

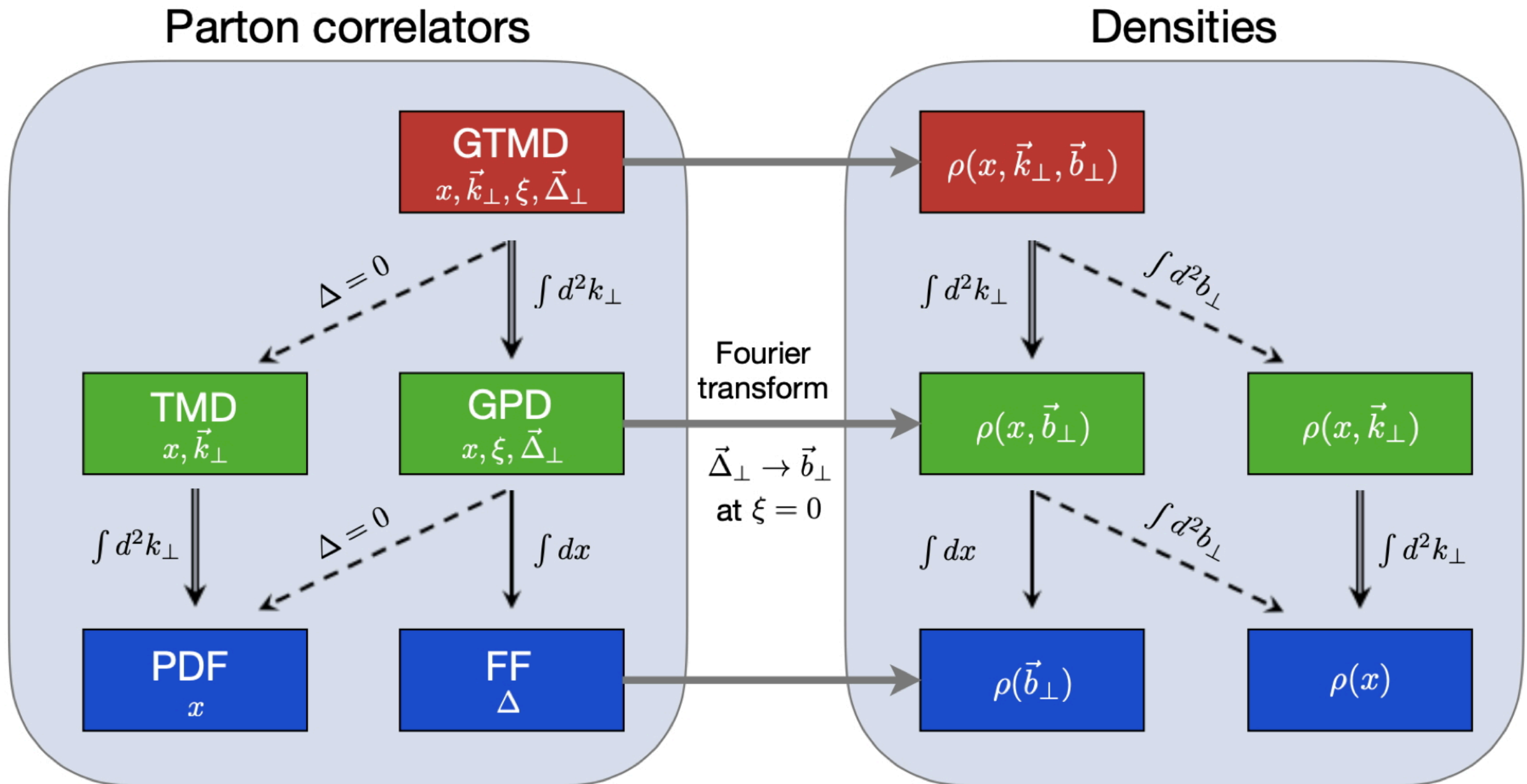


Bridging information from lattice and phenomenology on the Collins-Soper kernel

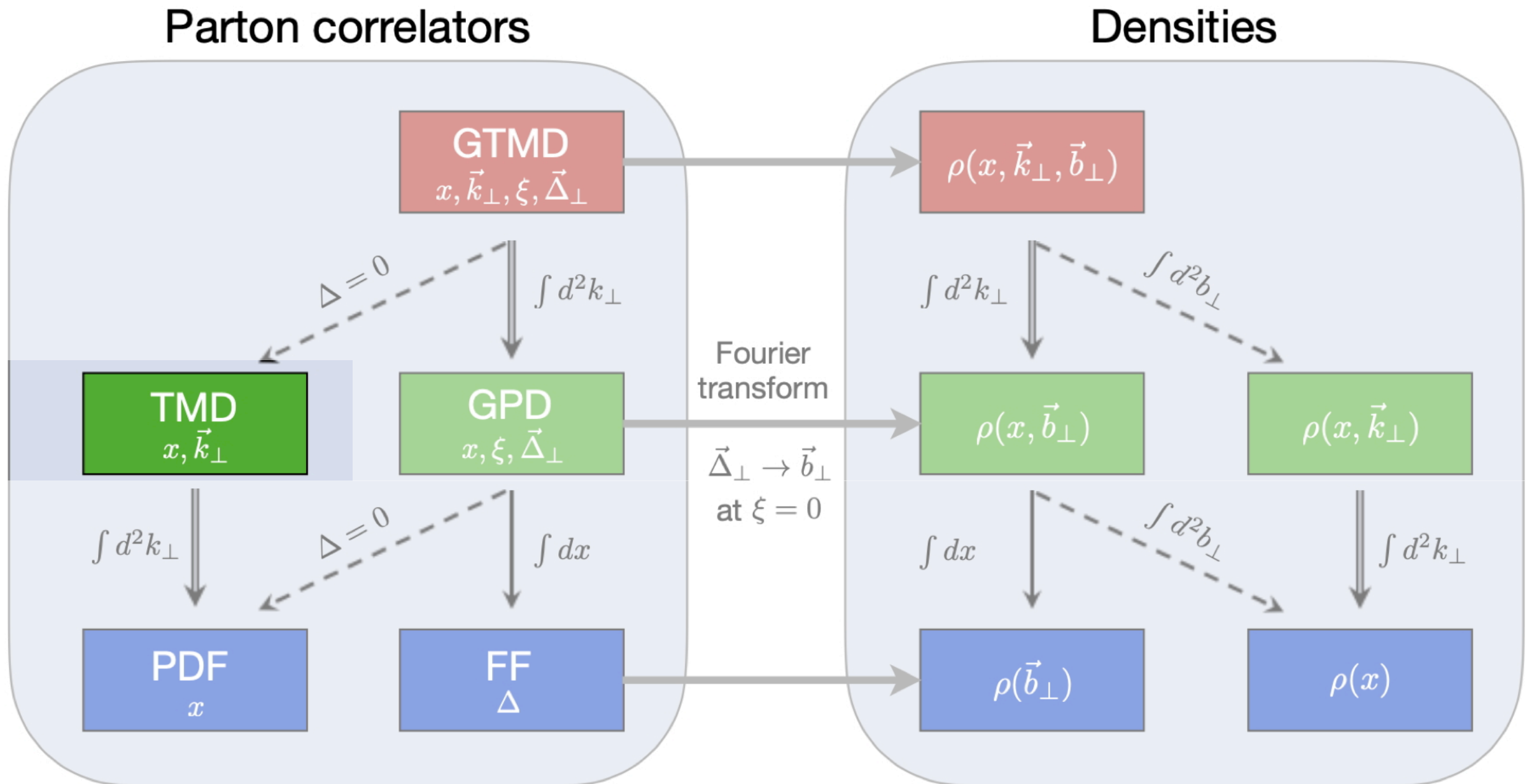
Matteo Cerutti



Imaging of hadron structure



Imaging of hadron structure



Transverse Momentum Distributions (TMDs)

3D Map of the internal structure of the proton

Non-Collinear framework

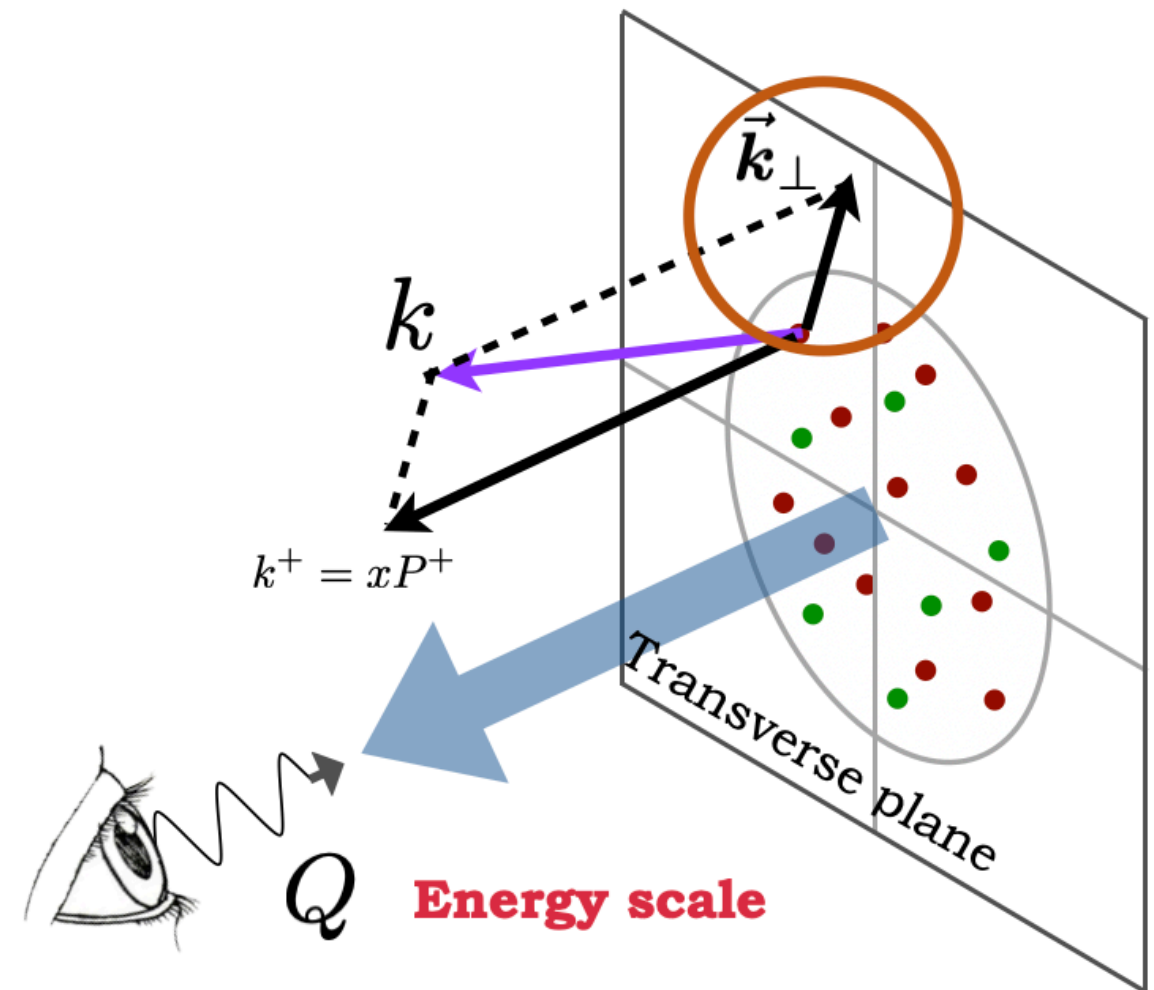
Quark Polarization

Nucleon Pol.

