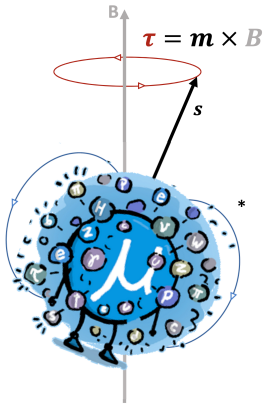
Probing the Standard Model to eleven digits with the muon magnetic moment

Laurent Lellouch

CNRS & Aix-Marseille U.

[Budapest-Marseille-Wuppertal [BMW] + Davier-Malaescu-Zhang [DMZ] collaboration]



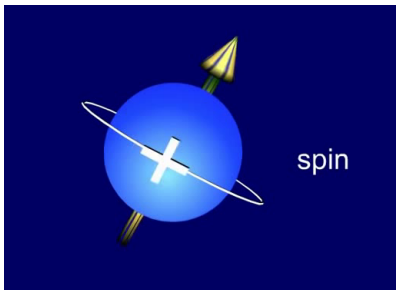
© Jorge Cham & David Tarazona

2407.10913 → This work
Nature 593 (2021) → BMW '20
PRL 121 (2018) 022002 (Editors' Selection) → BMW '17
Aoyama et al., Phys. Rep. 887 (2020) 1-166 → WP '20
Aliberti et al., 2505.21476 → WP '25

Charged lepton magnetic moments

Muons are tiny magnets

A massive elementary particle w/ electric charge and spin behaves like a tiny magnet



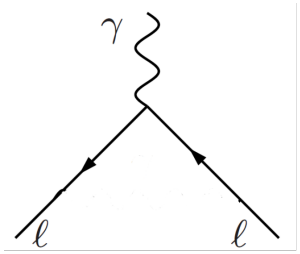
[← Silver Swan]

Magnetic moment of charged leptons

$$\vec{\mu}_\ell = \pm g_\ell \frac{e}{2m_\ell} \vec{S}$$

g_ℓ = Landé factor

- 1928 : Dirac's new theory predicts the existence of the positron and

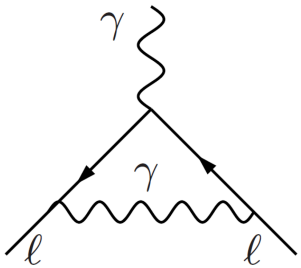


$$g_e|_{\text{Dirac}} = 2$$

"That was really an unexpected bonus for me" (P.A.M. Dirac)



- 1947 : Schwinger understands that same year observed $g_e \neq 2$ comes from Dirac's neglect ofs **quantum fluctuations** and computes them to obtain the **"anomalous"** contribution

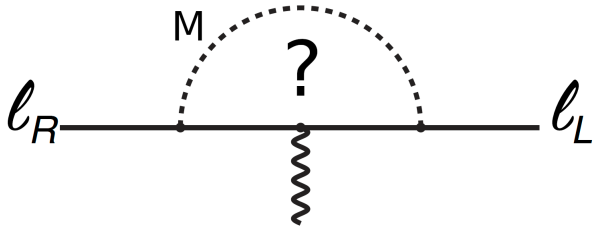


$$a_e = \frac{g_e - 2}{2} = \frac{\alpha}{2\pi} = 0.00116\ldots$$



→ birth of QED and relativistic quantum field theory

Why are a_ℓ special?



$$\longrightarrow \qquad \mathcal{L}_{\text{eff}} = -\frac{Qe}{2}\frac{a_\ell}{2m_\ell} F^{\mu\nu}[\bar{\ell}_L \sigma_{\mu\nu} \ell_R] + \text{hc}$$

- $a_{e,\mu}$ are rigorous predictions of the SM that can be measured very precisely $\Rightarrow$ excellent tests of SM
- Loop induced $\Rightarrow$ sensitive to new dofs that may be too heavy or too weakly coupled to be produced directly
- Flavor and CP conserving, chirality flipping $\Rightarrow$ probes mass generating mechanism and complementary to: EDMs, FCNCs (e.g. s and b decays), LHC direct searches, ...
- Chirality flipping $\Rightarrow$ generic contribution of particle w/ $M \gg m_\ell$

$$a_\ell^{\text{M}} = C\left(\frac{\Delta_{LR}}{m_\ell}\right)\left(\frac{m_\ell}{M}\right)^2$$

- In EW theory, $M = M_W$, chirality flipping from Yukawa, i.e.

$$\Delta_{LR} = m_\ell \qquad \text{and} \qquad C \sim \frac{\alpha}{4\pi \sin^2\theta_W}$$

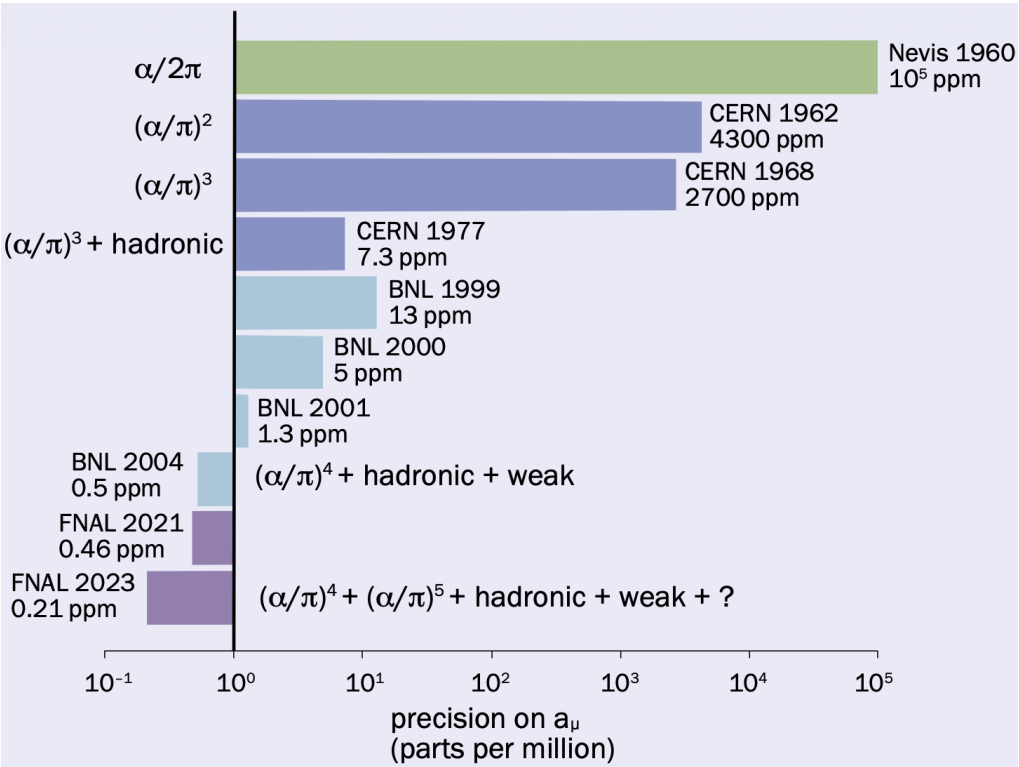
- In BSM, can have enhancements: e.g. SUSY $M = M_{\text{SUSY}}$ and $C \sim \alpha/(4\pi \sin^2\theta_W)$ & $\Delta_{LR} = (\mu/M_{\text{SUSY}}) \times \tan\beta \times m_\ell$; or radiative m_ℓ model, $\Delta_{LR} \simeq m_\ell$, $C \sim 1$ and $M = M_{\text{N}\Phi}$

Why is a_μ special?

