

Fast localization of massive black hole binaries using the coronagraphic method for the LISA mission

Hugo Alexandre

Dixième Assemblée Générale du GdR Ondes Gravitationnelles

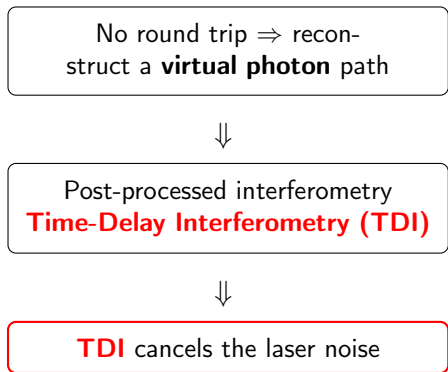


Groupement de recherche
Ondes gravitationnelles

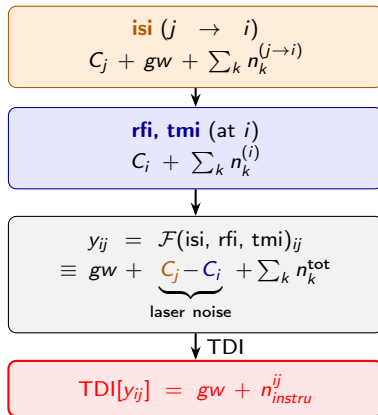


Time Delay Interferometry for LISA

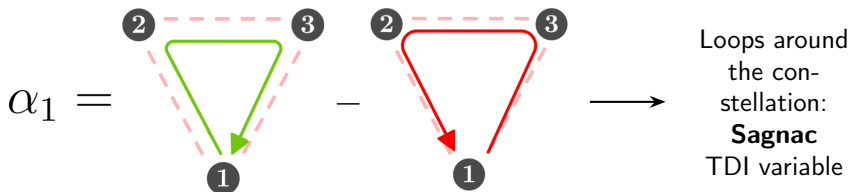
Time-Delay Interferometry (1/2): The Principle



From phasemeter measurements (isi, rfi, tmi), each y_{ij} is built as:



Time-Delay Interferometry (2/2): Building the Variables



$$\tilde{\alpha}_1 = \underbrace{(y_{12} + D_{12}y_{23} + D_{12}D_{23}y_{31})}_{\text{clockwise}} - \underbrace{(y_{13} + D_{13}y_{32} + D_{13}D_{32}y_{21})}_{\text{counter-clockwise}}$$

Starting the construction at SC1 $\Rightarrow \tilde{\alpha}_1$ at SC2 $\Rightarrow \tilde{\beta}_1$ at SC3 $\Rightarrow \tilde{\gamma}_1$

Complete laser noise suppression

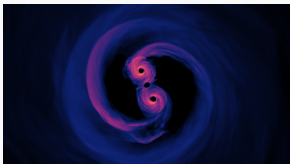
LISA: Expected Detections

Galactic Binaries (GBs)



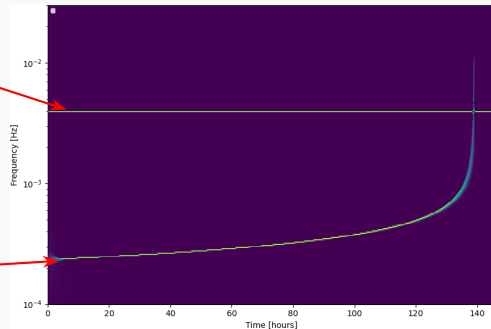
M. Weiss

Massive Black Hole Binaries (MBHBs)



NASA Goddard/Ascoli 2018

Frequency signature of sources



Other, yet unforeseen, sources may also exist

The Coronagraphic TDI Variable

We aim to build a linear combination of the TDI variables that **cancels** the GW signal, as a function of the position and frequency parameters characterizing the source:

$$\tilde{\kappa}(\alpha, \delta, f) = \vec{A}(\alpha, \delta, f) \cdot (\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})^T(f)$$

The Coronagraphic TDI Variable

We aim to build a linear combination of the TDI variables that **cancels** the GW signal, as a function of the position and frequency parameters characterizing the source:

$$\tilde{\kappa}(\alpha, \delta, f) = \vec{A}(\alpha, \delta, f) \cdot (\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})^T(f)$$

Objective: find $(\alpha_*, \delta_*, f_*)$ such that

$$\vec{A}(\alpha_*, \delta_*, f_*) \cdot \begin{pmatrix} \tilde{\alpha} \\ \tilde{\beta} \\ \tilde{\gamma} \end{pmatrix} = 0$$

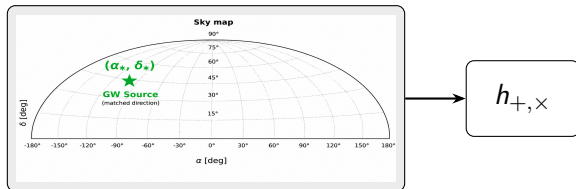
Coronagraphic Method : The Nullstream Pipeline

Principle

How to localize a source with LISA ?