	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1 h_{1T}^\perp$

Time-reversal odd

Time-reversal even



TMD PDFs

$$F(x, \mathbf{k}_\perp^2, \mu, \zeta)$$

Structure of a TMD: matching onto PDFs

Collins, “Foundation of Perturbative QCD”

Echevarria, Scimemi, et al., JHEP 09 (2016)

TMDs at the operator level

$$\Phi_{i,j}(x, \mathbf{k}_\perp) = \int \frac{db^- d\mathbf{b}_T}{(2\pi)^3} e^{-xk^+b^- + \mathbf{k}_\perp \cdot \mathbf{b}_T} \langle P | \bar{\psi}_j(0) \mathcal{U}(0, b) \psi_i(b) | P \rangle \Big|_{b^+=0}$$

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TMD operator

$b_T \rightarrow 0$

short-displacement limit

Collinear operator

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Matching onto collinear PDFs $b_T \ll 1/\Lambda_{\text{QCD}}$

$$\hat{f}_1^q(x, b_T, \mu, \zeta) = \sum_{i=q', \bar{q}', g} \int_x^1 \frac{dy}{y} C_{q/i}(y, b_T, \mu, \zeta) f_1^i\left(\frac{x}{y}, \mu\right) + \mathcal{O}(b_T^2 \Lambda_{\text{QCD}}^2)$$

Structure of a TMD: evolution

Collins and Soper, Nucl. Phys. B193 (1981)
J. Collins, “Foundation of Perturbative QCD”

Evolution in 2 dimensions

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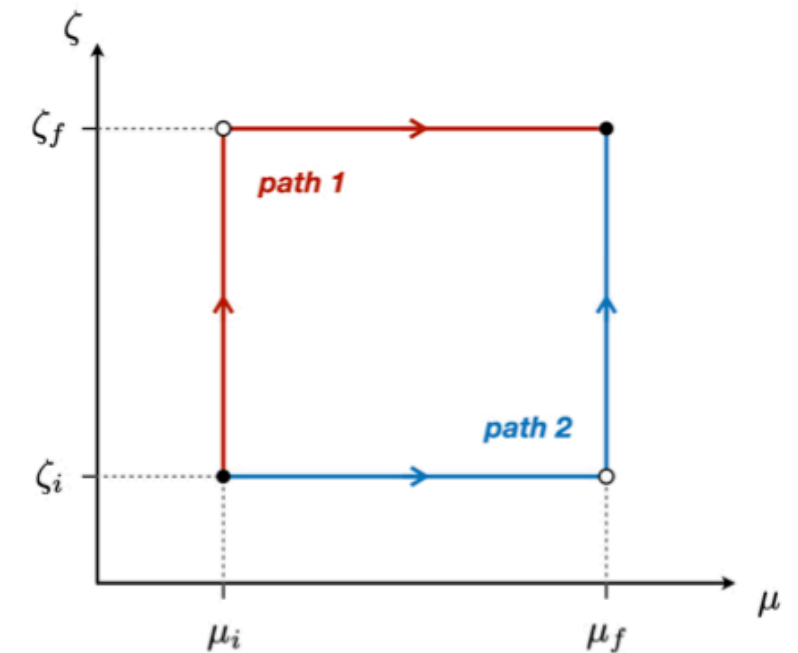
Collins-Soper kernel

$$\frac{\partial \ln \hat{f}_1(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \textcircled{K(\mu)}$$

$$\frac{\partial K(\mu)}{\partial \ln \mu} = -\gamma_K(\alpha_S(\mu))$$

$$\frac{\partial \ln \hat{f}_1(x, b_T; \mu, \zeta)}{\partial \ln \mu} = \gamma(\mu, \zeta)$$

$$\frac{\partial \gamma(\mu, \zeta)}{\partial \ln \sqrt{\zeta}} = -\gamma_K(\alpha_S(\mu))$$



Structure of a TMD: evolution

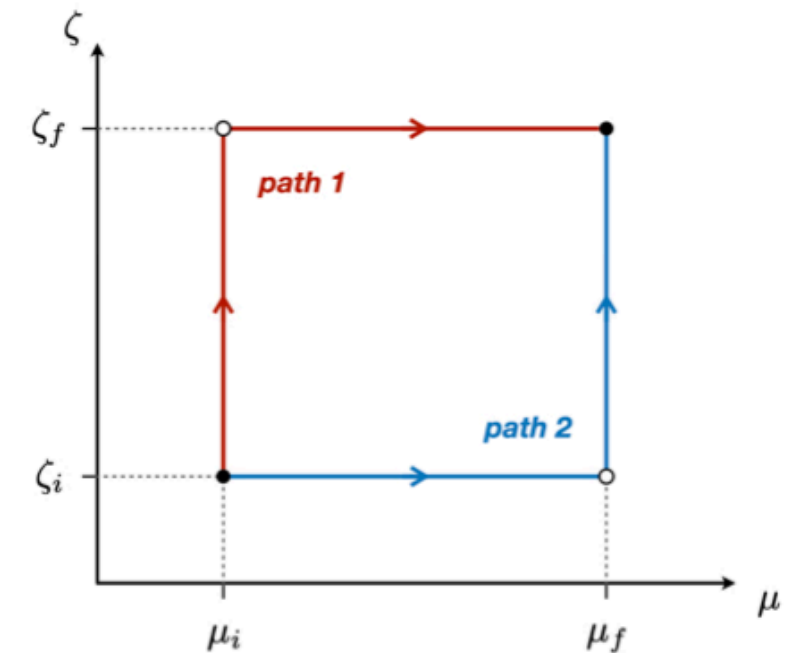
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Sudakov form factor = solution of evolution equations

$$\hat{f}_1(x, b_T; \mu, \zeta) = e^{S(b_T; \mu, \zeta)} \hat{f}_1(x, b_T; \mu_0, \zeta_0)$$



$$e^{S(b_T; \mu, \zeta)} = \exp \left\{ K(\mu_0) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} + \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F(\alpha_S(\mu')) - \gamma_K(\alpha_S(\mu')) \ln \frac{\sqrt{\zeta}}{\mu'} \right] \right\}$$

$$\gamma_F(\alpha_S(\mu)) = \gamma(\mu, \mu^2)$$

The emergence of nonperturbative physics

Initial scale: $\mu_b = \frac{2e^{-\gamma_E}}{b_T}$

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b_* prescription

$$b_*(b_T) = b_{\max} \left(\frac{1 - \exp\left(-\frac{b_T^4}{b_{\max}^4}\right)}{1 - \exp\left(-\frac{b_T^4}{b_{\min}^4}\right)} \right)^{\frac{1}{4}}$$

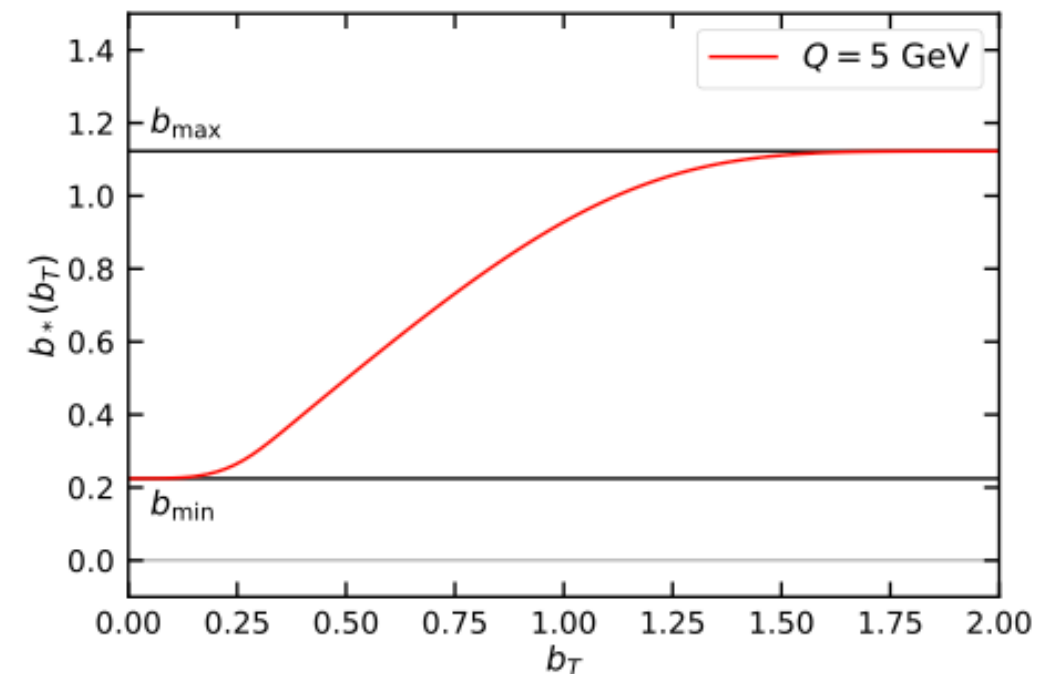
$$b_{\min} = \frac{2e^{-\gamma_E}}{\mu} \quad b_{\max} = \frac{2e^{-\gamma_E}}{1 \text{ GeV}}$$

Collins, Soper, and Sterman, Nucl.Phys. B250,199 (1985)

Collins, Gamberg, et al., Phys.Rev.D 94 (2016)

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MC and A. Simonelli, EPJ C 86 (2026)



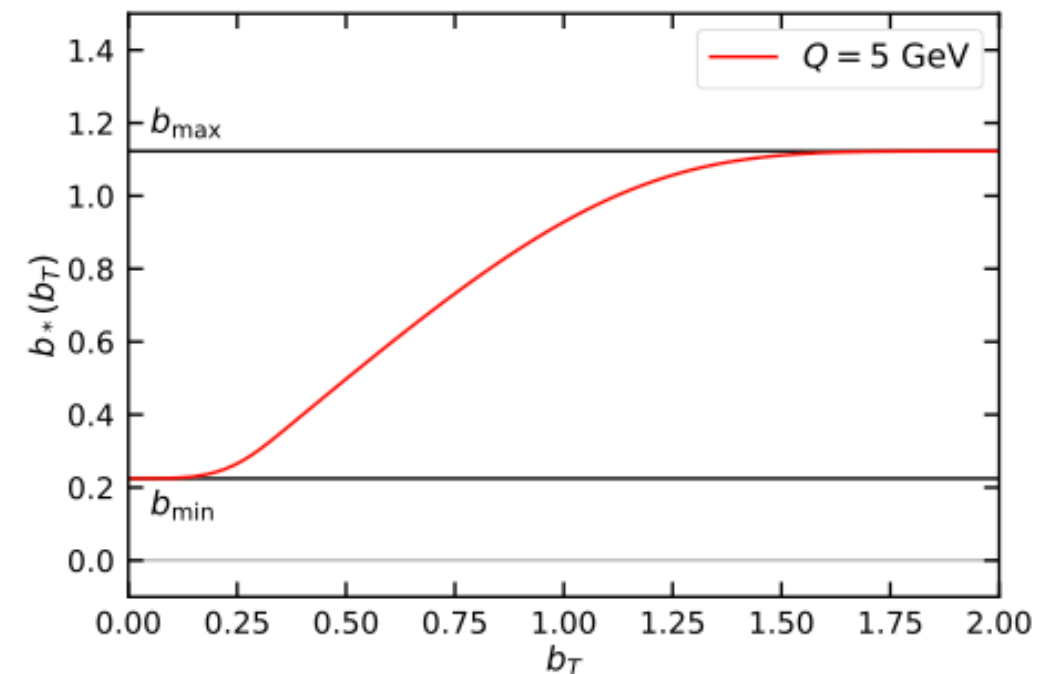
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$$\hat{f}_1(x, b_T^2; \mu, \zeta) = \left[\frac{\hat{f}_1(x, b_T^2; \mu, \zeta)}{\hat{f}_1(x, b_*(b_T^2); \mu, \zeta)} \right] \hat{f}_1(x, b_*(b_T^2); \mu, \zeta) \equiv \boxed{f_{\text{NP}}(x, b_T^2; \zeta)} \hat{f}_1(x, b_*(b_T^2); \mu, \zeta)$$

The complete expression of TMDs

TMD in Fourier space

$$\hat{F}(x, b_T^2; \mu, \zeta) = \int \frac{d^2 \mathbf{k}_\perp}{(2\pi)^2} e^{i \mathbf{b}_T \cdot \mathbf{k}_\perp} F(x, k_\perp^2; \mu, \zeta)$$

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Perturbative TMD at the initial scale

The complete expression of TMDs

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$$\times \exp \left\{ K(b_*; \mu_{b_*}) \ln \frac{\sqrt{\zeta}}{\mu_{b_*}} + \int_{\mu_{b_*}}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta}}{\mu'} \right] \right\} \quad : B$$

Evolution to final scale (of the process)

The complete expression of TMDs

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Evolution to final scale (of the process)

$$\times f_{NP}(x, b_T^2) \exp \left\{ g_K(b_T^2) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \right\} \quad : C$$

Non-perturbative part of the TMD

The complete expression of TMDs

TMD in Fourier space

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Non-perturbative part of the TMD

Parameterization

The complete expression of TMDs

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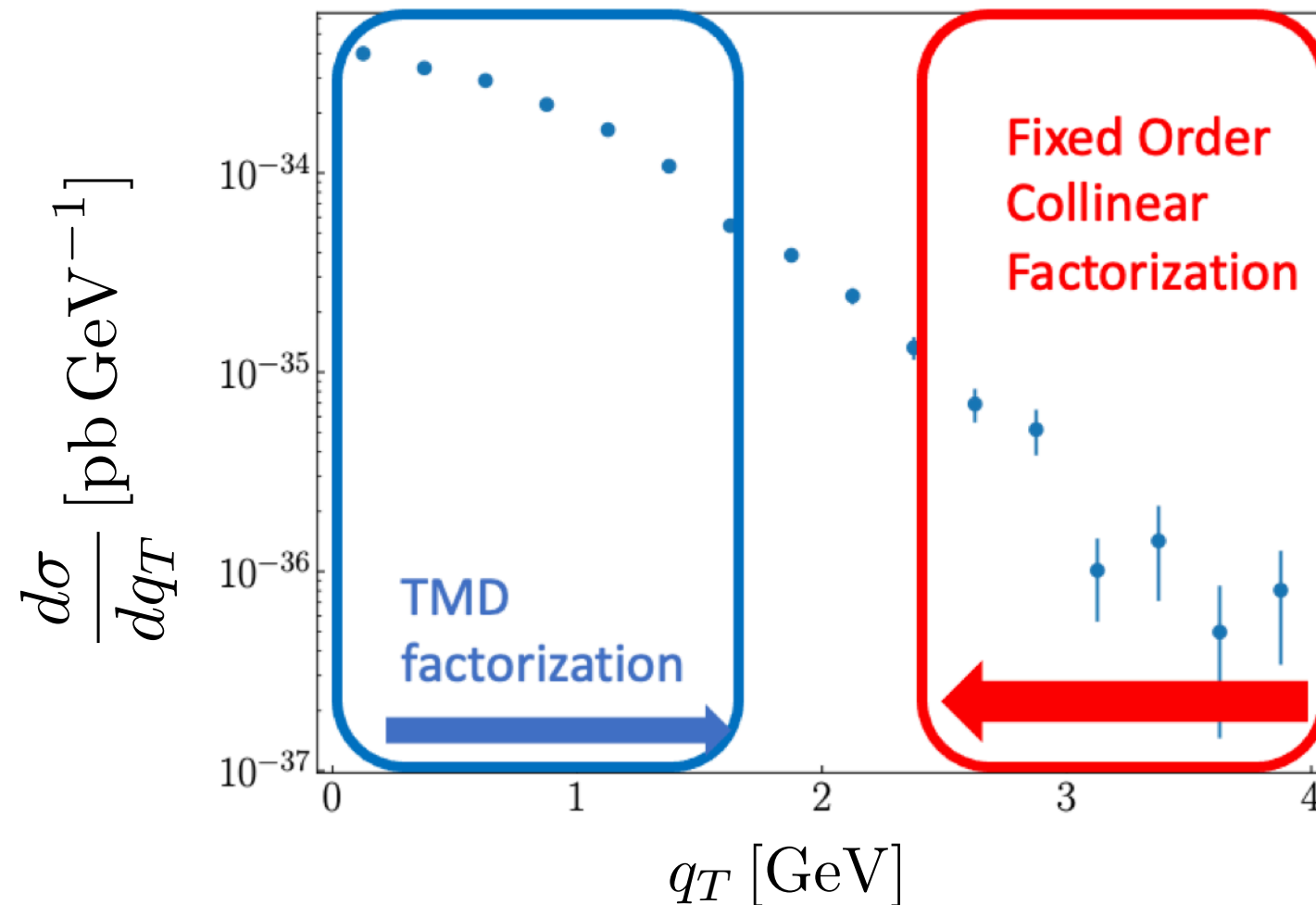
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Non-perturbative part of the TMD

