The most general Two-Higgs-Doublet Model

Audrey Degee

University of Liege

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The Standard Model of particle physics

The fundamental particles :

- **Fermions** : quarks and leptons (spin $\frac{1}{2}$)
- **Bosons** : gauge bosons (spin 1), Higgs boson (spin 0)

The four interactions in nature:

Interactions	Gravity	**Weak**	**Electromagnetic**	Strong
Carried by	1 Graviton (not yet observed)	W^+, W^-, Z	Photon	8 Gluons
Masses (GeV)	0	80, 91	0	0

- The Standard Model : **electromagnetic, weak** and strong interactions based on local gauge invariance :

$$G_{SM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

Particle masses in the SM ?

Mass term for gauge bosons breaks gauge invariance.

The flavour sector is rather unique :

- **Flavour conversion** proceeds through **weak charged current** interactions (via W^+ and W^- but never via Z).
- Flavour conversion involves **only left-handed currents** :

$$j^i_\mu = (\bar{u}_L \bar{d}_L)\gamma_\mu \sigma^i \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

$\rightarrow SU(2)_L$ invariant

The **fermion mass term** mixes left- and right-handed fermion component:

$$m\bar{u}u = m(\bar{u}_L u_R + \bar{u}_R u_L)$$

$\rightarrow$ **breaks gauge symmetry** $SU(2)_L$

Higgs mechanism of spontaneous symmetry breaking

- In the Standard Model: one SU(2) Higgs doublet of scalar fields

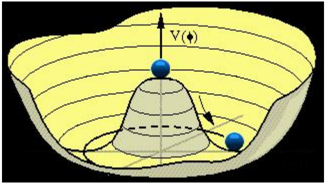
$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

→ Introduction of 4 degrees of freedom:

After symmetry breaking ($G_{SM} \rightarrow SU(3)_C \times U(1)_{EM}$) :

- 3 absorbed by W^+, W^-, Z
- 1 physical Higgs boson

- Higgs potential has **2 parameters**: $V = \mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2$:



$$\langle 0|\phi|0\rangle \neq 0$$

Motivations to expand the Higgs sector

- Higgs sector still not discovered in experiment
- Requirements of higher scale symmetries (supersymmetry, grand unification theory...)
- New source of CP violation
- Dark matter
- Fermion mass spectrum

Two Higgs doublets

- Various models: one of the simplest is the **Two-Higgs-Doublet Model** (T.D.Lee 1973)
- Two Higgs doublets

$$\phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}$$

$\longrightarrow$ Introduction of **8 degrees of freedom**

After symmetry breaking:

- 3 absorbed by W^+, W^-, Z
- 5 physical Higgs bosons

- Most general gauge-invariant and renormalizable potential contains **14 parameters** $\rightarrow$ many variants of 2HDM with different symmetries (CP, $U(1)$, Z_2...) and rich new phenomenologies

Motivations for the most general two-Higgs-doublet model (2HDM)

- Various sets of parameters $\longrightarrow$ similar phenomenologies $\longrightarrow$ there must be some structure in the 8-d space of Higgs fields
- What is the full spectrum of possible symmetries and their phenomenological consequences offered?
- Recovering all the particular models as limiting cases and establishing relations among them
- Building models with predefined symmetries
- Minimal supersymmetry with loop corrections
- Understanding the future LHC data

Difficulties with the most general 2HDM

The first step is to find the vacuum state ≡ minimum of the most general Higgs potential:

$$\begin{aligned} V_H \quad = \quad & -\frac{1}{2}[m_{11}^2(\phi_1^\dagger\phi_1) + m_{22}^2(\phi_2^\dagger\phi_2) + m_{12}^2(\phi_1^\dagger\phi_2) + m_{12}^{2*}(\phi_2^\dagger\phi_1)] \\ & \frac{\lambda_1}{2}(\phi_1^\dagger\phi_1)^2 + \frac{\lambda_2}{2}(\phi_2^\dagger\phi_2)^2 + \frac{\lambda_3}{2}(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \\ & \frac{\lambda_4}{2}(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \frac{1}{2}[\lambda_5(\phi_1^\dagger\phi_2)^2 + \lambda_5^*(\phi_2^\dagger\phi_1)^2] + \\ & \{[\lambda_6(\phi_1^\dagger\phi_1) + \lambda_7(\phi_2^\dagger\phi_2)](\phi_1^\dagger\phi_2) + h.c.\} \end{aligned} \tag{1}$$