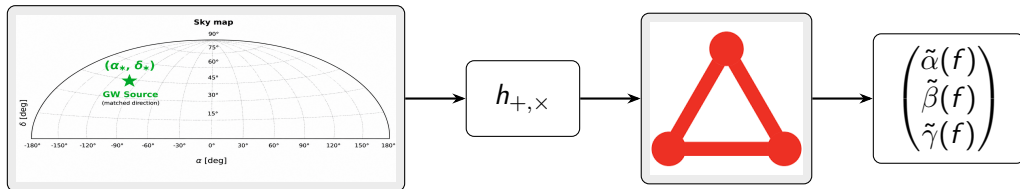
Principle

How to localize a source with LISA ?



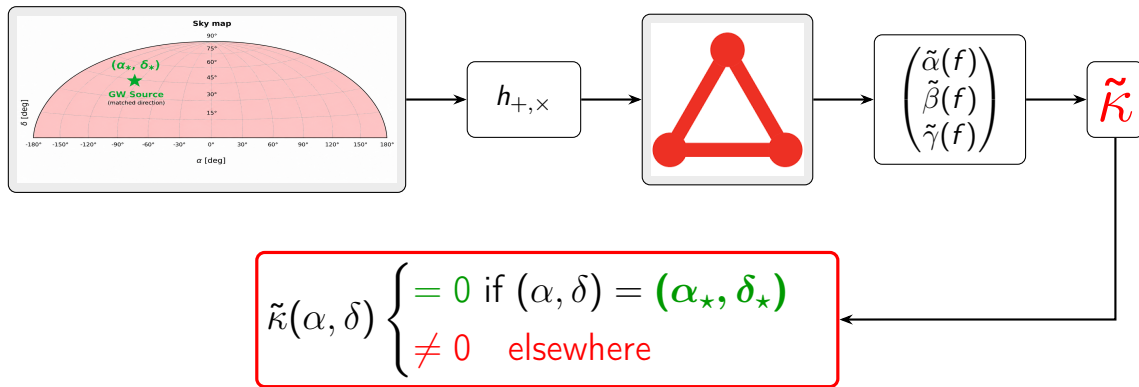
Principle

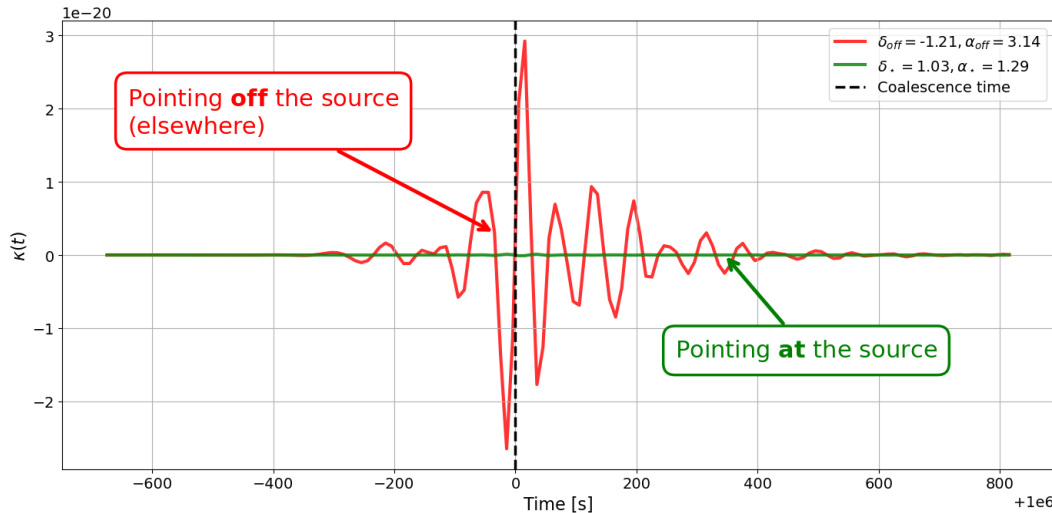
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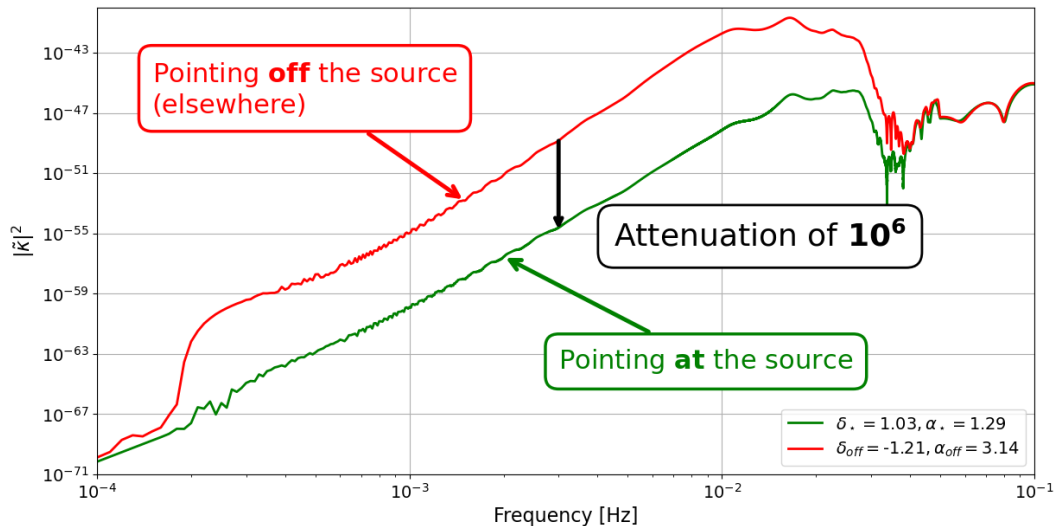


