

Gluon production in Proton-Nucleus collisions from BCFW recursion

François Gelis

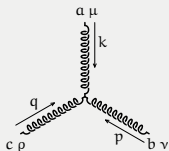
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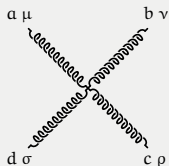
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QCD FEYNMAN RULES

$$\frac{p}{\text{gluon}} = \frac{-i g^{\mu\nu} \delta_{ab}}{p^2 + i0^+} + \frac{i \delta_{ab}}{p^2 + i0^+} \left(1 - \frac{1}{\xi}\right) \frac{p^\mu p^\nu}{p^2}$$



$$= -g f^{abc} \{ g^{\mu\nu} (k - p)^\rho + g^{\nu\rho} (p - q)^\mu + g^{\rho\mu} (q - k)^\nu \}$$



$$= -i g^2 \{ f^{abe} f^{cde} (g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}) + f^{ace} f^{bde} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\rho}) + f^{ade} f^{bce} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma}) \}$$

5-GLUON TREE AMPLITUDE

On peut écrire l'amplitude de 5 gluons en arbre en fonction des amplitudes de 3 et 4 gluons en arbre. On a par exemple :

$$A(1,2,3,4,5) = \frac{1}{2} \left(A(1,2,3) A(4,5) + A(1,2,4) A(3,5) + A(1,2,5) A(3,4) + A(1,3,4) A(2,5) + A(1,3,5) A(2,4) + A(1,4,5) A(2,3) + A(2,3,4) A(1,5) + A(2,3,5) A(1,4) + A(2,4,5) A(1,3) + A(3,4,5) A(1,2) \right)$$

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$k_1 \cdot k_4 \varepsilon_2 \cdot k_1 \varepsilon_1 \cdot \varepsilon_3 \varepsilon_4 \cdot \varepsilon_5$

Slide from Zvi Bern (~ 2004)

5-GLUON TREE AMPLITUDE

Parke-Taylor formula (1986):

$$\mathcal{A}_5(1^-2^+3^-4^+5^+) = -2\sqrt{2}g^3 \frac{\langle 13 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 51 \rangle}$$

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$k_1 \cdot k_4 \varepsilon_2 \cdot k_1 \varepsilon_1 \cdot \varepsilon_3 \varepsilon_4 \cdot \varepsilon_5$

For scattering amplitudes with only **on-shell** external gluons with **physical polarizations** :

- LO: Tools for obtaining directly the simple expressions
 - spinor-helicity formalism
 - on-shell recursion (BCFW)
- NLO: Generalized unitarity for one-loop amplitudes

Could these tools be used for CGC calculations?

- In the deep saturated regime, $f(\mathbf{p}) \sim \alpha_s^{-1}$, $A^\mu \sim g^{-1}$
- Commutators have a comparatively small effect, and the dynamics is approximately classical $\implies$ Gauge fields behave as classical solutions of Yang-Mills equations
- Classical solutions $\equiv$ tree Feynman diagrams
- The color charges in the projectiles act as sources for the color field. The CGC describes these sources as color currents in the direction of motion of the projectiles,

$$[D_\mu, F^{\mu\nu}] + J^\nu = 0, \quad J^\nu = \delta^{\nu+} \rho_1 + \delta^{\nu-} \rho_2$$

- The CGC is invariant under simultaneous gauge transformations of the field and the current:

$$A^\mu \rightarrow \Omega^\dagger A^\mu \Omega + \frac{i}{g} \Omega^\dagger \partial^\mu \Omega, \quad J^\mu \rightarrow \Omega^\dagger J^\mu \Omega$$

- A gauge must be chosen at $x^0 = -\infty$ (initial conditions) and at all subsequent times
- The complexity of A^μ depends a lot on the gauge
- Observables built from A^μ are gauge independent

PROTON-NUCLEUS COLLISIONS

- The proton is considered to be a dilute hadron
⇒ The solution of YM is calculated at order one in the proton color density, and to all orders in that of the nucleus
- At this order, the YM equations can be solved analytically
This has been done in at least three gauges: Landau ($\partial_\mu A^\mu = 0$), Fock-Schwinger ($x^+ A^- + x^- A^+ = 0$), Light-cone ($A^+ = 0$)
- The A^μ 's in these gauges look extremely different (and the derivation is particularly tedious in some gauges)
- It would be desirable to have a method that gives directly the gauge invariant spectrum, and bypasses all gauge dependent intermediate steps