$m_e : m_\mu : m_\tau = 0.0005 : 0.106 : 1.777\,\text{GeV}$ $\quad$ $\tau_e : \tau_\mu : \tau_\tau = \text{“}\infty\text{”} : 2.\cdot 10^{-6} : 3.\cdot 10^{-13}\,\text{s}$

- a_μ is $(m_\mu/m_e)^2 \sim 4.3 \times 10^4$ times more sensitive to new Φ than a_e
- a_τ is even more sensitive to new Φ, but measurement limited by short lifetime
- τ_μ small but manageable

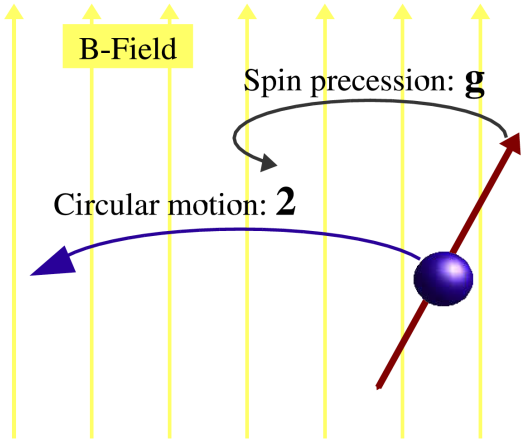
$\rightarrow$ focus on the muon



Big question:

$$a_\mu^{\text{exp}} = a_\mu^{\text{SM}}?$$

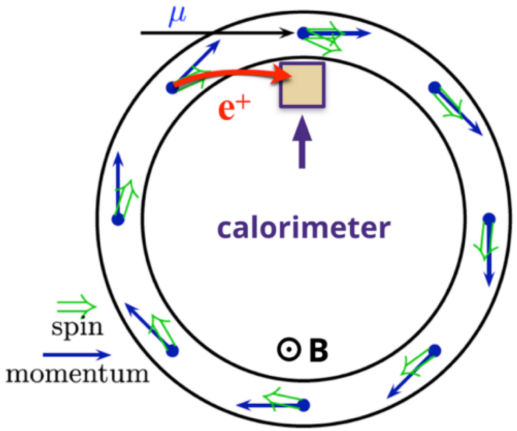
Measurement principle for a_μ



Precession determined by

$$\vec{\mu}_\mu = 2(1 + a_\mu)\frac{Qe}{2m_\mu}\vec{S}$$

$$\vec{d}_\mu = \eta_\mu \frac{Qe}{2m_\mu}\vec{S}$$



$$\vec{\omega}_{a\eta} = \vec{\omega}_a + \vec{\omega}_\eta \simeq -\frac{Qe}{m_\mu}\left[a_\mu \vec{B} - a_\mu\left(\frac{\gamma}{\gamma+1}\right)(\vec{\beta}\cdot\vec{B})\vec{\beta} - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\vec{\beta}\times\vec{E} + \frac{\eta_\mu}{2}\left(\vec{E} + \vec{\beta}\times\vec{B}\right)\right]$$

- Experiment measures very precisely $\vec{B}$ with $|\vec{B}| \gg |\vec{E}|$ &

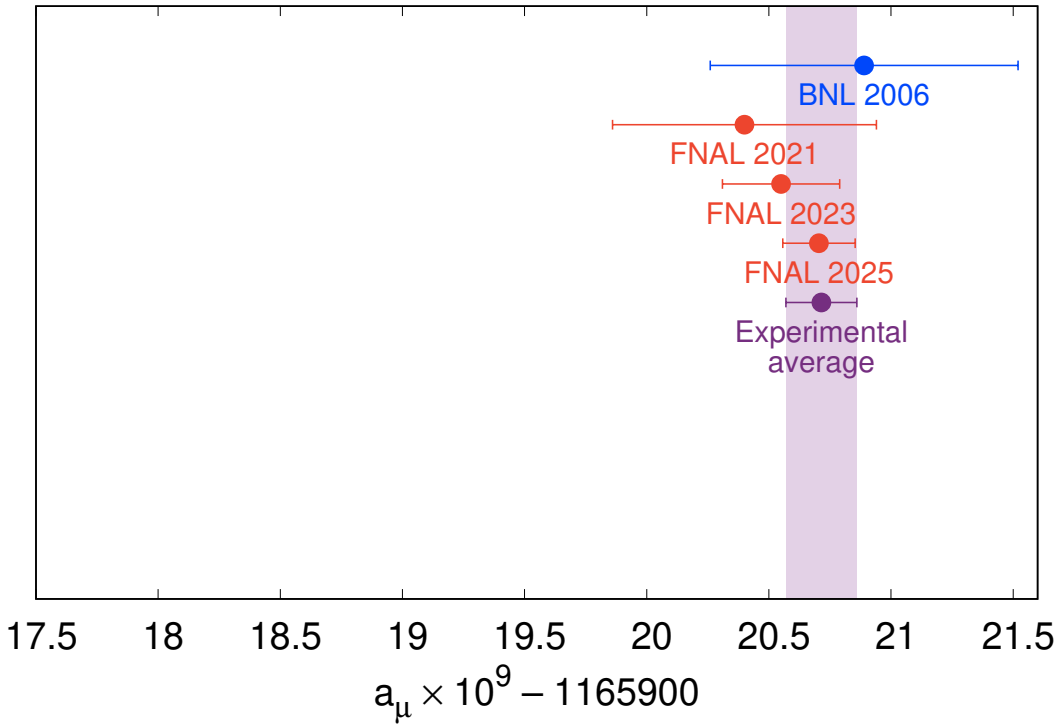
$$\Delta\omega \equiv \omega_S - \omega_C \simeq \sqrt{\omega_a^2 + \omega_\eta^2} \simeq \omega_a$$

since $d_\mu = 0.1(9) \times 10^{-19} e \cdot \text{cm}$ [Bennett et al '09]

- $\vec{\beta}\cdot\vec{B} \simeq 0$, and consider either magic $\gamma = 29.3$ (CERN/BNL/Fermilab) or $\vec{E} = 0$ (J-PARC)

$$\rightarrow \Delta\omega \simeq a_\mu |\vec{B}| \frac{e}{m_\mu}$$

a_μ: present experimental status



$$a_\mu|_{\text{expt}} = [11659207.15 \pm 1.45] \times 10^{-10} \qquad [0.124\,\text{ppm}]$$



vs



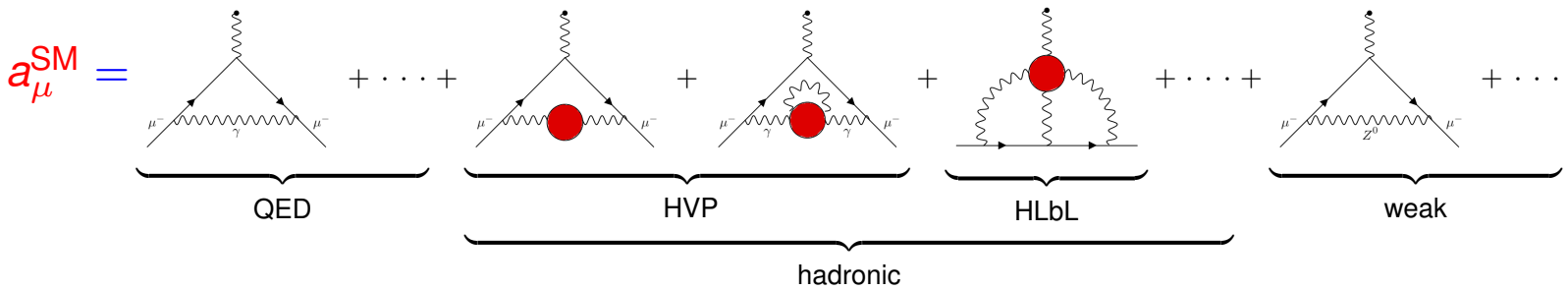
Standard model calculation of a_μ

[Aoyama et al '20 = WP '20, Aliberti et al '25 = WP '25]

$$
\begin{aligned}
a_\mu^{\text{SM}} &= a_\mu^{\text{QED}} + a_\mu^{\text{had}} + a_\mu^{\text{EW}} \\
&= O\left(\frac{\alpha}{2\pi}\right) + O\left(\left(\frac{\alpha}{\pi}\right)^2 \left(\frac{m_\mu}{M_\rho}\right)^2\right) + O\left(\left(\frac{\alpha}{16\pi \sin^2\theta_W}\right)\left(\frac{m_\mu}{M_W}\right)^2\right) \\
&= O\left(10^{-3}\right) + O\left(10^{-7}\right) + O\left(10^{-9}\right)
\end{aligned}
$$