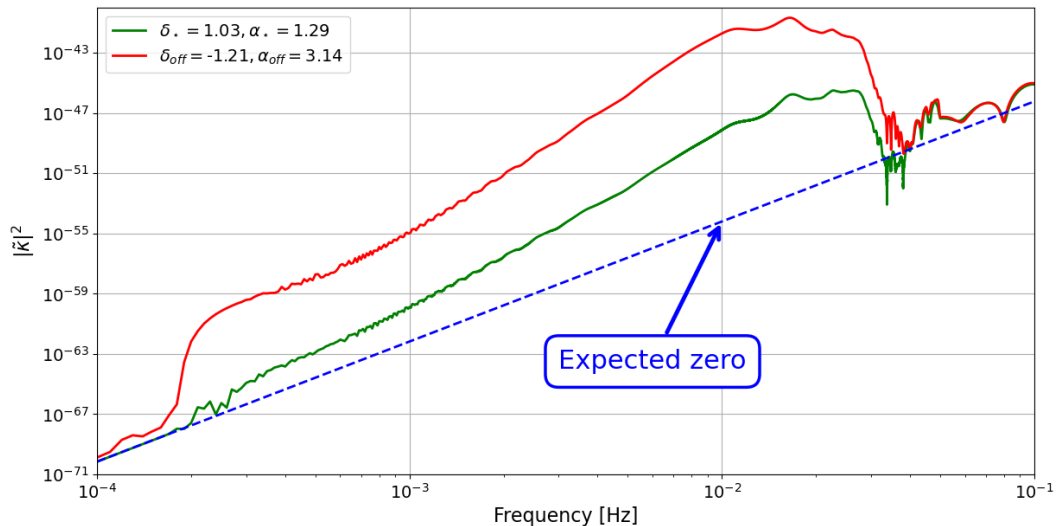
Principle

How to localize a source with LISA ?

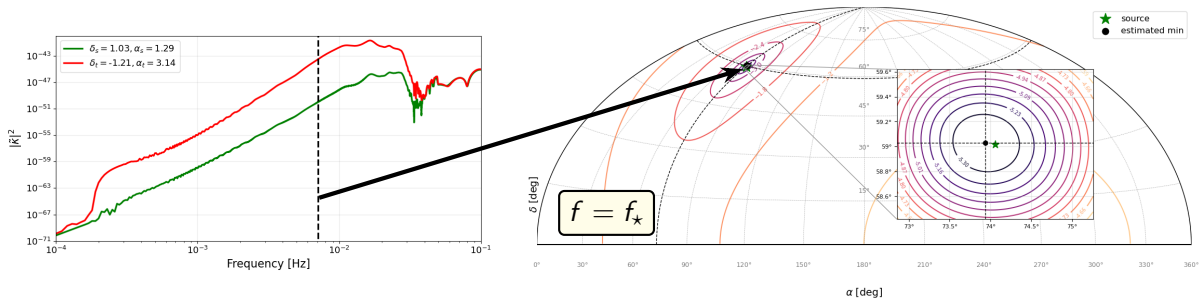


Time VS Frequency with $\tilde{\kappa}$ TDI (1/2)

Time VS Frequency with $\tilde{\kappa}$ TDI (2/2)

Time VS Frequency with $\tilde{\kappa}$ TDI (2/2)

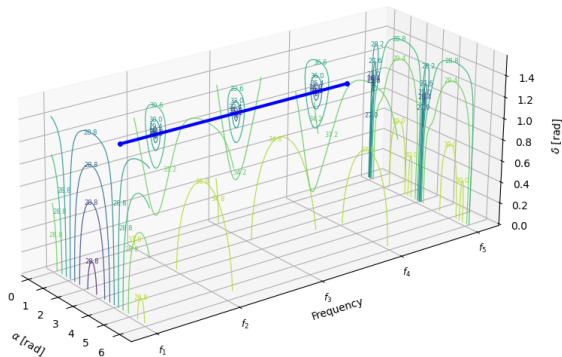
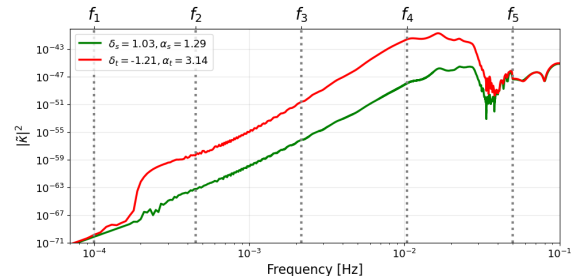
Sky Localization with κ



Localization done at a **single frequency** but the MBHB chirps across many frequencies

