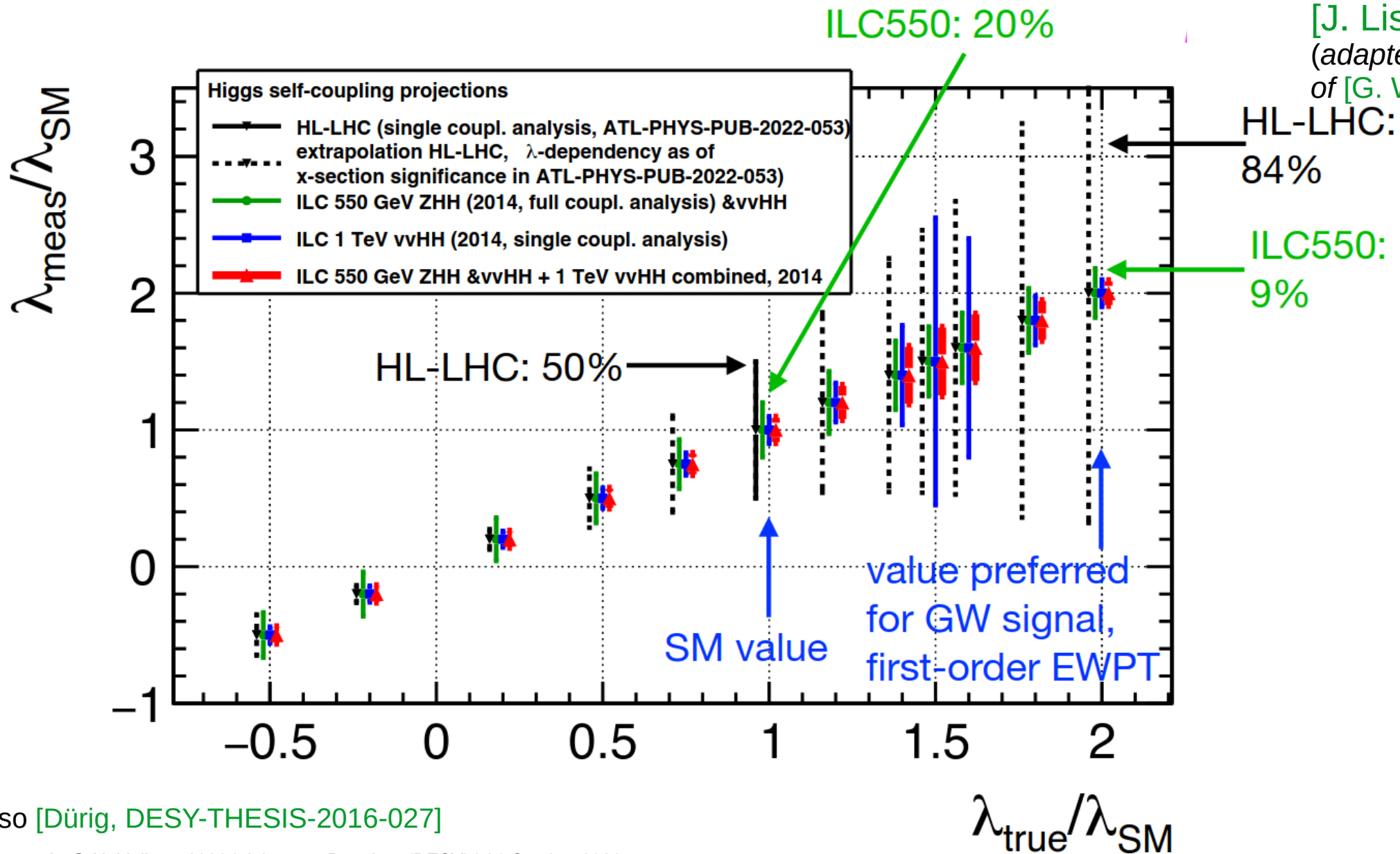


$\lambda_3/\lambda_3^{SM} = \kappa_\lambda$ [Reuter '19]

- **BSM deviation in κ_λ modifies the interference between different contributions to di-Higgs production**
- Strong impact on total cross-sections (and also on differential distributions, see later slides)

Precision on the determination of λ_{hhh} as a function of λ_{hhh}

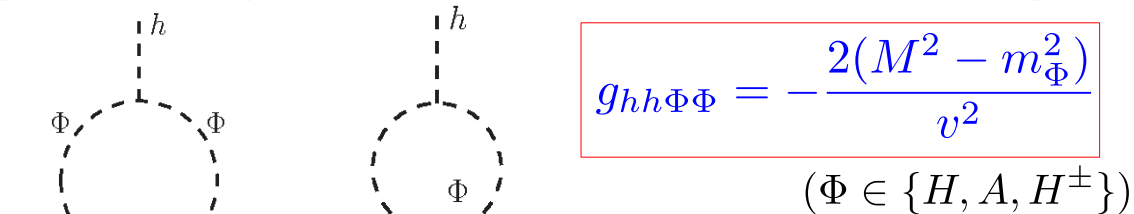
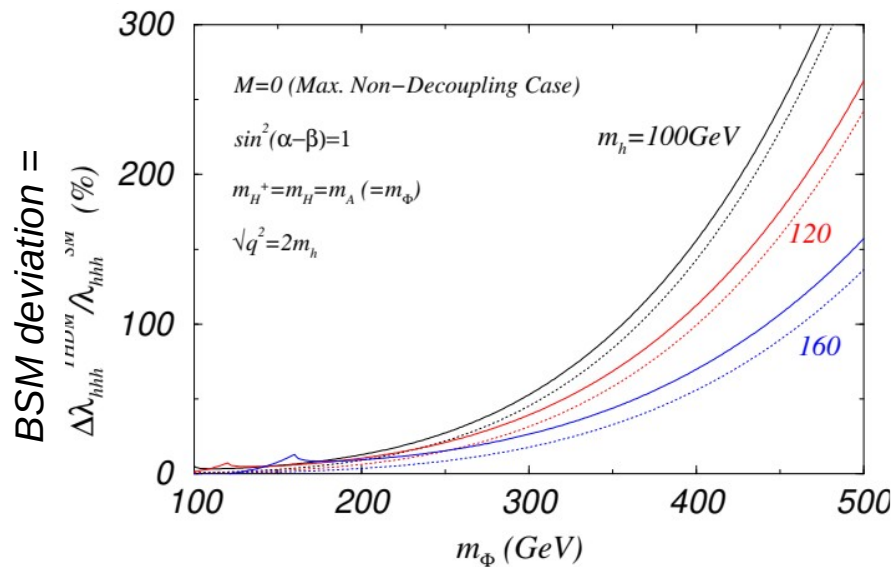
[J. List et al '24]
(adapted from slide
of [G. Weiglein '24])



See also [Dürig, DESY-THESIS-2016-027]

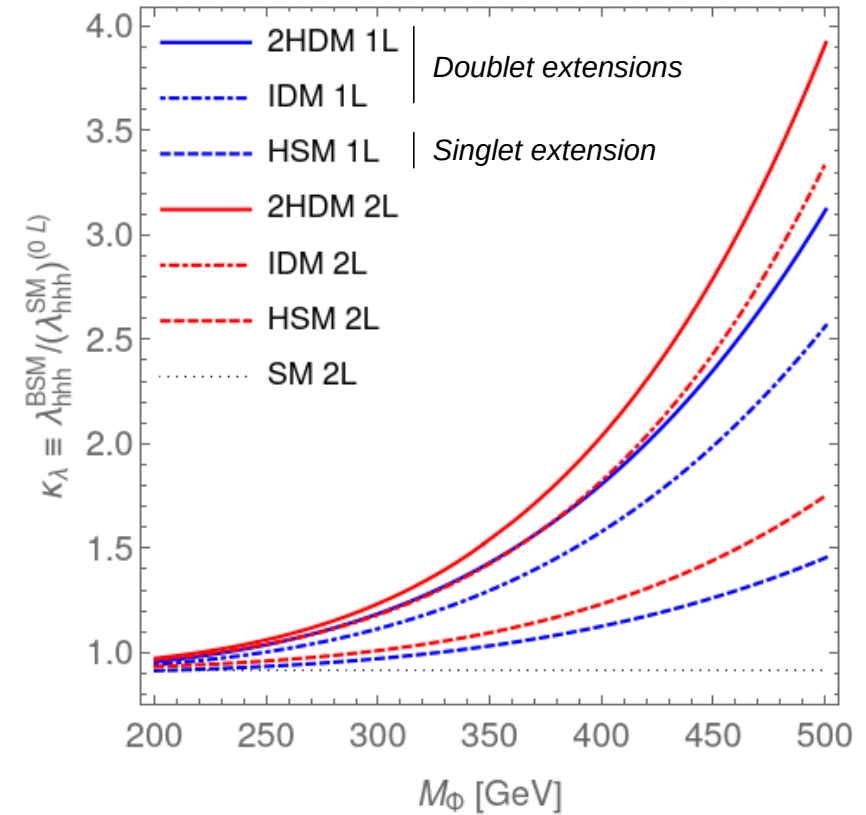
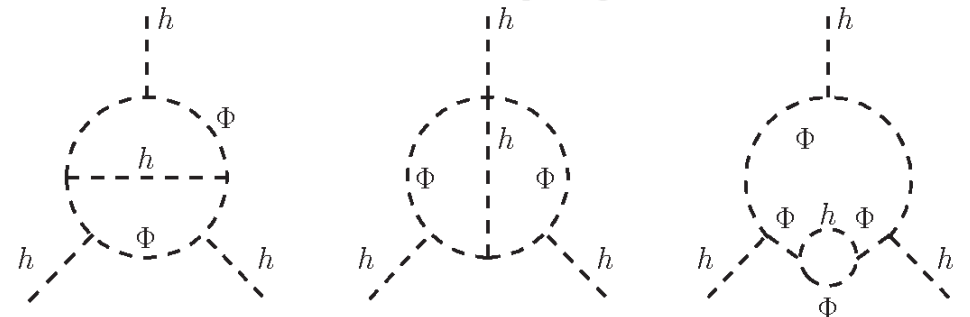
Mass splitting effects in λ_{hhh}

- First investigation of 1L BSM contributions to λ_{hhh} in 2HDM: [Kanemura, (Kiyoura), Okada, Senaha, Yuan '02, '04]



- Deviations of tens/hundreds of % from SM possible, for large $g_{h\Phi\Phi}$ or $g_{hh\Phi\Phi}$ couplings
- Mass splitting effects, now found in various models (2HDM, inert doublet model, singlet extensions, etc.)

