



Overview of precision QCD calculations

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- The motivation of pushing higher-order perturbative calculations is to improve the accuracy and precision of predictions in QFTs
- We know that the perturbative series is asymptotic and not convergent Dyson (PR'52)

Divergence of perturbation theory in quantum electrodynamics

F.J. Dyson (Cornell U., LNS)

1952

2 pages

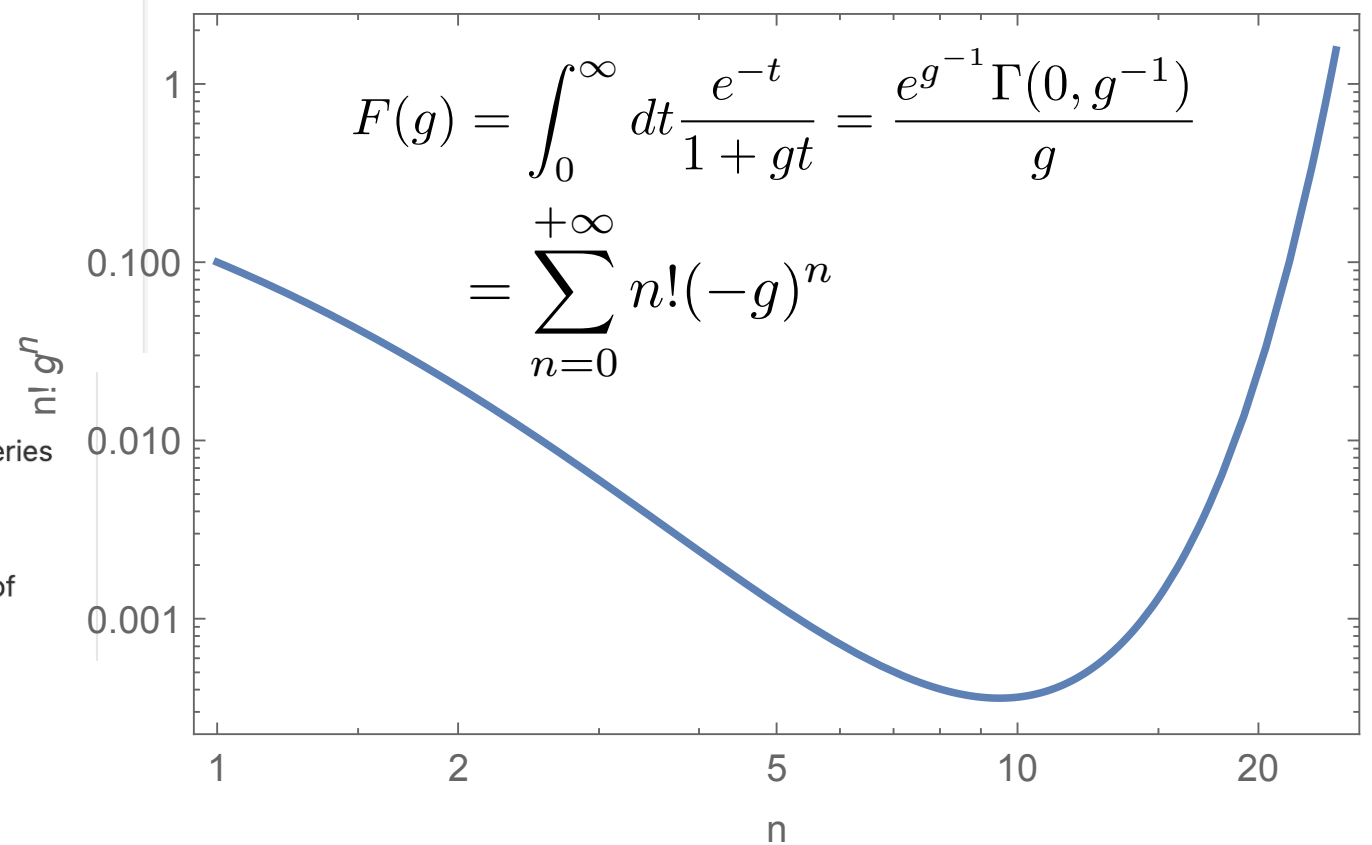
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DOI: [10.1103/PhysRev.85.631](https://doi.org/10.1103/PhysRev.85.631)

View in: [OSTI.GOV](https://osti.gov)

Abstract: (APS)

An argument is presented which leads tentatively to the conclusion that all the power-series expansions currently in use in quantum electrodynamics are divergent after the renormalization of mass and charge. The divergence in no way restricts the accuracy of practical calculations that can be made with the theory, but raises important questions of principle concerning the nature of the physical concepts upon which the theory is built.



- Disclaimer: I have personal bias in presenting results, and references cited in this talk are also selective. I apologize for any omissions.

- In a standard NLO QCD computation:

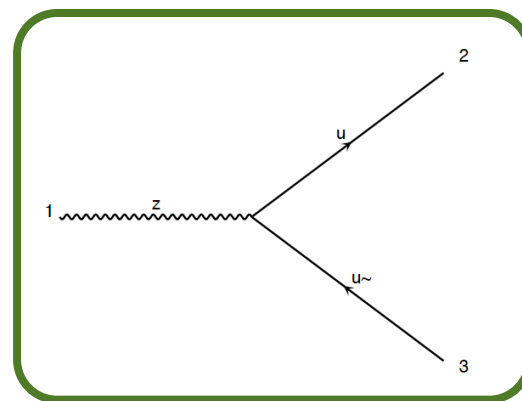
$$\sigma_{\text{NLO}} = \int_{\mathcal{O}(\alpha_s^b)} d\phi_n \mathcal{B} + \int_{\mathcal{O}(\alpha_s^{b+1})} d\phi_n \mathcal{V} + \int_{\mathcal{O}(\alpha_s^{b+1})} d\phi_{n+1} \mathcal{R}$$

Prominent issues:

1. Number of Feynman diagrams explodes as n !
2. IR divergences in real phase space integration
3. Virtual evaluation is (too) slow



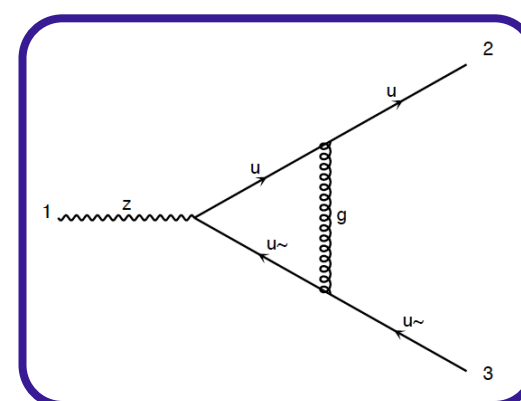
Born
cross section



Finite



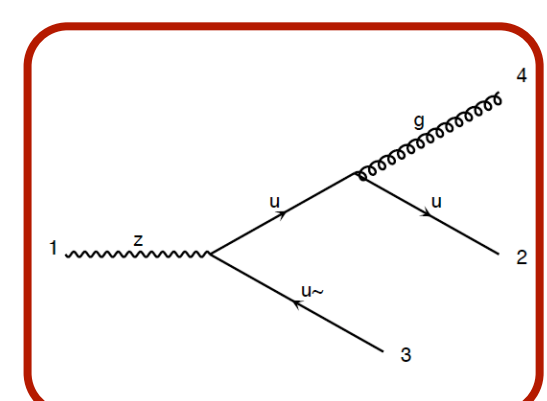
Virtual
correction



Divergent



Real
correction



Divergent

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$$\sigma_{\text{NLO}} = \int_{\mathcal{O}(\alpha_s^b)} d\phi_n \mathcal{B} + \int_{\mathcal{O}(\alpha_s^{b+1})} d\phi_n \mathcal{V} + \int_{\mathcal{O}(\alpha_s^{b+1})} d\phi_{n+1} \mathcal{R}$$

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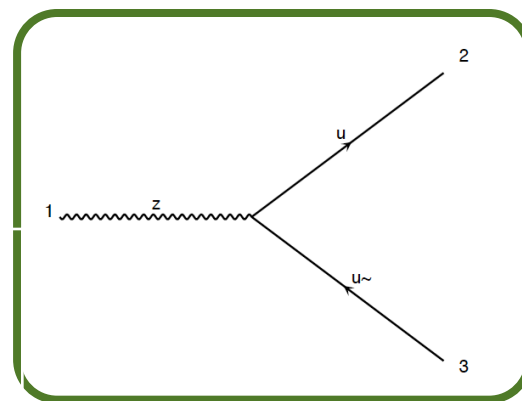
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✓ Automated !

~ 2010s



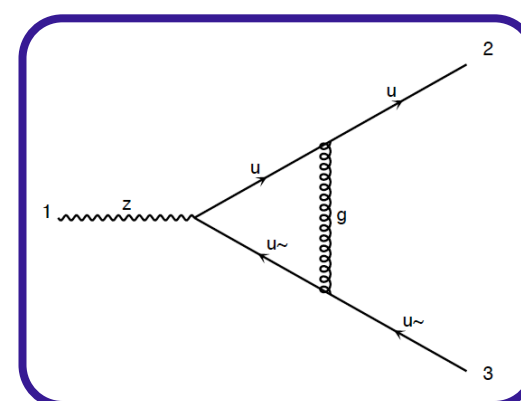
Born
cross section



Finite



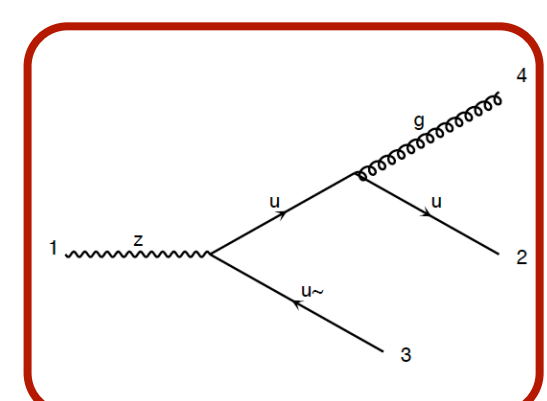
Virtual
correction



Divergent



Real
correction



Divergent

- **N^kLO (k>1) challenges in traditional/standard approaches**

$$\Delta\sigma_{\text{NNLO}} = \int d\Phi_{n+2} \mathcal{M}_{RR} + \int d\Phi_{n+1} \mathcal{M}_{RV} + \int d\Phi_n \mathcal{M}_{VV}$$

- N^kLO (k>1) challenges in traditional/standard approaches

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- Feynman integral reduction: *intermediate expression swell*
 - Most integrals can be reduced to a (much) smaller basis of master integrals via integration-by-parts identities
Laporta (hep-ph/0102033)
 - Such as Laporta's algorithm (an optimized Gauss elimination method)
 - ... simple and powerful but high computational costs
 - Active developments (syzygy equations, intersection numbers, finite field etc)
 - Many public tools (AIR, FIRE, Reduze, KIRA, FiniteFlow, LiteRed, NeatIBP, Blade etc)
 - A bottleneck for multiple-scale problems

- N^kLO (k>1) challenges in traditional/standard approaches

$$\Delta\sigma_{\text{NNLO}} = \int d\Phi_{n+2} \mathcal{M}_{RR} + \int d\Phi_{n+1} \mathcal{M}_{RV} + \boxed{\int d\Phi_n \mathcal{M}_{VV}}$$

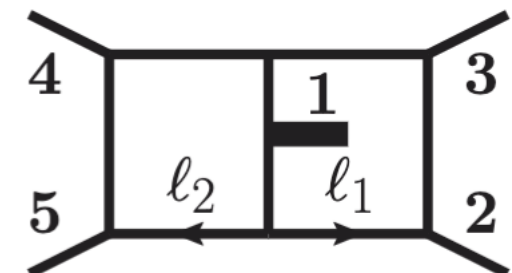
- Feynman integral reduction: *intermediate expression swell*

Badger et al. (arXiv:2409.08146)

family	deg.	# IBPs	# integrals	IBP disk size	running time
DPmz	5	26673	27432	71.4 MB	7h5m
DPzz	5	61777	63880	375.8 MB	21h54m*
HBmzz	5	15428	15916	17.0 MB	5h38m
HBzmz	5	10953	11289	13.4 MB	5h45m
HBzzz	5	21126	21766	38.0 MB	9h32m
PBmzz	5	10224	10329	7.8 MB	2h23m
PBzmz	5	11610	11791	6.5 MB	2h50m
PBzzz	5	8592	8752	5.8 MB	2h50m
HTmzzz	4	3120	3176	1.5 MB	1h12m
HTzmzz	4	6594	6650	2.5 MB	1h31m
HTzzzz	4	4680	4631	4.0 MB	2h31m

- Two-loop QCD for $q\bar{q}' \rightarrow W\gamma\gamma$

- DPzz family with NeatIBP



- 56 CPU threads & 1.5 TB RAM

- N^kLO (k>1) challenges in traditional/standard approaches

$$\Delta\sigma_{\text{NNLO}} = \int d\Phi_{n+2} \mathcal{M}_{RR} + \int d\Phi_{n+1} \mathcal{M}_{RV} + \boxed{\int d\Phi_n \mathcal{M}_{VV}}$$

- Feynman integral reduction: *intermediate expression swell*
- Master integrals: *special functions* (not even mathematically well understood)
 - Analytic structure becomes richer at two loops and beyond
 - Number of classes of special functions grows, which are not known fully.