$a_\mu^{\rm SM}$: standard model prediction, compiled in WP '25

To match experimental precision → all three interactions and all SM particles



SM contribution	$a_\mu^{\rm contrib.} \times 10^{10}$
QED [5 loops]	11658471.88 ± 0.02
EW [2 loops]	15.44 ± 0.04
HVP LO [lattice]	713.2 ± 6.1
HVP HO [lattice+pheno]	-8.72 ± 1.3
HLbL Tot. [lattice+data+pheno]	11.3 ± 1.0
SM [0.53 ppm]	11659203.3 ± 6.2
Exp [0.124 ppm]	11659207.15 ± 1.45

$\text{HVP} \sim a_\mu^{\rm SM}/(2 \times 10^4) \sim 65 \times \text{HLbL}$

$\sigma^2_{\rm HVP}/\sigma^2_{a_\mu} \sim 97\%$

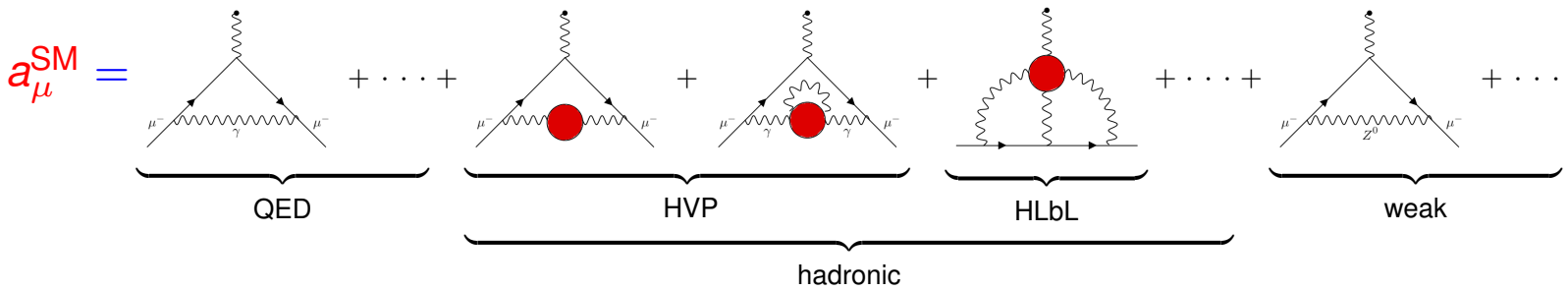
$\sigma^2_{\rm HLbL}/\sigma^2_{a_\mu} \sim 3\%$

Hadronic contributions involve low-energy, nonperturbative QCD:

- Data-driven: unitarity, analyticity, short-distance pQCD and data
  → dominated WP '20 averages
  → still very important for HLbL
  → making a comeback for HVP
- Lattice: massively-parallel numerical simulations of QCD
  → on a par w/ data-driven for HLbL
  → currently determines HVP → WP '25
- AdS QCD : → see talk by A. Rebhan for HLbL

$a_\mu^{\rm SM}$: standard model prediction, compiled in WP '25

To match experimental precision $\rightarrow$ all three interactions and all SM particles



SM contribution	$a_\mu^{\rm contrib.} \times 10^{10}$
QED [5 loops]	11658471.88 ± 0.02
EW [2 loops]	15.44 ± 0.04
HVP LO [This work]	715.1 ± 3.4
HVP HO [lattice+pheno]	-8.72 ± 1.3
HLbL Tot. [lattice+data+pheno]	11.3 ± 1.0
SM [0.31 ppm]	11659205.2 ± 3.6
Exp [0.124 ppm]	11659207.15 ± 1.45

HVP$\sim a_\mu^{\rm SM}/(2\times 10^4)\sim 65\times$HLbL

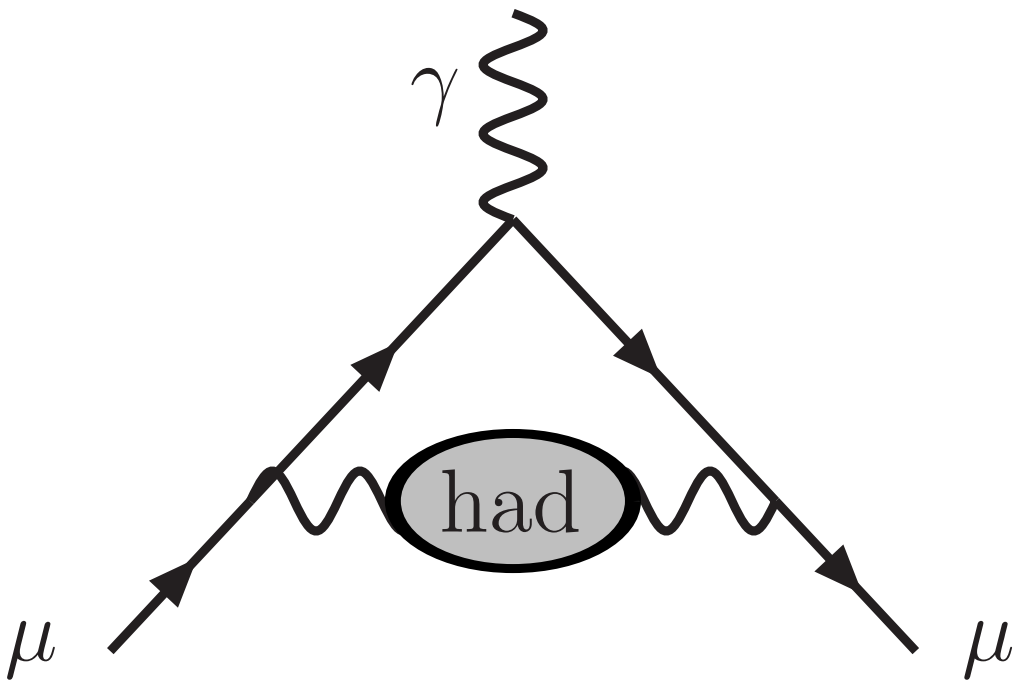
$\sigma^2_{\rm HVP}/\sigma^2_{a_\mu}\sim 89\%$

$\sigma^2_{\rm HLbL}/\sigma^2_{a_\mu}\sim 8\%$

Hadronic contributions involve low-energy, nonperturbative QCD:

- Data-driven: unitarity, analyticity, short-distance pQCD and data
  $\rightarrow$ dominated WP '20 averages
  $\rightarrow$ still very important for HLbL
  $\rightarrow$ making a comeback for HVP
- Lattice: massively-parallel numerical simulations of QCD
  $\rightarrow$ on a par w/ data-driven for HLbL
  $\rightarrow$ currently determines HVP $\rightarrow$ WP '25
- AdS QCD : $\rightarrow$ see talk by A. Rebhan for HLbL

Hybrid calculation of $a_\mu^{\text{LO-HVP}}$: taking the best from both approaches



All quantities related to a_μ will be given in units of 10^{-10}

$a_\mu^{\text{LO-HVP}}$ from LQCD: introduction



Compute on $T \times L^3$ Euclidean-time lattice w/ spacing a [Bernecker et al '11]

$$C_L(t) = \frac{a^3}{3} \sum_{i=1}^{3} \sum_{\vec{x}} \langle J_i(x) J_i(0) \rangle$$

w/ $J_\mu = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s + \frac{2}{3}\bar{c}\gamma_\mu c + \cdots$

