



Earth rotation impact for long-duration signals

GdR 2026 - Lyon
07/10/26



Mathieu SCHOOR
Damir BUSKULIC

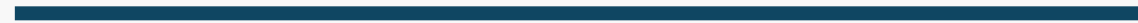


-
- *Motivations*
 - *Mismatch and SNR study*
 - *ET in 3G network*
 - *Conclusion - Discussion*
-





Motivations



$$h(t) = h_+(t)F_+(\theta, \phi, \psi) + h_\times F_\times(\theta, \phi, \psi)$$

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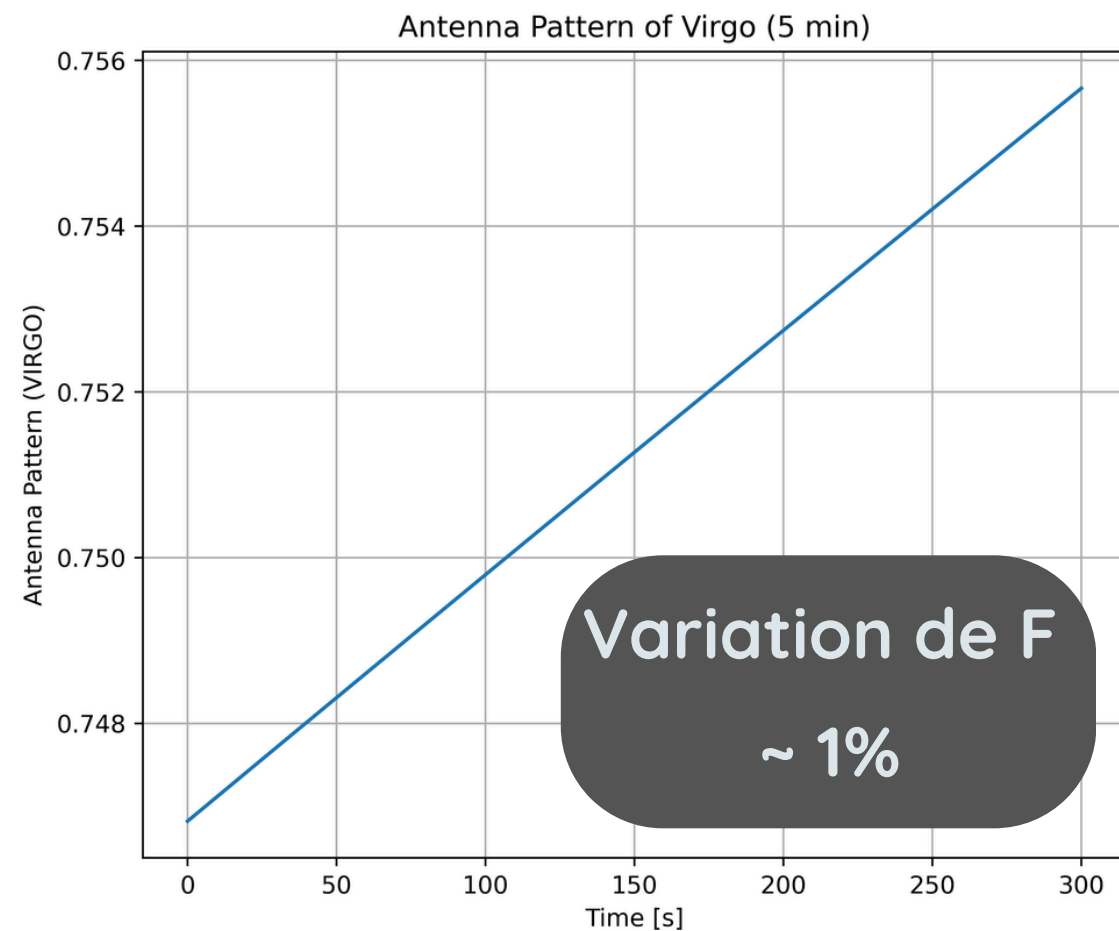
Signal duration

- Virgo : ~5 min

Random example

RA = 3.45 [rad]

DEC = -0.408 [rad]



$$h(t) = h_+(t)F_+(\theta, \phi, \psi) + h_\times F_\times(\theta, \phi, \psi)$$

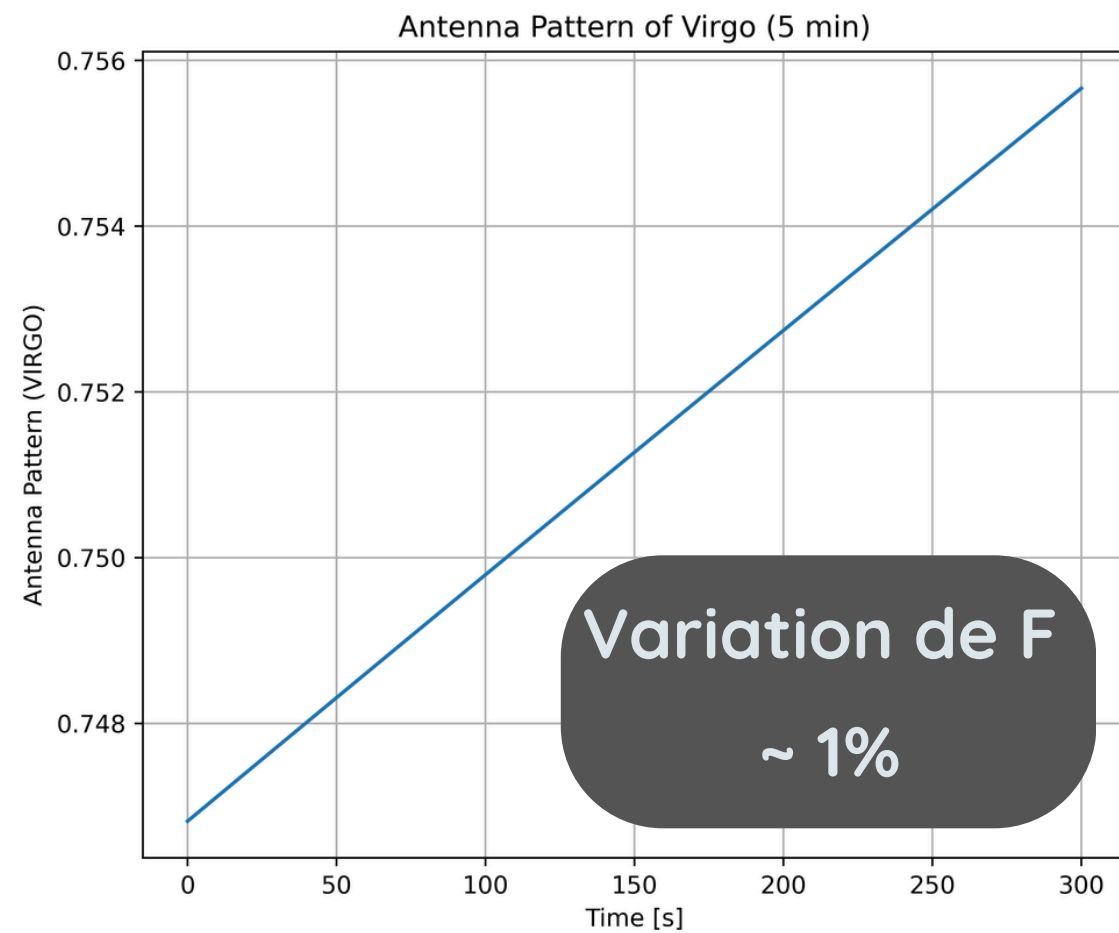
ET-Xylophone

Signal duration

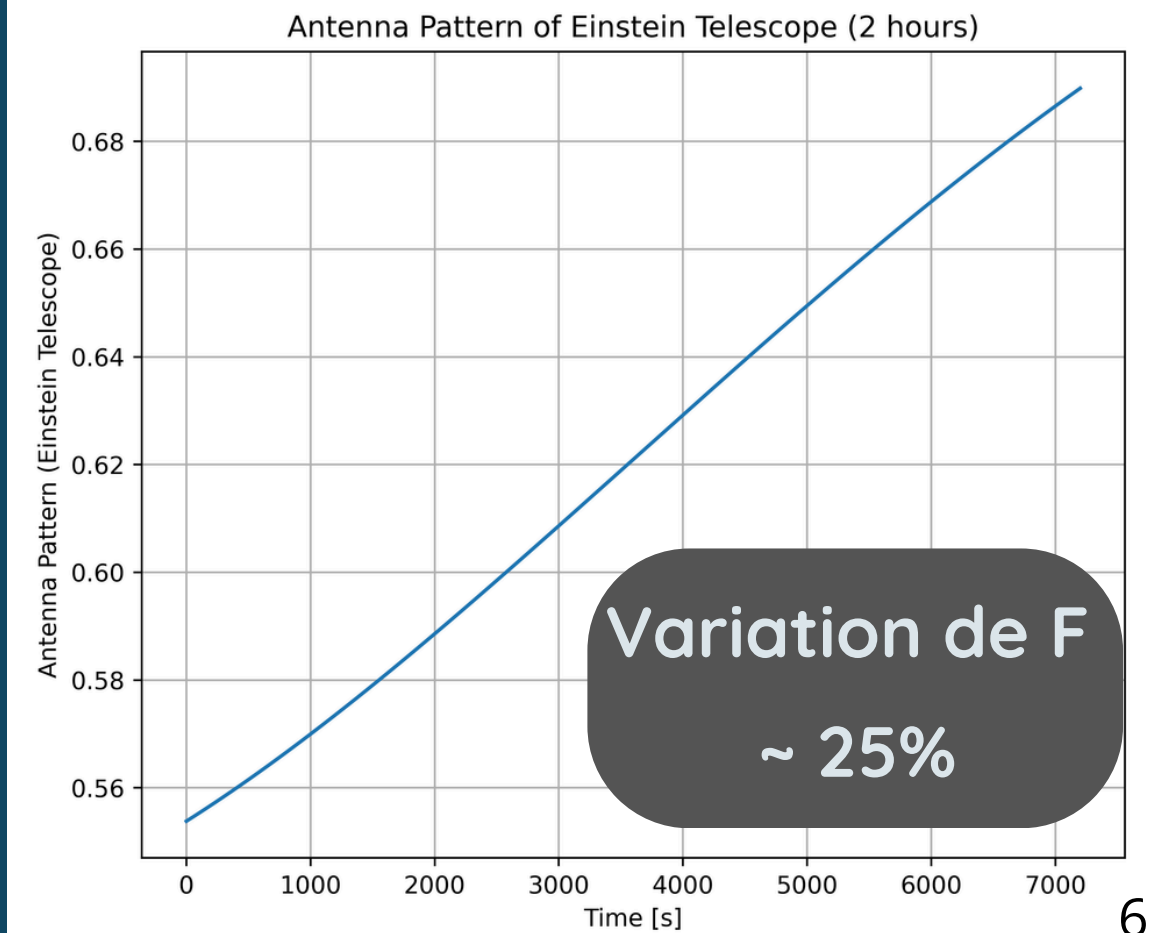
- Virgo : ~5 min
- ET : ~1-2h

Random example

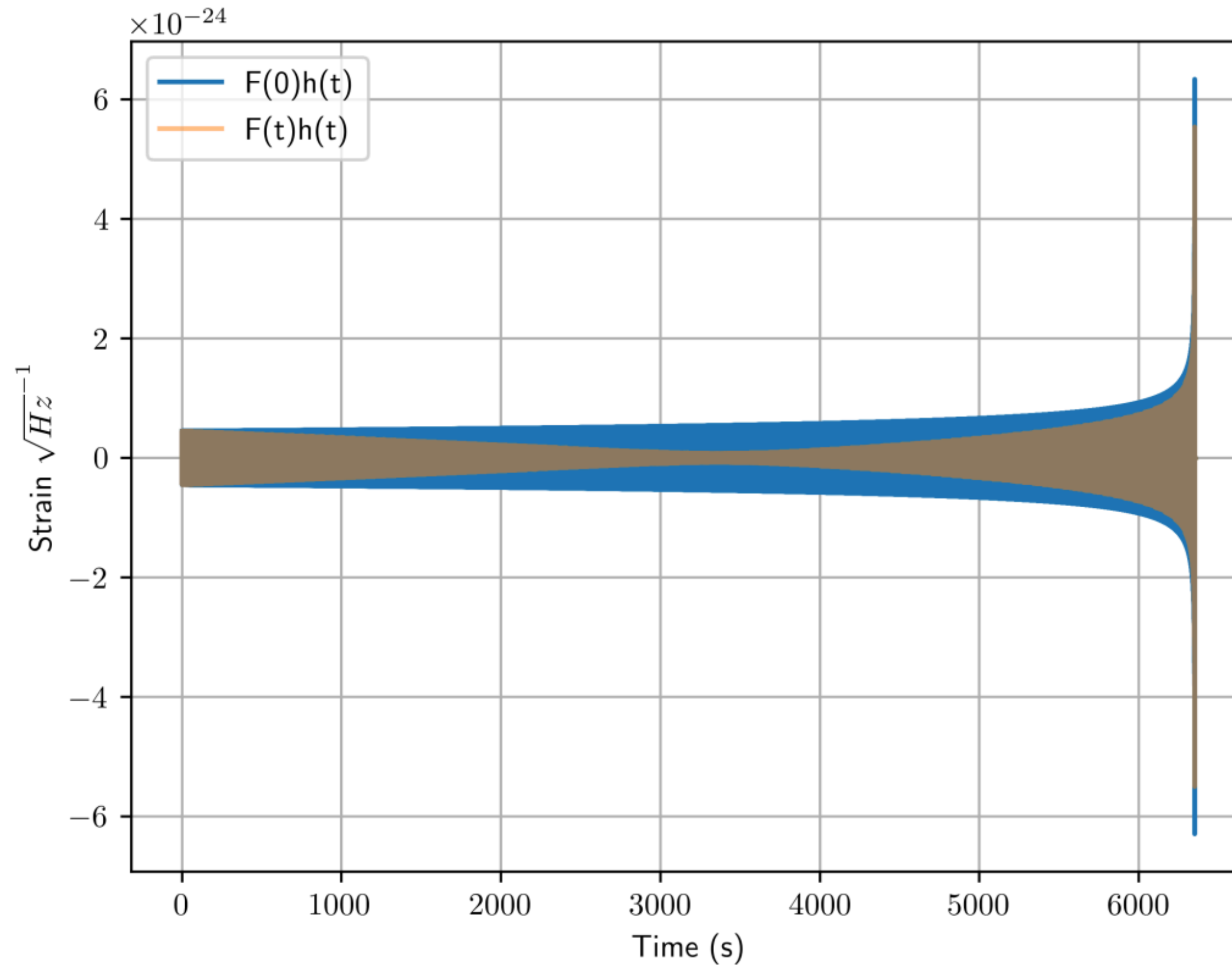
RA = 3.45 [rad]
DEC = -0.408 [rad]



The reason why Earth keeps moving btw:

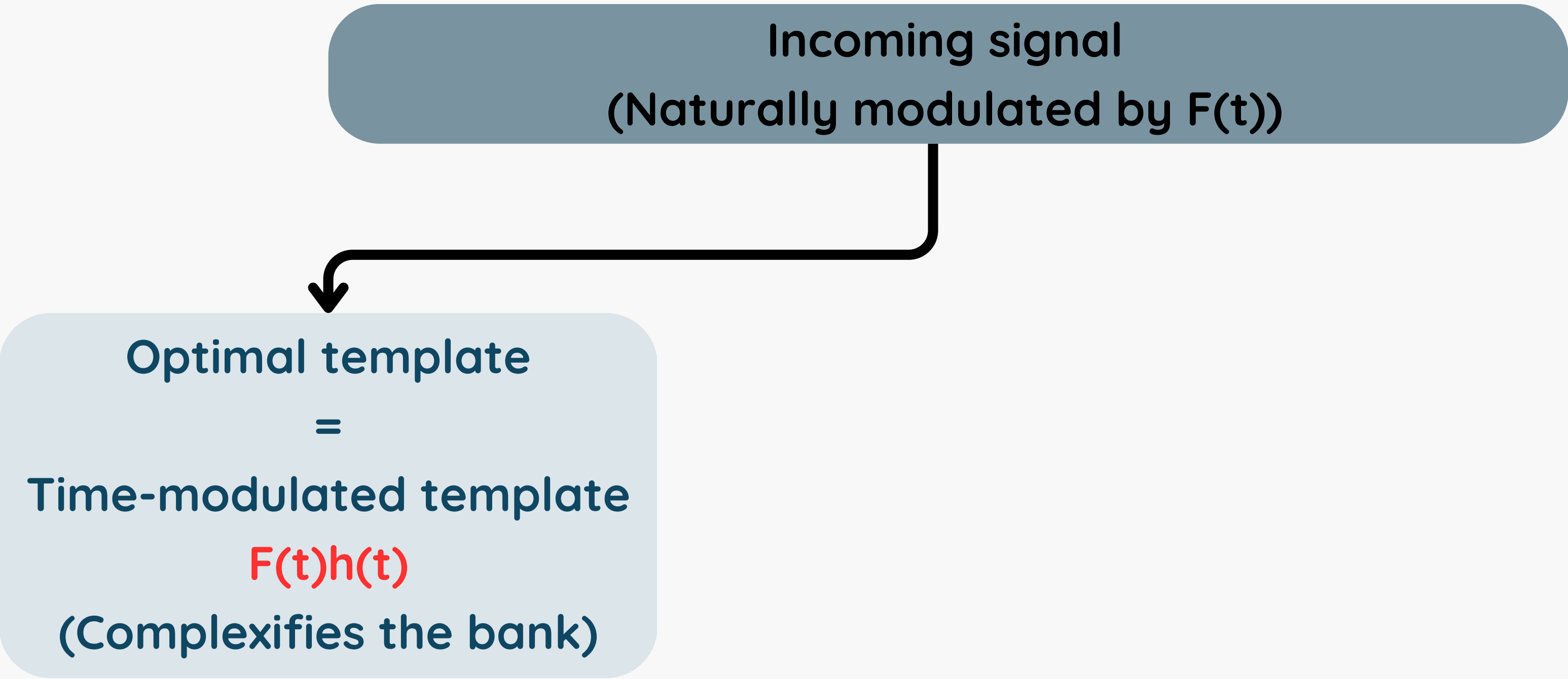


Earth Rotation can lead to highly disformed signals



Incoming signal
(Naturally modulated by $F(t)$)

Incoming signal
(Naturally modulated by $F(t)$)



Optimal template
=
Time-modulated template
 $F(t)h(t)$
(Complexifies the bank)

Incoming signal
(Naturally modulated by $F(t)$)

```
graph TD; A["Incoming signal  
(Naturally modulated by F(t))"] --> B["Optimal template  
=  
Time-modulated template  
F(t)h(t)  
(Complexifies the bank)"]; A --> C["Simple template  
=  
Non time-modulated template  
F(0)h(t)  
(Virgo-like case)"];
```