Parameterization
GLOBAL FITs

How to get information on TMDs

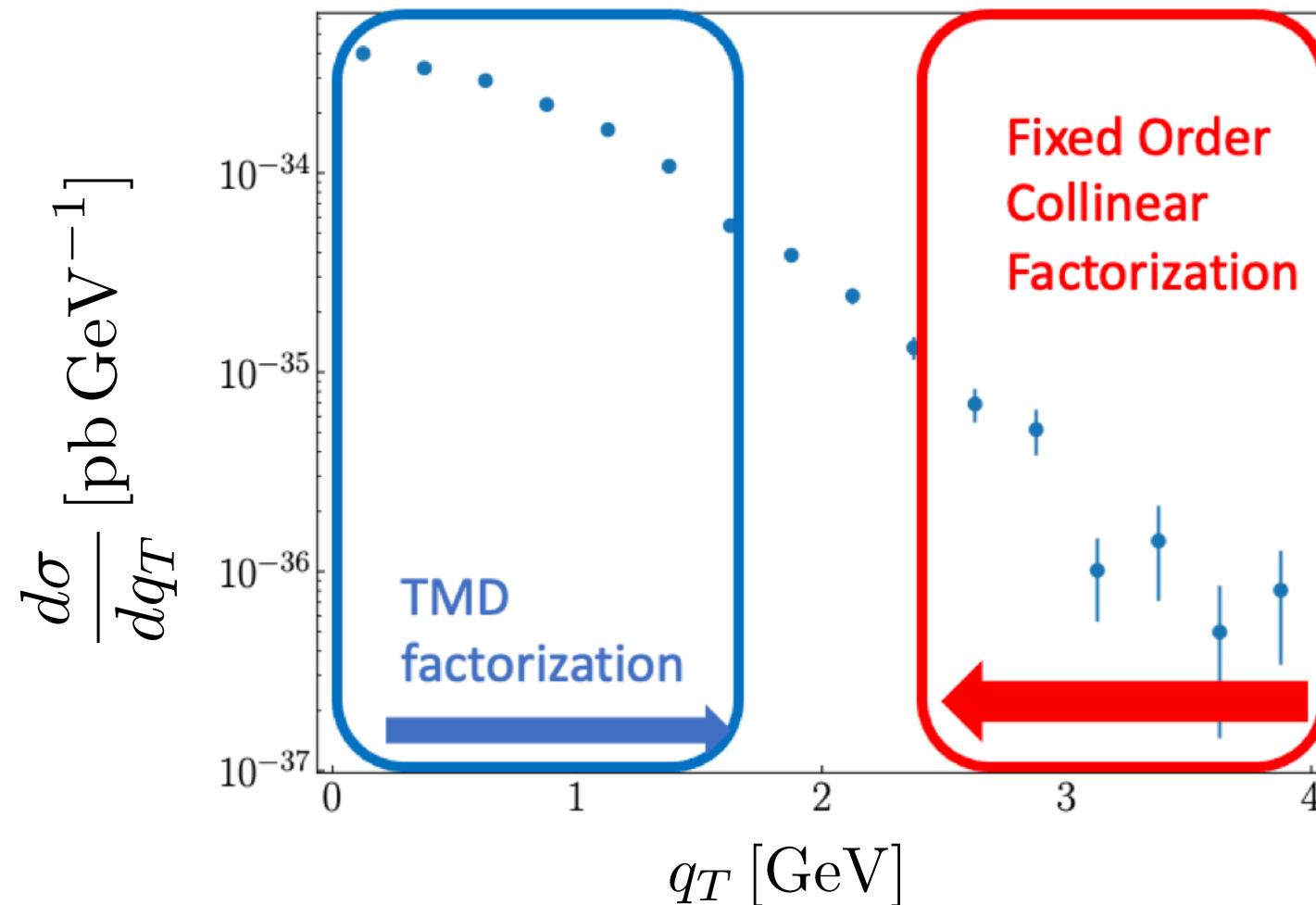
Cross section differential in transverse momentum



TMD factorisation: $Q^2 \gg M^2$

How to get information on TMDs

Cross section differential in transverse momentum



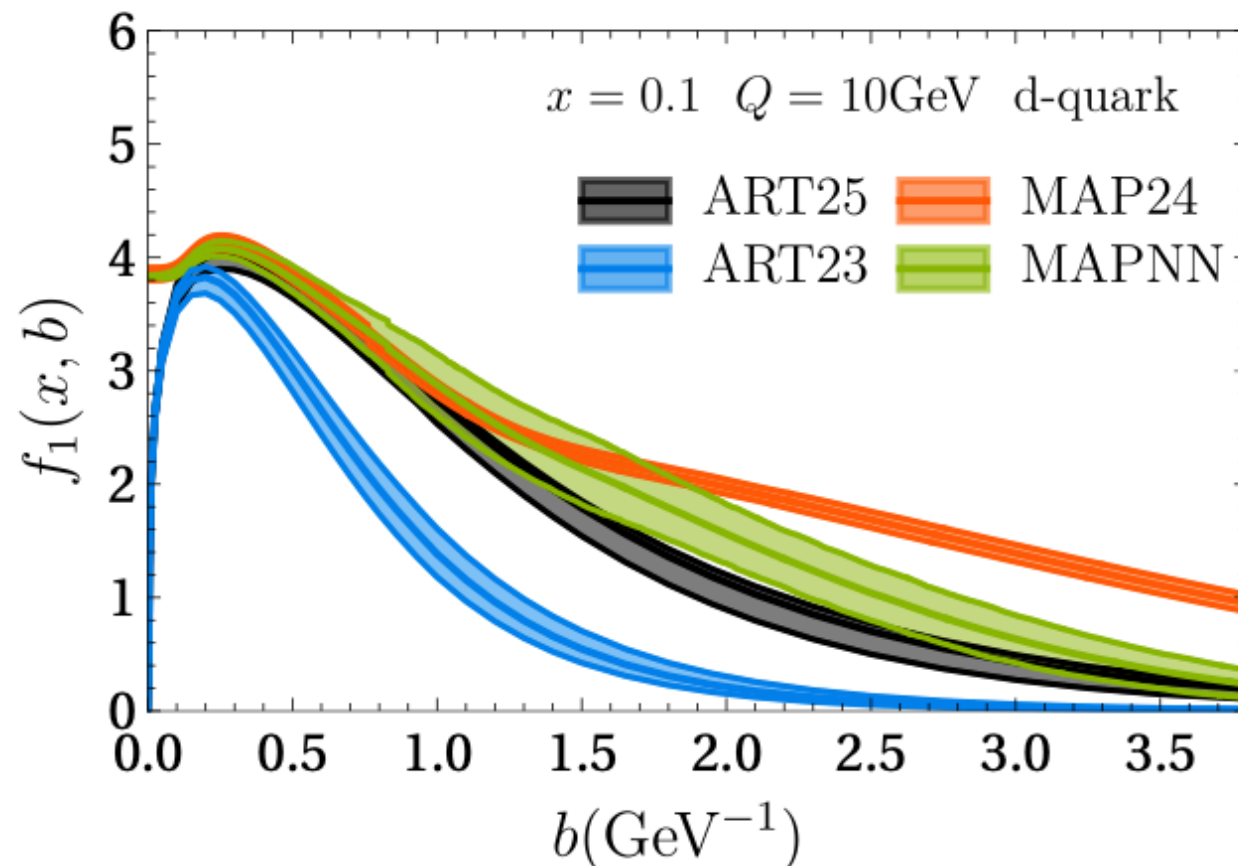
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 $q_T^2 \ll Q^2$

Available Global Fits

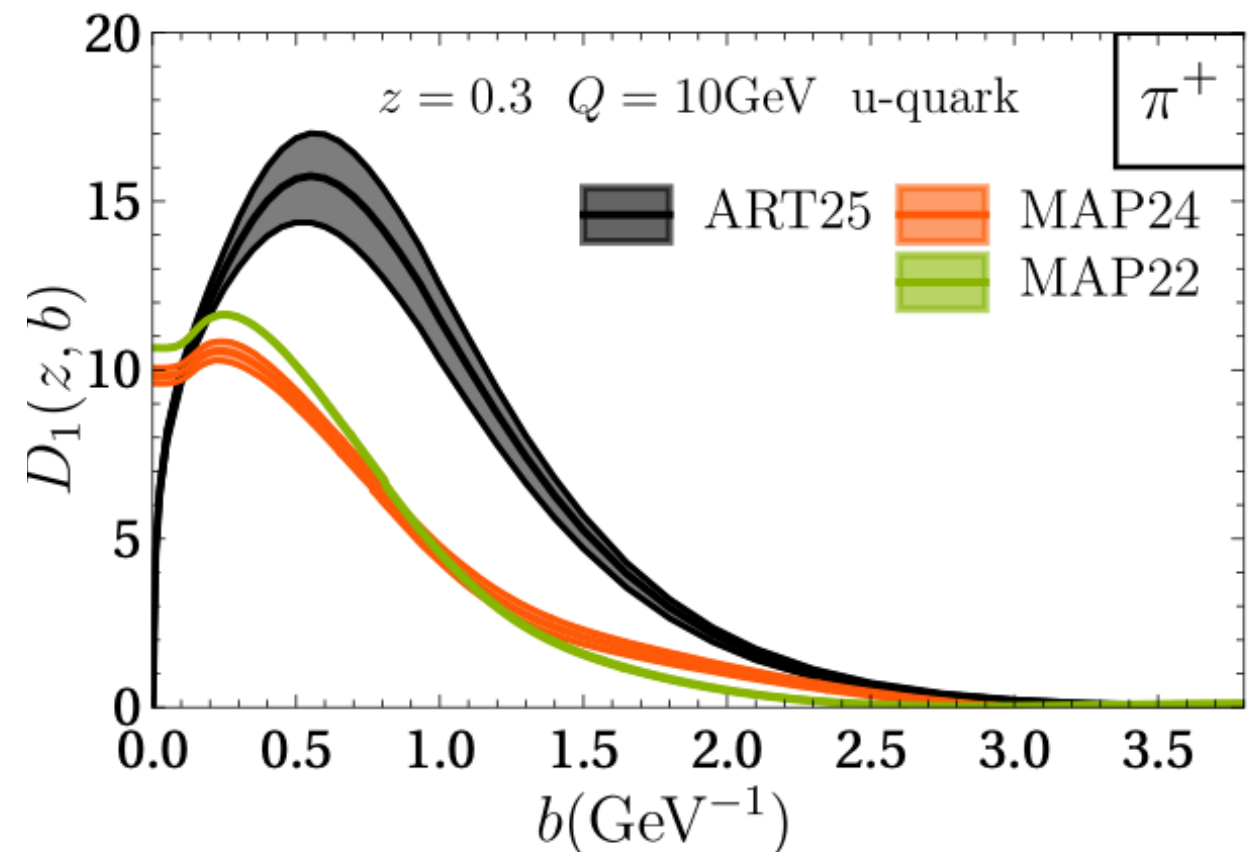
	<i>Accuracy</i>	<i>SIDIS</i>	<i>DY</i>	<i>N of points</i>	χ^2/N_{data}	<i>Flavor Dependence</i>
Pavia 2017 Bacchetta, Delcarro, et al., JHEP 06 (2017)	NLL	✓	✓	8059	1.55	✗
SV 2019 Scimemi, Vladimirov, JHEP 06 (2020)	N^3LL^-	✓	✓	1039	1.06	✗
MAPTMD22 Bacchetta, Bertone, et al., JHEP 10 (2022)	N^3LL^-	✓	✓	2031	1.06	✗
MAPTMD24 Bacchetta, Bertone, et al., JHEP 08 (2024)	N^3LL	✓	✓	2031	1.08	✓
ART25 Moos, Scimemi, et al., arXiv:2503.11201	N^3LL^+	✓	✓	1209	1.05	✓

