

Plug-and-Play methods for reconstructing individual polarizations of CBC signals

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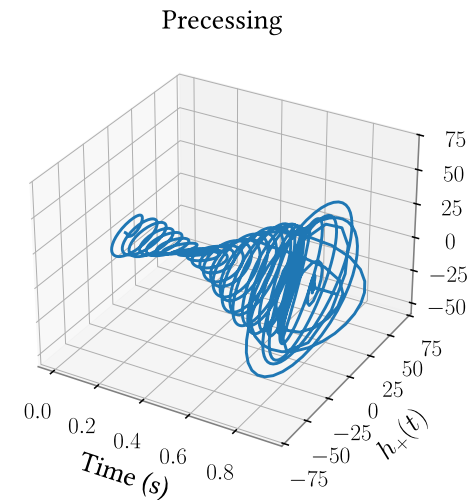
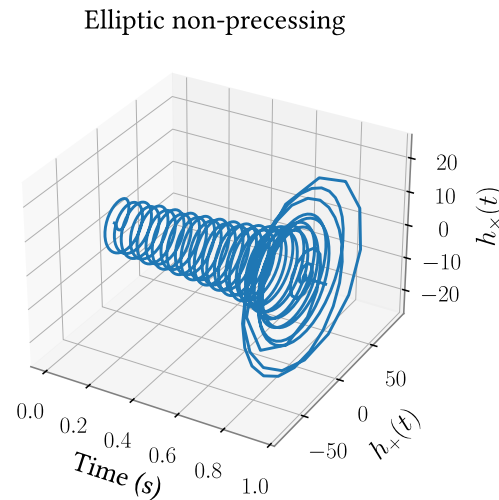
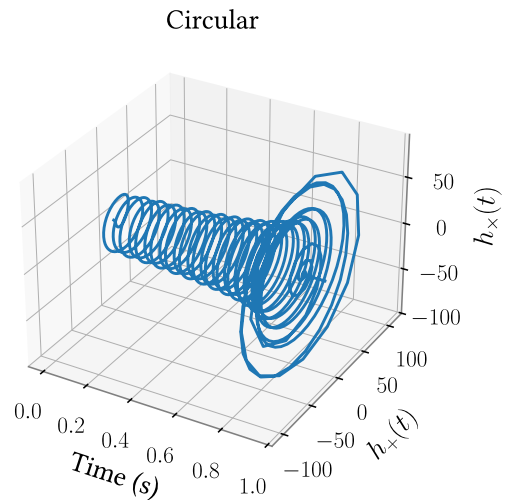
7 October 2026



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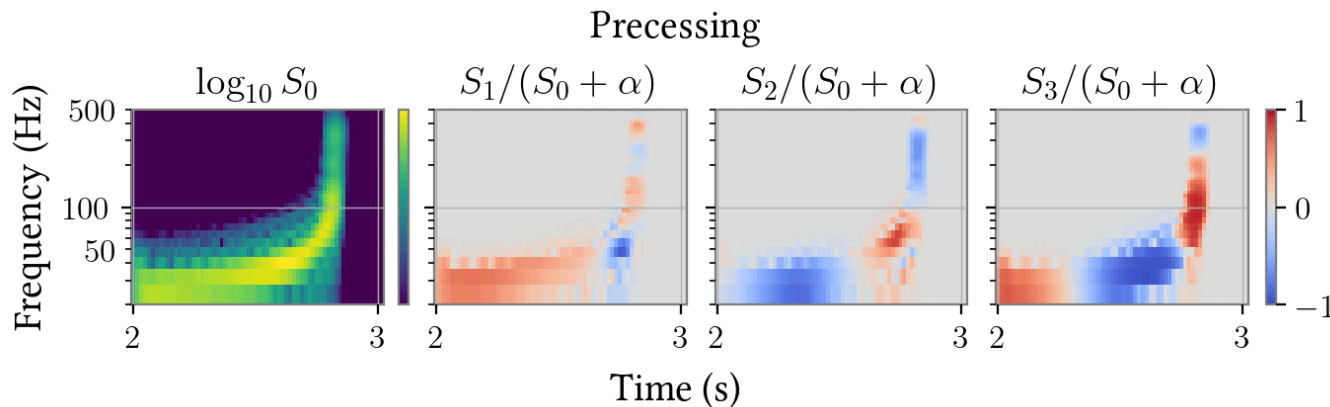
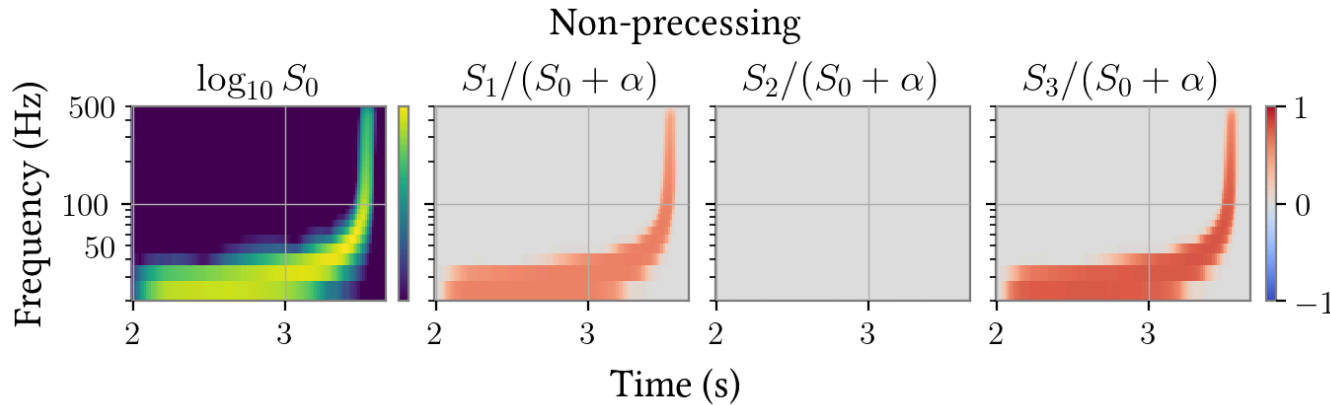
- Studying bivariate signals
 - Gravitational waves are a prime example: h_+ , $h_\times$
 - Here studying CBC signals
 - Polarization can be fairly complex for precessing signals!