PERTURBATIVE SOLUTION OF YM

- Classical solutions can be represented as a sum of tree diagrams

$$A_a^\mu(x) = \sum_{\text{trees}} \chi^{\mu,a} \text{---} J_b^\nu(y)$$

- Ingredients: YM rules + color sources
- Missing:** rules to account for $[D_\mu, J^\mu] = 0$

EXTRA RULES FOR CURRENT CONSERVATION

$$\supset = \rho^{(0)} \quad \frac{\overleftarrow{p}}{a \quad b} = \frac{\delta_{ab}}{v \cdot p}$$

$$\frac{a \quad b}{\text{wavy line}} = -gf^{abc}v^\mu$$

- Initial value problem: all propagators should be retarded
- The external legs attached to the sources $\rho^{(0)}$ are not amputated. They have v^μ instead of a polarization vector, and they carry a space-like momentum, that obeys $v \cdot p = 0$

- $\mathfrak{su}(N)$ **Fierz identity**: any tree level amplitude of Yang-Mills theory can be decomposed as

$$\mathcal{M}_n(1_{a_1} \cdots n_{a_n}) = 2 \sum_{\sigma \in \mathfrak{S}_n / \mathbb{Z}_n} \text{tr}(t^{a_{\sigma_1}} \cdots t^{a_{\sigma_n}}) \mathcal{A}_n(\sigma_1 \cdots \sigma_n)$$

- $\mathcal{A}_n(\sigma_1 \cdots \sigma_n)$ is a *color ordered* (or *partial*) amplitude
Given by color-stripped Feynman rules; only planar diagrams

- Define $\sigma^\mu \equiv (1, \sigma^i)$, $\bar{\sigma}^\mu \equiv (1, -\sigma^i)$

Given p^μ , define $P \equiv p_\mu \sigma^\mu$ and $\bar{P} \equiv -p_\mu \bar{\sigma}^\mu$ (2×2 matrices)

- $p^2 = 0$ is equivalent to $\det(P) = \det(\bar{P}) = 0$, i.e., $P, \bar{P}$ are factorizable as direct products of 2-component vectors,

$$P = |\mathbf{p}\rangle \langle \mathbf{p}|, \quad \bar{P} = |\mathbf{p}\rangle [\mathbf{p}|$$

PARAMETERIZATION OF OFF-SHELL MOMENTA

- Off-shell momenta inside graphs are completely generic (but they are seldom needed)
- Off-shell momenta attached to *initial* sources $p^{(0)}$ must obey $v \cdot p = 0$. We consider only sources moving at the speed of light, i.e., v^μ is light-like and $v_\mu \sigma^\mu = |\mathbf{v}\rangle\langle\mathbf{v}|$
- Given an auxiliary momentum q^μ , this momentum can be parameterized as

$$\bar{p} = \frac{p \cdot q}{v \cdot q} |\mathbf{v}\rangle[\mathbf{v}] - \kappa \frac{|\mathbf{v}\rangle[\mathbf{q}]}{[\mathbf{v}\mathbf{q}]} + \bar{\kappa} \frac{|\mathbf{q}\rangle[\mathbf{v}]}{\langle\mathbf{q}\mathbf{v}\rangle}$$

- Deformation of two external momenta in an amplitude, controlled by a complex parameter z
- **At all z :** must preserve momentum conservation, on-shell momenta must remain on-shell
- Assume $\mathcal{A}(\cdots; z) \rightarrow 0$ when $|z| \rightarrow \infty$
Not always true, must be checked!
- Take a circle $\mathcal{C}_R$ of radius $R \rightarrow \infty$, and integrate $\mathcal{A}(\dots, z)/z$

$$0 = \oint_{\mathcal{C}_R} \frac{dz}{2\pi i} \frac{\mathcal{A}(\cdots, z)}{z} = \underbrace{\mathcal{A}(\cdots, z=0)}_{\text{original amplitude}} + \sum_{\substack{\text{poles} \\ \neq 0}} \text{Residue} \left(\frac{\mathcal{A}(\dots, z)}{z} \right)$$