Extracted TMDs

TMD PDFs



TMD FFs



Many extractions are present in the literature

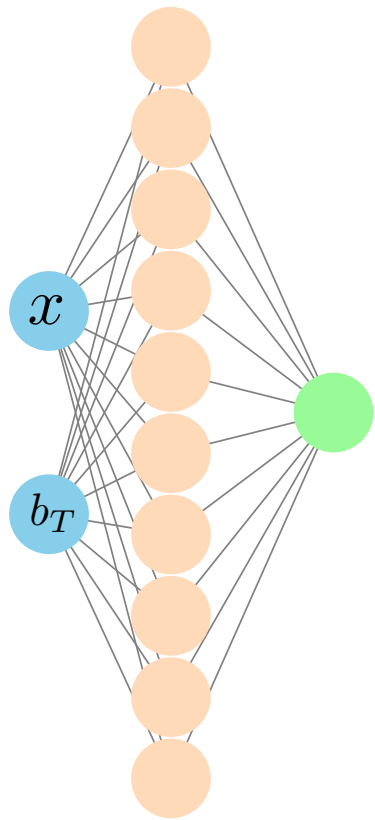
Incompatibilities can be ascribed to different phenomenological setup
(fitted dataset, parametrization bias, collinear distributions)

My talk at GdR QCD 2025

MAP Collaboration, PRL 135 (2025)

Neural-Network-based model $\Rightarrow$ reduction of the parametrisation bias

$$f_{\text{NP}}(x, b_T; \zeta) = \frac{\text{NN}(x, b_T, \{p_i\})}{\text{NN}(x, 0, \{p_i\})} \exp\left[-g_2^2 b_T^2 \ln\left(\frac{\zeta}{Q_0^2}\right)\right]$$



physically required constraints

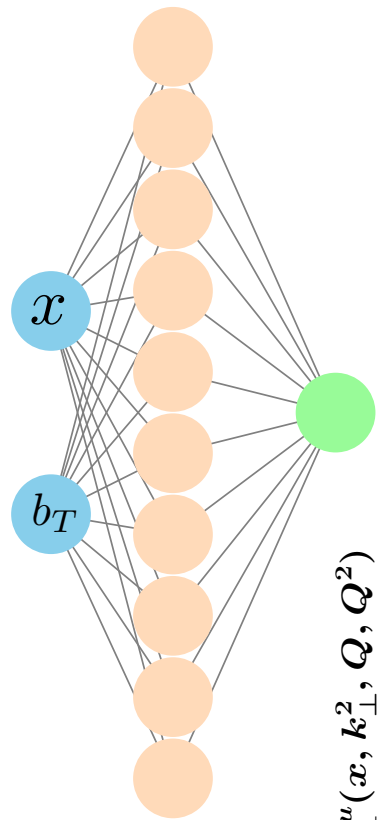
$$f_{\text{NP}} \rightarrow 1 \quad \text{for} \quad b_T \rightarrow 0$$

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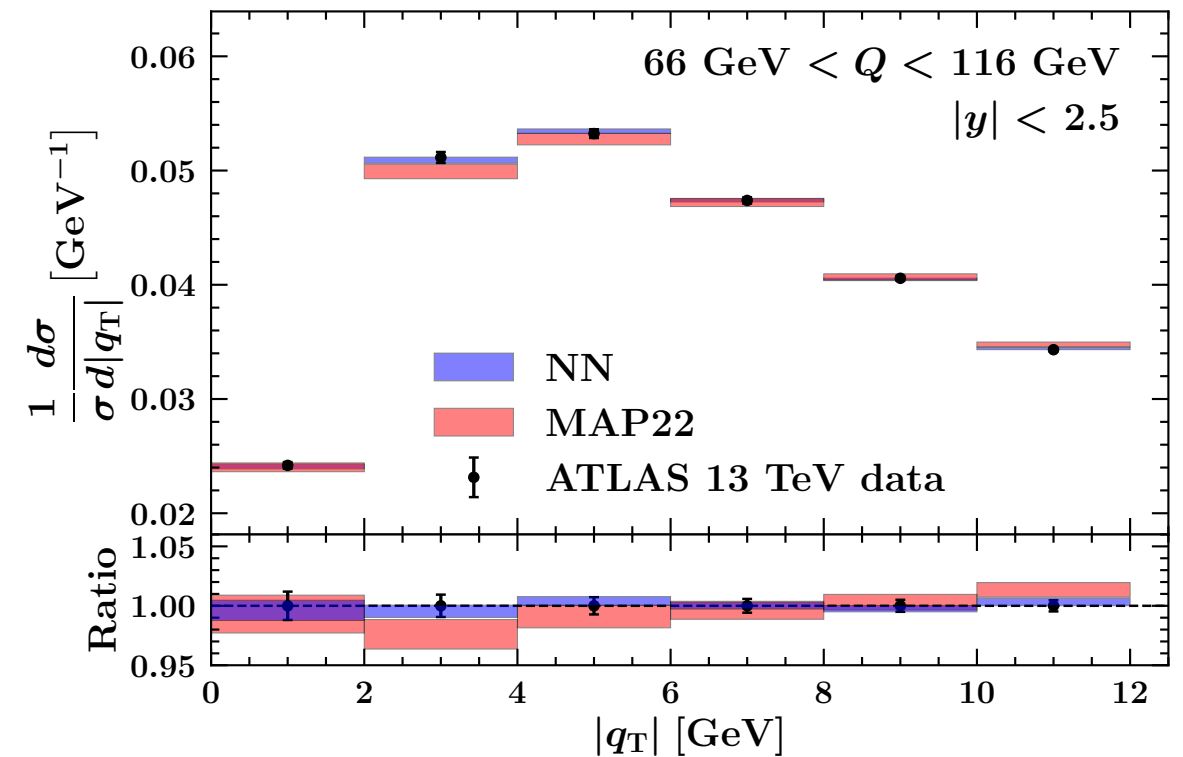
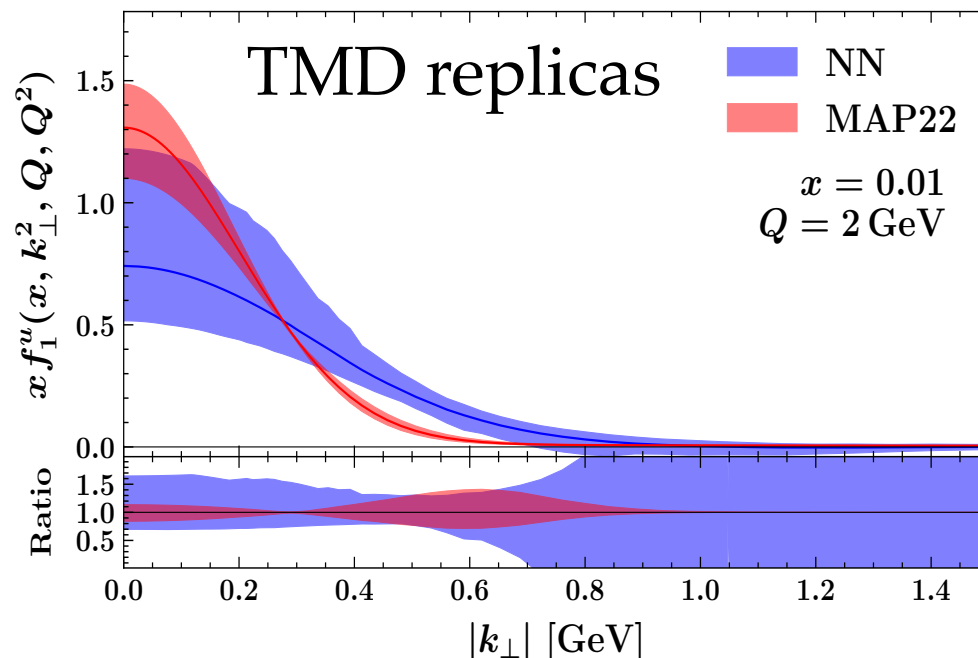
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Significant improvement
of data/theory agreement

Today: the Collins-Soper kernel

A. Vladimirov, PRL 125 (2020)

CSK: anomalous dimension of rapidity RGE

$$\frac{d \ln f(x, b_T, \mu, \zeta)}{d \ln \zeta} = K(b_T, \mu)$$

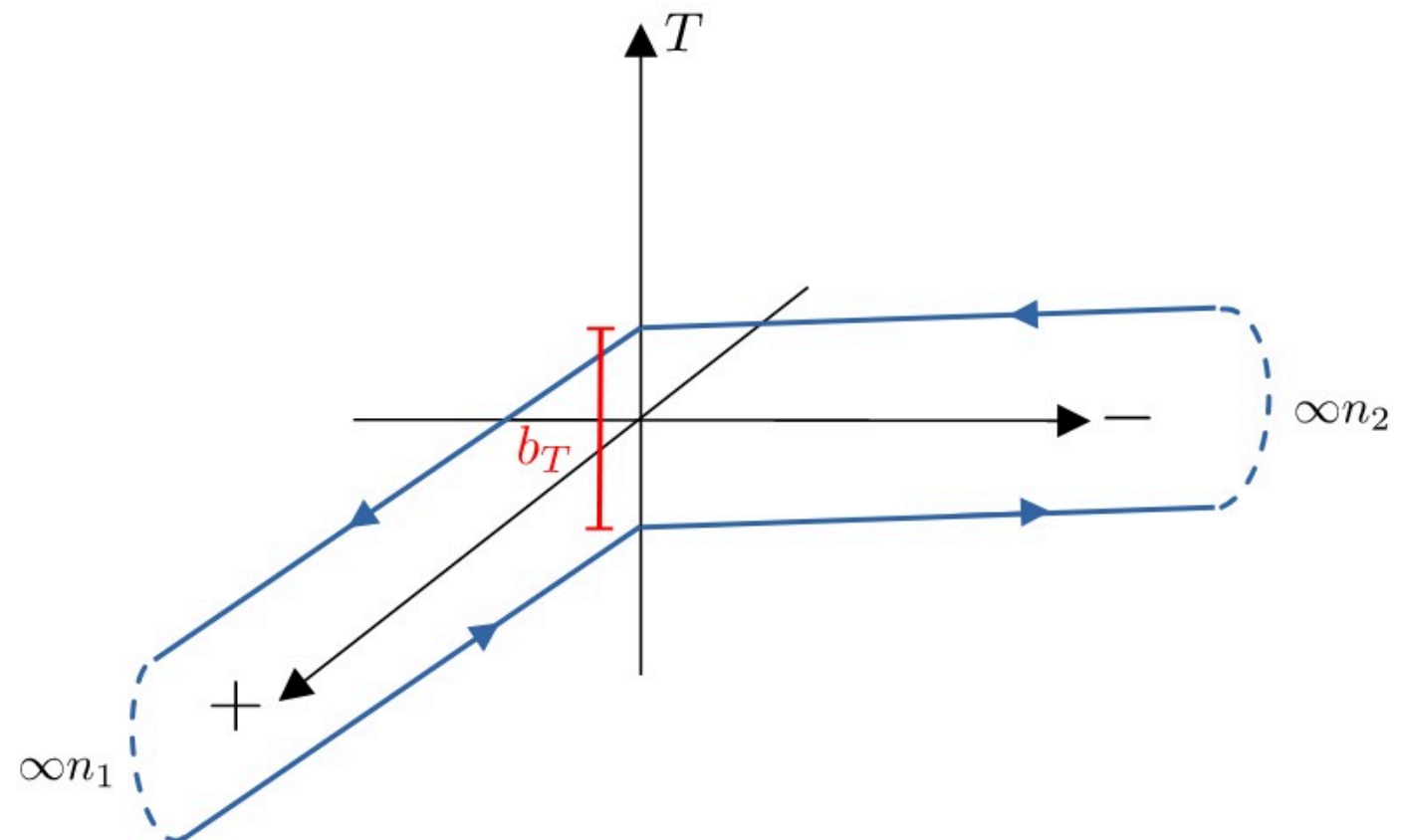
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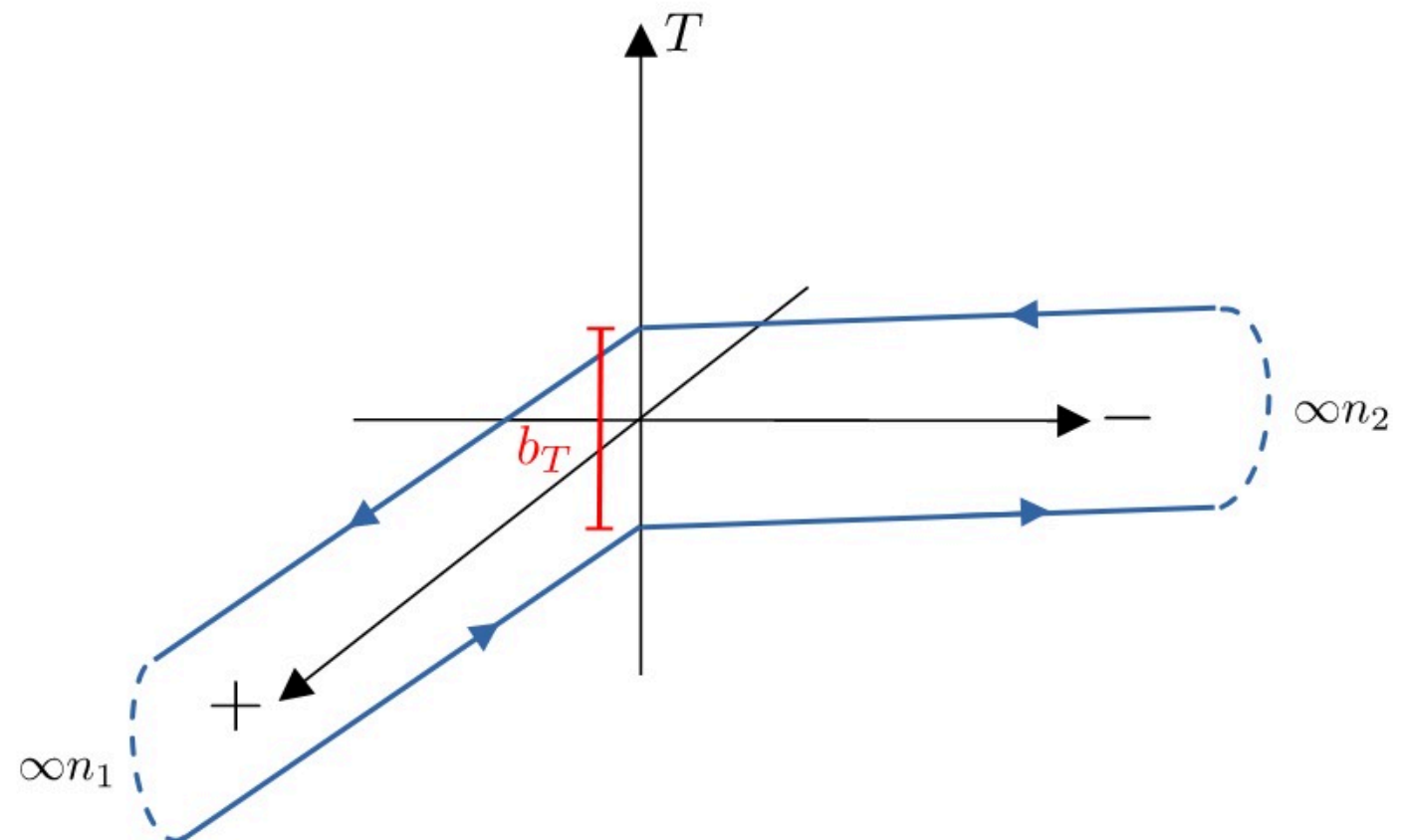
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It encodes info on QCD vacuum rather than on the actual structure of hadrons