$\mathcal{M} \ni$	multiple polylogarithms	<i>Two-loop di-jet</i>
	+ dlog one-forms	<i>Two-loop light-by-light</i>
	+ elliptic polylogarithms	<i>Two-loop top-quark pair</i>
	+ iterated integral on K3 surfaces	<i>Three-loop photon self-energy</i>
	+ ...	

- N^kLO (k>1) challenges in traditional/standard approaches

$$\Delta\sigma_{\text{NNLO}} = \int d\Phi_{n+2} \mathcal{M}_{RR} + \int d\Phi_{n+1} \mathcal{M}_{RV} + \boxed{\int d\Phi_n \mathcal{M}_{VV}}$$

- Feynman integral reduction: *intermediate expression swell*
- Master integrals: *special functions* (*not even mathematically well understood*)
 - Analytic structure becomes richer at two loops and beyond
 - Number of classes of special functions grows, which are not known fully.
 - Many new approaches (active subfield)
 - Fully numerical: sector decomposition, loop-tree duality, integrand-level sub.
 - Semi-numerical via DEs: DiffExp, Seasyde, AMFlow, LINE, ...
 - Full analytic: differential equations, Mellin-Barnes, etc

- N^kLO (k>1) challenges in traditional/standard approaches

$$\Delta\sigma_{\text{NNLO}} = \boxed{\int d\Phi_{n+2} \mathcal{M}_{RR} + \int d\Phi_{n+1} \mathcal{M}_{RV}} + \boxed{\int d\Phi_n \mathcal{M}_{VV}}$$

- Feynman integral reduction: *intermediate expression swell*
- Master integrals: *special functions* (not even mathematically well understood)
- IR divergences in phase space: *IR subtraction counter-terms*
 - Because of IR divergences, cannot do Monte Carlo integration over PS directly
 - The common wisdom is to introduce IR counter-terms
 - Matching IR singularities of real MEs but simpler to integrate in dim reg.
 - Challenges
 - Separation of overlapping singularities
 - Too many counter-terms to be constructed
 - Very high numerical accuracy required for phase-space mapping
 - Methods & tools: STRIPPER, antenna (NNLOjet), nest soft-collinear (history), CoLoRFulNNLO (NNLOCal), Torino, P2B, ..., also efforts to N³LO

“Numbers” of N^kLO Papers per year

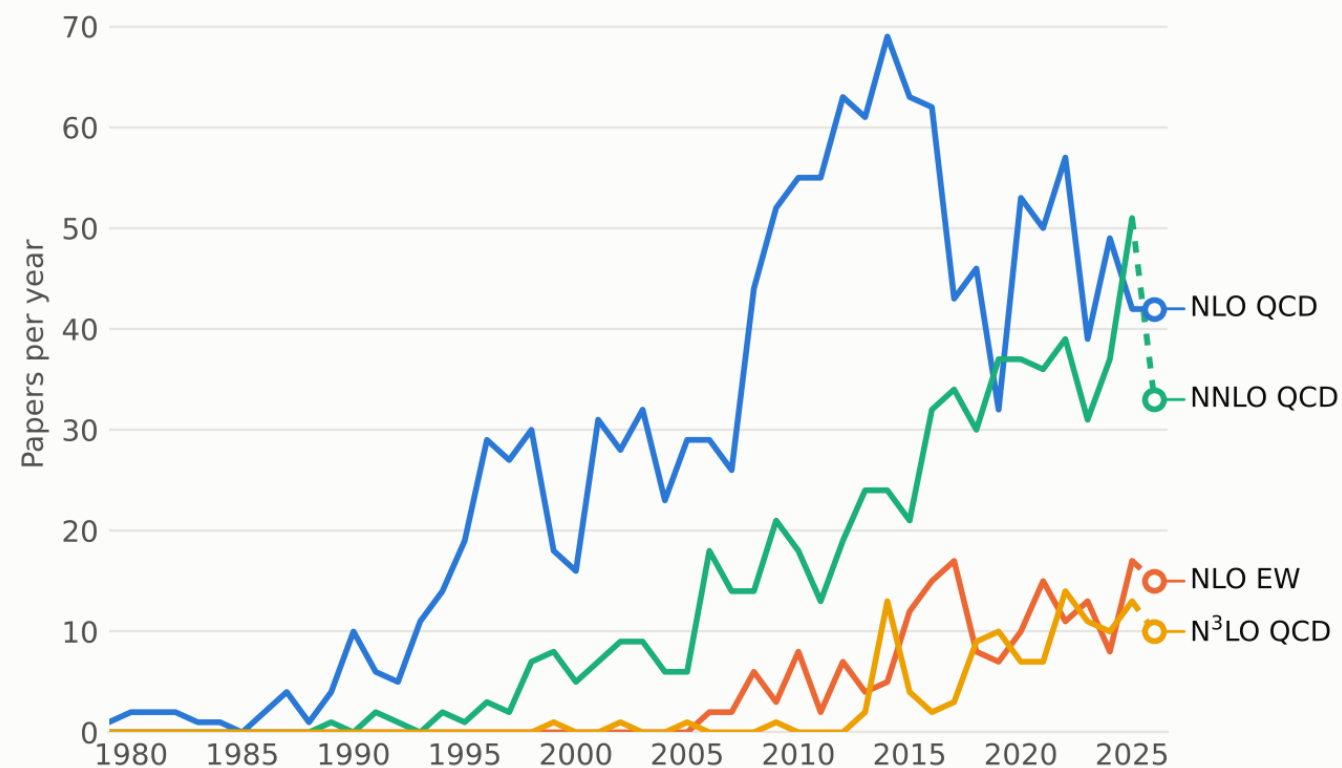
HEP papers whose main result is a higher-order calculation, by year

INSPIRE-HEP theory articles where the order is in the title, or the abstract says the authors computed it

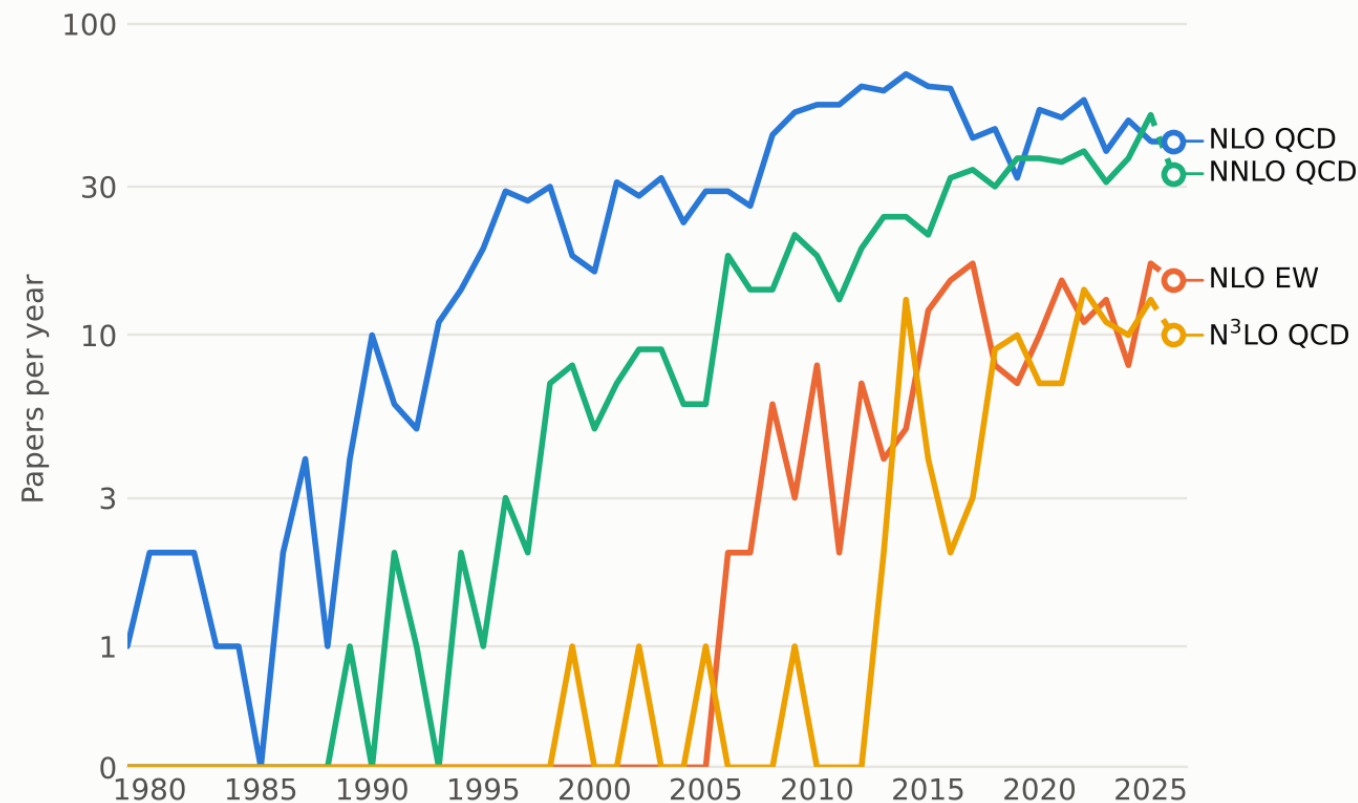
Plots made by Claude Opus 5.5 (Extra)

— NLO QCD — NLO EW — NNLO QCD — N³LO QCD

Linear scale



Log scale



Source: INSPIRE-HEP API, retrieved 2 Oct 2026. Of 10,318 records mentioning an order with QCD/EW, the 5,521 non-experimental articles and preprints were kept (no proceedings, theses, reviews). A paper counts if the order is in its title and tied to QCD/EW in title or abstract, or the abstract states the calculation ("we compute the NLO QCD corrections"). A paper can count in several series. Open marker and dashed segment: 2026 is incomplete (to 2 Oct). Totals: NLO QCD 1,380 · NLO EW 187 · NNLO QCD 652 · N³LO QCD 119.

