# Muon $g-2$: an overview of hadronic contributions

## and of open issues in hadronic light-by-light scattering

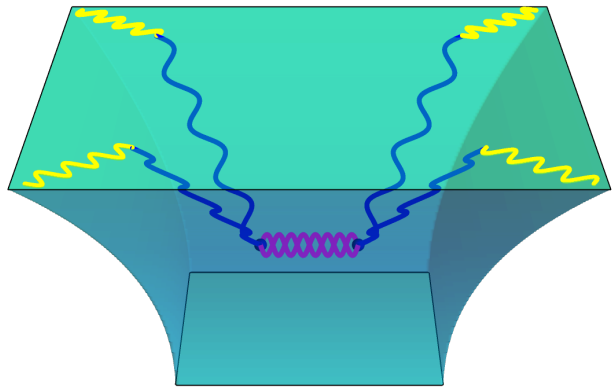
### where holographic QCD may provide clues

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TU Wien, Vienna, Austria

Assemblée Générale 2026 du GdR QCD, Nantes, 8 Oct 2026



# Muon $g-2$ SM prediction

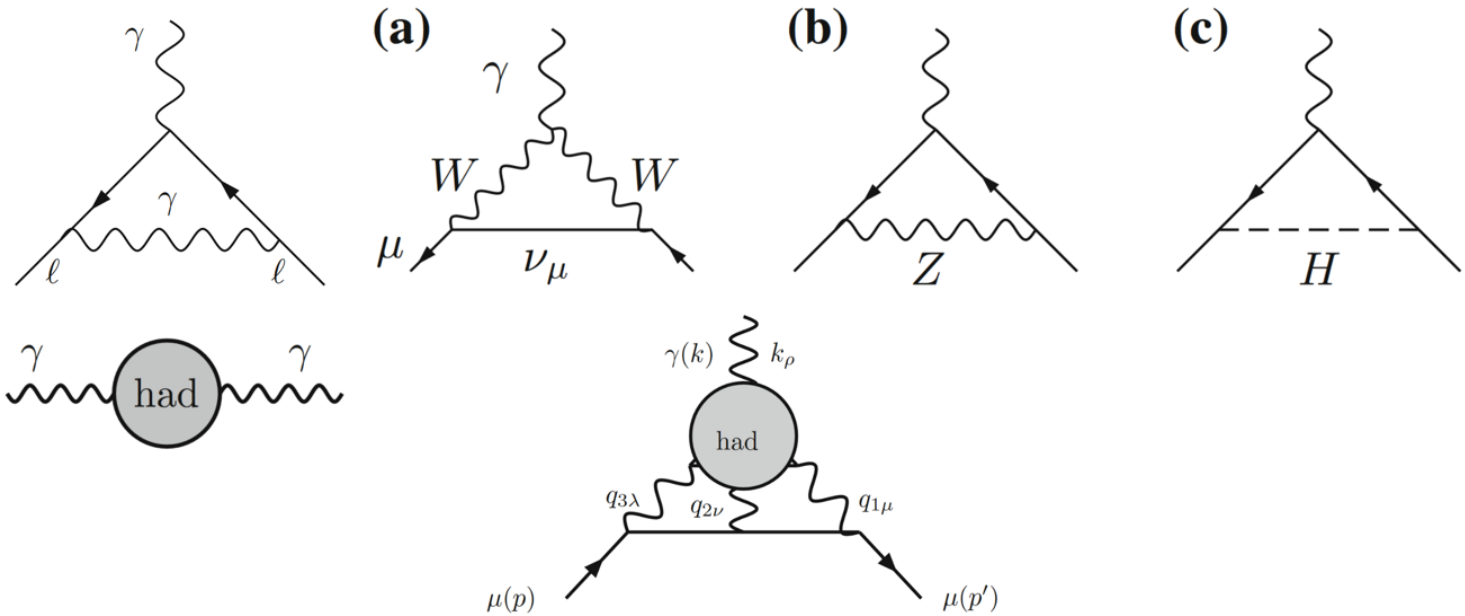
[TI "White Paper 2025", Aliberti et al., 2025]

Muon 200× heavier than electron ⇒ more sensitive to non-QED physics

Prediction limited by control over hadronic interactions at $\lesssim 10^{-9}$

$$a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{HVP}} + a_\mu^{\text{HLbL}} = 116\,592\,033(62) \times 10^{-11}$$

$$\begin{aligned} a_\mu^{\text{QED}} &= 116\,584\,718.8(2) \times 10^{-11} \\ a_\mu^{\text{EW}} &= 154.4(4) \times 10^{-11} \\ a_\mu^{\text{HVP}} &= 7\,045(61) \times 10^{-11} \\ a_\mu^{\text{HLbL}} &= 115.5(9.9) \times 10^{-11} \end{aligned}$$



# Hadronic contributions to $a_\mu = (g-2)/2$

Hadronic Vacuum Polarization (HVP) and Hadronic Light-by-Light Scattering (HLbL)

in units of $10^{-11}$:

| | WP25 | WP20 |
|---|---|---|
| Experimental world average | 116 592 071.5(14.5) | 116 592 089(63) |
| Standard Model prediction | 116 592 033(62) | 116 591 810(43) |
| HVP contribution | 7 045(61) | 6845(40) |
| HLbL contribution | 115.5(9.9) | 92(18) |

SM predictions formed by average of lattice QCD and data-driven (dispersive) results