- Large effects **confirmed at 2L** in [JB, Kanemura '19] → **leading 2L corrections involving BSM scalars ($H, A, H^\pm$) and top quark**, computed in effective potential approximation



Examples of scalar contributions to λ_{hhh} in aligned 2HDM

BSM scalars:
 $\Phi \in \{H, A, H^\pm\}$
 $m_\Phi^2 = M^2 + \tilde{\lambda}_\Phi v^2$

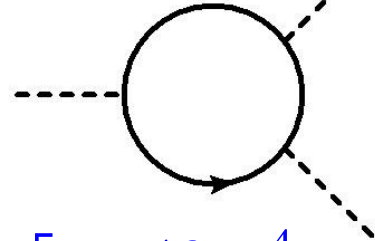
Coupling/Order	0L	1L	2L	3L
g_{hhhh}		<i>subleading</i> 	<i>subleading</i>	<i>subleading</i>
$g_{(h)h\Phi\Phi}$ $\left[g_{hh\Phi\Phi} = -\frac{2(M^2 - m_\Phi^2)}{v^2} \right]$	-			
$g_{(h)H\Phi\Phi'}$ [$g_{(h)G\Phi\Phi'}$ case similar]	-	-		
$g_{\Phi\Phi\Phi'\Phi'}$ [2 BSM scalars of species Φ , 2 of species Φ']	-	-		

[NB: 1 h can be replaced by a VEV] $\rightarrow$ no further type of coupling entering after 2L
 $\rightarrow$ for each class of diagrams, perturbative convergence can be verified!

One-loop mass-splitting effects

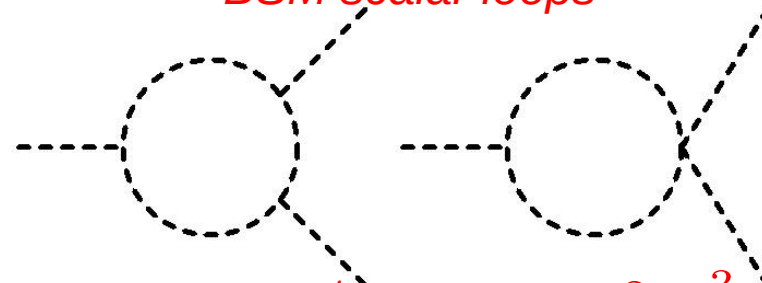
- **Leading one-loop** corrections to λ_{hhh} in models with extended sectors (like 2HDM):

SM top quark loop



$$\delta^{(1)}\lambda_{hhh} \supset \frac{1}{16\pi^2} \left[-\frac{48m_t^4}{v^3} \right]$$

BSM scalar loops



$$+ \sum_{\Phi} \frac{4n_{\Phi}m_{\Phi}^4}{v^3} \left(1 - \frac{\mathcal{M}^2}{m_{\Phi}^2} \right)^3$$

*First found in 2HDM:
[Kanemura, Kiyoura,
Okada, Senaha, Yuan '02]*

$\mathcal{M}$: **BSM mass scale**, e.g. soft breaking scale M of Z_2 symmetry in 2HDM

n_{Φ} : # of d.o.f of field Φ

- Size of new effects depends on how the BSM scalars acquire their mass: $m_{\Phi}^2 \sim \mathcal{M}^2 + \tilde{\lambda}v^2$

$$\left(1 - \frac{\mathcal{M}^2}{m_{\Phi}^2} \right)^3 \longrightarrow \begin{cases} 0, & \text{for } \mathcal{M}^2 \gg \tilde{\lambda}v^2 \\ 1, & \text{for } \mathcal{M}^2 \ll \tilde{\lambda}v^2 \end{cases}$$

**Huge BSM
effects possible!**

Model-independent higher-order effects in λ_{hhh}

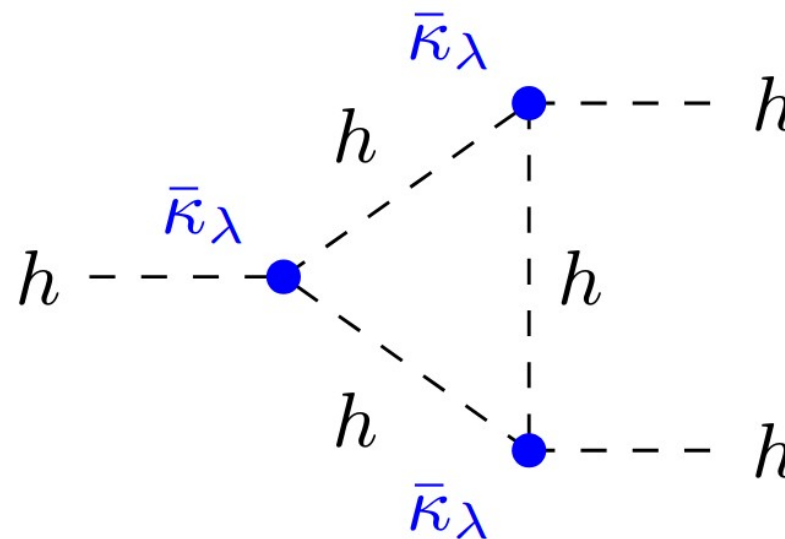
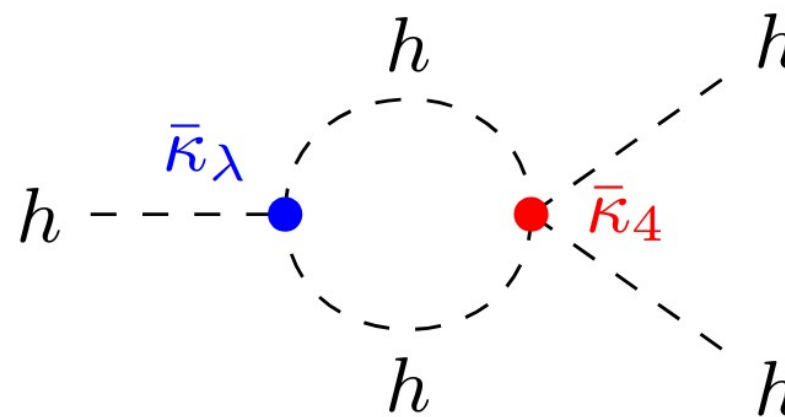
Based on

[JB, Naskar, Weiglein '26]

Model-independent higher-order corrections to λ_{hhh}

[JB, Naskar, Weiglein '26]

- Started from the pheno question:
For large κ_λ , what happens when calculating contributions involving κ_λ itself (or κ_4)?
- Large effects in κ_λ and κ_4 come from **BSM loops**
→ considering an SM-like diagram dressed with powers of κ_λ or κ_4
reminiscent of **effective approach**



Model-independent higher-order corrections to λ_{hhh}

[JB, Naskar, Weiglein '26]

Feynman diagrams and effective approach

Upper lines:

example contributions to λ_{hhh} from light SM-like Higgs boson h (*thin lines*) and heavy BSM scalar(s) Φ (*bold lines*)

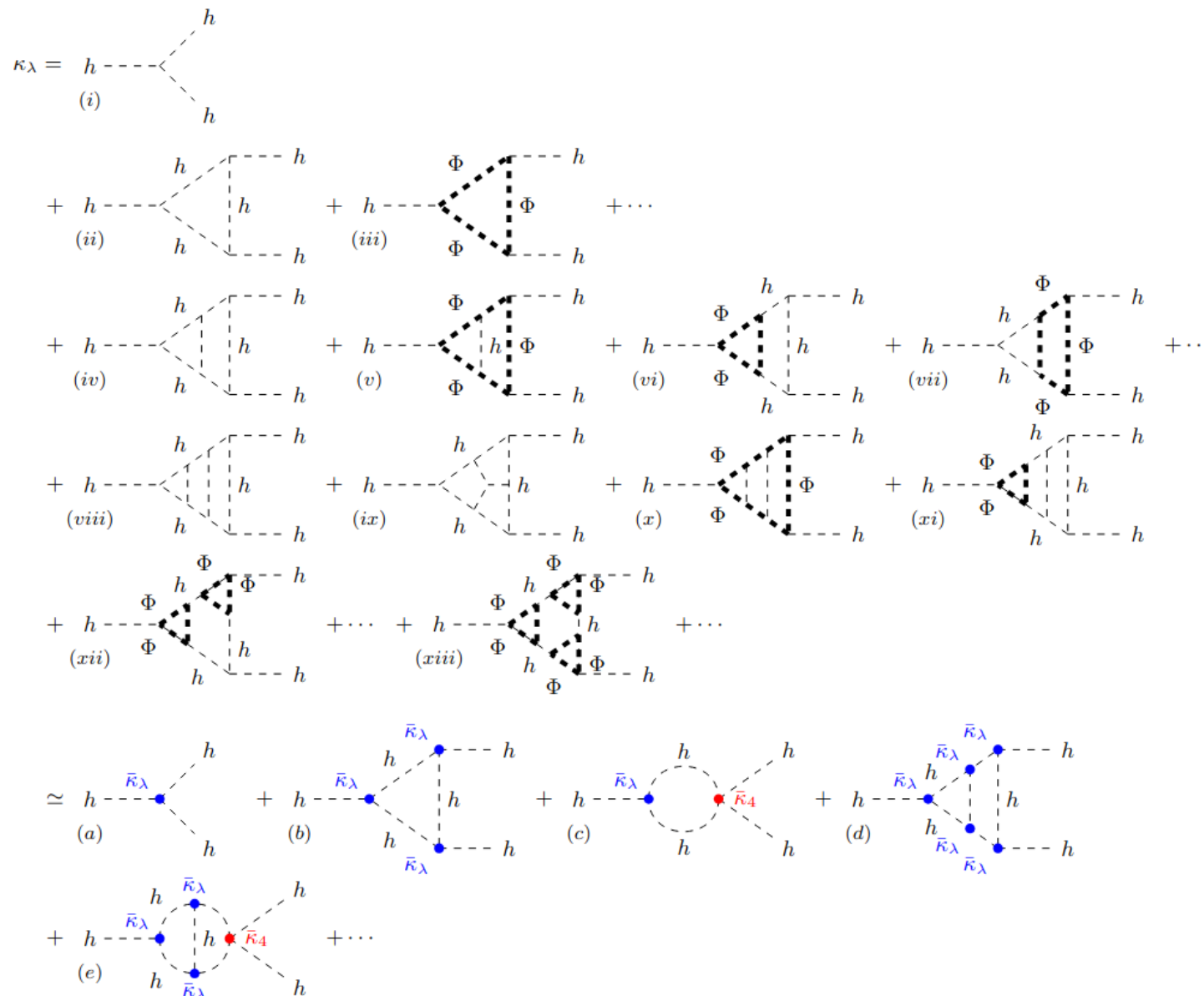
Lower lines:

example contributions in our effective setup, with higher-order corrections captured via **loop-corrected values of $\bar{\kappa}_\lambda$ and $\bar{\kappa}_4$** (using bars to distinguish from calculated quantity)

- Our approach: inspired by Higgs Effective Field theory, we consider a simple effective model

$$V^{(0)}(h) = \frac{1}{2}m_h^2 h^2 + \frac{1}{6}\bar{\kappa}_\lambda \left(\frac{3m_h^2}{v} \right) h^3 + \frac{1}{4!}\bar{\kappa}_4 \left(\frac{3m_h^2}{v^2} \right) h^4.$$

and compute λ_{hhh} up to two loops



Model-independent higher-order corrections to λ_{hhh}

[JB, Naskar, Weiglein '26]