$\Rightarrow$ localizations can be accumulated over many frequencies

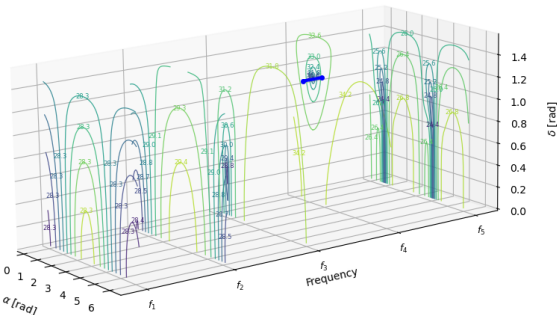
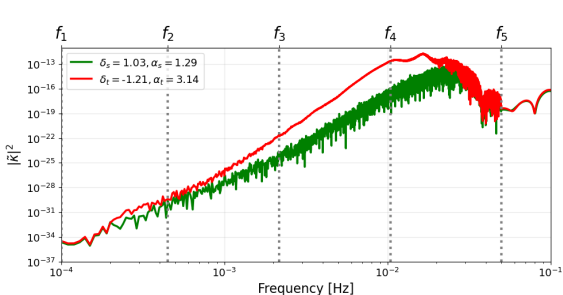
Scanning the MBHB Band (Noiseless)



$f_2, f_3, f_4 \in f_{\text{MBHB}} \rightarrow$ **localization** but $f_1, f_5 \notin f_{\text{MBHB}} \rightarrow$ **no localization**

The 3 localizations align into a segment $\Rightarrow$ pattern to identify and characterize

Scanning the MBHB Band (Noisy)



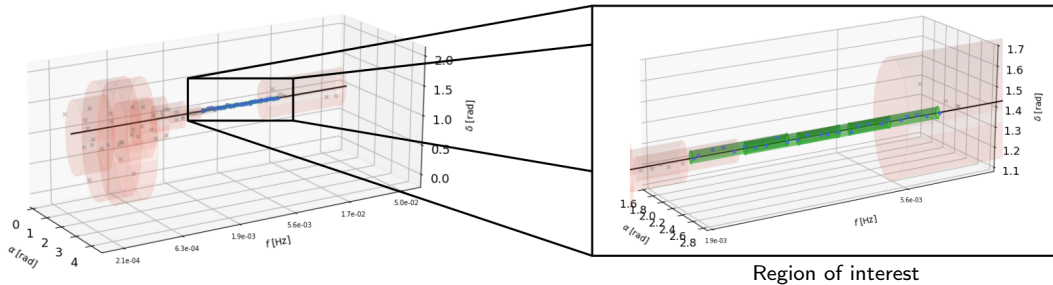
With noise, f_2, f_3 **lose their localization** only f_4 **still localizes** (highest spectral power)

must search $f_i \in f_{\text{MBHB}}$ for the frequencies that localize **and** align into a segment

Preliminary Approach

- ▶ Center: $(\hat{\mu}_\alpha, \hat{\mu}_\delta)$ mean of the sample
- ▶ Radius: $\hat{\sigma}$ std of the sample

Smallest $\hat{\sigma}$ cylinder(s) $\Rightarrow$
localization, with its uncertainty

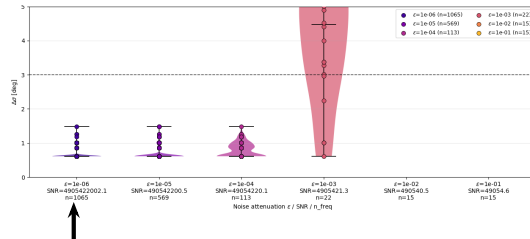
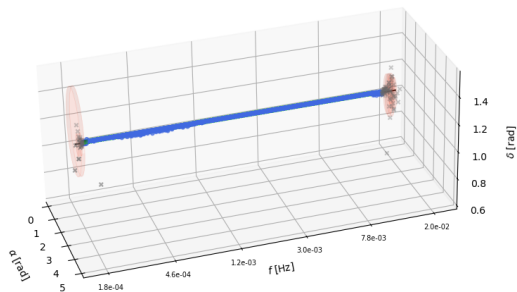


$$\Delta\alpha = 0.6013^\circ$$

$$\Delta\delta = 0.0442^\circ$$

$$\Delta\sigma = 0.600^\circ$$

Precision of the method

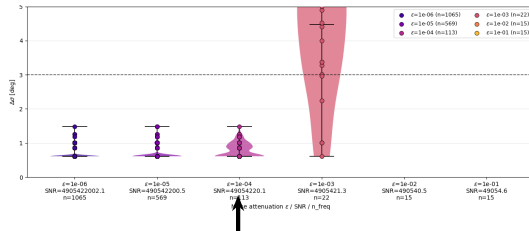
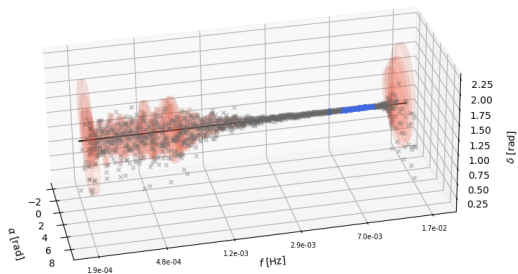