Goal: reconstruct polarizations with a “semi-modelled” approach

Mathematical description

We describe polarization using the so-called “Stokes parameters” $S_0 \dots S_3$

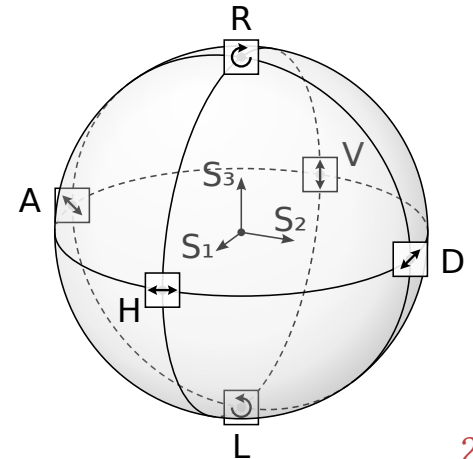


$$S_0 = h_+^2 + h_\times^2$$

$$S_1 = h_+^2 - h_\times^2 \quad \updownarrow \leftrightarrow$$

$$S_2 = 2 \operatorname{Re}(h_+ \overline{h_\times}) \quad \nearrow \searrow$$

$$S_3 = -2 \operatorname{Im}(h_+ \overline{h_\times}) \quad \circlearrowleft \circlearrowright$$



We model the signal as

$$\mathbf{y}(t + \boldsymbol{\tau}_\theta) = \mathbf{F}_\theta \mathbf{h}(t) + n$$

$$\mathbf{y}(t + \boldsymbol{\tau}_\theta) = \begin{pmatrix} y^H(t + \tau_\theta^H) \\ y^L(t + \tau_\theta^L) \\ y^V(t + \tau_\theta^V) \end{pmatrix}; \mathbf{F}_\theta = \begin{pmatrix} F_\theta^{+,H} & F_\theta^{\times,H} \\ F_\theta^{+,L} & F_\theta^{\times,L} \\ F_\theta^{+,V} & F_\theta^{\times,V} \end{pmatrix}; \mathbf{h}(t) = \begin{pmatrix} h^+(t) \\ h^\times(t) \end{pmatrix}$$

n is supposed to be Gaussian noise; in practice, we whiten the signal data and the $\mathbf{F}$ matrix.

A naive reconstruction of the polarizations can be achieved by maximizing the log-likelihood

$$\log \mathcal{L} = \log \pi(\mathbf{y}|\tilde{\mathbf{h}}) = \|\mathbf{y}(t + \tau_\theta) - \mathbf{F}_\theta \tilde{\mathbf{h}}\|^2$$

giving

$$\mathbf{h} = \mathbf{F}_\theta^+ \mathbf{y}(t + \tau_\theta)$$

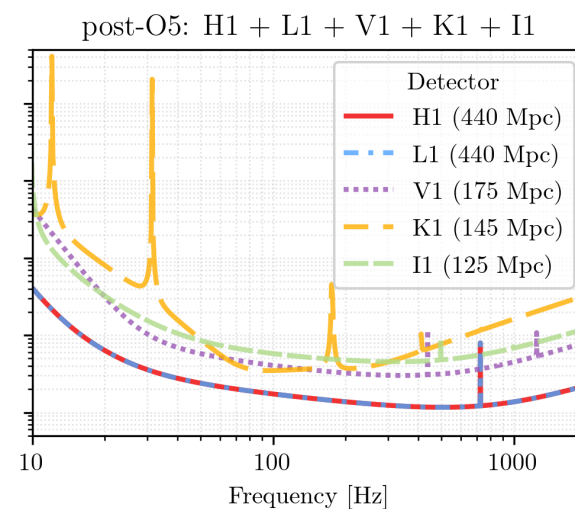
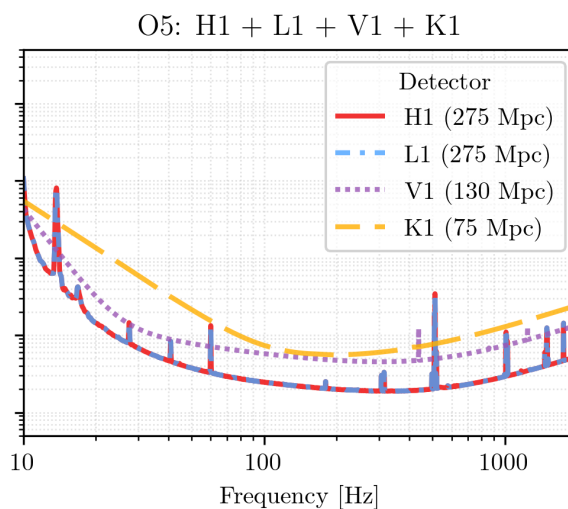
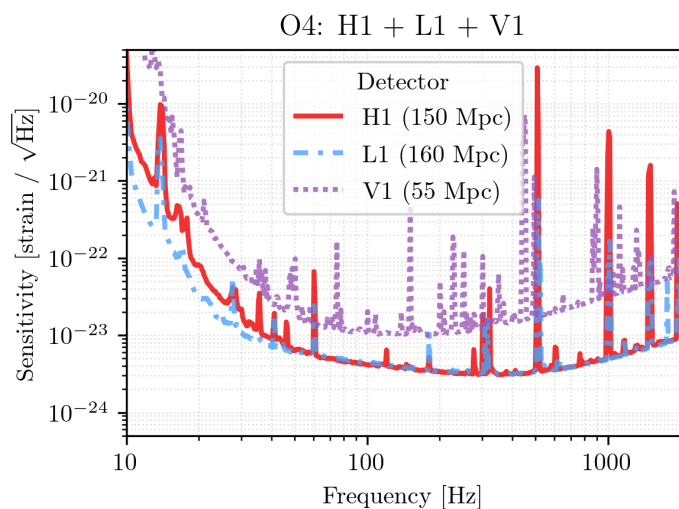
where $\mathbf{F}^+$ is the pseudoinverse of $\mathbf{F}$.