CS kernel: TMD extractions

CSK in phenomenological analyses

$$\hat{f}_1^q(x, b_T^2; \mu, \zeta) = \sum_i (C_{qi} \otimes f_1^i)(x, b_*; \mu_{b_*}) e^{S_{\text{pert}}(b_*; \mu_{b_*}, \mu, \zeta)} e^{S_{\text{NP}}(b_T^2; \zeta)} \hat{f}_{NP}^q(x, b_T^2)$$

$$K(b_T, \mu) = \left[K(b_*, \mu_{b_*}) - \int_{\mu_{b_*}}^{\mu} \frac{d\mu'}{\mu'} \gamma_K(\alpha_s(\mu')) \right] g_K(b_T)$$

Perturbative

Nonperturbative

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Nonperturbative

There are no experimental data providing direct information on it

Limited range in Q of current experimental measurements (~10-100 GeV)

It is parametrised by very simple functions (1-2 parameters)

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$$K(b_T, \mu) = \left[K(b_*, \mu_{b_*}) - \int_{\mu_{b_*}}^{\mu} \frac{d\mu'}{\mu'} \gamma_K(\alpha_s(\mu')) \right] - g_K(b_T)$$

Perturbative

Nonperturbative

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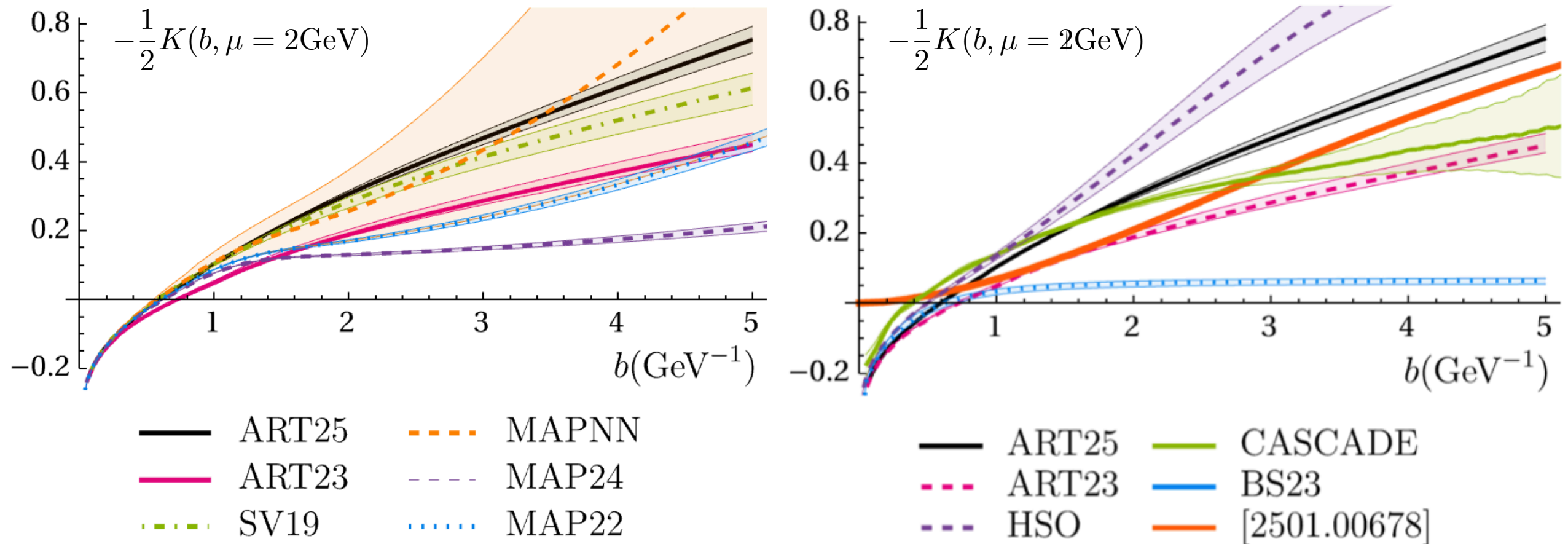
It is parametrised by very simple functions (1-2 parameters)

$$g_0 \left(1 - \exp \left[- \frac{C_F \alpha_s(\mu_{b_*}) b_T^2}{\pi g_0 b_{max}^2} \right] \right) \quad g_2 b_T^2 \quad b_T b_* (c_0 + c_1 \ln b_*)$$

CS kernel: TMD extractions

Moos, Scimemi, et al., JHEP 11 (2025)

CSK in phenomenological analyses



Many determinations with different formalisms

NN fit provides much larger error bands

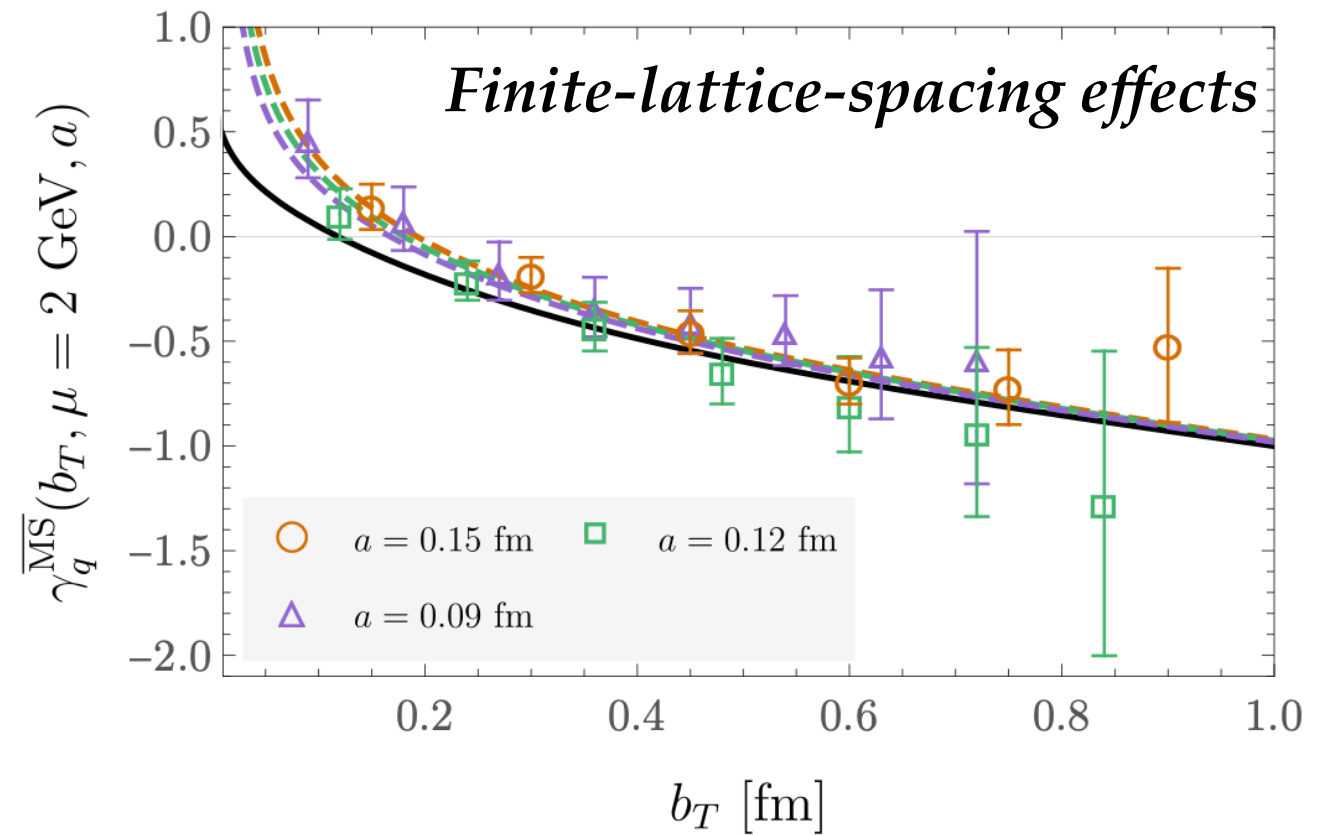
Discrepancies are a consequence of limited constraining power by exp. data

CS kernel: lattice QCD

Avkhadiev, Shanahan, Wagman, Zhao, PRL 132 (2024)

$$\begin{aligned} \hat{\gamma}_{\Gamma}^{\overline{\text{MS}}}(b_T, \mu, x, P_1^z, P_2^z, a) \\ = \frac{1}{\ln(P_1^z/P_2^z)} \ln \left[\frac{W_{\Gamma}^{\overline{\text{MS}}}(b_T, \mu, x, P_1^z, a)}{W_{\Gamma}^{\overline{\text{MS}}}(b_T, \mu, x, P_2^z, a)} \right] \\ + \delta\gamma_q^{\overline{\text{MS}}}(\mu, x, P_1^z, P_2^z). \end{aligned}$$

A “clean” TMD-related quantity
that can be meaningfully
calculated on the lattice