Unfortunately,

- We cannot minimize the most general 2HDM potential with straighforward algebra
- Numerical analysis does not help

⟶ **Other method is needed**

Solution

Method to avoid this computational difficulty was recently suggested by I.Ivanov (2008):

- Establishment of the geometric structure behind 2HDM
- Use of group theory and tensorial algebra

  $\longrightarrow$ description of the vacuum without the need to compute the exact position of the global minimum of the potential
- Explicit description of the phase diagram of 2HDM

  $\longrightarrow$ we can study the geometry and symmetries of this space

Main idea

If we combine two spinors

$$\phi_1 = |\uparrow\rangle \qquad \phi_2 = |\downarrow\rangle$$

we get :

$$\begin{pmatrix} \uparrow\uparrow \\ \uparrow\downarrow - \downarrow\uparrow \\ \downarrow\downarrow \end{pmatrix}$$ vector

$$\uparrow\downarrow + \downarrow\uparrow$$ scalar

Structure behind 2HDM

This method is based on two main properties:

- The reparamatrization symmetry: any linear transformations between the two doublets preserve the generic form of the potential but with reparametrized coefficients

  $\longrightarrow$ **reparametrization freedom with the group** $GL(2,C)$

- The orbit space: potential depends on 4 combinations $(\phi_i^\dagger \phi_j)$, $i,j = 1,2$ (electroweak orbits), which can be organized into a 4-vector $r^\mu = (r^0, r^i)$ with:

$$r_0 = (\phi_1^\dagger \phi_1) + (\phi_2^\dagger \phi_2)$$

and

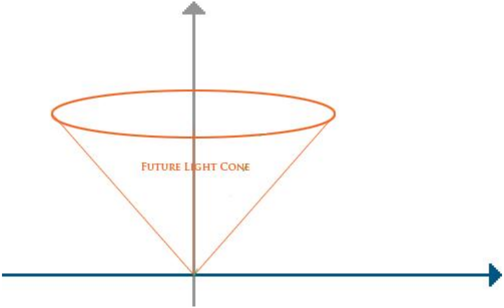
$$r_i = \begin{pmatrix} (\phi_2^\dagger \phi_1) + (\phi_1^\dagger \phi_2) \\ -i((\phi_1^\dagger \phi_2) - (\phi_2^\dagger \phi_1)) \\ (\phi_1^\dagger \phi_1) - (\phi_2^\dagger \phi_2) \end{pmatrix}$$

$\longrightarrow$ **reparametrization freedom with the Lorentz group** $SO(3,1)$

Structure behind 2HDM

Degrees of freedom	(ϕ_i, ϕ_j)	$\longrightarrow$	r_μ
Reparametrization group	$GL(2, C)$	$\longrightarrow$	$SO(3,1)$

- Possible r^μ lie on the surface and inside the **future light cone**: $r^\mu r_\mu \geq 0$ and $r^0 \geq 0$



Structure behind 2HDM

The Higgs potential in the r^μ space:

$$V_H = -M_\mu r^\mu + \frac{1}{2}\Lambda_{\mu\nu} r^\mu r^\nu$$

where

$$M_\mu = \frac{1}{4}(m_{11}^2 + m_{22}^2, -2Re\ m_{12}^2, 2Im\ m_{12}^2, -m_{11}^2 + m_{22}^2),$$

$$\Lambda_{\mu\nu} = \frac{1}{2}\begin{pmatrix} \frac{\lambda_1+\lambda_2}{2} + \lambda_3 & -Re(\lambda_6+\lambda_7) & Im(\lambda_6+\lambda_7) & -\frac{\lambda_1-\lambda_2}{2} \\ -Re(\lambda_6+\lambda_7) & \lambda_4 + Re\lambda_5 & -Im\lambda_5 & Re(\lambda_6-\lambda_7) \\ Im(\lambda_6+\lambda_7) & -Im\lambda_5 & \lambda_4 - Re\lambda_5 & -Im(\lambda_6-\lambda_7) \\ -\frac{\lambda_1-\lambda_2}{2} & Re(\lambda_6-\lambda_7) & -Im(\lambda_6-\lambda_7) & \frac{\lambda_1+\lambda_2}{2} - \lambda_3 \end{pmatrix}$$

Positivity constraint : The Higgs potential has to be bounded from below : the tensor $\Lambda_{\mu\nu}$ is positive definite on LC^+.