Issue: solution is polluted by noise $\rightarrow$ introduce a regularizing prior $\pi(\tilde{\mathbf{h}})$ and maximize $\log \pi(\tilde{\mathbf{h}}|\mathbf{y}) \propto \log \pi(\mathbf{y}|\tilde{\mathbf{h}}) + \log \pi(\tilde{\mathbf{h}})$

Polarization restorability using MLE

We evaluate the error in the polarization restoration with the future detector network using basic least-square inversion (no regularization)

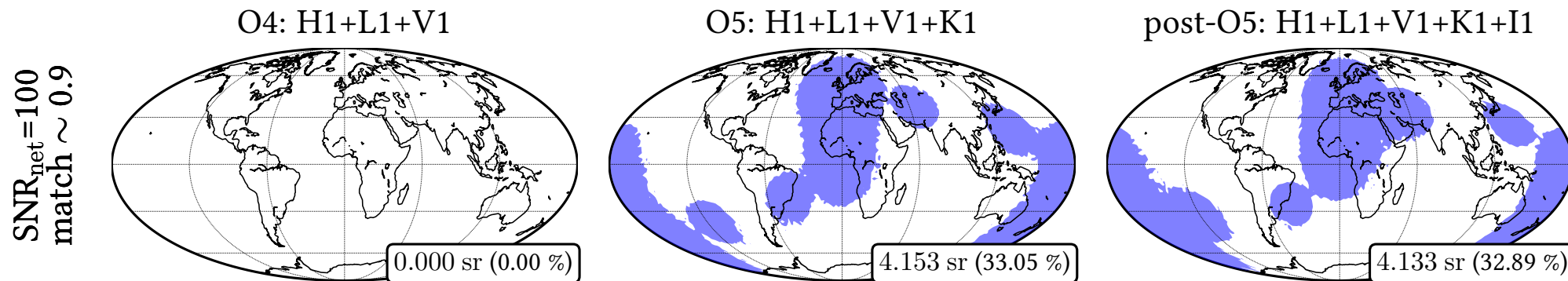
3 phases considered for the next decade (3G not included in this study)



Polarization restorability using MLE

Solid angle of sky regions with match > 0.9 at fixed $\text{SNR}_{\text{net}} = 100$

Measured empirically using randomly drawn polarizations; valid for any kind of transient!



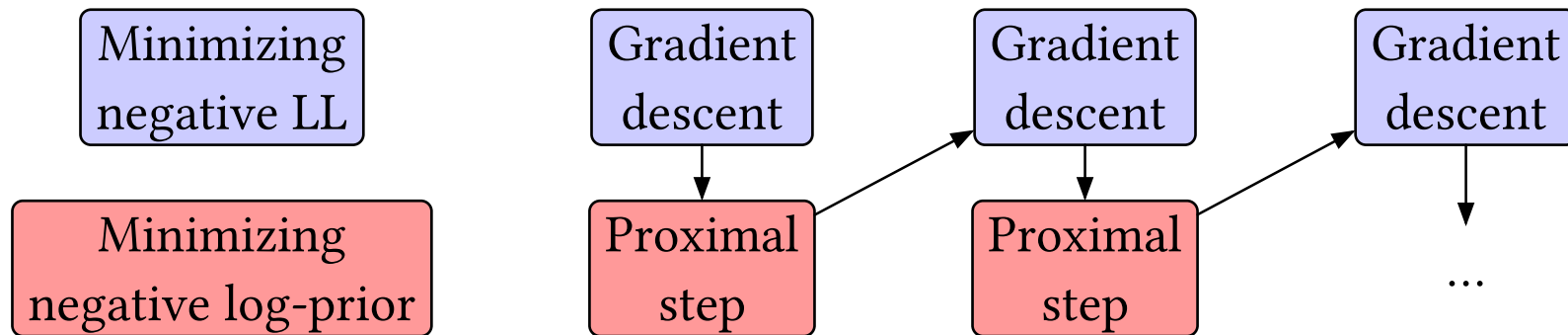
Large fraction of the sky with limited accuracy $\rightarrow$ need for regularization using knowledge of the signal.

Plug-and-Play method

A prior encoding characteristics of CBC signals will quickly become very complicated and not necessarily differentiable...

Solution (forward-backward algorithm): alternate maximizing the (differentiable) likelihood with gradient descent and the prior with the proximal operator

$$\text{prox}_{\pi}(\tilde{\mathbf{h}}) = \underset{\tilde{\mathbf{h}}'}{\operatorname{argmin}} \|\tilde{\mathbf{h}}' - \tilde{\mathbf{h}}\|^2 - \log \pi(\tilde{\mathbf{h}}')$$