Large band of frequency: $\Delta f = 1.6 \times 10^{-2}$ Hz

Mean $\mu(\Delta\sigma) = 0.4^\circ$

Precise localization

Precision of the method

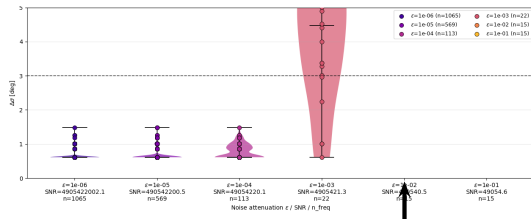
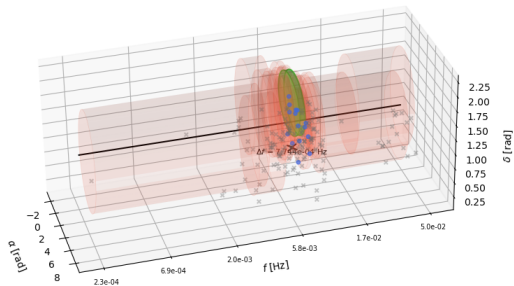


Narrower band of localization: $\Delta f = 4.1 \times 10^{-3}$ Hz

Mean $\mu(\Delta\sigma) = 0.8^\circ$

Still precise localization (fewer points, so slightly less precise)

Precision of the method

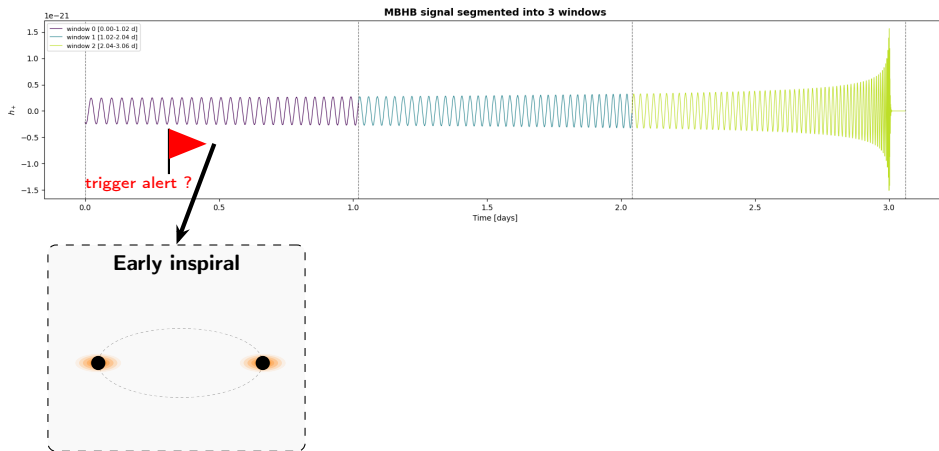


Even narrower frequency band: $\Delta f = 7.8 \times 10^{-4}$ Hz

Mean $\mu(\Delta\sigma) = 17.8^\circ$

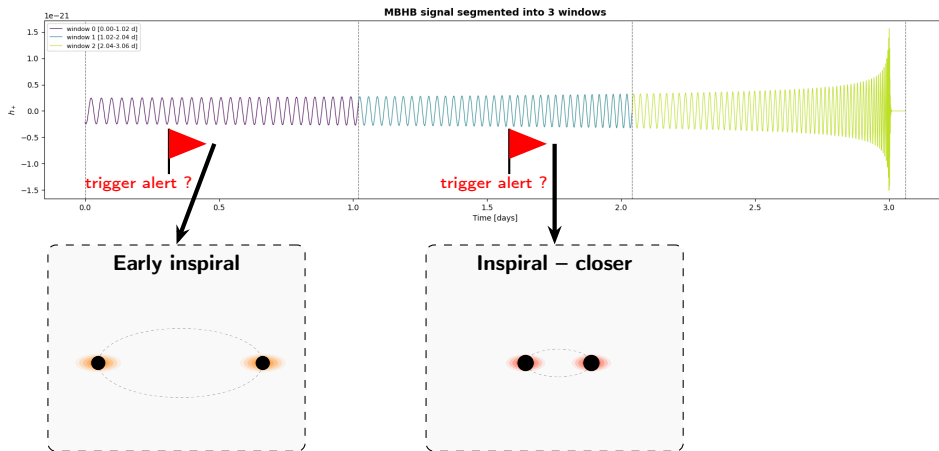
Not precise, above the required 10 deg^2 for localization

L2A - Catching EM Counterparts



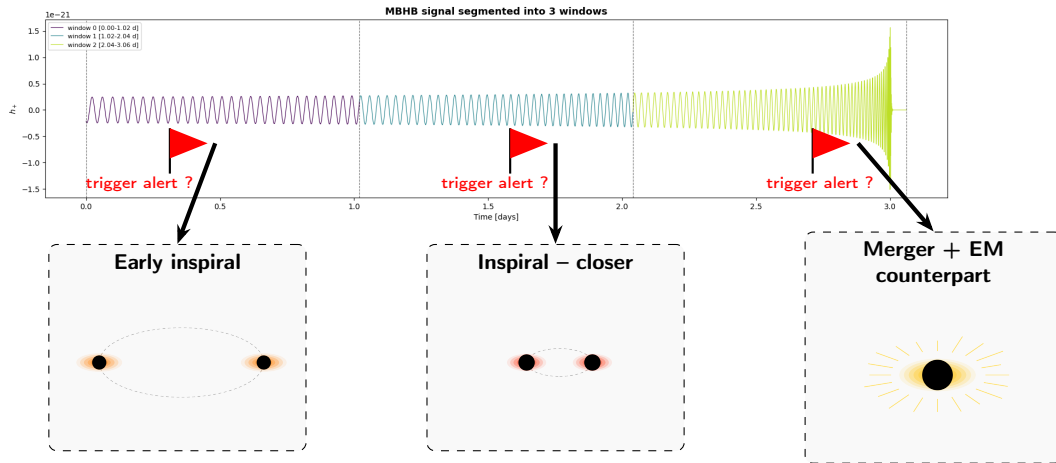
Early inspiral: signal just emerging $\Rightarrow$ a first, coarse alert