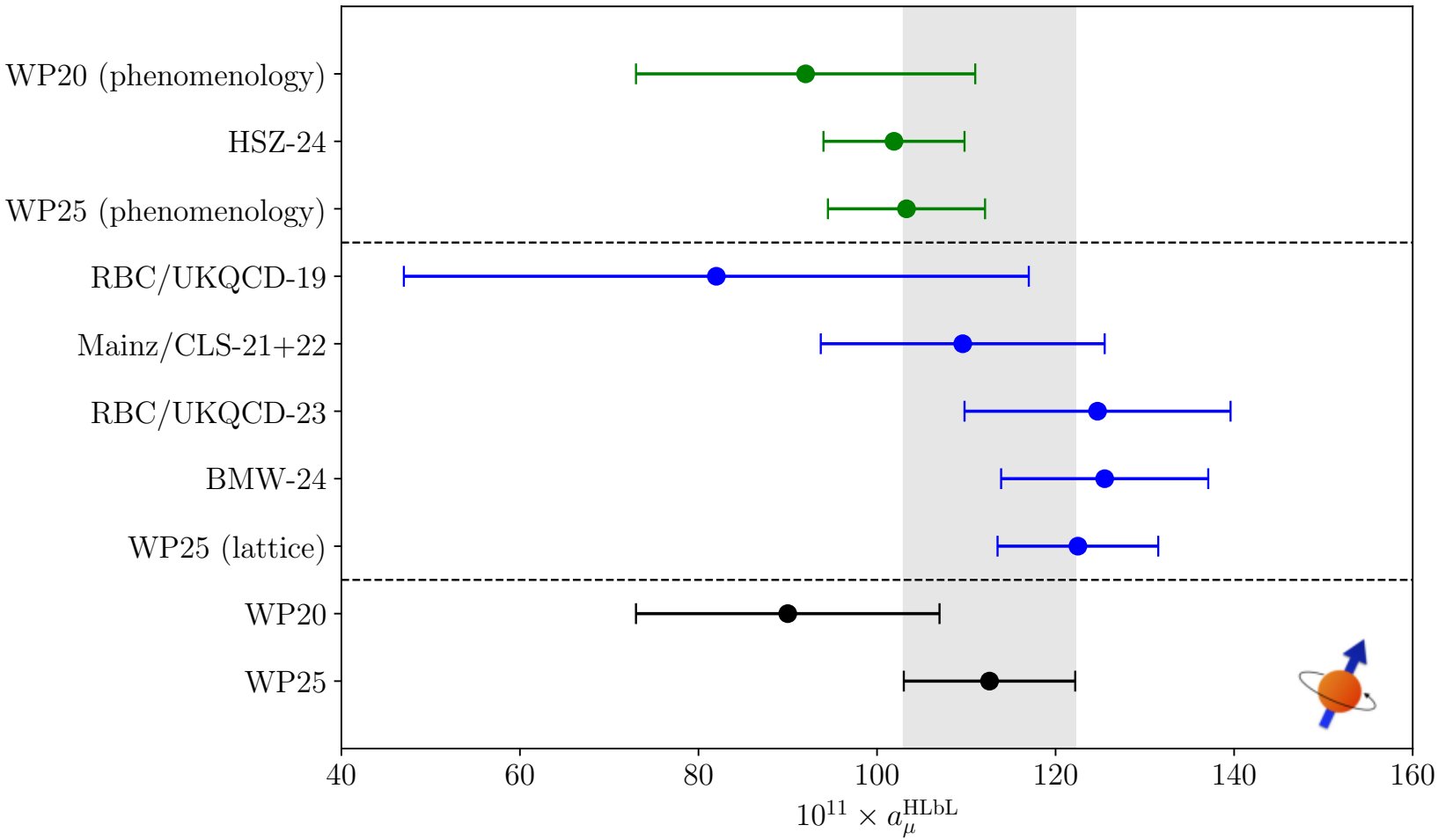
but errors in data-driven approach to HVP would have increased so much with new CMD-3 experiment that only lattice QCD results were averaged for HVP in WP25

(see talk by Laurent Lellouch for hybrid approach with LO-HVP errors reduced to 34)

[2407.10913] $a_\mu^{\rm th} \times 10^{-11} = 116\,592\,052(36)$

HLbL: long thought to be the most troublesome, now seemingly well consolidated

# HLbL: Status according to WP25




1.5$\sigma$ tension between WP25 dispersive result and WP25 lattice average

$\rightarrow$ if resolved, HLbL error could be reduced by $\sim 33\%$

# HLbL contributions where hQCD might be of interest

Holographic QCD (hQCD) makes interesting(*) predictions
where WP error estimates from phenomenology are the largest:

(*) not too model-dependent in the class of models that match
$m_\rho$, $f_\pi$, and longitudinal short-distance constraints (LSDC)

| Contribution | WP20–$a_\mu^{\rm HLbL} \times 10^{11}$ | $\to$ WP25 | hQCD [LMR22+CLMR25] |
|---|---|---|---|
| $\pi^0, \eta, \eta'$-poles | $93.8(4.0)$ | $91.2^{+2.9}_{-2.4}$ | $95.9(3.8)$ |
| $\pi, K$-loops/boxes | $-16.4(0.2)$ | $-16.4(0.2)$ | |
| $S$-wave $\pi\pi$ rescattering | $-8(1)$ | $-9.1(1.0)$ | |
| scalars & tensors | $-1(3)$ | | |
| axial vectors | $6(6)$ | | see below |
| $u, d, s$-loops / short-distance | $15(10)$ | | |
| — of which LSDC: $13(6)$ | | | |
| $c$-loop | $3(1)$ | $3(1)$ | |
| total | $92(19)$ | $103.3(8.8)$ | $114^{+10}_{-4}$ |

WP20 $\to$ WP25:
Subleading contributions from axials, scalars, tensors, and SDCs in new complete (up to tensors)
dispersive analysis by Hoferichter, Stoffer & Zillinger [HSZ24, 2412.00178]

# Chiral Lagrangians with resonances from holographic QCD

**(Axial) vector mesons and pions** are described by **5-d YM fields** $\mathcal{F}_{MN}^{L,R}$ for global $U(N_f)_L \times U(N_f)_R$ chiral symmetry of boundary theory

$$S_{\rm YM} \propto \frac{1}{g_5^2}\,\mathrm{tr}\int d^4x \int_0^{z_0} dz\, e^{-\Phi(z)}\sqrt{-g}\, g^{PR} g^{QS}\left(\mathcal{F}^{(L)}_{PQ}\mathcal{F}^{(L)}_{RS} + \mathcal{F}^{(R)}_{PQ}\mathcal{F}^{(R)}_{RS}\right),$$

where $P, Q, R, S = 0, \ldots, 3, z$ and $\mathcal{F}_{MN} = \partial_M \mathcal{B}_N - \partial_N \mathcal{B}_M - i[\mathcal{B}_M, \mathcal{B}_N]$

with conformal boundary at $z = 0$, and

either sharp cut-off of $AdS_5$ at $z_0$ (HW) or with nontrivial dilaton $z_0 = \infty$ (SW)
(SS: not asymptotically $AdS_5$, finite $z_0$ corresponding to point where D8 branes join)

**Chiral symmetry breaking** either from

- extra bifundamental scalar field [Erlich-Katz-Son-Stephanov 2005] (HW1), or
- through different boundary conditions for vector/axial-vector fields at $z_0$ [Hirn-Sanz 2005] (HW2), [Sakai-Sugimoto 2004] (SS)

**Vector meson dominance** (VMD) naturally built in:
photons couple through *bulk-to-boundary propagators of vector gauge fields* whose normalizable modes give (infinite tower of!) vector mesons ($\rho$, $\omega$, $\phi$, ...)

# Short distance constraints on TFFs

Crucially, hQCD models with asymptotic $AdS_5$ geometry reproduce
**asymptotic momentum dependence of LC expansion** [Brodsky-Lepage 1979-81]
thanks to VMD with infinite tower of vector meson resonances

- **Pseudoscalars** [Grigoryan & Radyushkin, PRD76,77,78 (2007-8)]:

$$F_{\pi^0\gamma^*\gamma^*}(Q_1^2,Q_2^2) \quad \to \quad \frac{2f_\pi}{Q^2}\sqrt{1-w^2}\int_0^\infty d\xi\,\xi^3 K_1(\xi\sqrt{1+w})K_1(\xi\sqrt{1-w})$$
$$= \frac{2f_\pi}{Q^2}\left[\frac{1}{w^2}-\frac{1-w^2}{2w^3}\ln\frac{1+w}{1-w}\right],$$

with $Q^2=\frac{1}{2}(Q_1^2+Q_2^2)\to\infty$, $w=(Q_1^2-Q_2^2)/(Q_1^2+Q_2^2)$,
corresponding to asymptotic behavior

$$F^\infty(Q^2,0)=\frac{2f_\pi}{Q^2}, \qquad F^\infty(Q^2,Q^2)=\frac{2f_\pi}{3Q^2} \quad (\Leftarrow \text{OPE}).$$