- The recursion reads

$$\mathcal{A}(\dots) = \sum_{\substack{\text{poles} \\ \neq 0}} \sum_{h=\pm} \mathcal{A}_L(\dots \widehat{k}^h) \frac{i}{k^2} \mathcal{A}_R(-\widehat{k}^{-h} \dots)$$

- $\mathcal{A}_{L,R}$ are amplitudes with **fewer** external legs
- **Method:**
 - Choose the pair of lines i, j and the shift vector e^μ
 - List the internal propagators that can have a pole
 - For each pole, calculate $z_*, \widehat{k}, \mathcal{A}_L$ and $\mathcal{A}_R$

- The method of shifting external momenta to split an amplitude into a pair of simpler amplitudes works also with off-shell external legs [[van Hameren, 2014](#)]
- When the line i is off-shell, use the shift $e^\mu = \frac{1}{2} \langle \mathbf{v}_i | \bar{\sigma}^\mu | j \rangle$, with v_i^μ the light-like direction of the source producing the gluon i
 - Poles in internal propagators: exactly as before
 - There can be a pole in the external off-shell propagator

THREE ON-SHELL GLUONS

- With BCFW recursion, everything starts with 3-point amplitudes
- Obtained directly from Feynman rules (1 graph), or from *little group scaling* (Feynman rule only used to get the prefactor)
- Non-zero 3-gluon on-shell amplitudes:

$$\mathcal{A}_3(0^+ 1^+ 2^-) = -g\sqrt{2} \frac{[01]^3}{[12][20]}$$

$$\mathcal{A}_3(0^- 1^- 2^+) = g\sqrt{2} \frac{\langle 01 \rangle^3}{\langle 12 \rangle \langle 20 \rangle}$$

(Parke-Taylor formulas for $n = 3$)

TWO ON-SHELL, ONE OFF-SHELL

- Perturbation theory: 2 graphs

- $0^+1^+2^*$: shift $e^\mu = \frac{1}{2}\langle \mathbf{v}_2 | \bar{\sigma}^\mu | 1 \rangle$ $\mathcal{A}_3(0^+1^+2^*) = 0$

- $0^+1^-2^*$: shift $e^\mu = \frac{1}{2}\langle 1 | \bar{\sigma}^\mu | \mathbf{v}_2 \rangle$

$$\mathcal{A}_3(0^+1^-2^*) = \frac{ig}{\kappa_2} \frac{\langle 1 \mathbf{v}_2 \rangle^3}{\langle \mathbf{v}_2 0 \rangle \langle 01 \rangle}$$

- $0^-1^+2^*$: shift $e^\mu = \frac{1}{2}\langle \mathbf{v}_2 | \bar{\sigma}^\mu | 1 \rangle$

$$\mathcal{A}_3(0^-1^+2^*) = \frac{ig}{\kappa_2} \frac{[1 \mathbf{v}_2]^3}{[\mathbf{v}_2 0] [01]}$$

- Perturbation theory: 3 graphs

- $0^+1^*2^*$: shift $e^\mu = \frac{1}{2} \langle \mathbf{v}_2 | \bar{\sigma}^\mu | \mathbf{v}_1]$

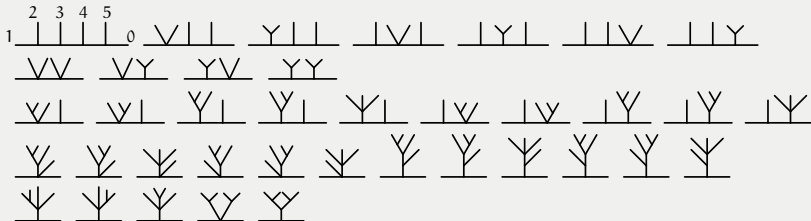
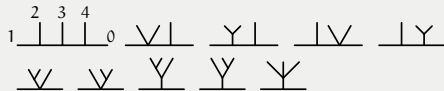
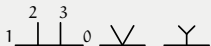
$$\mathcal{A}_3(0^+1^*2^*) = -\frac{g}{\sqrt{2} \bar{\kappa}_1 \bar{\kappa}_2} \frac{\langle \mathbf{v}_1 \mathbf{v}_2 \rangle^3}{\langle \mathbf{v}_2 0 \rangle \langle 0 \mathbf{v}_1 \rangle}$$