Kinetic term

The **kinetic term** in the reparametrization covariant form:

$$K = K_\mu \rho^\mu, \quad \rho^\mu = (\partial_\alpha \Phi)^\dagger \sigma^\mu (\partial^\alpha \Phi)$$

- Transformations from the reparametrization group $GL(2, C)$ modify the Higgs kinetic term $\rightarrow$ **non-diagonal kinetic terms**
- Energy density positive $\rightarrow$ in any basis K^μ lies inside the future light-cone
- The standard $K_\mu = (1, 0, 0, 0)$

Physically observable quantities must be reparametrization invariant. $\rightarrow$ full contractions of K_μ, $\Lambda_{\mu\nu}$ and M_μ and invariant tensors $g_{\mu\nu}$ and $\epsilon_{\mu\nu\rho\sigma}$

Vacua

There are three types of **minima** of the potential:

- **Electroweak (EW) symmetry conserving vacuum**: all bosons remain massless

  $\rightarrow r^\mu = 0$

- **Neutral vacuum**: breaks the EW symmetry, conserves the electric charge and the photon remains massless

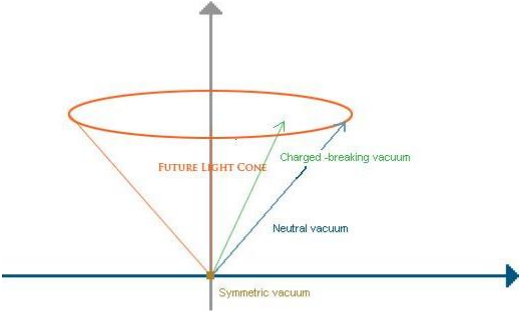
  $\rightarrow r^\mu r_\mu = 0, \quad r^\mu \neq 0$

- **Charge-breaking vacuum**: breaks the EW symmetry, does not conserve the electric charge and the photon becomes massive

  $\rightarrow r^\mu r_\mu \neq 0, \quad r^\mu \neq 0$

Vacua

- These three types of vacuum have a geometric meaning: the **singular point, the surface** or **the interior** of the r^μ space



- Interpretations of various models become easier

Vacua

With this method, one can understand **the spectrum of the extrema** of the potential **withtout the explicit algebraic solution** of the minimization problem:

- How many extrema can the potential have?
- What is the nature of the extrema? (neutral/charged-breaking and saddle/minimum nature)
- Can the global minimum be degenerated and when does it happen? How is it related to the symmetries of the model?
- What is the phase diagram of the model?

This approach relies on **geometric properties**.

Masses of Higgs bosons

Aim of this work

Study of Higgs boson masses in all these types of vacua in the most general 2HDM without the explicit solution of the minimization

- Physical quantities = reparametrization-invariant quantities
- Mass matrix $\neq$ reparametrization-invariant quantity $\rightarrow$ in two different basis: $\mathcal{M} \neq \mathcal{M}'$
- But the eigenvalues of the mass matrices (masses) are invariant $\rightarrow$ calculated from
    - $Tr(\mathcal{M}) = Tr(\mathcal{M}')$,
    - $Tr(\mathcal{M}^n) = Tr(\mathcal{M}'^n)$,
    - $Det(\mathcal{M}) = Det(\mathcal{M}')$

Masses of Higgs bosons

Steps to get masses

1. Expression of the mass matrix as a function of the parameters
2. $Tr(\mathcal{M})$, $Tr(\mathcal{M}^2)$, $Det(\mathcal{M})$
3. Convert unknown ϕ into known r^μ at the minimum
4. Masses