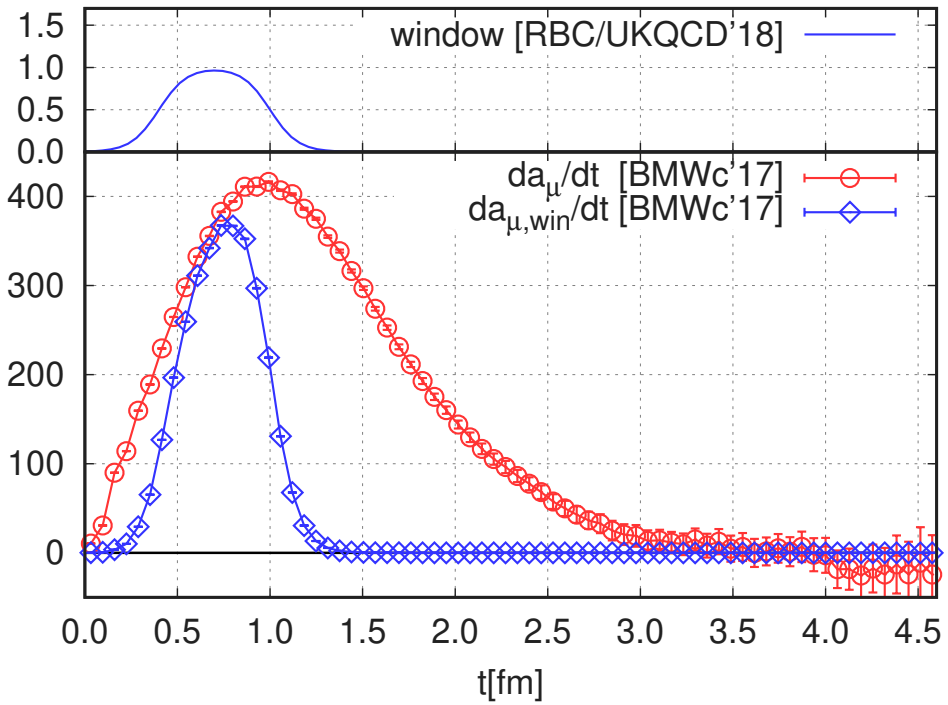
Decompose

$$C_L(t) = C_L^{ud}(t) + C_L^{s}(t) + C_L^{c}(t) + C_L^{\text{disc}}(t) + C_L^{\text{QED}}(t) + C_L^{\text{SIB}}(t)$$

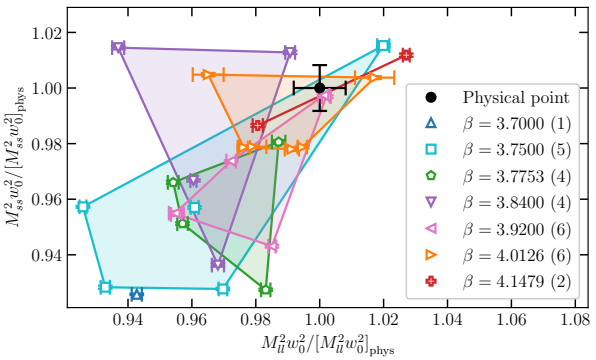
Then get

$$a_{\mu,f}^{\text{LO-HVP}} = \lim_{\substack{a \to 0 \\ L,\, T \to \infty}} \left(\frac{\alpha}{\pi}\right)^2 \left(\frac{a}{m_\mu^2}\right) \sum_{t=0}^{T/2} K(t m_\mu)\ \text{Re} C_L^f(t)$$

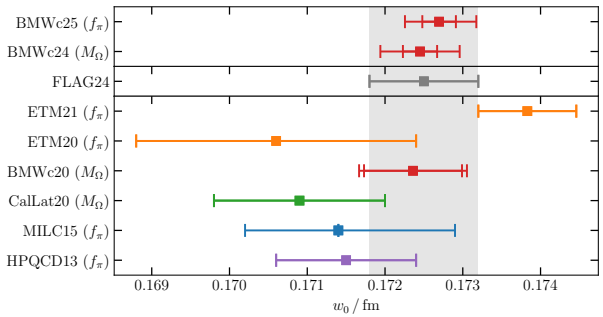
Define "windows" [RBC/UKQCD '18]
$K(\tau) \to W(\tau; \tau_i, \tau_f, \bar{\Delta}) K(\tau)$



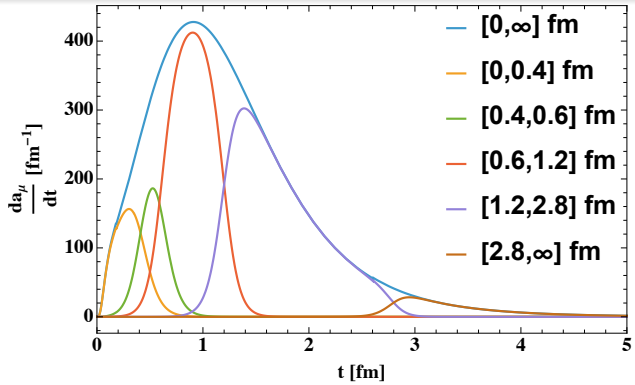
Strategy for improvement over BMW '20



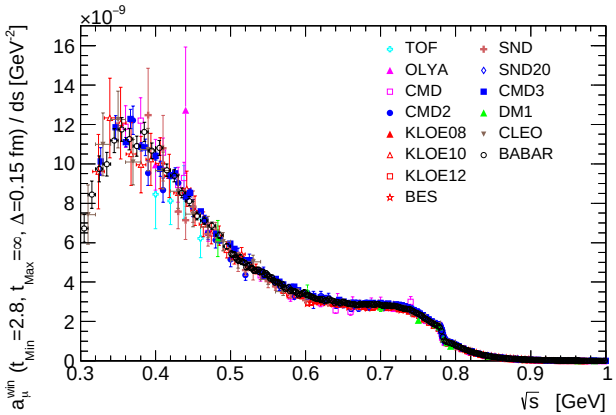
New finer ("Monster") lattices:
$a = 0.064\,\text{fm}\ [96^3 \times 144] \longrightarrow$
$a = 0.048\,\text{fm}\ [128^3 \times 192]$
$\rightarrow$ distance to continuum limit $\div 1.8$



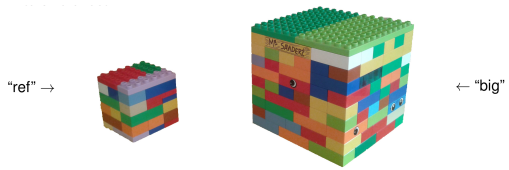
Significantly improve tuning of QCD parameters
$\rightarrow$ (scale uncertainty)^2 $\div 2.1$



Break up calculation into set of 5 windows
$\rightarrow$ optimize calculation in small distance scale intervals



Use $e^+e^- \rightarrow$ hadrons (and τ) data for long-distance $2.8\,\text{fm} \rightarrow \infty$ contribution [$1\,\text{fm} \rightarrow \infty$ proposed in RBC/UKQCD '18]



Dedicated simulations on huge $L = 11\,\text{fm}$ lattice to correct finite-V effects



[ericwilliambarnum.wordpress.com]

