

Quarkonia (production in AA collisions), Quo vadis ?

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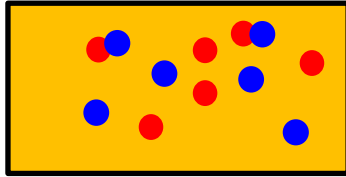
With: A. Daddi Hammou, S. Delorme, Binu Muraleedharan Nair, J. Aichelin, J.P. Blaizot,
Th. Gousset, Vincenzo Greco, M. Ruggieri, K. Werner

Prix GDR QCD 2025

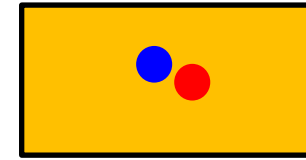
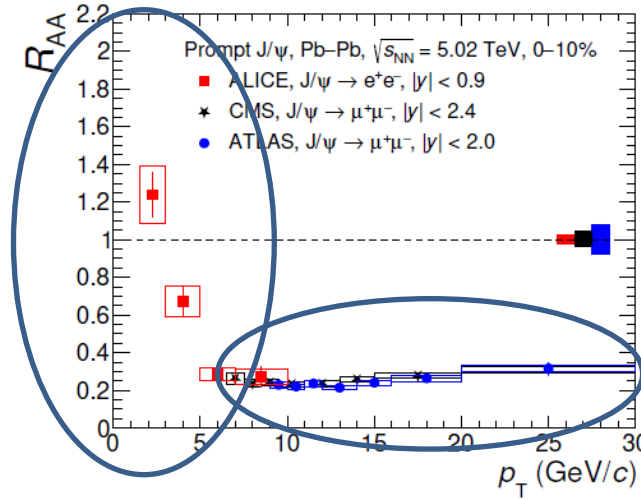


Quarkonia in AA: what experiment tells us

ALICE Collab. JHEP02 (2024)

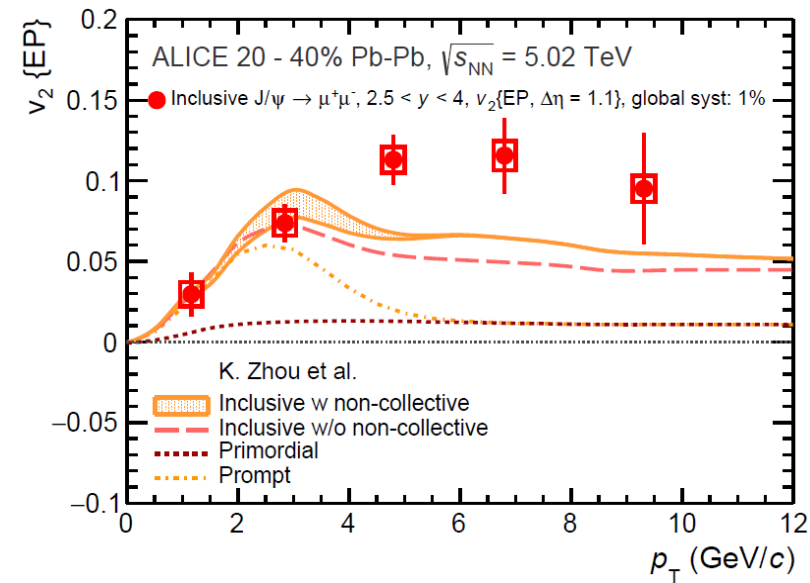


Dense (in phase space)
=> recombination



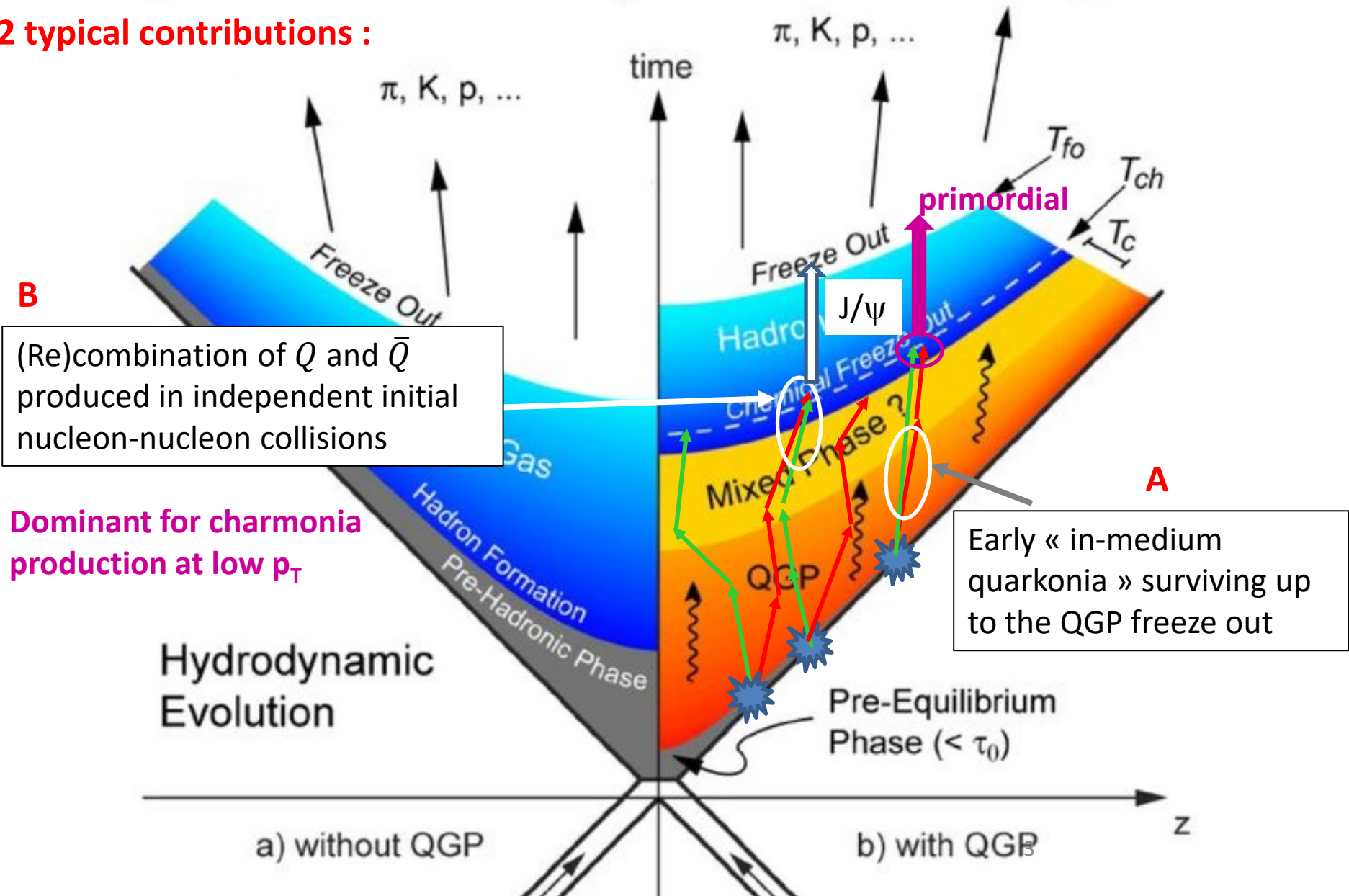
Dilute (in phase space)

Alternate possible explanation : p_T -dependent absorption cross section : not excluded, but not favored by the finite v_2 observed for J/ψ by ALICE



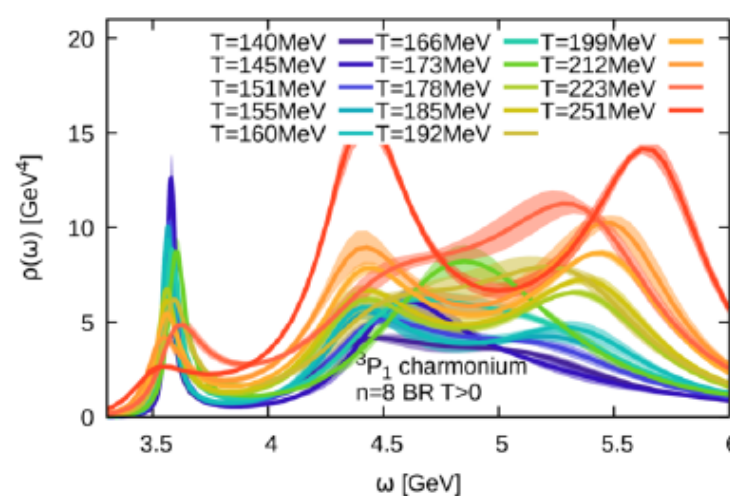
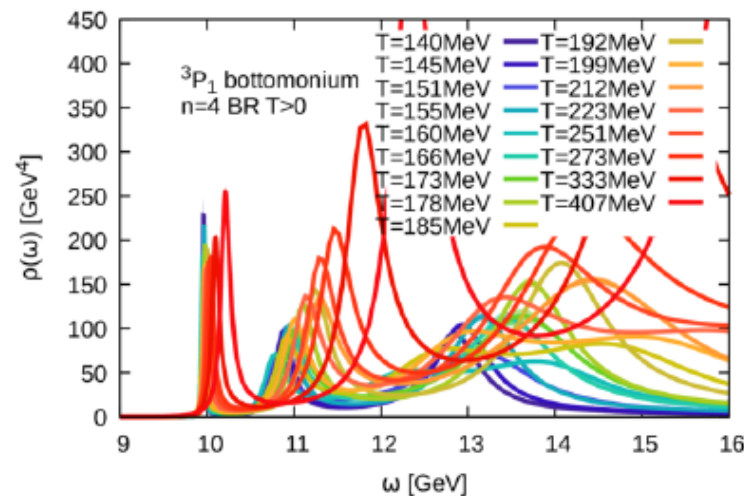
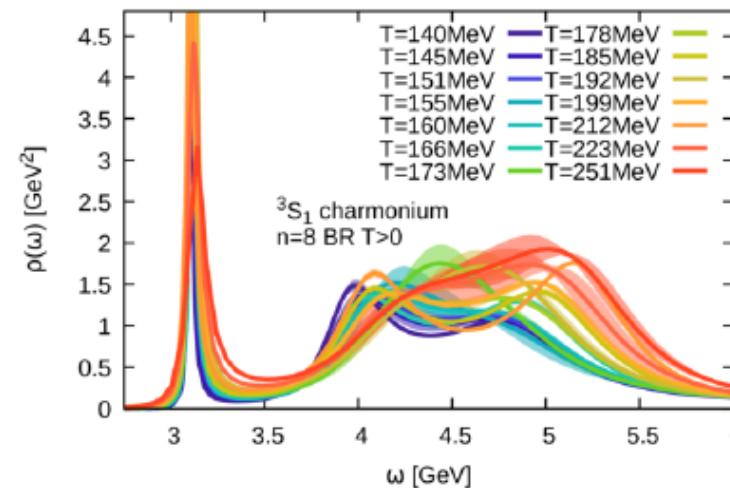
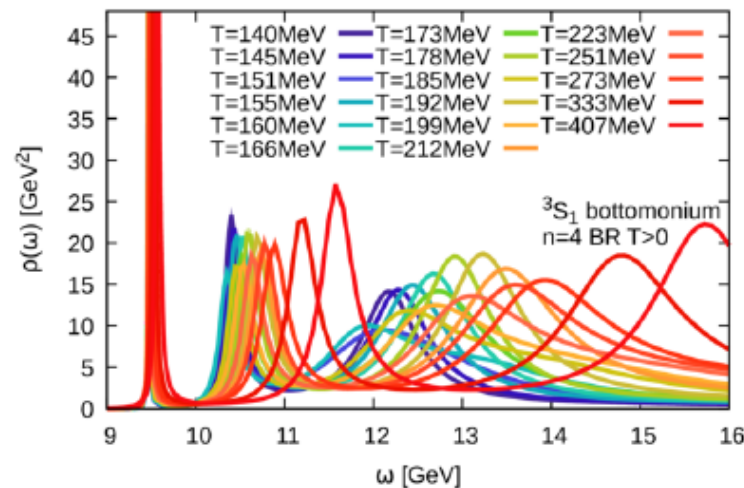
Probing URHIC with quarkonia production

2 typical contributions :



IQCD perspective on quarkonia in hot medium : spectral function

Kim et al, JHEP11(2018)088



Many such kind of results in the literature

Rich structure : broadening and mass shift. What are the underlying “ingredients” ?