• NNLO, N³LO & LI@NLO calculations between Sep 2025 - Oct 2026

Collider processes

Tables made by Claude Opus 5.5 (Extra)

Fixed-order NNLO and N³LO QCD calculations, September 2025 – October 2026

DATE	PROCESS	ORDER		AUTHORS	ARXIV
Sep 2026	Heavy-quark flavour flux correlator in e^+e^-	N ³ LO	analytic; forward-backward asymmetry	Lee, Moulton	2609.13366
Jun 2026	Boosted WH (Higgs-strahlung)	N ³ LO		Gehrmann-De Ridder et al.	2606.31286
Apr 2026	Photon-pair production	N ³ LO		Czakon et al.	2604.12613
Mar 2026	Electroweak Higgs-pair production (VBF, VHH)	N ³ LO	+ NLO EW for VBF; NNLO for VHH	Gröber et al.	2603.02764
Jan 2026	Higgs-pair production in gluon fusion, fully differential	N ³ LO	heavy-top limit with top-mass effects	Chen et al.	2601.19990
Mar 2026	$H \rightarrow b\bar{b}$ decay with massive b quarks	O(α_s^4)	top-Yukawa-induced contribution	Wang, Wang, Wang	2603.18576
Aug 2026	WW γ production	NNLO		Garbarino, Grazzini, Kallweit	2608.14419
Jun 2026	$t\bar{t}W$ production	NNLO	generalised leading-colour two-loop	Becchetti et al.	2606.09503
May 2026	Inclusive $c\bar{c}$ and $b\bar{b}$ cross sections at hadron colliders	NNLO	new MaunaKea code	d'Enterria et al.	2605.16019
Feb 2026	Four-jet rate in e^+e^- annihilation	NNLO	O(α_s^4)	Chen et al.	2602.18185
Feb 2026	Hadron-in-jet production in $e^+e^- \rightarrow 2$ and 3 jets	NNLO		Bonino et al.	2602.12344
Dec 2025	Photon and jet final states at the LHC	NNLO	subleading-colour two-loop effects	Czakon, Poncelet	2512.17591
Dec 2025	Dijets in e^+e^- and in $H \rightarrow b\bar{b}$ decays	NNLO	new non-local subtraction scheme	Buonocore et al.	2512.03954
Nov 2025	W $\gamma\gamma$ production	NNLO		Garbarino et al.	2511.20581
Nov 2025	Top-quark pair plus jet	NNLO	two-loop amplitudes in leading colour	Badger et al.	2511.11431
Oct 2025	Doubly polarised WZ production	NNLO	QCD + NLO EW	Pelliccioli, Poncelet	2510.25898
Oct 2025	W boson plus charm meson	NNLO		Generet, Poncelet, Muškinja	2510.24525
Oct 2025	Jet observables in ZH production at e^+e^- colliders	NNLO		Caletti, Gehrmann-De Ridder, Marcoli	2510.20485
Sep 2025	$\gamma\gamma \rightarrow Q\bar{Q}$ heavy-quark pair production	NNLO	with Coulomb resummation and NLO EW	Capatti et al.	2509.13439

Source: INSPIRE-HEP, records retrieved 2 Oct 2026

• NNLO, N³LO & LI@NLO calculations between Sep 2025 - Oct 2026

Lepton–hadron scattering

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Fixed-order NNLO and N³LO QCD calculations, September 2025 – October 2026

DATE	PROCESS	ORDER	AUTHORS	ARXIV
Sep 2026	Polarised semi-inclusive DIS	N ³ LO	Dong et al.	2609.13743
Mar 2026	Semi-inclusive DIS via two-dimensional q_T subtraction	N ³ LO	Dong et al.	2603.29673
Sep 2025	Single-mass heavy-flavour contributions to F_2 and g_1 at large Q^2	N ³ LO three-loop	Ablinger et al.	2509.16124
Jul 2026	Pion- and kaon-induced exclusive Drell-Yan	NNLO	Jia et al.	2607.15214
Jun 2026	Heavy-quark contributions to charged-current DIS	NNLO	Caola, Gambuti, Link	2606.21702
Apr 2026	Deeply virtual pion production	NNLO	Chen et al.	2604.28164
Mar 2026	Electroweak structure functions in semi-inclusive DIS	NNLO	Goyal et al.	2603.30012
Mar 2026	Dimuon production in neutrino–nucleus collisions	NNLO	Helenius, Paukkunen, Yrjänheikki	2603.09646
Feb 2026	Hadron production in DIS at finite transverse momentum	NNLO	Dong et al.	2602.22972
Feb 2026	Semi-inclusive DIS via q_T subtraction, fully differential	NNLO	Gao et al.	2602.06364
Jan 2026	Heavy-quark-initiated charged-current DIS coefficient functions	NNLO	Kudashkin	2601.02916
Oct 2025	QCD⊗QED corrections to semi-inclusive DIS	NNLO	Goyal et al.	2510.18872
Sep 2025	Polarised neutral- and charged-current semi-inclusive DIS	NNLO	Bonino et al.	2510.00100

Source: INSPIRE-HEP, records retrieved 2 Oct 2026

• NNLO, N³LO & LI@NLO calculations between Sep 2025 - Oct 2026

Heavy-flavour decays and quarkonium

Tables made by Claude Opus 5.5 (Extra)

Fixed-order NNLO and N³LO QCD calculations, September 2025 – October 2026

DATE	PROCESS	ORDER	AUTHORS	ARXIV
Mar 2026	$B \rightarrow X_s \gamma$ photon-energy spectrum, light-fermion and large- N_c parts	N ³ LO	Fael et al.	2603.15751
Feb 2026	Triple-differential heavy-to-light semileptonic decays	N ³ LO	Chen et al.	2602.11537
Feb 2026	Heavy-to-light decay structure functions at $O(\alpha_s^3)$	N ³ LO	Chen et al.	2602.11879
Sep 2026	Toponium decay $\eta_t \rightarrow HZ$	NNLO	Jia et al.	2609.34364
Aug 2026	$B \rightarrow X_s \gamma$ with exact charm-mass dependence	NNLO	Czaja et al.	2608.29864
Aug 2026	Inclusive $B \rightarrow X_s \gamma$ decay rate, upgraded analysis	NNLO	Misiak et al.	2608.29863
Jul 2026	b-baryon lifetimes	NNLO	Lenz, Piscopo, Rusov	2607.06538
Jun 2026	S-wave quarkonium production and decay, soft contributions	NNLO	Maxia et al.	2606.28494
Jun 2026	D-wave quarkonium decay $\eta_{Q2} \rightarrow \gamma\gamma$, with PMC scale setting	NNLO	Zhou et al.	2606.24455
Jun 2026	$^3S_1^{(8)}$ gluon fragmentation function for quarkonium	NNLO	Feng, Jia, Sang	2606.23594
Jun 2026	$e^+e^- \rightarrow J/\psi + \chi_{cJ}$ at B factories	NNLO two-loop	Liu, Li, Sang	2606.02182
Apr 2026	B and D meson lifetime ratios	NNLO	Moretti et al.	2604.24841
Mar 2026	$J/\psi \rightarrow 3\gamma$ decay	NNLO	Zeng et al.	2603.26199
Jan 2026	Inclusive semileptonic b-quark decay structure functions	NNLO	Broggio et al.	2601.15447
Dec 2025	$B \rightarrow \pi$ form factors from light-cone sum rules	NNLO	Huang et al.	2512.18866
Dec 2025	B-meson mixing decay matrix at leading power	NNLO	Nierste et al.	2512.07949
Dec 2025	$e^+e^- \rightarrow \eta_c/\chi_{cJ} + \gamma$ at B factories	NNLO	Li, Sang, Zhang	2512.04758
Oct 2025	$e^+e^- \rightarrow Z^* \rightarrow J/\psi + J/\psi$	NNLO two-loop	Chen et al.	2510.10261
Sep 2025	η_Q transition form factor, with all-order v^2 resummation	NNLO	Babiarz et al.	2509.15310

Source: INSPIRE-HEP, records retrieved 2 Oct 2026

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Loop-induced processes

Fixed-order calculations at NLO QCD, NLO EW and beyond, September 2025 – October 2026

DATE	PROCESS	ORDER	AUTHORS	ARXIV
Sep 2026	$gg \rightarrow \gamma\gamma$, complete electroweak corrections	NLO EW	Chen et al.	2609.18828
Mar 2026	Light-by-light scattering $\gamma\gamma \rightarrow \gamma\gamma$	NLO QCD + NLO QED, with Coulomb resummation	Abdul Hameed, Chaubey, Shao	2604.00099
Mar 2026	Light-by-light scattering $\gamma\gamma \rightarrow \gamma\gamma$, massless	NNLO three-loop QCD and QED	Bargiela et al.	2603.22423
Mar 2026	$gg \rightarrow ZH$ with leptonic Z decays	NLO QCD with parton-shower interface	Davies et al.	2603.15762
Mar 2026	$\eta_b \rightarrow J/\psi \gamma$ decay	NLO QCD two-loop for the QCD-initiated amplitude	Yang, Zuo, Long	2603.14847
Mar 2026	$gg \rightarrow HH$ at high energies	NLO EW form factors, high-energy expansion	Davies et al.	2603.08789
Jan 2026	$q\bar{q} \rightarrow HH$ via electroweak loops	NLO QCD	Bonetti et al.	2601.16924
Dec 2025	$gg \rightarrow HH$	NLO EW top-Yukawa- and light-quark-induced parts	Bhattacharya et al.	2512.14823
Nov 2025	$gg \rightarrow ZH$ in the large- m_t limit	NNLO three-loop virtual corrections only	Davies et al.	2512.00156
Oct 2025	$H \rightarrow \gamma\gamma$ decay	QCD×EW three-loop, $O(\alpha\alpha_s)$	Sang, Feng, Jia	2510.25516
Oct 2025	$gg \rightarrow \gamma\gamma$, first two quark generations	NLO EW	Fiore, Williams	2510.21643
Sep 2025	$gg \rightarrow ZH$ and off-shell $gg \rightarrow ZZ$	NLO QCD two-loop virtual corrections only	Davies et al.	2509.07072

Source: INSPIRE-HEP, records retrieved 2 Oct 2026

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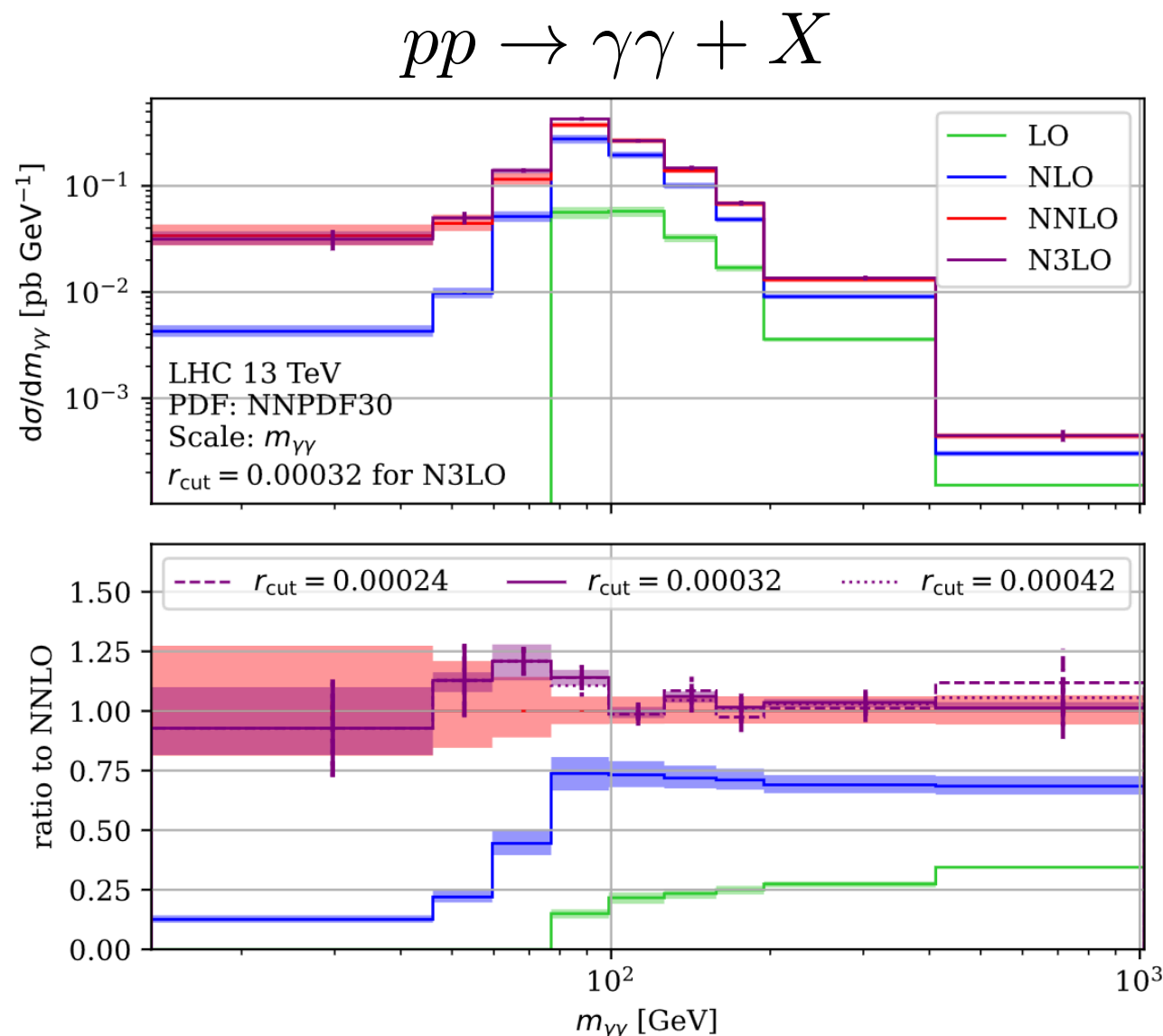
Source: INSPIRE-HEP, records retrieved 2 Oct 2026

Disclaimer: It is an active subfield. I will not go through one by one, but instead ...