- **Axial vector mesons** [J. Leutgeb & AR, 1912.01596]
(confirmed by pQCD/LCE result of Hoferichter & Stoffer 2004.06127):

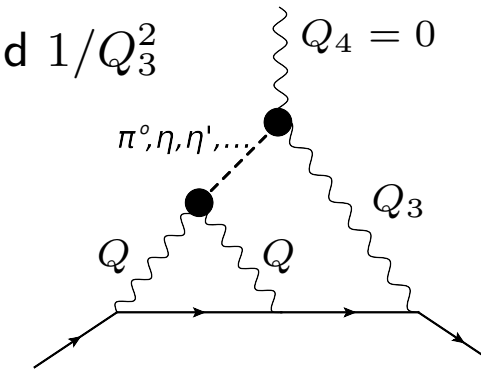
$$A_n(Q_1^2,Q_2^2)\to\frac{12\pi^2F_n^A}{N_cQ^4}\frac{1}{w^4}\left[w(3-2w)+\frac{1}{2}(w+3)(1-w)\ln\frac{1-w}{1+w}\right]$$

# Melnikov-Vainshtein short-distance constraint

Melnikov and Vainshtein [hep-ph/0312226, PRD70(2004)]:
nonrenormalization theorem for axial anomaly implies
short-distance constraint for 4-photon-amplitude (in BTT basis w/ 54 structure functions):

$$\lim_{Q_3\to\infty}\lim_{Q\to\infty} Q^2 Q_3^2 \hat{\Pi}_1(Q,Q,Q_3) = -\frac{2}{3\pi^2}$$

each single meson exchange contribution gives 0
because propagator $\sim 1/Q_3^2$ and the two form factors $\sim 1/Q^2$ and $1/Q_3^2$




hQCD models: *infinite* tower of axial-vector mesons saturates MV-SDC exactly!

with $g_5^2 = 12\pi^2/N_c$ as required by OPE of vector-vector correlator
[Leutgeb & AR, 1912.01596; Cappiello, Cata, D'Ambrosio, Greynat & Iyer, 1912.02779]

# Axial vectors and SDCs contributions updated

**WP 2020 estimate**: $a_\mu^{\text{axials+SDC}+P^*} = 21(16) \times 10^{-11}$

- **hQCD model [LMR22] prediction** (with $m_s > m_{u,d}$ and U(1)$_A$ anomaly) considerably larger:

$$a_\mu^{\text{axials+MV-SDC}+P^*} = 32.7^{+3.0}_{-0.0} \times 10^{-11}$$

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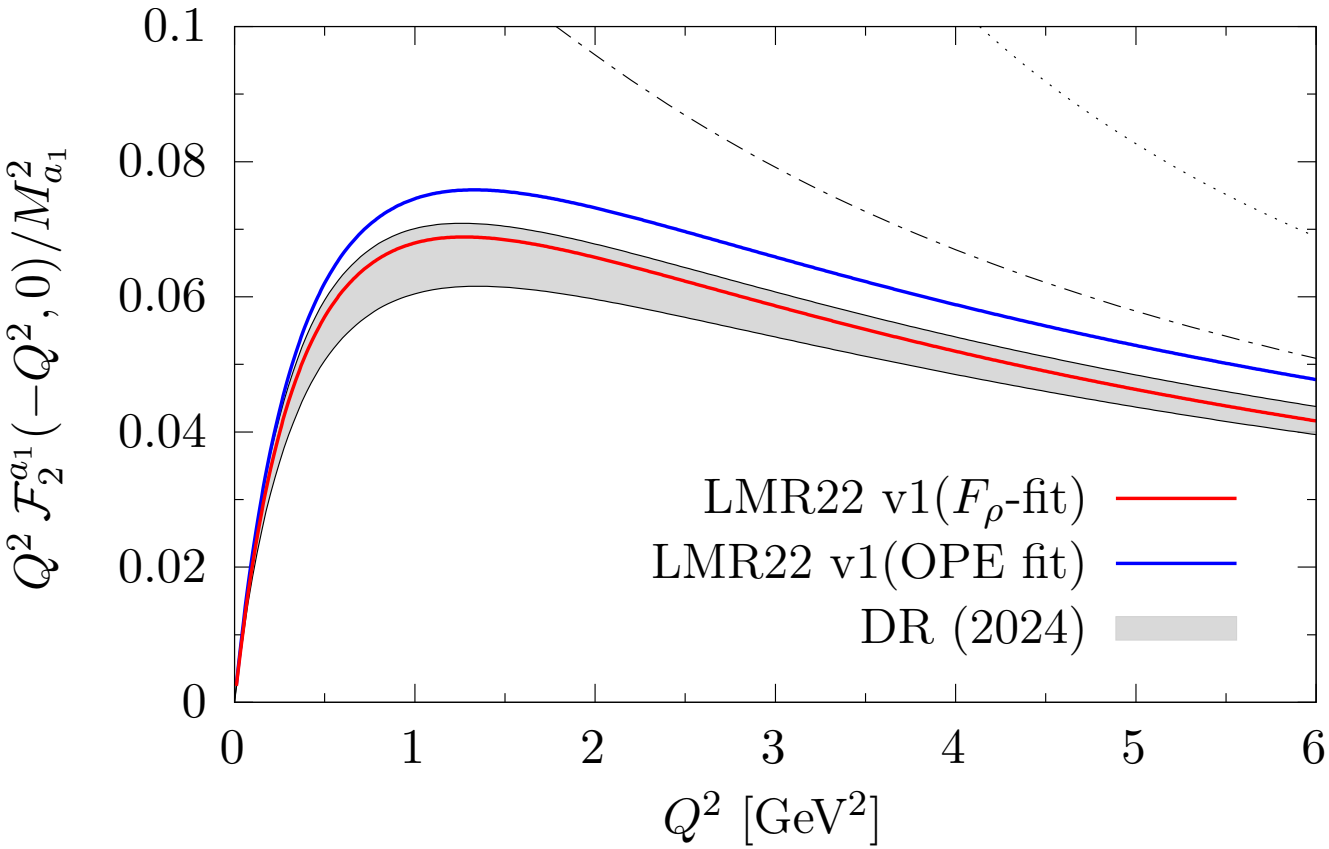
- confirmed by **new dispersive results** of Hoferichter, Stoffer & Zillinger [HSZ 2412.00178 w/o $\{S,T\}_{IR}$]:

$$a_{\mu,\text{dispersive}}^{\text{axials+SDC}+P^*} = 36.4(4.6) \times 10^{-11}$$

[1] Range of models considered in LMR24 around "best-guess" model LMR22($F_\rho$-fit)