The present challenges for Quarkonium modelling in URHIC

Meet the higher and higher precision
of experimental data (already beyond
the present model uncertainties)

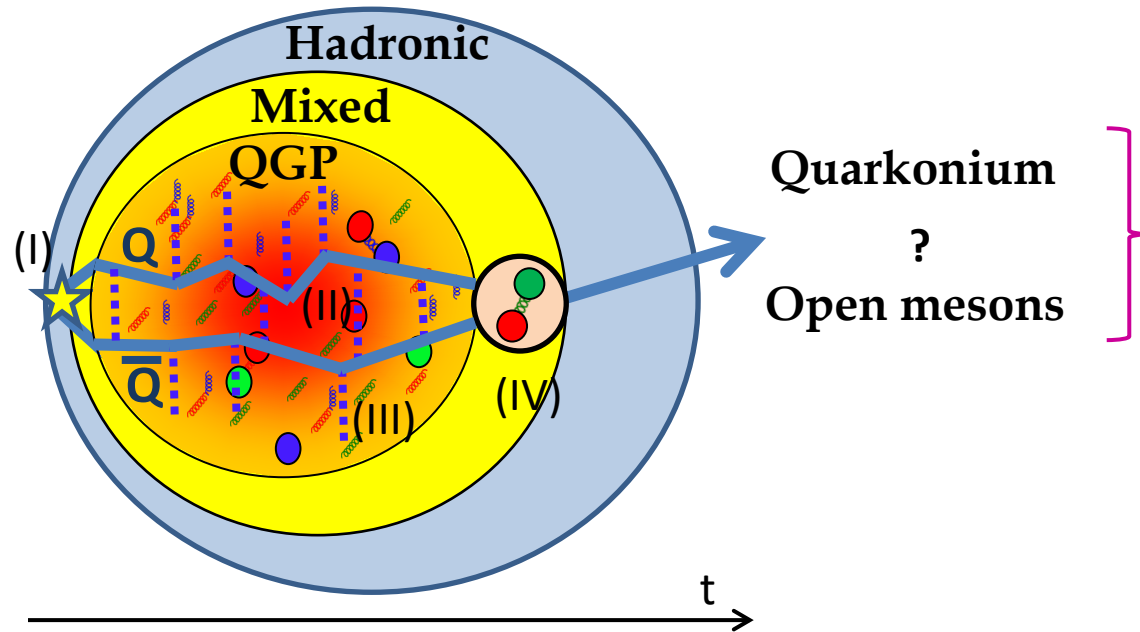
Unravel the Q - $\bar{Q}$ interactions under
the influence of the surrounding QGP
and with the QGP



**Develop a scheme able to deal with the evolution
of one (or many) $Q\bar{Q}$ pair(s) in a QGP, fulfilling all
fundamental principles (quantum features, gauge
invariance, equilibration,...)**

Need for IQCD constraints / inputs

The full transport scheme



Complicated QFT problem (also due to the evolving nature of the QGP that mixes several scales)... only started to be addressed at face value recently

- 1) Initial state
- 2) (Screened) interaction between both HQ
- 3) Interactions with surrounding QGP partons
- 4) Projection on the final quarkonia

How to proceed ?

Strictly speaking, only
resolved at the end of the
evolution



Beware of
quantum coherences
during the whole
evolution !



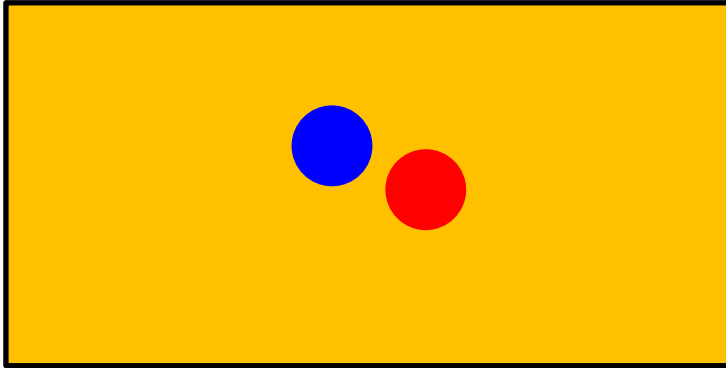
Especially at early time...

In practice, what counts is the so-called
decoherence time, not the "Heisenberg time"

First incomplete QM treatments dating back to Blaizot & Ollitrault, Thews, Cugnon and Gossiaux; early 90's

Regeneration: Dilute vs Dense

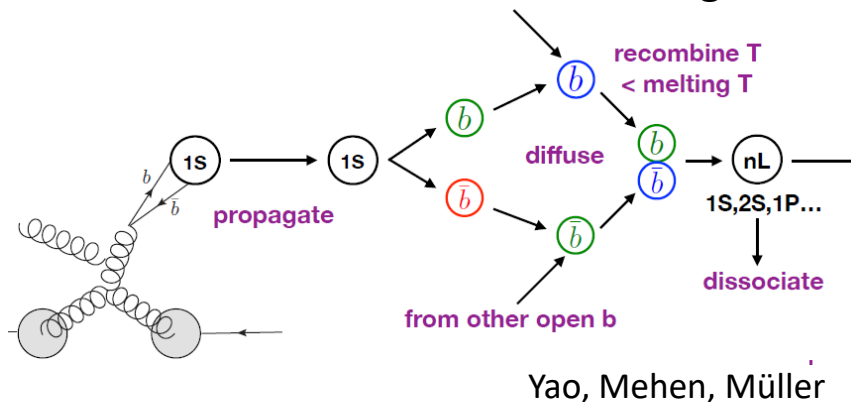
Bottomia



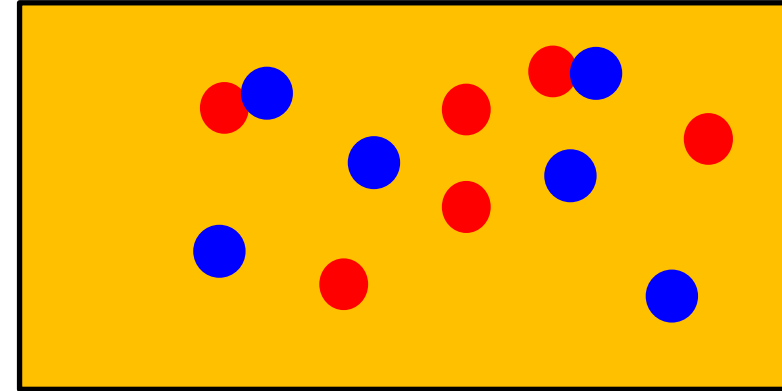
No exogenous recombination : only the b - $\bar{b}$ pairs... initially close together will emerge as bottomia states

Full quantum treatment affordable

In some SC formalisms : intermediate regeneration



Charmonia



Exogenous recombination : c & $\bar{c}$ initially far from each other may recombine and emerge as charmonia states

No full quantum treatment possible => need semi-classical approximation(s)



Level of accuracy ?

Key question : when does the recombination (dominantly) happen ? Crucial role of the binding force.

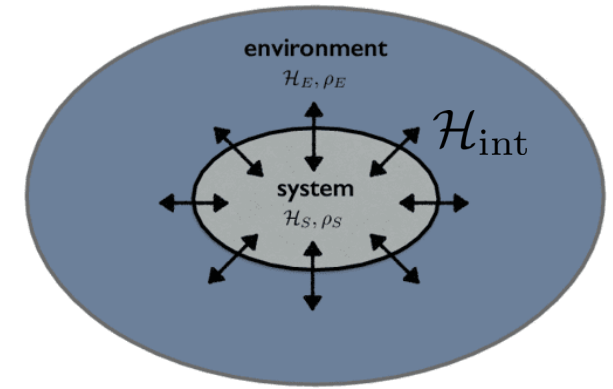
One extreme viewpoint : regeneration happens at the end of the QGP (Statistical Hadronization Model)

Open Quantum Systems & Quantum Master Equations

Quite generally, system (Q-Qbar pair) builds correlation with the environment thanks to the Hamiltonian $\hat{H} = \hat{H}_{Q\bar{Q}}^{(0)} + \hat{H}_E + \hat{H}_{\text{int}}$ with $\hat{H}_E = \hat{H}_{\text{QGP}}$

Von Neumann equation for the total

density operator ρ



System + environment (QGP)
 $\rho(t=0) = \rho_{Q\bar{Q}} \otimes \rho_{\text{QGP}}$

$$\frac{d}{dt}\rho = -i[H, \rho]$$

Evolution of the total system

$$\rho(t) = U(t, 0) [\rho_{Q\bar{Q}} \otimes \rho_{\text{QGP}}] U(t, 0)^\dagger$$

Trace out QGP degrees of freedom =>
 Reduced density operator $\rho_{Q\bar{Q}}$

System ($Q\bar{Q}$ pair)
 $\rho_{Q\bar{Q}}(t=0)$

Evolution of the system

$$\rho_{Q\bar{Q}}(t) = \text{Tr}_{\text{QGP}} [U(t, 0) \rho(t=0) U(t, 0)^\dagger]$$

Can be formulated differentially ./ time :

$$\frac{d\rho_{Q\bar{Q}}}{dt} = \mathcal{L}[\rho_{Q\bar{Q}}]$$

Definition of $\mathcal{L}[\cdot]$

However, $\mathcal{L}[\cdot]$ is generically a non local super-operator in time (linear map)

A special Quantum Master Equation: The Lindblad Equation

There are many different QME... a special one :

$$\frac{d}{dt}\rho_{Q\bar{Q}}(t) = \boxed{-i[H_{Q\bar{Q}}, \rho_{Q\bar{Q}}(t)]} + \sum_i \boxed{\gamma_i} \left[L_i \rho_{Q\bar{Q}}(t) L_i^\dagger - \frac{1}{2} \left\{ L_i L_i^\dagger, \rho_{Q\bar{Q}}(t) \right\} \right]$$

γ_i Characterize the coupling of the system (Q-Qbar) with the environment

$H_{Q\bar{Q}} : \{Q, \bar{Q}\}$ $\underbrace{\text{kinetics + Vacuum potential } V}_{\hat{H}_{Q\bar{Q}}^{(0)}} + \text{Lamb shift / screening}$ (every unitary term that is generated by tracing out the environment $\Leftrightarrow$ Von Neumann)

L_i : Collapse (or Lindblad) operators, depend on the properties of the medium

3 important conservation properties :

$$\rho_{Q\bar{Q}}^\dagger = \rho_{Q\bar{Q}}$$

(Hermiticity)

$$\text{Tr}[\rho_{Q\bar{Q}}] = 1$$

(Norm)

$$\langle \varphi | \rho_{Q\bar{Q}} | \varphi \rangle > 0, \forall |\varphi\rangle$$

(Positivity)

... but in general, non unitary !!! (relaxation)

Nice feature : Can be brought to the form of a stochastic Schroedinger equation (quantum jump method : QTRAJ) => much more effective numerically

A special QME: The Lindblad Equation

Non unitary / dissipative evolution $\equiv$ decoherence

$$\frac{d}{dt}\rho_{Q\bar{Q}}(t) = -i[H_{Q\bar{Q}}, \rho_{Q\bar{Q}}(t)] + \sum_i \gamma_i \left[L_i \rho_{Q\bar{Q}}(t) L_i^\dagger - \frac{1}{2} \{ L_i L_i^\dagger, \rho_{Q\bar{Q}}(t) \} \right]$$

Genuine transitions :

- ✓ Singlet $\leftrightarrow$ octet
- ✓ Octet $\leftrightarrow$ octet

Can be reshuffled into non
Hermitic effective hamiltonian

$$\hat{H}_{Q\bar{Q},\text{eff}} = \hat{H}_{Q\bar{Q}} - i \left[\sum_j \gamma_j \frac{L_j L_j^\dagger}{2} \right] \equiv \text{Dissociation width}$$

Indeed, starting from a singlet density matrix : $\rho_{Q\bar{Q}}^s = |s\rangle\langle s|$, one generates an octet component :

$$\frac{d}{dt}\langle o|\rho_{Q\bar{Q}}|o\rangle = \sum_i \gamma_i \langle o|L_i|s\rangle\langle s|L_i^\dagger|o\rangle = \sum_i \gamma_i |\langle o|L_i|s\rangle|^2$$

Usual transition rate

Open Quantum System approach for Quarkonia

Starting from generic $\hat{H} = \hat{H}_{Q\bar{Q}}^{(0)} + \hat{H}_E + \hat{H}_{\text{int}}$ with $\hat{H}_{\text{int}} = \sum A_{\text{QGP}} \times S$ how to obtain a Linblad equation ?