Feynman diagrams and effective approach

Upper lines:

example contributions to λ_{hhh} from light SM-like Higgs boson h (*thin lines*) and heavy BSM scalar(s) Φ (*bold lines*)

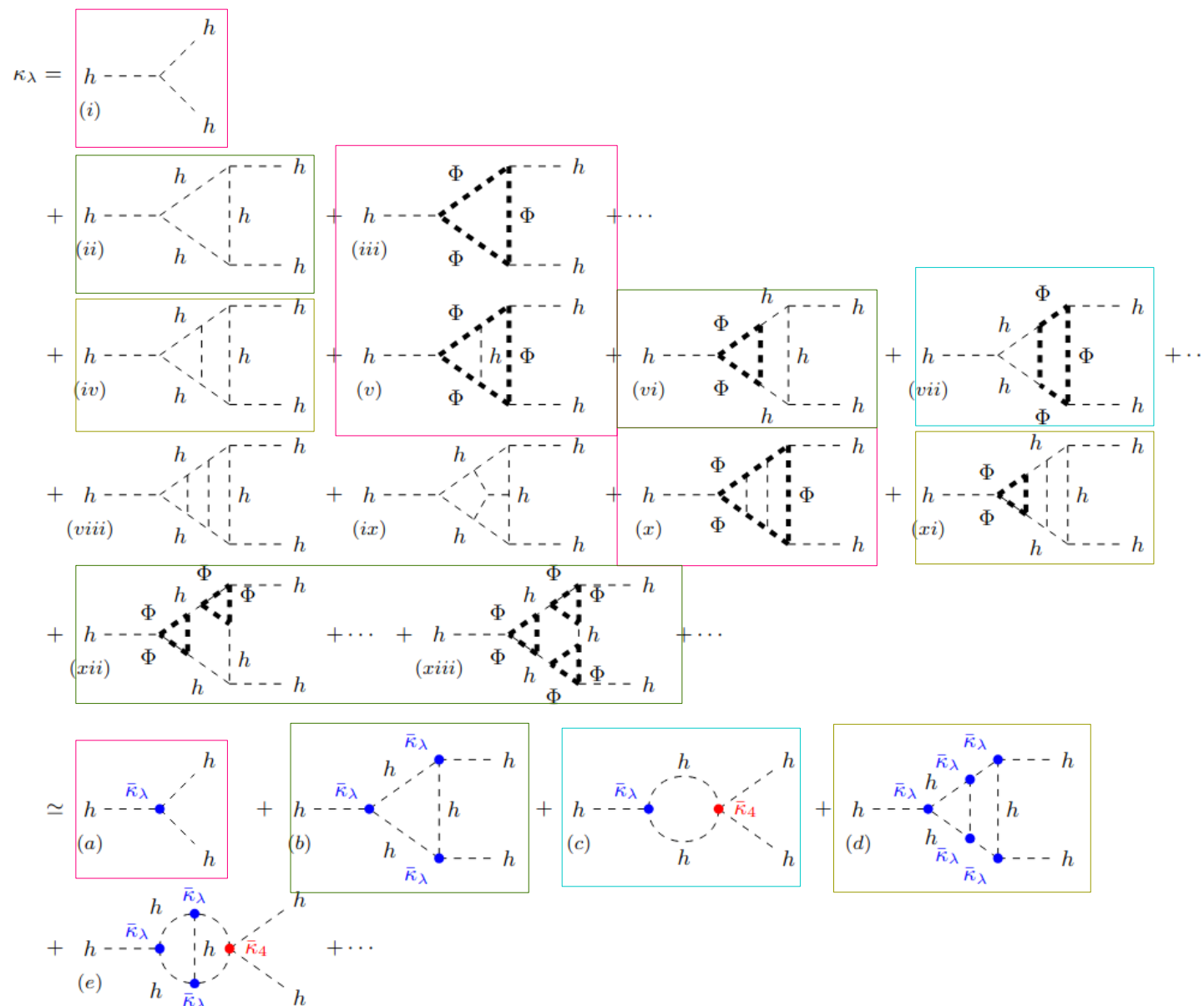
Lower lines:

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and compute λ_{hhh} up to two loops

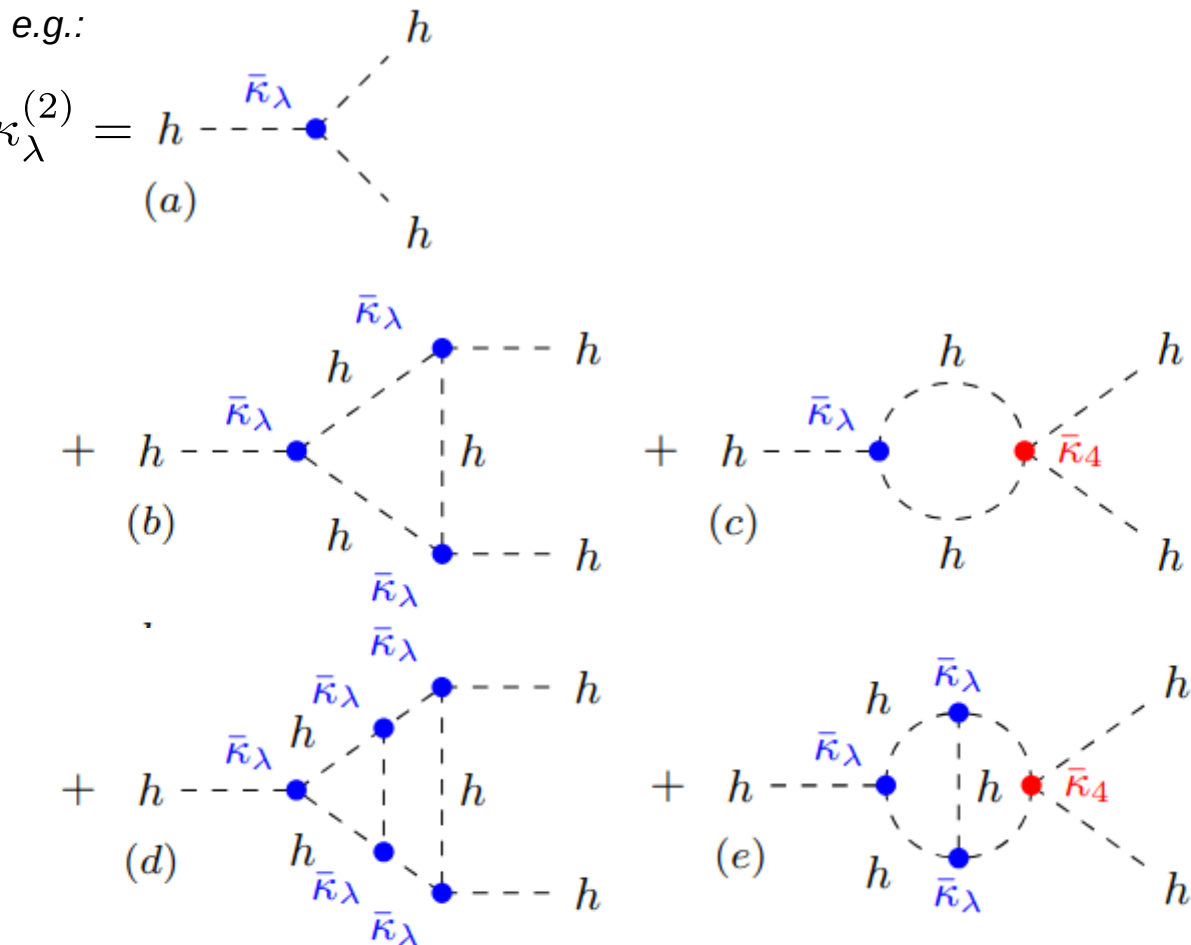


Model-independent higher-order corrections to λ_{hhh}

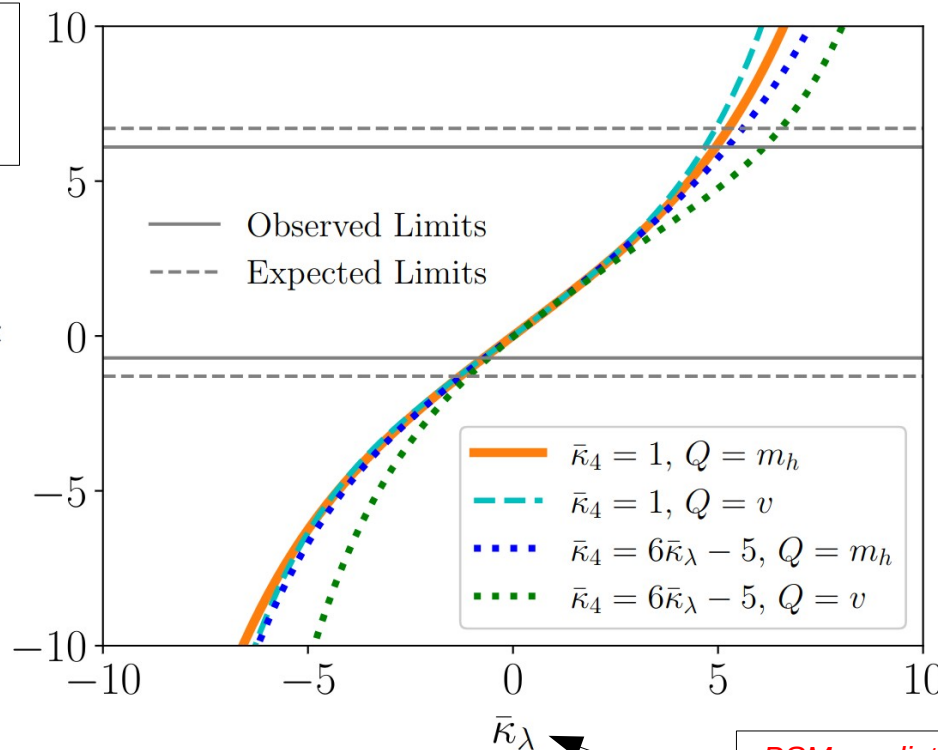
[JB, Naskar, Weiglein '26]

- Following **effective approach**, inspired by scalar sector of **Higgs Effective Field Theory**, compute diagrams with insertions of $\bar{\kappa}_\lambda$ and $\bar{\kappa}_4$ (denoted with bars to indicate inputs), up to two loops

- Incorporates well-defined set of higher-order corrections
- Modifies theoretical interpretation of experimental bounds



BSM prediction including also higher order effects



Higher-order effects can still be significant for $\kappa_\lambda \gg 1$

Public code available at:

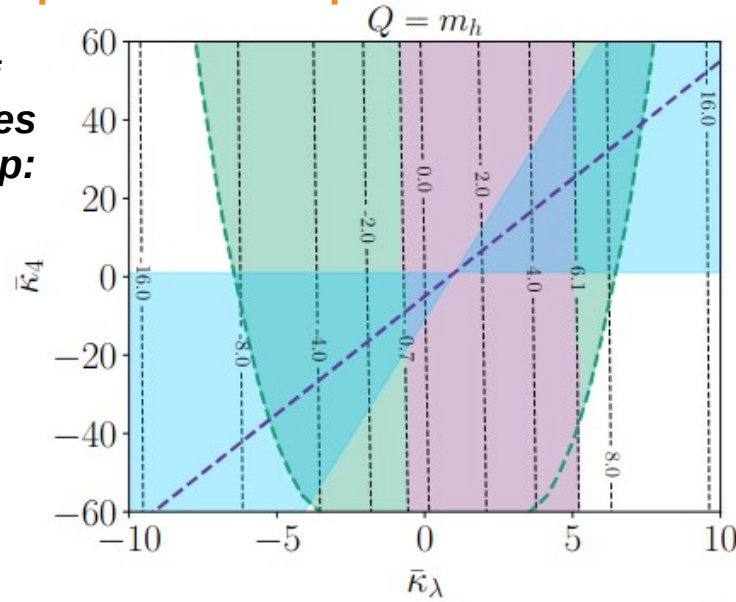
https://gitlab.com/wrishik/check_kappas.git

Model-independent higher-order corrections to λ_{hhh} and λ_{hhhh}

Impact on interpretation of experimental bounds

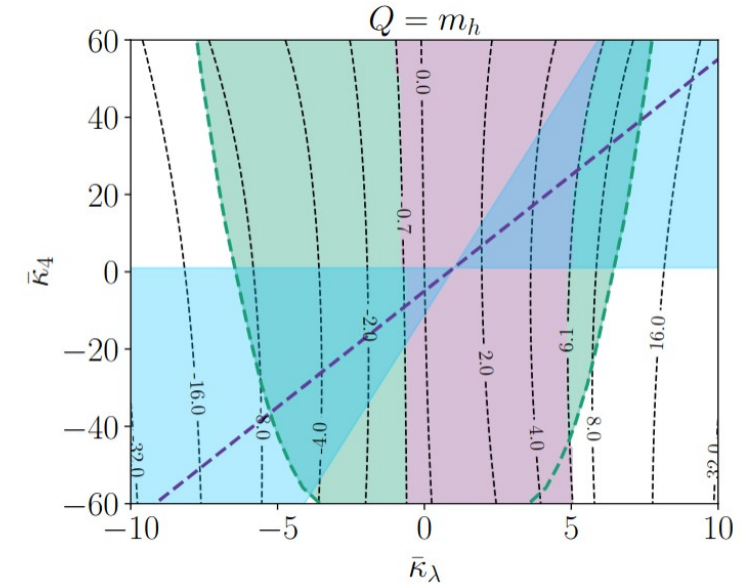
[JB, Naskar, Weiglein '26]

Interpretation of di-Higgs searches on κ_λ at one loop:

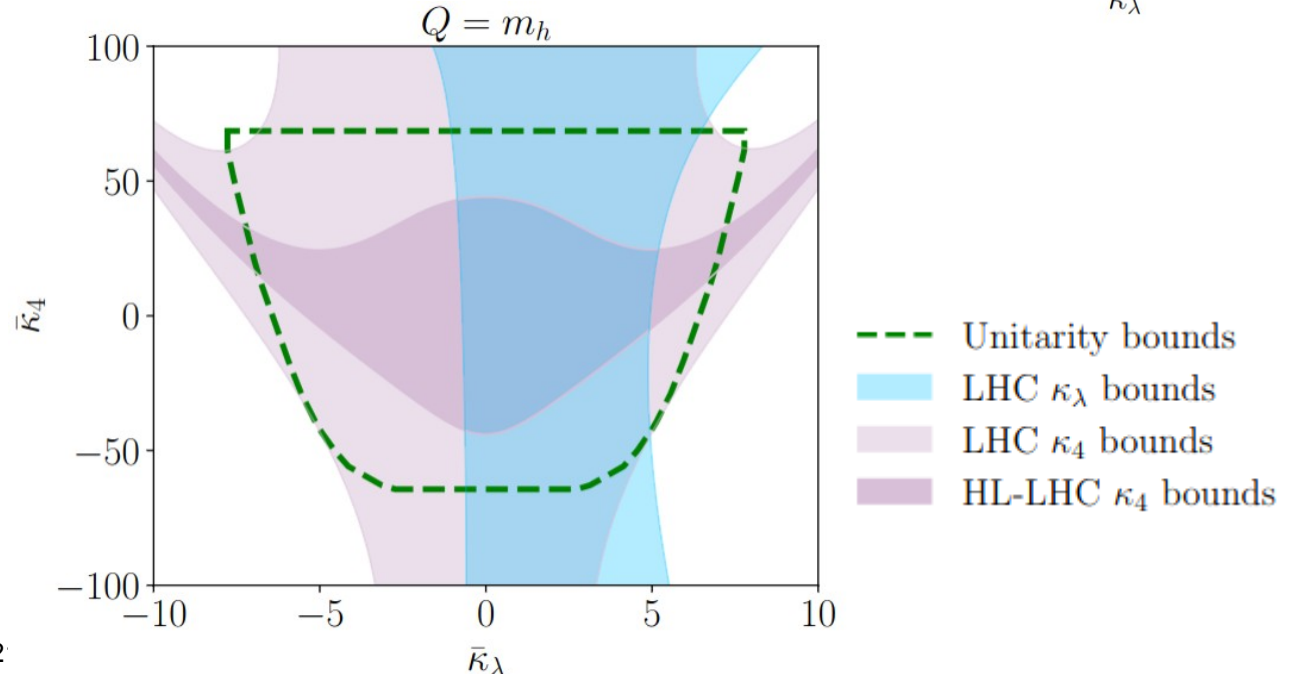


Interpretation of di-Higgs searches on κ_λ at two loops:

- $\bar{\kappa}_4 = 6\bar{\kappa}_\lambda - 5$
- SMEFT region
- Unitarity bounds
- Exp. bounds



Interpretation of di-Higgs searches on κ_λ and triple-Higgs searches on κ_4 at two loops:



Constraints on the $(\bar{\kappa}_\lambda, \bar{\kappa}_4)$ plane

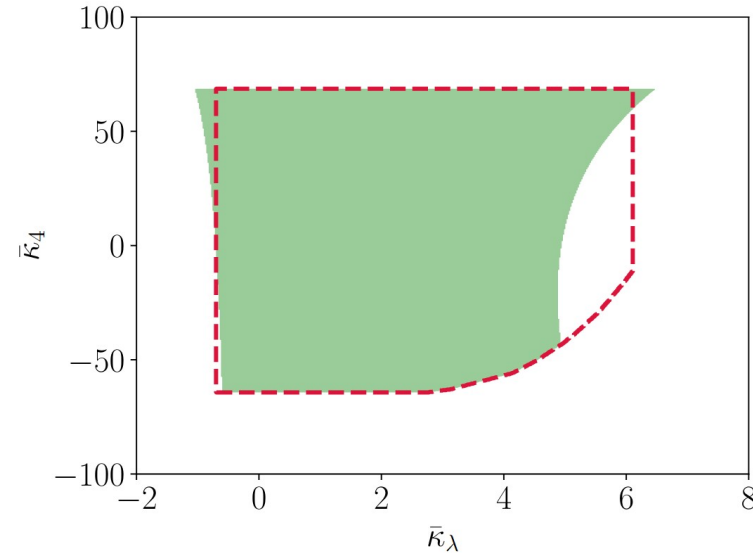
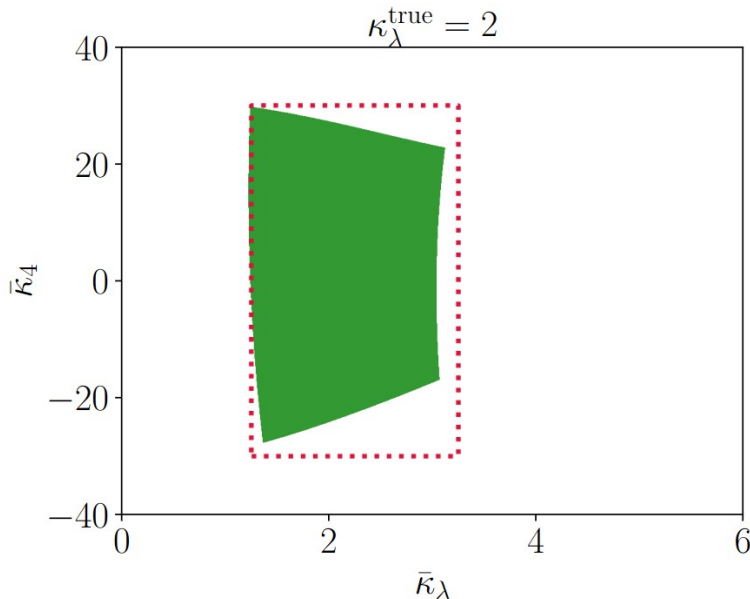
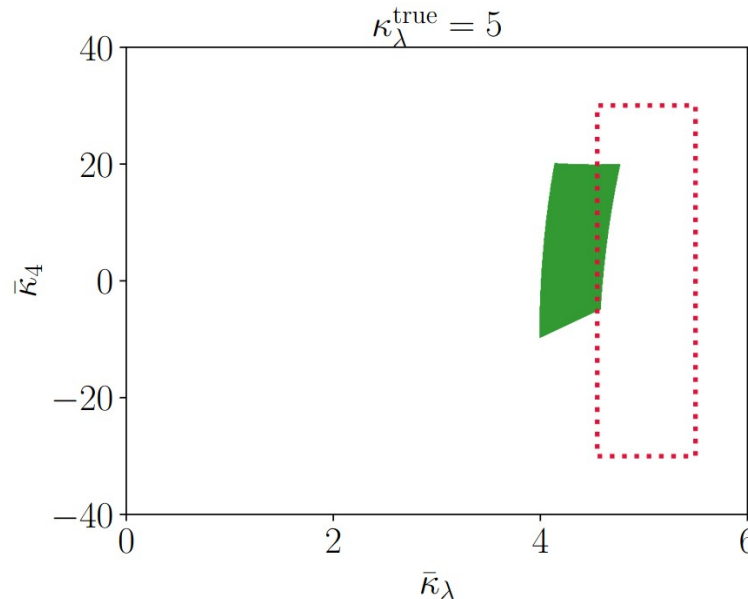
[JB, Naskar, Weiglein '26]

Impact on interpretation of current experimental bounds and expected future limits

Current limits on κ_λ [ATLAS+CMS Run 2 combination '26] and κ_4 [CMS '26], and perturbative unitarity [Stylianou, Weiglein '23]

Red dashed line: constraints directly applied to $\bar{\kappa}_\lambda, \bar{\kappa}_4$

Green shaded area: higher-order $(\bar{\kappa}_\lambda, \bar{\kappa}_4)$ corrections taken into account



Expected future limits at HL-LHC on κ_λ [ATLAS+CMS HL-LHC ESPPU highlights '25] and κ_4 [Stylianou, Weiglein '23], [ATLAS '25], assuming $\kappa_\lambda=2$ (left) or $\kappa_\lambda=5$ (right) is realised in Nature

Red dotted line: constraints directly applied to $\bar{\kappa}_\lambda, \bar{\kappa}_4$

Green shaded area: higher-order $(\bar{\kappa}_\lambda, \bar{\kappa}_4)$ corrections taken into account

Constraining BSM models with λ_{hhh}

- i. Can we apply the limits on κ_λ , extracted from experimental searches for di-Higgs production, for BSM models?*
- ii. Can large BSM deviations occur for points still allowed in light of theoretical and experimental constraints? If so, how large can they become?*

As a concrete example, we consider an aligned 2HDM

Based on

arXiv:2202.03453 (Phys. Rev. Lett.) in collaboration with Henning Bahl and Georg Weiglein

Can we apply di-Higgs results for the aligned 2HDM?