I will only select a few examples to stress some of my points.

- At the borderline of available computing resources for research
 - A fully differential N³LO calculation for an LHC process usually takes > million CPU hours
 - For example, the first fully differential 2->2 N³LO calculation

Czakon, Eschment, Generet & Poncelet (2604.12613)



- CPU hours:
 - RRV: 2.7M
 - RRR: 5.2M
 - RVV: 0.6M
- Statistical uncertainties are comparable as NNLO scale uncertainties and are larger than N³LO scale uncertainties.
- Improvements are possible, such as using a local N³LO IR subtraction scheme.

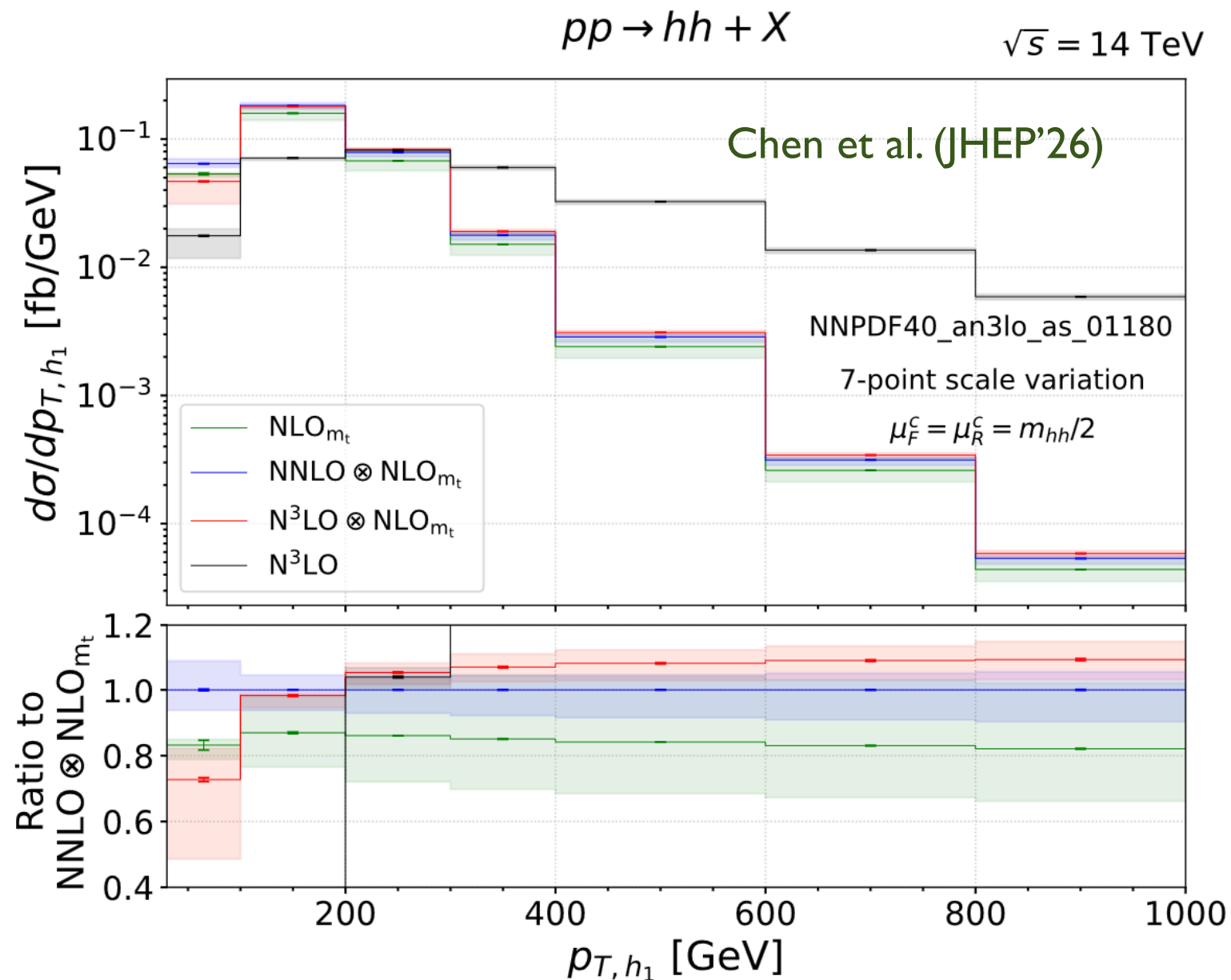
Challenges: e.g., huge expressions for CTs

Del Duca et al. (1912.06425, 2007.05345)

https://people.phys.ethz.ch/~pheno/quadruple_collinear/

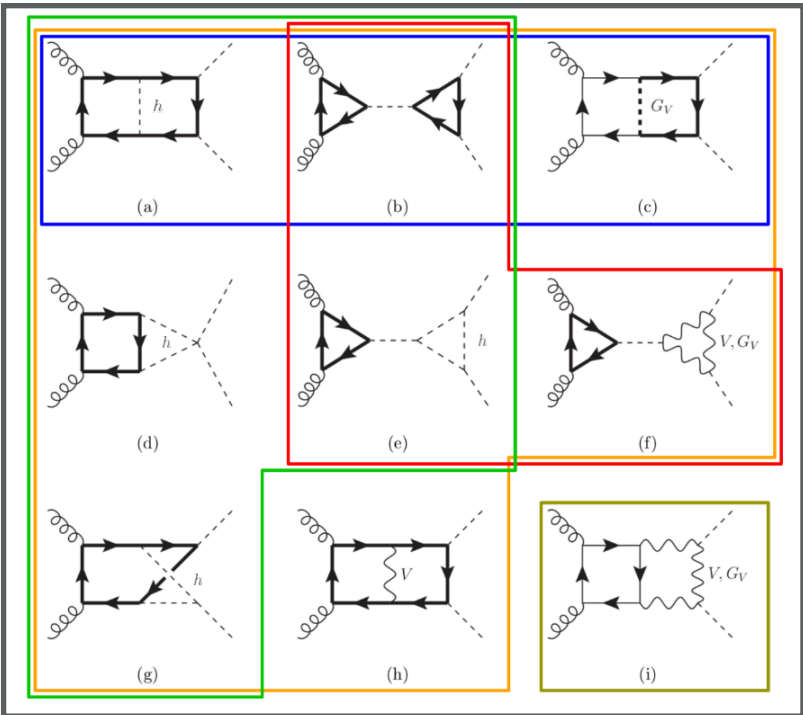
- How can a given computation be useful in experiments ?

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- Inclusive N³LO and N³LO+N³LL known earlier
Chen et al. (PLB'20,JHEP'20),
Ajjath et al. (JHEP'23)
- Fully differential N³LO in heavy-top limit
- NNLOJet+MG5aMC+ggxy+qT slicing
- 1-2 week run w/ ~6000 threads

- How can a given computation be useful in experiments ?



Electroweak corrections to $gg \rightarrow hh$ at two loops

Breakdown of the individual contributions

SECTION	DIAGRAM SETS	KEY REFERENCES
● Full SM electroweak corrections	(a) to (i)	Bi, Huang, Huang, Ma, Yu Phys. Rev. Lett. 132 (2024) 231802 · arXiv:2311.16963
● Top Yukawa-induced electroweak corrections	(a), (b), (c)	Mühlleitner, Schlenk, Spira JHEP 10 (2022) 185 · arXiv:2207.02524 Bhattacharya, Campanario, Carlotti, Chang, Mazzitelli, Mühlleitner, Ronca, Spira arXiv:2512.14823
● Top quark contributions	(a) to (h)	Davies, Schönwald, Steinhauser, Zhang JHEP 10 (2023) 033 · arXiv:2308.01355 Davies, Schönwald, Steinhauser, Zhang arXiv:2603.08789
● Factorizable contributions	(b), (e), (f)	Mühlleitner, Schlenk, Spira JHEP 10 (2022) 185 · arXiv:2207.02524 Zhang, Schönwald, Steinhauser, Davies PoS LL2024 (2024) 014 · arXiv:2407.05787
● Top Yukawa and Higgs boson self-coupling corrections	(a), (b), (d), (e), (g)	Davies, Mishima, Schönwald, Steinhauser, Zhang JHEP 08 (2022) 259 · arXiv:2207.02587 Davies, Schönwald, Steinhauser, Zhang JHEP 04 (2025) 193 · arXiv:2501.17920 Heinrich, Jones, Kerner, Stone, Vestner JHEP 11 (2024) 040 · arXiv:2407.04653
● Light-quark contributions	(i)	Bonetti, Rendler, Torres Bobadilla JHEP 07 (2025) 024 · arXiv:2503.16620 Bhattacharya, Campanario, Carlotti, Chang, Mazzitelli, Mühlleitner, Ronca, Spira arXiv:2512.14823

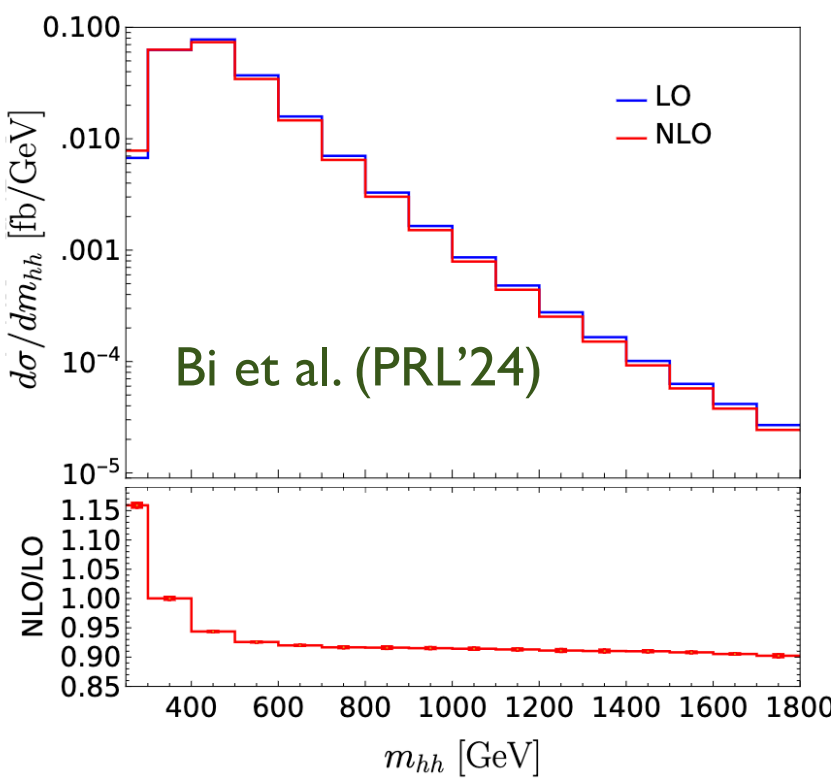


Table remade by Claude Opus 5.5 (Extra)

Adapted from Table 8 of arXiv:2609.04868; diagram sets refer to Fig. 21 of that paper

- How can a given computation be useful in experiments ?