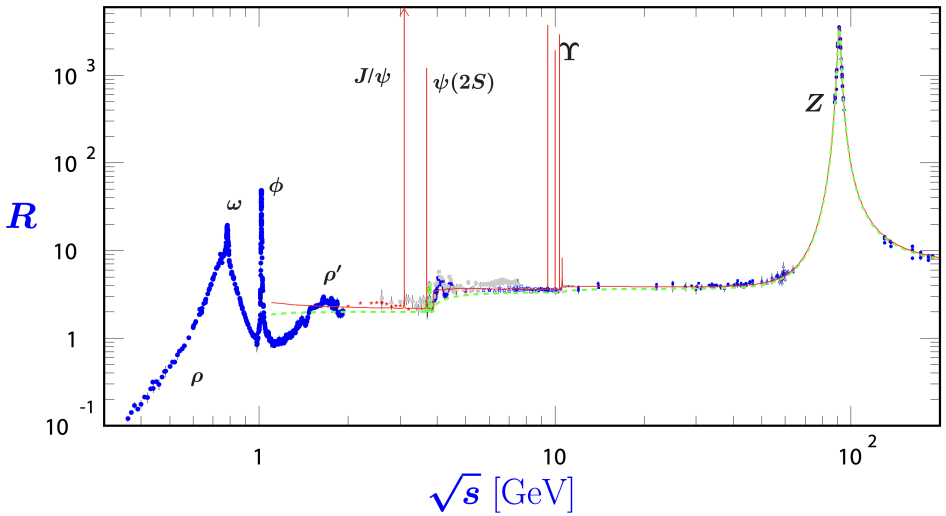
Calculation fully blinded

Re-evaluated all QED and SIB corrections

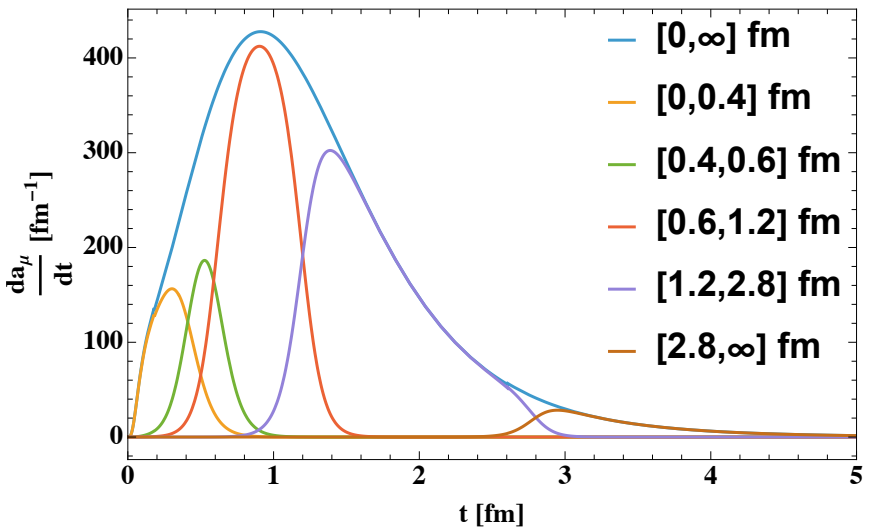
Five windows: lattice w/ a touch of e^+e^-

$$a_{\mu,win}^{\text{LO-HVP}}(t_i,t_f) = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \frac{dt}{m_\mu^2} K(tm_\mu) W(t;t_i,t_f) C(t)$$

$$R(s) \equiv \frac{\sigma(e^+e^- \to \text{had})}{\sigma(e^+e^- \to \mu^+\mu^-)} \quad \longrightarrow \quad C(t) = \frac{1}{24\pi^2} \int_{s_{\text{th}}}^\infty ds \sqrt{s}\, R(s)\, e^{-|t|\sqrt{s}}$$



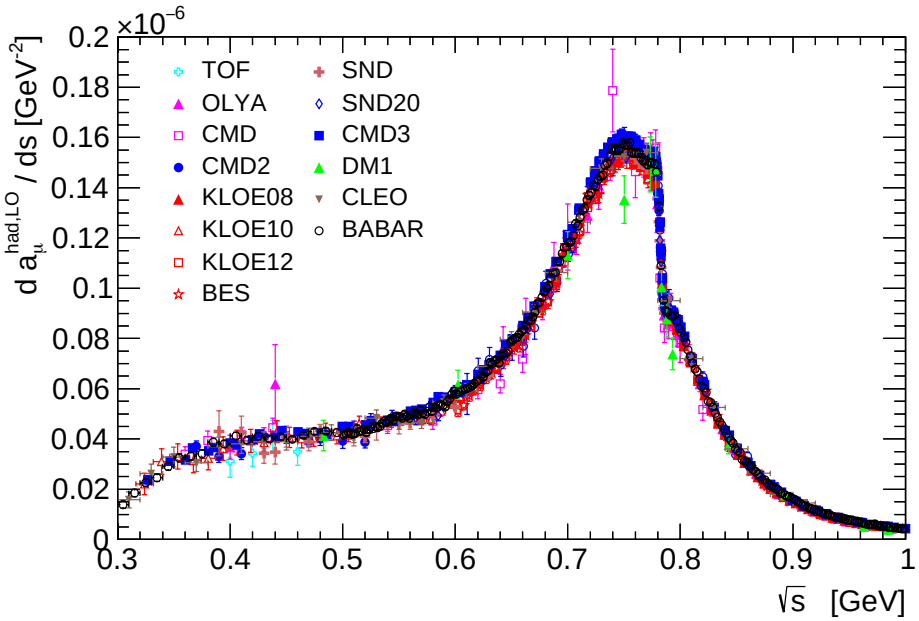
[PDG '18 compilation]



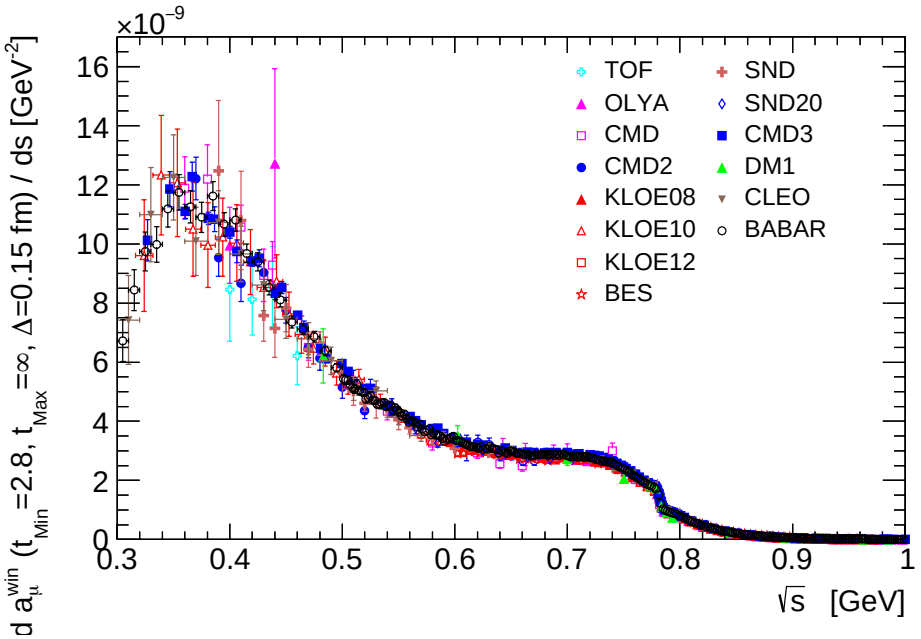
- Compute 4 windows from $0 \to 2.8\,\text{fm}$ on lattice
- Obtain small contribution from $2.8\,\text{fm} \to \infty$ using $e^+e^- \to$ hadrons data