L2A - Catching EM Counterparts



Binary tightens: signal grows $\Rightarrow$ localization sharpens $\Rightarrow$ updated alert

L2A - Catching EM Counterparts

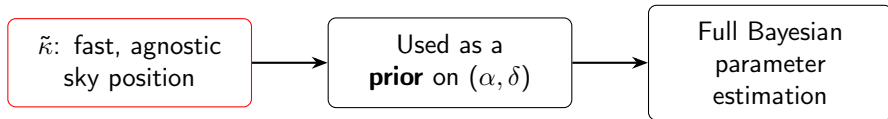


Merger: disk interactions may radiate $\Rightarrow$ **final, sharpest alert** for EM follow-up

Perspectives & Conclusion

Perspectives

- ▶ Numerical analysis work needed to characterize the **residual signal**
- ▶ The coronagraphic method can be used as a **prior** to accelerate parameter estimation (PE)



- ▶ **Machine Learning (Clustering?)** → automate the tracking of aligned localization points across frequencies

Summary

- ▶ The coronagraphic method is an agnostic method to localize GW sources for the LISA mission.
- ▶ Validated on **noiseless** and **low-noise** MBHB simulations.
- ▶ A residual signal prevents a perfect cancellation, but localization remains possible as long as the noise stays strongly attenuated.

Backup

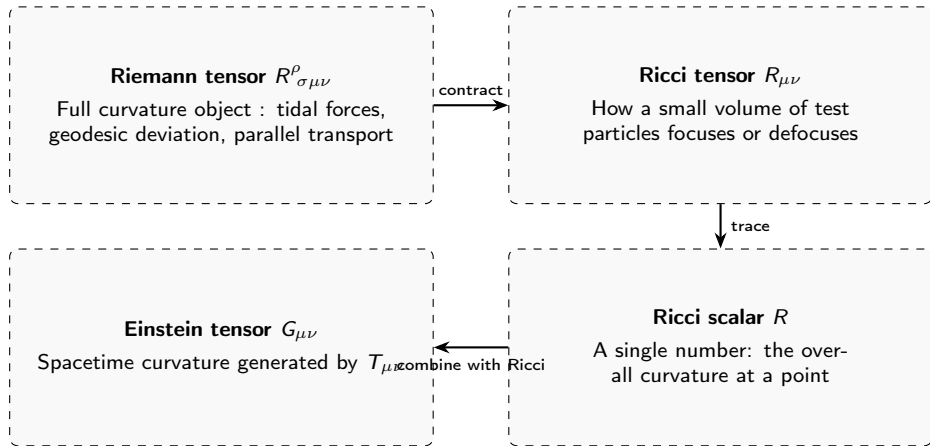
Angular Localization Error

The sky localization error is defined as the angular separation between the true and estimated source positions:

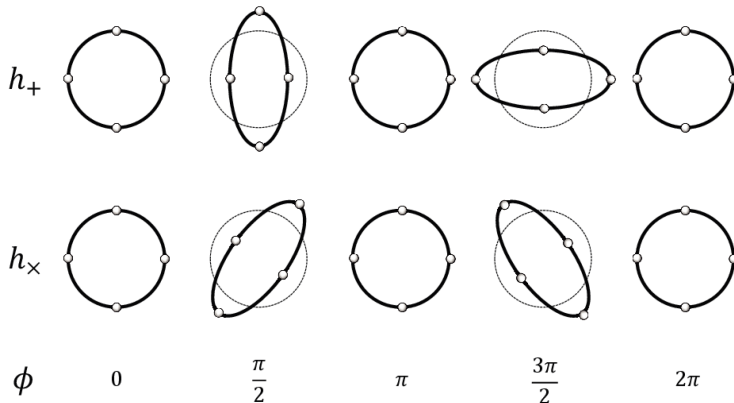
$$\Delta\sigma = \arccos \left[\sin(\delta_{\text{test}}) \sin(\delta_{\star}) + \cos(\delta_{\text{test}}) \cos(\delta_{\star}) \cos(\alpha_{\text{test}} - \alpha_{\star}) \right]$$

⇒ measures the angular distance on the celestial sphere between the estimated position $(\alpha_{\text{test}}, \delta_{\text{test}})$ and the true position $(\alpha_{\star}, \delta_{\star})$

Curvature Tensors: Telling Them Apart



Quadrupole Radiation Pattern



A ring of test particles stretches and squeezes along the $+$ and $\times$ polarizations as the wave passes

Standard Sirens: Measuring H_0 with GW170817

A **standard siren** is the GW analogue of a standard candle: the waveform directly encodes the luminosity distance D_L