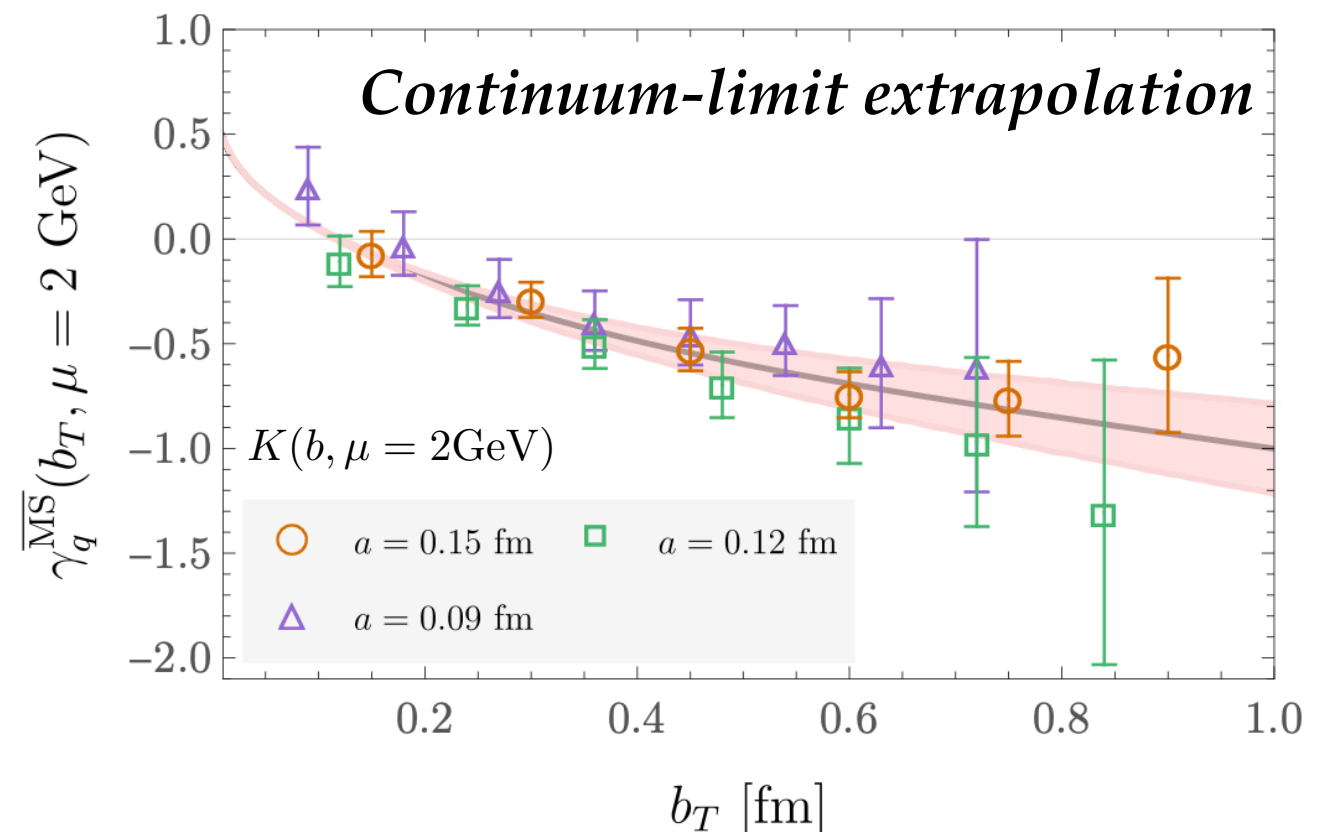
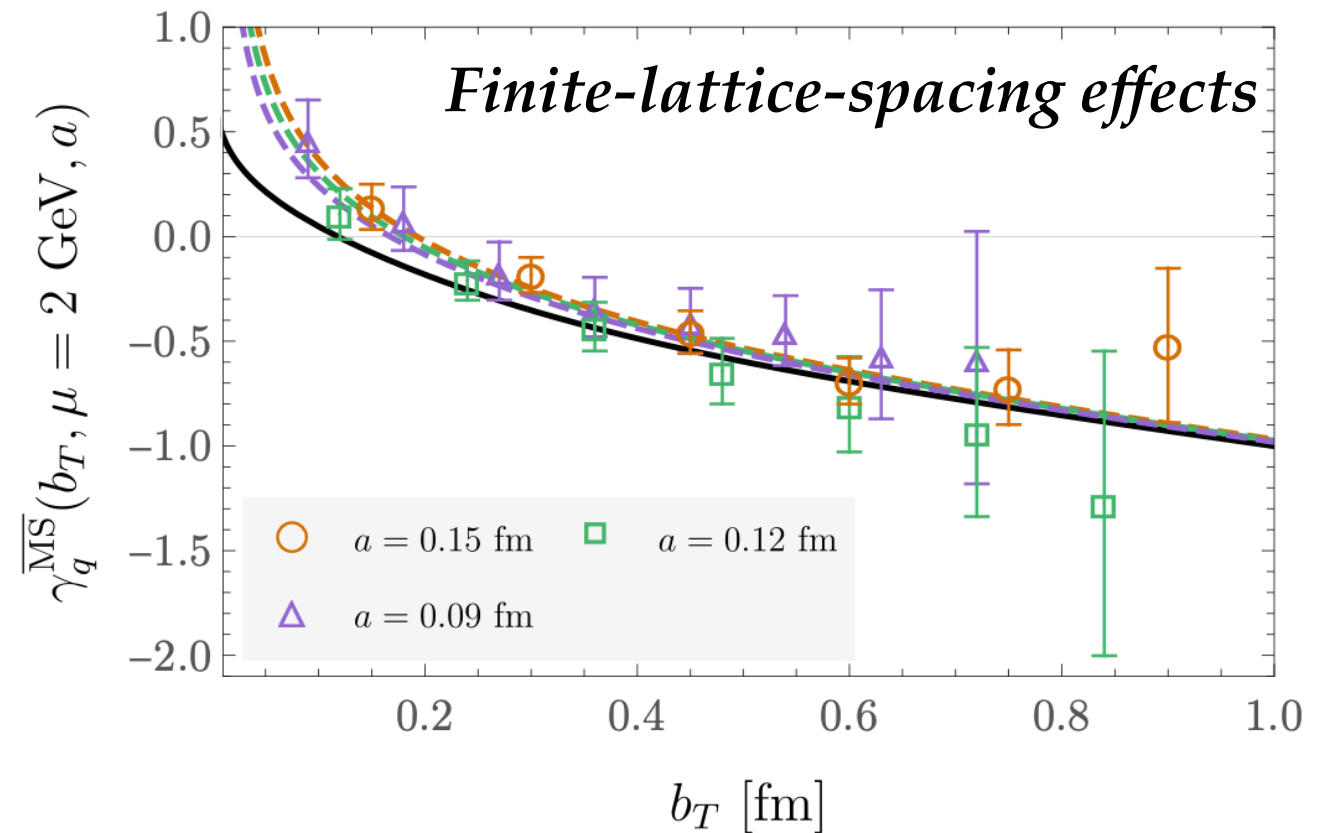
CS kernel: lattice QCD

Avkhadiev, Shanahan, Wagman, Zhao, PRL 132 (2024)

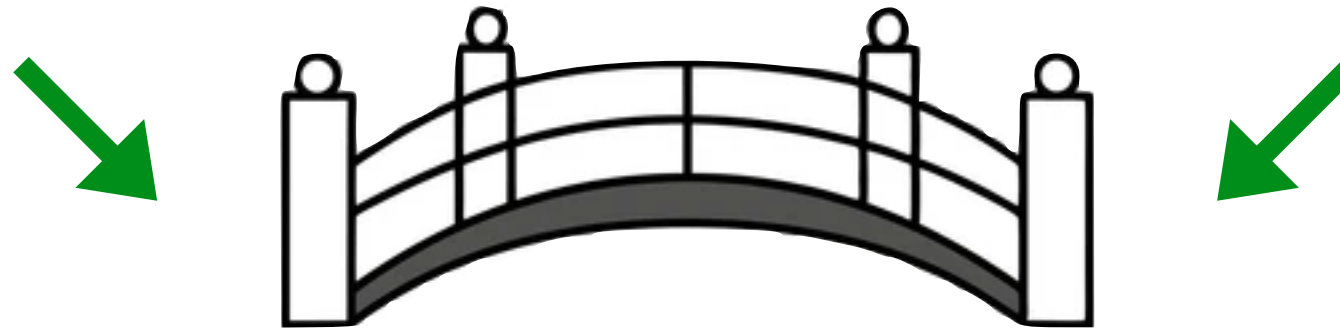
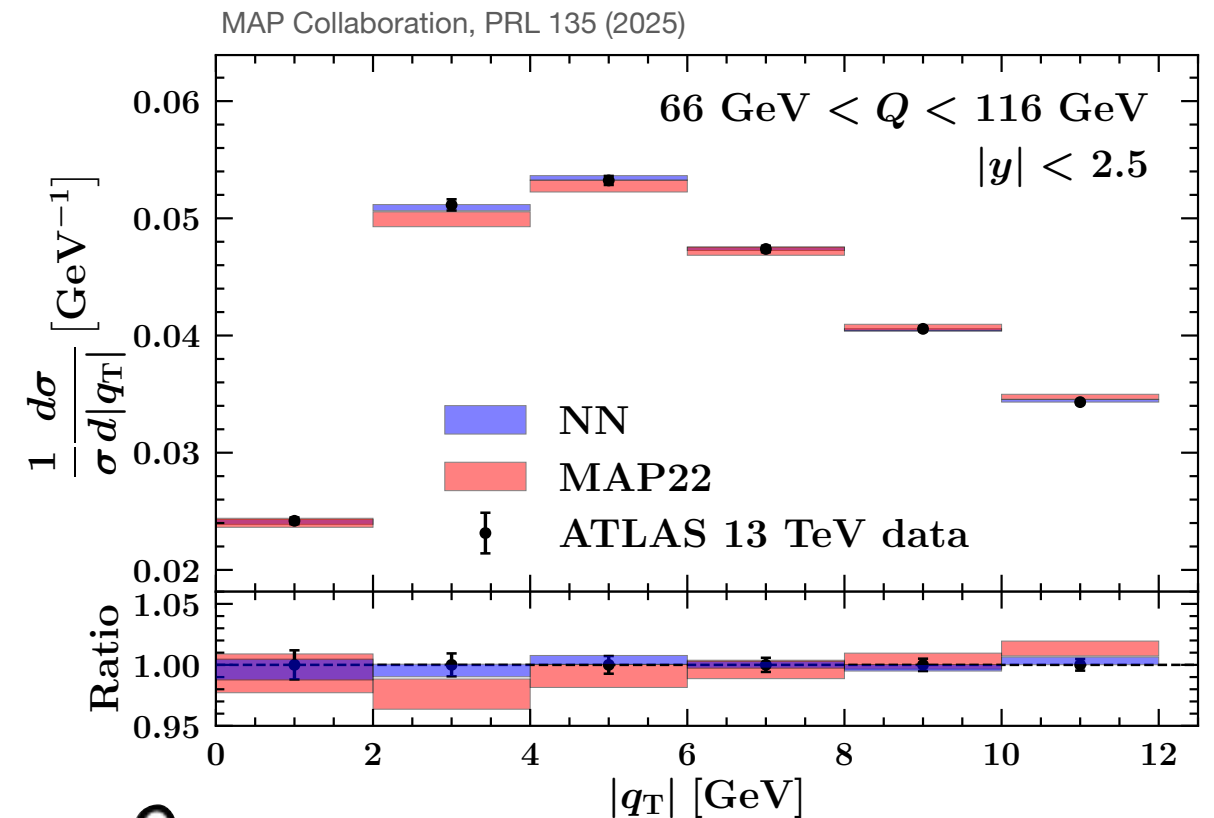
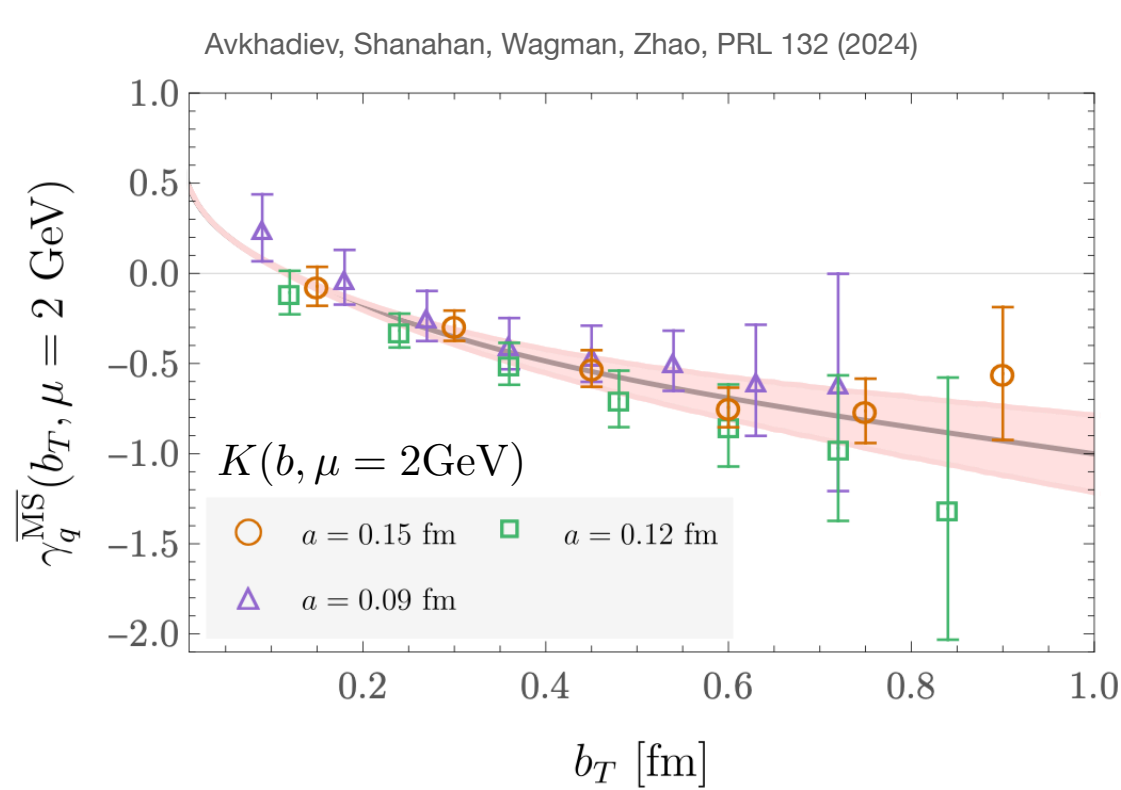
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A “clean” TMD-related quantity
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For the first time, control has been
achieved for quark mass, operator
renormalization and discretization
effects!!