Masses of Higgs bosons

Constraints on the parameters of the potential from positivity of the Higgs masses

$$\begin{aligned} V_H &= -\frac{1}{2}[m_{11}^2(\phi_1^\dagger\phi_1) + m_{22}^2(\phi_2^\dagger\phi_2) + m_{12}^2(\phi_1^\dagger\phi_2) + m_{12}^{2*}(\phi_2^\dagger\phi_1)] \\ & \frac{\lambda_1}{2}(\phi_1^\dagger\phi_1)^2 + \frac{\lambda_2}{2}(\phi_2^\dagger\phi_2)^2 + \frac{\lambda_3}{2}(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \\ & \frac{\lambda_4}{2}(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \frac{1}{2}[\lambda_5(\phi_1^\dagger\phi_2)^2 + \lambda_5^*(\phi_2^\dagger\phi_1)^2] + \\ & \{[\lambda_6(\phi_1^\dagger\phi_1) + \lambda_7(\phi_2^\dagger\phi_2)](\phi_1^\dagger\phi_2) + h.c.\} \\ &= -M_\mu r^\mu + \frac{1}{2}\Lambda_{\mu\nu} r^\mu r^\nu \qquad (2) \end{aligned}$$

Masses of Higgs bosons

Electroweak-symmetric vacuum

- Position:

$$\langle r_{\mu sym} \rangle = 0$$

- $m^2_{1,2} = -(K^\mu M_\mu) \pm \sqrt{(K^\mu M_\mu)^2 - M^\mu M_\mu}$

  reparametrization invariant expression

  $\rightarrow$ positive if M^μ lies inside the backward lightcone

- $m^2_1, m^2_2 = -\frac{1}{4}(m^2_{11} + m^2_{22}) \pm \frac{1}{4}(4|m^2_{12}|^2 + m^4_{11} + m^4_{22} - 2m^2_{11}m^2_{22})^{\frac{1}{2}}$

  $\rightarrow$ positive if $m^2_{11} < 0\,, \quad m^2_{22} < 0\,, \quad m^2_{11}m^2_{22} > |m^2_{12}|^2$

Masses of Higgs bosons

Charge-breaking vacuum

1. Position:

$$\Lambda^{\mu\nu}\langle r_{\nu\,ch}\rangle = M^{\mu}$$

2. If $\Lambda^{\mu\nu}$ is not singular, $\langle r_{\mu\,ch}\rangle = (\Lambda^{-1})_{\mu\nu}M^{\nu}$.
3. $\prod_i m_i^2 = 16\Lambda_0(-\Lambda_1)(-\Lambda_2)(-\Lambda_3)\cdot\langle r_{\nu\,ch}\rangle^2(K^{\nu}\langle r_{\nu\,ch}\rangle)^2$

   $\rightarrow$ minimum only if all $\Lambda_i < 0$, $i = 1, 2, 3$, $\langle r_{\nu\,ch}\rangle$ and K^{μ} lie inside the future lightcone

Masses of Higgs bosons

Neutral vacuum

1. Positions:

$$\Lambda^{\mu\nu}\langle r_\nu\rangle - \zeta\langle r^\mu\rangle = M^\mu ,$$

   where ζ is a **Lagrange multiplier**

2. **Charged modes** : $m^2_{H^\pm} = 2(K^\mu \zeta_\mu)$.

   $\rightarrow \zeta$ must lie on the surface of the future lightcone

3. **Neutral modes** : one golstone mode, no simple solution for the masses,

   a new symmetry: $g_{\mu\nu} r^\mu r^\nu = 0 \longrightarrow \Lambda_{\mu\nu} \rightarrow \Lambda_{\mu\nu} + C g_{\mu\nu}$

Conclusion

- There is now a method to analyze the most general 2HDM
- This method was used to study the general structure of the 2HDM vacuum without explicit minimization, without manipulation with high-order algebraic equations
- This work consisted in developing it further to study the **masses of the Higgs bosons**: we worked out a formalism to compute the traces of any power of the mass matrix in any type of minimum in a general 2HDM

Outlook

What is in progress :

- Extensions to the **N Higgs doublet model**
- 3HDM and frustrated symmetries (octahedral and tetrahedral)

What remains to be studied:

- **Dynamics** of the general 2HDM (Feynman rules) as well as **Yukawa interactions** with fermions in the reparametrization-invariant description.
- **Loop corrections** to the potential and their impact on the geometry

Thank you. Any questions ?

Questions ?