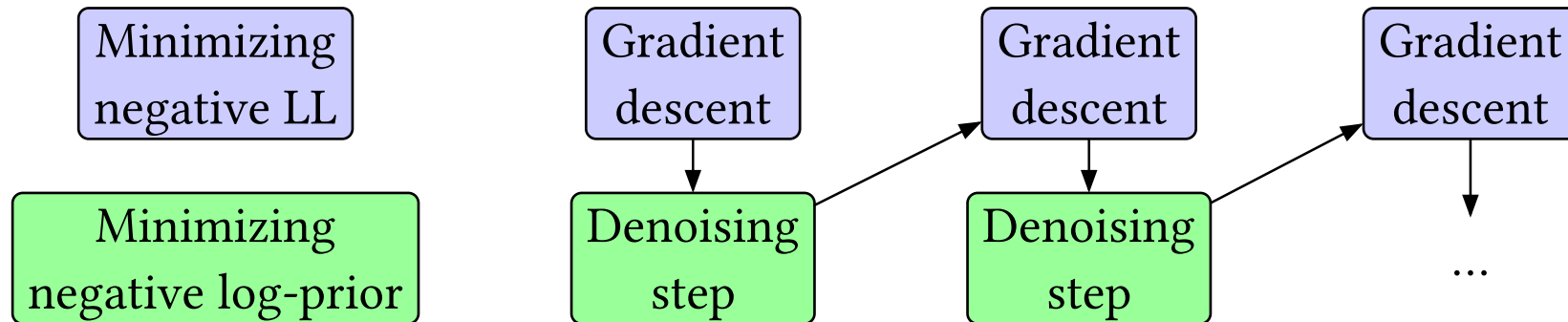


$$x_{n+1} = \text{prox}_{\pi}(x_n - \tau \nabla \log \mathcal{L})$$

Plug-and-Play method

Problem: How do we compute this proximal operator?

Key observation (Plug-and-Play algorithm): the proximal step is the same thing as a denoising step $\rightarrow$ replace it with an “off-the-shelf” denoiser D



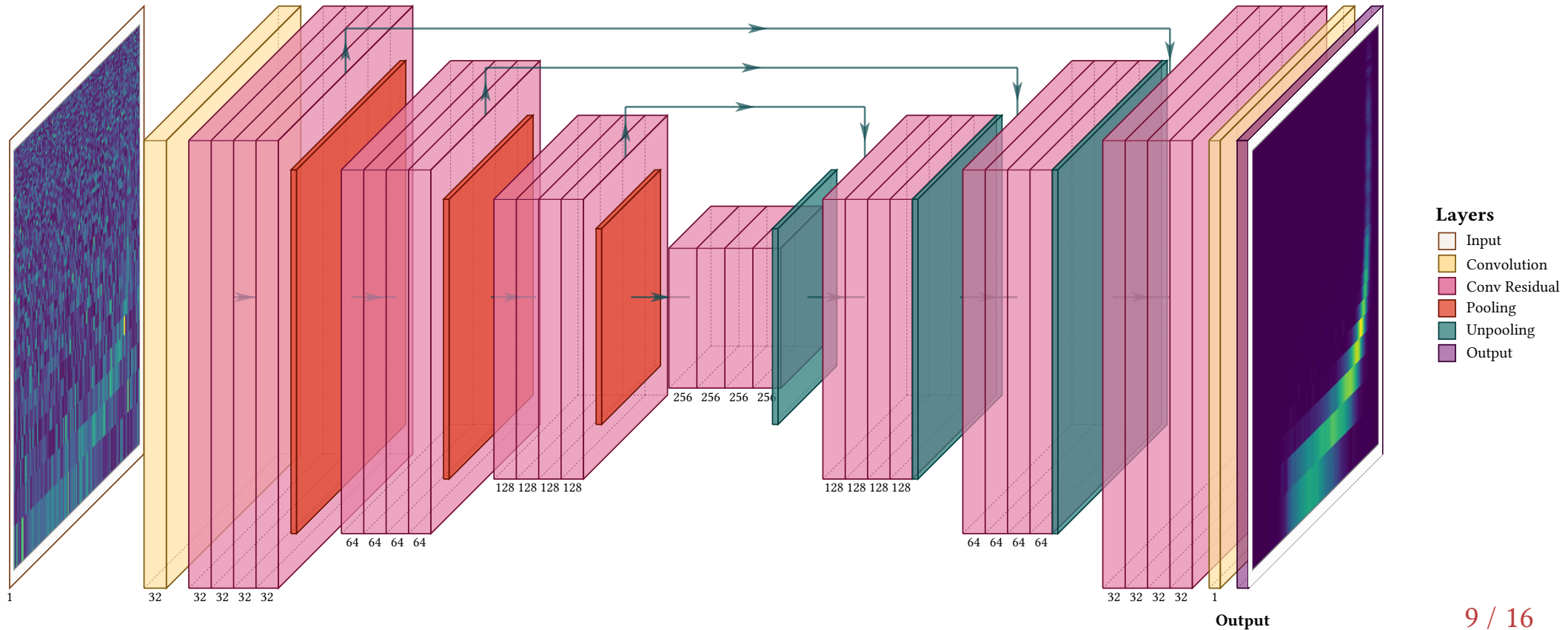
$$x_{n+1} = D(x_n - \tau \nabla \log \mathcal{L})$$

In our case, we train a neural network for the denoising operation.