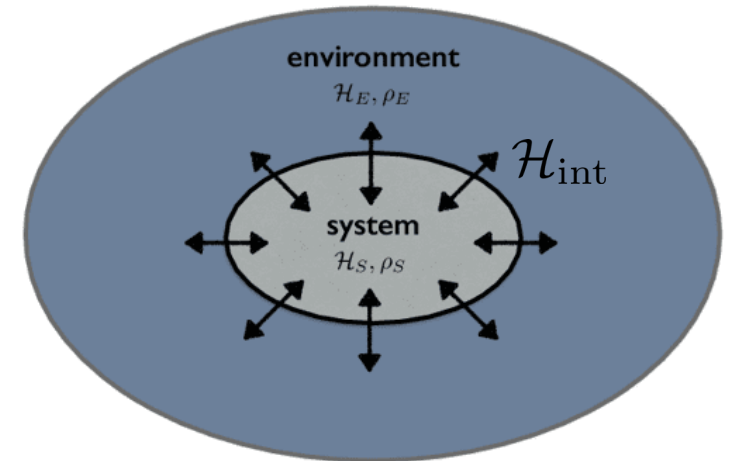
QGP operator (field) $Q\bar{Q}$ operator (charge)

Also named n^a in the rest of the talk

System : a $Q\bar{Q}$ pair



Environment
 $\hat{H}_E = \hat{H}_{\text{QGP}}$



Strategy : trace out the environment and establish a Quantum Master Equation (QME) for the reduced density matrix ρ

Pictorial summary for a $Q\bar{Q}$ pair

τ_{Env} : environment autocorrelation time $\approx \frac{1}{T}$

τ_{Relax} : system (quarkonium) relaxation time $\approx \frac{\alpha_s m_Q^2}{T^3}$

Subsystem + environment: von Neumann equation

Iterate Von Neumann eq. + Trace out environment

Subsystem: non-unitary, time-irreversible evolution

Weak sys-environment coupling + Markovian limit $\tau_E \ll \tau_R$ Ok for HQ

Redfield equation

field-field correlator

$$\frac{\partial}{\partial t} \rho_I(t) = -\frac{1}{\hbar^2} \sum_{m,n} \int_0^{+\infty} d\tau (\Delta_{mn}(\tau) [S_{m,I}(t), S_{n,I}(t-\tau) \rho_I(t)] - \Delta_{mn}^*(\tau) [S_{m,I}(t), \rho_I(t) S_{n,I}(t-\tau)])$$

Some similitude with the Lindblad equation but with time delay effects => **Not Lindbladian**

$$\frac{d}{dt} \rho_{Q\bar{Q}}(t) = -i[H_{Q\bar{Q}}, \rho_{Q\bar{Q}}(t)] + \sum_i \gamma_i \left[L_i \rho_{Q\bar{Q}}(t) L_i^\dagger - \frac{1}{2} \{ L_i L_i^\dagger, \rho_{Q\bar{Q}}(t) \} \right]$$

Pictorial summary

Subsystem + environment: von Neumann equation

Iterate Von Neumann eq. + Trace out environment

Subsystem: non-unitary, time-irreversible evolution

Weak syst-environment coupling + Markovian limit $\tau_E \ll \tau_R$

Redfield equation

Quantum Optical Regime

Quantum Brownian Motion

Lindblad equation

Lindblad equation

Eigenstates of the HQ Hamiltonian

Phase space densities

Caldeira Leggett model

Dissociation of well identified levels by scarce "high-energy" modes (dilute medium => cross section ok) .

Multiple scattering on quasi free states. Best described in position-momentum space

$$\tau_{\text{Env}} \approx \frac{1}{T}$$

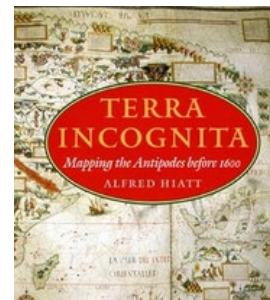
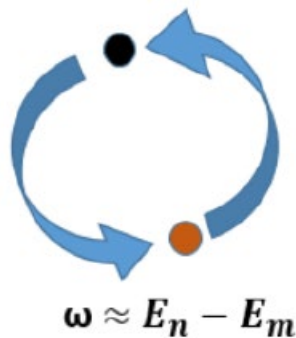
$$\tau_{\text{Relax}} \approx \frac{\alpha_s m_Q^2}{T^3}$$

$$\tau_{\text{Syst}} \approx \frac{1}{E_n - E_m} \approx \frac{1}{\alpha_s m_Q}$$

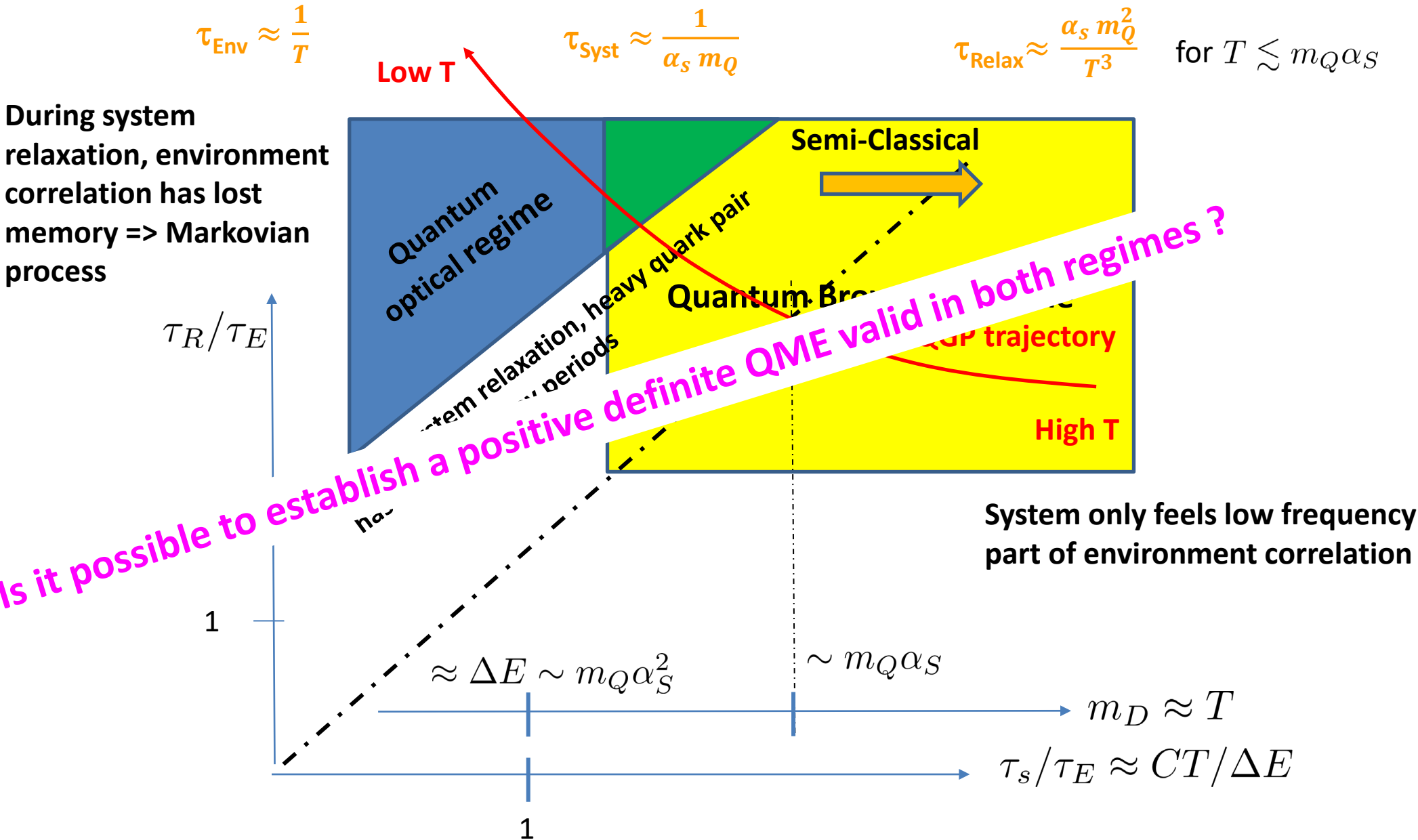
system intrinsic time scale

Smallest time scales wins it all !

Davies secular equation



QCD time scales



The core of the Universal Lindblad Equation derivation

Lindblad structure $\frac{d\rho(t)}{dt} = -i[H_Q + H_{LS}, \rho(t)] + \sum_n \gamma_n \left(L_n \rho(t) L_n^\dagger - \frac{1}{2} \{ L_n^\dagger L_n, \rho(t) \} \right)$

Starting from the NRQCD Hamiltonian $H_{\text{tot}} = \left(\frac{\mathbf{p}_Q^2}{2M} + \frac{\mathbf{p}_{\bar{Q}}^2}{2M} \right) \otimes I_{\text{QGP}} + I_{Q\bar{Q}} \otimes H_{\text{QGP}} + \int_x n_x^a \otimes g A_0^a(x)$

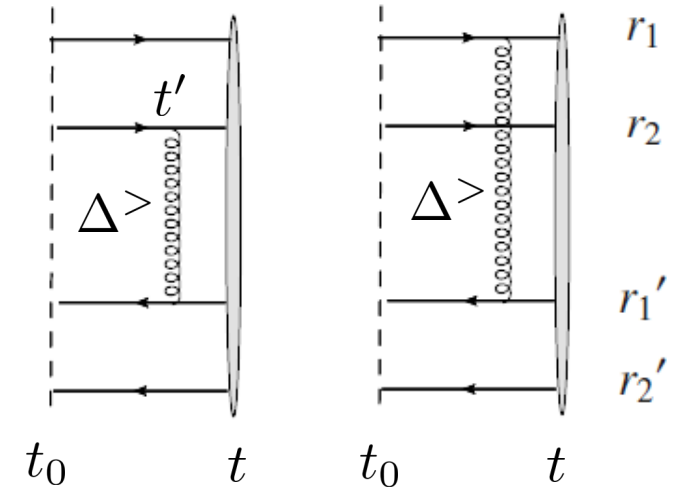
Iterating the Von Neumann equation, tracing out the QGP d.o.f. and assuming a slow evolution of ρ^I in the **interaction representation** (Born - Markov), one arrives at the Redfield equation

$$\frac{d\rho^I(t)}{dt} = - \int_{t_0}^t dt' \int_{xx'} [n^a(t, x), n^a(t', x') \rho^I(t)] \Delta^>(t - t', x - x') + h.c.$$

Where the thermal-gluon propagator is

$$\underbrace{\delta^{ab} \Delta^>(t_1 - t_2, x - x')} = g^2 \left\langle T_C \left[A_0^a(t_1, x) A_0^b(t_2, x') \right] \right\rangle_0$$

IMPORTANT : localized on $t_1 - t_2 \lesssim \tau_E \sim \frac{1}{T}$



The core of the ULE derivation

Lindblad structure $\frac{d\rho(t)}{dt} = -i[H_Q + H_{LS}, \rho(t)] + \sum_n \gamma_n \left(L_n \rho(t) L_n^\dagger - \frac{1}{2} \{L_n^\dagger L_n, \rho(t)\} \right)$

Jumps -> other states

Redfield equation : $\frac{d\rho^I(t)}{dt} = - \int_{t_0}^t dt' \int_{xx'} [n^a(t, x), n^a(t', x') \rho^I(t)] \Delta^>(t - t', x - x') + h.c.$

$$\delta^{ab} \Delta^>(t_1 - t_2, x - x') = g^2 \left\langle T_C [A_0^a(t_1, x) A_0^b(t_2, x')] \right\rangle_0$$

Jump contribution : $\frac{d\rho^I(t)}{dt} = \int_{t_0}^t dt' \int_{xx'} \Delta^>(t - t', x - x') n^a(t', x') \rho^I(t) n^a(t, x) + h.c. + \dots$

Not a symetrized form ! => no Lindblad structure

For $t \ll \tau_R$:

(slow evolution
of ρ^I)

$$\begin{aligned} \rho^I(t) - \rho^I(t_0) &= \int_{t_0}^t dt' \int_{t_0}^{t'} dt'' \int_{xx'} \Delta^>(t' - t'', x - x') n^a(t'', x') \rho^I(t) n^a(t', x) + h.c. + \dots \\ &= \int_{t_0}^t dt' \int_{t_0}^{+\infty} dt'' \theta(t' - t'') \int_{xx'} \Delta^>(t' - t'', x - x') n^a(t'', x') \rho^I(t) n^a(t', x) + h.c. + \dots \end{aligned}$$

$$\theta(t' - t'') = \frac{1}{2} + \frac{1}{2} \text{sign}(t' - t'')$$

« Lamb shift »

The core of the ULE derivation

Lindblad structure $\frac{d\rho(t)}{dt} = -i[H_Q + H_{LS}, \rho(t)] + \sum_n \gamma_n \left(L_n \rho(t) L_n^\dagger - \frac{1}{2} \{L_n^\dagger L_n, \rho(t)\} \right)$

$$\rho^I(t) - \rho^I(t_0) = \frac{1}{2} \int_{t_0}^t dt' \int_{t_0}^{+\infty} dt'' \int_{x' x''} \Delta^>(t' - t'', x' - x'') n^a(t'', x'') \rho^I(t) n^a(t', x') + h.c. + \dots$$

Jumps -> other states

t' and t'' still intricated

Trick : $\Delta^>(t' - t'', x' - x'') = \int_{-\infty}^{+\infty} dv \int_y g(v - t'', y - x'') g(t' - v, x' - y)$ g : jump correlators

with $g(t' - v, x' - y) = g^*(v - t', y - x')$ Can be interpreted as a « mid-time »

$$\begin{aligned} \rho^I(t) &= \underbrace{\frac{1}{2} \int_y \int dv}_{\sim \sum_n \gamma_n} \int_{t_0}^t dt' \int_{t_0}^{+\infty} dt'' \int_{x''} g(v - t'', y - x'') \hat{n}^a(t'', x'') \rho^I(t) \int_{x'} g^*(v - t', y - x') \hat{n}^a(t', x') + \dots \\ &= \underbrace{\frac{1}{2} \int_y \int dv}_{\sim \sum_n \gamma_n} \underbrace{\int_{t_0}^{+\infty} dt'' \int_{x''} g(v - t'', y - x'') \hat{n}^a(t'', x'') \rho^I(t)}_{\equiv L_n ???} \underbrace{\int_{t_0}^t dt' \int_{x'} g^*(v - t', y - x') \hat{n}^a(t', x') + h.c. + \dots}_{\equiv L_n^\dagger ???} \end{aligned}$$

Not quite ! As time derivation wrt t ``obviously'' breaks this structure !!! **Redfield : NOT LINDBLAD**

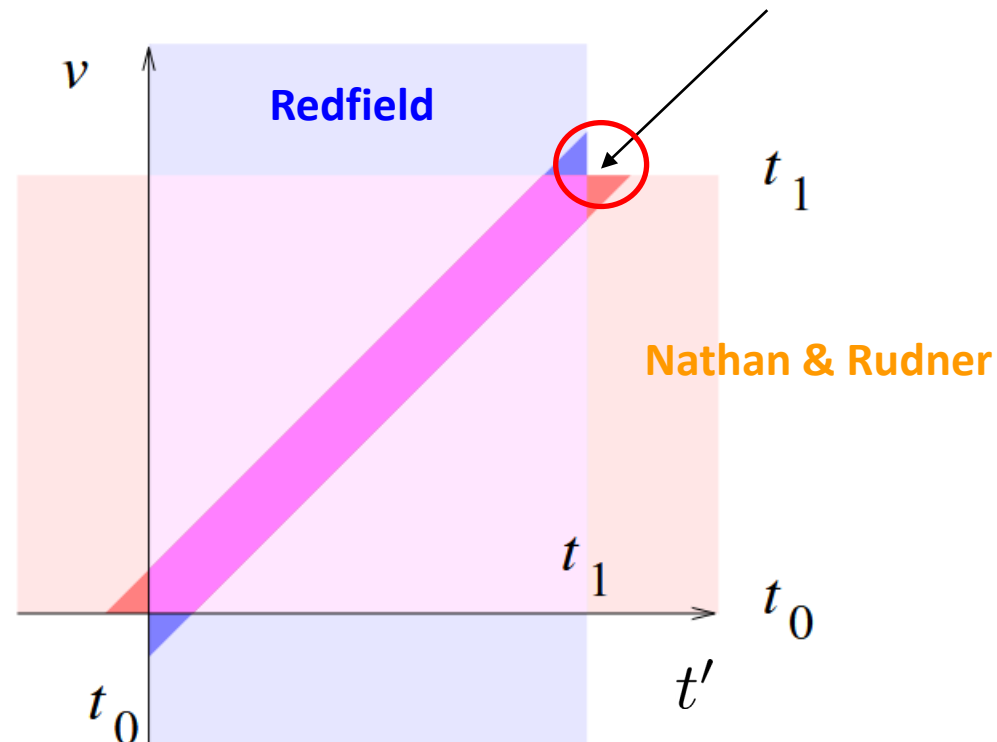
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Lindblad structure $\frac{d\rho(t)}{dt} = -i[H_Q + H_{LS}, \rho(t)] + \sum_n \gamma_n \left(L_n \rho(t) L_n^\dagger - \frac{1}{2} \{ L_n^\dagger L_n, \rho(t) \} \right)$

$$\rho^I(t) = \frac{1}{2} \int_y \int_{-\infty}^{+\infty} dv \int_{t_0}^t dt' \int_{t_0}^{+\infty} dt'' \int_{x''} g(v - t'', y - x'') \hat{n}^a(t'', x'') \rho^I(t) \int_{x'} g^*(v - t', y - x') \hat{n}^a(t', x') + \dots$$

How to proceed without further hierarchies (than $\tau_E \ll \tau_R$) ? **In fact, there is an $\{\}$ of equivalent QMEs encoding the same Φ !**

Nathan and Rudner (2020): In significant contributions to the integrals stem from regions where the 3 times v, t' and t'' are separated by τ_E at most $\Rightarrow$ one can **permute the boundaries on v and t'** (at the price of a small correction in τ_E / τ_R)



The core of the ULE derivation

Lindblad structure $\frac{d\rho(t)}{dt} = -i[H_Q + H_{LS}, \rho(t)] + \sum_n \gamma_n \left(L_n \rho(t) L_n^\dagger - \frac{1}{2} \{ L_n^\dagger L_n, \rho(t) \} \right)$

$$\rho^I(t) = \frac{1}{2} \int_y \int_{-\infty}^{+\infty} dv \int_{t_0}^t dt' \int_{t_0}^{+\infty} dt'' \int_{x''} g(v - t'', y - x'') \hat{n}^a(t'', x'') \rho^I(t) \int_{x'} g^*(v - t', y - x') \hat{n}^a(t', x') + \dots$$

How to proceed without further hierarchies (than $\tau_E \ll \tau_R$) ? In fact, there is an **{}** of equivalent QMEs encoding the same Φ !

Nathan and Rudner (2020): In significant contributions to the integrals stem from regions where the 3 times v, t' and t'' are separated by τ_E at most $\Rightarrow$ one can **permute the boundaries on v and t'** (at the price of a small correction in τ_E / τ_R)

$$\rho_{\text{NR}}^I(t) = \frac{1}{2} \int_y \int_{t_0}^t dv \int_{-\infty}^{+\infty} dt' \int_{t_0}^{+\infty} dt'' \int_{x''} g(v - t'', y - x'') \hat{n}^a(t'', x'') \rho^I(t) \int_{x'} g^*(v - t', y - x') \hat{n}^a(t', x') + \dots$$

Then proceeding with time derivative wrt t :

$$\begin{aligned} \frac{d\rho_{\text{NR}}^I(t)}{dt} &= \frac{1}{2} \int_y \int_{-\infty}^{+\infty} dt' \int_{-\infty}^{+\infty} dt'' \int_{x''} g(t - t'', y - x'') \hat{L}_y^a \hat{n}^a(t'', x'') \rho^I(t) \int_{x'} g^*(t - t', y - x') \hat{n}^a(t', x') + h.c. + \dots \\ &= \frac{1}{2} \int_y \left[\int_{-\infty}^{+\infty} dt'' \int_{x''} g(t - t'', y - x'') \hat{n}^a(t'', x'') \right] \rho^I(t) \left[\int_{-\infty}^{+\infty} dt' \int_{x'} g^*(t - t', y - x') \hat{n}^a(t', x') \right] + h.c. + \dots \end{aligned}$$

Markovianity

Lindblad structure, with $\hat{L}_y^a = \int_{-\infty}^{+\infty} dt'' \int_{x''} g(t - t'', y - x'') \hat{n}^a(t'', x'')$ **Contains all spectral information of the QGP environment !**

Same structure in the Schroedinger representation

Dealing with colour

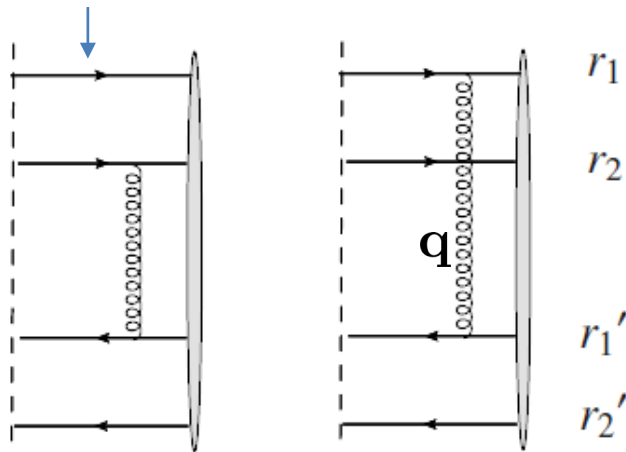
In the Schroedinger representation:

$$\frac{d\rho(t)}{dt} = -i[H_Q + \Lambda, \rho(t)] + \int_{\mathbf{y}} \left(L^A(\mathbf{y}) \rho(t) L^{A\dagger}(\mathbf{y}) - \frac{1}{2} \left\{ L^{A\dagger}(\mathbf{y}) L^A(\mathbf{y}), \rho(t) \right\} \right) \quad A : 1 \rightarrow N_c^2 - 1$$

$$\rho_{Q\bar{Q}} = \rho_s(t) |s\rangle \langle s| + \rho_o(t) \sum_B |o^B\rangle \langle o^B|$$

with $\hat{L}^A(\mathbf{y}) := \int_{-\infty}^{t_+} dt'' \int_{x''} g(t-t'', \mathbf{y}-\mathbf{x}'') \hat{n}^A(t'', \mathbf{x}'')$

$Q\bar{Q}$ pair



Scatterings with gluons modify the color representation : $o \leftrightarrow s$

$$\begin{aligned} \frac{d\rho_o}{dt} &= \dots + \frac{1}{N_c^2 - 1} \int_{\mathbf{y}} \sum_C \langle O^C | \hat{L}^A(\mathbf{y}) \rho \hat{L}^{A\dagger}(\mathbf{y}) | O^C \rangle + \dots \\ &= \dots + \frac{1}{N_c^2 - 1} \int_{\mathbf{y}} \sum_C \langle O^C | \hat{L}^A(\mathbf{y}) | s \rangle \rho_s \langle s | \hat{L}^{A\dagger}(\mathbf{y}) | O^C \rangle + \dots \end{aligned}$$

singlet $\rightarrow$ octet transition

octet $\rightarrow$ octet transition

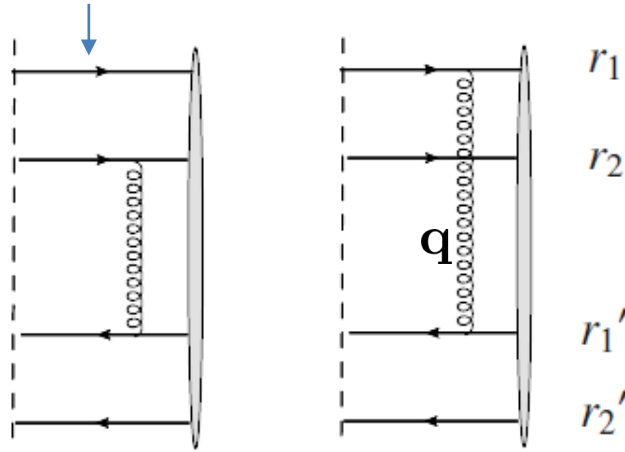
One next derives :

$$\langle o^C | L^A(\mathbf{y}) | s \rangle = \frac{\delta^{AC}}{\sqrt{2N_c}} \int_{v\mathbf{X}} g_-(t-v, \mathbf{y}-\mathbf{X}) U_o^\dagger(v-t) P_{\mathbf{X}} U_s(v-t)$$