Optimal template

=

Time-modulated template

$F(t)h(t)$

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“Simple” template

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Non time-modulated template

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Non time-modulated template  
F(0)h(t)  
(Virgo-like case)"]; B --> D["Which one do we  
choose ?"]; C --> E["How much Match and SNR do we  
lose if taking F(0)h(t) ?"];
```

Optimal template

=

Time-modulated template

$F(t)h(t)$

(Complexifies the bank)

“Simple” template

=

Non time-modulated template

$F(0)h(t)$

(Virgo-like case)

Which one do we
choose ?

How much Match and SNR do we
lose if taking $F(0)h(t)$?

$$\frac{\Delta\rho}{\rho} := \frac{\rho_{mod} - \rho_{const}}{\rho_{mod}} \approx 1 - \mathcal{M}[F(t)h(t), F(0)h(t)]$$

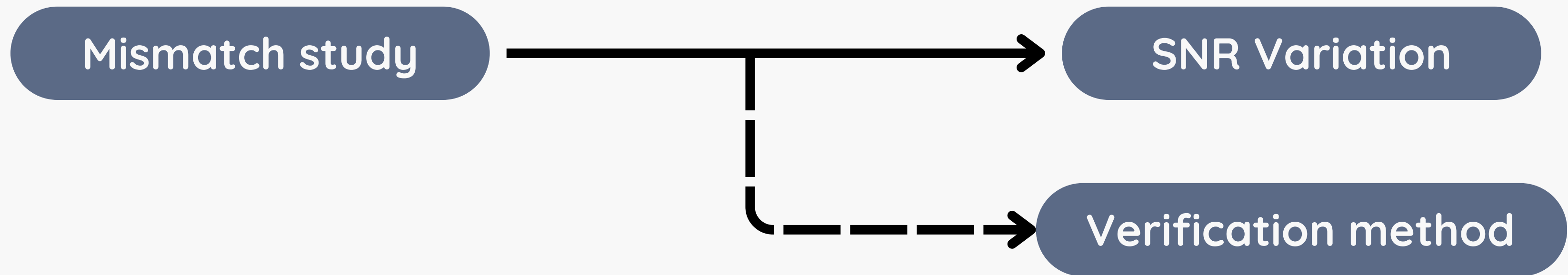
$$\frac{\Delta\rho}{\rho} := \frac{\rho_{mod} - \rho_{const}}{\rho_{mod}} \approx 1 - \mathcal{M}[F(t)h(t), F(0)h(t)]$$

Mismatch study



SNR Variation

$$\frac{\Delta\rho}{\rho} := \frac{\rho_{mod} - \rho_{const}}{\rho_{mod}} \approx 1 - \mathcal{M}[F(t)h(t), F(0)h(t)]$$



For each (RA, DEC) point in the sky

Coded in Python
(with PyCBC)

For each (RA, DEC) point in the sky



Generating $h(t)$ (+ and x) (SPA)

Coded in Python
(with PyCBC)

Coded in Python
(with PyCBC)

For each (RA, DEC) point in the sky



Generating $h(t)$ (+ and x) (SPA)

Computation of $F(t)$ and $F(0)$ (+ and x)

Coded in Python
(with PyCBC)

For each (RA, DEC) point in the sky