Ajjath et al. (arXiv:2609.04868)

SciPost Physics Community Reports

Submission

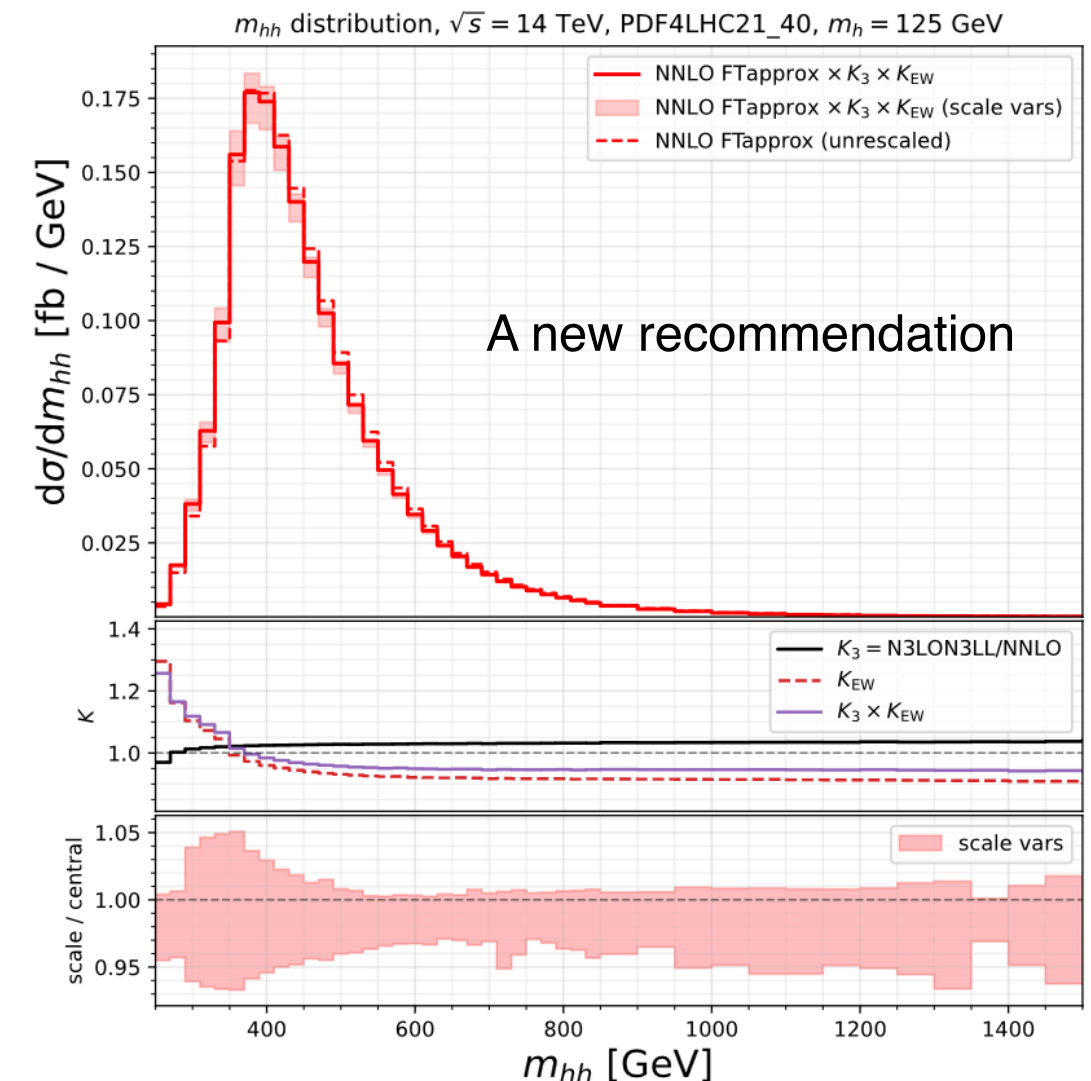
LHCHWG-2026-010

Higgs Boson Pair Production via Gluon Fusion: Higher-Order Corrections and Theoretical Uncertainties

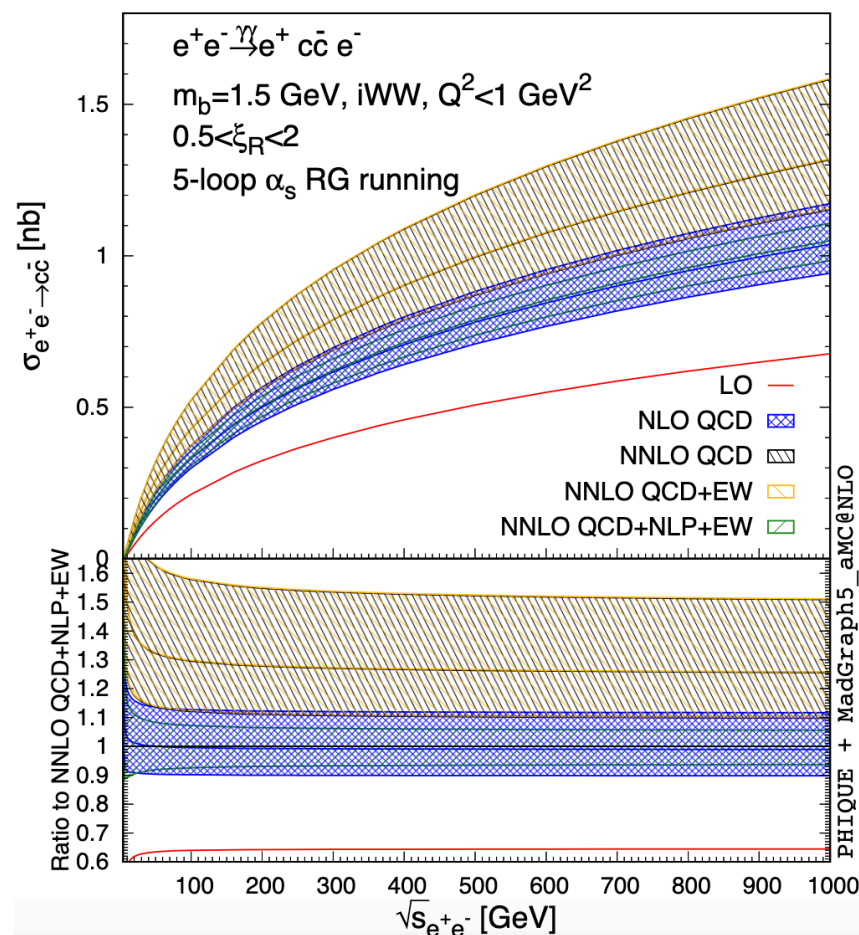
Ajjath A H^{1,2}, Simone Alioli³, Emanuele Bagnaschi⁴, Arunima Bhattacharya⁵, Huan-Yu Bi⁶, Roberto Bonciani⁷, Marco Bonetti^{8,9}, Francisco Campanario⁵, Sauro Carlotti⁹, Jamie Chang^{10,11}, Long-Bin Chen¹², Xuan Chen¹³, Yuesheng Dai¹³, Joshua Davies¹⁴, Giuseppe Degraassi¹⁵, Pier Paolo Giardino¹⁶, Massimiliano Grazzini¹⁷, Ramona Gröber¹⁸, Gudrun Heinrich⁹, Li-Hong Huang¹⁹, Rui-Jun Huang¹⁹, Sebastian Jaskiewicz²⁰, Stephen Jones¹, Stefan Kallweit¹⁷, Matthias Kerner⁹, Hai Tao Li¹³, Shi-Yuan Li¹³, Jonas M. Lindert²¹, Yan-Qing Ma¹⁹, Giulia Marinelli²², Javier Mazzitelli¹⁰, Margarete Mühlleitner⁹, Davide Napoletano³, Philipp Rendler⁹, Jonathan Ronca¹⁸, Johannes Schlenk²³, Kay Schönwald^{17,28}, Hua-Sheng Shao²⁴, Michael Spira¹⁰, Thomas Stone^{1,25}, Daniel Stremmer²⁶, Robert Szafron²⁷, William J. Torres Bobadilla¹⁴, Yannick Ulrich¹⁴, Augustin Vestner⁹, Marco Vitti²⁶, Jian Wang¹³, Huai-Min Yu³ and Hantian Zhang²⁸

Editors: Fabio Monti²⁹, Arantxa Ruiz Martínez³⁰, Angela Taliencio³¹, Andreas Papaefstathiou³² and Ludovic Scyboz³³

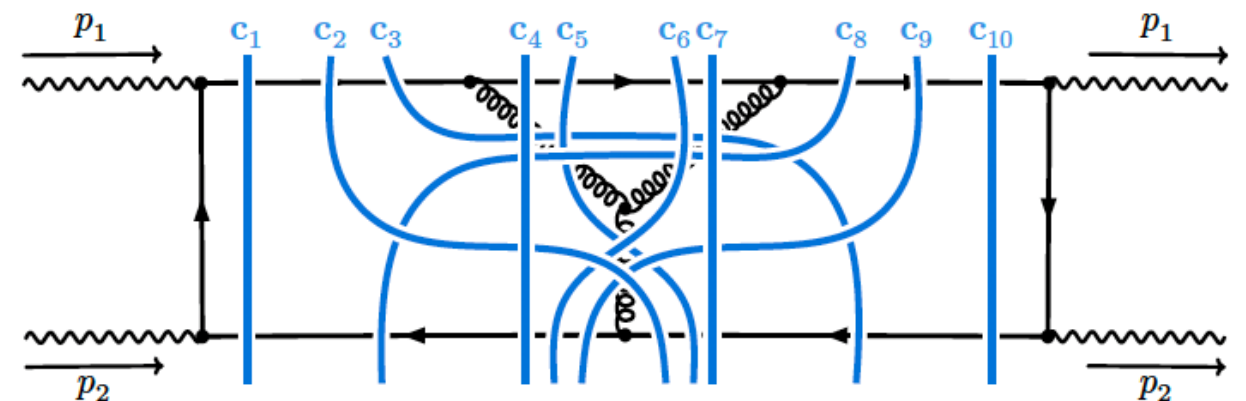
$\sqrt{s}$ [TeV]	13	13.6	14
$\sigma_{\text{rec.}}$ [fb]	30.40	33.62	35.81
$\pm$ PDF unc. [%]	1.96	1.92	1.89
$\pm \alpha_s$ unc. [%]	1.51	1.49	1.47
$\pm$ PDF + α_s unc. [%]	2.47	2.43	2.39
QCD scale unc. [%]	+2.11 -4.98	+2.07 -4.85	+2.01 -4.78
m_t scheme unc. [%]	+5 -16	+5 -16	+5 -16
$\pm$ TH _{num} [%]	0.14	0.19	0.21
Scale + TH _{num} + m_t scheme unc. [%]	+7.25 -21.12	+7.26 -21.04	+7.22 -20.99



- **A promising alternative: Local Unitarity** Capatti et al. (JHEP'20, JHEP'22)
Talk by Valentin Hirschi at MadGraph 2025 Meeting
- Leveraging the Kinoshita-Lee-Nauenberg theorem at integrand level
- One stone three birds
 - No need of Feynman integral reduction
 - No need of special functions (Monte Carlo integration instead)
 - No need of (final-state) IR divergence subtraction counter-terms
- Fully differential (in principle, if threshold singularities are under control)

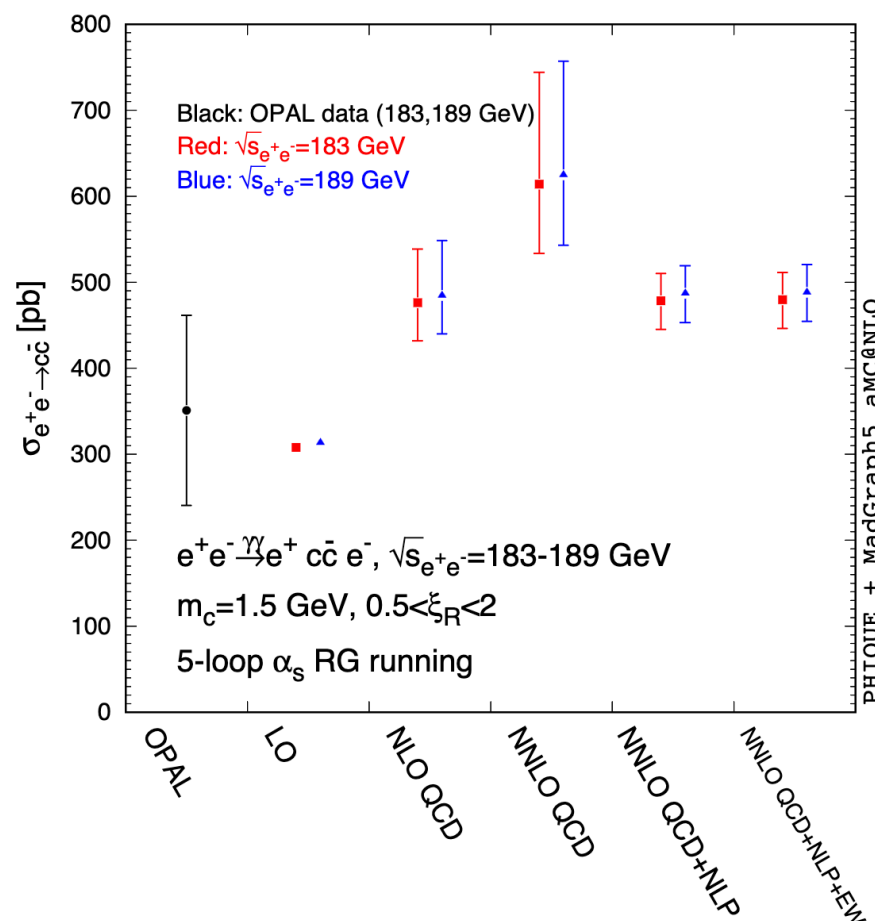


Capatti et al. (JHEP'26, arXiv:2509.13439)