Combining lattice QCD and exp. data



Avkhadiev, Bertone, Bissolotti, MC, et al., PRL 136 (2026)

Two strategies to combine the information:

1- Reweighting

2- Global fit experimental data + lattice QCD

Combining lattice QCD and exp. data

Avkhadiev, Bertone, Bissolotti, MC, et al., PRL 136 (2026)

1- Reweighting

Combining lattice QCD and exp. data

Avkhadiev, Bertone, Bissolotti, MC, et al., PRL 136 (2026)

1- Reweighting

Continuum-Extrapolated data

$$\chi_{\alpha}^2 = \sum_{j=1}^n \left(\frac{K_j - K^{(\alpha)}(b_j)}{\sigma_j} \right)^2$$

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Avkhadiev, Bertone, Bissolotti, MC, et al., PRL 136 (2026)

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Finite-lattice-spacing data

$$\begin{aligned} \chi_{\alpha}^2 &= \sum_{i=1}^3 \sum_{j_i, l_i=1}^{n_i} \left(K_{j_i} - \overline{K}^{(\alpha)}(b_{j_i}) \right) \\ &\times (\Sigma_i^{-1})_{j_i, l_i} \left(K_{l_i} - \overline{K}^{(\alpha)}(b_{l_i}) \right) \end{aligned}$$

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$$w_{\alpha} = \mathcal{N} \left(\chi_{\alpha}^2 \right)^{n/2-1} e^{-\chi_{\alpha}^2/2}$$

NNPDF collab., NP B 855 (2012)

$$\langle g_2 \rangle = \sum_{\alpha=1}^N w_{\alpha} g_2^{(\alpha)}, \quad \sigma_{g_2}^2 = \sum_{\alpha=1}^N w_{\alpha} (g_2^{(\alpha)} - \langle g_2 \rangle)^2$$

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$$g_2^{\text{Baseline}} = 0.186 \pm 0.033$$

$$g_2^{\text{Rew-cont}} = 0.164 \pm 0.020$$

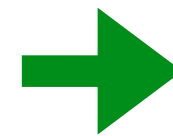
$$g_2^{\text{Rew-fin}} = 0.165 \pm 0.020$$

Combining lattice QCD and exp. data

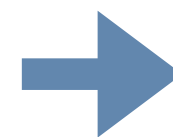
Avkhadiev, Bertone, Bissolotti, MC, et al., PRL 136 (2026)

2- Global fit experimental data + lattice

Experiment	N_{dat}	$\langle\chi^2\rangle$		
		Baseline	Reweighting	Fit
Fixed-target	233	239.67	234.09	242.29
RHIC	7	7.06	7.11	7.62
Tevatron	71	61.33	59.91	65.31
LHCb	21	22.94	21.88	22.77
CMS	78	30.74	29.85	29.48
ATLAS	72	95.35	95.60	95.71
Lattice a_1	6	<i>31.13</i>	11.5	12.56
Lattice a_2	7	<i>6.30</i>	4.67	1.77
Lattice a_3	8	<i>8.50</i>	3.88	6.01



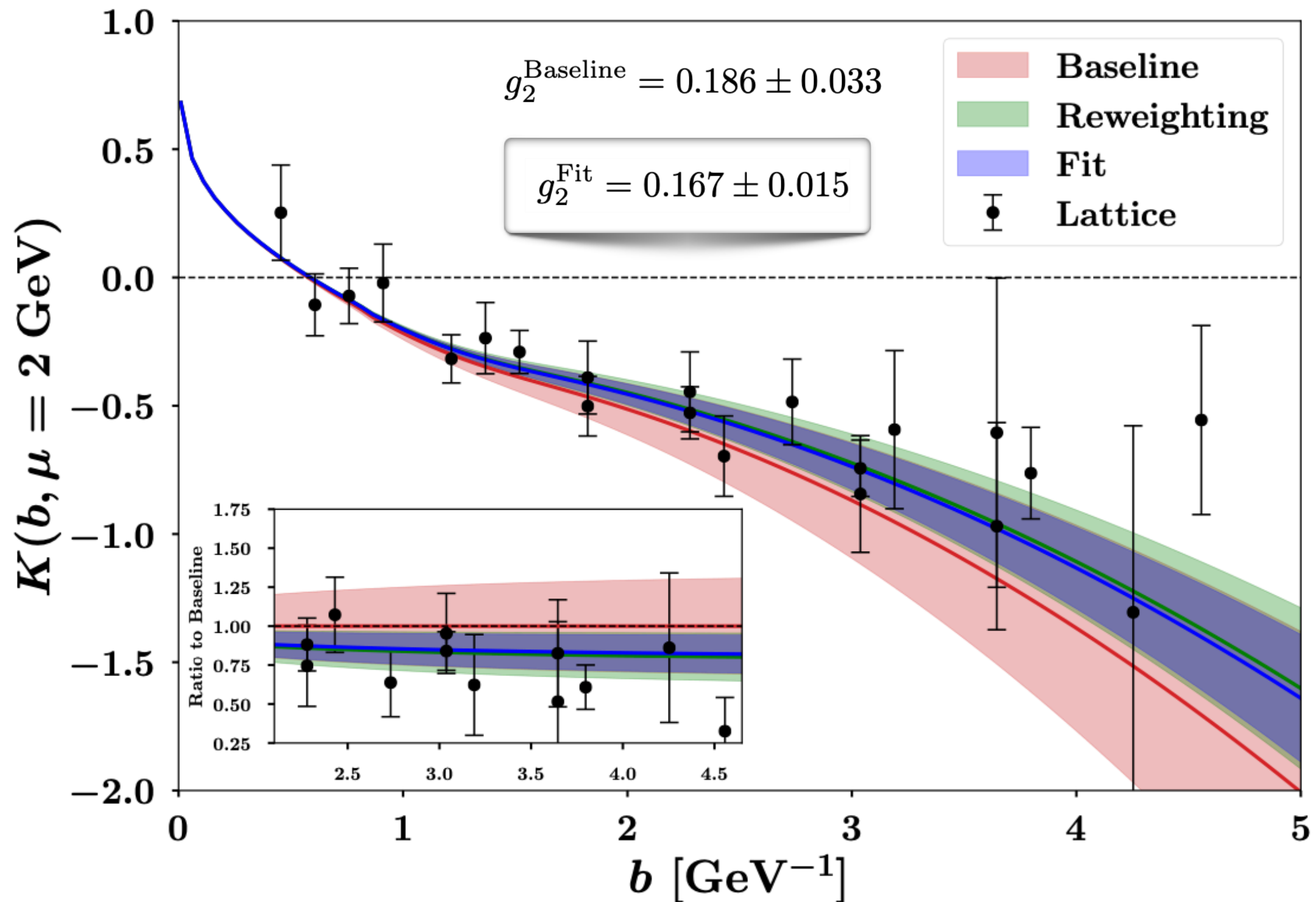
The combined fit
is stable



Improved
agreement with
lattice data

Combining lattice QCD and exp. data

Avkhadiev, Bertone, Bissolotti, MC, et al., PRL 136 (2026)



Conclusions

- ★ CSK is a fundamental quantity in QCD
 - It can be extracted by TMD fits, but exp. data provide limited constraints
 - It can be calculated on the lattice, but systematics must be under control
- ★ We performed the first extraction of the CSK combining exp. data and lattice calculation in the same framework
- ★ We found a significantly consistent improvement using two different methodologies
- ★ This shows the potential of lattice QCD inputs in TMD fits

Backup

Theory/data agreement

Decomposition of the χ^2

experimental data

$$m_i \pm \sigma_{i,\text{unc}} \pm \sigma_{i,\text{corr}}^{(1)} \pm \cdots \pm \sigma_{i,\text{corr}}^{(k)}$$

uncorrelated

correlated

$$\chi^2 = \sum_{i=1}^n \left(\frac{m_i - \bar{t}_i}{s_i} \right)^2 + \sum_{\alpha=1}^k \lambda_{\alpha}^2$$

uncorrelated contribution

penalty term

nuisance parameters $\frac{\partial \chi^2}{\partial \lambda_{\alpha}} = 0$

$$\bar{t}_i = t_i + d_i$$

shifted prediction

shift

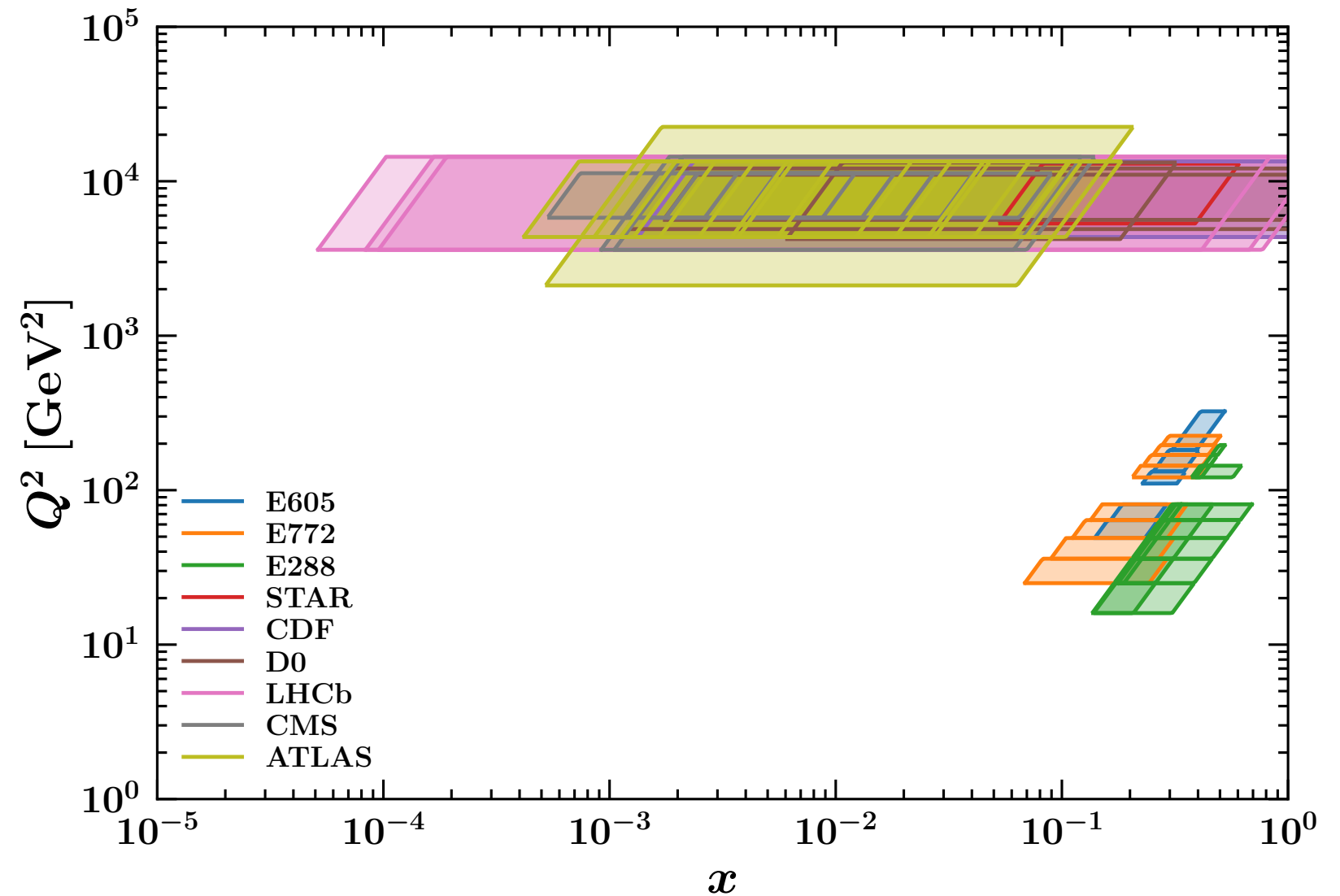
$$d_i = \sum_{\alpha=1}^k \lambda_{\alpha} \sigma_{i,\text{corr}}^{(\alpha)}$$

Included data sets

Drell-Yan data

Fixed-target:
E288, E605, E772

Collider mode:
RHIC, Tevatron, LHC



Included data sets

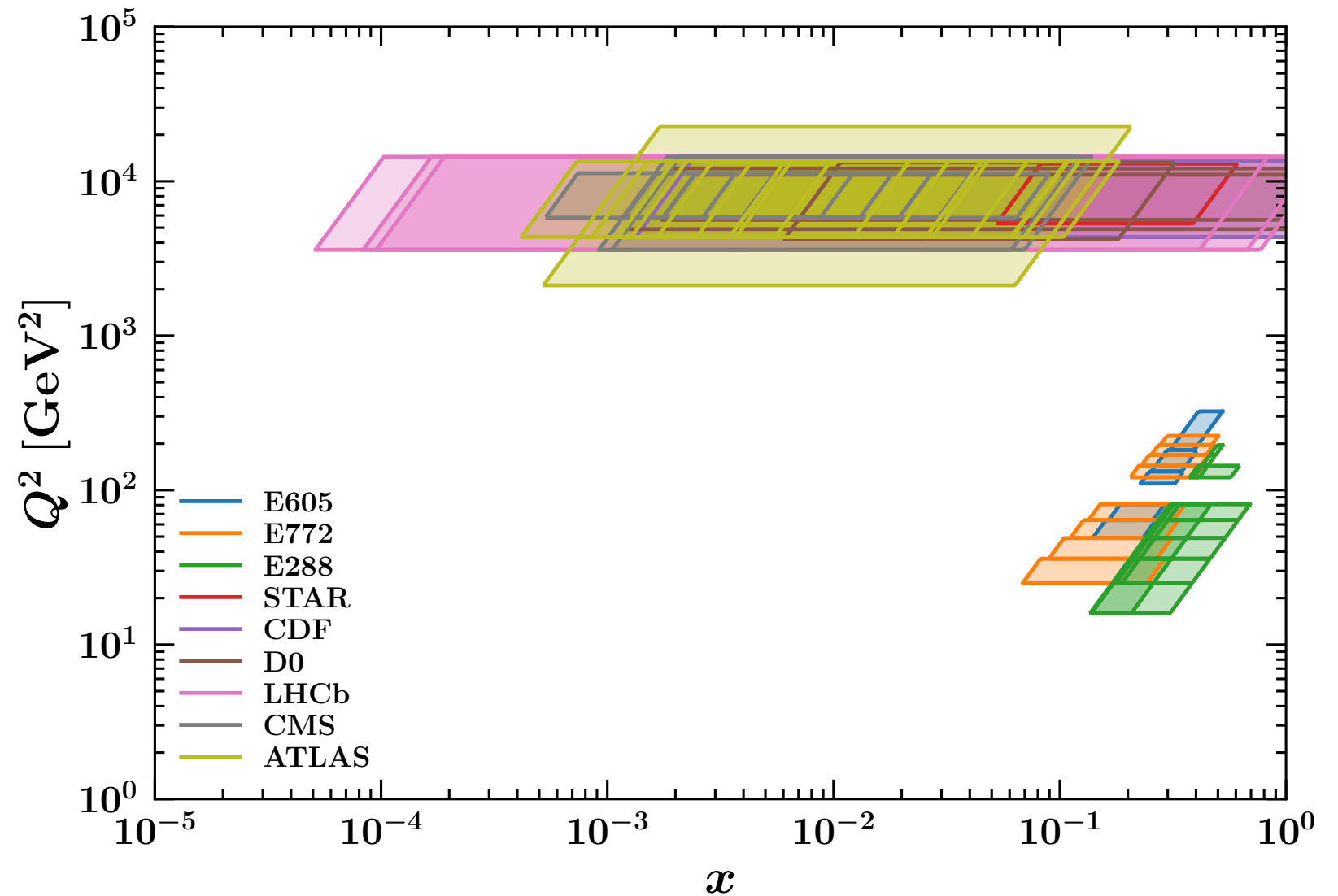
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Theory cut