Generating $h(t)$ (+ and x) (SPA)

Computation of $F(t)$ and $F(0)$ (+ and x)

Computation of $F(t)h(t)$ and $F(0)h(t)$

For each (RA, DEC) point in the sky

Generating $h(t)$ (+ and x) (SPA)

Computation of $F(t)$ and $F(0)$ (+ and x)

Computation of $F(t)h(t)$ and $F(0)h(t)$

Mismatch computation between
 $F(t)h(t)$ and $F(0)h(t)$

Coded in Python
(with PyCBC)

$$\mathcal{M}(s, T) = \max_{\Delta t, \Delta \phi} \frac{\langle s, T \rangle}{||s|| \cdot ||T||}$$

Simple case



$$M1 = M2 = 1.4 M_{\odot}$$

No spin

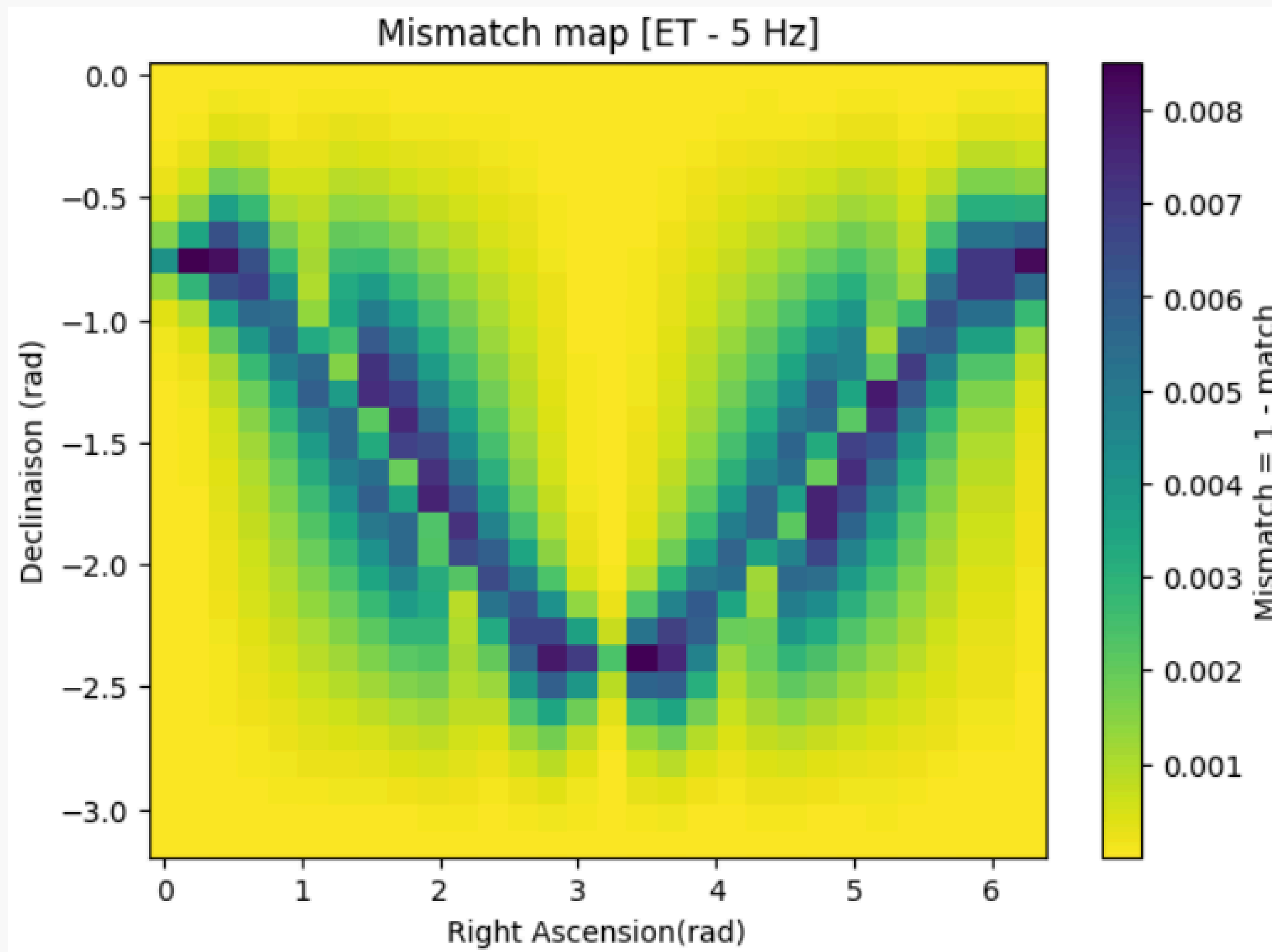
$$f_{\text{low}} = 5 \text{ Hz}$$

$$\text{Inclination} = 0^{\circ}$$

$$\text{Distance} = 100 \text{ Mpc}$$

$$\text{PSI} = 0^{\circ} *$$

*Equal contribution of F_{+} and $F_{\times}$
in the Match



Max ~ 0.8%

Earth rotation can
be neglected here

What can we expect
in average ?



*Global study of the Mismatch
Monte-Carlo simulation*



Sky position

$$\alpha, \cos(\delta) \sim \mathcal{U}[0, 2\pi], \mathcal{U}[0, 1]$$

Polarisation

$$\Psi \sim \mathcal{U}[0, 2\pi]$$

Inclination

$$\cos(\iota) \sim \mathcal{U}[0, 1]$$

Individual masses

$$M_i \sim \mathcal{N}(1.35, 0.1)$$

(Low) frequency cutoff

$$f_{low} = 5 \text{ Hz}$$

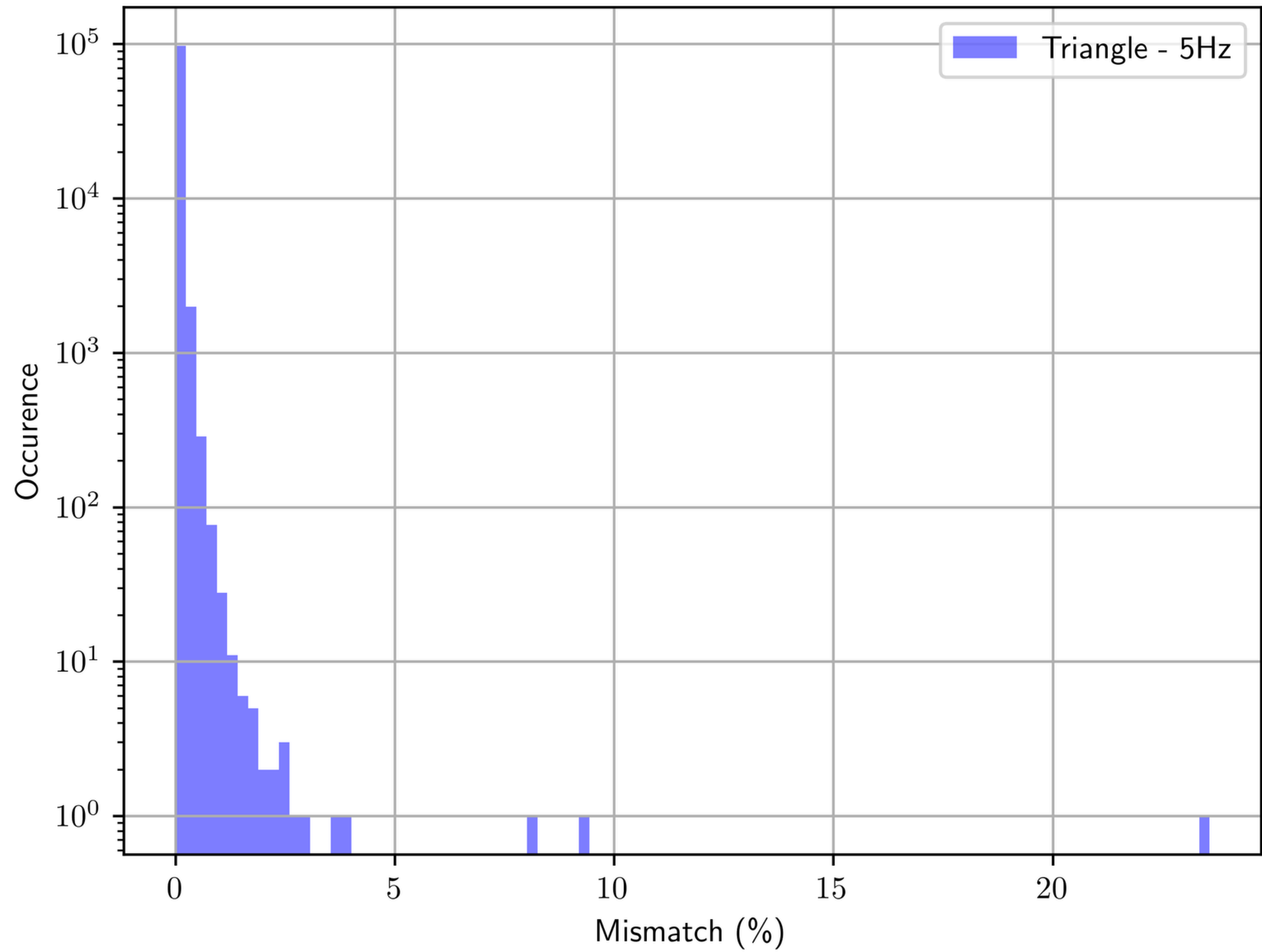
Coalescence time

$$t_c \sim \mathcal{U}[0, 86400]$$

Redshift

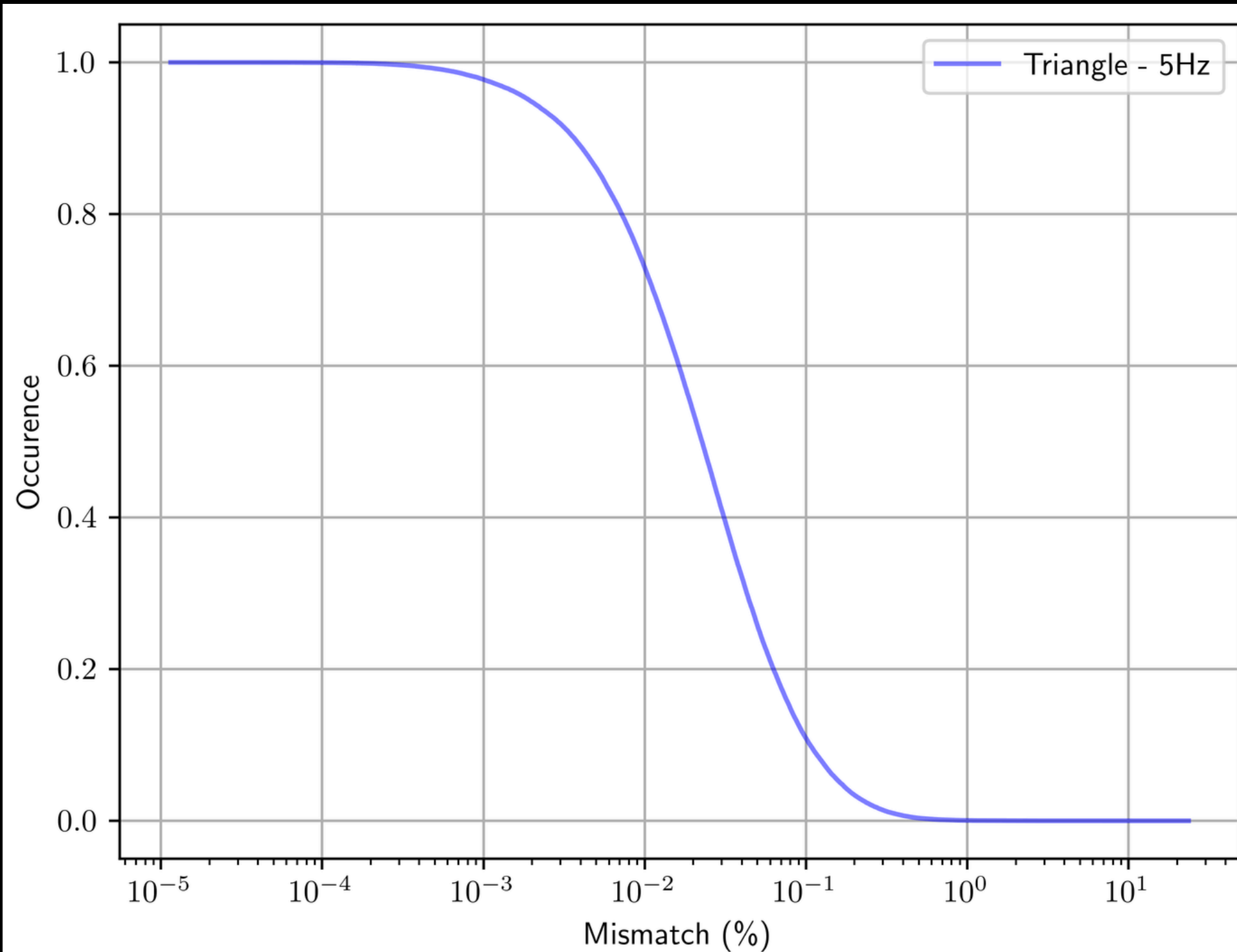
$$p(z) \propto \frac{dV_c}{dz} \mathcal{R}(z)$$

Tidal and spin
neglected



(100.000 sources)

Mismatch histogram



Up to **20%** Mismatch

~**0.005%** of sources
have Mismatch > **5%**