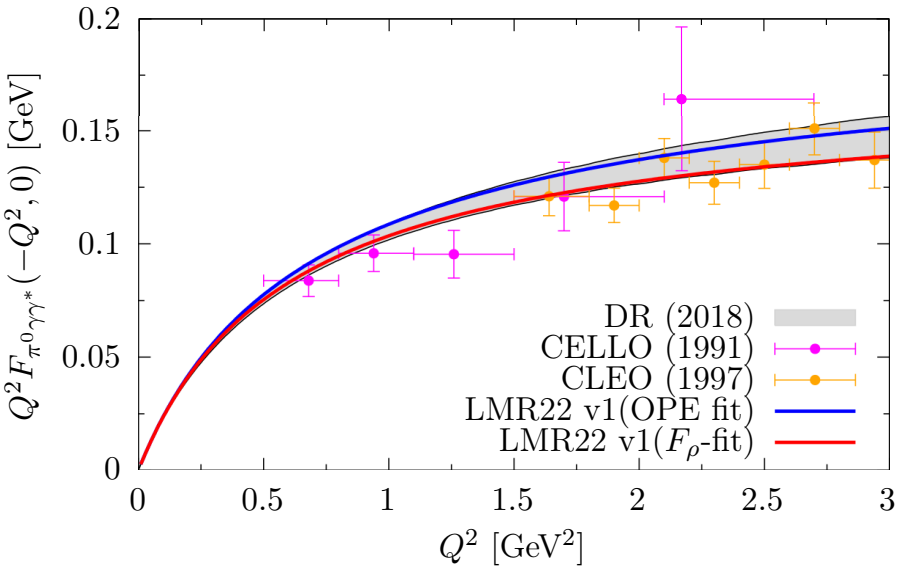
# Axial vector TFF (singly virtual)

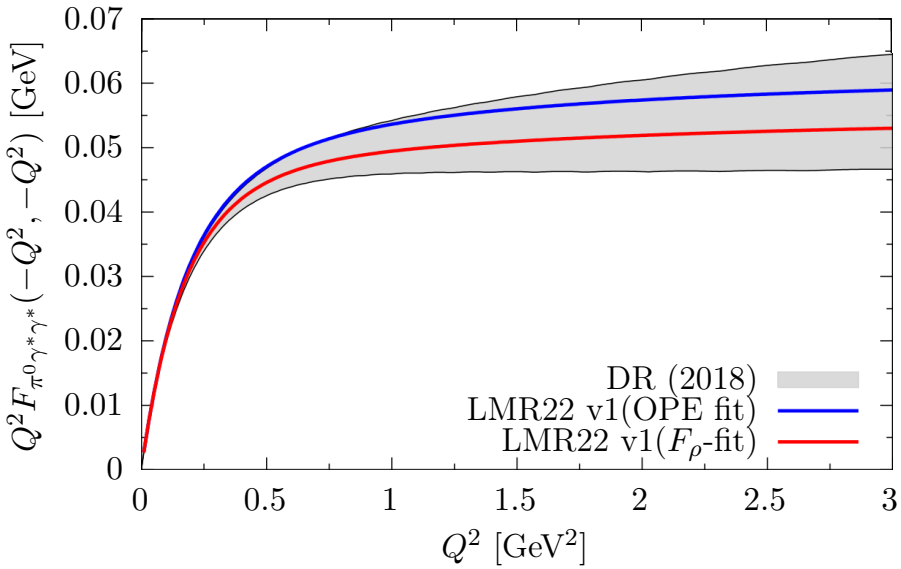
Surprisingly good agreement of $a_1$ TFF comparison of best guess hQCD model with dispersive results from Lüdtke et al.[2410.11946]



# Pion TFFs (singly and doubly virtual)

Similarly good agreement for leading HLbL contribution of $\pi^0$:




$\pi^0$ TFF comparison of hQCD model [LMR22] with dispersive results (DR) from Hoferichter et al.[1808.04823] (Fig. 53 in WP2).

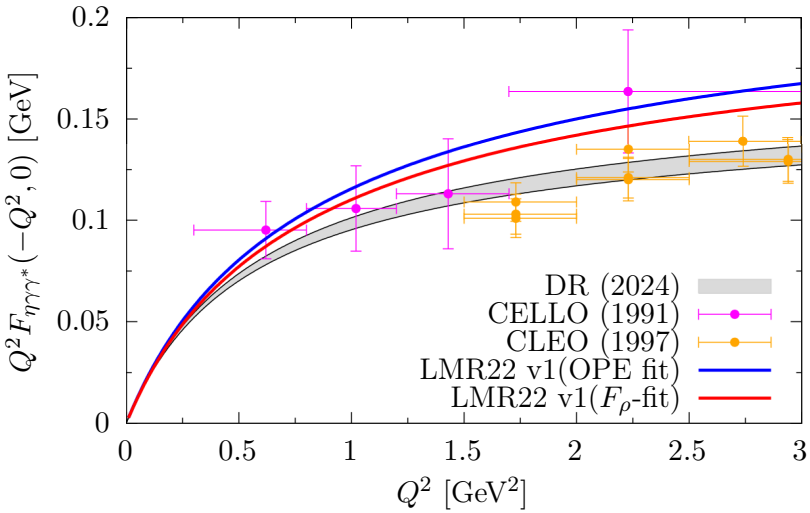
# $\eta$ and $\eta'$ TFFs (singly and doubly virtual)

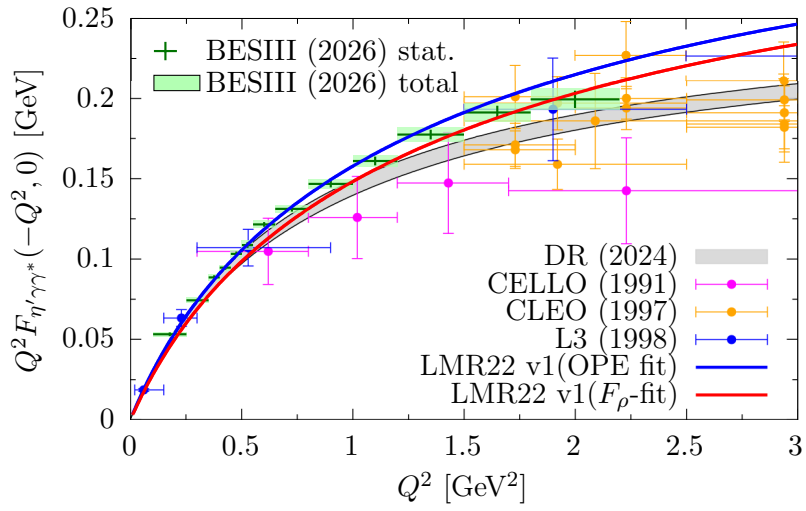
hQCD model [LMR22] predicts somewhat larger values for TFFs and $a_\mu$ (in units of $10^{-11}$):

| | $a_\mu$ dispersive [WP2] | $a_\mu$ hQCD |
|---|---|---|
| $\pi^0$ | $63.0^{+2.7}_{-2.1}$ | $63.4(2.7)$ |
| $\eta$ | $14.7(9)$ | $17.6(1.7)$ |
| $\eta'$ | $13.5(7)$ | $14.9(2.0)$ |
| Sum | $91.2^{+2.9}_{-2.4}$ | $95.9(3.8)$ |