GW signal

GR gives the luminosity distance directly:

$$D_L \approx 40 \text{ Mpc}$$

EM counterpart

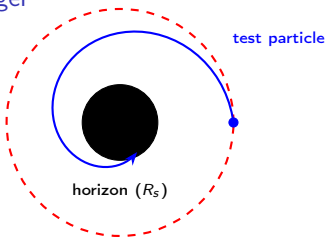
Host galaxy identified (NGC 4993) $\Rightarrow$ redshift from its spectrum:

$$z \approx 0.0970$$

From Hubble's law

$$D_L \approx \frac{cz}{H_0} \implies H_0 = \frac{cz}{D_L} \approx 70 \text{ km/s/Mpc}$$

The Merger Frequency: f_{merger}



Beyond the ISCO, circular orbits are no longer stable – the particle plunges in. The GW frequency at that point sets f_{merger} :

$$f_{\text{merger}} \approx f_{\text{ISCO}} = \frac{c^3}{6^{3/2} \pi GM}$$

Building the Vector $\vec{A}$

For a generator, e.g.

$$\tilde{\alpha}_{gw} = \alpha_+ \tilde{h}_+ + \alpha_\times \tilde{h}_\times$$

where $\alpha_{+,\times} = \sum_{ij} \alpha_{ij} \xi_{ij}^{+,\times}$

$\xi_{ij}^{+,\times}$: **antenna response** – how each link ij responds to $h_+, h_\times$

$$\vec{P}_{+,\times} = (\alpha_{+,\times}, \beta_{+,\times}, \gamma_{+,\times})^T$$

Cancelling $\tilde{k}_{gw}$ for any $\tilde{h}_{+,\times}$:

$$\vec{A} \cdot \vec{P}_+ = 0 \quad \vec{A} \cdot \vec{P}_\times = 0$$

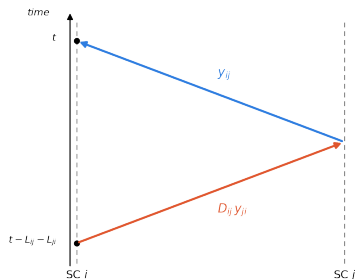
(orthogonality condition)

$$\vec{A} = \vec{P}_+ \times \vec{P}_\times = \begin{pmatrix} \beta_+ \gamma_\times - \gamma_+ \beta_\times \\ \gamma_+ \alpha_\times - \alpha_+ \gamma_\times \\ \alpha_+ \beta_\times - \beta_+ \alpha_\times \end{pmatrix} \quad (3.1.27)$$

Building the TDI Operators (1/2): The Delay Operator

The **delay operator** D_{ij} delays a measurement by the light travel time $\tau_{ij}(t)$ along link ij :

$$D_{ij} x(t) := x(t - L_{ij}), \quad \text{chained: } D_{ijk} := D_{ij} D_{jk}$$



Telescoping: a simple round trip already cancels part of the laser noise

$$y_{ij}|_{\text{laser}} = C_j(t - L_{ij}) - C_i(t)$$

$$D_{ij} y_{ji}|_{\text{laser}} = C_i(t - L_{ij} - L_{ji}) - C_j(t - L_{ij})$$

The **same term** appears in both, with opposite sign, it cancels (telescopes) in the sum

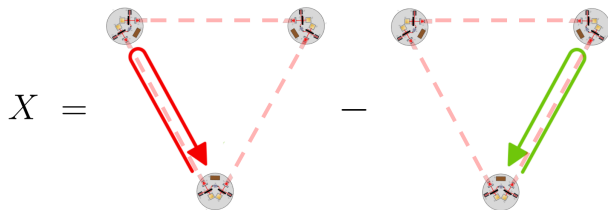
$$y_{ij} + D_{ij} y_{ji} = -C_i(t) + C_i(t - L_{ij} - L_{ji})$$

Only the **local** laser noise C_i remains, at two different times a full TDI variable pairs *two* such round trips of equal delay so this residual cancels too

Building the TDI Operators (2/2): Example – X

X is the difference of two round trips from SC1, one on each adjacent arm:

$$X = \underbrace{[y_{12} + D_{12} y_{21}]}_{\text{round trip SC1} \leftrightarrow \text{SC2}} - \underbrace{[y_{13} + D_{13} y_{31}]}_{\text{round trip SC1} \leftrightarrow \text{SC3}}$$



Using the round-trip result from (1/2), with $L_{12} = L_{21} = \ell_3$ and $L_{13} = L_{31} = \ell_2$ (arm opposite the third satellite):

Red: $y_{12} + D_{12} y_{21}|_{\text{laser}} = -C_1(t) + C_1(t - 2\ell_3)$

Green: $y_{13} + D_{13} y_{31}|_{\text{laser}} = -C_1(t) + C_1(t - 2\ell_2)$

$\Rightarrow$ vanishes only under the **equal-arm** assumption
 $\ell_2 = \ell_3$ (first generation, simplified): both terms
 become $C_1(t - 2\ell)$ and cancel, $X|_{\text{laser}} = 0$

$$X|_{\text{laser}} = C_1(t - 2\ell_3) - C_1(t - 2\ell_2)$$

TDI Generations

First generation

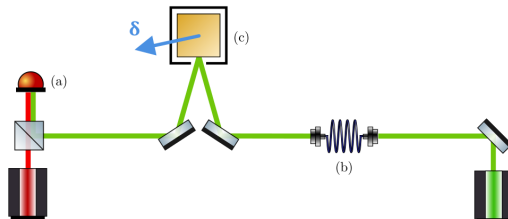
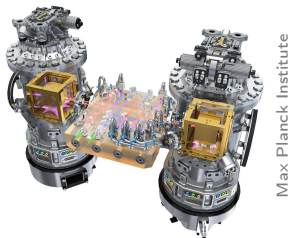
- ▶ **Static and symmetric** arms:
 $L_{ij}(t) = L_{ji}(t) = \text{const}$ (round-trip delay equal; the three arms can still differ from each other)
- ▶ Delay operators **commute** $\Rightarrow$ laser noise cancels exactly under this assumption
- ▶ Word length: 6 links (Sagnac), 8 links (Michelson)