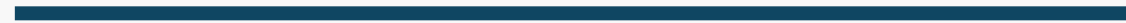
~**0.5%** of sources have
Mismatch > **1%**

~**10%** of sources have
Mismatch > **0.1%**

.....



Global study of SNR



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We necessarily lose SNR

$$\rho_{const} \leq ||h|| = \rho_{opt,mod}$$

We necessarily lose SNR

$$\rho_{const} \leq ||h|| = \rho_{opt,mod}$$

We keep the same MC parameters

Sky position

$$\alpha, \cos(\delta) \sim \mathcal{U}[0, 2\pi], \mathcal{U}[0, 1]$$

Polarisation

$$\Psi \sim \mathcal{U}[0, 2\pi]$$

Inclination

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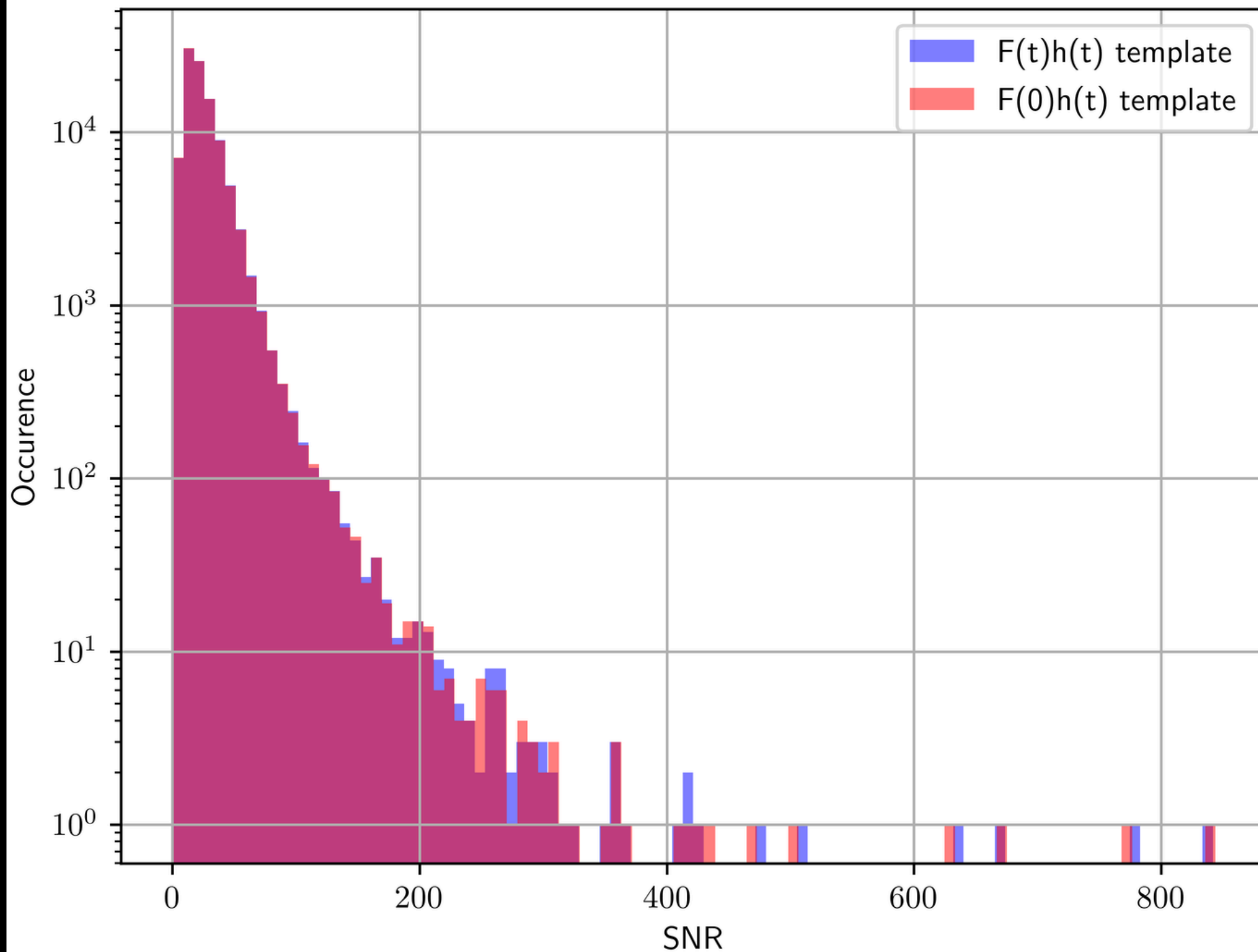
$$t_c \sim \mathcal{U}[0, 86400]$$

Redshift

$$p(z) \propto \frac{dV_c}{dz} \mathcal{R}(z)$$

Tidal and spin
neglected

SNR histogram



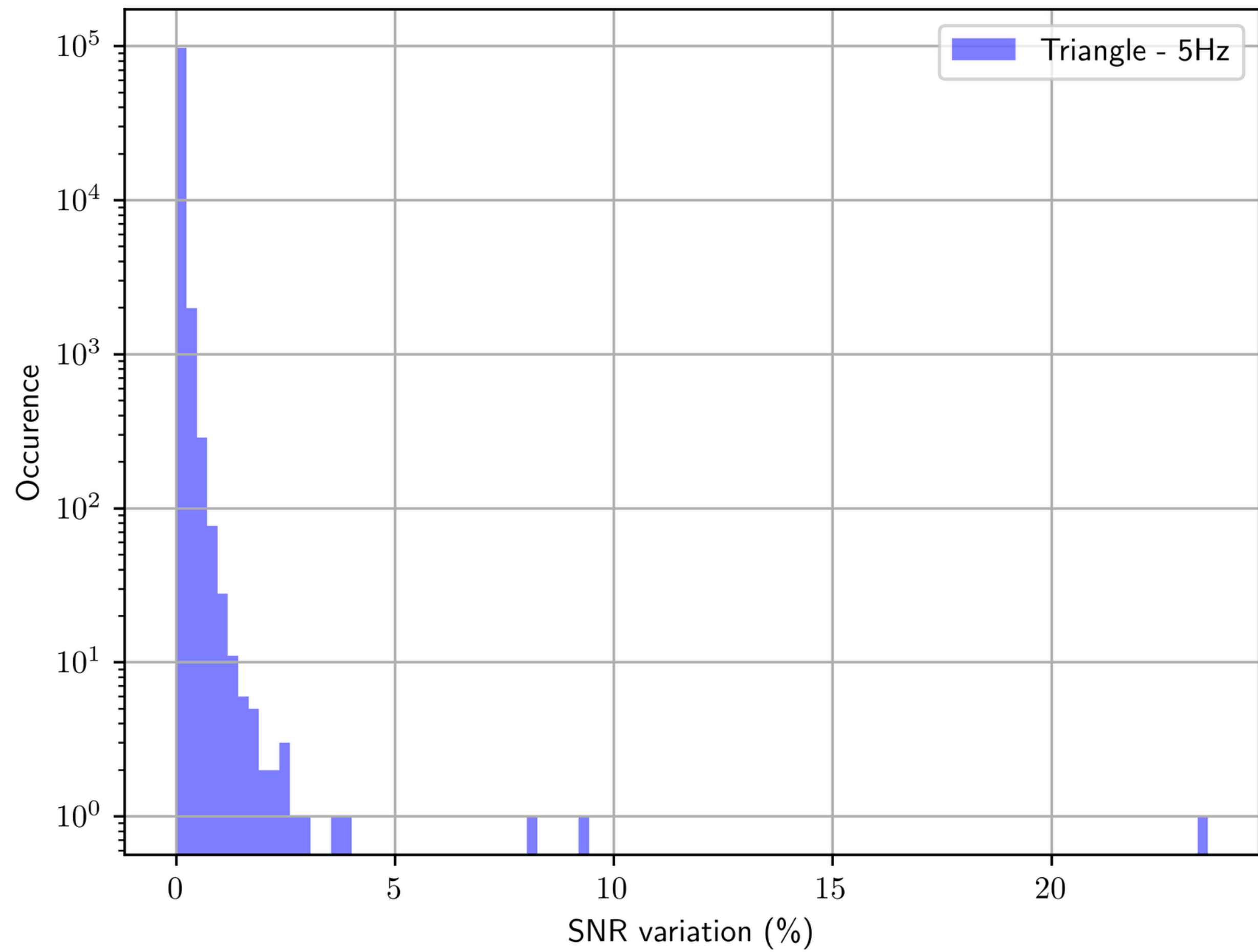
(100.000 sources)

SNR between 0 and 1000

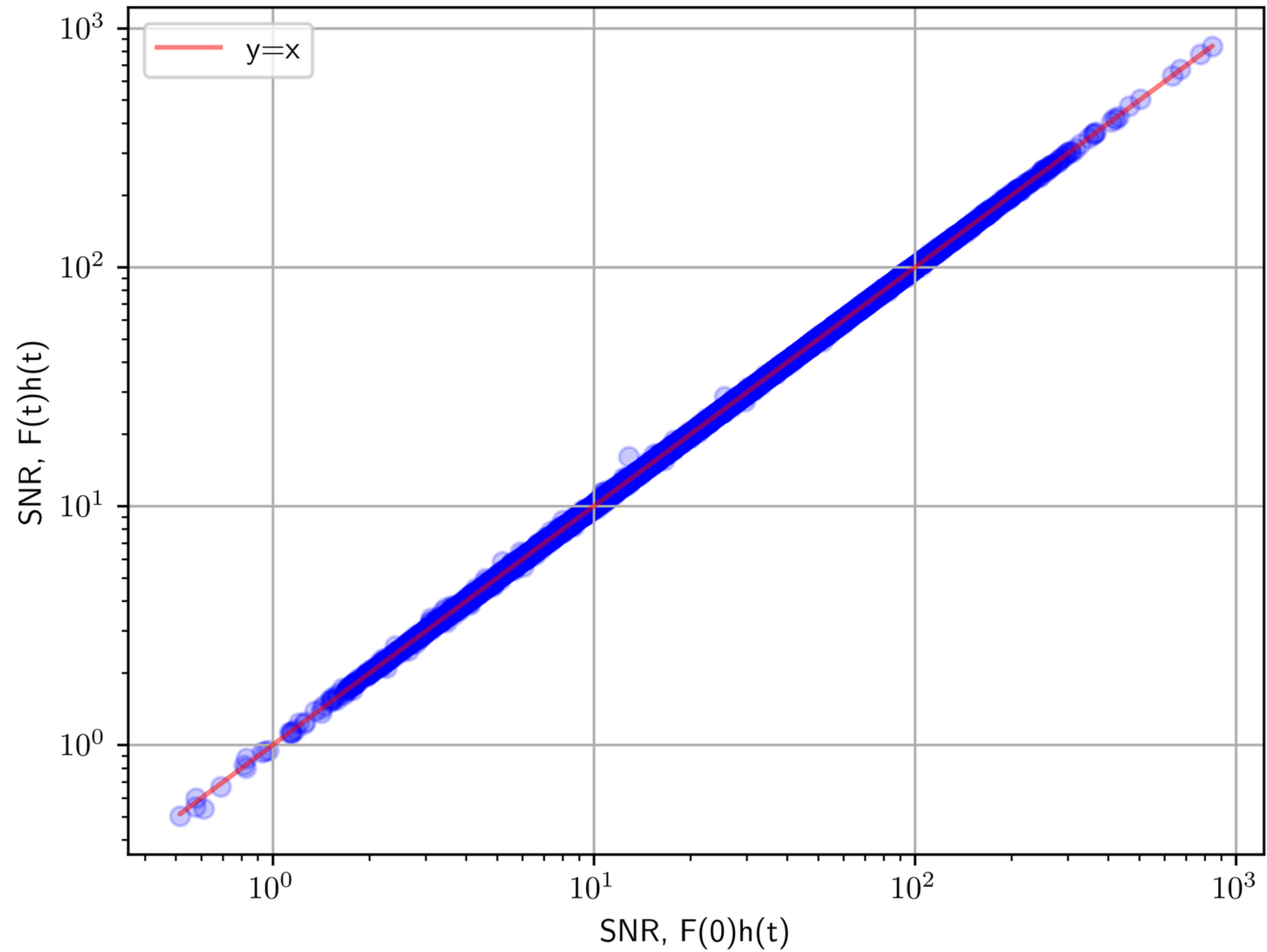
Mean SNR ~25

~90% of sources have
 $8 < \text{SNR} < 60$

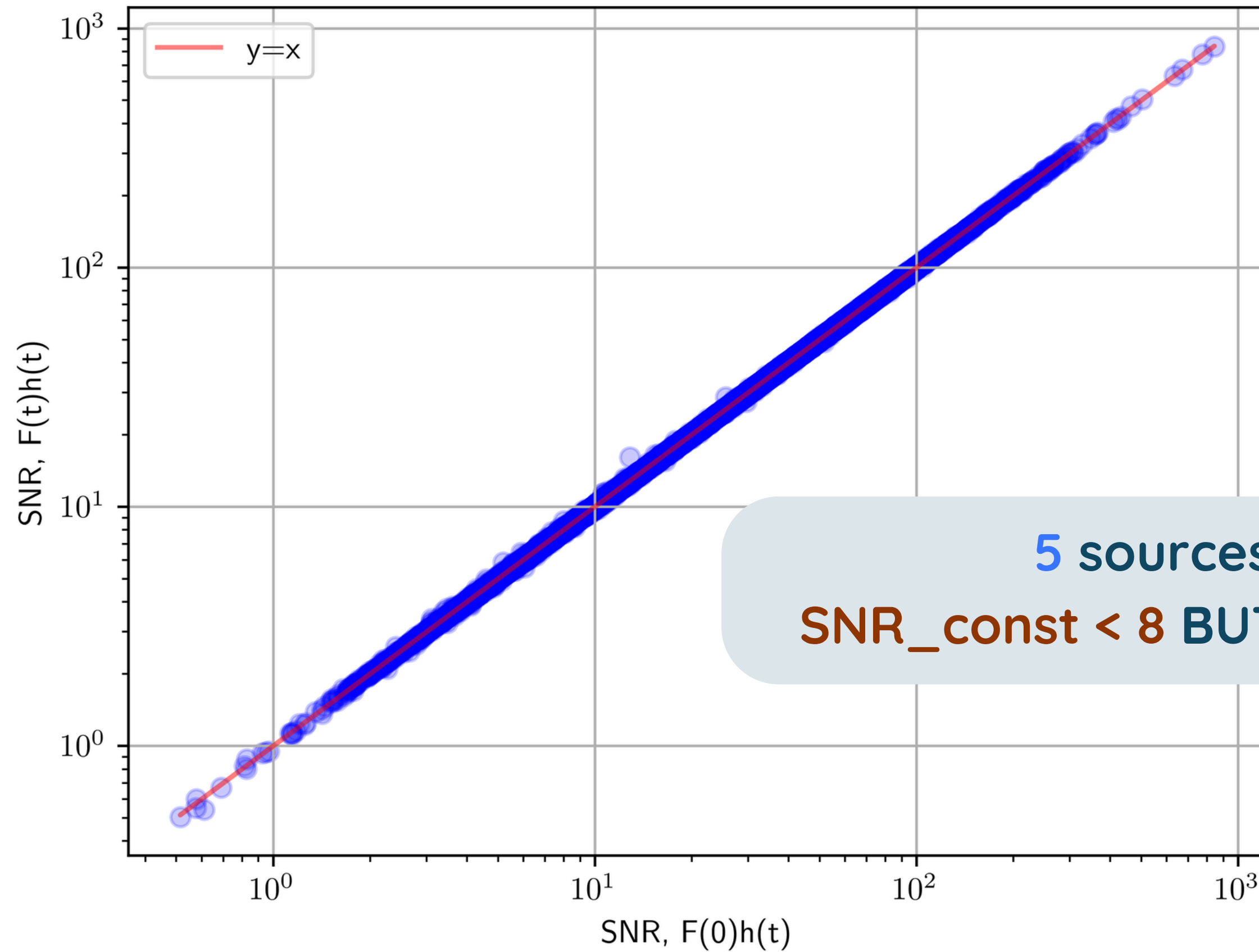
More optimistic than MDC



ρ_{mod} VS ρ_{const}

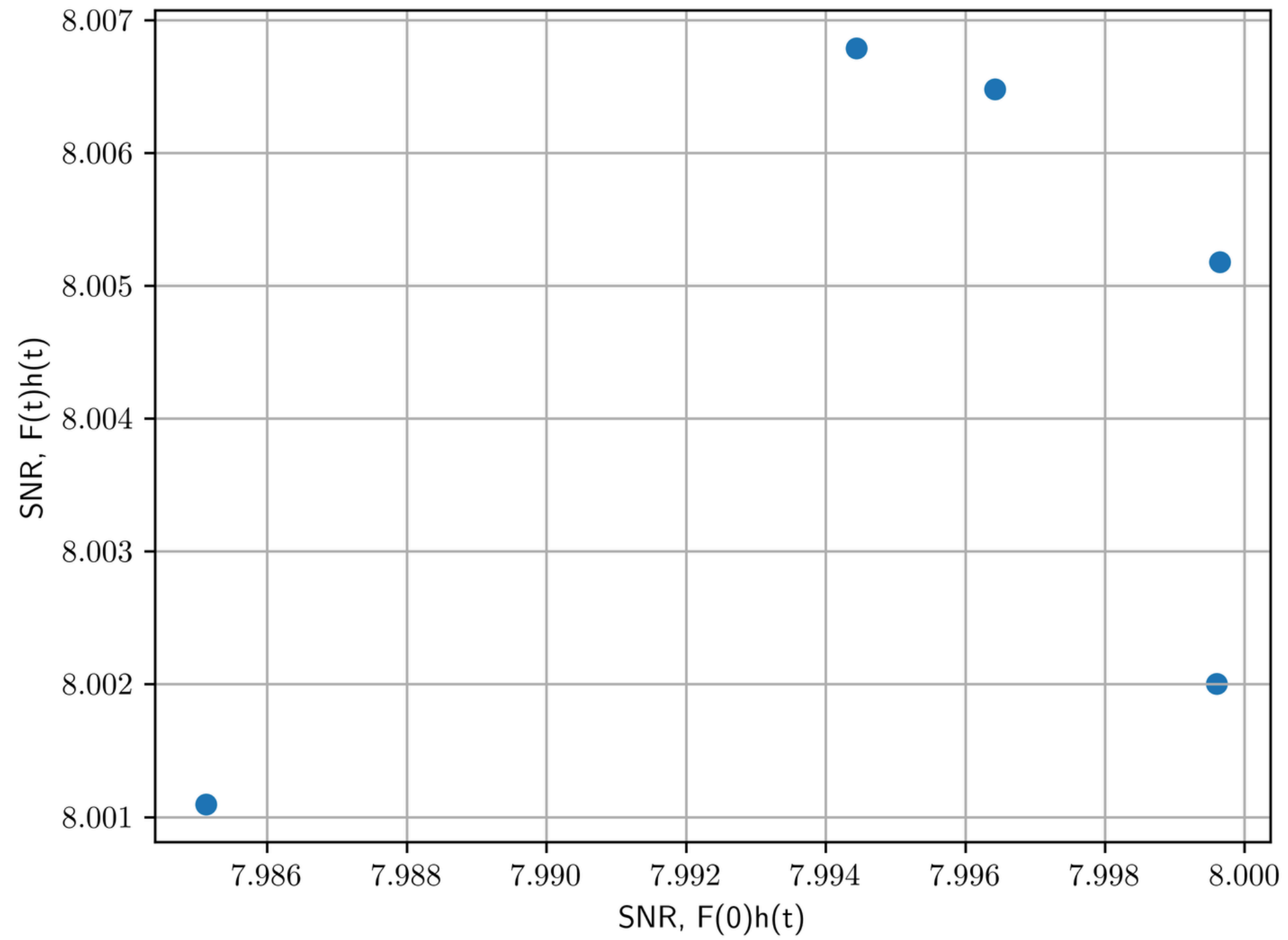


ρ_{mod} VS ρ_{const}



5 sources with
 $\text{SNR}_{\text{const}} < 8$ BUT $\text{SNR}_{\text{mod}} > 8$

Recovered SNR stay around SNR = 8



Conclusion #1 (ET-Triangle, 5 Hz)

MC simulation show that
Mismatch can be $> 5\%$

Earth rotation-induced SNR
variation can be neglected for
 $f_{low} > 5\text{Hz}$

Taking it into account would complexify the
Match Filtering for $<1\%$ of all detected sources

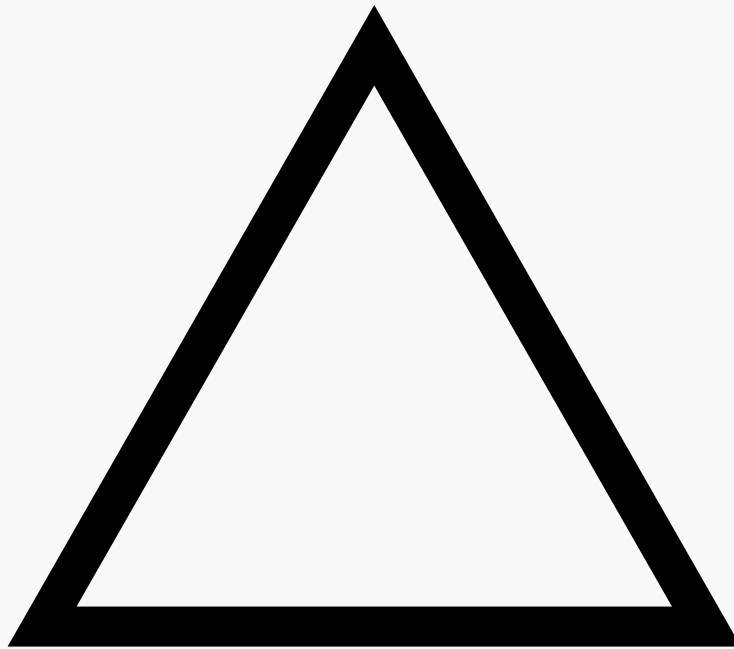