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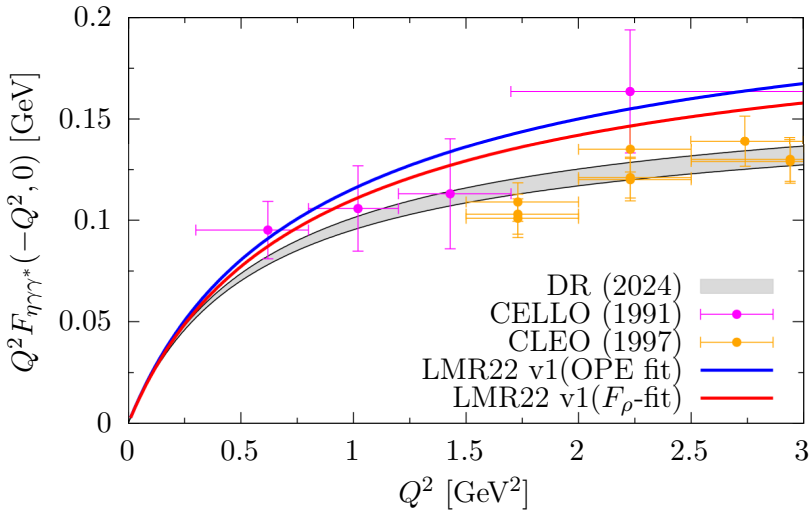


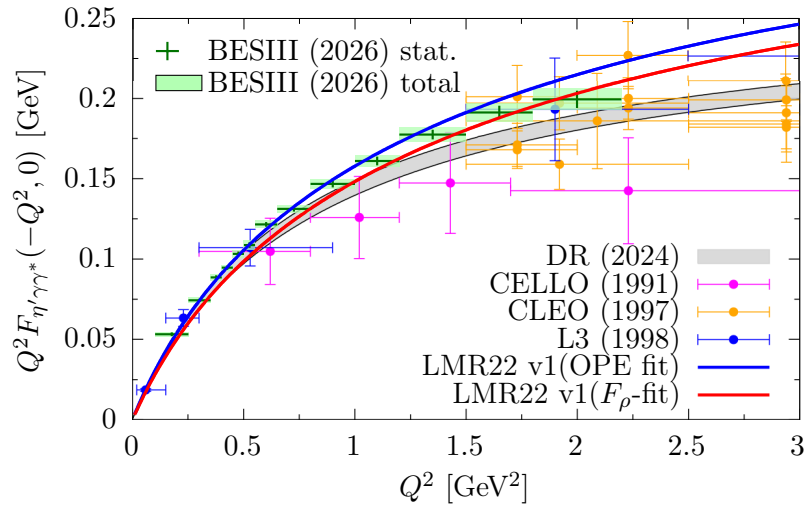




Singly virtual $\eta, \eta'$ TFF comparison of hQCD model with dispersive results [2411.08098]
and new high-precision results for $\eta'$ from BESIII [2608.12793]

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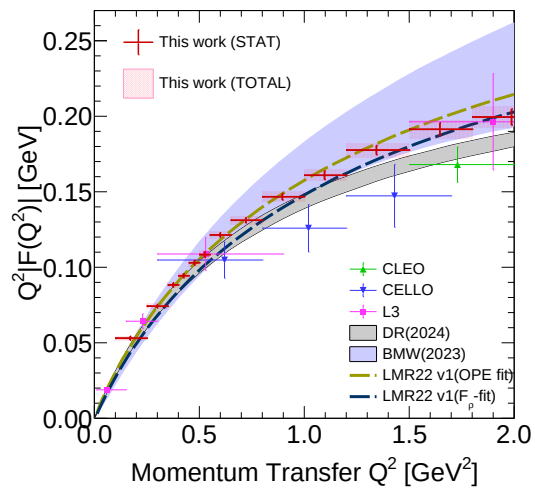
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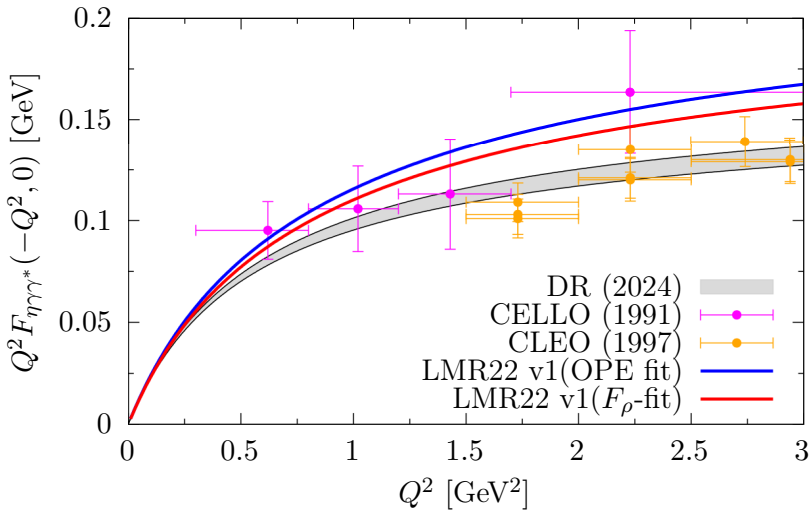
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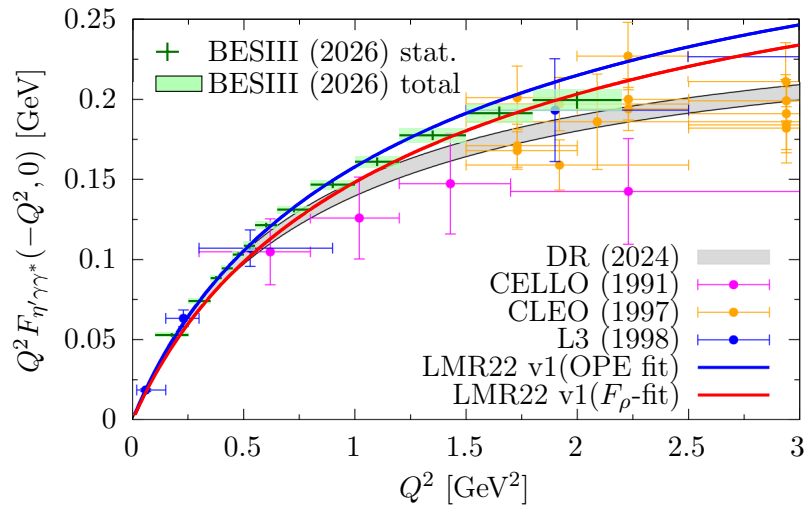



consistent with BMW, but tension with DR(2024)!