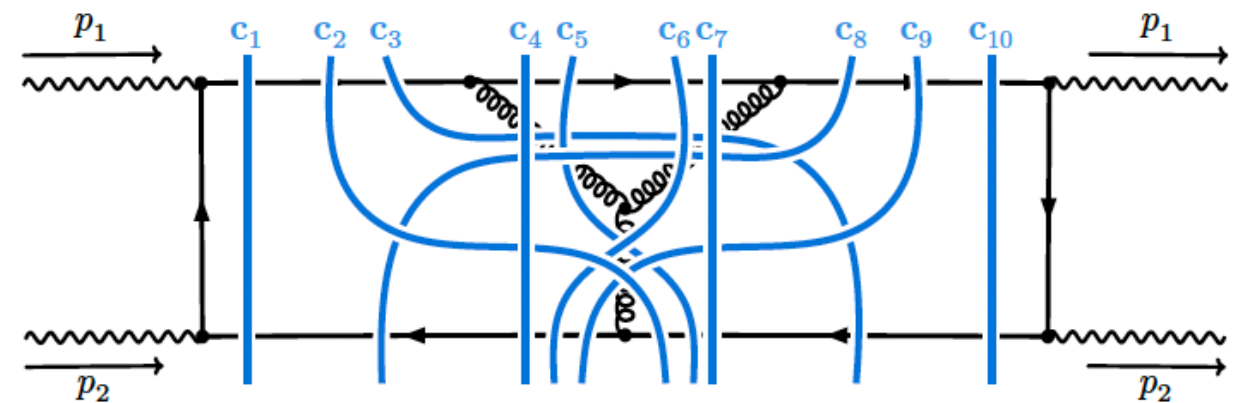


+ 137 other forward scattering graphs

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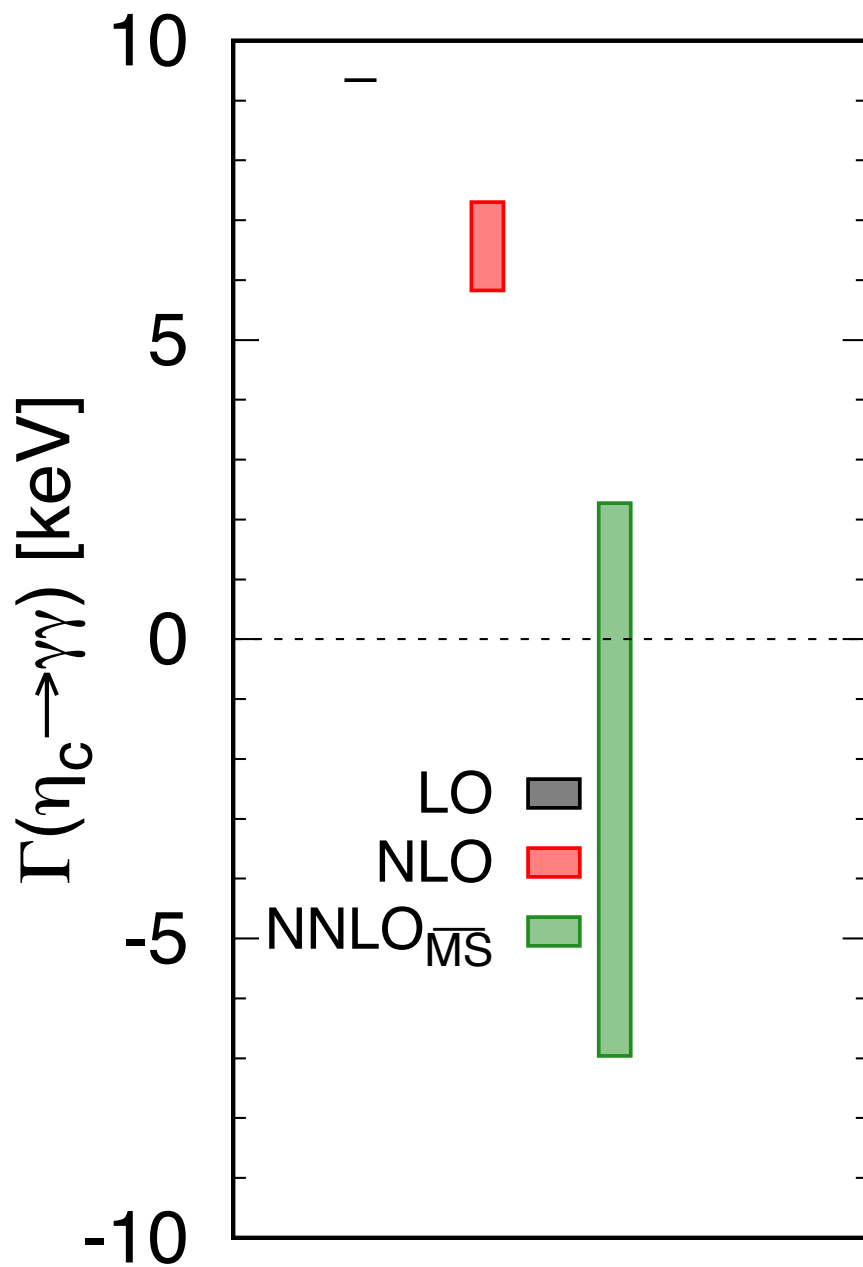
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What if $N^k\text{LO}$ behaves surprisingly

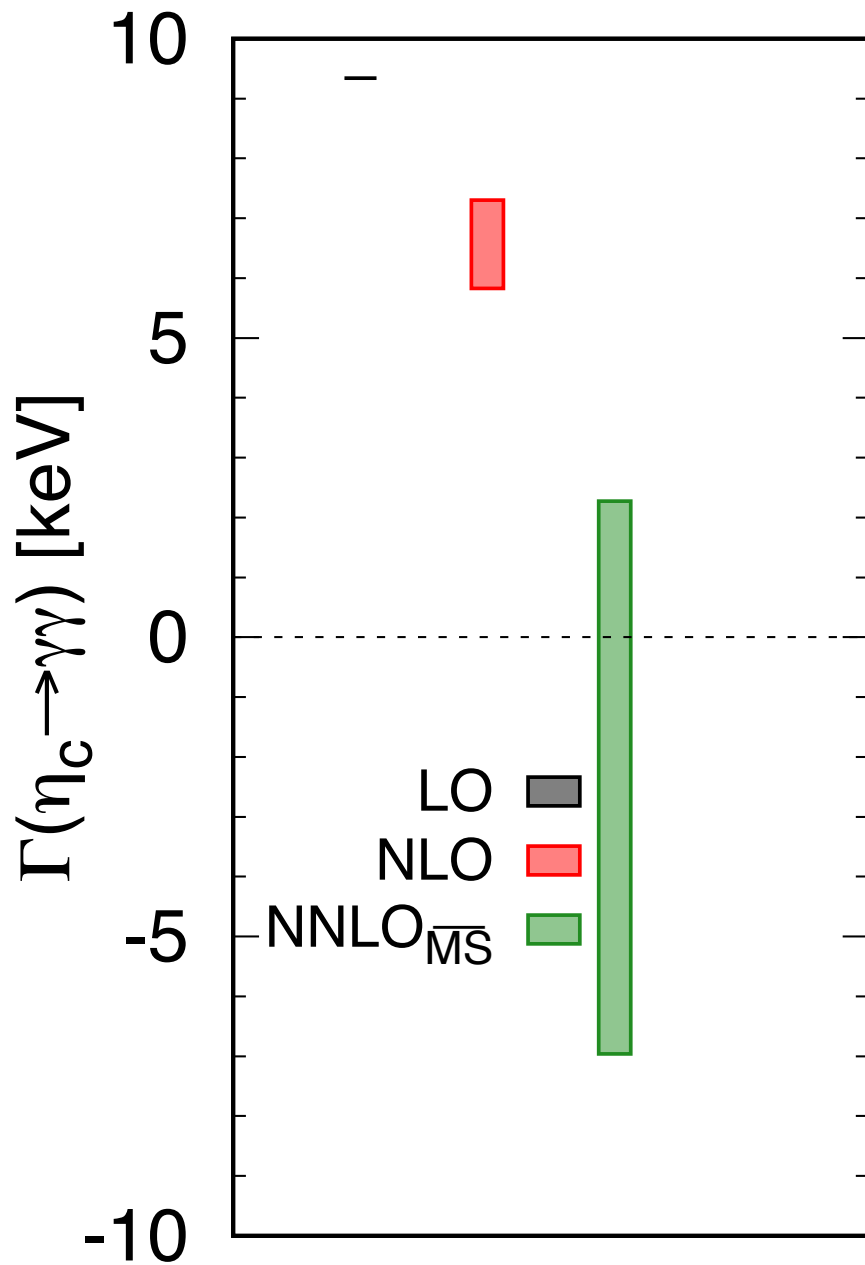
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$$\alpha_s(2m_c) \approx 0.25$$

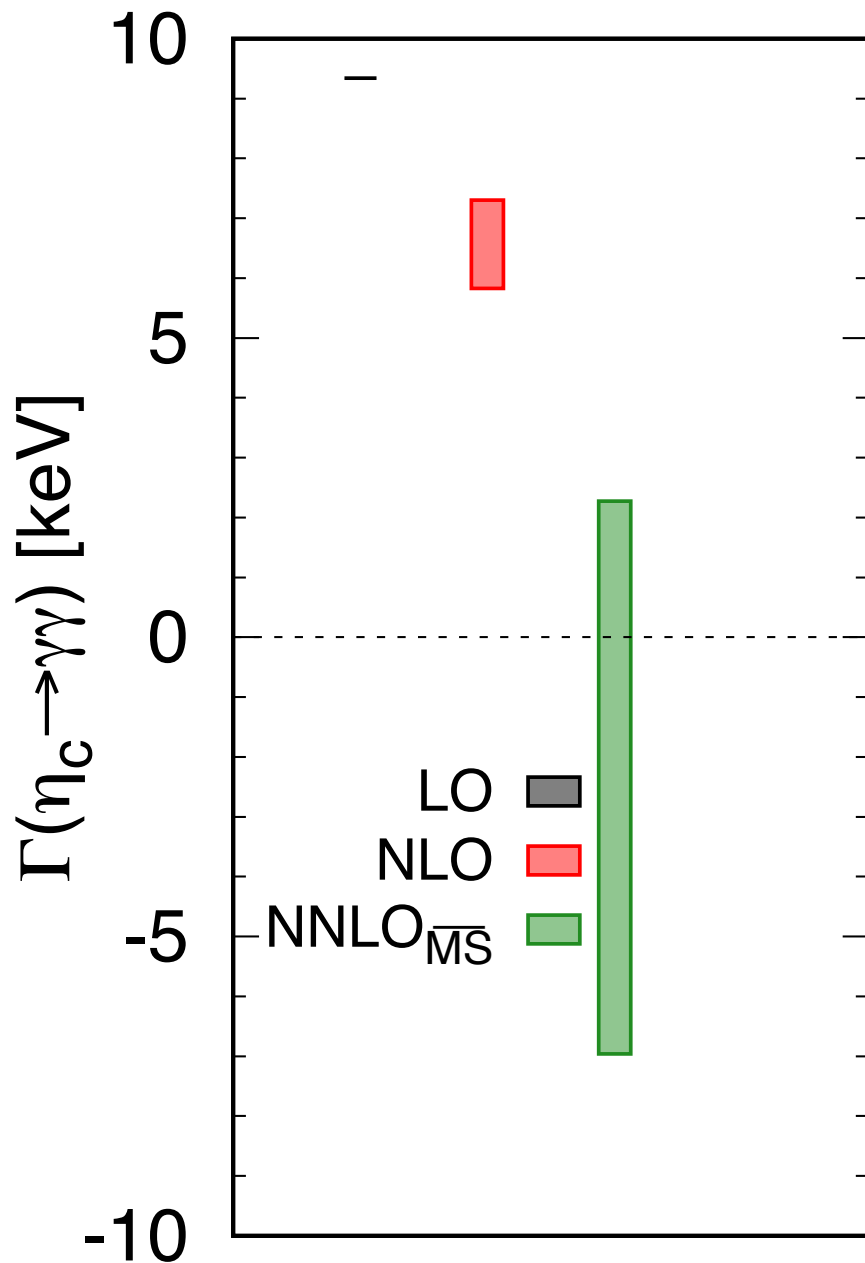


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$$\alpha_s(2m_b) \approx 0.18$$