$$q_T/Q < 0.2$$



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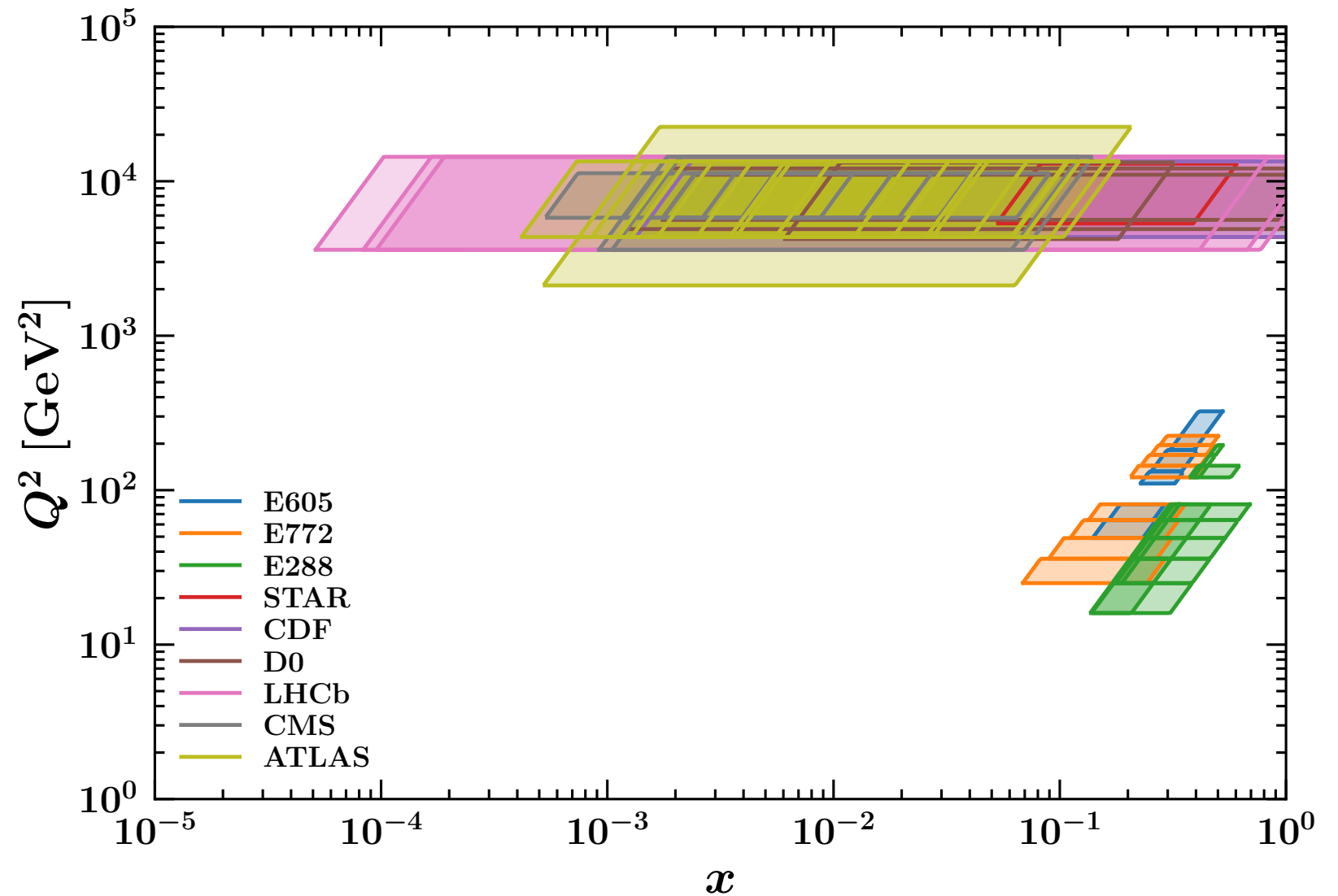
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Total number of data: 482

Results of the analysis

Data / Theory agreement

$$\chi^2 = \sum_{i=1}^n \left(\frac{m_i - \bar{t}_i}{s_i} \right)^2 + \sum_{\alpha=1}^k \lambda_{\alpha}^2$$

Experiment	N_{dat}	$\bar{\chi}^2 \ (\bar{\chi}_D^2 + \bar{\chi}_{\lambda}^2)$	
		NN	MAP22
Fixed-target	233	1.08 (0.98 + 0.10)	0.91 (0.70 + 0.21)
RHIC	7	1.11 (1.03 + 0.07)	1.45 (1.37 + 0.08)
Tevatron	71	0.80 (0.73 + 0.06)	1.20 (1.17 + 0.04)
LHCb	21	0.98 (0.88 + 0.10)	1.25 (1.05 + 0.20)
CMS	78	0.40 (0.38 + 0.02)	0.41 (0.35 + 0.06)
ATLAS	72	1.38 (1.09 + 0.29)	3.51 (3.03 + 0.49)
Total	482	0.97 (0.86 + 0.11)	1.28 (1.09 + 0.20)

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NN provide better quality than MAP22

Results of the analysis

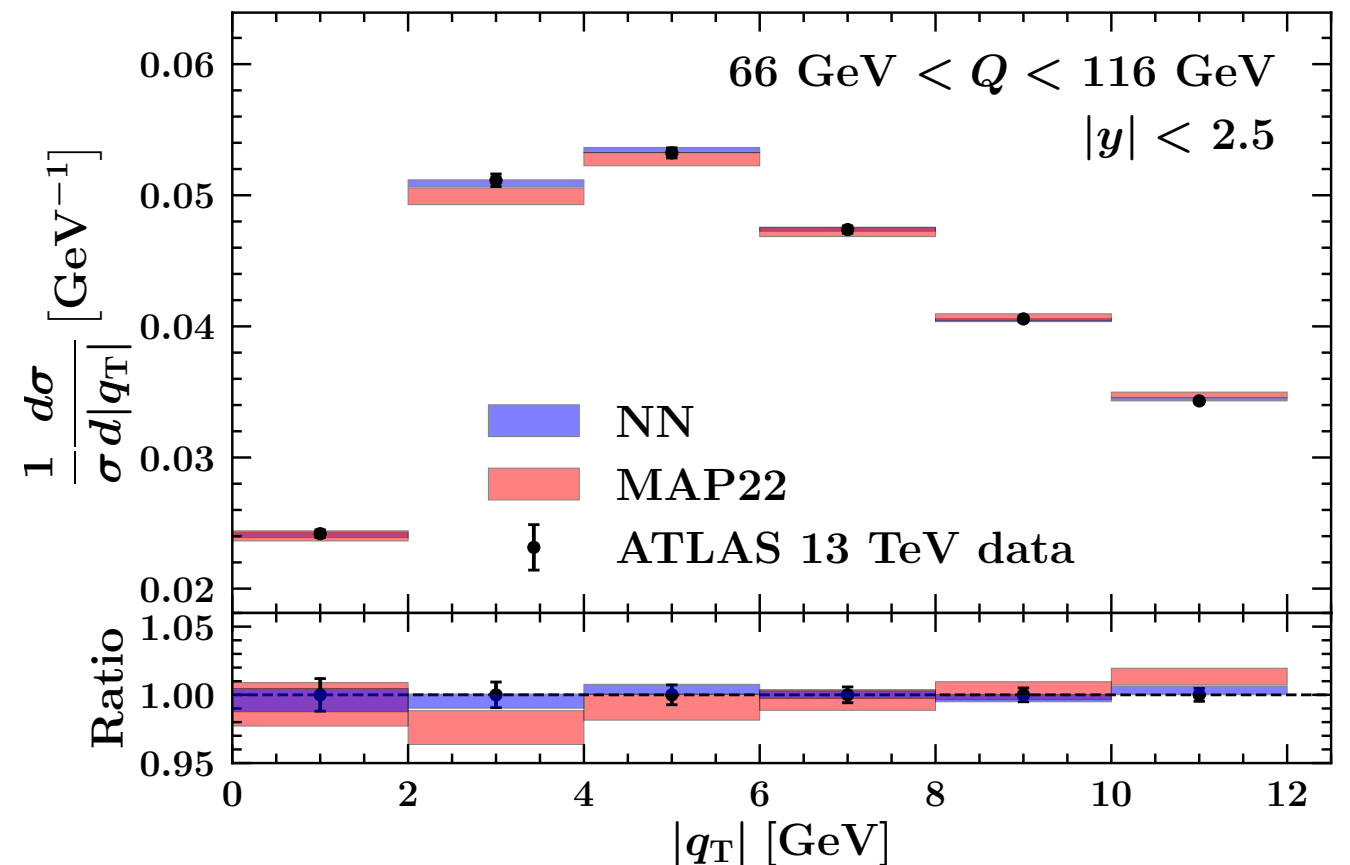
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NN provide better quality than MAP22

Significant improvement for ATLAS data

$$\chi^2 = \sum_{i=1}^n \left(\frac{m_i - \bar{t}_i}{s_i} \right)^2 + \sum_{\alpha=1}^k \lambda_\alpha^2$$

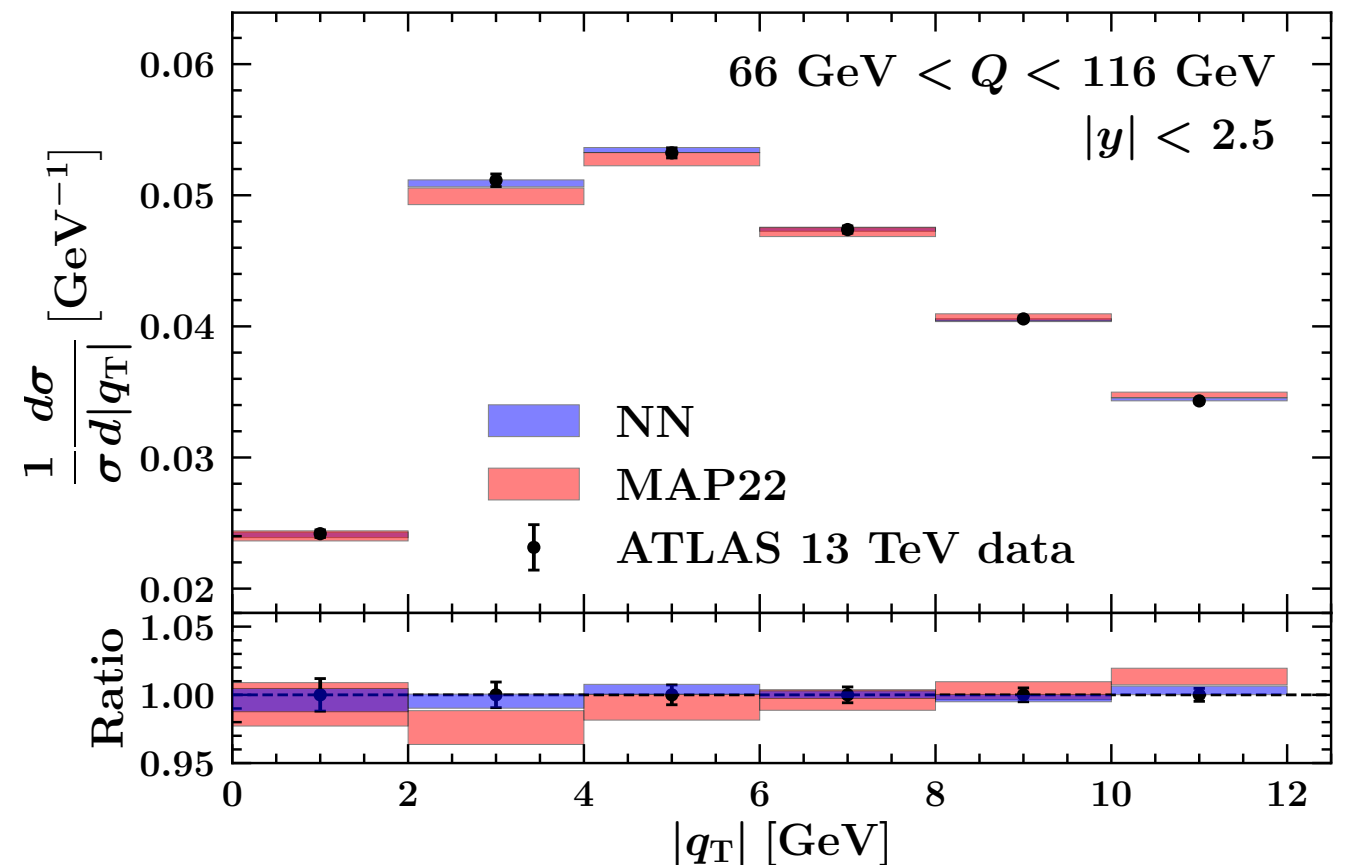


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NN provide better quality than MAP22

Significant improvement for ATLAS data

MAP22 relies more on systematic shifts $\Rightarrow$ large error bands