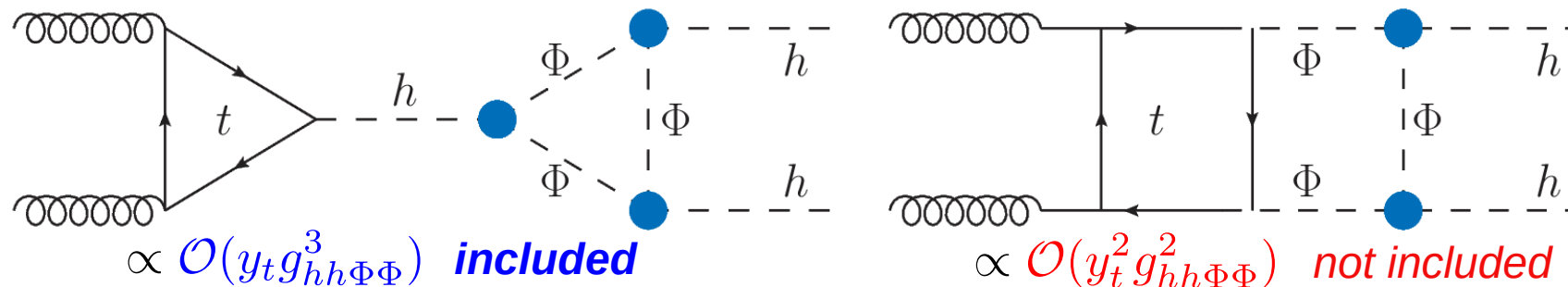
- Current strongest limits on κ_λ from ATLAS di-Higgs searches

$$-1.2 < \kappa_\lambda < 7.2 \text{ [ATLAS-CONF-2024-006]}$$

$$[\text{where } \kappa_\lambda \equiv \lambda_{hhh} / (\lambda_{hhh}^{(0)})^{\text{SM}}]$$

- What are the *assumptions* for the ATLAS limits?

- All other Higgs couplings (to fermions, gauge bosons) are SM-like
→ this is **ensured by the alignment** ✓
- The modification of λ_{hhh} is the only source of deviation of the *non-resonant Higgs-pair production cross section* from the SM



→ We **correctly include all leading BSM effects to di-Higgs production, in powers of $g_{hh\Phi\Phi}$, up to NNLO!** ✓

- **We can apply the ATLAS limits to our setting!**

A parameter scan in the aligned 2HDM

[Bahl, JB, Weiglein PRL '22]

- Our strategy:
 1. **Scan BSM parameter space**, keeping only points passing various theoretical and experimental constraints (*see below*)
 2. Identify regions with **large BSM deviations in λ_{hhh}**
 3. Devise a **benchmark scenario** allowing large deviations and investigate impact of experimental limit on λ_{hhh}
- Here: we consider an **aligned 2HDM of type-I**, but similar results expected for other 2HDM types, or other BSM models with extended Higgs sectors
- Constraints in our parameter scan:
 - experimental

 - 125-GeV Higgs measurements with HiggsSignals
 - Direct searches for BSM scalars with HiggsBounds
 - b-physics constraints, using results from [Gfitter group 1803.01853]
 - EW precision observables, computed at two loops with THDM_EWPOS [Hessenberger, Hollik '16, '22]

Checked with ScannerS
[Mühlleitner et al. 2007.02985]
 - theoretical

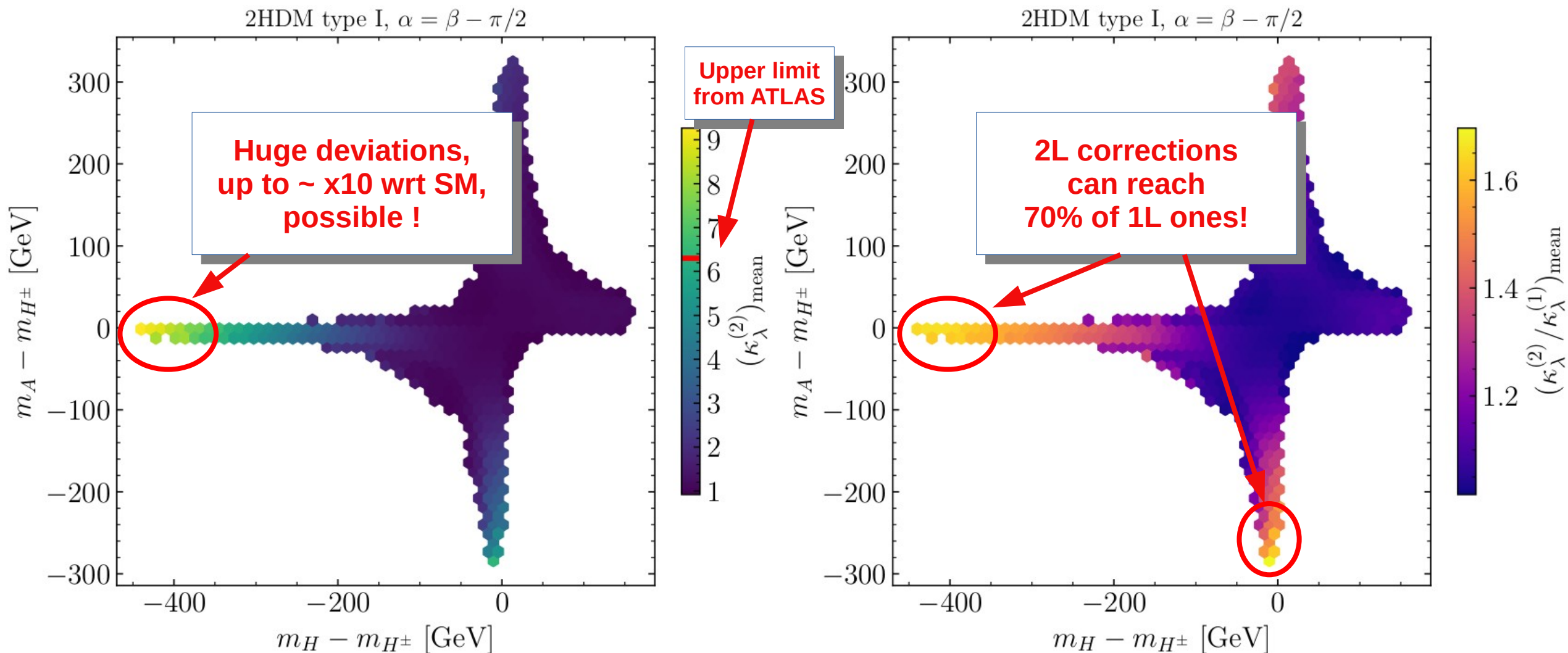
 - Vacuum stability
 - Boundedness-from-below of the potential
 - NLO perturbative unitarity, using results from [Grinstein et al. 1512.04567], [Cacchio et al. 1609.01290]

Checked with ScannerS
- For points passing these constraints, we **compute κ_λ at 1L and 2L**, using results from [JB, Kanemura '19]

Parameter scan results

[Bahl, JB, Weiglein PRL '22]

Mean value for $\kappa_\lambda^{(2)} = (\lambda_{hhh}^{(2)})^{2\text{HDM}} / (\lambda_{hhh}^{(0)})^{\text{SM}}$ [left] and $\kappa_\lambda^{(2)} / \kappa_\lambda^{(1)} = (\lambda_{hhh}^{(2)})^{2\text{HDM}} / (\lambda_{hhh}^{(1)})^{2\text{HDM}}$ [right] in $(m_H - m_{H^\pm}, m_A - m_{H^\pm})$ plane



- 2L corrections can become **significant** (up to $\sim 70\%$ of 1L)
- **Huge enhancements** (by a factor ~ 10) of λ_{hhh} possible for $m_A \sim m_{H^\pm}$ and $m_H \sim M$

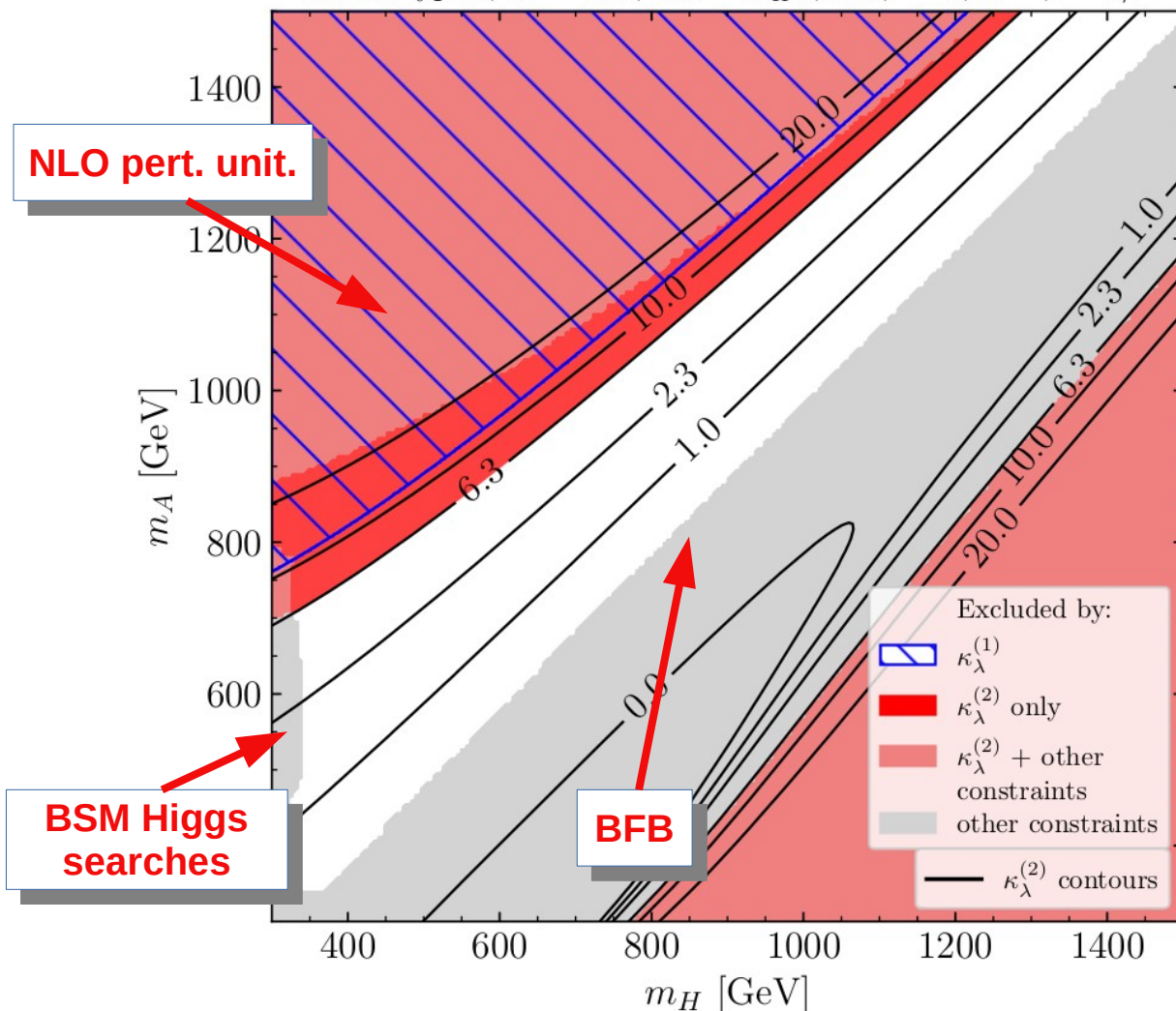
A benchmark scenario in the aligned 2HDM