Easy color summation preserving the $\hat{L}_{s \rightarrow o}(\mathbf{y}) \rho \hat{L}_{s \rightarrow o}^\dagger(\mathbf{y})$ structure essential for Lindblad

Dealing with colour

$Q\bar{Q}$ pair



Scatterings with gluons modify the color representation : $o \leftrightarrow s$

- Illustration for the singlet $\rightarrow$ octet transition (center of mass integrated)
- One assumes some binding potential in the singlet representation $\Rightarrow$ in-QGP Bound states ($|n\rangle, |m\rangle, \dots$)
- For the octet representation, no binding potential $\Rightarrow$ diffusion states $|\mathbf{k}\rangle \Rightarrow$ « large » energy gap.

$$\frac{d\rho_o}{dt} = \dots + \int_{\mathbf{y}} \hat{L}_{s \rightarrow o}(\mathbf{y}) \rho_s \hat{L}_{s \rightarrow o}^\dagger(\mathbf{y}) + \dots$$

$$\text{with } \hat{L}_{s \rightarrow o}(\mathbf{y}) = \int d\mathbf{k} \sum_n \tilde{L}_{s \rightarrow o}(\mathbf{k}, n, \mathbf{y}) |\mathbf{k}\rangle \langle n|$$

Transition $s \rightarrow o$

$$\tilde{L}_{s \rightarrow o}(\mathbf{k}, n, \mathbf{y}) = -i \sqrt{\frac{1}{2N_c}} \int d\mathbf{q} e^{i\mathbf{q} \cdot \mathbf{y}} \tilde{g} \left(q^0 = E_n - \frac{k^2}{M}, \mathbf{q} \right) \langle \mathbf{k} | \sin \frac{\mathbf{q} \cdot \mathbf{s}}{2} | n \rangle$$

Fourier transform of g , $\sqrt{\Delta}$ of the QGP correlator Δ

Transition matrix element from $|n\rangle \rightarrow |\mathbf{k}\rangle$

Illustrations of transition rates (charmonia)

Full width

Real potential : Vacuum

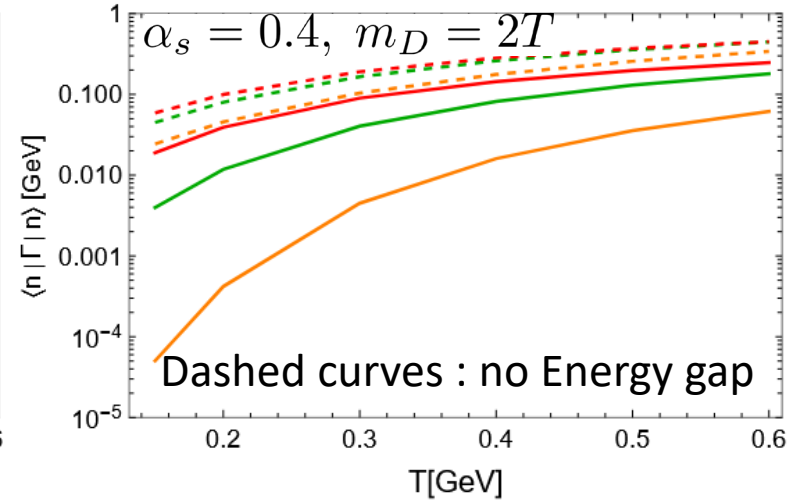
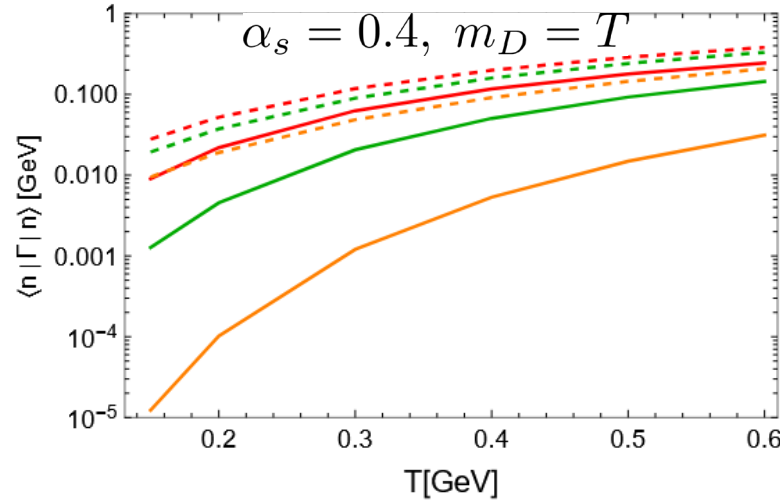


Figure 26: Γ_{3D} as a function of T for the 3D case (with $l_k \leq 5$ and $\tilde{k} \leq 40$). Same conventions as in Fig. 10; orange

Real potential : Screened (Lafferty – Rothkopf prescription)

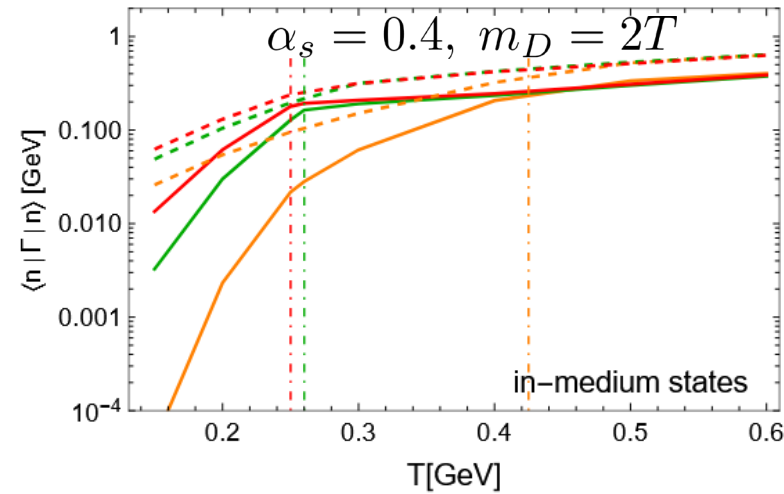


Figure 27: Same as Fig. 26 (right) with in-medium eigenstates obtained with a T -dependent real potential; the vertical dot-dashed lines indicates the temperatures beyond which a given eigenstate ceases to be bound.

Illustrations of spectral densities (charmonia)

Lindblad equation :

$$\frac{d\rho(t)}{dt} = \boxed{-i[H_Q + H_{LS}, \rho(t)]} + \sum_n \gamma_n \left(L_n \rho(t) L_n^\dagger - \frac{1}{2} \{ L_n^\dagger L_n, \rho(t) \} \right)$$

$\boxed{}$: Von Neumann equation with

$$H_{\text{eff}} = H_Q + H_{LS} - i\frac{\Gamma}{2} \quad \text{with} \quad \Gamma := \sum_n \gamma_n L_n^\dagger L_n$$

Non Hermitic

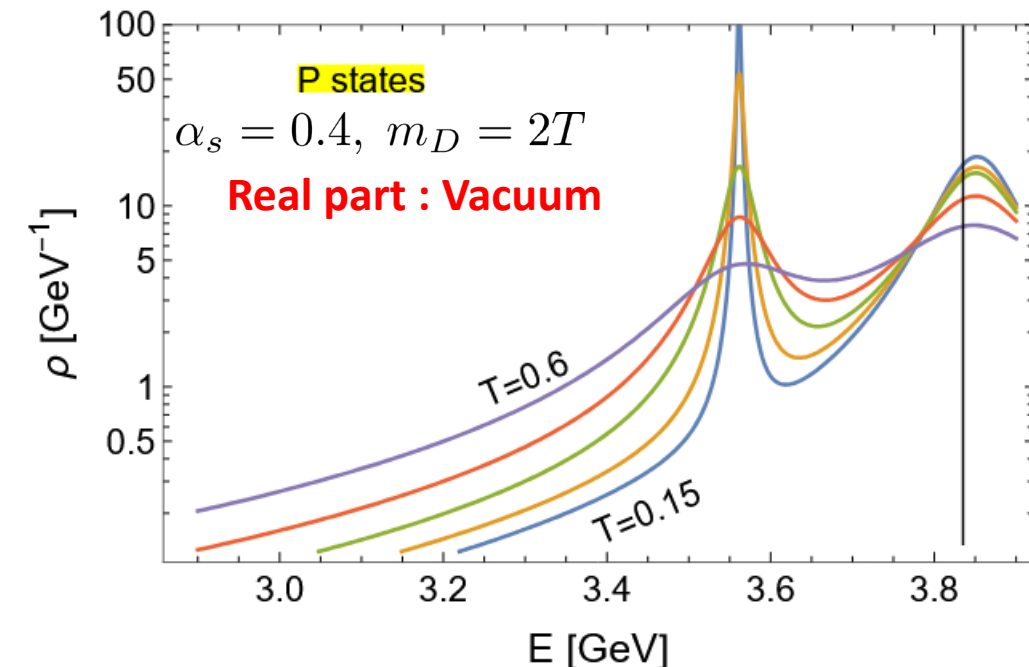
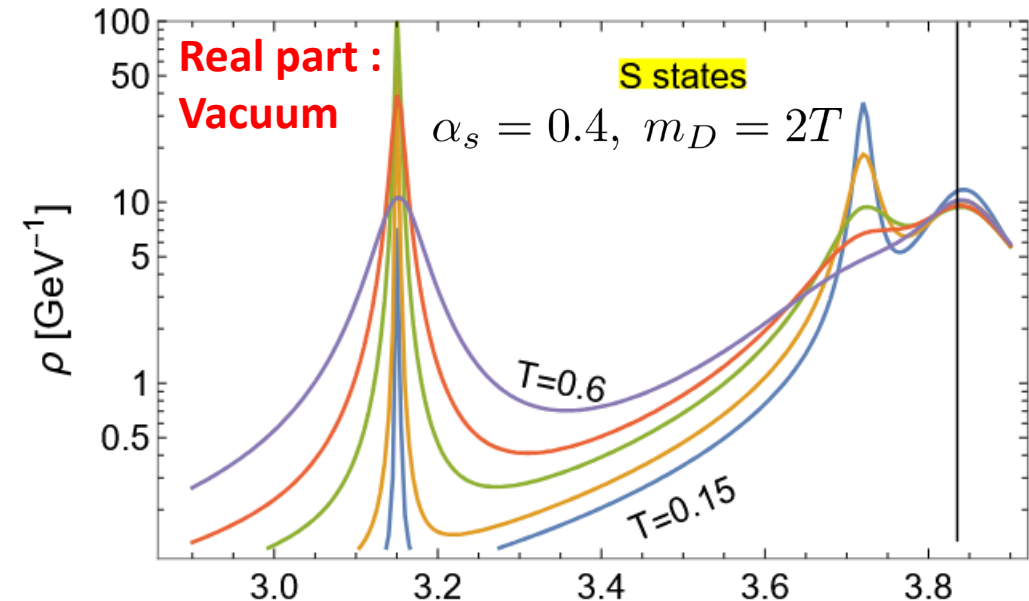
Definition of the spectral density :

$$\rho(E) = \frac{1}{\pi} \Im \text{tr} \frac{1}{E - H_{\text{eff}} - i\epsilon}$$

At T=0, a sum of Dirac peaks

Main result: fast melting of the χ_c state, progressive melting of the J/ ψ state beyond T_c...