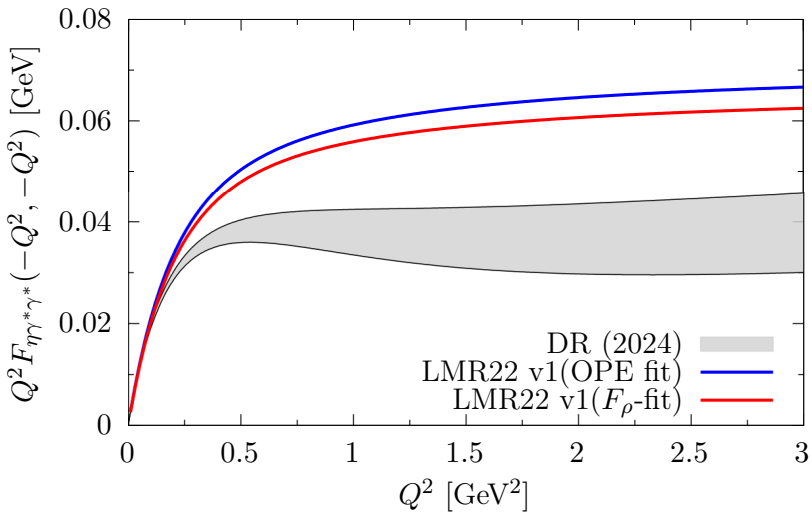
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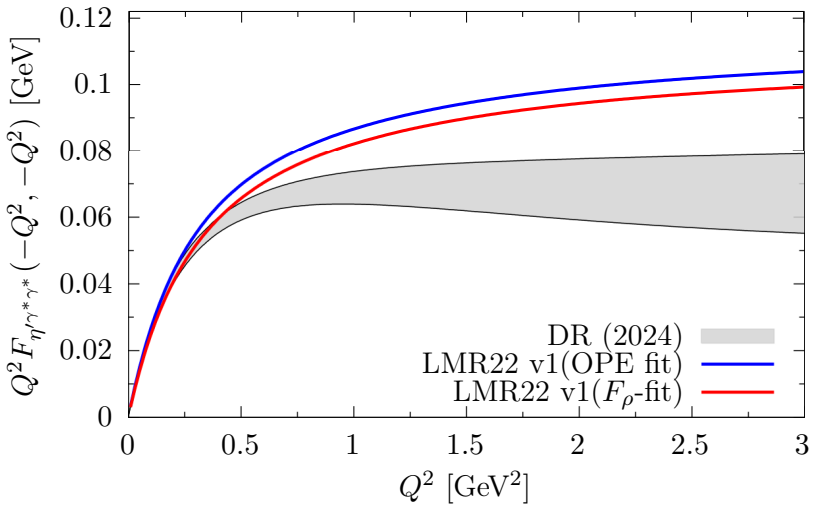
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Singly virtual $\eta, \eta'$ TFF comparison of hQCD model with dispersive results [2411.08098]




Doubly virtual $\eta, \eta'$ TFF comparison of hQCD model with dispersive results [2411.08098]

# New puzzle: Tensor mesons

**WP20:** Despite their prominence in photon collisions,
contribution from tensor mesons was considered to be almost negligibly small

With simple Quark Model (QM) ansatz for TFFs,

Danilkin & Vanderhaeghen [1611.04646] got $\boxed{a_\mu^{\mathrm{T}} = +0.64(13) \times 10^{-11}}$

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**HSZ24 [2412.00178]:** Previous QM result overlooked kinematical singularities!

Absent a dispersive analysis, the same QM (with different scale setting) yields

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**CLMR25 [2501.09699]:** hQCD gives instead

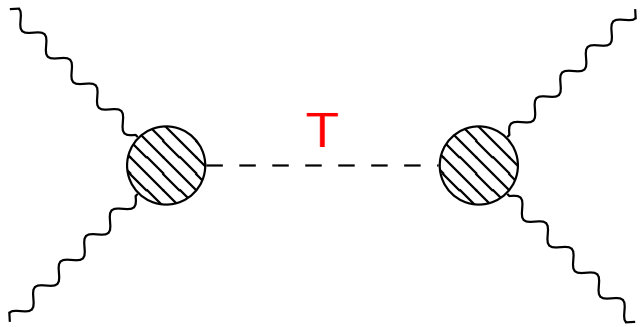
$$\boxed{a_\mu^{\mathrm{T}} = +2.9 \times 10^{-11} \text{ and additionally } a_\mu^{\mathrm{T}^*} = +5.6 \times 10^{-11}} \text{ !!}$$

T: groundstate tensor multiplet; $\mathrm{T}^*$: infinite tower of excited tensor modes

$\Rightarrow$ $a_\mu^T$ in **WP25** based on HSZ24 estimate, but with somewhat enhanced error

# Tensor mesons

T$= f_2(1270), a_2(1320), f_2'(1430), ..$




TFF decomposition:

$$\mathcal{M}^{\mu\nu\alpha\beta}(q_1^2, q_2^2) = \sum_{i=1}^{5} \frac{\mathcal{F}_i^T(q_1^2, q_2^2)}{m_T^{n_i}} T_i^{\mu\nu\alpha\beta},$$

QM: only $\mathcal{F}_1^T$ nonvanishing with scale $\Lambda_T$ either $\sim M_T$ (DV) or $M_\rho$ (HSZ24)
and match to real-photon decay rate

$$\mathcal{F}_1^T(-Q_1^2, -Q_2^2) = \frac{\mathcal{F}_1^T(0,0)}{(1 + (Q_1^2 + Q_2^2)/\Lambda_T^2)^2}, \quad \mathcal{F}_{2,3,4,5}^T = 0$$

# Tensor mesons in hQCD

hQCD describes tensors via metric fluctuations $h_{MN}$ [Katz et al. 0510388]

$$S = -2k_T \int d^5x \sqrt{g} \left(\mathcal{R} + 2\Lambda\right) + \frac{1}{2g_5^2} \mathrm{tr} \int d^5x \sqrt{g} F_{MN} F^{MN}$$

Only 1 new parameter $k_T$ compared to LMR22,

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Non-zero $\mathcal{F}_i^T$: [CLMR 2501.09699]

$$\mathcal{F}_1^T(-Q_1^2,-Q_2^2) = -m_T\frac{1}{g_5^2}\mathrm{tr}\mathcal{Q}^2\int\frac{dz}{z}h_n(z)\mathcal{J}(z,Q_1)\mathcal{J}(z,Q_2),$$
$$\mathcal{F}_3^T(-Q_1^2,-Q_2^2) = -m_T^3\frac{1}{g_5^2}\mathrm{tr}\mathcal{Q}^2\int\frac{dz}{z}h_n(z)\frac{\partial_z\mathcal{J}(z,Q_1)}{Q_1^2}\frac{\partial_z\mathcal{J}(z,Q_2)}{Q_2^2}.$$

$\mathcal{F}_1^T$ and $\mathcal{F}_3^T$ originate both from $5d$ covariant $F_{MN}F^{MN}$.
The quark model ansatz (QM) has only nonzero $\mathcal{F}_1^T$.