[Bahl, JB, Weiglein PRL '22]

Results shown for aligned 2HDM of type-I, similar for other types (*available in backup*)

We take $m_A = m_{H^\pm}$, $M = m_H$, $\tan\beta = 2$

2HDM type I, $M = m_H$, $m_A = m_{H^\pm}$, $\tan\beta = 2$, $\alpha = \beta - \pi/2$

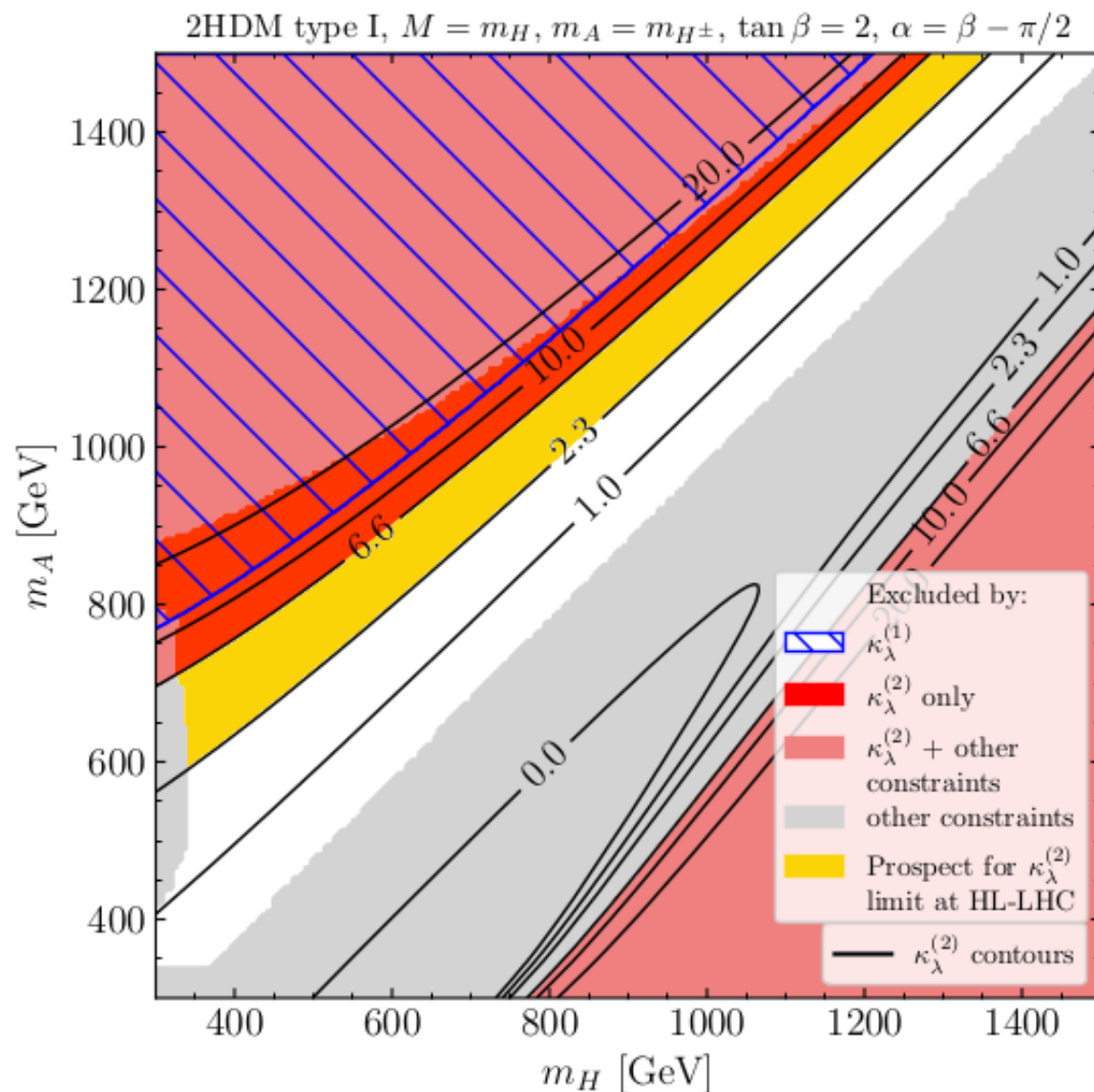


- **Grey area:** area excluded by other constraints, in particular BSM Higgs searches, boundedness-from-below (BFB), perturbative unitarity
- **Light red area:** area excluded both by other constraints (BFB, perturbative unitarity) and by $\kappa_\lambda^{(2)} > 6.3$ [in region where $\kappa_\lambda^{(2)} < -0.4$ the calculation isn't reliable]
- **Dark red area:** new area that is **excluded ONLY by $\kappa_\lambda^{(2)} > 6.3$** . Would otherwise not be excluded!
- **Blue hatches:** area excluded by $\kappa_\lambda^{(1)} > 6.3 \rightarrow$ impact of including 2L corrections is significant!

A benchmark scenario in the aligned 2HDM – future prospects

Suppose for instance the upper bound on κ_λ becomes $\kappa_\lambda < 2.3$

[Bahl, JB, Weiglein '23]



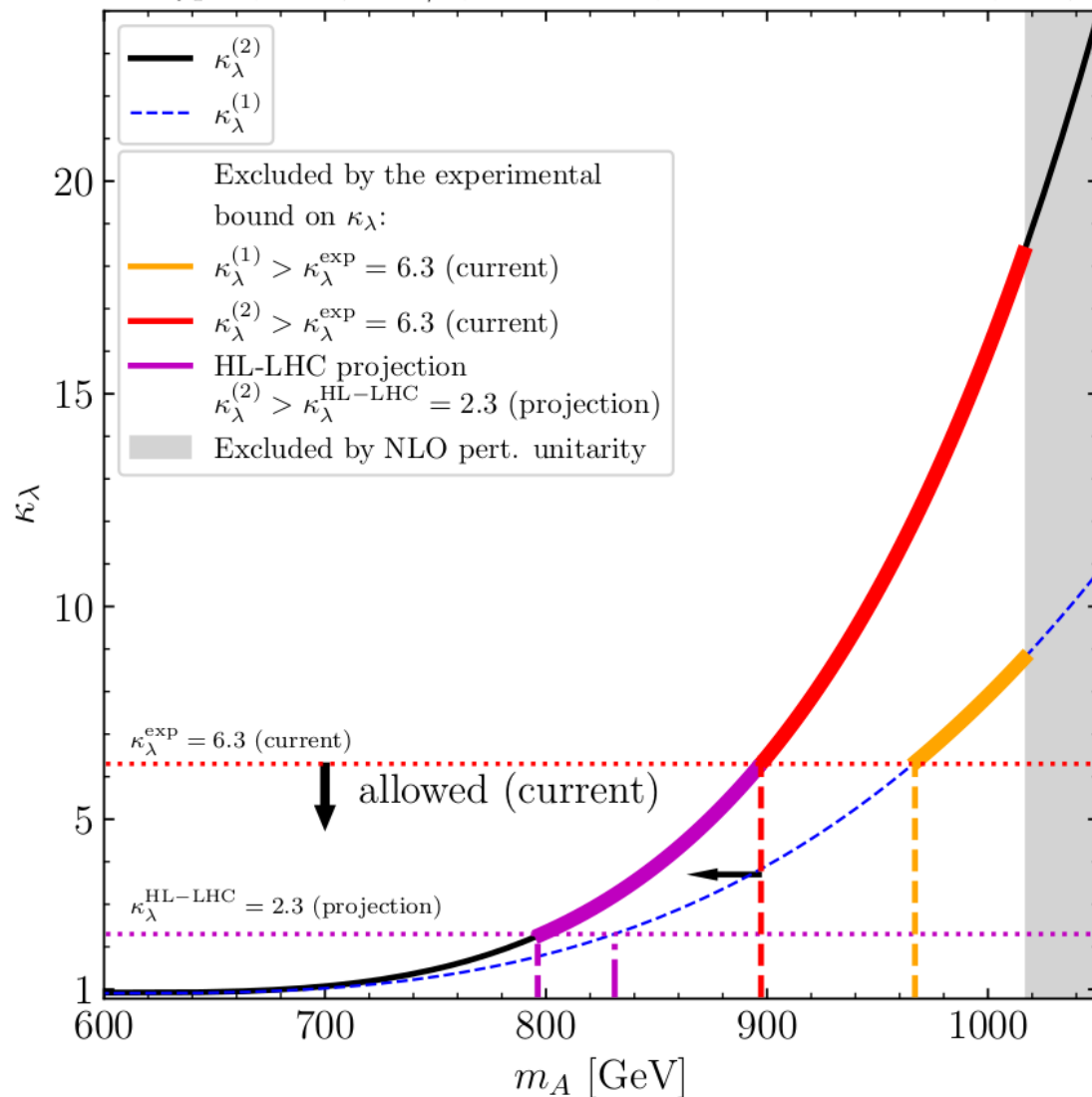
- **Golden area:** additional exclusion if the limit on κ_λ becomes $\kappa_\lambda^{(2)} < 2.3$ (achievable at HL-LHC)
- Of course, **prospects even better with an e⁺e⁻ collider!**
- Experimental constraints, such as Higgs physics, may also become more stringent, however **not** theoretical constraints (like BFB or perturbative unitarity)