... correct interpolation between the Quantum Optical regime at low T and the Quantum Brownian Motion regime at high T



Going Semi-Classical

τ_E : environment autocorrelation time τ_R : system relaxation time τ_S : system intrinsic time scale

Subsystem + environment: von Neumann equation

Iterate Von Neumann eq. + Trace out environment

Subsystem: non-unitary, time-irreversible evolution

Weak syst-environment coupling + Markovian limit $\tau_E \ll \tau_R$

Redfield equation

$\tau_S \ll \tau_R$

Quantum Optical Regime

Lindblad equation

Davies secular equation

Eigenstates of the HQ Hamiltonian

Boltzmann equation

Rate equations: $\Leftrightarrow$ transport models

$\tau_E \ll \tau_S$

Quantum Brownian Motion

Lindblad equation

Caldeira Leggett

Phase space densities

Fokker-Planck equation

Good method for many $c\bar{c}$ pairs

Comparison

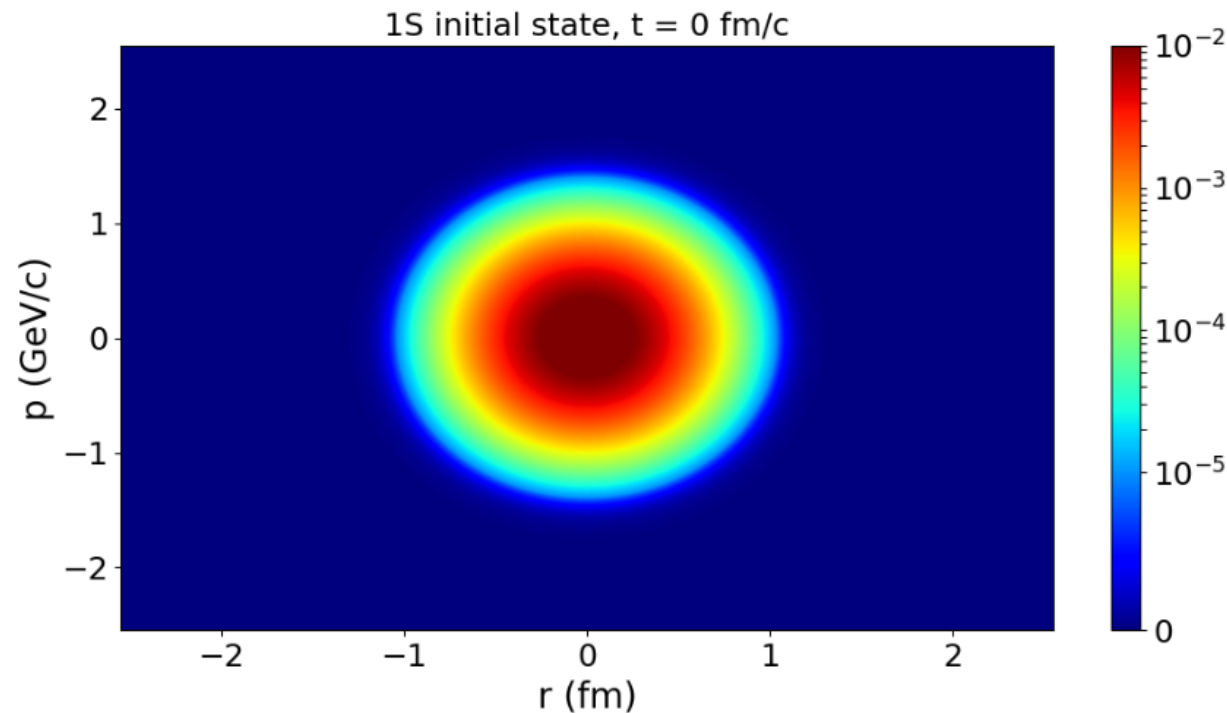
Not the same basis !

Wigner transform + gradient expansion

Semi-classical approx : density matrix $\approx$ diagonal

Concrete implementation in a simplified set up

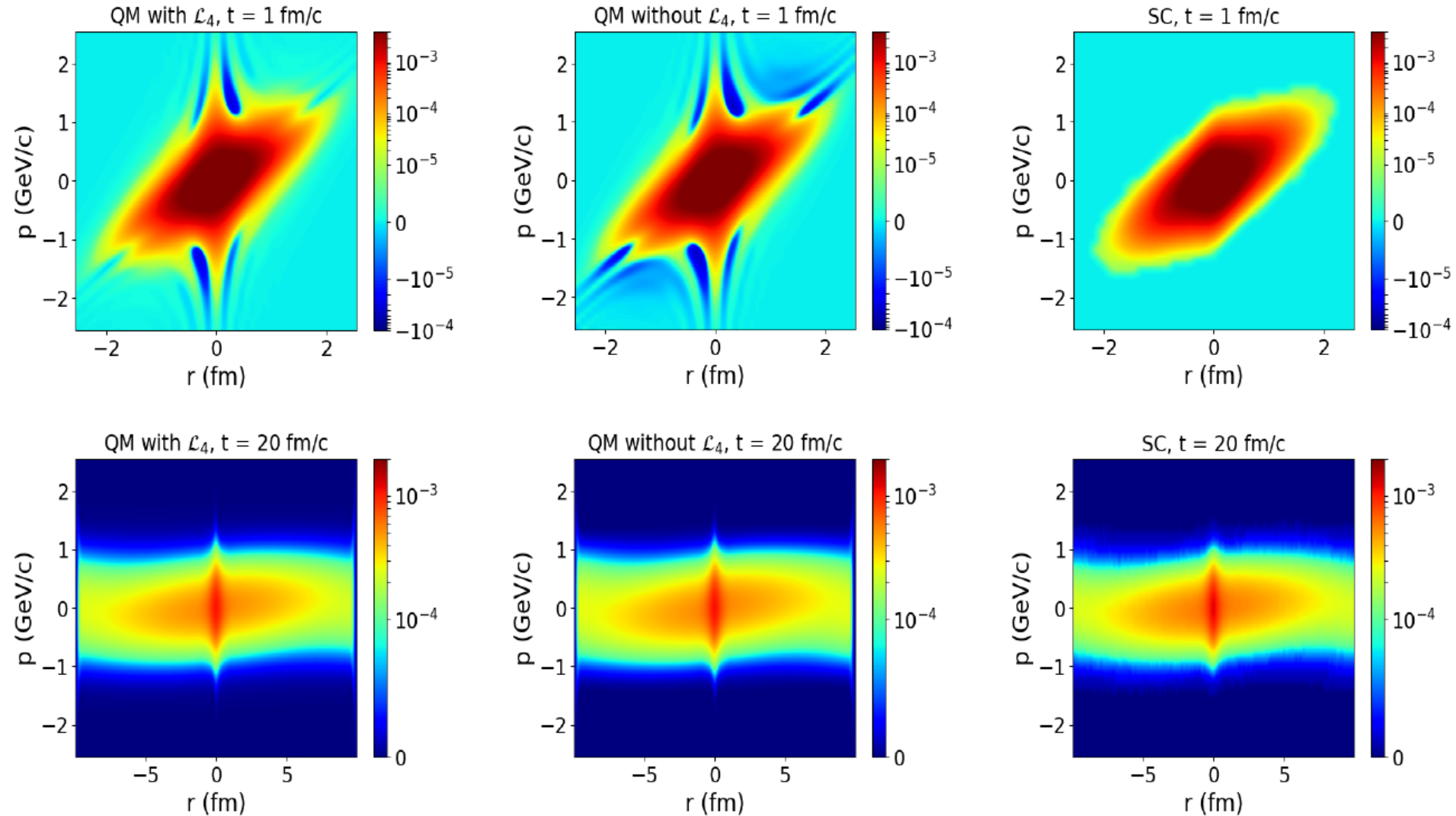
- 1D, 1 $c\bar{c}$ pair
- Same real potential, W in the QME $\Leftrightarrow \eta$ and γ (drag and diffusion) in the Fokker Planck
- **Abelian case** (for the time, not clear how to deal with the singlet $\leftrightarrow$ octet transition in a SC approach)
- Yet, not trivial...
- Initial state vacuum 1S state : $\psi(s) \propto e^{-\frac{s^2}{2\sigma^2}}$ $\sigma = 0.38 \text{ fm}$



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Non-unitary evolution

Full coupling with the QGP

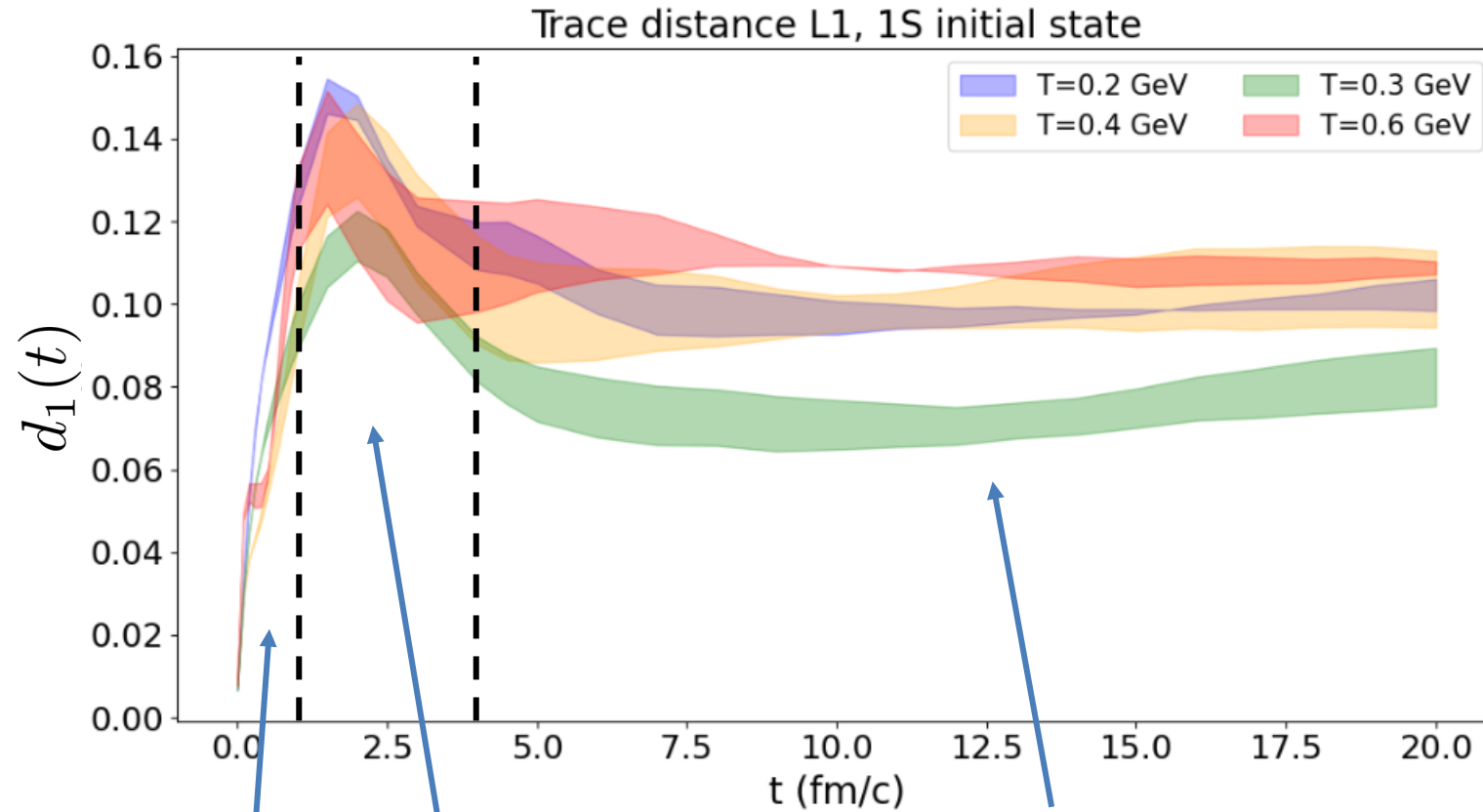


- Some specific signs of genuine QM evolution at small time although same trends observed
- Better agreement between the two descriptions at late times.