*What about the other
ET scenarios ?*

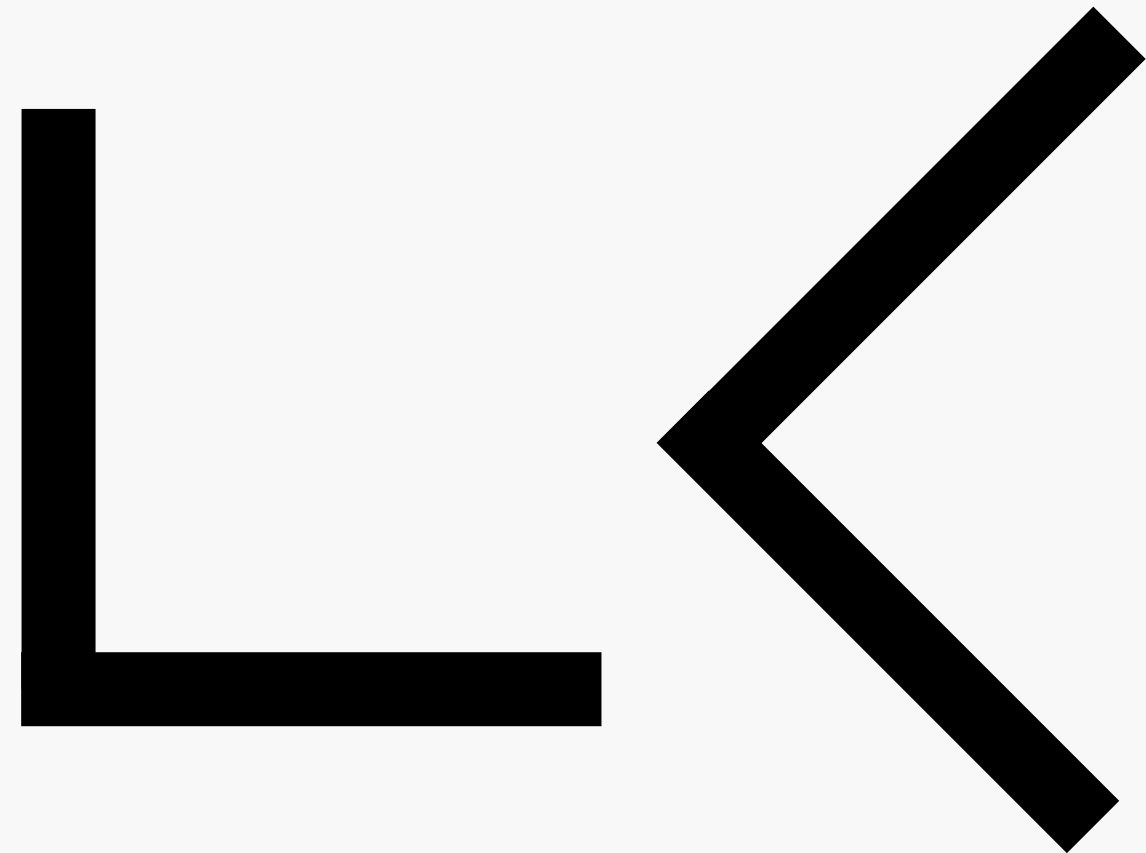


2 geometries...

Triangle

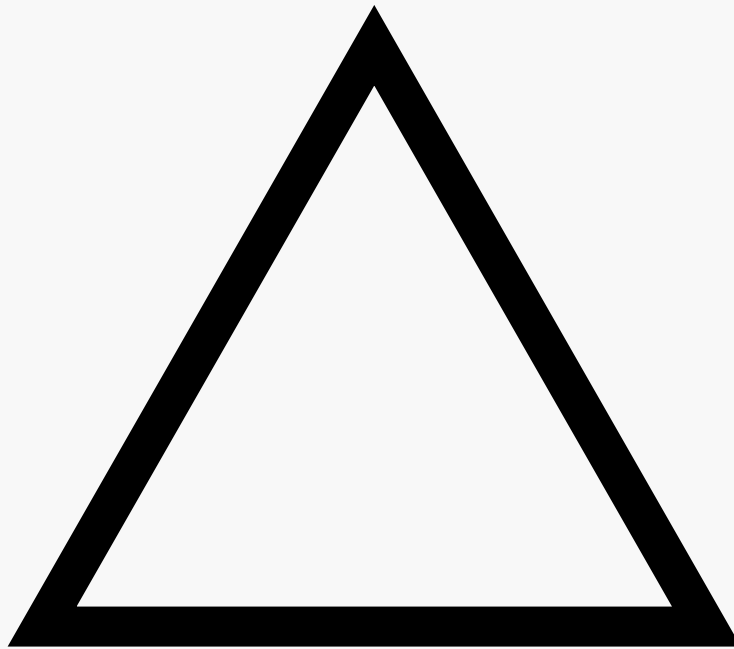


2L

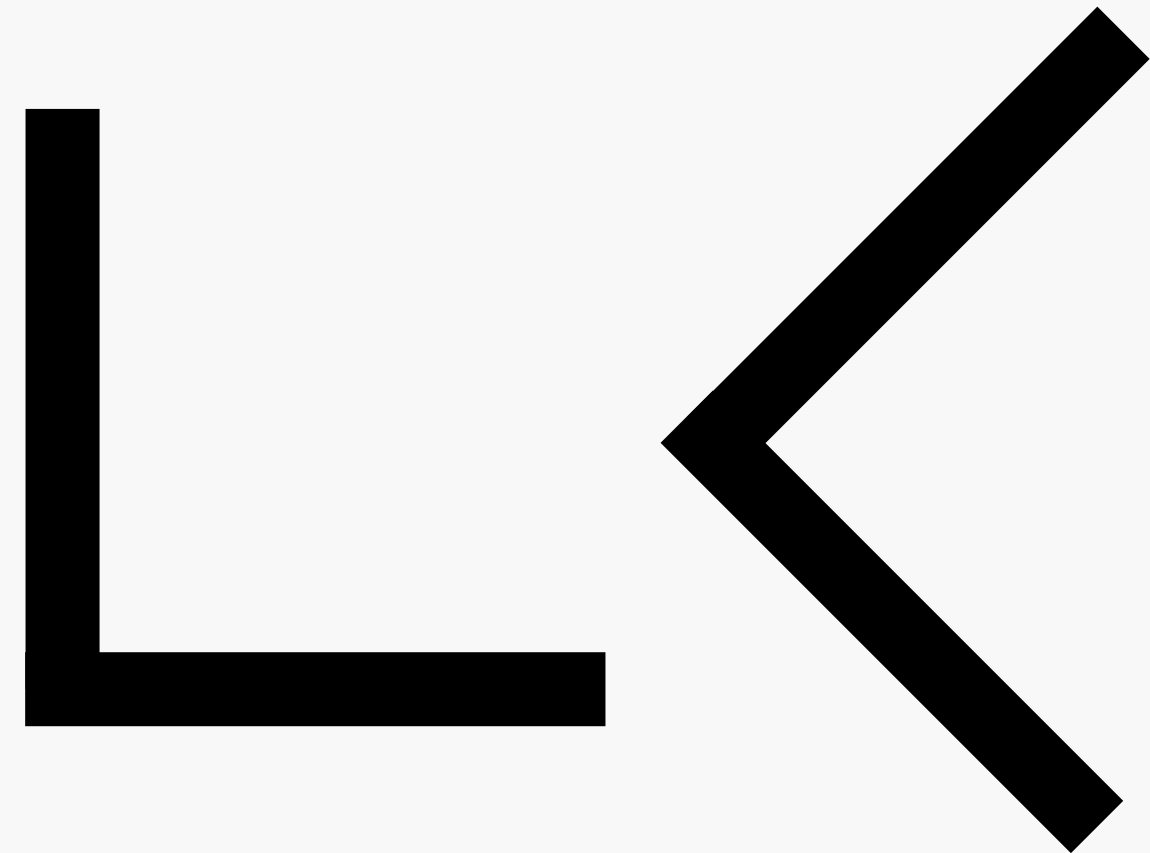


2 geometries... and 2 starting frequencies !

Triangle

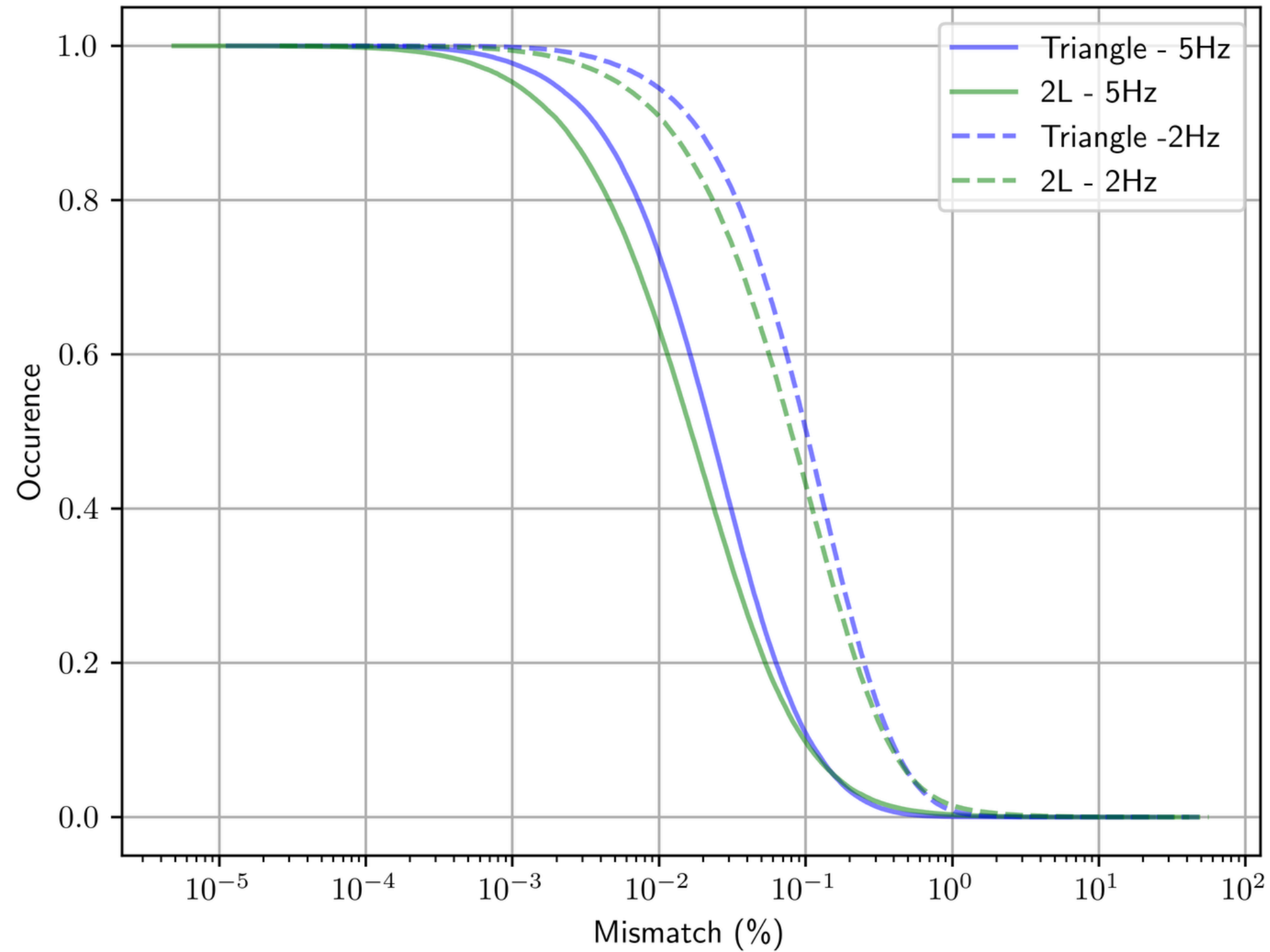


2L



Both starting either at 2 or 5 Hz → 4 scenarios !

Starting frequency matters more than the design



Retrieved sources (100.000 sources)

	Triangle	2L
5 Hz	5	11
2 Hz	14	31

Conclusion #2 (All ET scenarios)

MC simulation show that
Mismatch can be $> 5\%$

Starting frequency matters
more than the design

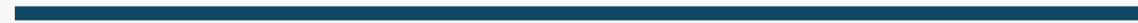
2L more sensitive to high
mismatches than Triangle

Taking E.R. allows to retrieve $< 1\%$
of the sources

• • • • •



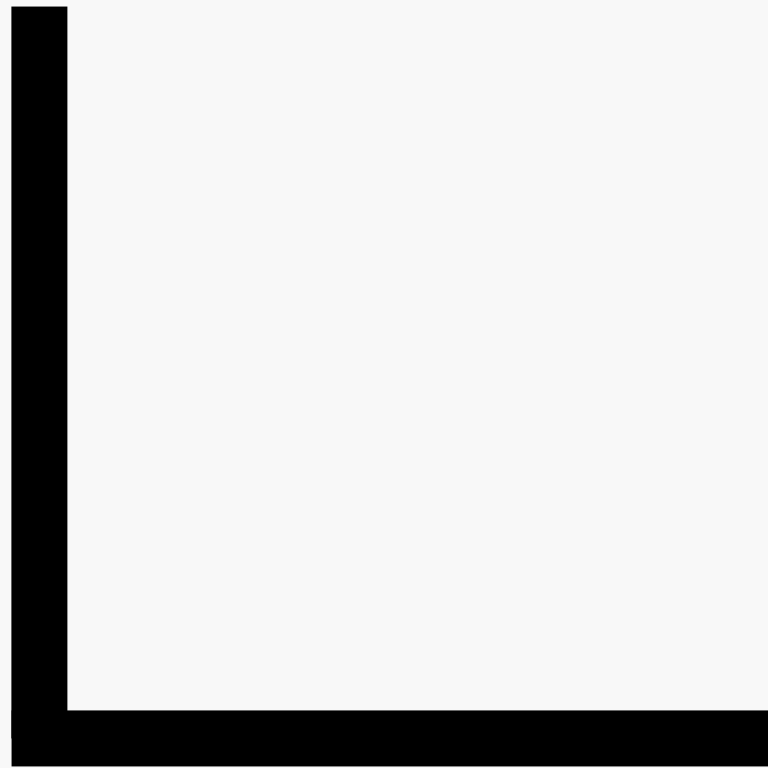
ET in 3G networks



• • • • •

2 geometries for CE

1 CE - 40 km

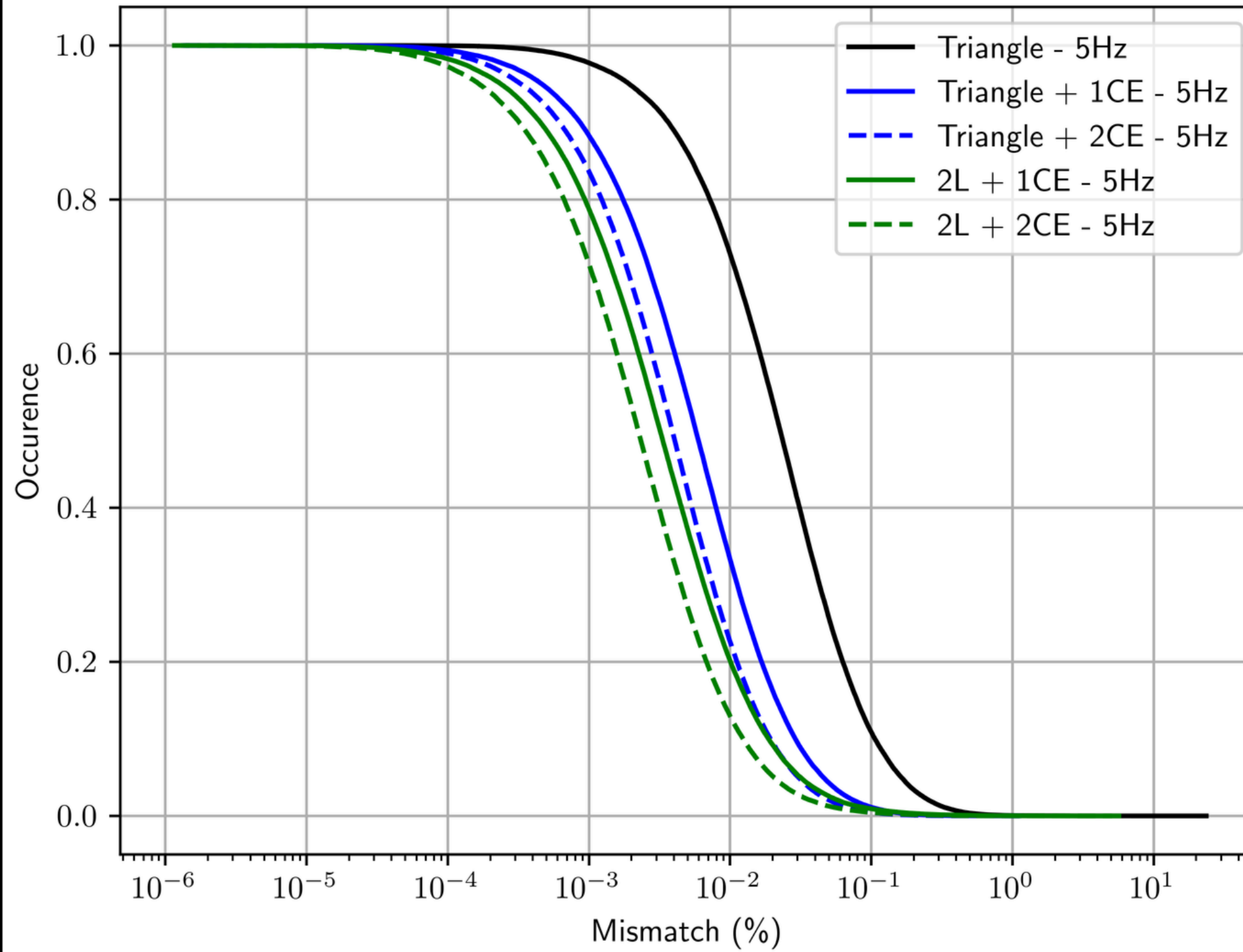


2 CE - 40 km + 20 km

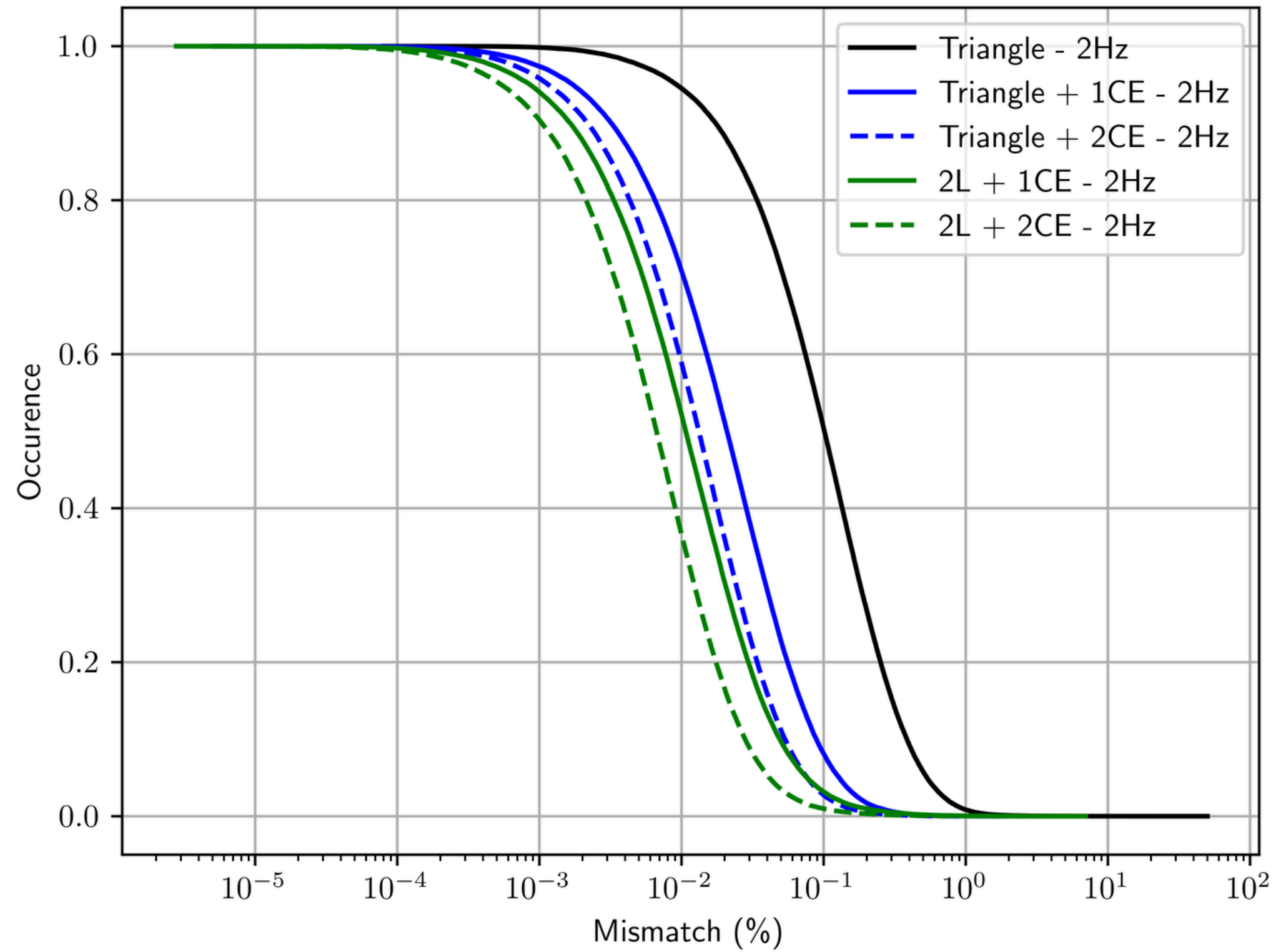


Both starting either at 2 or 5 Hz with ET → 8 scenarios !

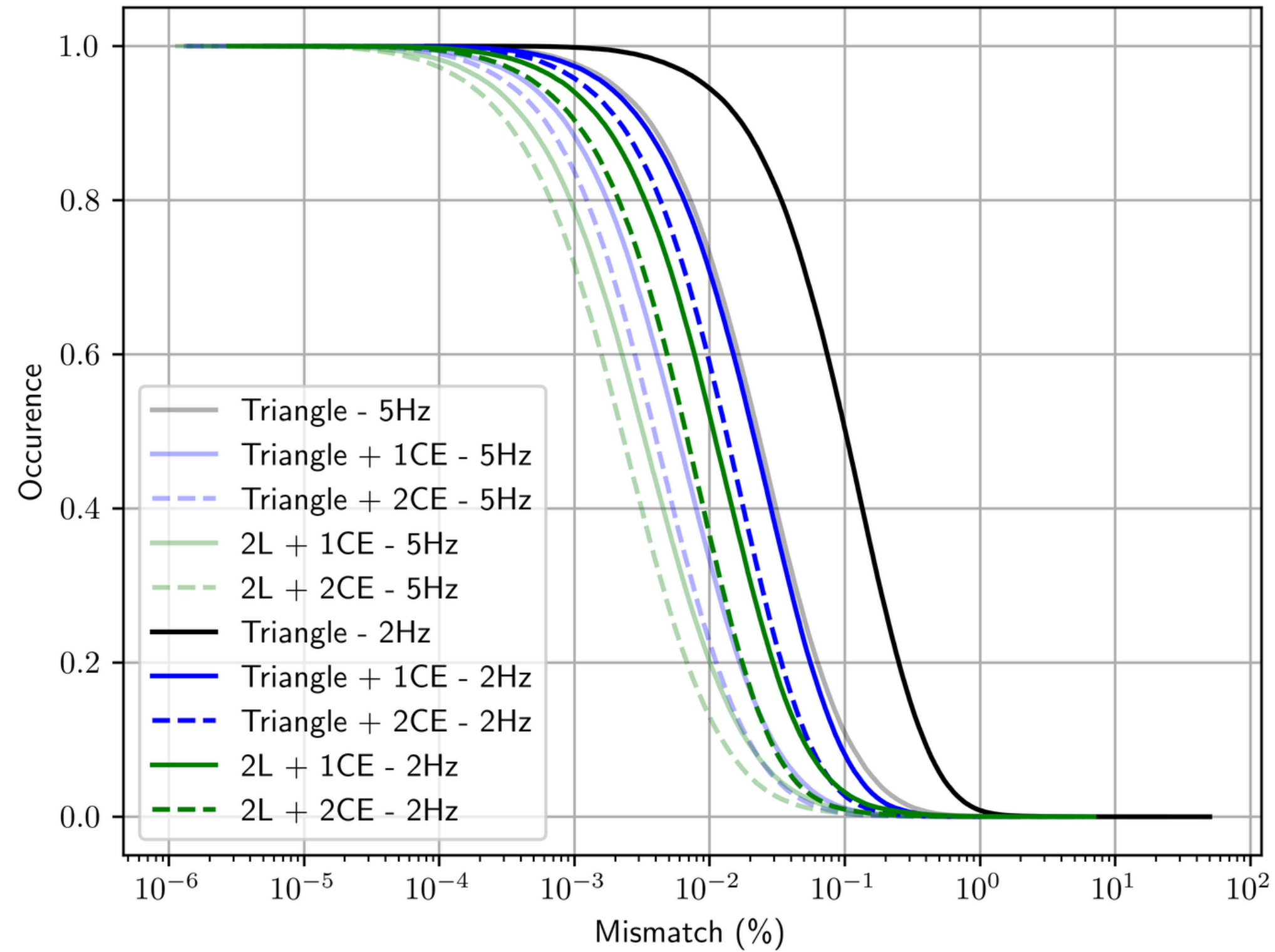
Starting at 5 Hz



Starting at 2 Hz



Starting at 2 Hz and 5 Hz



Conclusion #3

MC simulation show that
Mismatch can be $> 5\%$

Starting frequency matters more
than the design

Having Cosmic Explorer kills the
effect of Earth rotation

Taking E.R. allows to retrieve $< 1\%$
of the sources

.....



Thank you !



.....

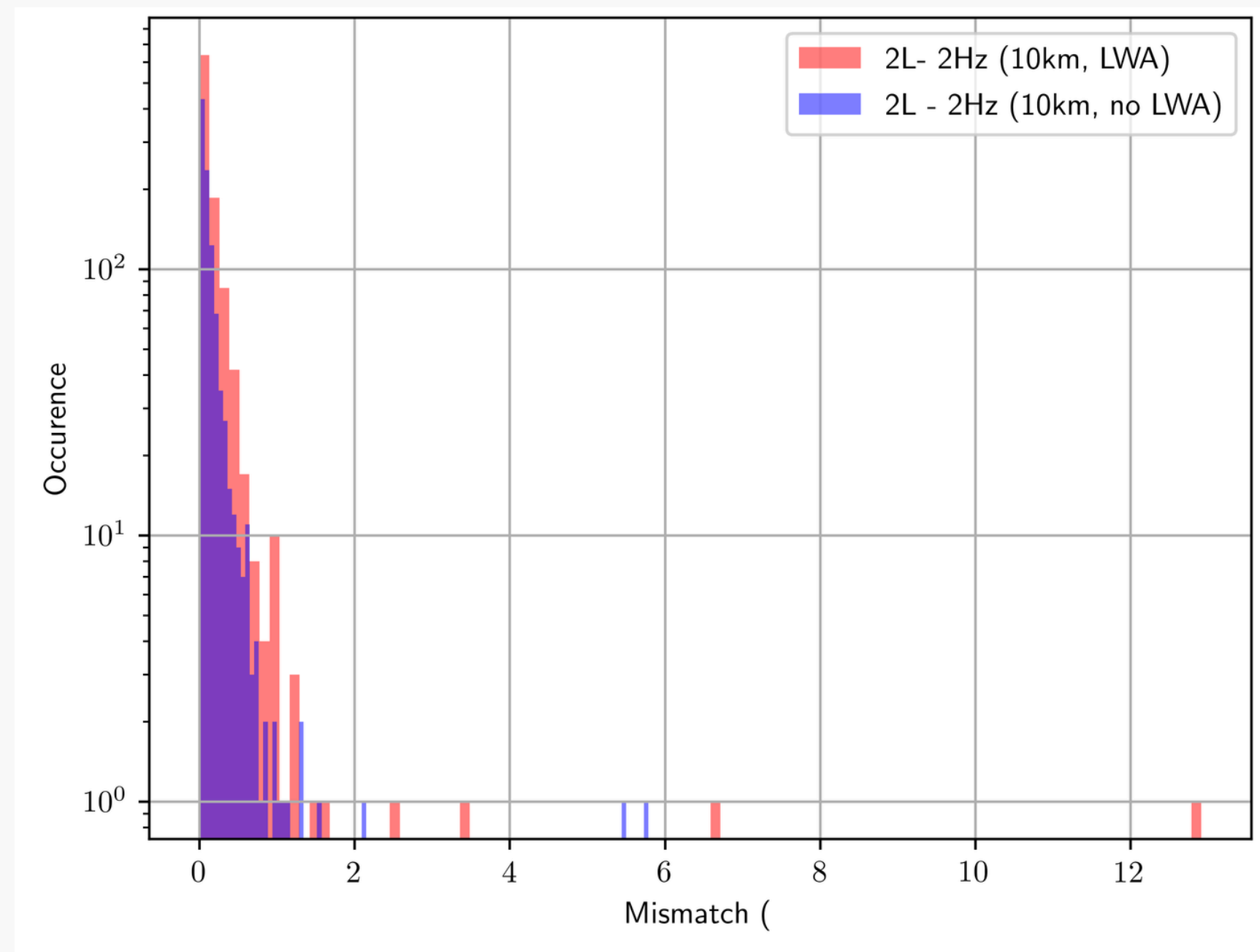
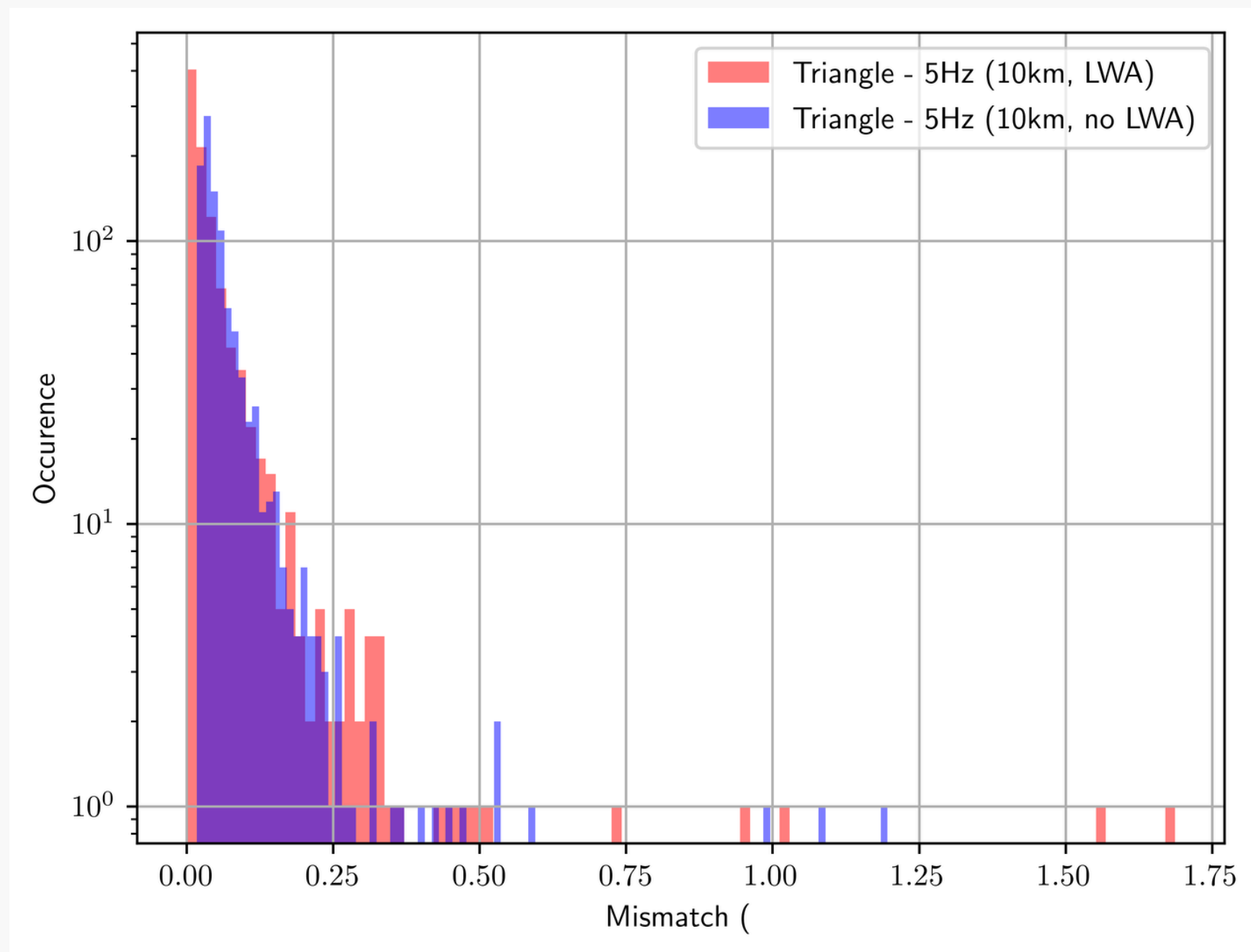
.....