A benchmark scenario in the aligned 2HDM – 1D scan

Within the previously shown plane, we fix $M=m_H=600$ GeV, and vary $m_A=m_{H^\pm}$

2HDM type I, $\alpha = \beta - \pi/2$, $m_A = m_{H^\pm}$, $M = m_H = 600$ GeV, $\tan \beta = 2$

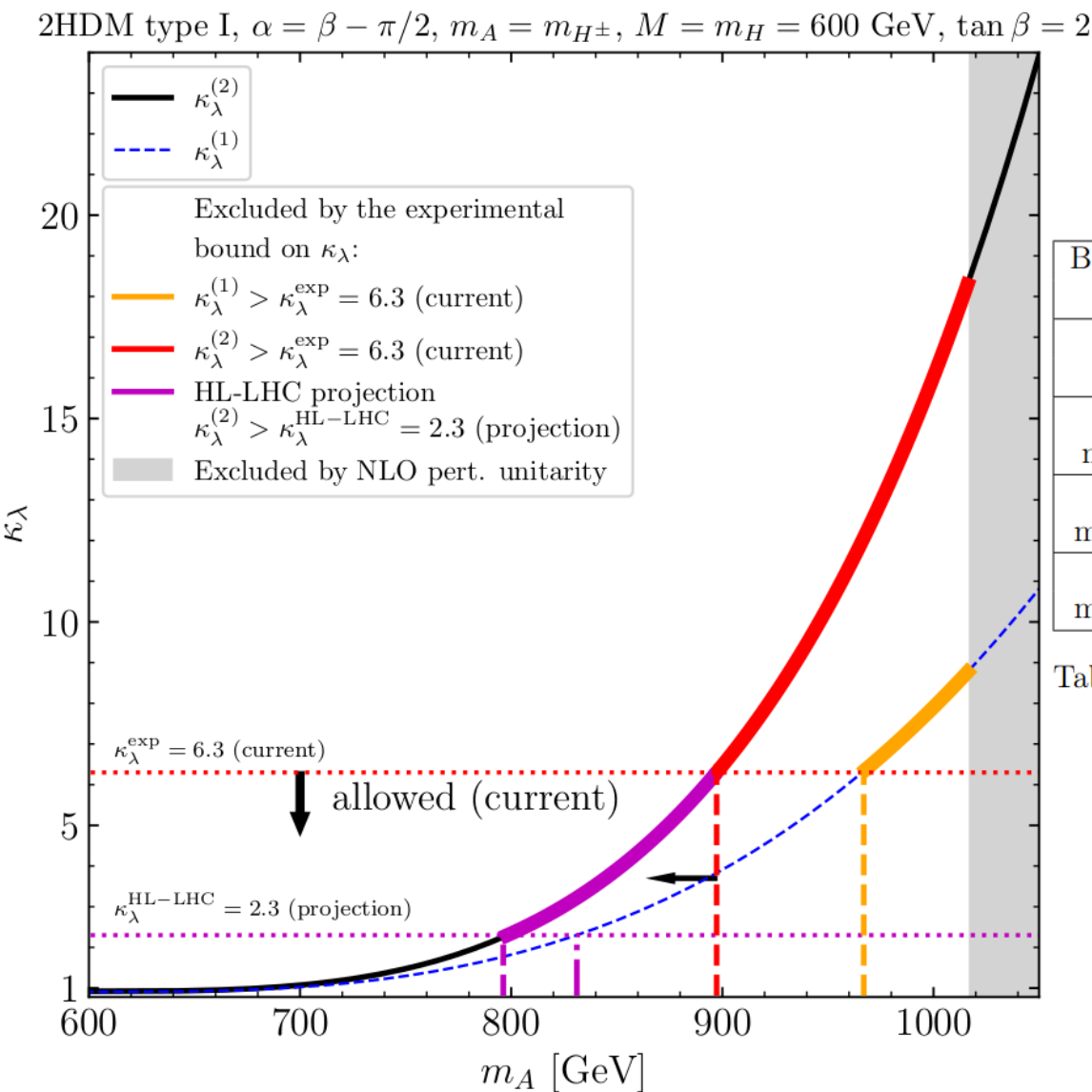
[Bahl, JB, Weiglein PRL '22]



- Illustrates the significantly improved reach of the experimental limit when including **2L corrections** in calculation of κ_λ
- A stricter choice for the perturbative unitarity constraint (grey) does not significantly change the region excluded by $\kappa_\lambda^{(2)}$

A benchmark scenario in the aligned 2HDM – 1D scan

[Bahl, JB, Weiglein PRL '22]

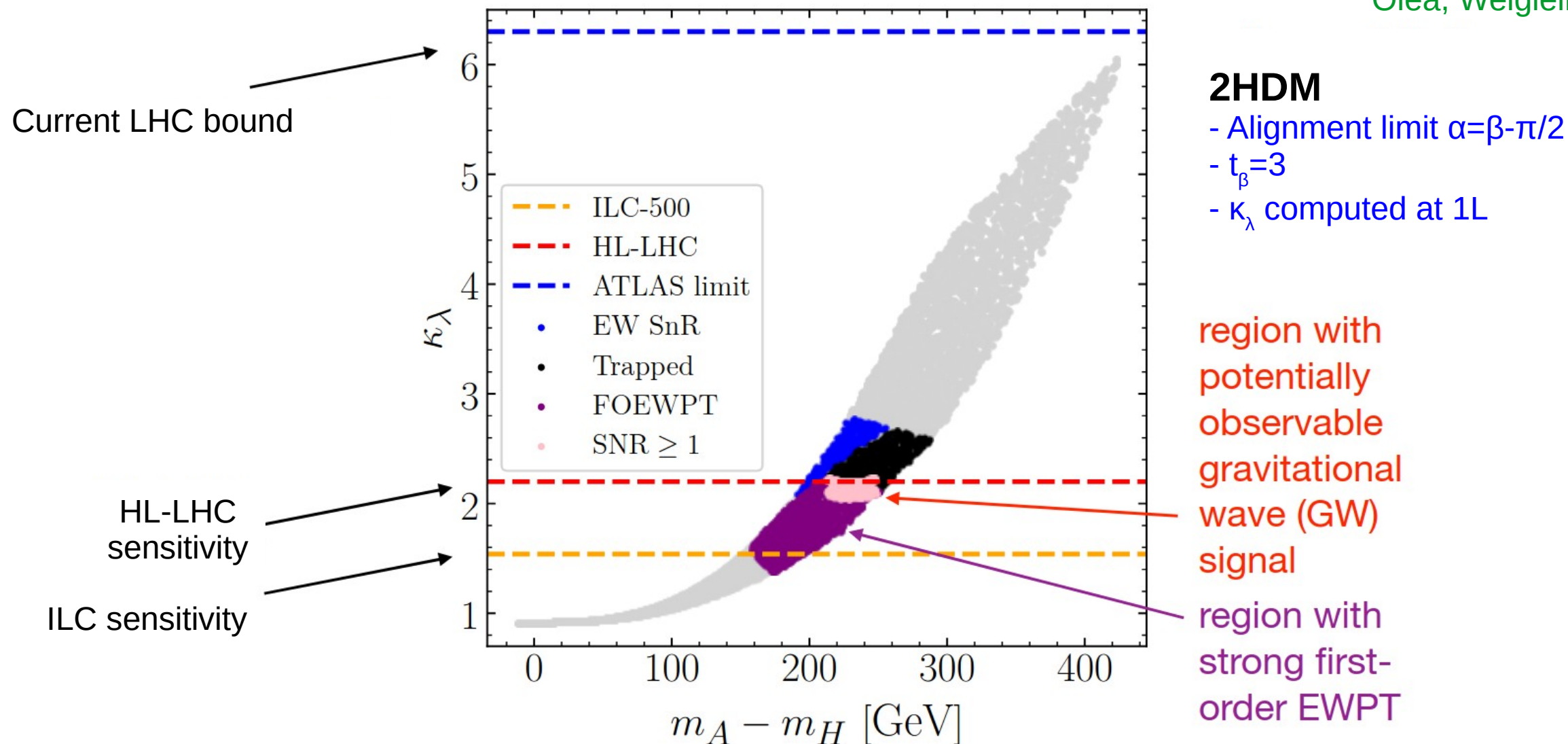


Bound on eigenvalues	$\max(m_A)$ with LO pert. unit.	$\max(m_A)$ with NLO pert. unit.	$\max(m_A)$ with finite $\sqrt{s} \in [3 \text{ TeV}, 10 \text{ TeV}]$
$\max(a_i) < 1$ $\max(\Re(a_i)) < 1$	1161 GeV 1161 GeV	1017 GeV 1033 GeV	– 1260 GeV
$\max(a_i) < 0.5$ $\max(\Re(a_i)) < 0.5$	917 GeV 917 GeV	937 GeV 958 GeV	– 929 GeV
$\max(a_i) < 0.49$ $\max(\Re(a_i)) < 0.49$	911 GeV 911 GeV	933 GeV 956 GeV	– 922 GeV
$\max(a_i) < 0.45$ $\max(\Re(a_i)) < 0.45$	889 GeV 889 GeV	912 GeV 948 GeV	– 897 GeV

Table 1: Maximal values of m_A allowed in the benchmark scenario under the constraint of perturbative unitarity, at LO and NLO, and for different upper bounds on the $2 \rightarrow 2$ scattering eigenvalues used in the perturbative unitarity constraint. Note that tree-level scattering eigenvalues are all real, so there is no difference between using $\max$ or $\Re(\max)$ for the left column.