Non-unitary evolution

Full coupling with the QGP

$$d_1 = \int dr dp |W_{\text{QME}} - W_{\text{SC}}|$$

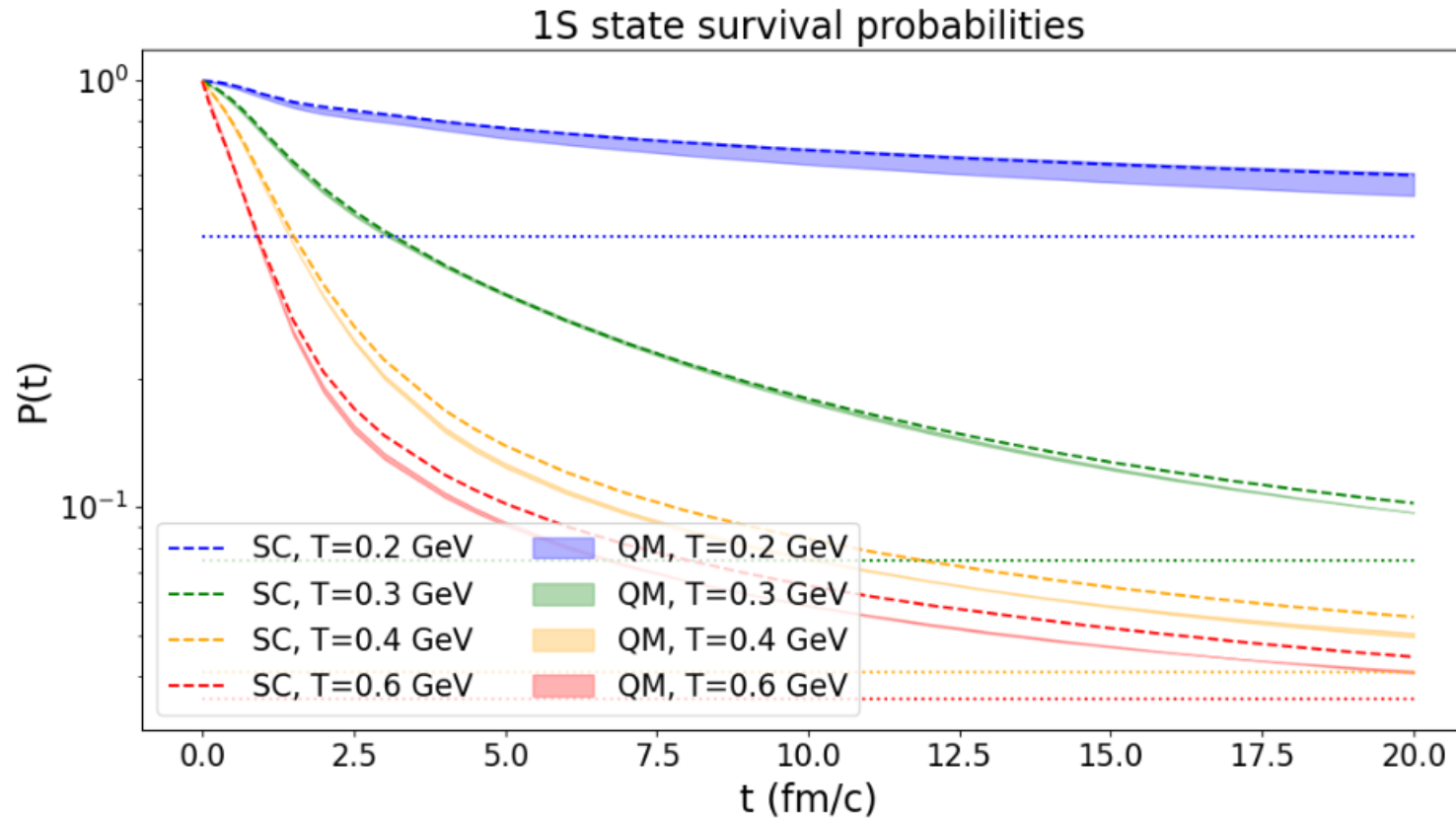


I. Growth of the $\neq$ between QME and SC evolution : genuine QM features

II. Saturation and decrease of the $\neq$ between QME and SC evolution : **classicalization**

III. Late stage evolution: the norm difference d_1 ceases to decrease and saturates ?!

- More concrete observable : **Survival probability of the 1S initial state:**

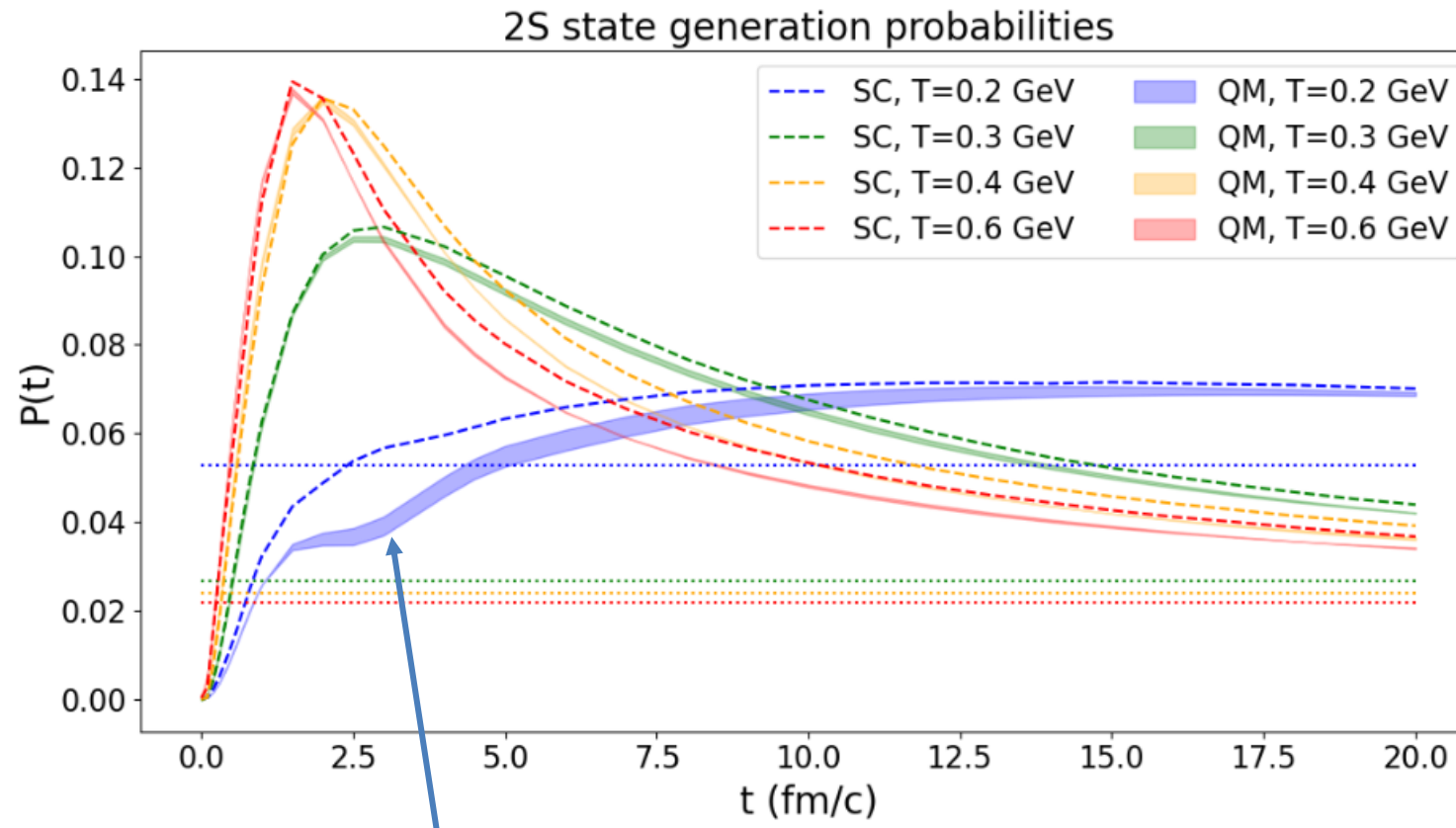


- Good agreement of the SC calculation with the QME benchmark
- Slight over suppression for the QME (overheating)

Non-unitary evolution

Full coupling with the QGP

- More concrete observable : **(Re)generate 2S state**
- Good agreement of the SC calculation with the QME benchmark, especially at large T

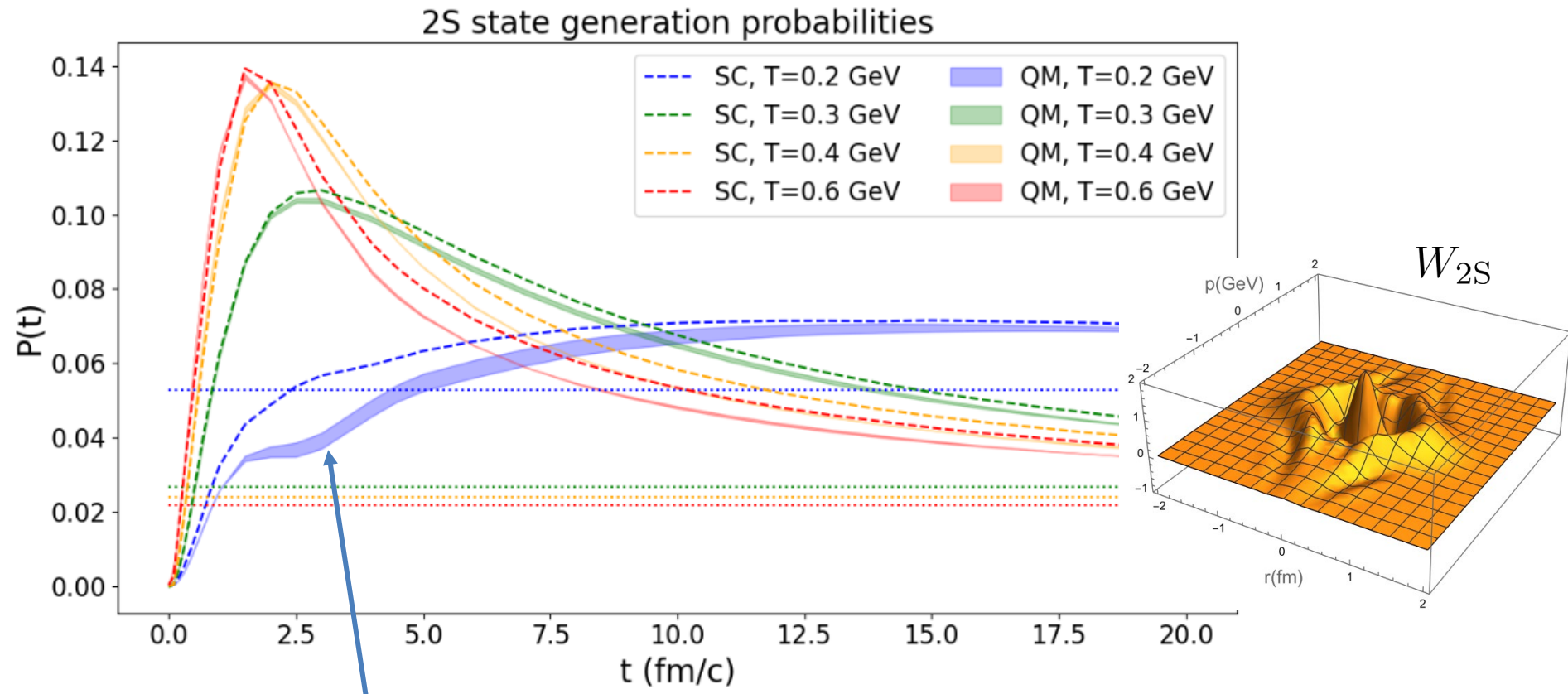


- Most significant disagreement for low T , around $t = 2.5$ fm/c (beginning of the classicalization)