Data-driven tail: 2.8 fm $\rightarrow \infty$

$a_\mu^{\text{LO-HVP}}$

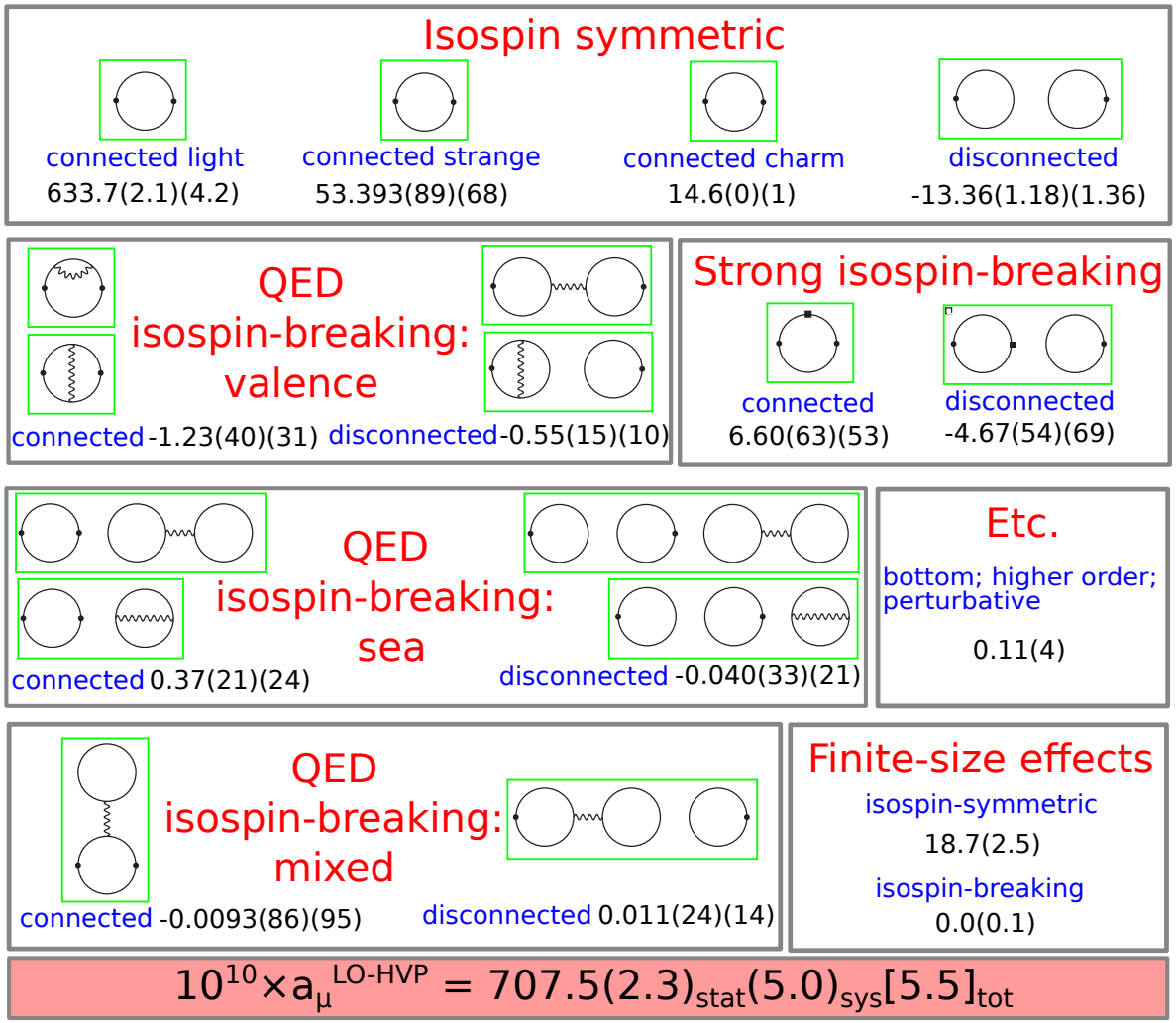


$a_{\mu,28-\infty} \equiv a_\mu^{\text{LO-HVP}}\big|_{\text{tail}}$



- Over 70% of tail comes from below ρ peak
- All datasets agree well in that range
- Good agreement with lattice for $a_{\mu,28-35}$
- Contributes only 4% to $a_\mu^{\text{LO-HVP}}$
- 96% comes lattice computation
- Helps reduce many of our uncertainties

Summary of contributions to $a_\mu^{\text{LO-HVP}}$: BMW 2020

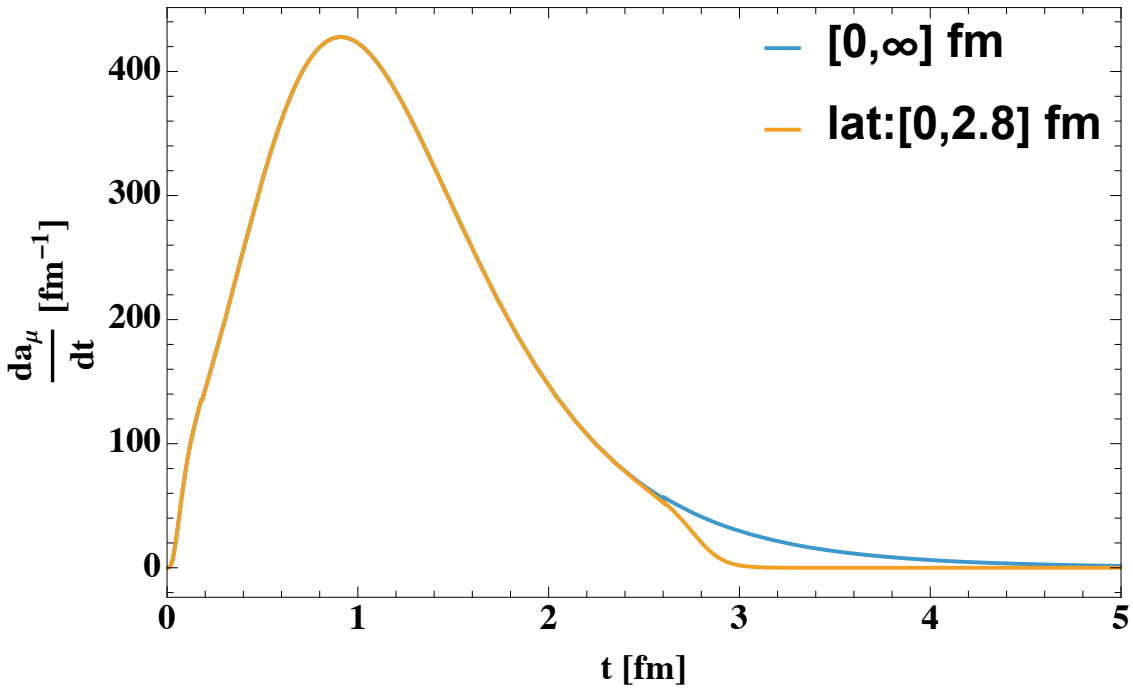


$a_\mu^{\text{LO-HVP}}$ to 0.77%

$\rightarrow$ 2024 Frontiers of Science Award in Theoretical Physics

HVP contribution result

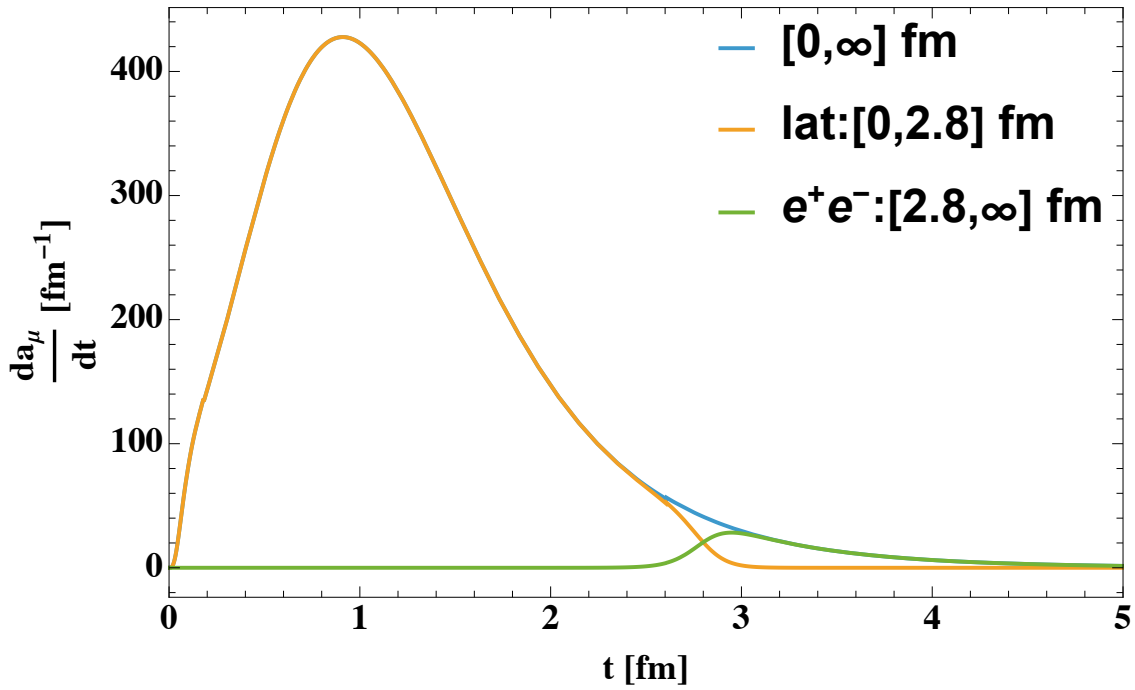
Re-computed all lattice HVP contributions [This work]



$$a_\mu^{\text{LO-HVP}} \equiv a_{\mu,0\text{-}2.8\,\text{fm}}^{\text{LO-HVP}}\big|_{\text{lat}} + a_{\mu,2.8\text{-}\infty\,\text{fm}}^{\text{LO-HVP}}\big|_{e^+e^-,\tau}$$
$$= 687.51(3.36) + 27.59(52)$$
$$= 96\% + 4\%$$
$$= 715.1(3.4) \quad [0.48\%]$$

HVP contribution result

Re-computed all lattice HVP contributions [This work]