Neural network architecture

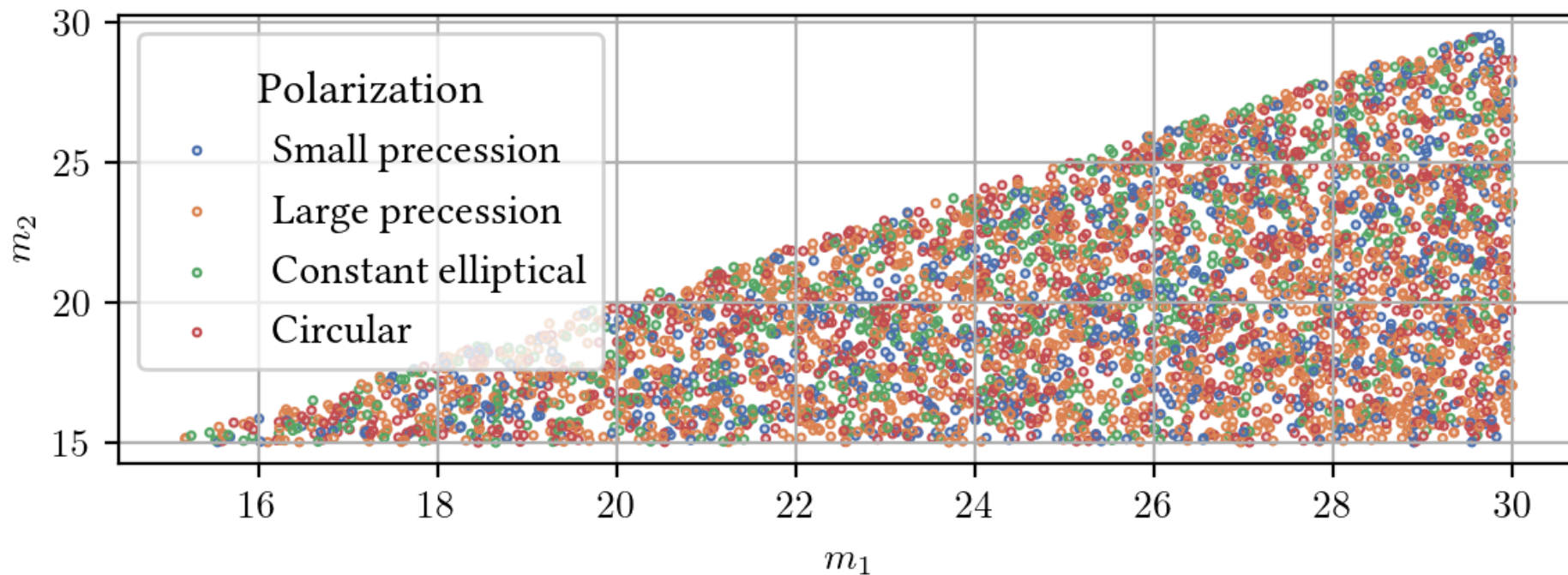
DRUNet: variation of the U-Net using residual blocks

~ 8M parameters

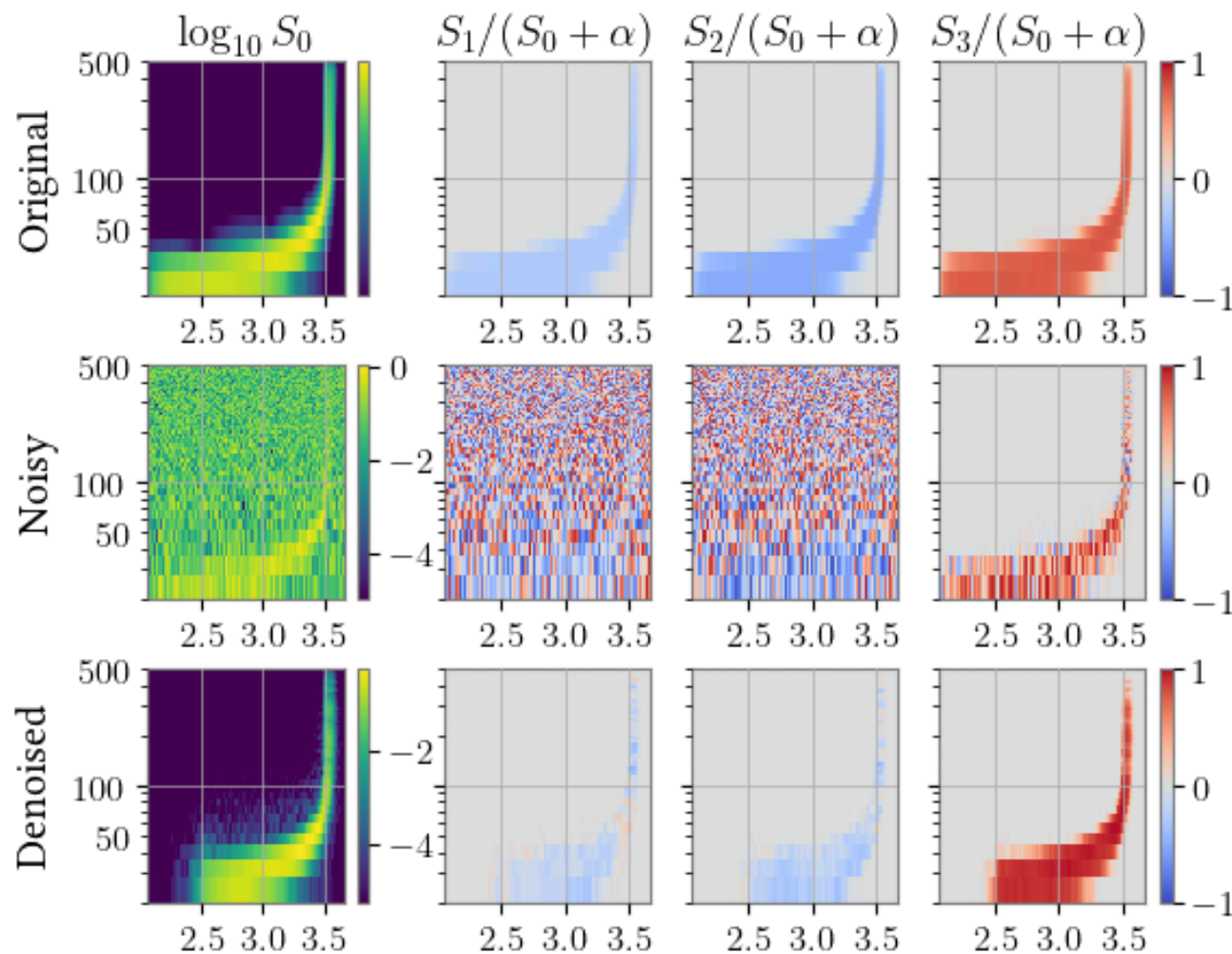


Neural network training

- Synthetic dataset of 4096 waveforms, using SEOBNRv4PHM
- $[15, 30]M_{\odot}$
- Mix of circular, elliptic and changing polarizations
 - Also introduce empty images to avoid hallucinations