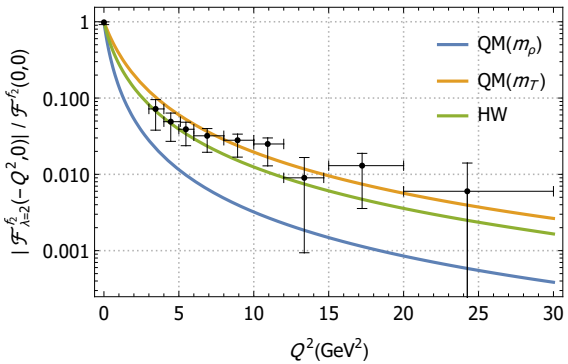
- Neither QM nor hQCD tensor TFFs reproduce the LC asymptotics, which involves *all* $\mathcal{F}_i^T$ simultaneously.
- Comparison with resonance chiral theory suggests that $\mathcal{F}_1^T$ and $\mathcal{F}_3^T$ may be equally important at low energies.

# Comparison of singly virtual f2 TFFs with data

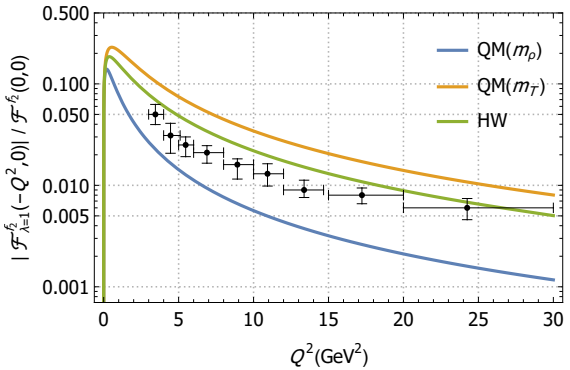
$\mathcal{F}_1^T$ from hQCD (HW) in between QM($M_\rho$) (DV 2016) and QM($M_T$) (HSZ24) (all w/o $\mathcal{F}_2^T$)

vs. BELLE $(3.45 \leq Q^2[\mathrm{GeV}^2] \leq 24.25)$:

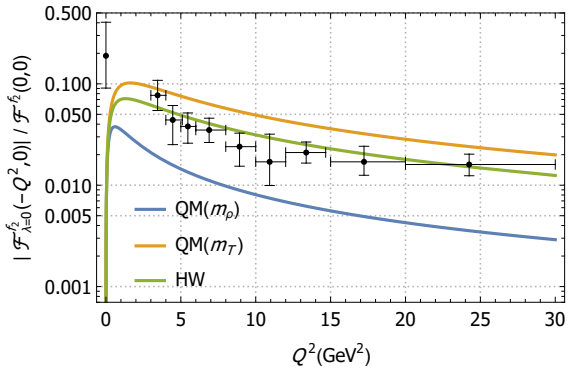
$\lambda = 2$




$\lambda = 1$

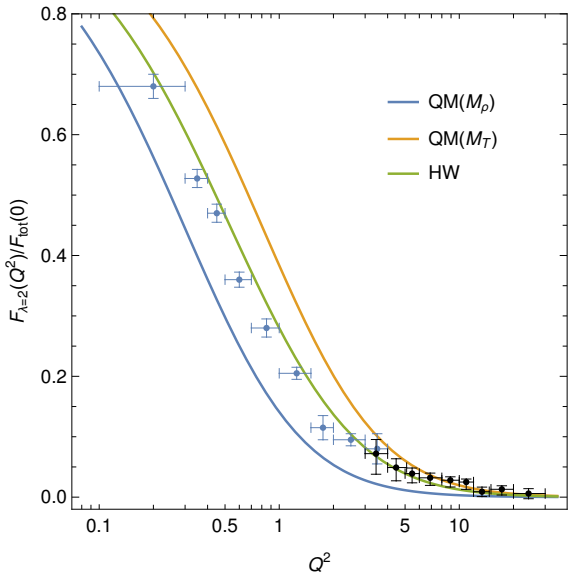



$\lambda = 0$

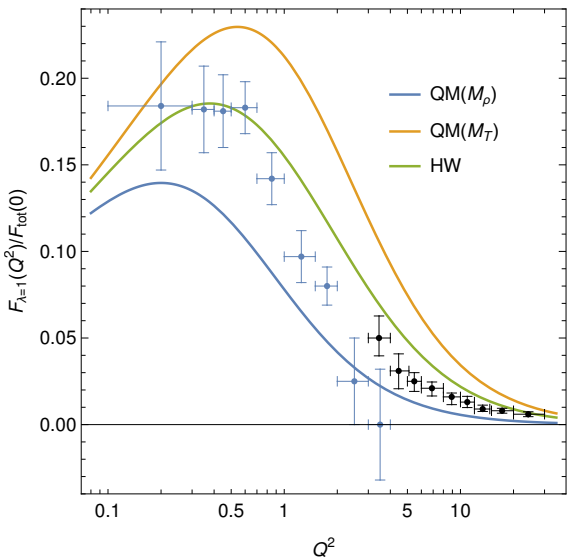



vs. BELLE and BESIII preliminary (PhD thesis Max Lellmann, U. Mainz 2026):

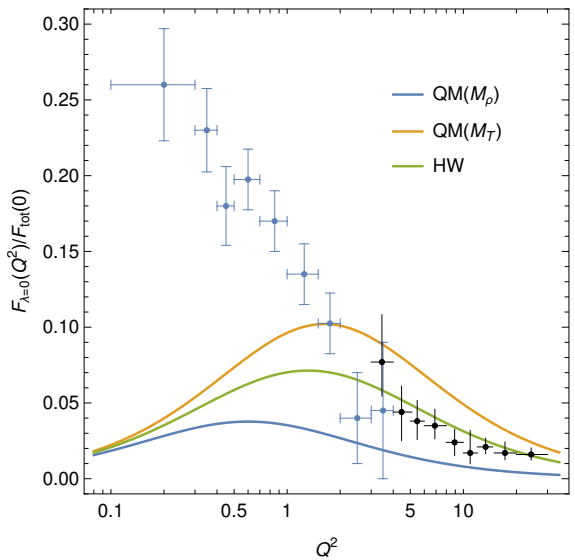
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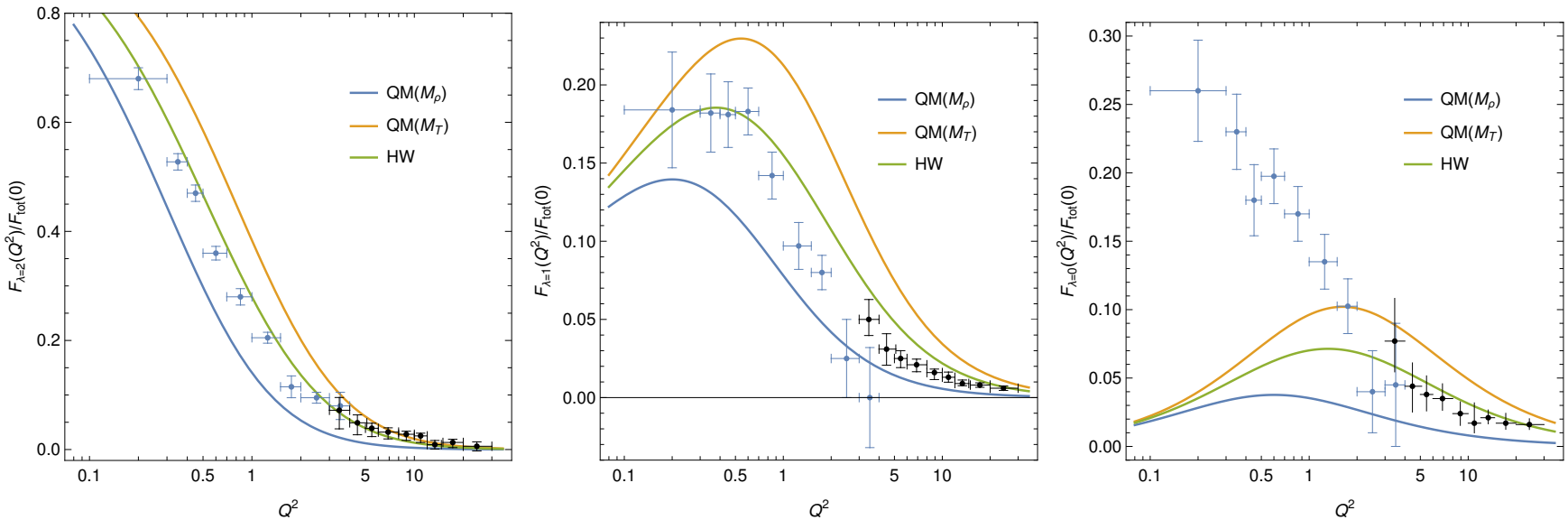