Second generation

- ▶ Accounts for the real, **time-varying** arm lengths: $\dot{L}_{ij}/c \sim 10^{-8}$ (rotation, flexing)
- ▶ Delay operators no longer commute $\Rightarrow$ each virtual beam retraces its loop a second time
- ▶ Word length doubles: 12 links (Sagnac), 16 links (Michelson) – residual laser noise drops from $\mathcal{O}(\dot{L})$ to $\mathcal{O}(\dot{L}^2)$

LISA's constellation slowly rotates and flexes – second-generation TDI is required to meet the noise budget

The Test Masses



J.B. Bayle (PhD Thesis - 2019)

- ▶ Each spacecraft hosts **two** free-falling test masses (one per MOSA), shielded from every non-gravitational force
- ▶ **Drag-free control**: the spacecraft itself is steered to follow the test mass, not the other way around
- ▶ The TMI (test-mass interferometer) measures the test mass's displacement δ **along the optical axis**, relative to its own optical bench isolating the local test-mass motion in the data

Residual Noise: Test-Mass & Optical Metrology

Once TDI cancels the laser noise, two secondary noises set LISA's final sensitivity

Test-mass acceleration noise

$$\sqrt{S_a} \sim 3 \frac{\text{fm/s}^2}{\sqrt{\text{Hz}}}$$

Physically: a residual **acceleration noise** disturbing free-fall (equivalent to a force smaller than the weight of a virus acting on the 2 kg test mass)

Dominates **low frequencies**

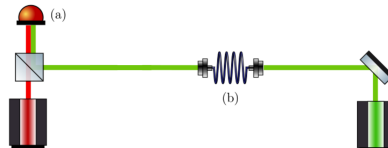
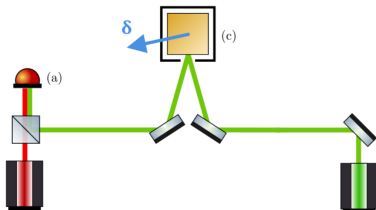
Optical metrology noise (OMS)

$$\sqrt{S_x} \sim 15 \frac{\text{pm}}{\sqrt{\text{Hz}}}$$

Physically: the shot-noise-limited **measurement precision** of the optical readout ($\sim 1/7$ of a hydrogen atom's diameter)

Dominates **high frequencies**

The Three Interferometers: ISI, TMI, RFI



- ▶ **ISI** (Inter-Spacecraft Interferometer): local laser $\times$ beam received from the distant spacecraft (~ 8.3 s later). The *only* one of the three that carries the GW signal
- ▶ **TMI** (Test-Mass Interferometer): local laser reflected off the local test mass. It measures the test mass's motion relative to the bench
- ▶ **RFI** (Reference Interferometer): local laser $\times$ the other on-board laser, via an internal backlink fiber, touches no test mass

Backup: Cancel Out the Laser Noise

$$\tilde{\alpha}_1 = (y_{12} + D_{12}y_{23} + D_{12}D_{23}y_{31}) - (y_{13} + D_{13}y_{32} + D_{13}D_{32}y_{21})$$

Clockwise

$$\begin{aligned} y_{12}|_{\text{laser}} &= C_2(t - L_{12}) - C_1(t) \\ D_{12}y_{23}|_{\text{laser}} &= C_3(t - L_{12} - L_{23}) - C_2(t - L_{12}) \\ D_{12}D_{23}y_{31}|_{\text{laser}} &= \\ C_1(t - L_{12} - L_{23} - L_{31}) - C_3(t - L_{12} - L_{23}) \\ \Rightarrow C_1(t - L_{12} - L_{23} - L_{31}) - C_1(t) \end{aligned}$$

Counter-clockwise

$$\begin{aligned} y_{13}|_{\text{laser}} &= C_3(t - L_{13}) - C_1(t) \\ D_{13}y_{32}|_{\text{laser}} &= C_2(t - L_{13} - L_{32}) - C_3(t - L_{13}) \\ D_{13}D_{32}y_{21}|_{\text{laser}} &= \\ C_1(t - L_{13} - L_{32} - L_{21}) - C_2(t - L_{13} - L_{32}) \\ \Rightarrow C_1(t - L_{13} - L_{32} - L_{21}) - C_1(t) \end{aligned}$$

$$\tilde{\alpha}_1|_{\text{laser}} = \text{clockwise} - \text{counter-clockwise} = C_1(t - L_{12} - L_{23} - L_{31}) - C_1(t - L_{13} - L_{32} - L_{21})$$

1st generation TDI: equal arm $\Rightarrow L_{ij} = L_{ji}$

$$\tilde{\alpha}_1|_{\text{laser}} = 0$$

Perspectives & Future Work

How to localize MBHBs **in realistic conditions** in the context of the **L2A** ?