- $0^-1^*2^*$: shift $e^\mu = \frac{1}{2} \langle \mathbf{v}_1 | \bar{\sigma}^\mu | \mathbf{v}_2]$

$$\mathcal{A}_3(0^-1^*2^*) = \frac{g}{\sqrt{2} \kappa_1 \kappa_2} \frac{[\mathbf{v}_1 \mathbf{v}_2]^3}{[\mathbf{v}_2 0] [0 \mathbf{v}_1]}$$

SCATTERING OFF AN EXTERNAL FIELD: PERTURBATION THEORY

- Graphs in the solution of $[D_\mu, F^{\mu\nu}] + J^\nu = 0$:



- One should also add the graphs that account for $[D_\mu, J^\mu] = 0$

SCATTERING OFF AN EXTERNAL FIELD: BCFW

- BCFW with shift $e^\mu = \frac{1}{2} \langle \mathbf{v}_2 | \bar{\sigma}^\mu | \mathbf{v}_2 \rangle$ on lines $(n-1, n)$:

$$\mathcal{A}_{n+1}(0^+ 1^- 2^* \dots n^*) = \underbrace{\mathcal{A}_3(0^+ - \widehat{1}^- \widehat{n}^*)}_{\text{known}} \frac{i}{(p_0 + p_n)^2} \underbrace{\mathcal{A}_n(\widehat{1}^+ 1^- 2^* \dots \widehat{n-1}^*)}_{\text{previous order}}$$

$\implies$ gives a recursion on the number of scatterings, directly at the level of scattering amplitudes (we just need to work out the values of z_* , $|\widehat{1}\rangle$ and $|\widehat{1}]$ at the pole)

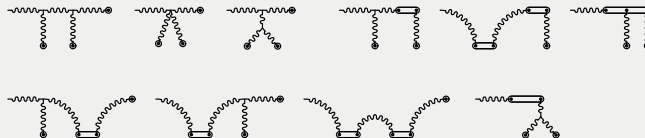
- This leads to

$$\mathcal{A}_{n+1}(0^+ 1^- 2^* \dots n^*) = i \frac{[0\nu_2]^2}{[1\nu_2]^2} \frac{(g[1\nu_2]\langle 1\nu_2 \rangle)^{n-1}}{|\kappa_2|^2 \dots |\kappa_n|^2} \times \frac{1}{(p_0 + p_n)^2 (p_0 + p_n + p_{n-1})^2 \dots (p_0 + p_n + \dots + p_3)^2}$$

- Remarkably compact compared to what one might expect from the list of Feynman graphs
- These amplitudes are in fact the successive terms in the expansion of a Wilson line

- We need the partial amplitudes $\mathcal{A}_{n+1}(0^+ 1^* \underbrace{2^* \dots n^*}_{\text{nucleus}})$
- Gluon production requires color sources with two distinct directions, v_1^μ (proton) and v_2^μ (nucleus)
- For pA collisions: one source of direction v_1^μ and many sources of direction v_2^μ
- Shift $e^\mu = \frac{1}{2} \langle \mathbf{v}_2 | \bar{\sigma}^\mu | \mathbf{v}_2 \rangle = v_2^\mu$

- Perturbation theory:



- BCFW with the shift $e^\mu = v_2^\mu$ on the lines (2, 3) gives:

$$\mathcal{A}_4(0^+ 1^* 2^* 3^*) = -\frac{g^2}{|\kappa_1|^2 |\kappa_2|^2 |\kappa_3|^2} \frac{\langle 0v_2 \rangle [0v_2]}{(p_0 + p_3)^2} \frac{\kappa_1 (\kappa_2 + \kappa_3) \langle v_1 v_2 \rangle^3}{\sqrt{2} \langle 0v_1 \rangle \langle 0v_2 \rangle}$$