Relation between κ_λ and strong first-order EWPT

[Biekötter, Heinemeyer, No, Olea, Weiglein '22]



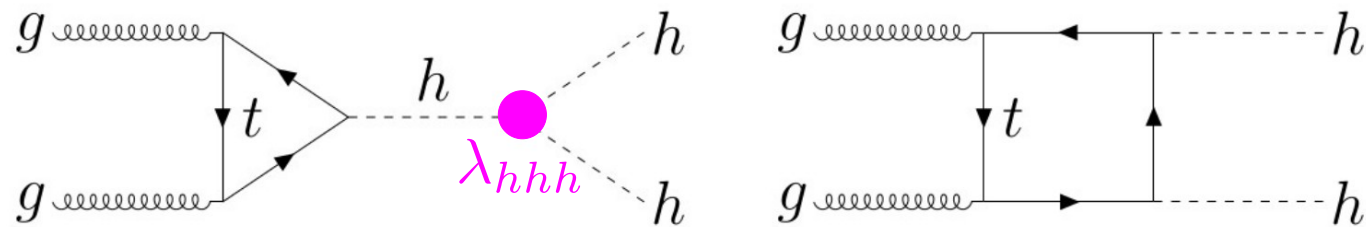
- **Region with a strong first-order EWPT and a potentially detectable GW signal is correlated with significant BSM deviation in κ_λ**

Interference effects in di-Higgs

Interference in non-resonant di-Higgs production

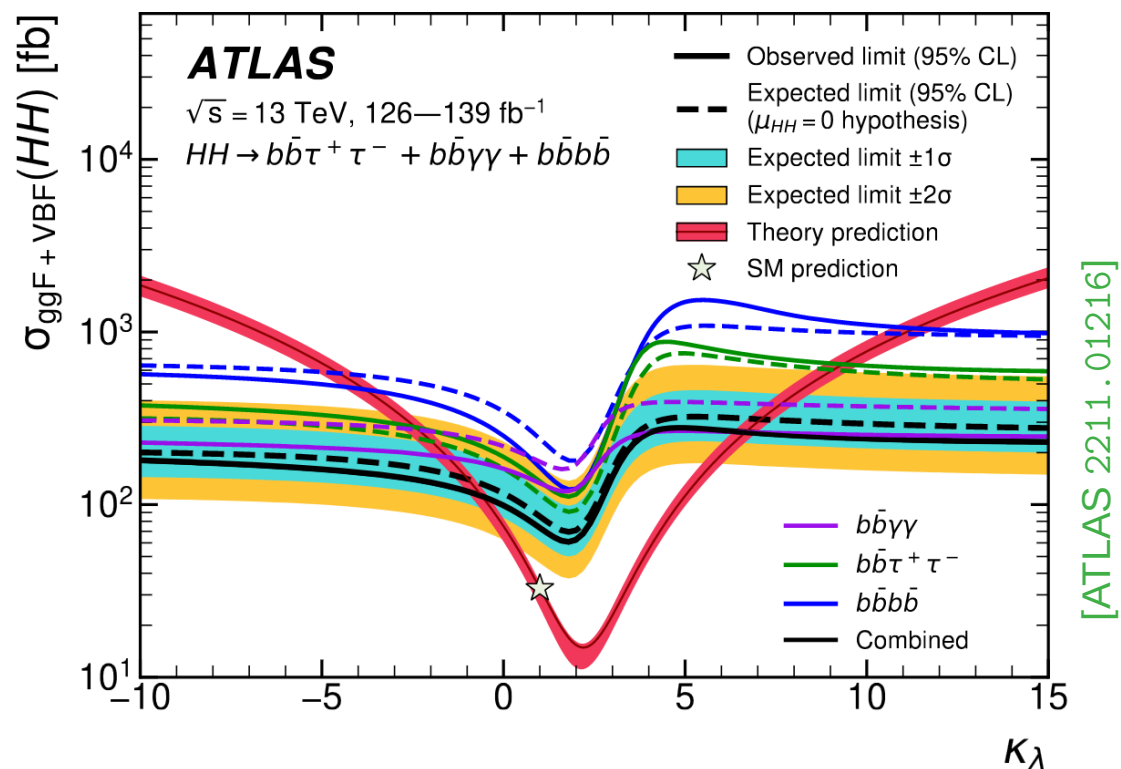
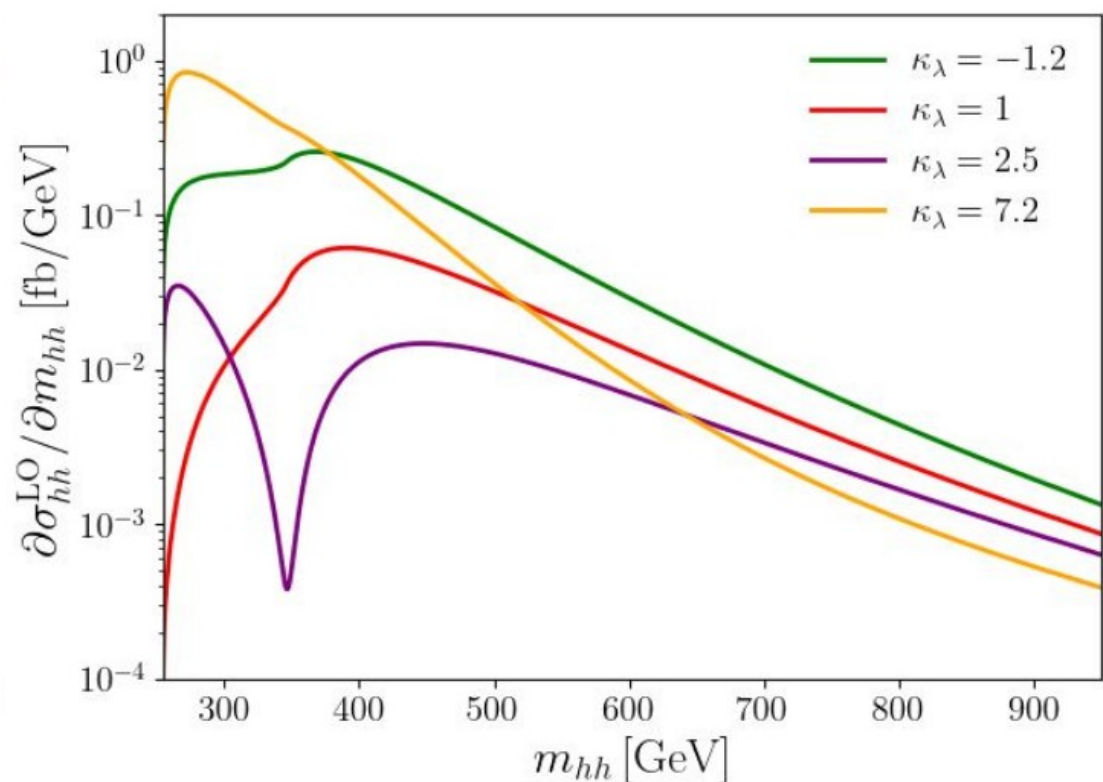
Coupling modifier:

$$\kappa_\lambda \equiv \frac{\lambda_{hhh}}{(\lambda_{hhh}^{(0)})^{\text{SM}}}$$



Relative change in total cross-section for varying κ_λ

Differential m_{hh} distributions for varied κ_λ



Plots by [K. Radchenko Serdula '24]

Note: impact of change in top Yukawa → overall shift (up/down) of distribution

Interference and reweighting in spontaneously broken Z_2 SSM

[Feuerstake, Fuchs, Robens, Winterbottom '24]
(see also talk by T. Robens on Monday)

- Simpler version of the singlet extension of SM, potential symmetric under $S \rightarrow -S$, only broken by singlet VEV

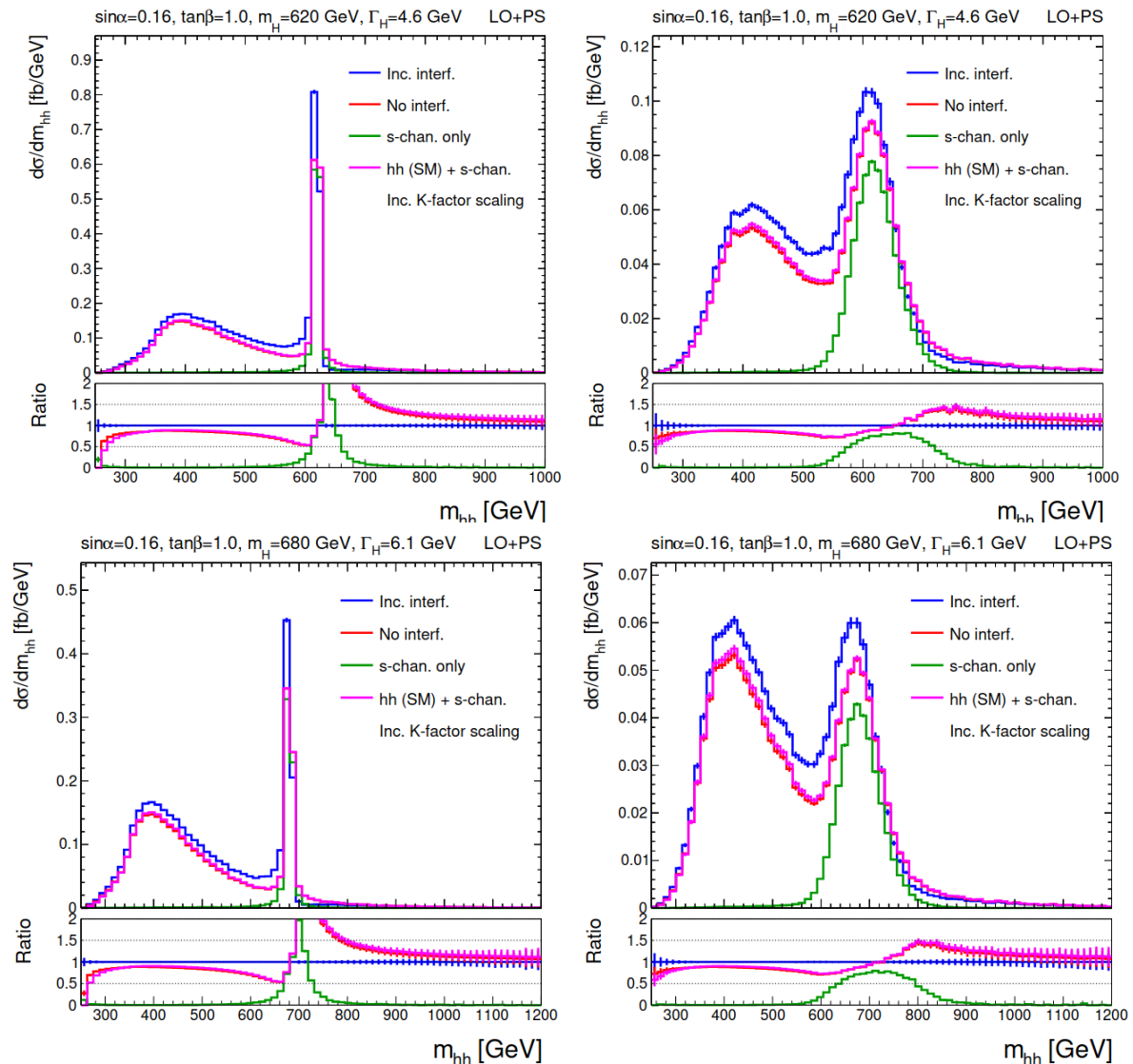
$$V(\Phi, S) = -m^2 \Phi^\dagger \Phi - \mu^2 S^2 + \lambda_1 (\Phi^\dagger \Phi)^2 + \lambda_2 S^4 + \lambda_3 \Phi^\dagger \Phi S^2$$

- Colours in distributions:
 - **blue**: total result
 - **green**: only resonant (s-channel) contribution
 - **red**: non-resonant and resonant contributions but without interference terms
 - **magenta**: SM result for non-resonant (i.e. $\kappa_\lambda=1$) part + resonant contributions, without interference terms

- Analysis with tree-level trilinears, but distributions considered over more kinematic variables (e.g. p_T)

- Here: **interference amounts to 12% of total result**

- Developed public tool **HHReweigher** for matrix-element reweighting (works for one BSM resonance)



Left/right: before/after experimental smearing

NB: y-axes here not in log-scale