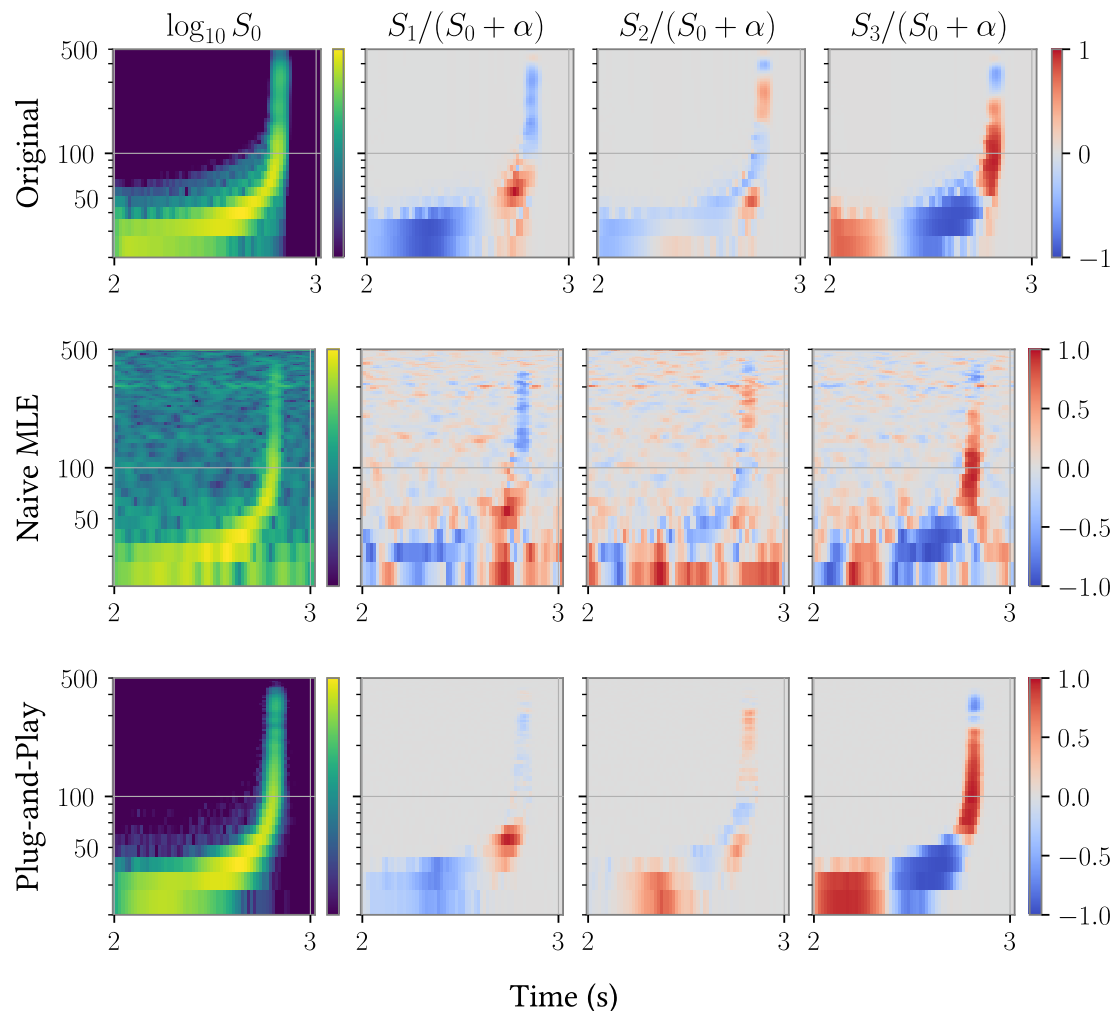


Denoising example

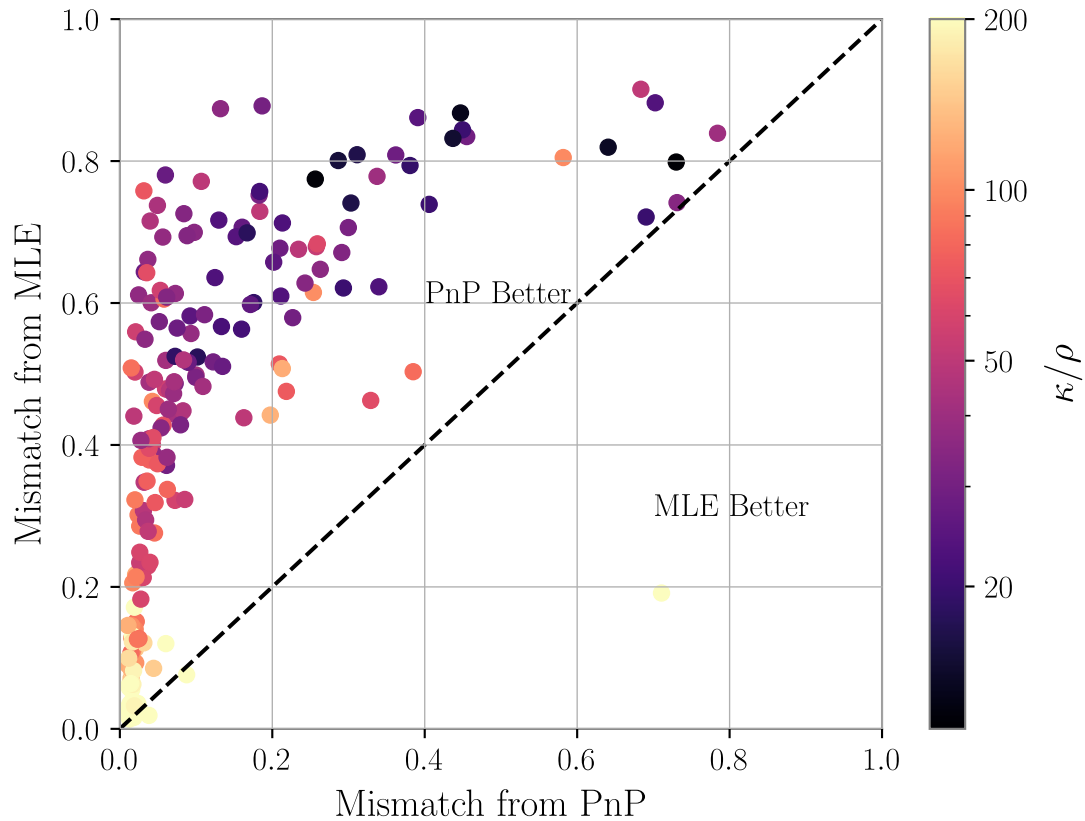


Plug-and-Play results

- Tested on injected signals
- Different SNR, sky localization
- Compared to naive MLE
- Good results, including with complex polarizations