- From the orders $\rho_1 \rho_2$ and $\rho_1 \rho_2^2$, we may guess

$$\mathcal{A}_{n+1}(0^+ 1^* \dots n^*) = - \frac{g^{n-1} \kappa_1 (\kappa_2 + \dots + \kappa_n)}{|\kappa_1|^2 |\kappa_2|^2 \dots |\kappa_n|^2} \frac{\langle \mathbf{v}_1 \mathbf{v}_2 \rangle^3}{\sqrt{2} \langle \mathbf{v}_2 0 \rangle \langle 0 \mathbf{v}_1 \rangle} \\ \times \frac{\langle \mathbf{v}_2 0 \rangle [0 \mathbf{v}_2]}{(p_0 + p_n)^2} \dots \frac{\langle \mathbf{v}_2 0 \rangle [0 \mathbf{v}_2]}{(p_0 + p_n + \dots + p_3)^2}$$

- **Strategy:**
 - This ansatz is true for $n = 2$ and $n = 3$
 - BCFW gives a recursion from $\mathcal{A}_n$ to $\mathcal{A}_{n+1}$, that we use in order to prove the validity of the ansatz at order $n + 1$, assuming it was true at order n

Summary

- On-shell methods (spinor-helicity formalism, BCFW recursion) can be generalized to handle the amplitudes with off-shell external gluons that arise in CGC calculations
- Does not require gauge fixing
- Provides recursion relations among gauge invariant amplitudes
- Simpler than solving YM equations, with the added bonus of being gauge agnostic
- **Possible extensions:**
 - Other tree level processes: $g + A \rightarrow gg + A$, $g + A \rightarrow ggg + A$
 - Amplitudes with $q\bar{q}$ pairs (for massless quarks)
 - *Generalized unitarity*: break down 1-loop amplitudes into products of up to four (five in dim. reg.) tree level amplitudes

Behavior at $|z| \rightarrow \infty$

NAIVE POWER COUNTING

- Consider the shift $e^\mu = \frac{1}{2} \langle \mathbf{v} | \bar{\sigma}^\mu | \mathbf{v} \rangle = v^\mu$ applied to two off-shell gluons produced by sources of direction v^μ
- With this shift, the external off-shell propagators do not depend on z
- Counting rules:
 - Inner propagator $\sim z^{-1}$
 - 3-gluon vertex $\sim z$
 - 4-gluon vertex $\sim z^0$
- Worst case: 3-gluon vertices only, one more vertex than internal propagators $\implies z$

IMPROVED POWER COUNTING

- The previous counting gives the graph-by-graph behavior.

Actual behavior may be better if cancellations happen

- Write $A^\mu \equiv \mathcal{A}^\mu + a^\mu$, with a^μ a hard component that carries the momentum $\sim z$ ($\partial a \sim z$), and a soft part $\mathcal{A}^\mu$ ($\partial \mathcal{A} \sim z^0$),

$$\mathcal{D}^\mu = \partial^\mu - ig\mathcal{A}^\mu, \quad -ig\mathcal{F}^{\mu\nu} = [\mathcal{D}^\mu, \mathcal{D}^\nu]$$

$$\mathcal{L} = -\text{tr} \left((\mathcal{D}^\mu a^\nu)(\mathcal{D}_\mu a_\nu) \right) + \text{tr} \left((\mathcal{D}_\mu a^\mu)(\mathcal{D}_\nu a^\nu) \right) + ig \text{tr} \left(\mathcal{F}_{\mu\nu} [a^\mu, a^\nu] \right) + \dots$$

- Background field gauge, $\mathcal{D}_\mu a^\mu = 0$,

$$\mathcal{L}_{\text{gauge fixed}} = -\text{tr} \left((\mathcal{D}^\mu a^\nu)(\mathcal{D}_\mu a_\nu) \right) + ig \text{tr} \left(\mathcal{F}_{\mu\nu} [a^\mu, a^\nu] \right) + \dots$$

- First term: extended Lorentz symmetry (a^ν and $\mathcal{D}_\mu$ can be transformed independently), contains derivatives of a^ν
- Second term: antisymmetric in the hard fields, no derivatives of the hard fields (i.e., no z)