Non-unitary evolution

Full coupling with the QGP

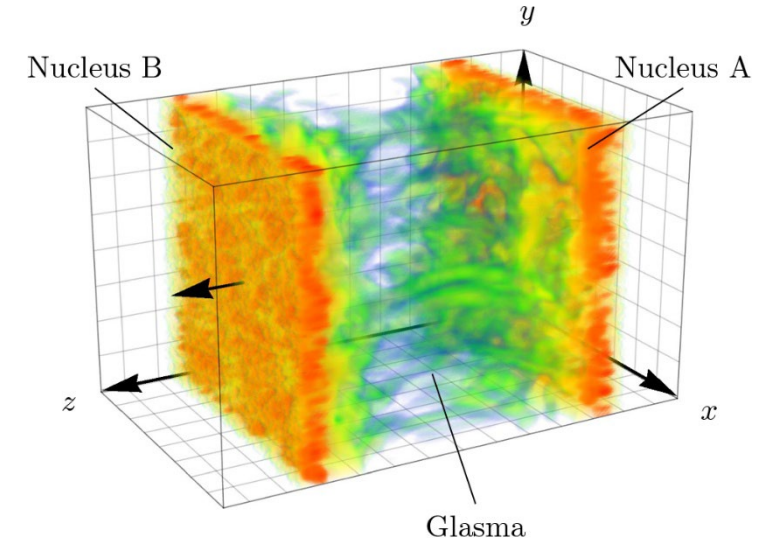
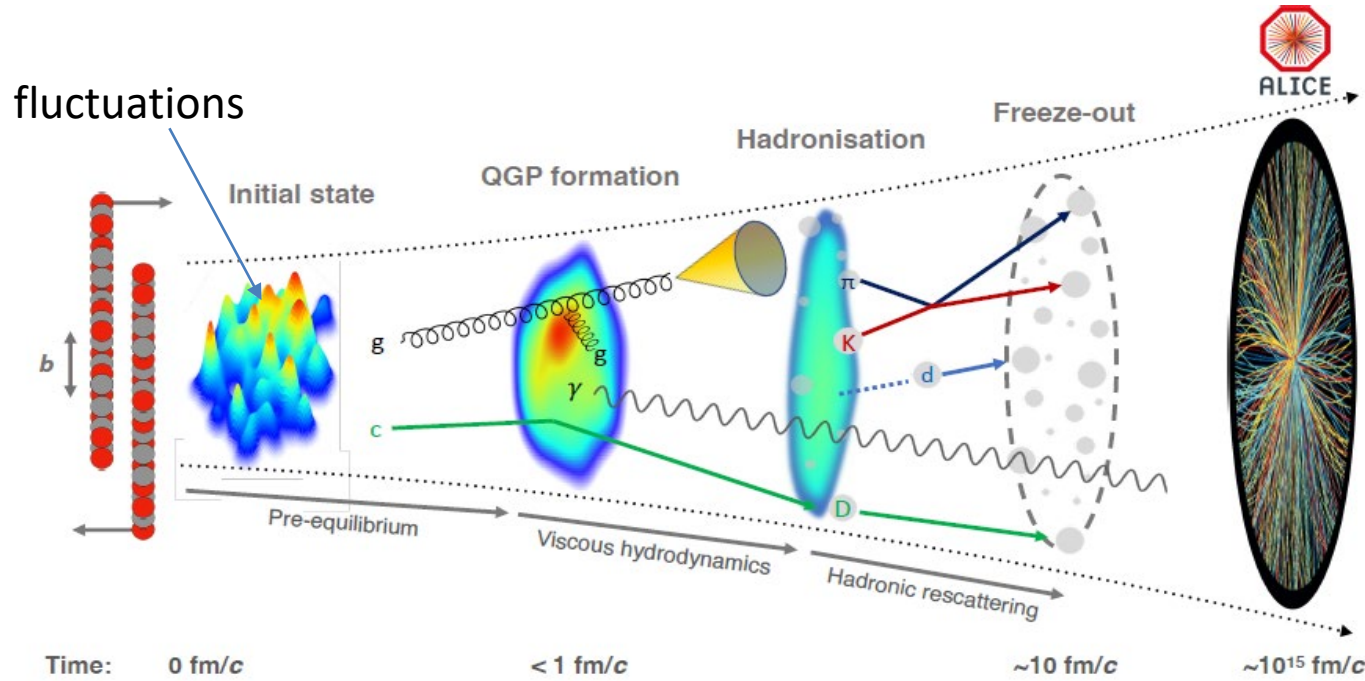
- More concrete observable : **(Re)generate 2S state**
- Good agreement of the SC calculation with the QME benchmark, especially at large T



- Higher effect of the genuine interference quantum effects due to the mixture of positive and negative regions in W_{2S} .

Life before QGP...

Binu Nair (Join PhD with Catania)

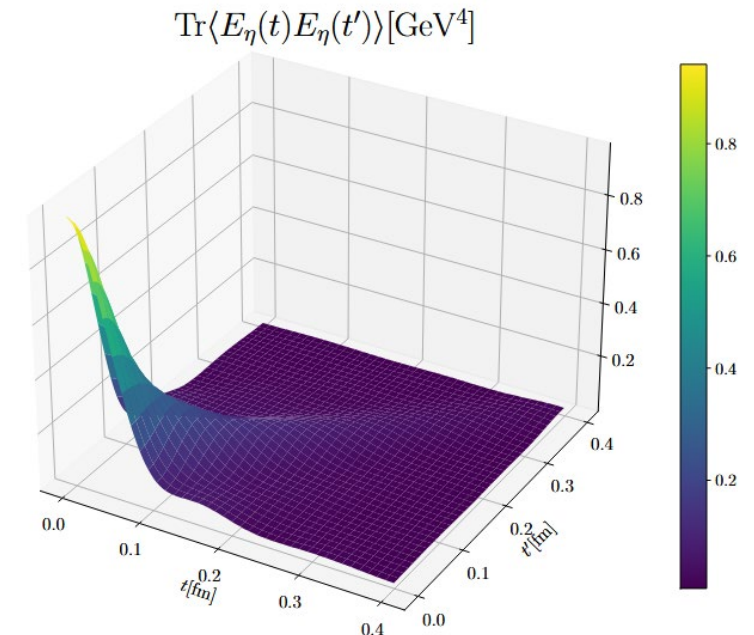


pNRQCD :

$$\mathcal{L}_{\text{int}} = \int d^3x \left[\text{Tr} \left(g \mathcal{O}^\dagger r \cdot ES + \frac{g}{2} \mathcal{O}^\dagger r \cdot E \mathcal{O} + h.c. \right) \right]$$

Octet weight evolution (fixed size):

$$\frac{d\rho_{OO}}{dt} = \frac{4\pi\alpha_s r_i^2}{N_c} \int_0^t dt' C_i(t, t') \cos[\omega(t - t')] [d_A - (1 + d_A)\rho_{OO}(t')] \quad d_A = 8$$



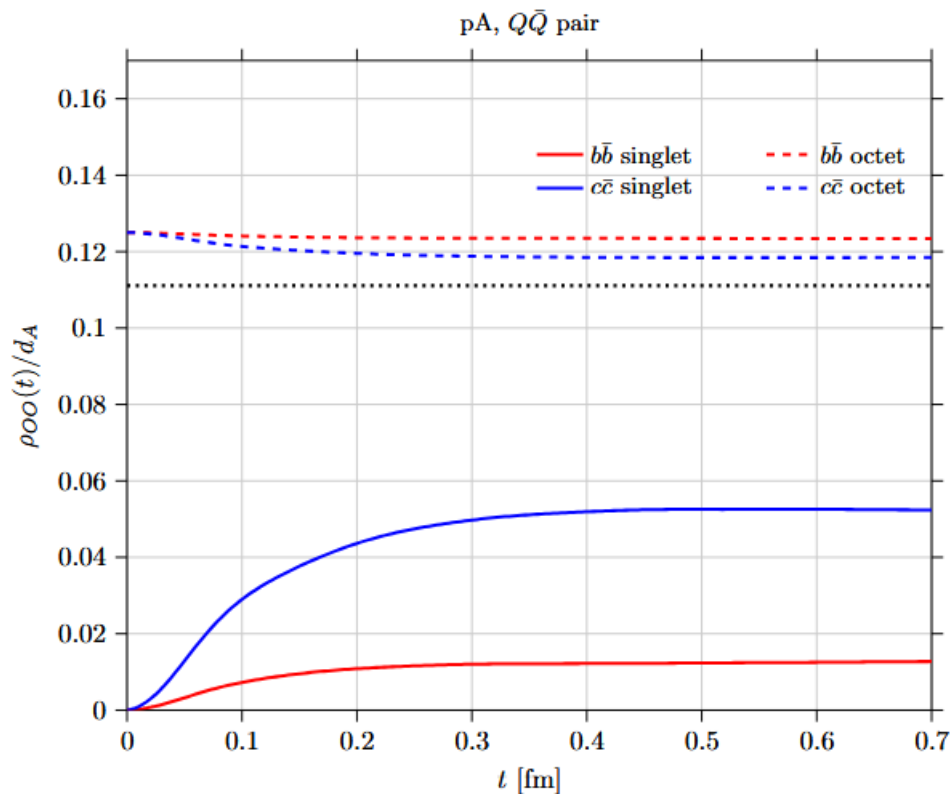


FIG. 5. Evolution of ρ_{OO}/d_A of a $Q\bar{Q}$ pair ($b\bar{b}$ in red and $c\bar{c}$ in blue) in glasma produced in pA collisions, up to $t = 0.7$ fm from different initial color states. The color dipole is in isotropic orientation in both the cases ($\langle x^2 \rangle = \langle y^2 \rangle = \langle \eta^2 \rangle = \langle r^2 \rangle/3$). The black dotted line indicates the color equilibration limit $1/9$.

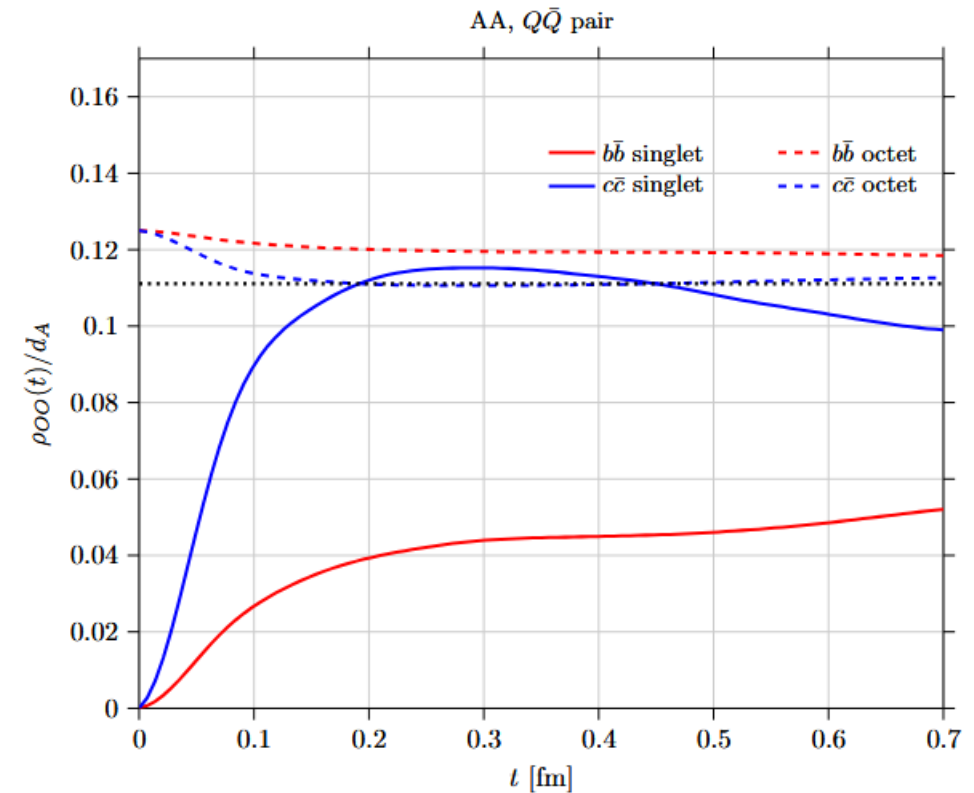


FIG. 6. Evolution of ρ_{OO}/d_A of a $Q\bar{Q}$ pair in glasma produced in an AA collision, up to $t = 0.7$ fm from different initializations for a $Q\bar{Q}$ pair ($b\bar{b}$ in red and $c\bar{c}$ in blue) in an isotropic orientation ($\langle x^2 \rangle = \langle y^2 \rangle = \langle \eta^2 \rangle = \langle r^2 \rangle/3$). The black dotted line corresponds to the color equilibration value $1/9$.

Conclusions



<https://arxiv.org/abs/2608.15347>

Track 1: Describe the entire time evolution of in-QGP quarkonia...

...through a general Lindblad equation, encompassing both the high and low T regimes.

- The proper QME was identified
- First numerical applications show the « interpolating » features of such Universal Lindblad equation
- Further numerical studies of dynamical evolution on the way...

Track 2: Evaluate the level of “precision loss” resulting from the semiclassical approximation (instead of a full quantum scheme)

- In the (unphysical) abelian representation, the Semi-classical approximation allows to reproduce the results from the exact Quantum Master Equation relevant for phenomenology.
- This is due to the coupling with the heat bath which “classicalizes” the subsystem.

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Track 3: Explore physics before QGP : some level of thermalization seen from early calculation