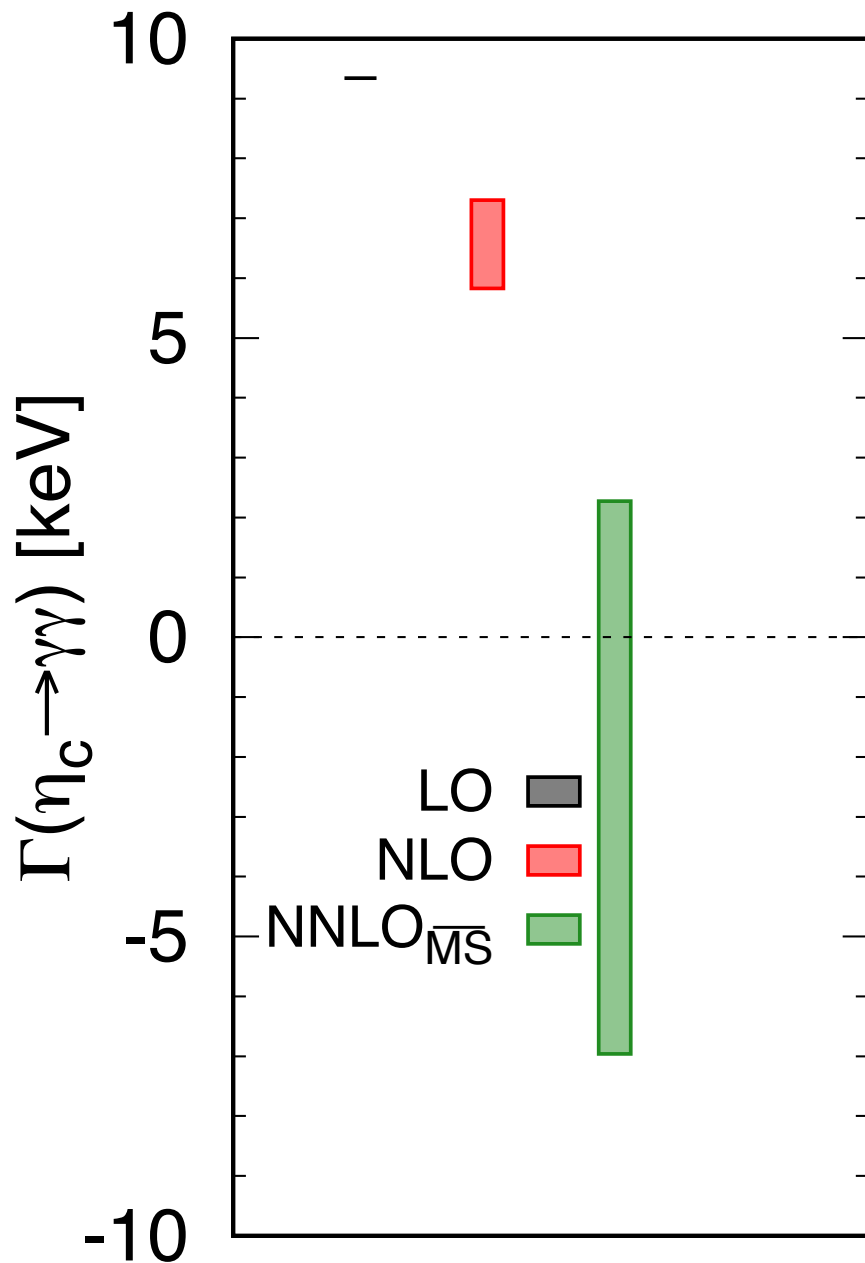
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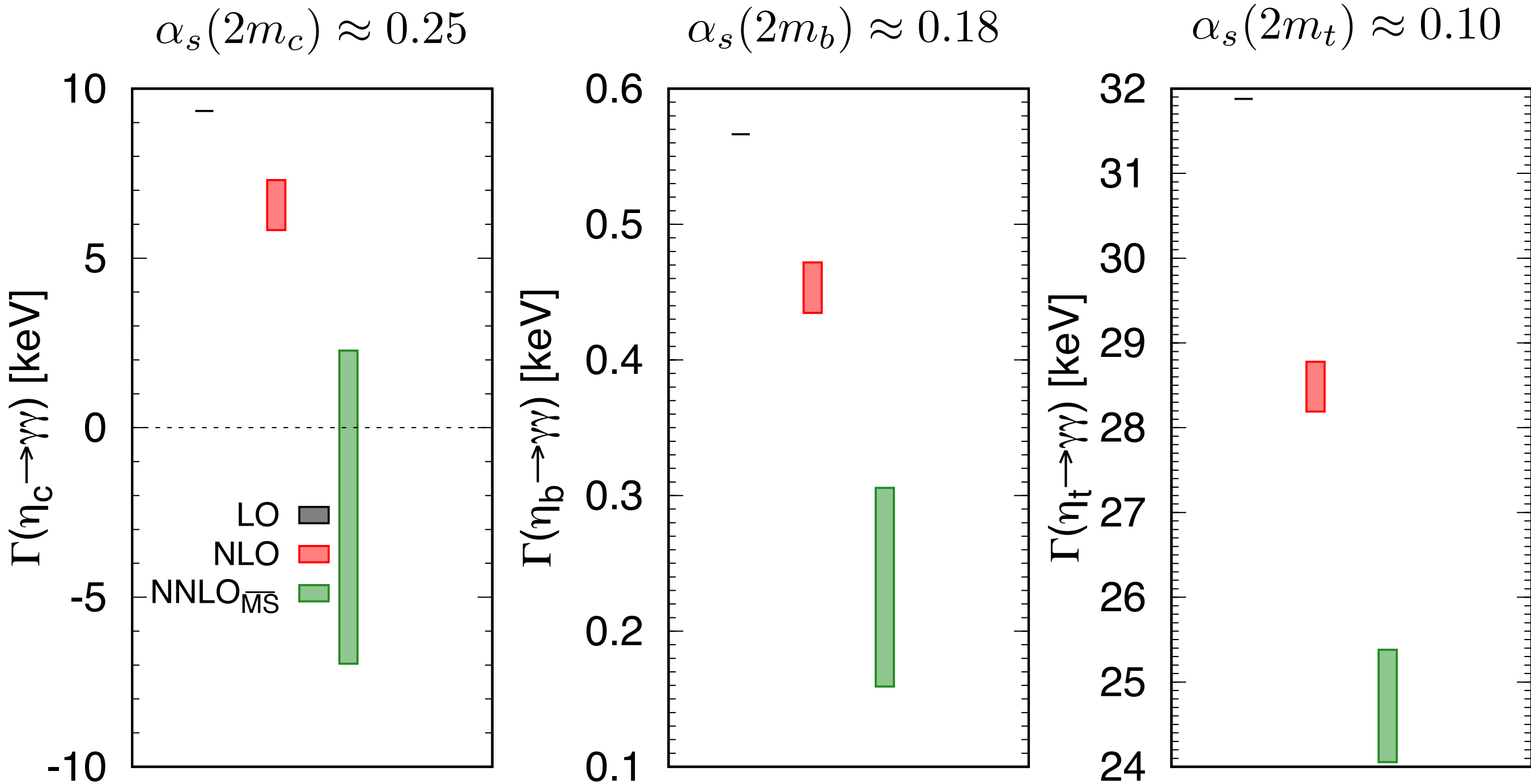
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$$\alpha_s(2m_t) \approx 0.10$$



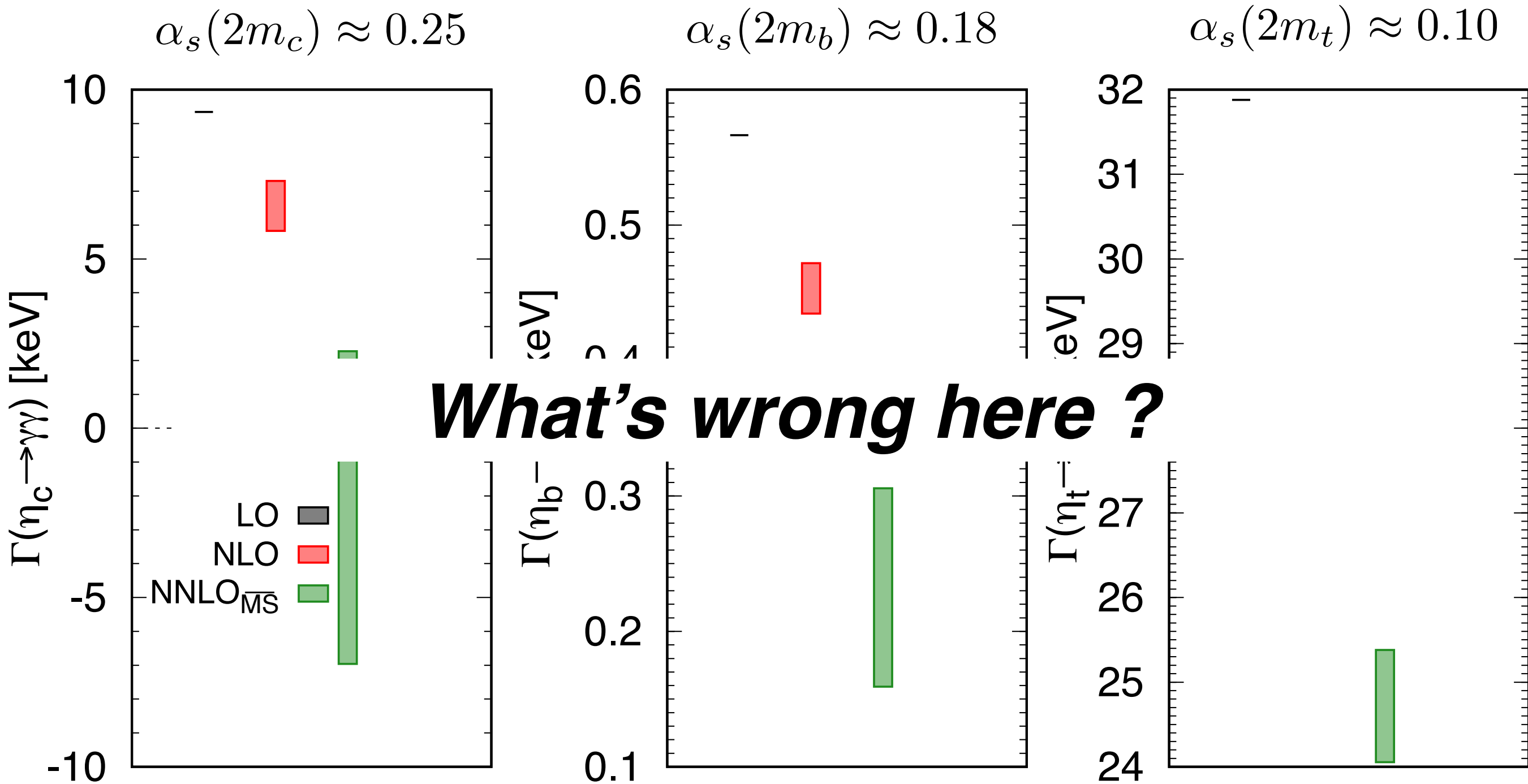
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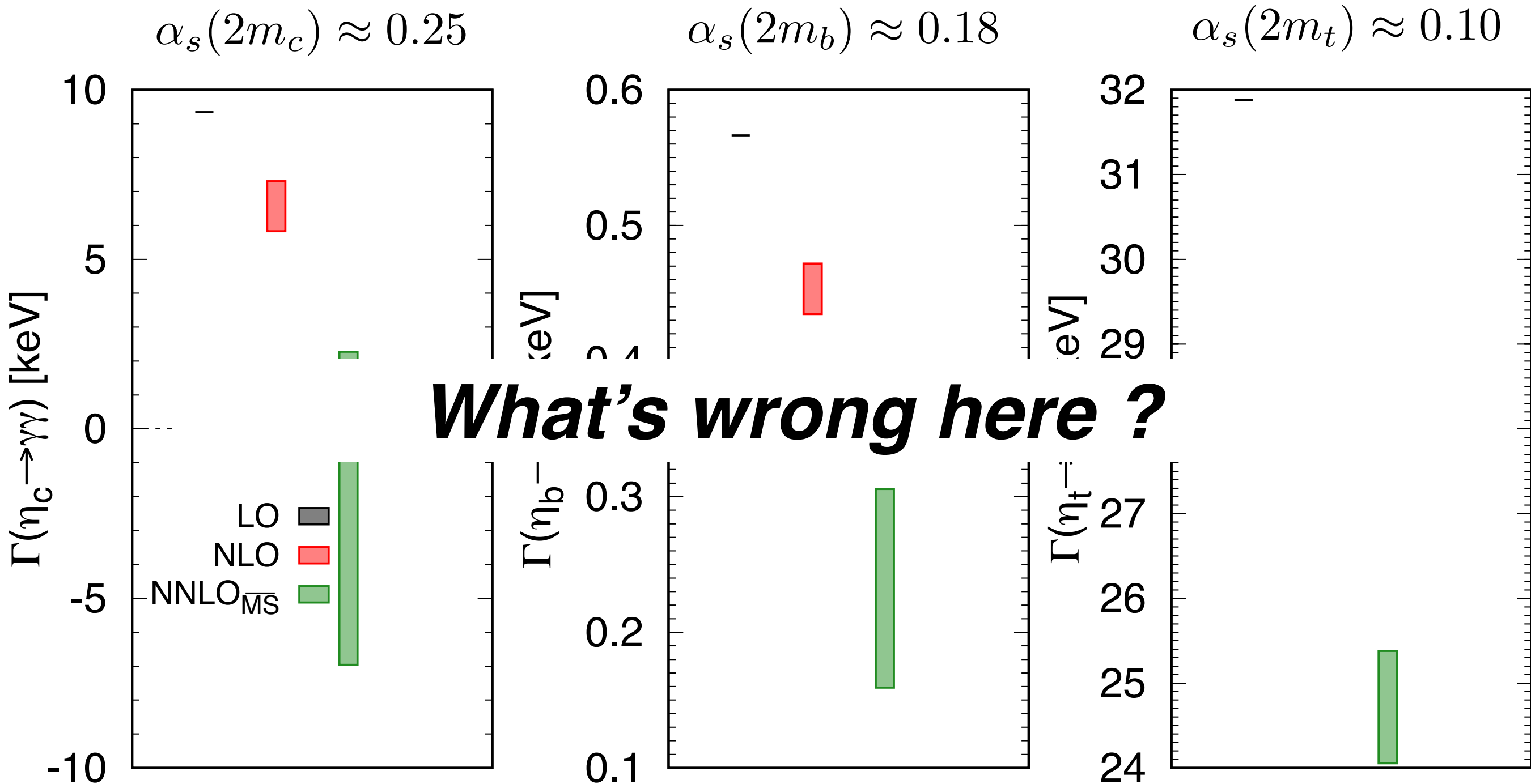
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Logs ? Renormalons ? N³LO ? Or we should abandon perturbation theory for quarkonium ?