IMPROVED POWER COUNTING

- Generic form of amputated graphs with two hard fields:

$$\mathcal{M}^{\mu\nu} = \underbrace{(c_1 z + c_0 + c_{-1} z^{-1} + \dots) g^{\mu\nu}}_{\text{first term only}} + A^{\mu\nu} + R^{\mu\nu}$$

- $A^{\mu\nu}$ comes from a single insertion of $[a^\mu, a^\nu] \mathcal{F}_{\mu\nu}$ (antisymmetric in (μ, ν) , order $\sim z^0$)
- $R^{\mu\nu}$ contains at least two 4-gluon vertices (order $\sim z^{-1}$)
- To obtain an amplitude, contract $\mathcal{M}^{\mu\nu}$ with the propagators of two off-shell gluons and with the directions of the sources,

$$\begin{aligned} (v^\alpha G_{\alpha\mu}) \mathcal{M}^{\mu\nu} (G_{\nu\beta} v^\beta) &\propto v_\mu \mathcal{M}^{\mu\nu} v_\nu \\ &= (c_1 z + c_0 + c_{-1} z^{-1} + \dots) \underbrace{v^2}_{=0} + \underbrace{A^{\mu\nu} v_\mu v_\nu}_{=0} + \underbrace{R^{\mu\nu} v_\mu v_\nu}_{\sim z^{-1}} \end{aligned}$$

Solution in Landau gauge

SOLUTION OF YM IN LANDAU GAUGE

$$A_a^\mu(p) = A_{1a}^\mu(p) + \frac{ig}{p^2} \int \frac{d^2 \mathbf{p}_{1\perp}}{(2\pi)^2} \left[C^\mu (U - 1)_{\mathbf{p}_{2\perp}} + \underline{D^\mu (\textcolor{red}{V} - 1)_{\mathbf{p}_{2\perp}}} \right]_{ab} \frac{\rho_{1b}^{(0)}(\mathbf{p}_{2\perp})}{\mathbf{p}_{2\perp}^2}$$

$$U(y^+, x^+; \mathbf{x}_\perp) = \text{P exp } ig \int_{x^+}^{y^+} dz^+ A_2^-(z^+, \mathbf{x}_\perp),$$

$$V(y^+, x^+; \mathbf{x}_\perp) = \text{P exp } \frac{ig}{\textcolor{red}{2}} \int_{x^+}^{y^+} dz^+ A_2^-(z^+, \mathbf{x}_\perp)$$

$$A_{1a}^\mu(p) = g\delta^{\mu+} \frac{\rho_{1a}^{(0)}(\mathbf{p}_\perp)}{\mathbf{p}_\perp^2}, \quad A_{2a}^\mu(p) = g\delta^{\mu-} \frac{\rho_{2a}^{(0)}(\mathbf{p}_\perp)}{\mathbf{p}_\perp^2}$$

$$C^+ = \frac{\mathbf{p}_{1\perp}^2}{p^-}, \quad C^- = \frac{\mathbf{p}_\perp^2 - \mathbf{p}_{2\perp}^2}{p^+}, \quad C^i = -2\mathbf{p}_{1\perp}^i$$

$$D^+ = -2p^+, \quad D^- = 2p^- - 2\frac{\mathbf{p}_\perp^2}{p^+}, \quad D^i = -2\mathbf{p}_\perp^i$$

SOLUTION OF YM IN LANDAU GAUGE

- D^μ is longitudinal when p^μ is on-shell
 $\implies V$ does not contribute to gluon production
- Quark-Antiquark production:
 - D^-/p^2 contains a term that does not go to zero when $p^- \rightarrow \infty$
Corresponds to a singular term $\sim \delta(x^+)$ in coordinate space
 - This term leads to $Q\bar{Q}$ production inside the nucleus shockwave
 $\implies$ does quark production depend on V ?
 - **NO**, but V cancels thanks to a rather weird identity,

$$\begin{aligned} \frac{ig}{2} \int_{-\infty}^{+\infty} dx^+ U(+\infty, x^+; \mathbf{x}_\perp) A_2^-(x^+, \mathbf{x}_\perp) V(\mathbf{x}^+, -\infty; \mathbf{x}_\perp) \\ = U(+\infty, -\infty; \mathbf{x}_\perp) - V(+\infty, -\infty; \mathbf{x}_\perp) \end{aligned}$$