- Example with SNR ~ 120



Large sample results

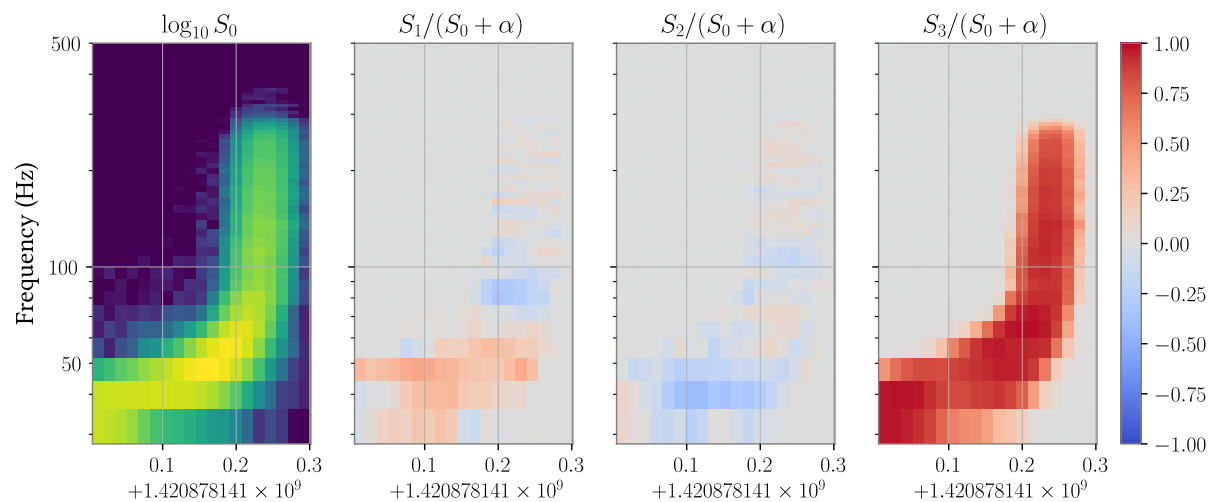
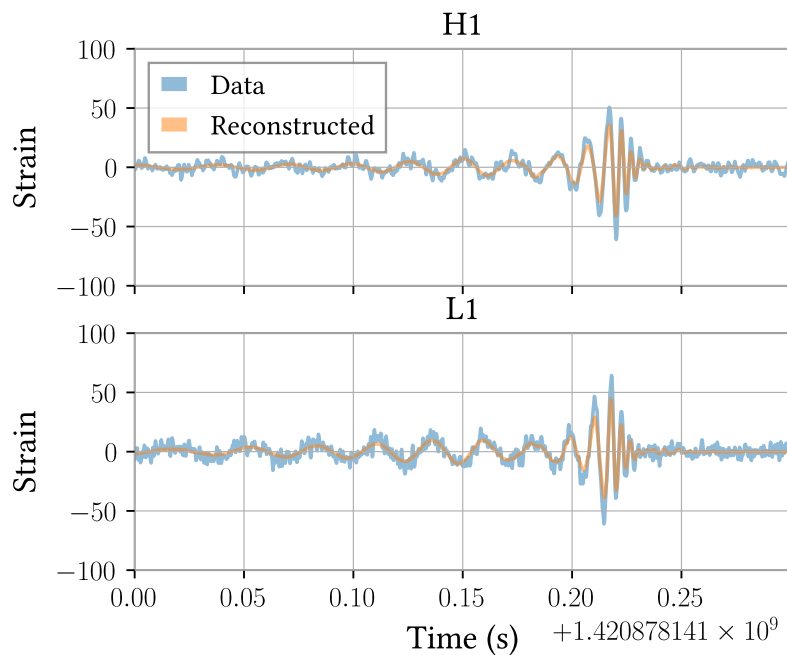
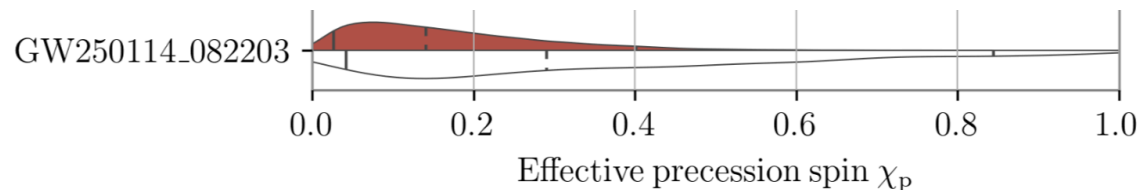


Stark improvement over regular MLE
on a larger sample of injections!