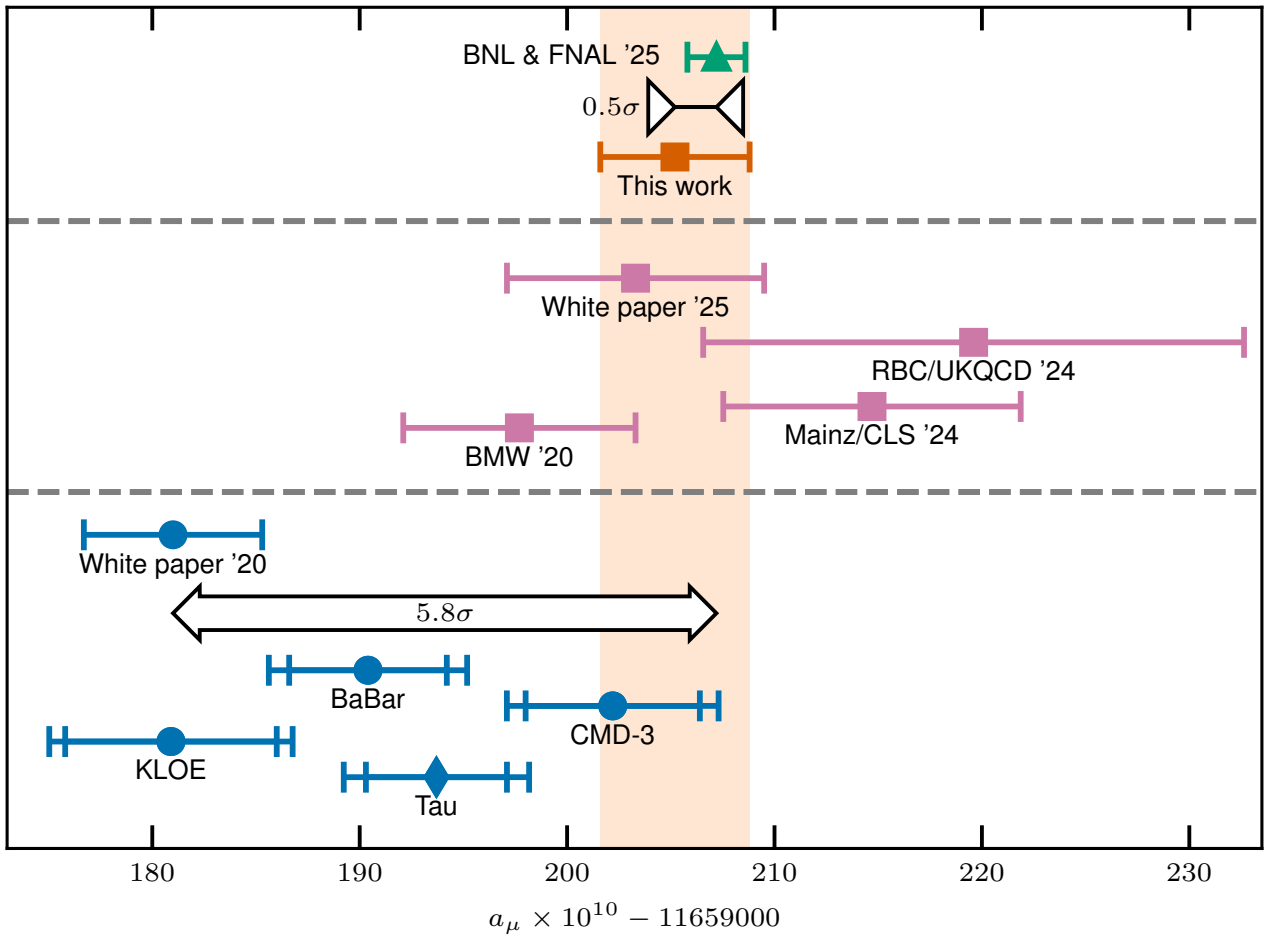
$$
\begin{aligned}
a_\mu^{\text{LO-HVP}} &\equiv a_{\mu,0\text{-}2.8\,\text{fm}}^{\text{LO-HVP}}\big|_{\text{lat}} + a_{\mu,2.8\text{-}\infty\,\text{fm}}^{\text{LO-HVP}}\big|_{e^+e^-,\tau} \\
&= 687.51(3.36) + 27.59(52) \\
&= 96\% + 4\% \\
&= 715.1(3.4) \quad [0.48\%]
\end{aligned}
$$

Experiment vs SM: October 2026

$$a_\mu|_{\text{expt}} = (11659207.15 \pm 1.45) \times 10^{-10}$$

$$a_\mu|_{\text{WP25}} = (11659203.3 \pm 6.2) \times 10^{-10}$$

$$a_\mu|_{\text{This work+WP25}} = (11659205.1 \pm 3.4) \times 10^{-10}$$



Conclusions

- New "hybrid" calculation (lattice + $e^+e^- \to$ hadrons) of $a_\mu^{\text{LO-HVP}}$ to 0.48%
- Decades of perturbative QED & EW calculations, combined w/ fully nonperturbative QCD (+QED) and data-driven ones, predict g_μ to 0.36 ppb !
- Decades of measurement give g_μ to 0.14 ppb ! $\to$ 2026 Breakthrough Prize
- With our result for $a_\mu^{\text{LO-HVP}}$, SM confirmed to 0.38 ppb
- Stringent, single test of the complete SM (all particles & interactions)
- Lattice calculation of $0 \to 2.8\,\text{fm}$ window $> 96\%$ of total
- $e^+e^- \to$ hadrons evaluation of $2.8 \to \infty\,\text{fm}$ window $\leq 4\%$ of total
- Fully blinded analysis
- $\sim$ 10 years of lattice work and billions of supercomputer core-hours
- Error reduction vs BMW '20:
  - $\sim 37\%$ from finer ensembles
  - additional $\sim 22\%$ from $e^+e^- \to$ hadrons data for tail
- Lattice calculation agrees w/ others in windows: $0 \to 0.4\,\text{fm}$, $0.4 \to 1.0\,\text{fm}$ & $1.5 \to 1.9\,\text{fm}$, $1.0\,\text{fm} \to \infty$