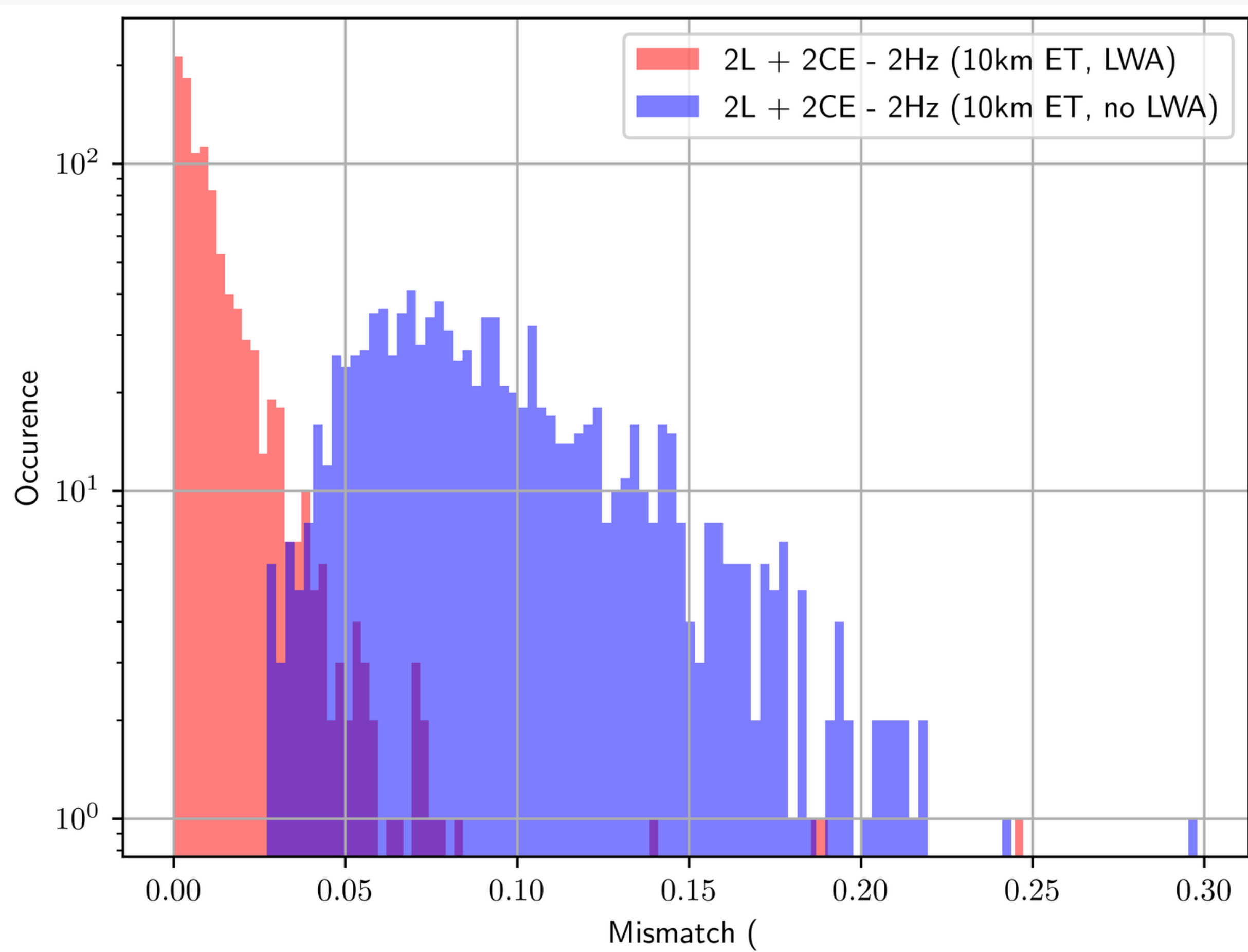


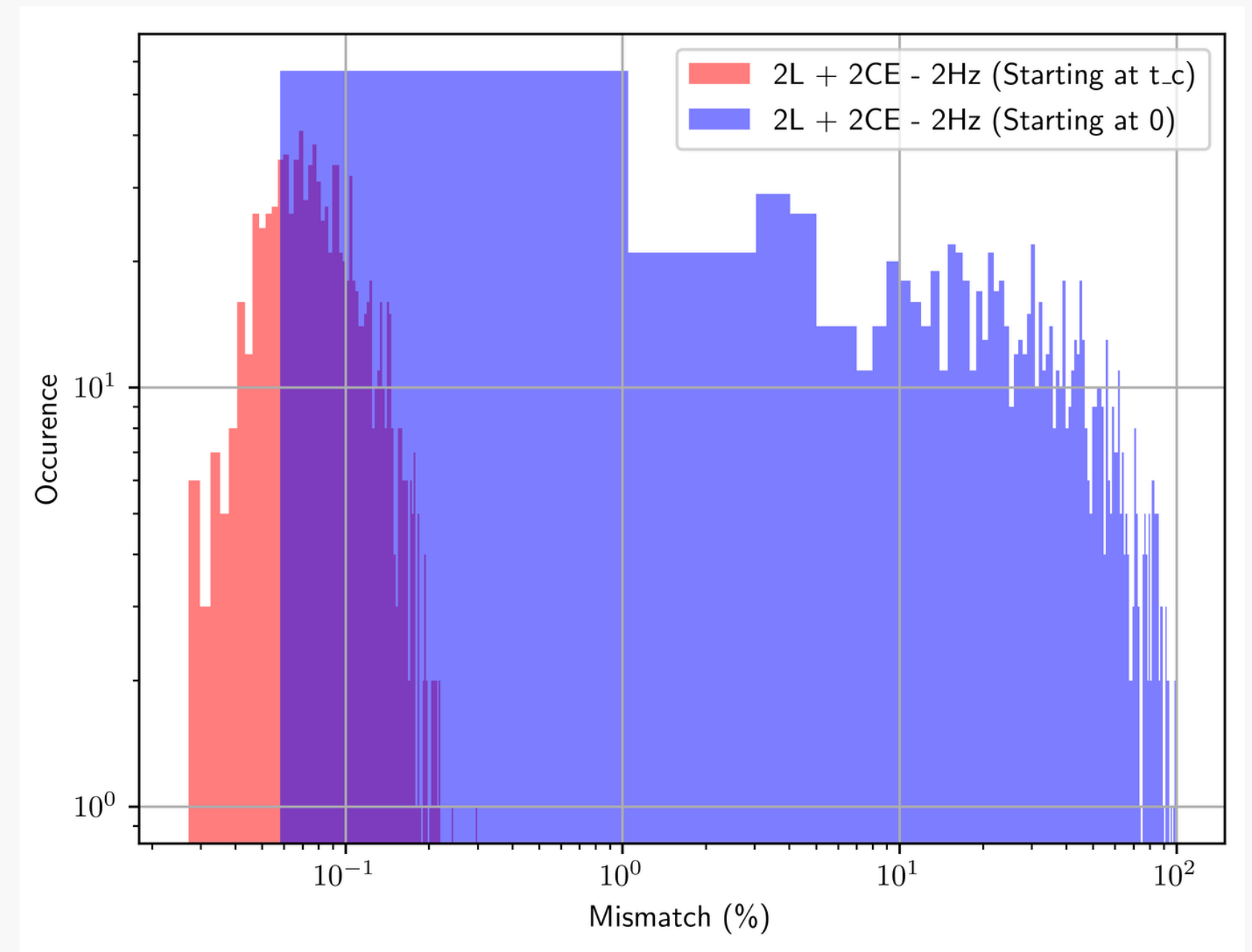
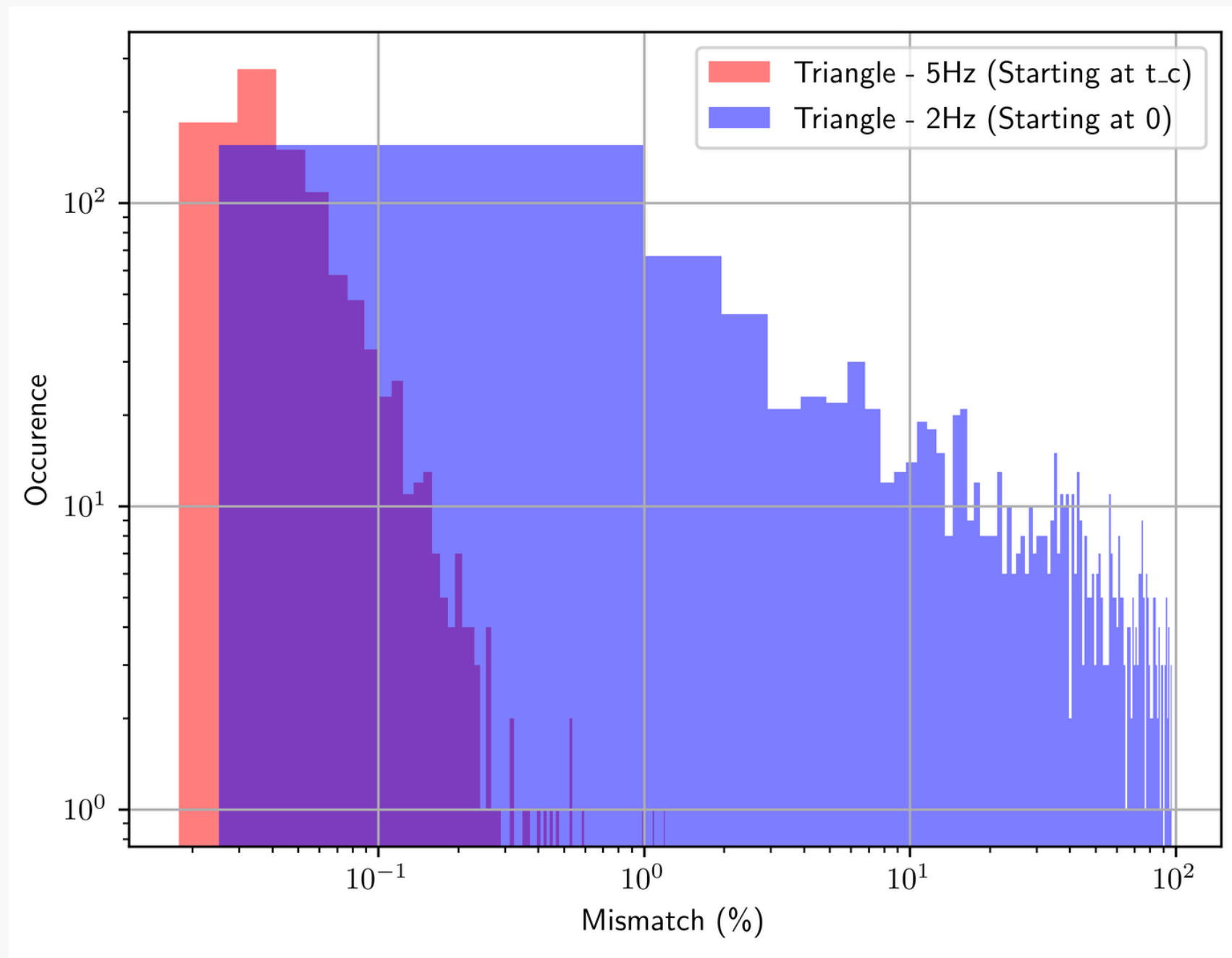
Backup



.....







$$\frac{\rho_{mod} - \rho_{const}}{\rho_{mod}} = 1 - \frac{\rho_{const}}{\rho_{mod}}$$

$$\frac{\rho_{mod} - \rho_{const}}{\rho_{mod}} = 1 - \frac{\rho_{const}}{||s||}$$

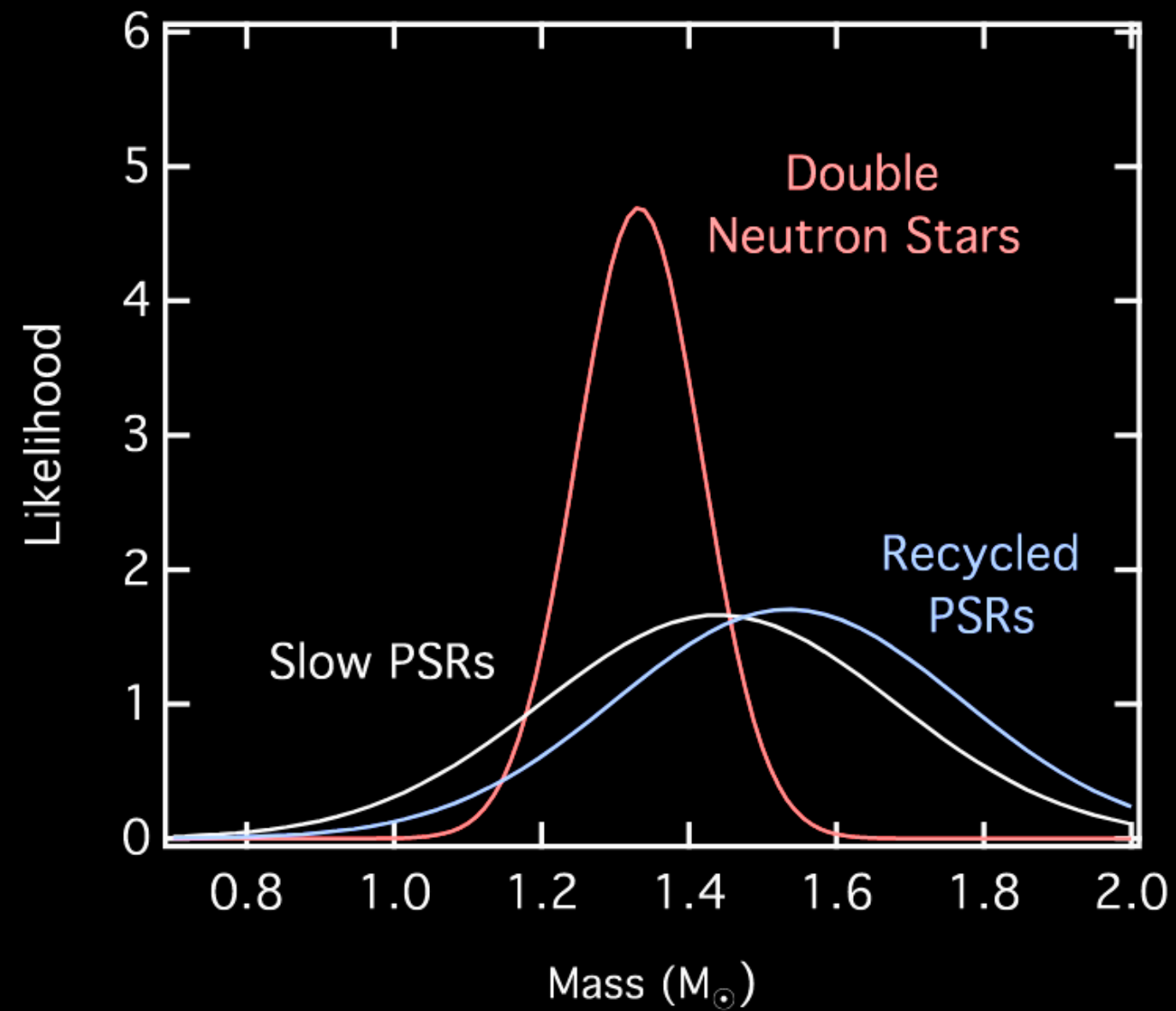
$$\mathcal{M}(s, T) = \max_{\Delta t, \Delta \phi} \frac{\langle s, T \rangle}{||s|| \cdot ||T||}$$

$$\mathcal{M}(s, T) \approx \frac{\langle s, T \rangle}{||s|| \cdot ||T||}$$

$$\mathcal{M}(s, T) \approx \frac{\rho_{const}}{||s||}$$

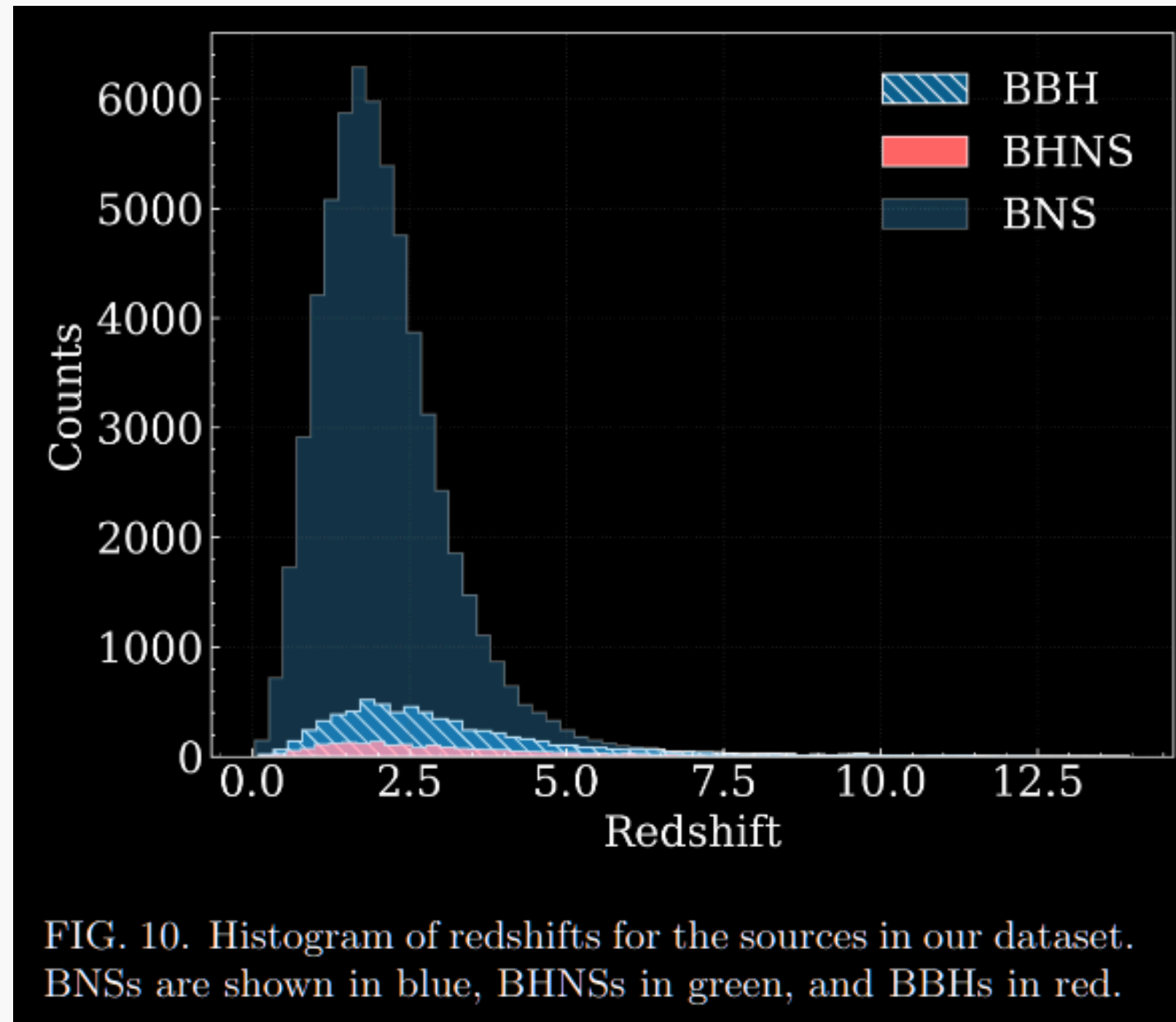
$$\rho_{const} = \frac{\langle h, T \rangle}{||T||}$$

<http://arxiv.org/abs/1603.02698>



<http://arxiv.org/abs/1403.0007>

$$\psi(z) = 0.015 \frac{(1+z)^{2.7}}{1 + [(1+z)/2.9]^{5.6}} M_{\odot} \text{ year}^{-1} \text{ Mpc}^{-3}.$$



- 61031 BNSs: 19% have $\text{SNR} > 8$, 7% have $\text{SNR} > 12$.

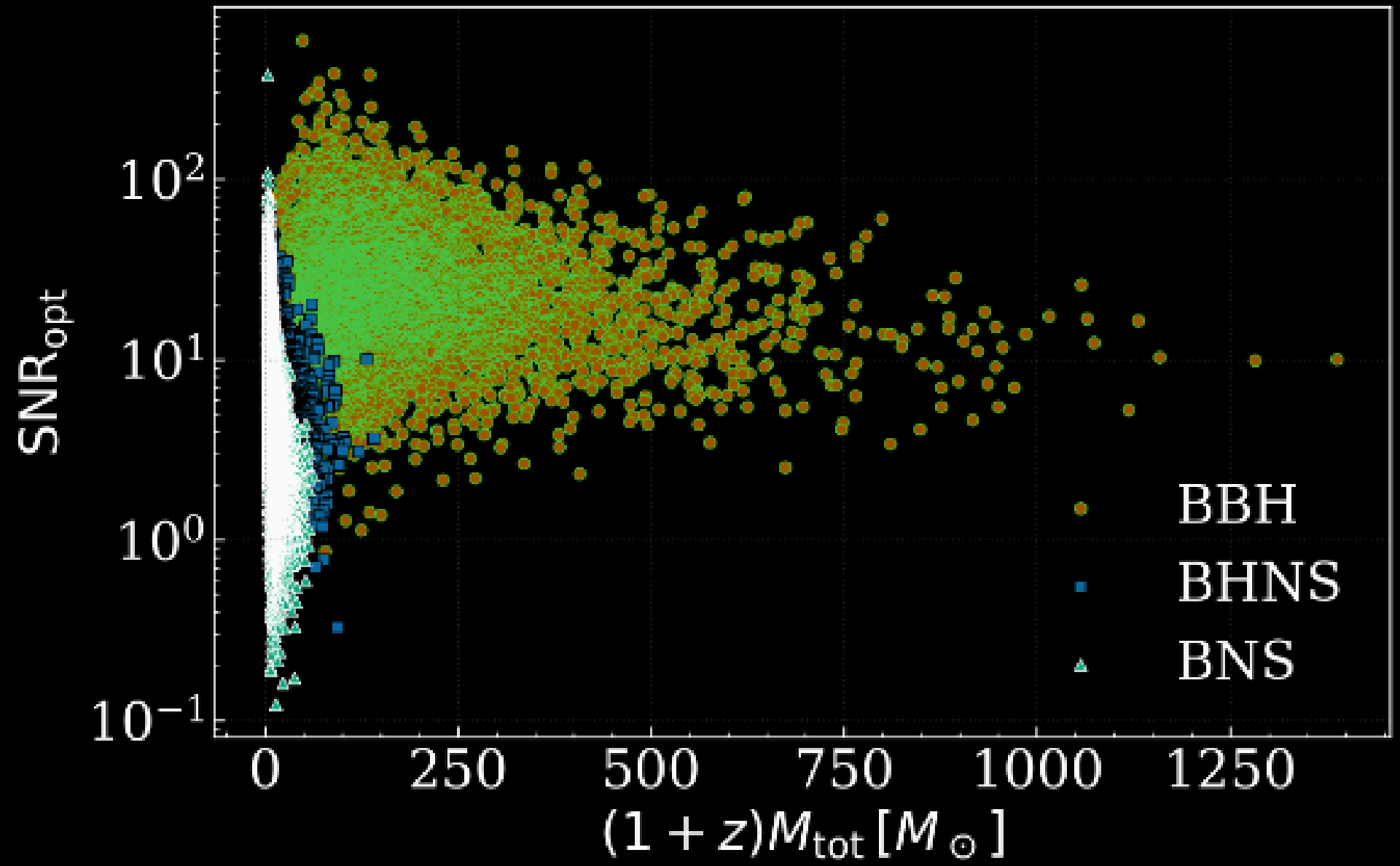
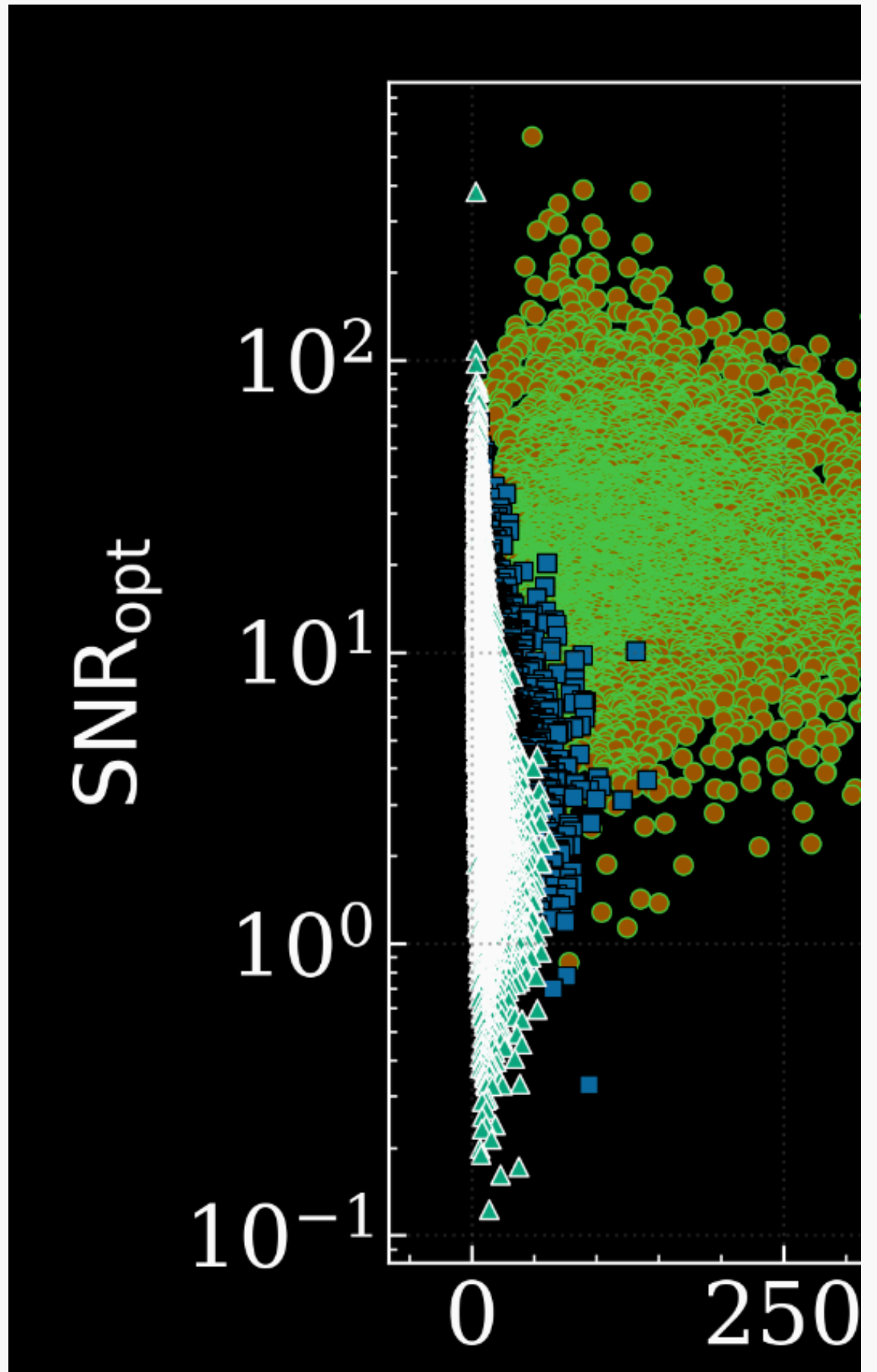
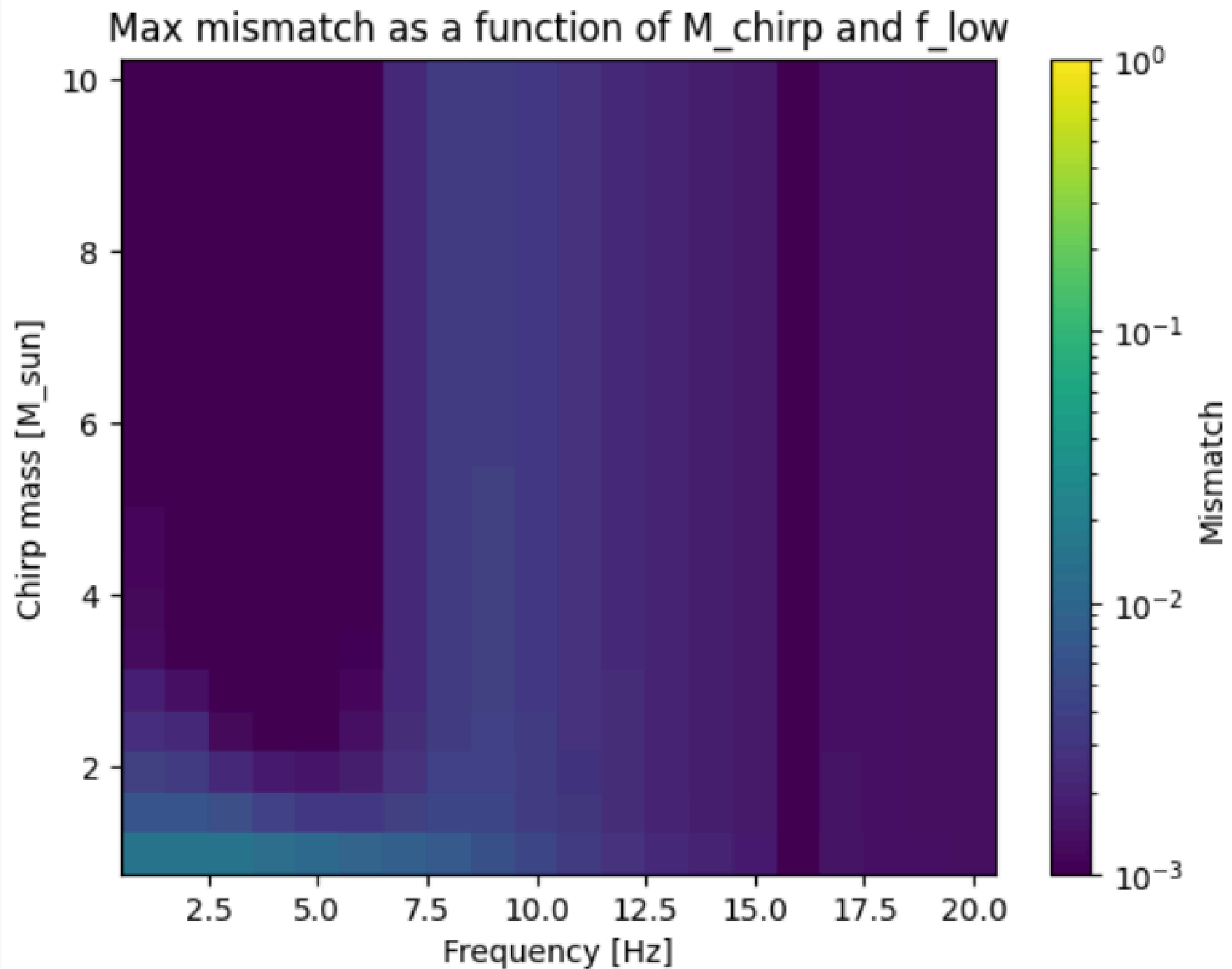
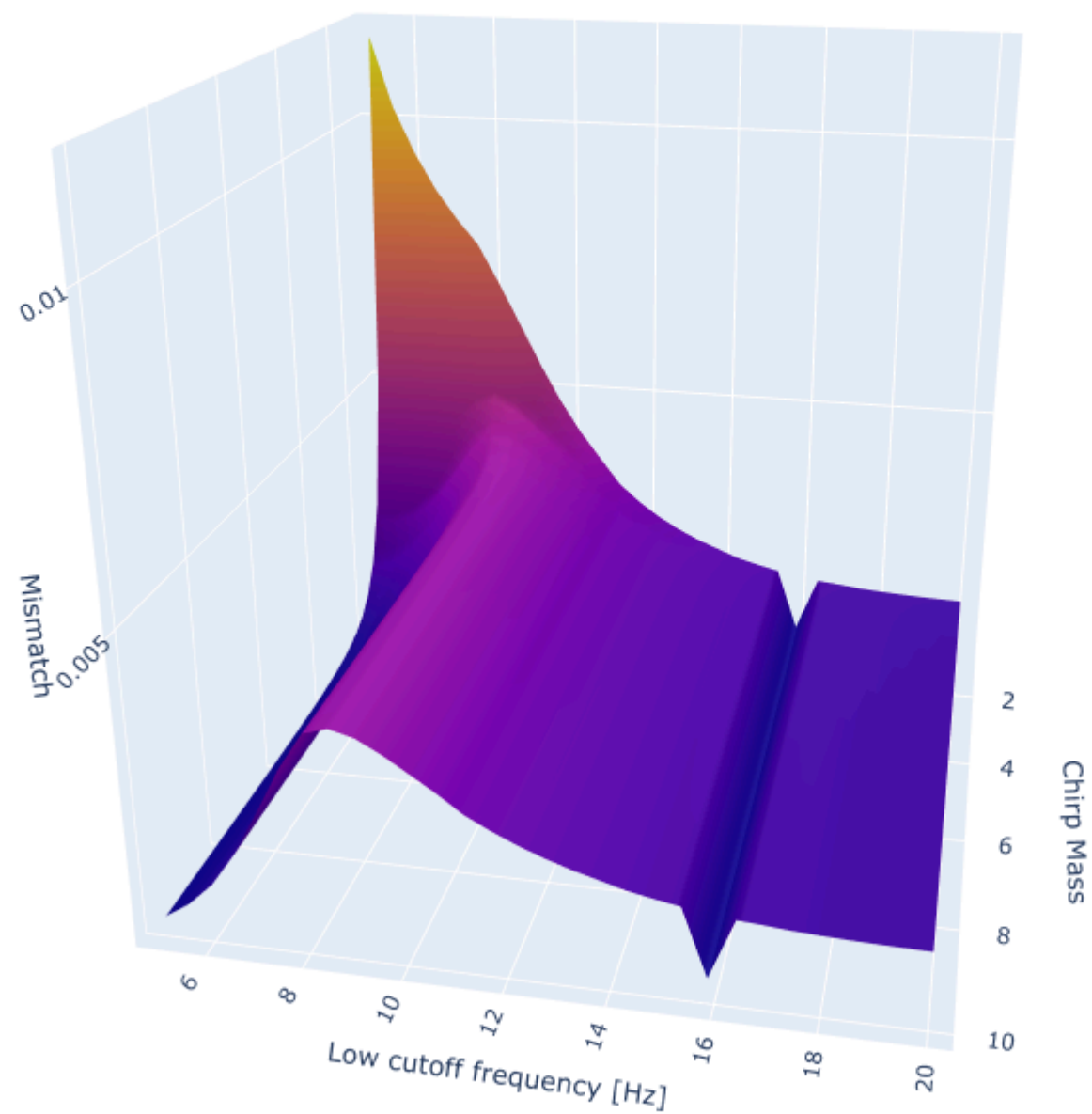


FIG. 13. Optimal signal-to-noise ratio (ρ_{opt}) as a function of the observed total mass $M_{z,\text{tot}}$.

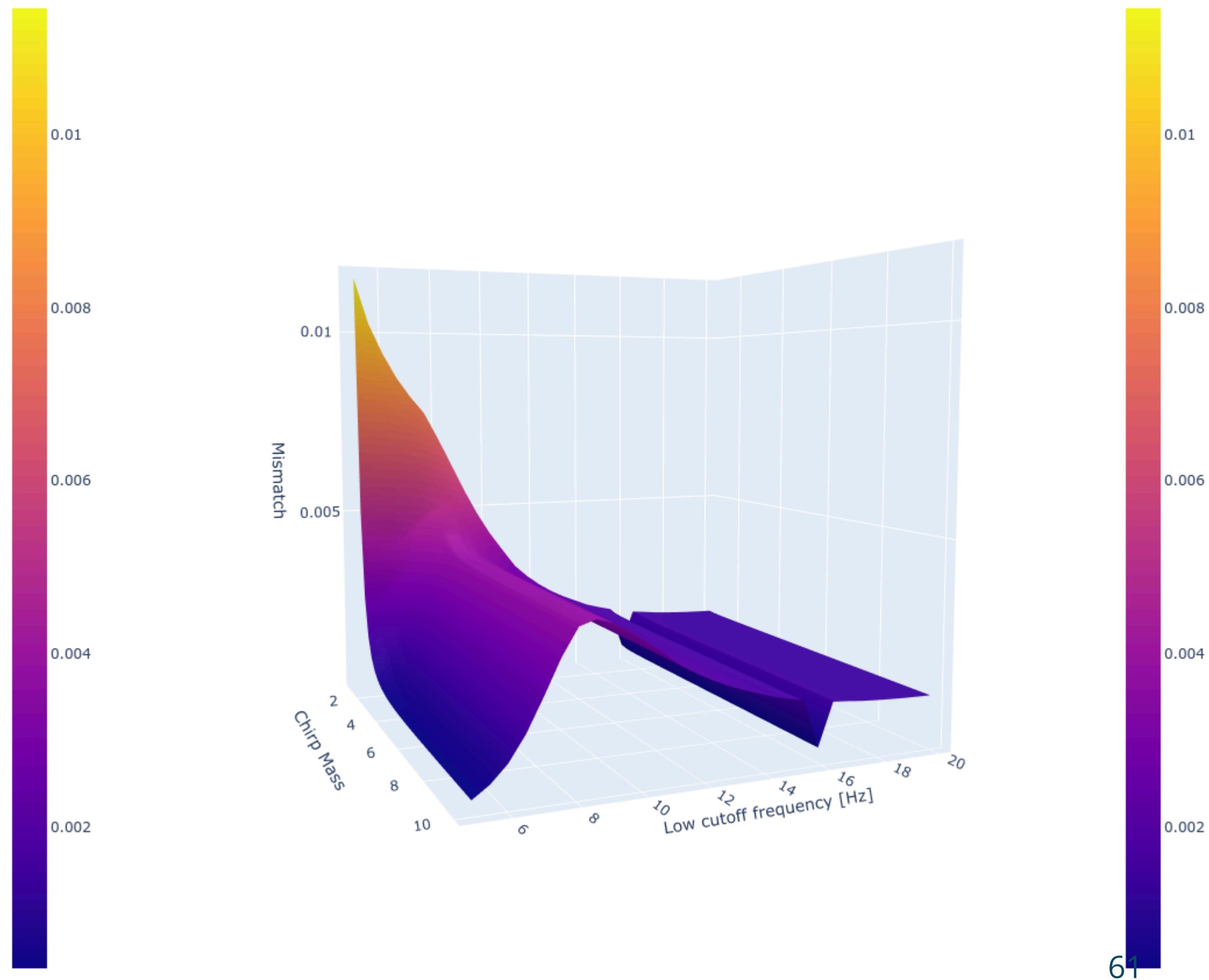




Mismatch as a function of M_{chirp} and f_{low}

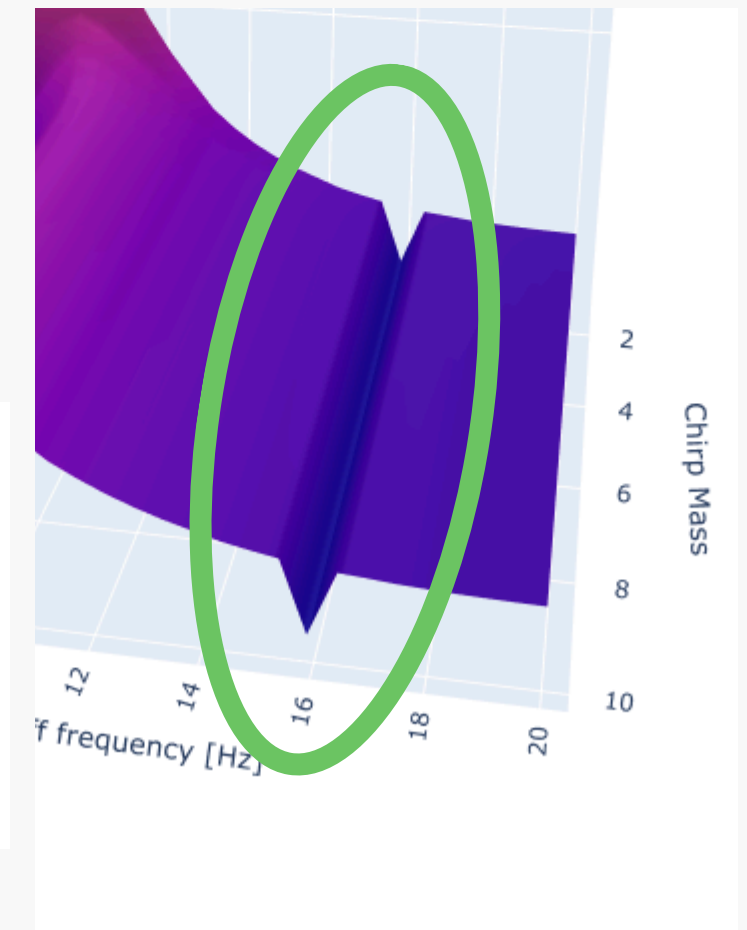


Mismatch as a function of M_{chirp} and f_{low}

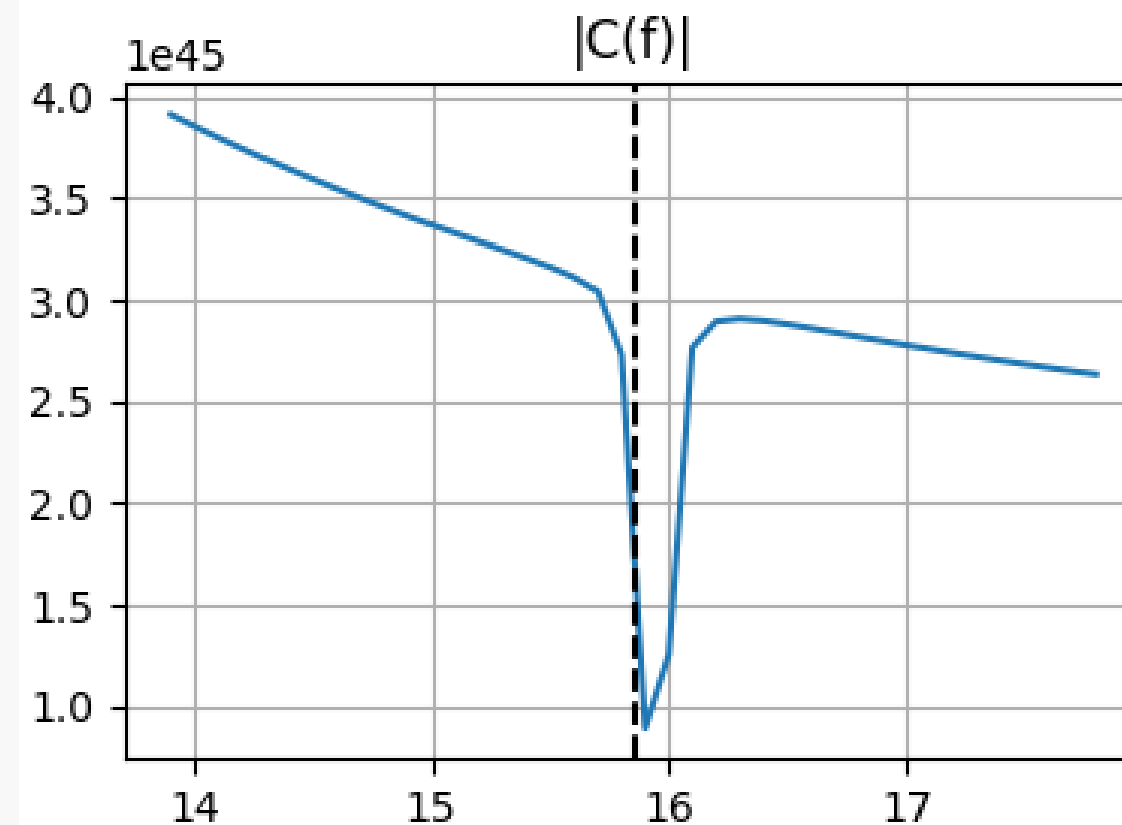


Caused by the PSD

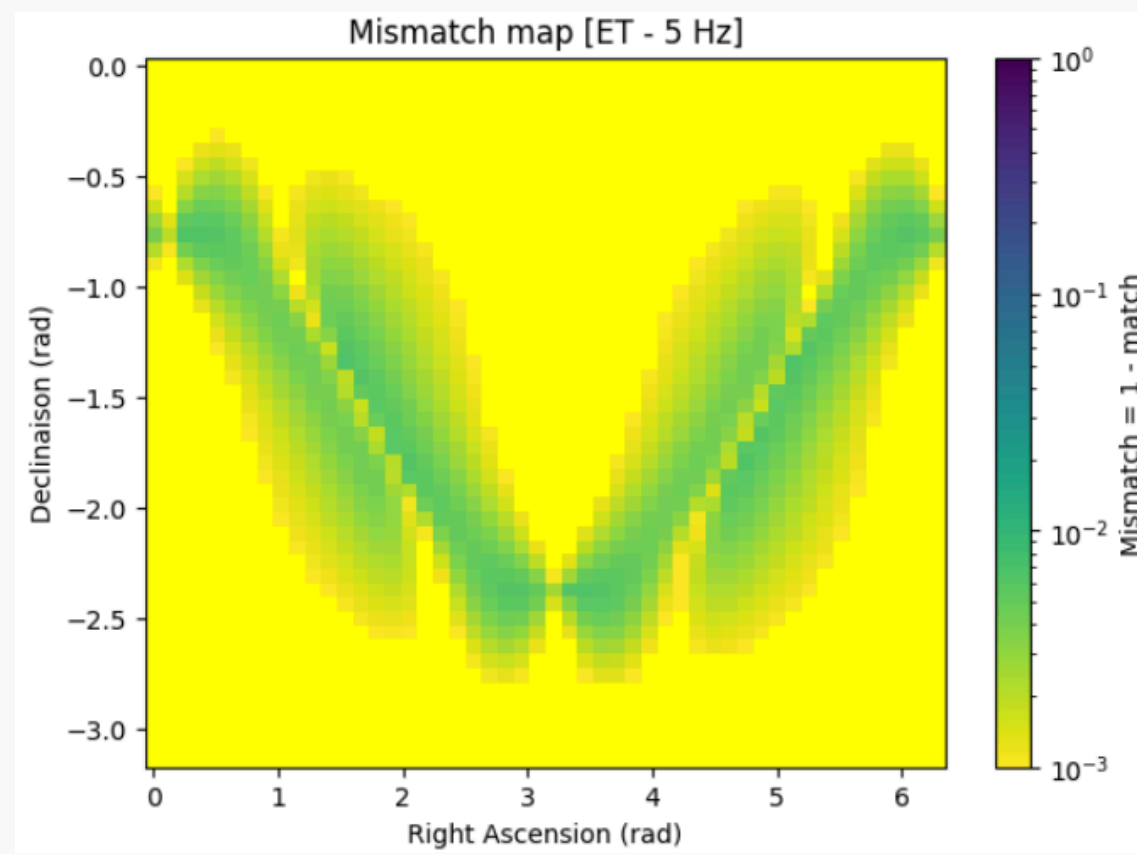