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$$\frac{\Gamma_{\text{NNLO}}^{\overline{\text{MS}}}(\eta_Q \rightarrow \gamma\gamma)}{\Gamma_{\text{LO}}(\eta_Q \rightarrow \gamma\gamma)} \bigg|_{\mu_R=m_Q} = 1 - 1.07\alpha_s + \left(\begin{array}{c} -11.16_{n_f=3} \\ -10.40_{n_f=4} \\ -11.19_{n_f=5} \end{array} \right) \alpha_s^2$$

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The pole of soft contributions from (p)NRQCD

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$$\delta^{\text{Soft}} |\psi_{s,n}(0)|^2 = |\psi_n^{\text{LO}}(0)|^2 \alpha_s^2 \left[C_F^2 \left(\frac{\log(\mu_\Lambda^2 a_n^2) - 2S_1(n)}{2s+1} + \frac{5}{3} + \frac{7}{3(2s+1)} - \frac{15}{8n^2} + \frac{4}{(2s+1)n} \right) + C_A C_F \left(\frac{\log(\mu_\Lambda^2 a_n^2) - 2S_1(n)}{2} + \frac{5}{4} + \frac{2}{n} \right) \right]$$

Kniesl et al. (PRL'03), Penin et al. (NPB'04),
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- We propose a new soft-improved matching (SIM) scheme

Maxia, HSS, Simon and Wang (arXiv:2606.28494)

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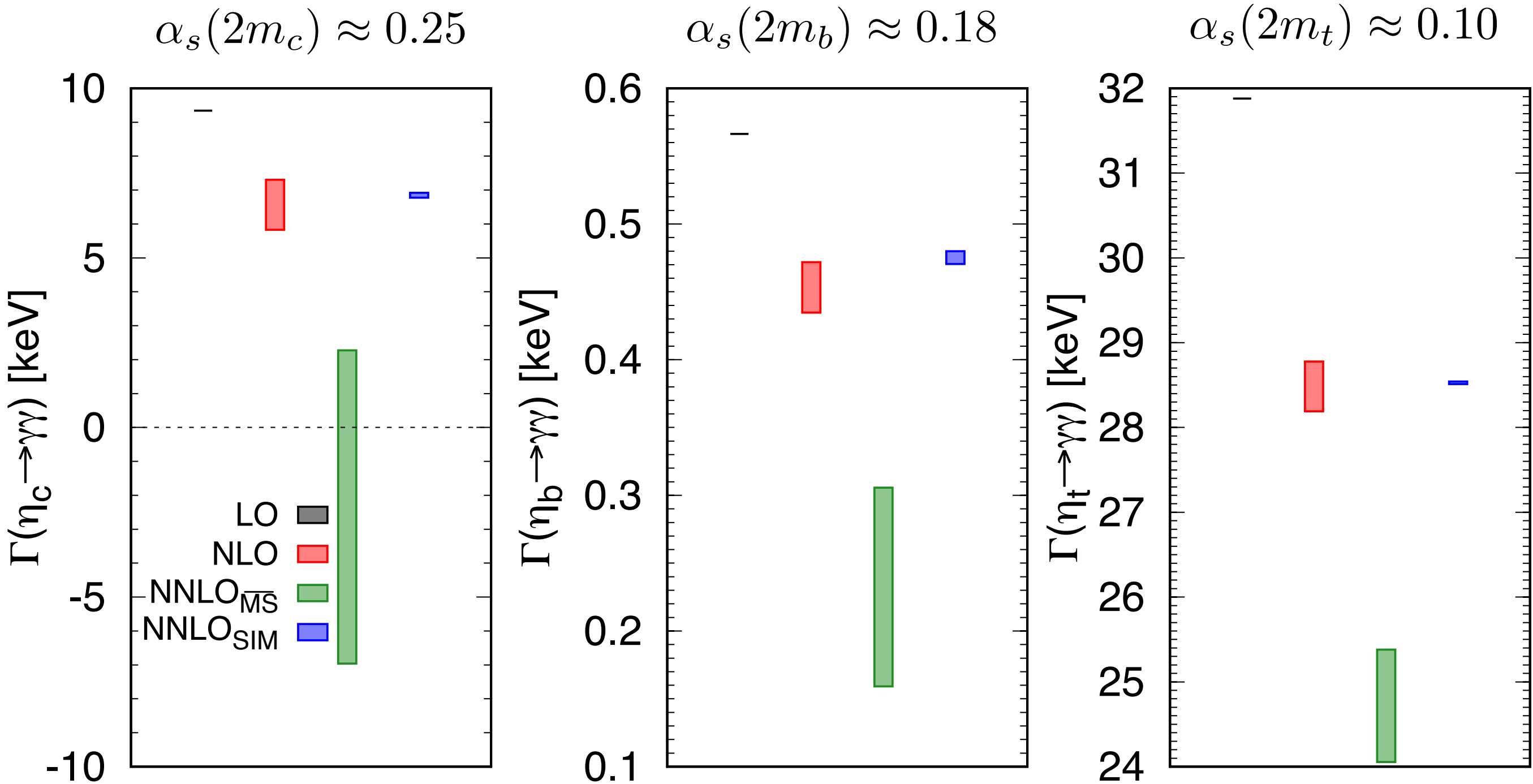
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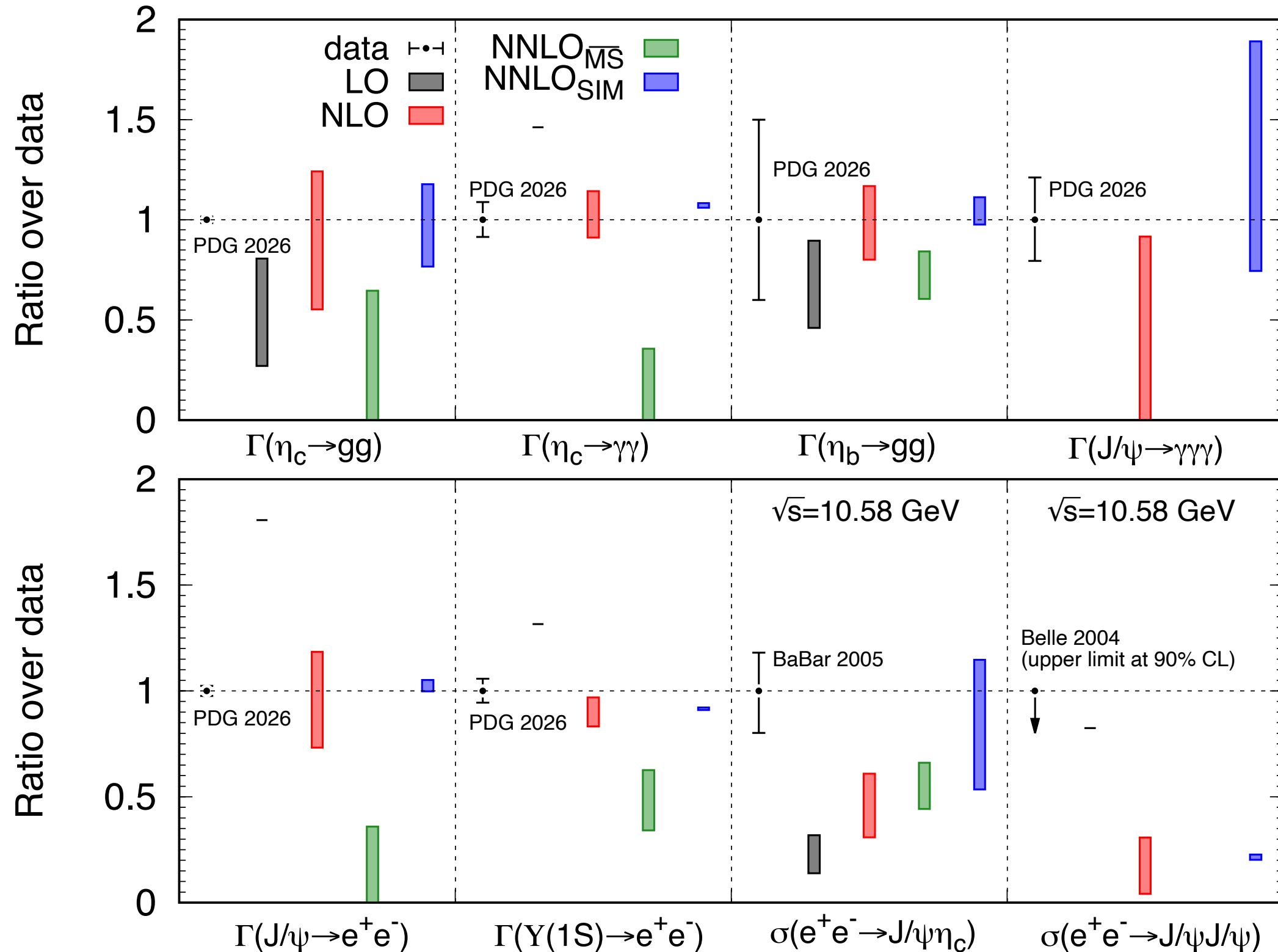
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Thank you for your attention !