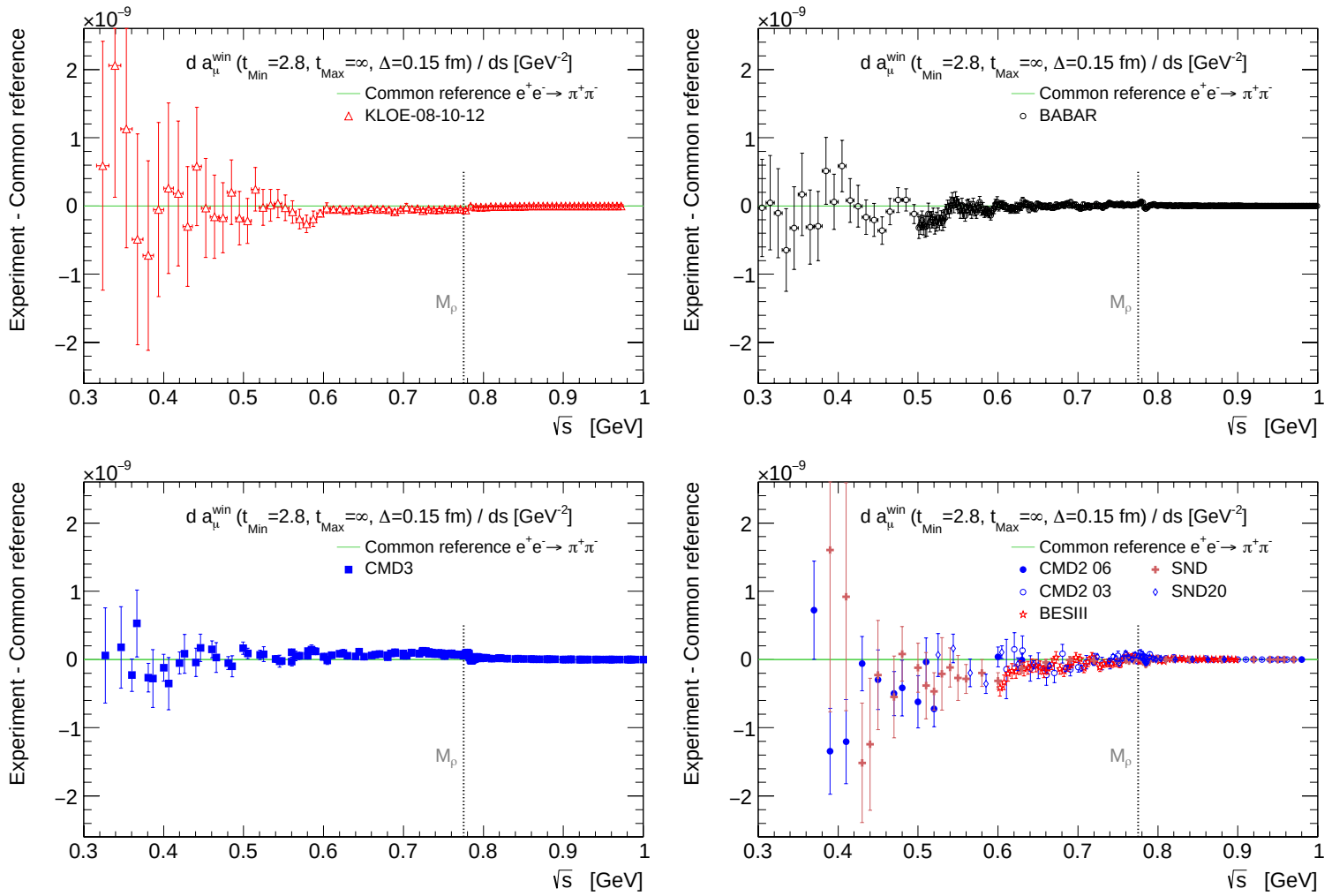
Conclusions

- Still need $\times 2$ improvement to match experiment !
- Significant work to understand tensions among e^+e^- measurements [BaBar '23, Colangelo et al, ...] and w/ lattice [ETM '22, BMW-DMZ '23, ...]
- BMW also has complete calculation of $a_\mu^{\rm HLbL}$ [BMW '24]
- Eagerly await
  - Lattice results for complete $a_\mu^{\text{LO-HVP}}$ by Fermilab/HPQCD/MILC expected soon
  - Next generation of 100% lattice calculations w/ precision $\sim$BMW '24
  - New BABAR $e^+e^- \to$ hadrons analysis
  - New KLOE analysis
  - New BES III, BELLE-II, CMD-3, SND-2 data and analysis
  - J-PARC entirely new method for a_μ measurement
  - MUonE @ CERN for spacelike HVP
- $\alpha(M_Z^2)$ for EWPO tests of SM @ FCC-ee
- ...

BACKUP

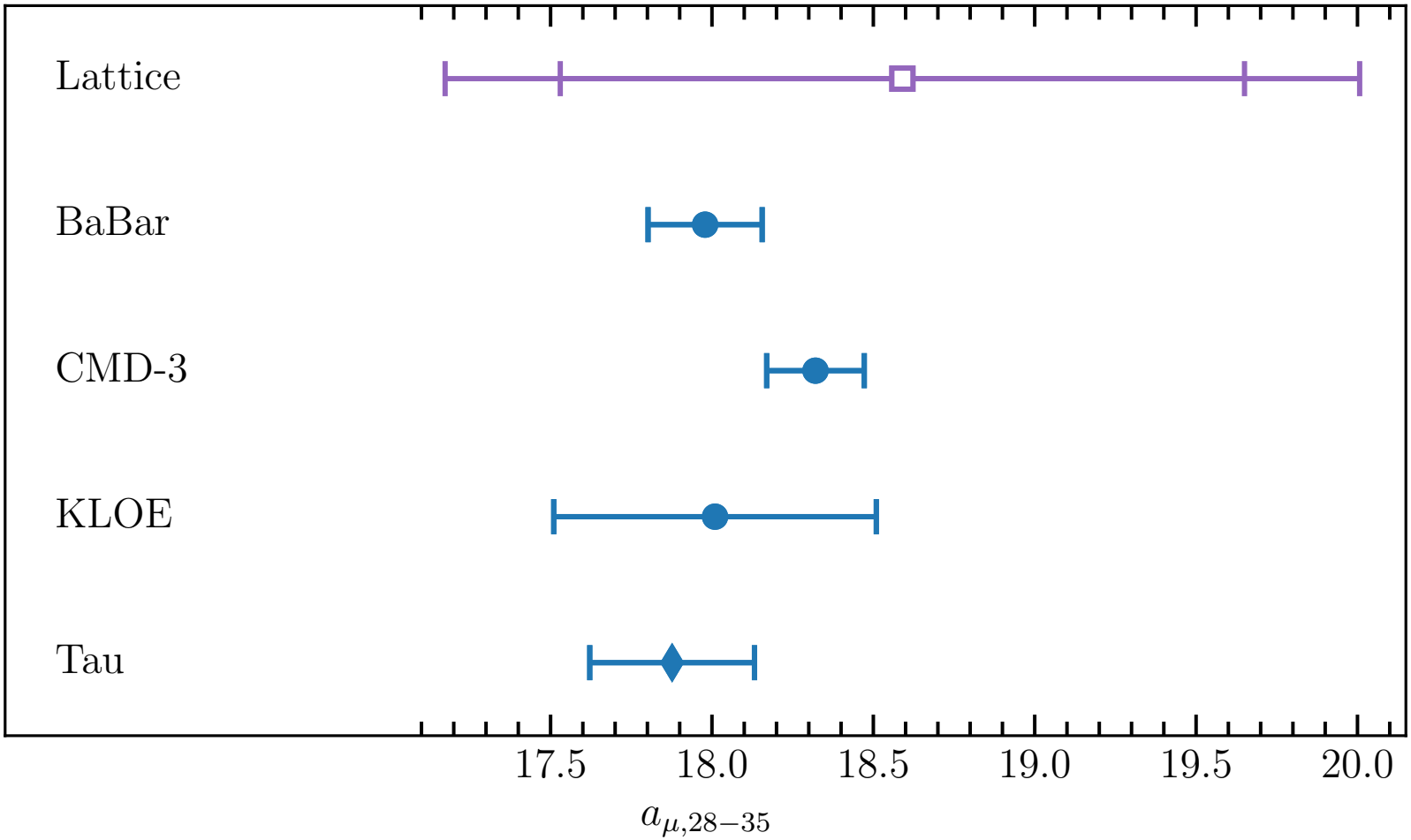
Data agreement in low-$\sqrt{s}$ region

- All datasets agree in low-$\sqrt{s}$ region that is relevant for $a_{\mu,28-\infty}$
- Contribution and uncertainties from low-$\sqrt{s}$ region overwhelms discrepancies on ρ peak



Lattice vs data-driven agreement for tail head

- $a_{\mu,28-35}$ represents $\sim$ 70% of tail
- Good agreement among experiments . . .
- . . . and lattice determination w/in $\sim$ 10% lattice uncertainty



All contributions to $a_\mu^{\text{LO-HVP}}$ [BMW-DMZ '24]

$00-28$	light and disconnected	619.6(2.3)(2.1)[3.1]	this work
$00-28$	strange	53.12(14)(11)[18]	this work
$00-28$	charm	14.68(24)(29)[38]	this work
$00-28$	light qed	$-1.86(53)(27)$	this work
$00-28$	light sib	7.06(64)(35)	this work
$00-28$	disconnected qed	$-1.00(46)(28)$	this work
$00-28$	disconnected sib	$-4.39(58)(63)$	this work
$00-28$	strange qed	$-0.0136(85)(77)$	this work
$00-\infty$	disconnected charm	0.0(1)	Borsanyi:2017zdw
$00-\infty$	charm qed	0.0182(36)	Giusti:2019xct
$00-\infty$	bottom	0.271(37)	Colquhoun:2014ica
$28-\infty$	tail from data-driven	27.59(26)(45)[52]	this work
$00-\infty$	total	715.1(2.5)(2.3)[3.4]	

Table: List of all contributions to a_μ.

$$a_\mu^{\text{LO-HVP}} \times 10^{10} = 715.1(2.5)(2.3)[3.4] \quad [0.48\%]$$