$$m(h_{\text{rot}}, h_{\text{const}}) = \max_{\Delta t_c, \Delta \phi_c} \frac{\Re \int \frac{f^{-7/3} |B(t_{\text{rot}}(f))| e^{-i\theta(t_{\text{rot}}(f))} e^{i(2\pi f \Delta t_c - \Delta \phi_c)} e^{i\phi_0}}{S_n(f)} df}{\sqrt{\int \frac{f^{-7/3} |B(t_{\text{rot}}(f))|^2}{S_n(f)} df \int \frac{f^{-7/3}}{S_n(f)} df}}$$



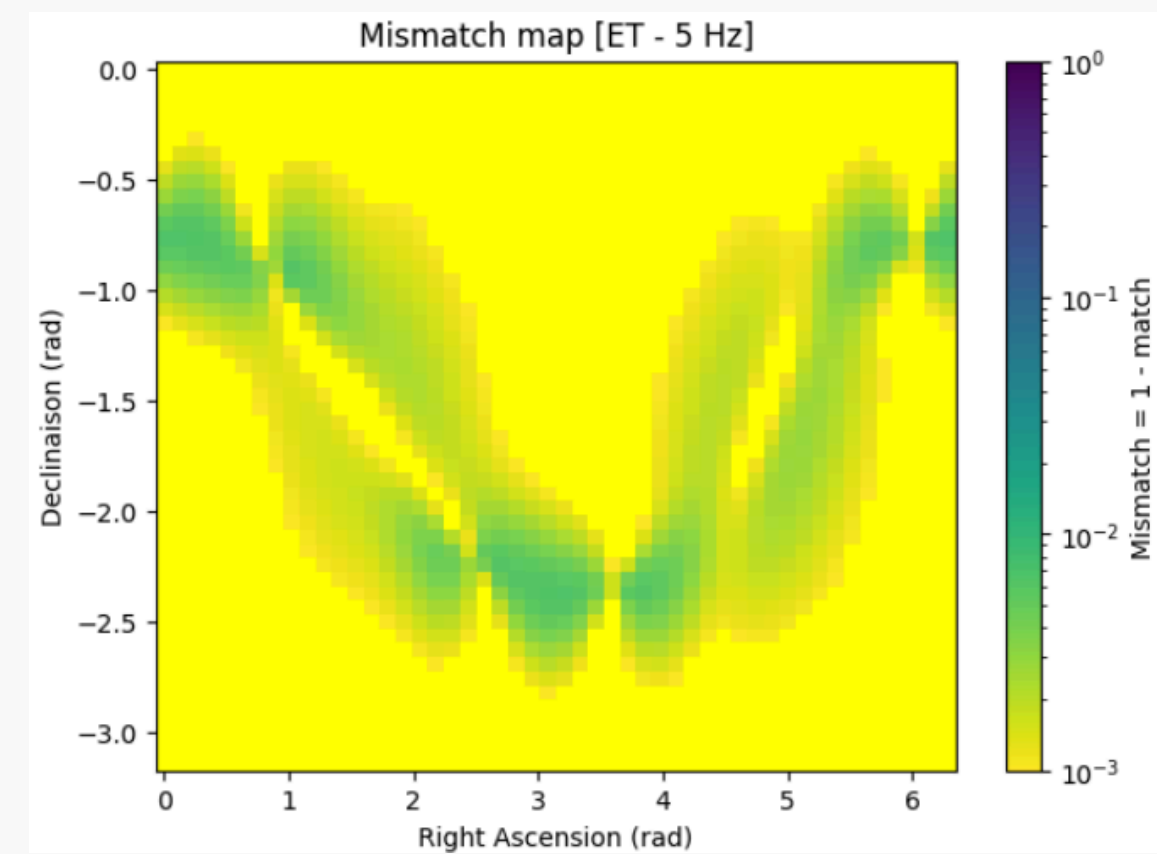
Modulus of
numerator's integrand



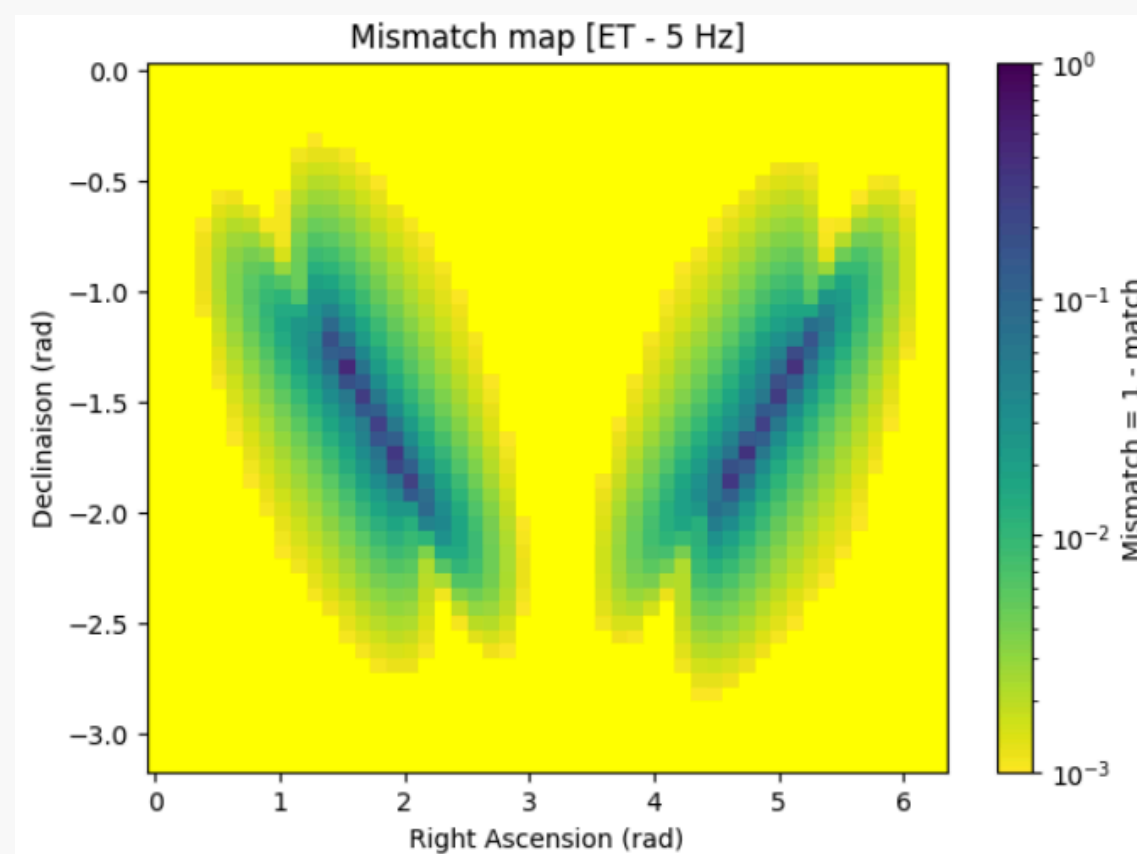
Physical
interpretation yet
to be given
(if not an error)



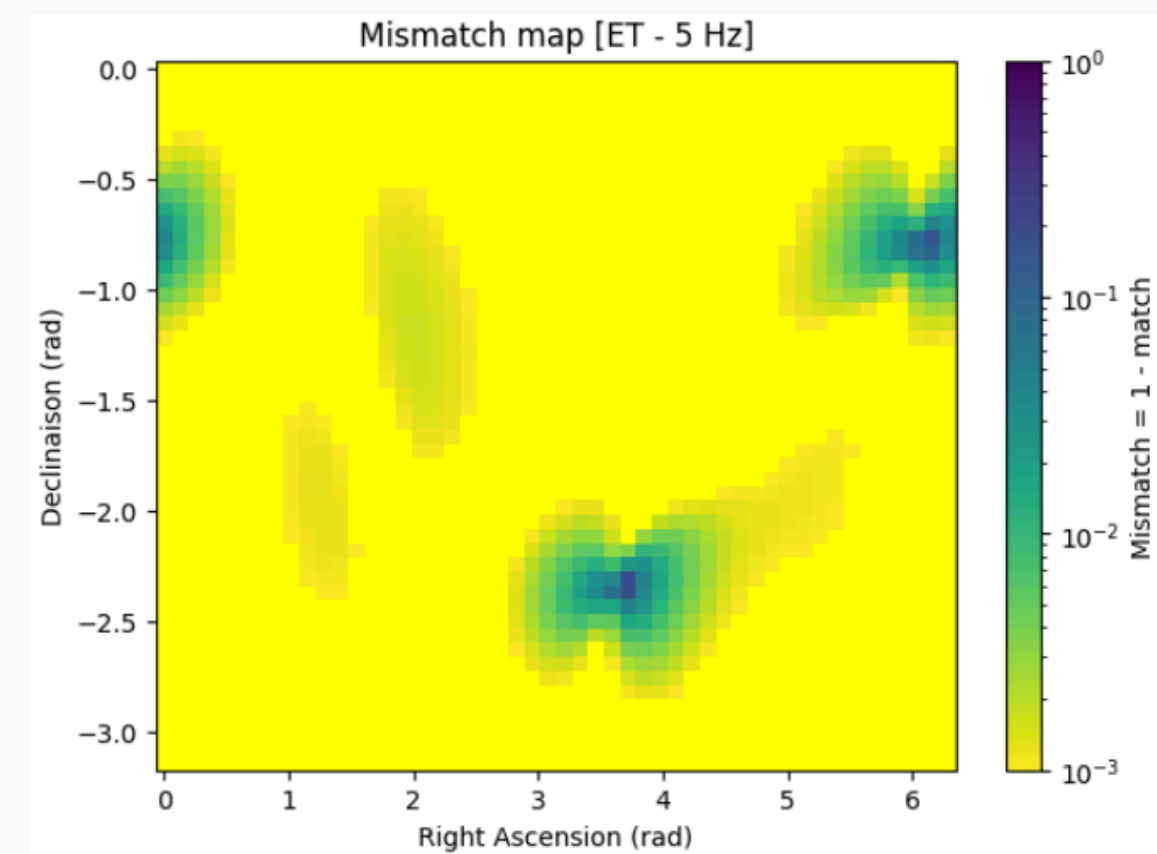
$\text{PSI} = 0^\circ, \text{incl} = 0^\circ$



$\text{PSI} = 30^\circ, \text{incl} = 0^\circ$



$\text{PSI} = 0^\circ, \text{incl} = 90^\circ$



$\text{PSI} = 30^\circ, \text{incl} = 90^\circ$

Development of equations using Marion's formalism

$$\mathcal{F}(\alpha, \delta, \psi, t) = F_+(\alpha, \delta, \psi, t) + iF_\times(\alpha, \delta, \psi, t)$$

$$\begin{cases} h_{mod}(\alpha, \delta, \psi, t) = f^{-\frac{7}{6}}(t)(F_+(\alpha, \delta, \psi, t) + iF_\times(\alpha, \delta, \psi, t)) = f^{-\frac{7}{6}}\mathcal{F}(\alpha, \delta, \psi, t) \\ h_{const}(\alpha, \delta, \psi) = f^{-\frac{7}{6}}(t)(F_+(\alpha, \delta, \psi, 0) + iF_\times(\alpha, \delta, \psi, 0)) = f^{-\frac{7}{6}}\mathcal{F}(\alpha, \delta, \psi, 0) \end{cases}$$

For 3 detectors (ET-Xylophone)
&
inclination = 0°

$$\mathcal{M}(\alpha, \delta, \psi) \propto \max_{\Delta\tau} \int_{f_{low}}^{f_{high}} \frac{f^{-\frac{7}{6}}}{S_n(f)} \Re([\mathcal{F}_{ET}(\alpha, \delta, \psi, 0)][\mathcal{F}_{ET}^*(\alpha, \delta, \psi, t)]) df$$