Testing on real events: GW250114

- Very preliminary!
- Mostly circular precession
 - consistent with LVK PE

From GWTC-5.0 results



Sky localization

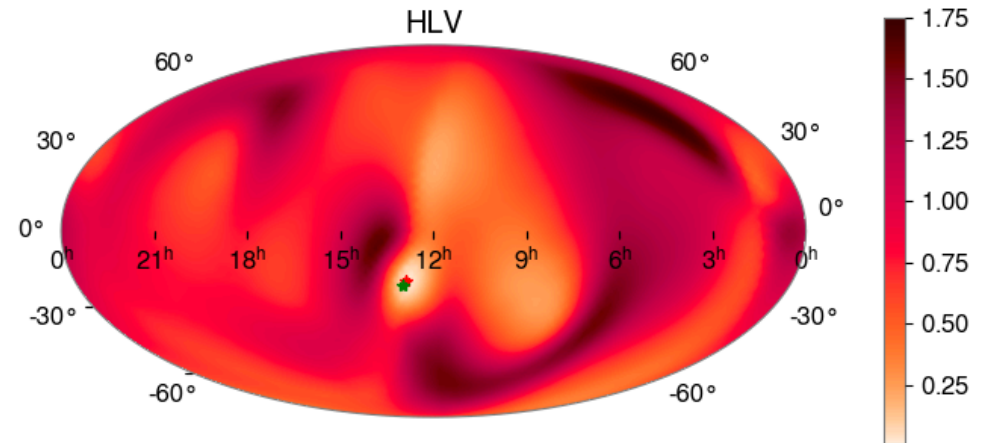
Up to now, we assumed that the sky localization θ is known $\rightarrow$ not the case in practice.

- Can we jointly estimate the position with the PnP process?
- How does incorrect localization affect reconstruction? Can we put error bars?

Current design (inspired by Burst pipelines):

- Apply PnP for many different localizations
- Pick the maximum likelihood
- Works well at high SNR
- Can use Bayesian Optimization to reduce the number of points needed

Work also in progress towards a more robust method for making the joint estimate.



Conclusion and perspectives

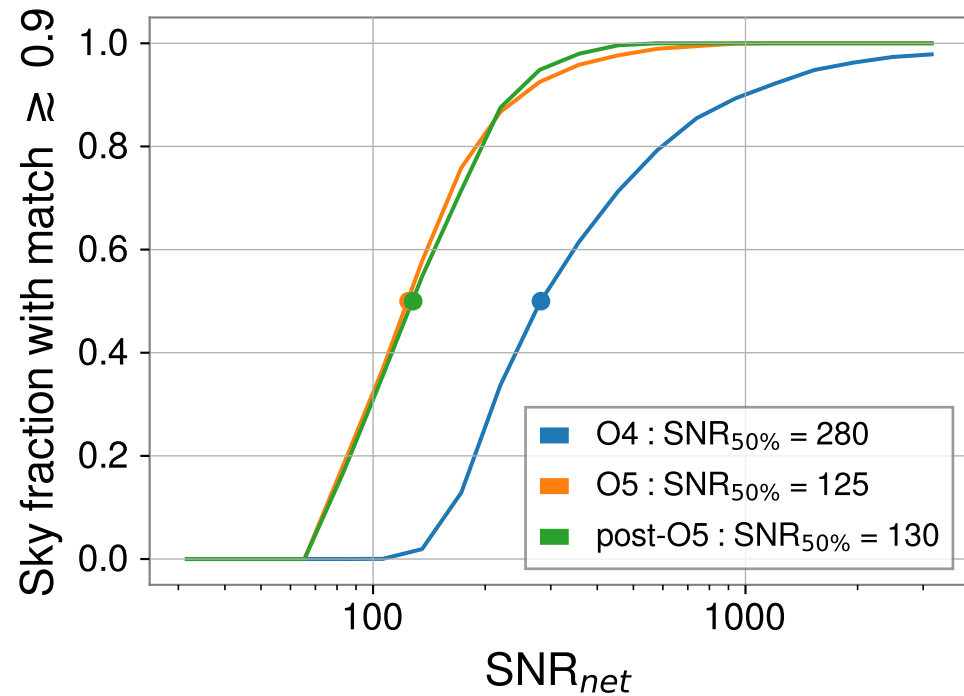
- Plug-and-Play is viable for reconstructing polarization under ideal conditions
- Currently wrapping up a long paper describing the method
- Future work:
 - Better handling of sky localization
 - Investigating flow matching methods

Thank you! Any questions?

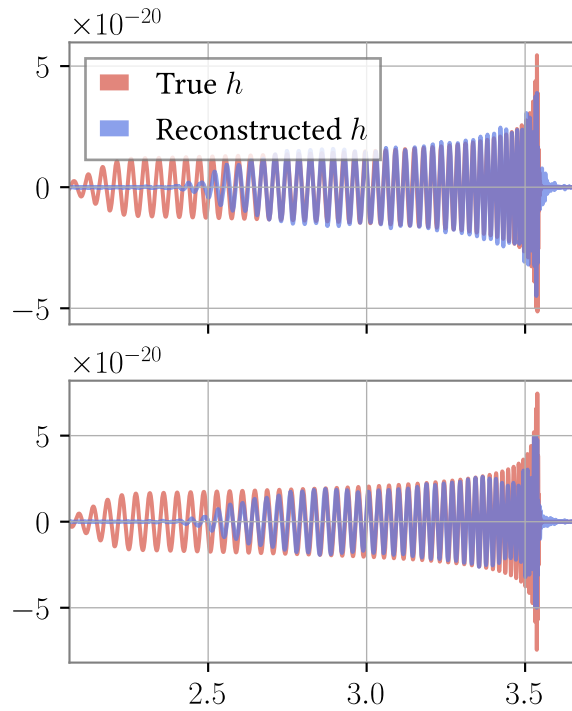
See more: Palud, P., Chassande-Mottin, E., & Pilavci, Y. Y. (2025, June). Synthetic-data-driven Plug-and-Play method for inverse problems on bivariate signals. *23rd IEEE Statistical Signal Processing Workshop (SSP 2025)*. <https://hal.science/hal-05126577>

Backup slides

MLE sky coverage



Denoiser result



PnP