# Comparison of singly virtual f2 TFFs with data

$\mathcal{F}_1^T$ from hQCD (HW) in between QM($M_\rho$) (DV 2016) and QM($M_T$) (HSZ24) (all w/o $\mathcal{F}_2^T$)

vs. BELLE and BESIII preliminary (PhD thesis Max Lellmann, U. Mainz 2026):

$\lambda = 2$ $\lambda = 1$ $\lambda = 0$




- Zero helicity ($\lambda = 0$) amplitude vanishes due to missing $\mathcal{F}_2^T$ in both HW and QM
- More important for $a_\mu$: Size and sign of $\mathcal{F}_3^T$, will be tested in forthcoming experiments at BESIII through doubly virtual TFFs for $f_2$ [A. Denig, M. Lellmann et al.]
- Meanwhile: will expand hQCD model by higher-derivative interactions of hQCD tensors (restricted by 5D diffeo invariance) with aim to match LCE asymptotics of all five $\mathcal{F}_i^T$ [CMLR, in progress]

# Conclusions

- *Biggest issue*: **HVP** needs reanalysis of data-driven dispersive approach

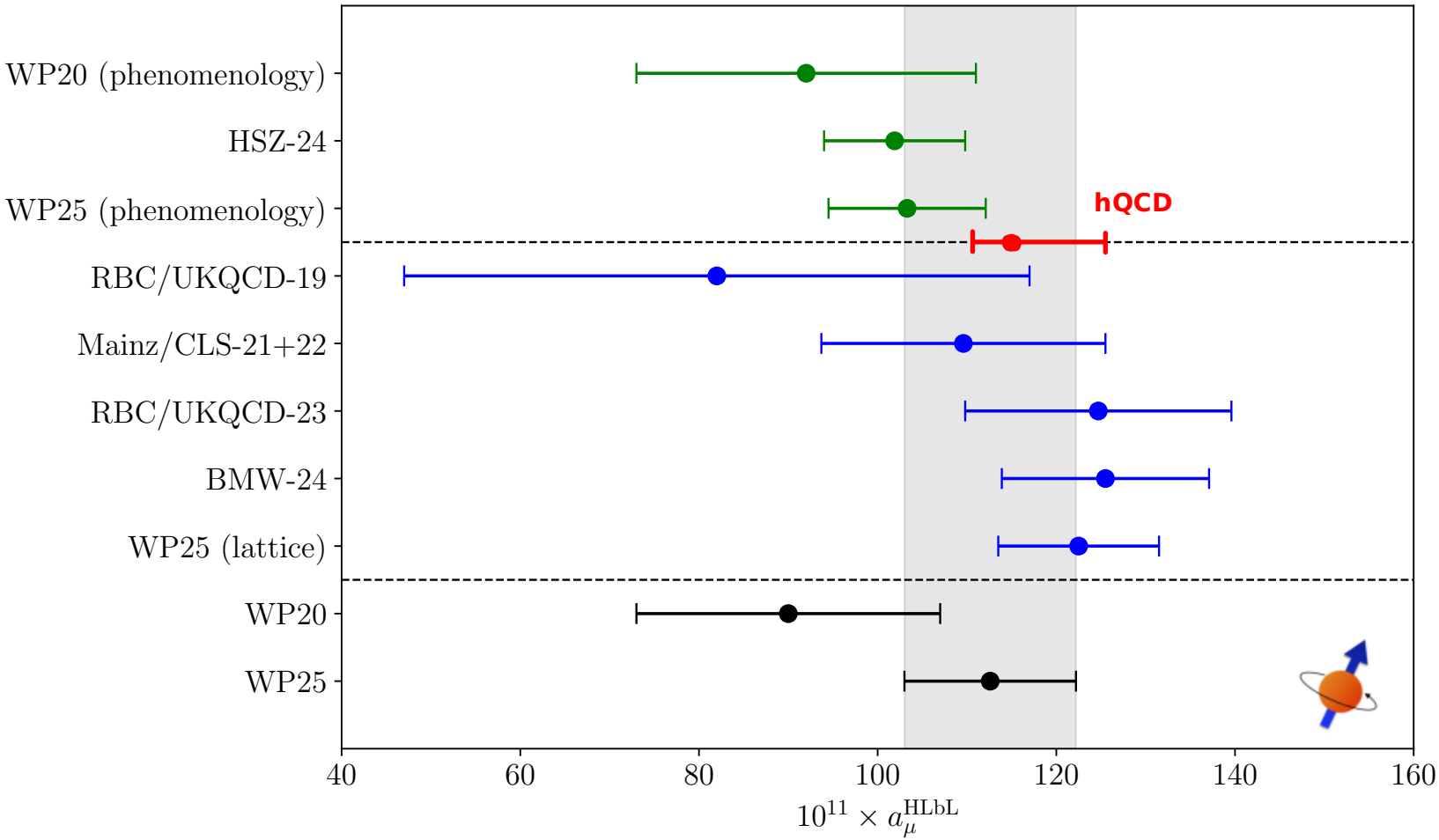
  Discrepancy with muon $g-2$ experiment resolved by accepting lattice QCD results (continued work will hopefully bring down errors further)

- **HLbL**: great progress WP20 $\rightarrow$ WP25

  *Smaller issue*: $S = 1.5$ tension between dispersive approach and lattice QCD in subleading HLbL contribution (will become important once HVP errors brought down)

  - according to hQCD: underestimated contributions from tensor mesons (even different sign)
  - according to new high-precision data from BESIII (and hQCD): also underestimated contributions from $\eta'$

# HLbL: Status according to WP2025 vs. hQCD




**hQCD:** completed by negative contribution from pseudoscalar boxes (dispersive)

favors lattice average over data-driven result

[Eq. (5.52) in WP25]