$$\mathcal{L}(\psi) := \Re([\mathcal{F}_{ET}(\alpha, \delta, \psi, 0)][\mathcal{F}_{ET}^*(\alpha, \delta, \psi, t)])$$

$$\begin{aligned} \mathcal{L}(\psi) &= \sqrt{A + \mathcal{B} \sin(4\psi + \Phi_1) + \mathcal{C} \sin(8\psi + \Phi_2)} + \sqrt{A - \mathcal{B} \sin(4\psi + \Phi_1) + \mathcal{C} \sin(8\psi + \Phi_2)} \\ &= f_+(\psi) + f_-(\psi) \end{aligned}$$

Match has a non-trivial
polarisation angle dependency

For 3 detectors (ET-Xylophone)
&
any inclination angle (ι)

$$\begin{aligned}\mathcal{F}_{ET}(\alpha, \delta, \psi, t) &= [1 + \cos^2(\iota)]F_{+,ET}(\alpha, \delta, \psi, t) + i[2 \cos(\iota)]F_{\times,ET}(\alpha, \delta, \psi, t) \\ &= \eta F_{+,ET}(\alpha, \delta, \psi, t) + i\xi F_{\times,ET}(\alpha, \delta, \psi, t)\end{aligned}$$

$$\mathcal{L}(\psi) = \eta^2 f_+(\psi) + \xi^2 f_{\times}(\psi)$$

Match has a non-trivial
polarisation angle dependency

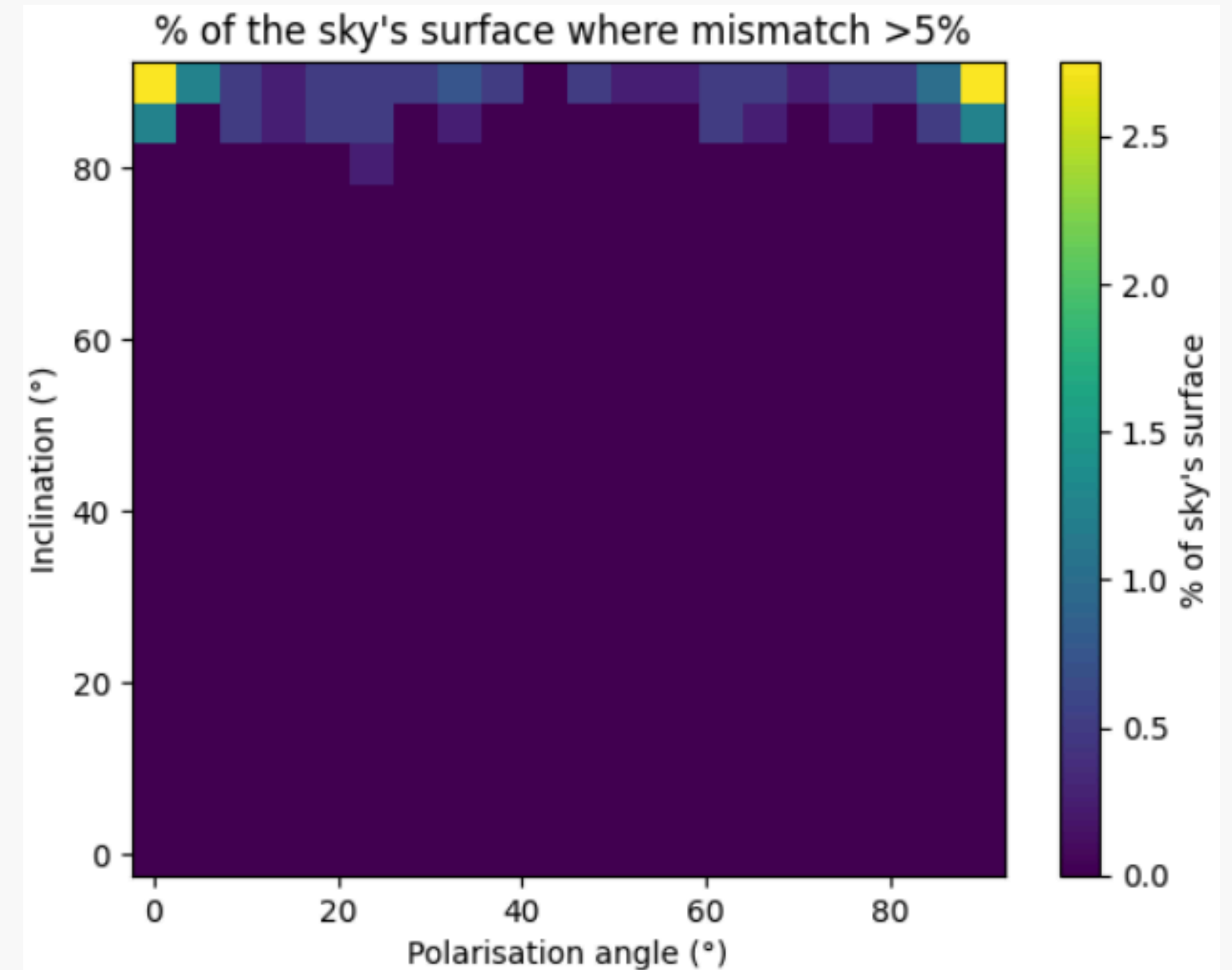
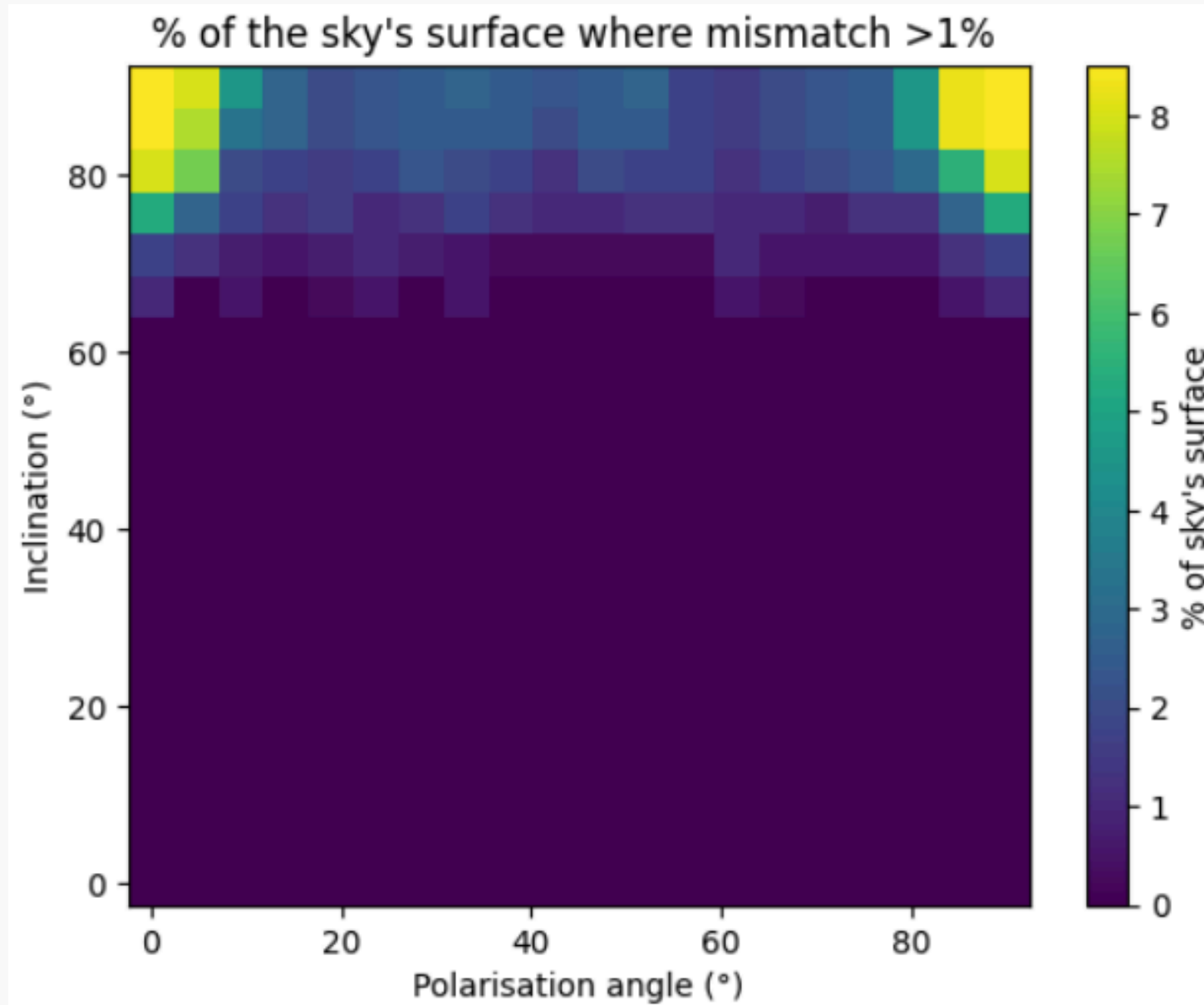
Studying the L function gives us the “geometric” behavior of the match

Extendable to any network of L-shaped detectors

$$\mathcal{L}(\psi) = \eta^2 f_+(\psi) + \xi^2 f_\times(\psi)$$

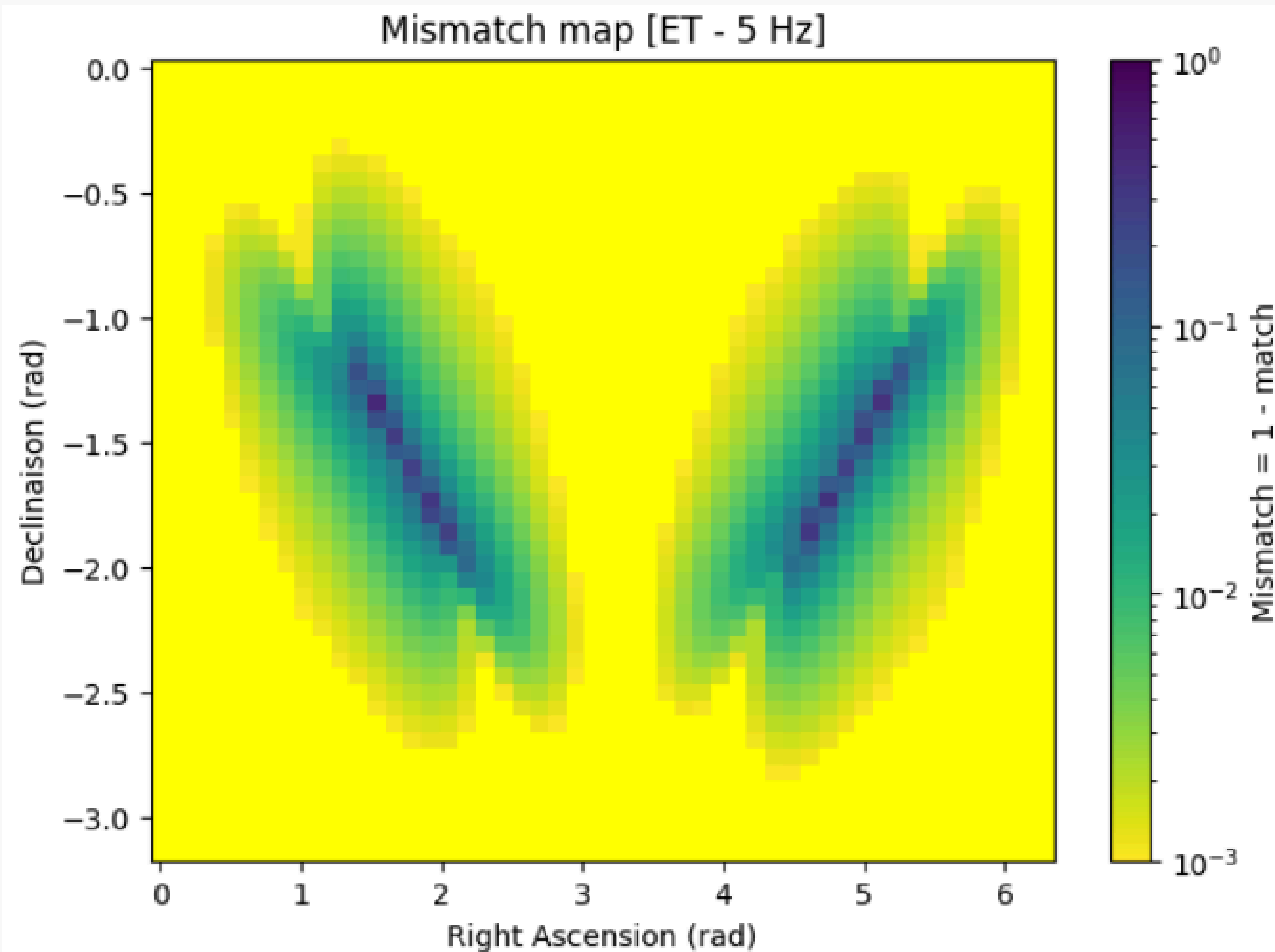
$$F_{+, \text{net}}(\alpha, \delta, \psi, t) = \sqrt{\sum_{i=1}^n F_{+, i}^2(\alpha, \delta, \psi, t)}$$
$$F_{\times, \text{net}}(\alpha, \delta, \psi, t) = \sqrt{\sum_{i=1}^n F_{\times, i}^2(\alpha, \delta, \psi, t)}$$

Areas where mismatch >1% and 5%



Confirmed !

Inclination = 90° , PSI = 90°



Max mismatch ~ 50%